

Massachusetts Institute of Technology  
Department of Electrical Engineering and Computer Science

6.002 – Circuits & Electronics  
Spring 2006

Problem Set #2

Issued 2/15/06 – Due 2/22/06

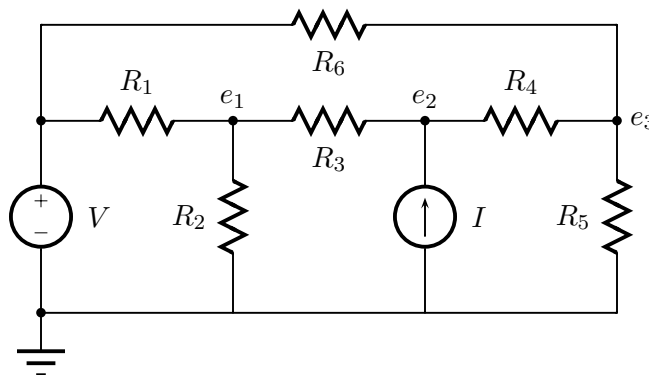
**Exercise 2.0:** Problem 2.4 makes use of WebLab, a physical remote-access laboratory that runs through the MIT iLab Service Broker. To obtain an iLab account, go to <http://ilab.mit.edu>, click on the *register here* link, provide the requested information, group yourself with Spring 2006 6.002 Students, and submit your request for an account. Account approval may take several hours to a day, so register early to avoid any last minute delays.

**Exercise 2.1:** A 1-k $\Omega$ , a 2-k $\Omega$  and a 5-k $\Omega$  resistor, all in parallel, are driven by a current source. If all resistors are thermally identical, which one runs hottest in steady state? That is, which resistor dissipates the greatest power? The same three resistors, now all in series, are driven by a voltage source. Now which one runs hottest in steady state? Finally, the 1-k $\Omega$  resistor is placed in series with the parallel combination of the other two resistors, and the resulting network is driven by a current source. Now which resistor runs hottest in steady state?

**Exercise 2.2:** Using the node method, develop a set of simultaneous equations for the network shown below that can be used to solve for the three unknown node voltages in the network. Express these equations in the form

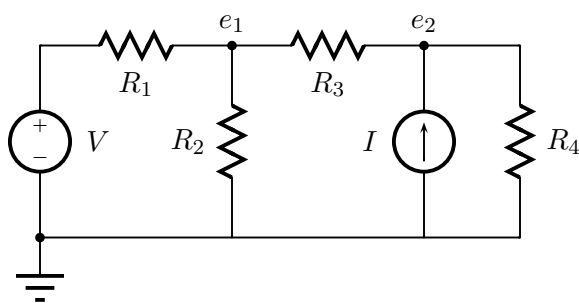
$$G \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = S$$

where  $G$  is a  $3 \times 3$  matrix of conductance terms and  $S$  is a  $3 \times 1$  vector of terms involving the sources. You need not solve the set of equations for the node voltages.



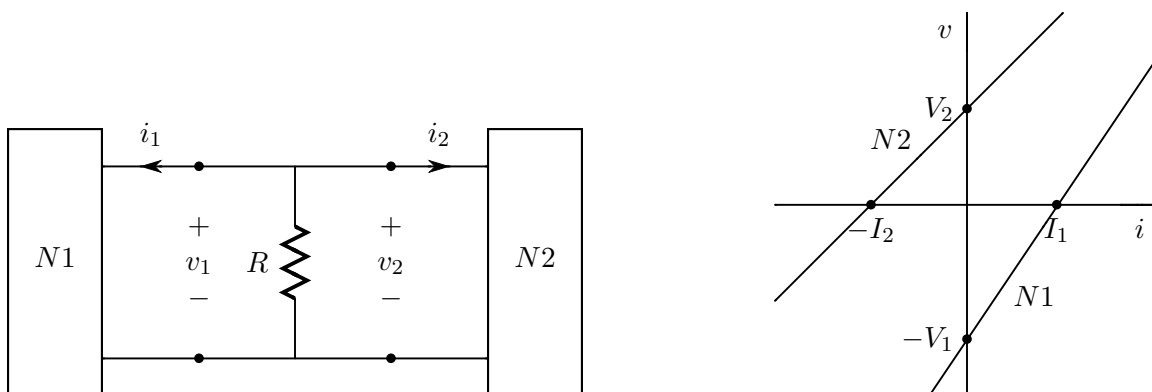
**Problem 2.1:** This problem analyzes the network shown below by two methods: superposition and the direct application of the node method. You should compare for yourself the work required to analyze the network by these two methods.

- (A) First, use superposition to determine  $e_1$  and  $e_2$ . That is, superpose the two partial node voltages obtained with only single sources active to find the total node voltages. Remember that a zero-valued voltage source is a short circuit, and a zero-valued current source is an open circuit. Hint: rather than employing the node method twice, once for each partial analysis, consider employing alternative simpler analyses involving the use of parallel and series resistor combinations, and voltage and current dividers.
- (B) Second, use the node method to directly determine  $e_1$  and  $e_2$  in total.
- (C) Compare the solutions to Parts (A) and (B). The two solutions should be the same.

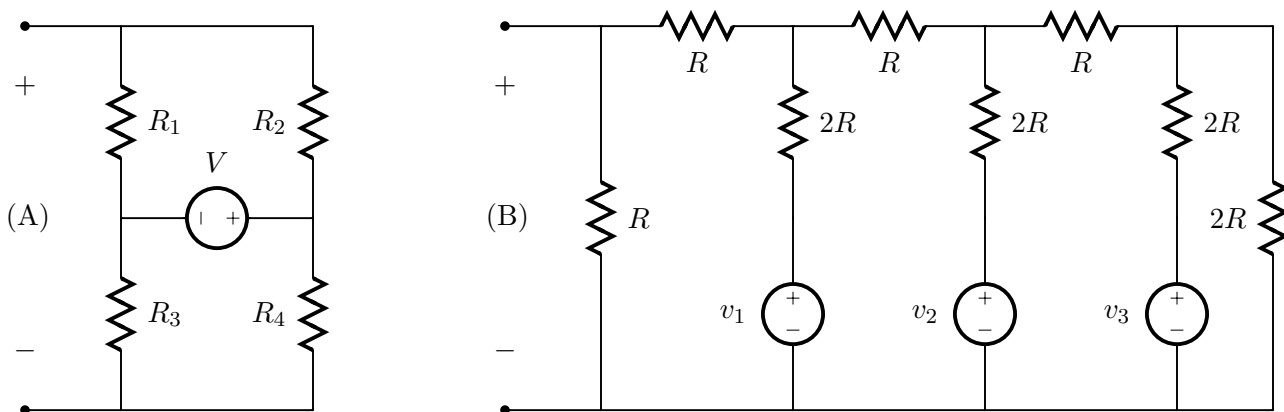


**Problem 2.2:** Two networks, N1 and N2, are described graphically in terms of their  $i$ - $v$  relations, and connected together through a single resistor, as shown below.

- (A) Find the Thevenin and Norton equivalents of N1 and N2.
- (B) Find the currents  $i_1$  and  $i_2$  that result from the interconnection of N1 and N2.



**Problem 2.3:** Find the Thevenin and Norton equivalents of the following networks, and graph their  $i$ - $v$  relations as viewed at their ports. Hint: use superposition for Network (B).



**Problem 2.4:** This problem makes use of WebLab, a remote-access laboratory that enables experimentation controlled through a Java-enabled web browser. The remote laboratory, actually located in Building 38 at MIT, offers access to more expensive and sophisticated equipment than is available in the 6.002 laboratory. For example, this problem uses an HP4155B Semiconductor Parameter Analyzer to measure the voltage-current relation of a real diode as follows.

- Start WebLab
  - Go to <http://ilab.mit.edu> and login to your account on the MIT iLab Service Broker. If your account does not open up under the *My Labs* section of iLab, move to that section by clicking on the *My Labs* tab on the top bar. If you have trouble running iLab on one computer, try switching to another computer, or try moving to an Athena workstation. You can request help and advice from iLab personnel, or report a bug, by clicking on the *Help* tab on the top bar.
  - Select the *Microelectronics WebLab Graphical Client* by clicking on its link. Afterwards, you can click on the *Documentation* link for detailed information about the WebLab client.
  - Start the WebLab client by clicking the *Launch Lab* link.

- Set Up Experiment

Experiments are run through the WebLab window that will pop up following the launch of the WebLab client. In this experiment, you will measure the voltage-current relation of a diode.

- From the drop-down *Devices* menu on the top bar, select the *p-n diode*. For this problem set, the diode is connected to the HP4155B through two Source Measurement Units (SMU). SMU #1 is connected to the positive terminal of the diode, and SMU #2 is connected to the negative terminal of the diode. Each SMU can source either the voltage across, or the current through, its terminals. It can also measure both its current and voltage.
- Click on SMU #1. Name its current and voltage, and select the download option for both variables. Select a voltage sweep (*Mode = V* & *Function = VAR1*) from -1.5 V to 1.5 V

with a 10-mV step. Set the compliance, or current limit, to 10 mA. Click on *Apply* and *OK* to complete the setup of SMU #1.

- Click on SMU #2. Set this SMU to ground (*Mode = COMM*). You must also name the voltage and current, but do not select them for downloading. (The voltage will always be zero, and the current will be the opposite of that measured by SMU #1. Thus, neither variable is interesting.) Click on *Apply* and *OK* to complete the setup of SMU #2.
- Run Experiment & View Measurements
  - From the drop-down *Measurement* menu on the top bar, select *Run Measurement*. Your experiment will be queued, and run on a first-come-first-served basis. Hint: if you wait until the last minute like everyone else, you may find the queue and the wait longer than desired.
  - The measured data will be displayed on a graph at the bottom of the WebLab window. It is probably best viewed with a linear X-axis display. Both a linear and a logarithmic Y-axis display are useful for answering the questions below. Note that the logarithmic display first takes the absolute value of the data.
  - Note the temperature of the diode. It is shown just above the measurement graph.
  - If for any reason you wish to clear your data and run the experiment again, you can first clear the data by selecting *Clear Data* from the *Results* drop-down menu on the top bar.
- Download Measurements

To download the measured voltage and current data to your local computer, select *Download Data* from the *Results* drop-down menu on the top bar.

Use your voltage and current data measured for the diode via WebLab to answer the following questions. As you view the data, remember that SMU #1 was set up to limit the diode current to 10 mA, even if the applied voltage attempts to drive a larger current.

- (A) In theory, the voltage-current ( $v_D$ - $i_D$ ) relation for a diode is

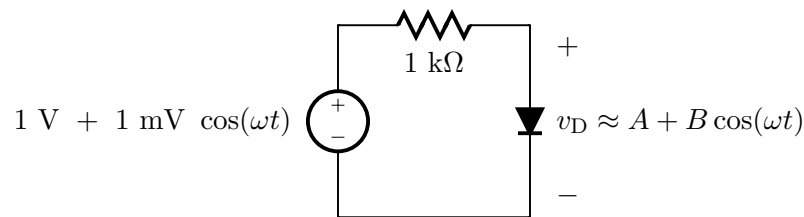
$$i_D = I_S(e^{v_D/V_T} - 1) .$$

The thermal voltage  $V_T$  is given by  $V_T = kT/q$ , where  $k = 1.38 * 10^{-23}$  J/K is Boltzmann's constant,  $T$  is the diode temperature in degrees Kelvin, and  $q = 1.60 * 10^{-19}$  C is the electron charge. Given the measured diode temperature, compute  $V_T$ .

- (B) Over what range of voltage does the measured diode data exhibit  $i_D \approx -I_S$ ? Is this as expected from the theoretical relation given above? Using the measured data, estimate  $I_S$ .
- (C) Over what range of voltage does the measured diode data exhibit the exponential relation  $i_D \approx I_S e^{V_D/V_T}$ ? Is this as expected from the theoretical relation given above?
- (D) A real diode often behaves like the theoretical diode described above in series with a linear resistor. Describe how this explains any discrepancies observed in Part (C). Using the measured data, determine the series resistance.
- (E) The real diode is connected to a Thevenin equivalent network as show below. In this case,  $v_D$  will take the form

$$v_D \approx A + B \cos(\omega t) ,$$

which is a linearized approximation. First, using graphical or numerical analysis, determine  $A$ ;  $A$  is the large-signal value of  $v_D$  that occurs when the Thevenin voltage is 1 V, that is, when the small-signal voltage  $1 \text{ mV} \times \cos(\omega t)$  vanishes. Second, using a linearized small-signal analysis, determine  $B$ ;  $B$  is the amplitude of the small-signal component of the linearized  $v_D$  that is driven by the small-signal cosinusoidal component of the Thevenin voltage. In doing so, linearize the measured diode data, either graphically or numerically, as opposed to the theoretical diode characteristic discussed in (A) above.



The following hints may help in answering Parts (B) through (E) above.

- Parts (B) through (E) can be answered either graphically or numerically. If you choose a graphical method, then you should plot the downloaded data using MatLab, Excel or another program, or print a screen-shot of the graph produced by WebLab. Capturing screen-shots is a platform specific activity; consult the WebLab documentation for Windows- and Athena-specific advice. If you choose a numerical method, then you could process the downloaded data using MatLab, Excel or another program.
- Before answering Parts (B) through (E), you may find it instructive to examine the measured data visually using WebLab. In this case, a logarithmic y-axis display is probably best used for Parts (B) and (C), while a linear y-axis display is probably best used for Parts (D) and (E).
- The series resistor mentioned in Part (D) is a parasitic resistor, and results from the resistance of the silicon that surrounds the diode. This resistance has a noticeable affect on only over a limited portion of the  $V_D$ - $i_D$  relation of the diode. You should first think about which portion of the relation is so affected and why.