SFB TRR55

BERGISCHE UNIVERSITÄT WUPPERTAL

JÜLICH FORSCHUNGSZENTRUM

Fluctuations from LQCD

In collaboration with: S. Borsanyi, Y. Delgado, S. Durr, Z. Fodor, S.D. Katz, T. Lippert, D. Nogradi, C. Ratti, C. Schroeder, K.K. Szabo
(Wuppertal-Budapest collaboration)

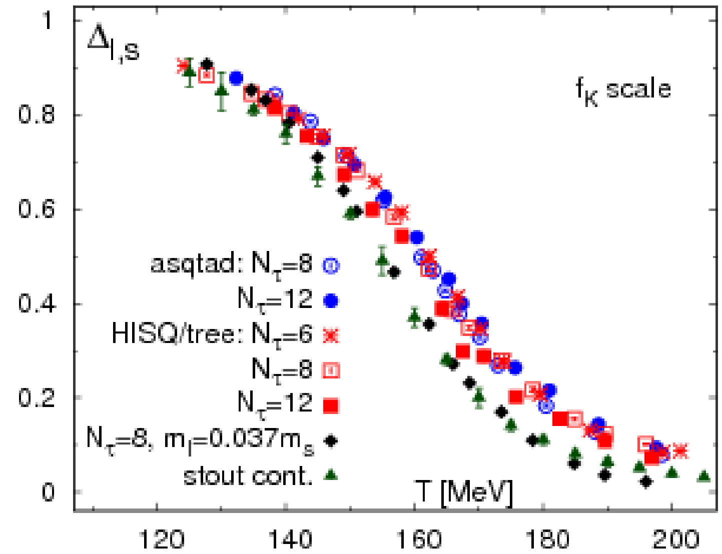
28.11.2012 | Stefan Krieg

Outline

1. Why do we believe our results are reliable?
   a. Emerging consensus between the collaborations
   b. Discussion of uncertainties in simulations
   c. Crosschecks with other actions
2. Results on fluctuations
   a. Lattice setup and techniques
   b. Error analysis
   c. Quadratic fluctuations, correllations
   d. kurtosis
3. Conclusions and outlook

Transition temperature

N_t	Budapest-Wuppertal	HotQCD	MILC
N_t=4,6			169(15)
N_t=8-16	147(4)-155(4)		
N_t=8-12		154(8)	

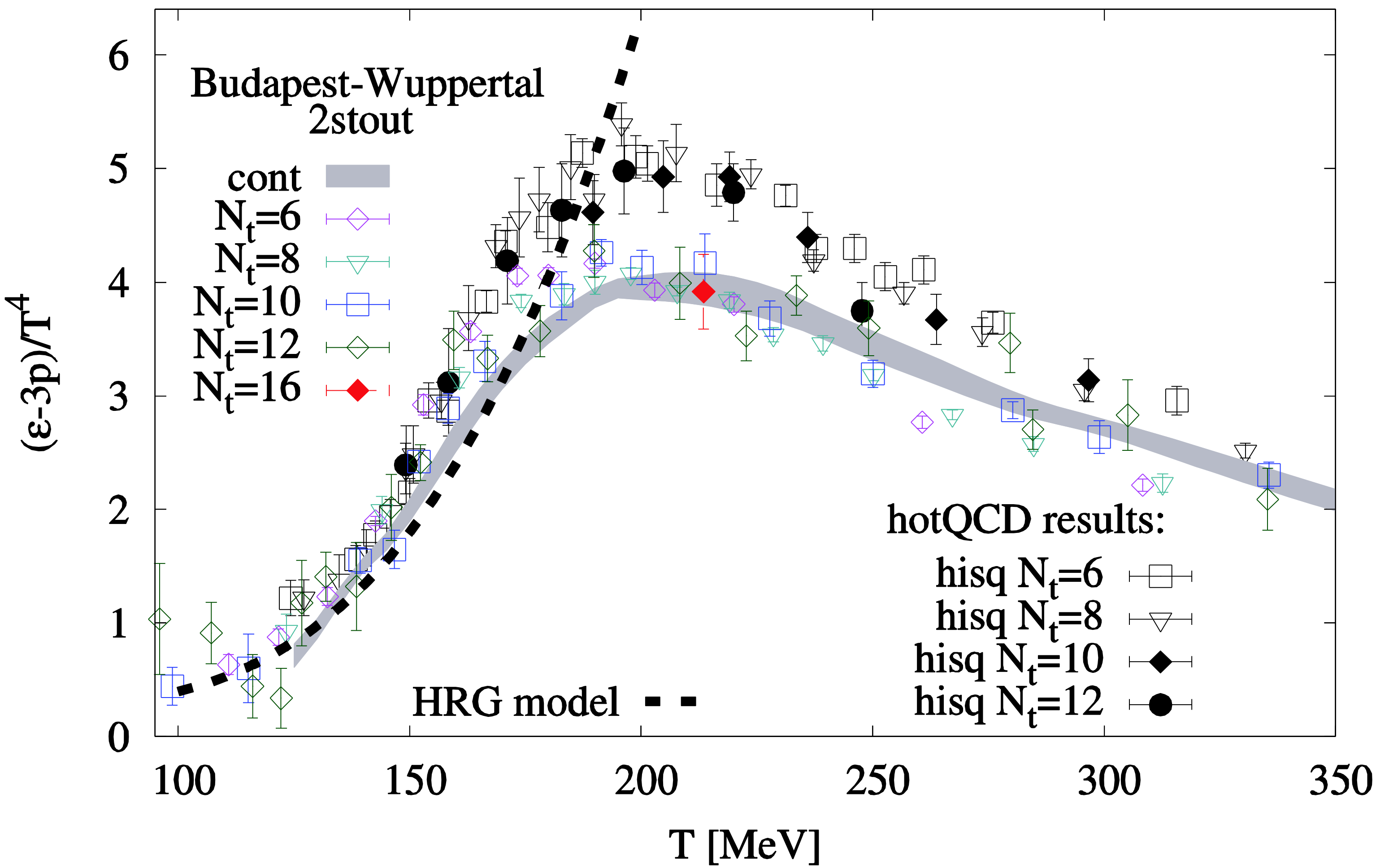


hotQCD:1111.1710

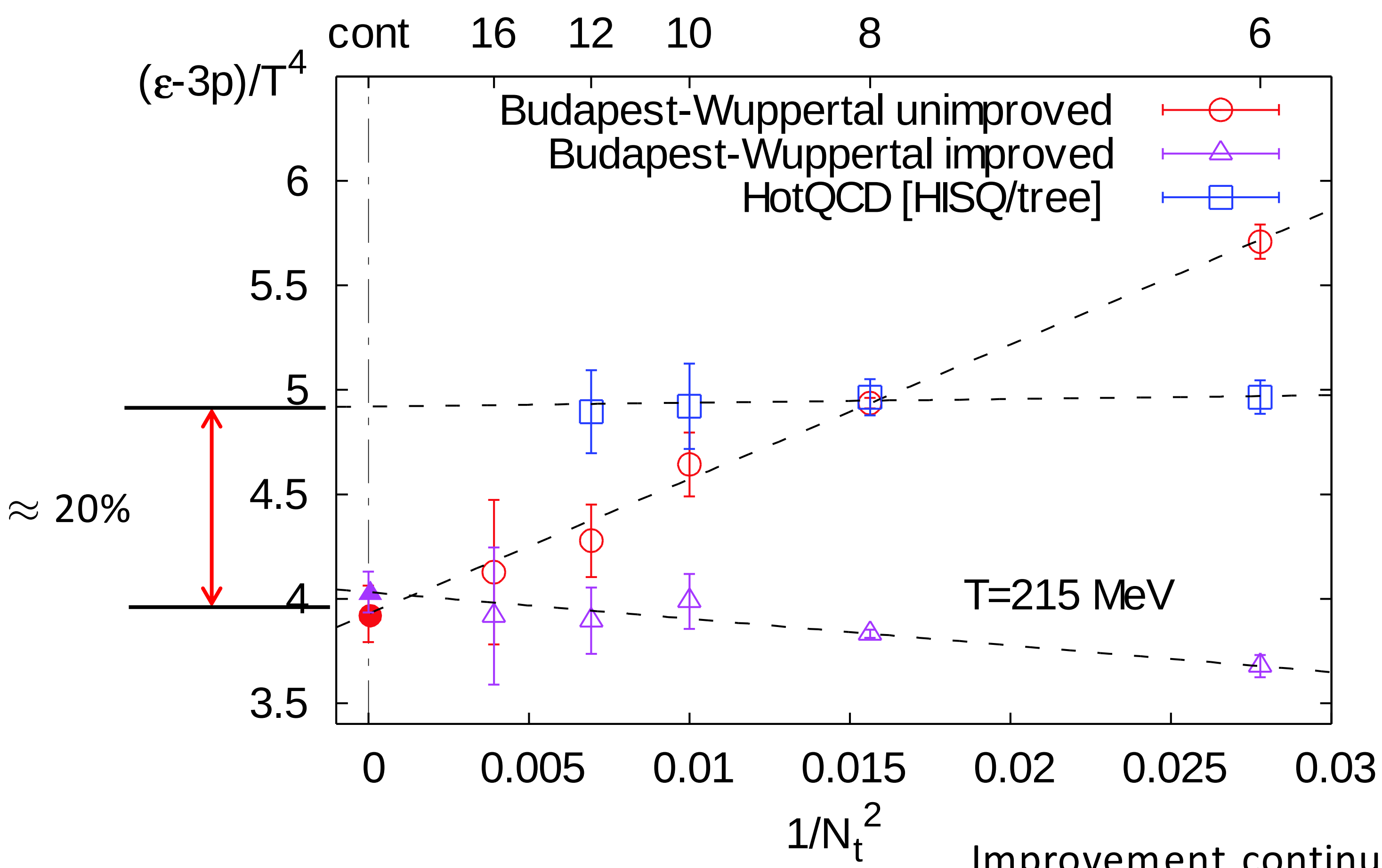
Collaboration	Reference
MILC	hep-lat/0608013
WB	arXiv:1005.3508
HotQCD	arXiv:1111.1710

hotQCD: Interpolation to physical point using O(2) or O(4) scaling

Equation of State

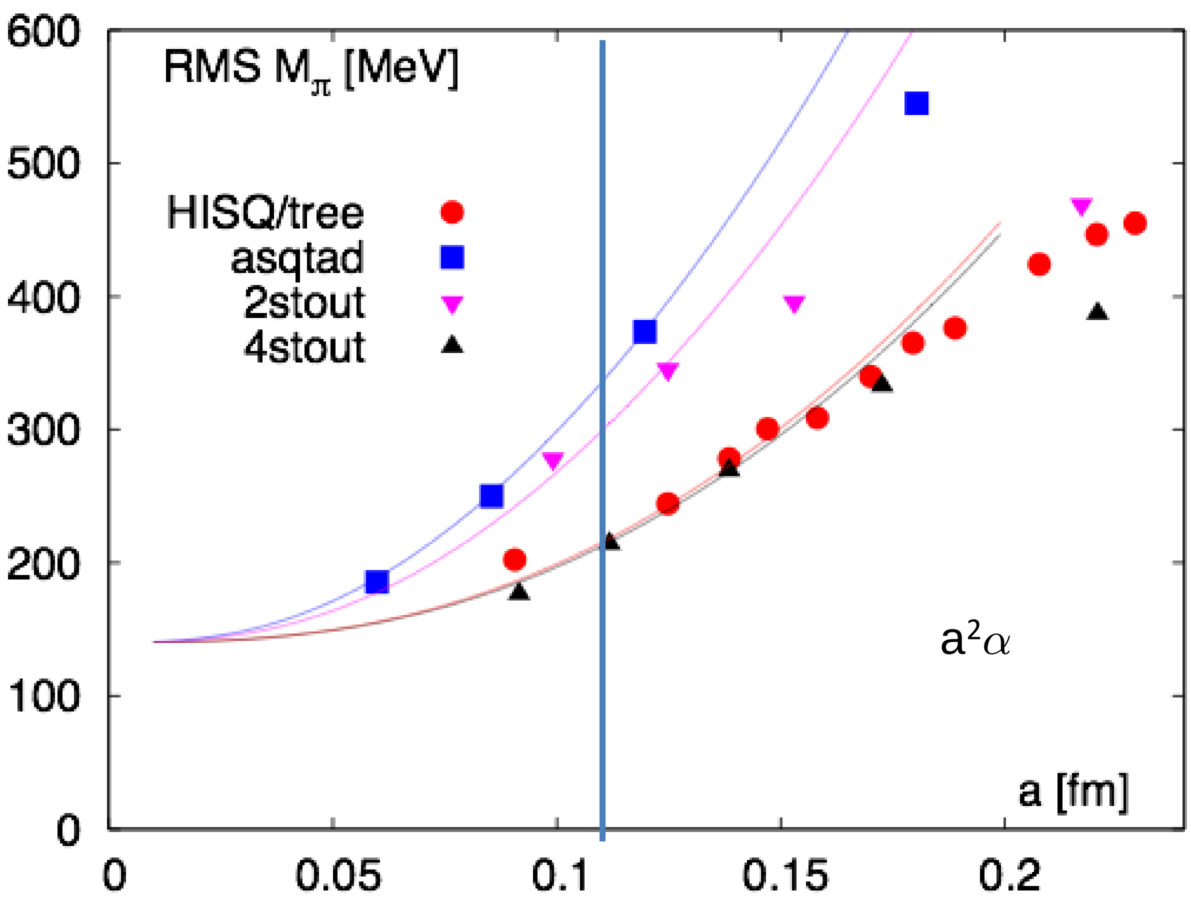


Equation of State



Improvement continuum irrelevant

Uncertainties: distorted pion sector



Staggered ferm's

- 4 tastes
- 1+15 "pions"

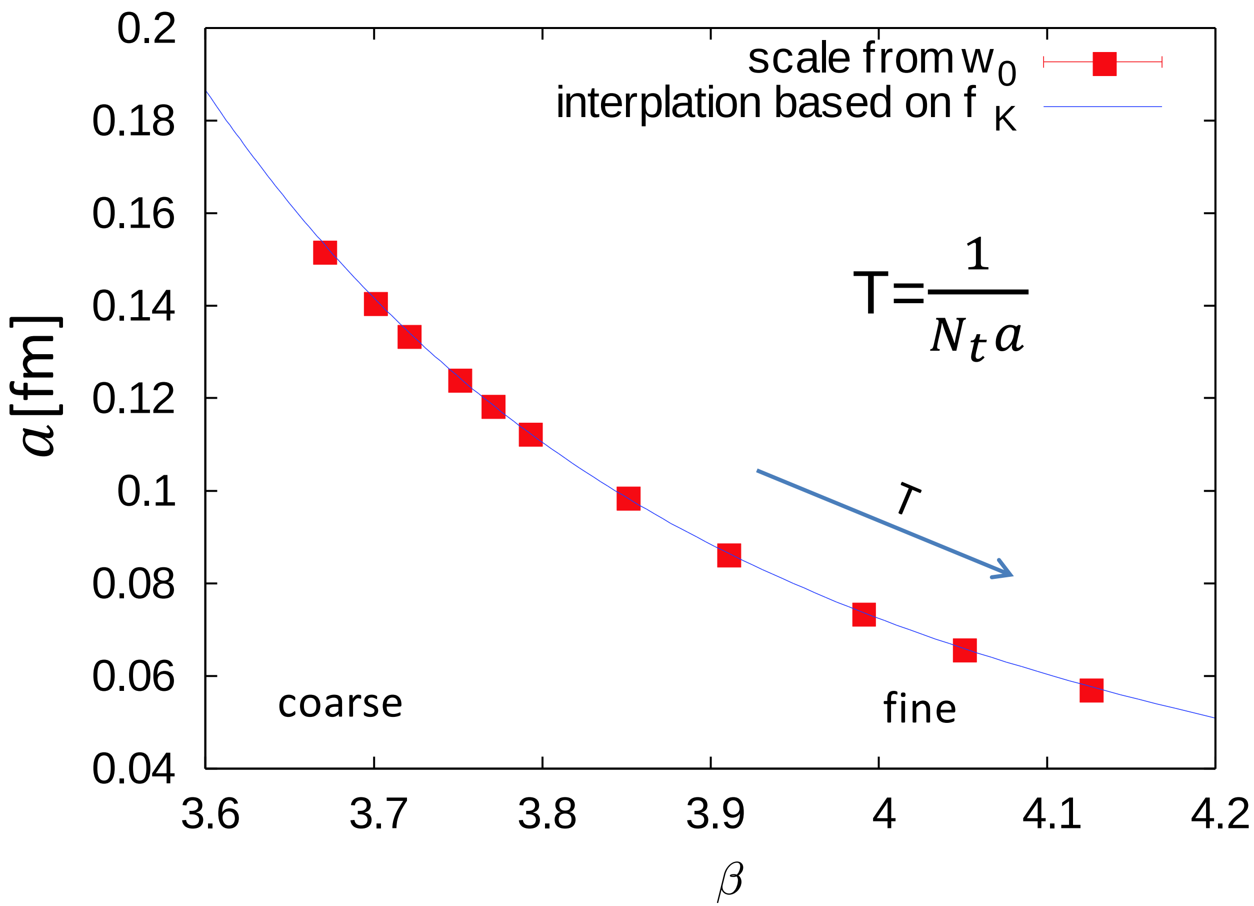
Taste breaking:

- extra "pions" heavy
- RMS large

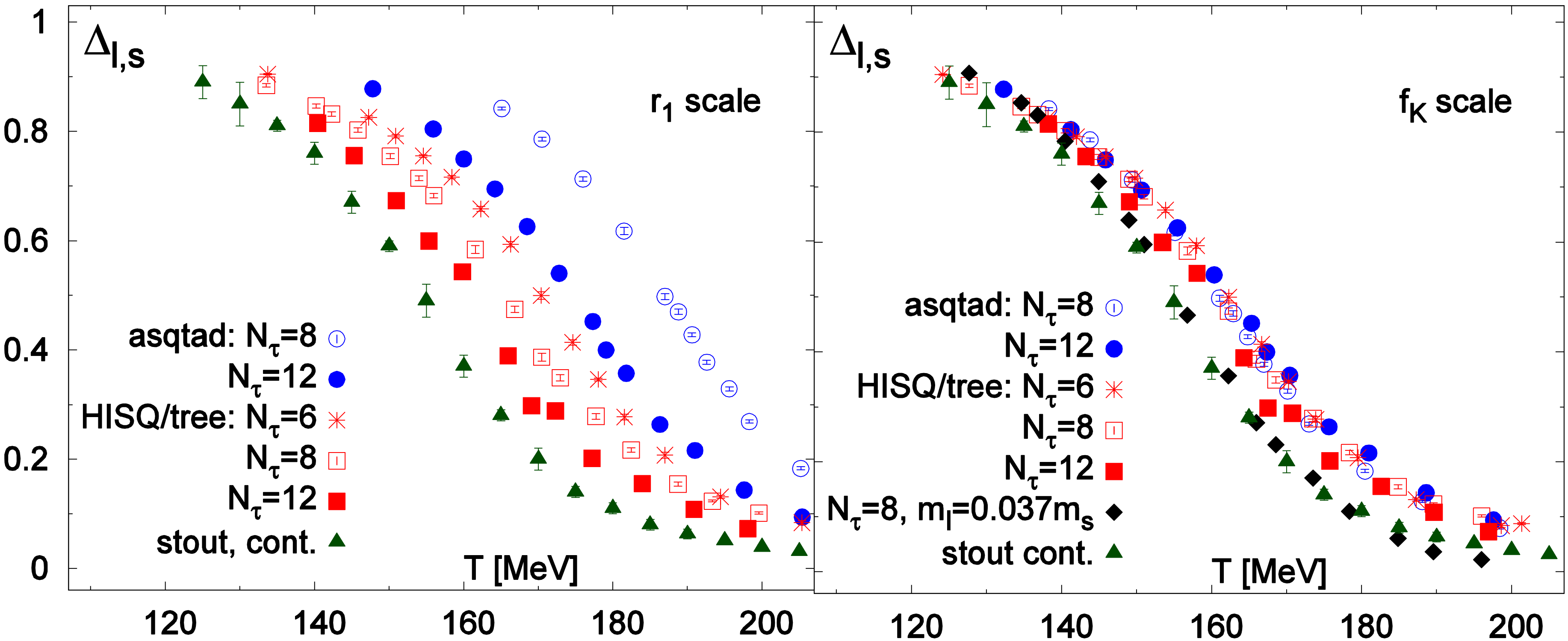
Recovery only in continuum limit!

Smearing req'd.

Uncertainties: Scale setting: LCP

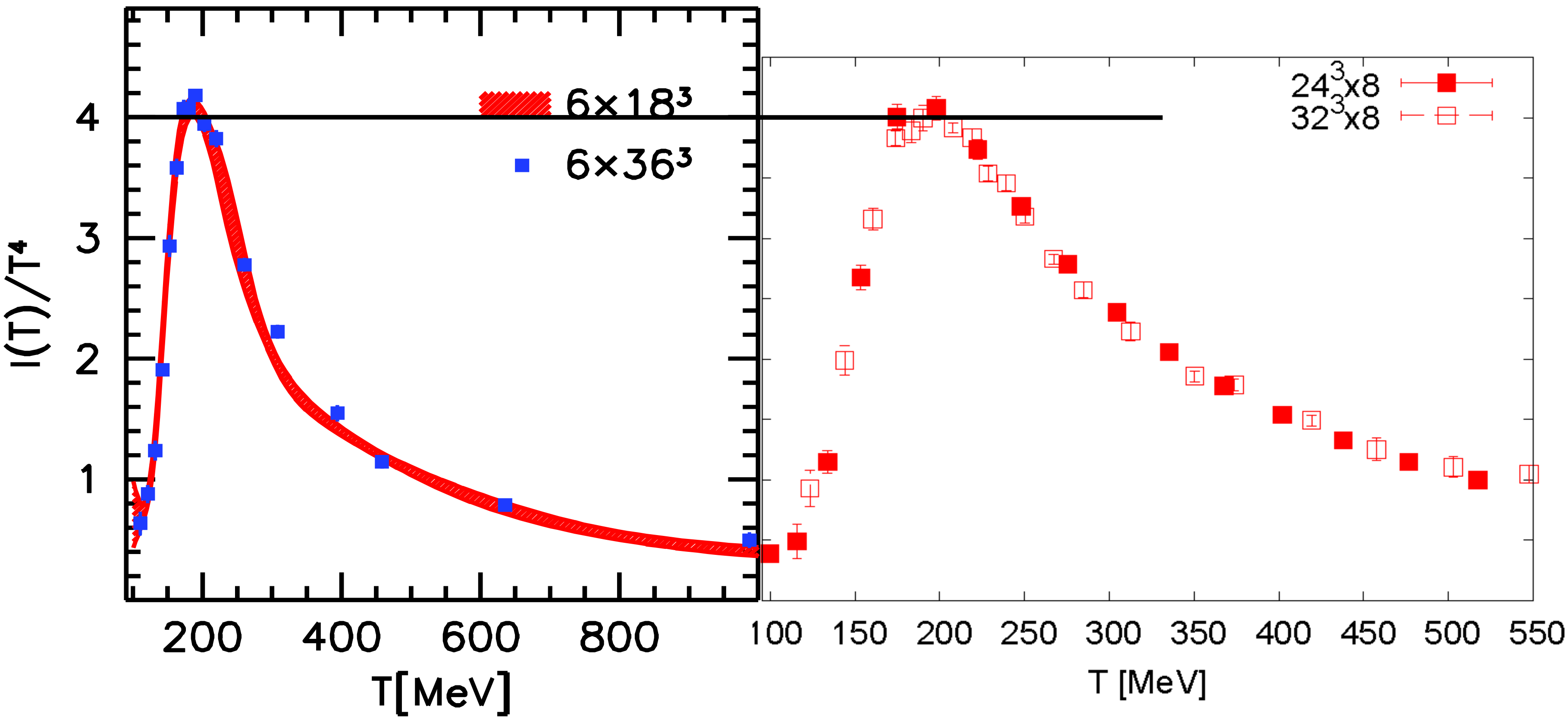


Uncertainties: scale setting

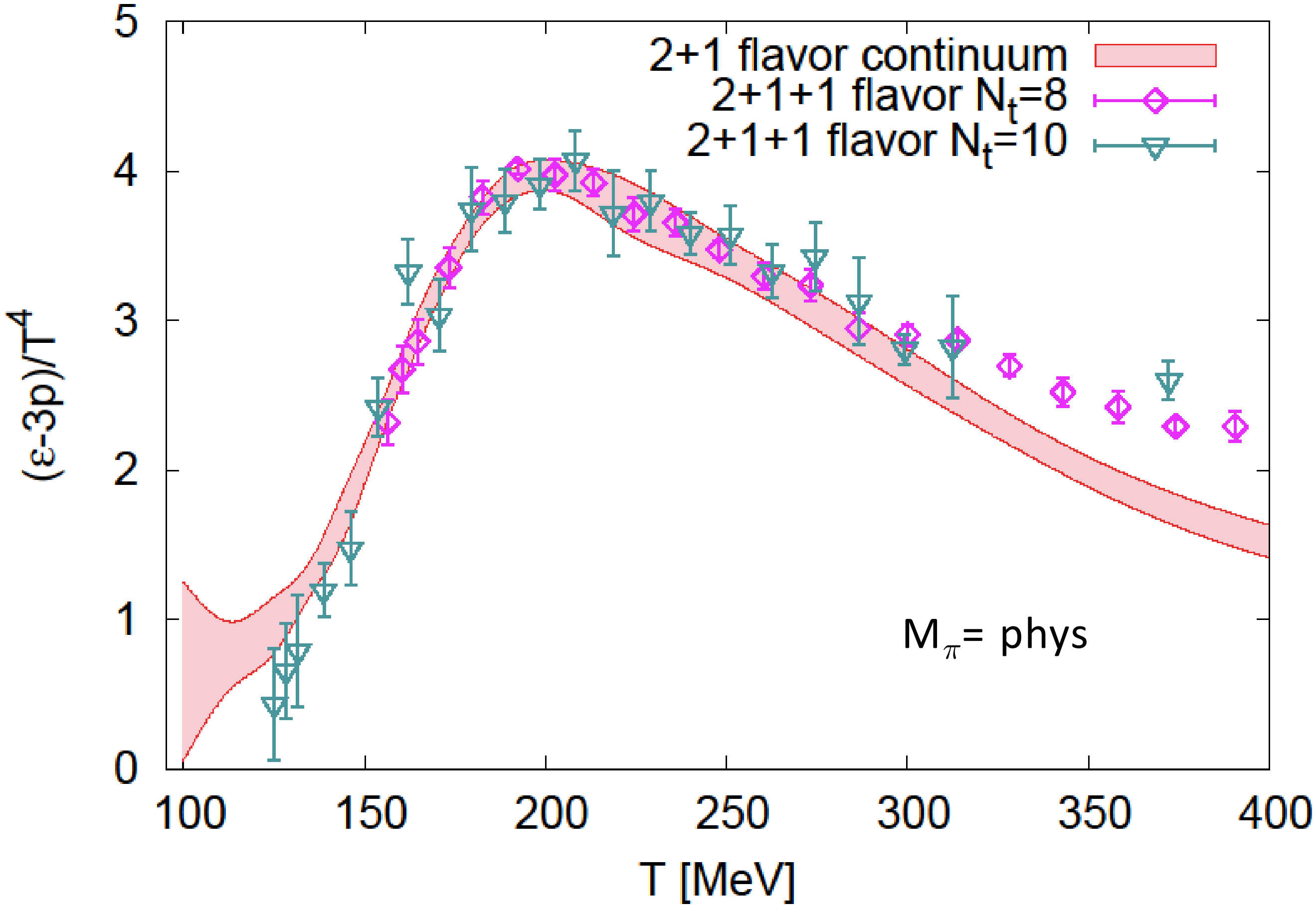


hotQCD: 1111.1710

Uncertainties: finite volume effects

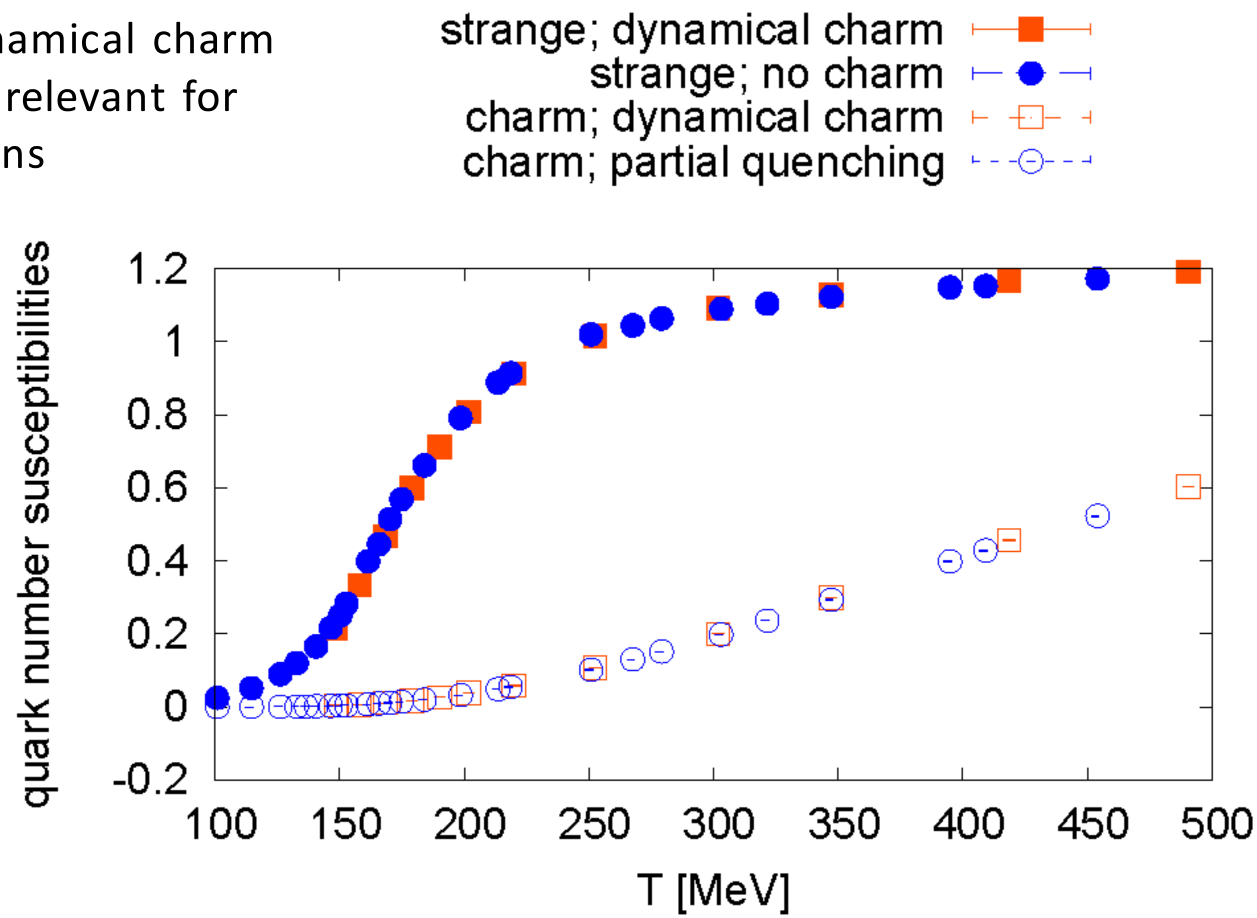


Uncertainties: dynamical charm



Uncertainties: dynamical charm

Note: dynamical charm likely not relevant for fluctuations



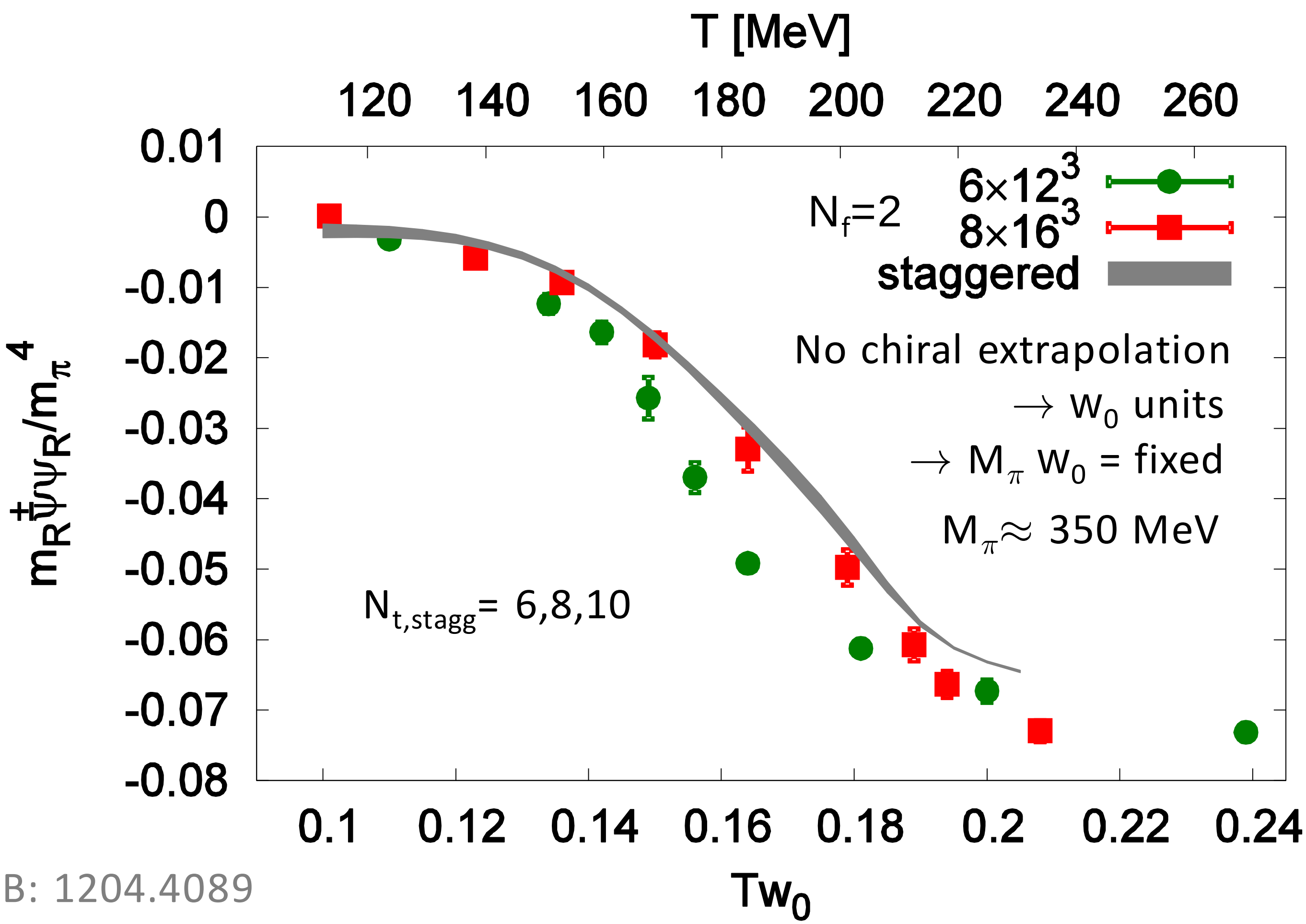
Mitglied der Helmholtz-Gemeinschaft

Uncertainties: summary

- Taste breaking: fine lattices and smearing required
- Proper scale setting procedure can help
- Finite volume effects need to be checked
- Rooting: perform crosschecks with different

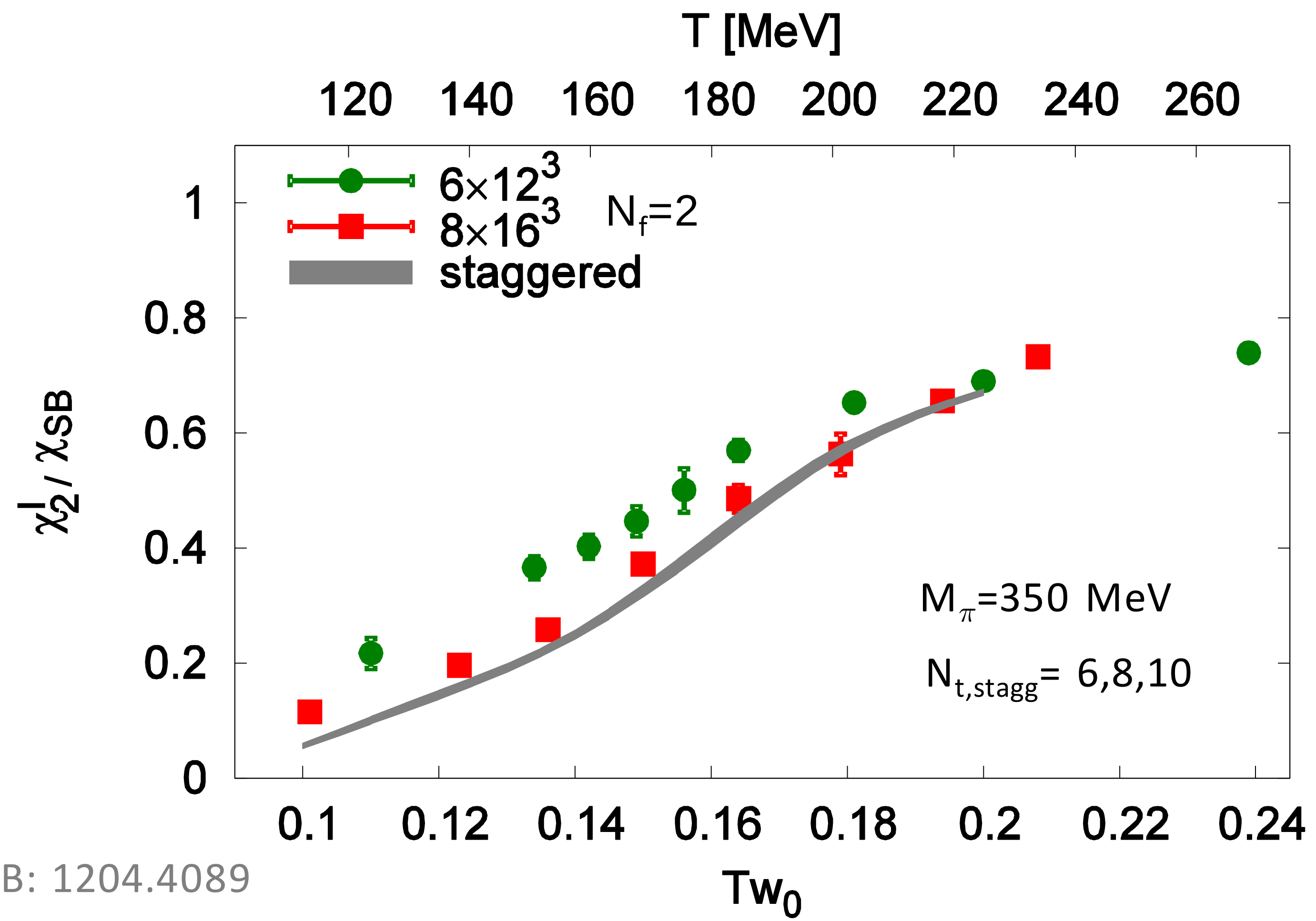
- For T>300 MeV a dynamical charm becomes relevant

Crosschecks: overlap



WB: 1204.4089

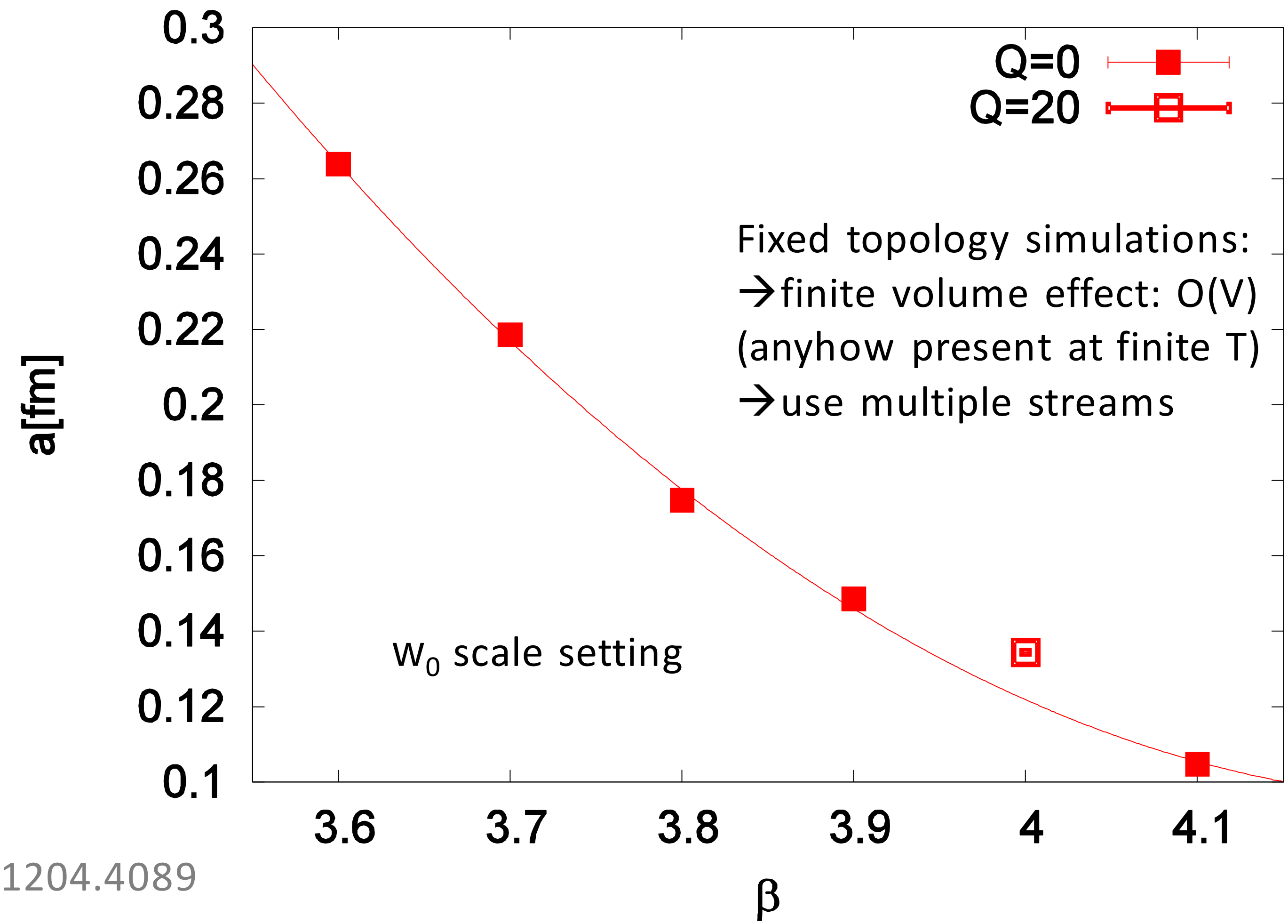
Crosschecks: overlap



WB: 1204.4089

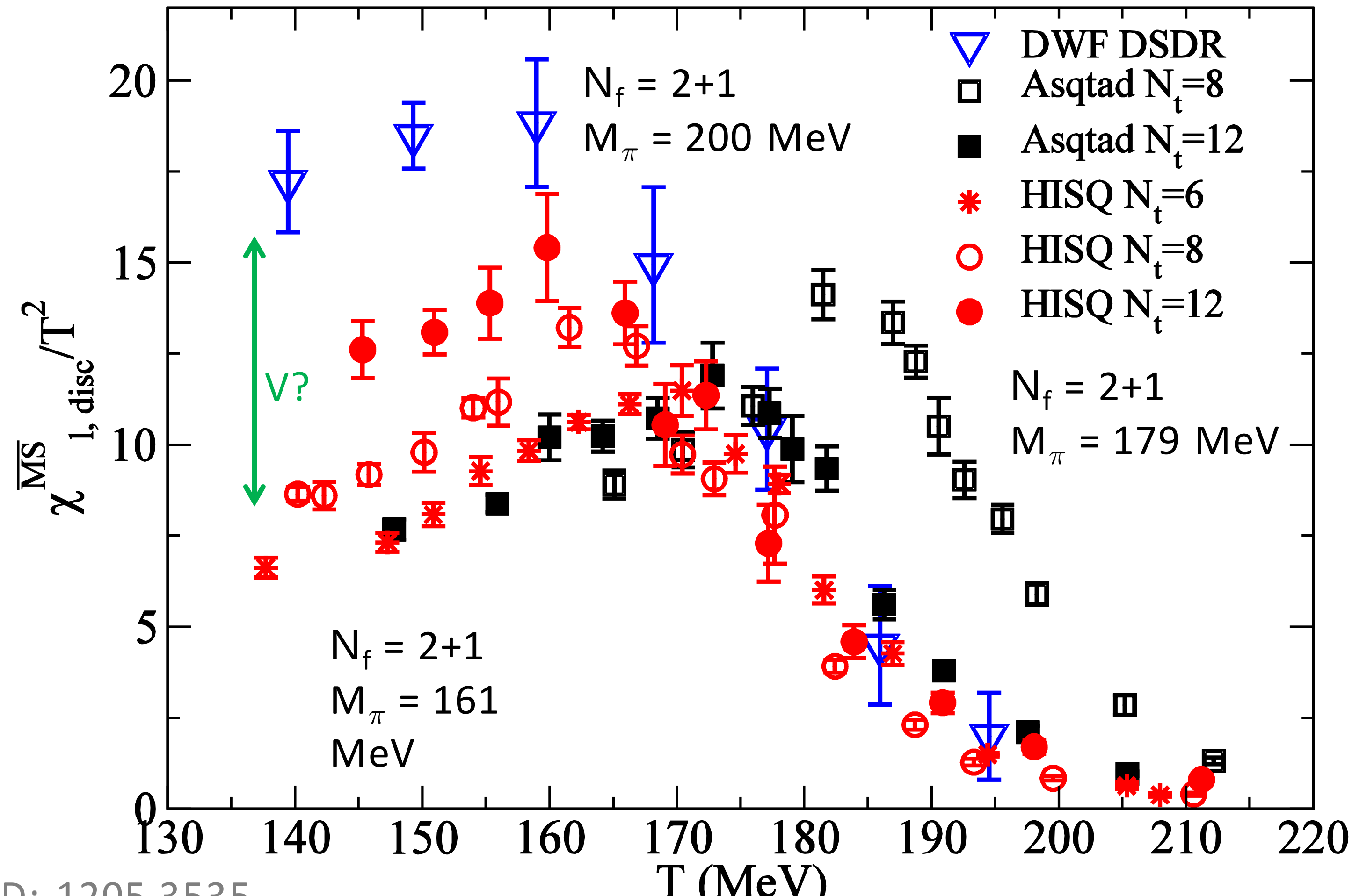
BERGISCHE UNIVERSITÄT WUPPERTAL
JÜLICH FORSCHUNGSZENTRUM

Crosschecks: overlap



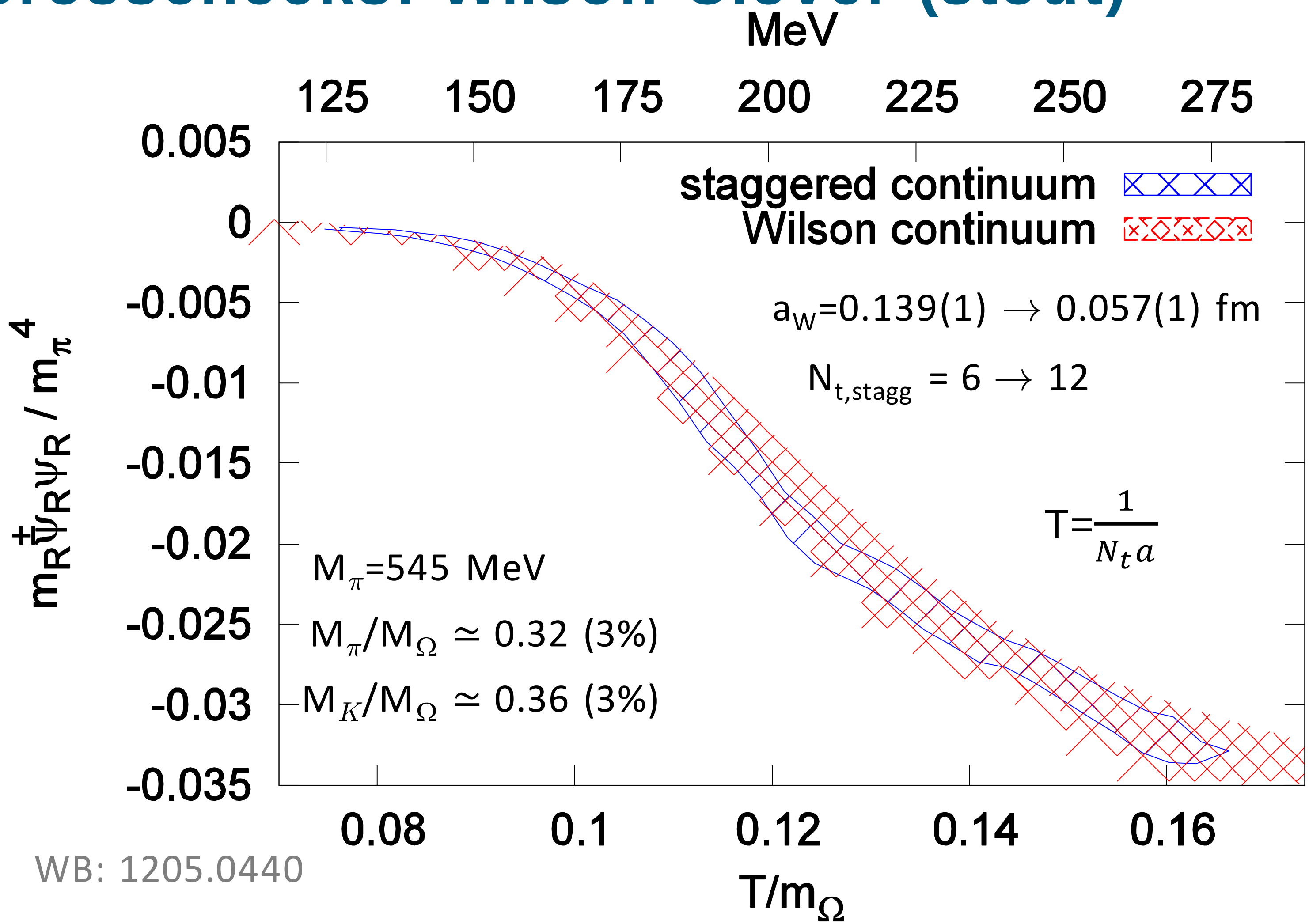
WB: 1204.4089

Crosschecks: Domain Wall Fermions



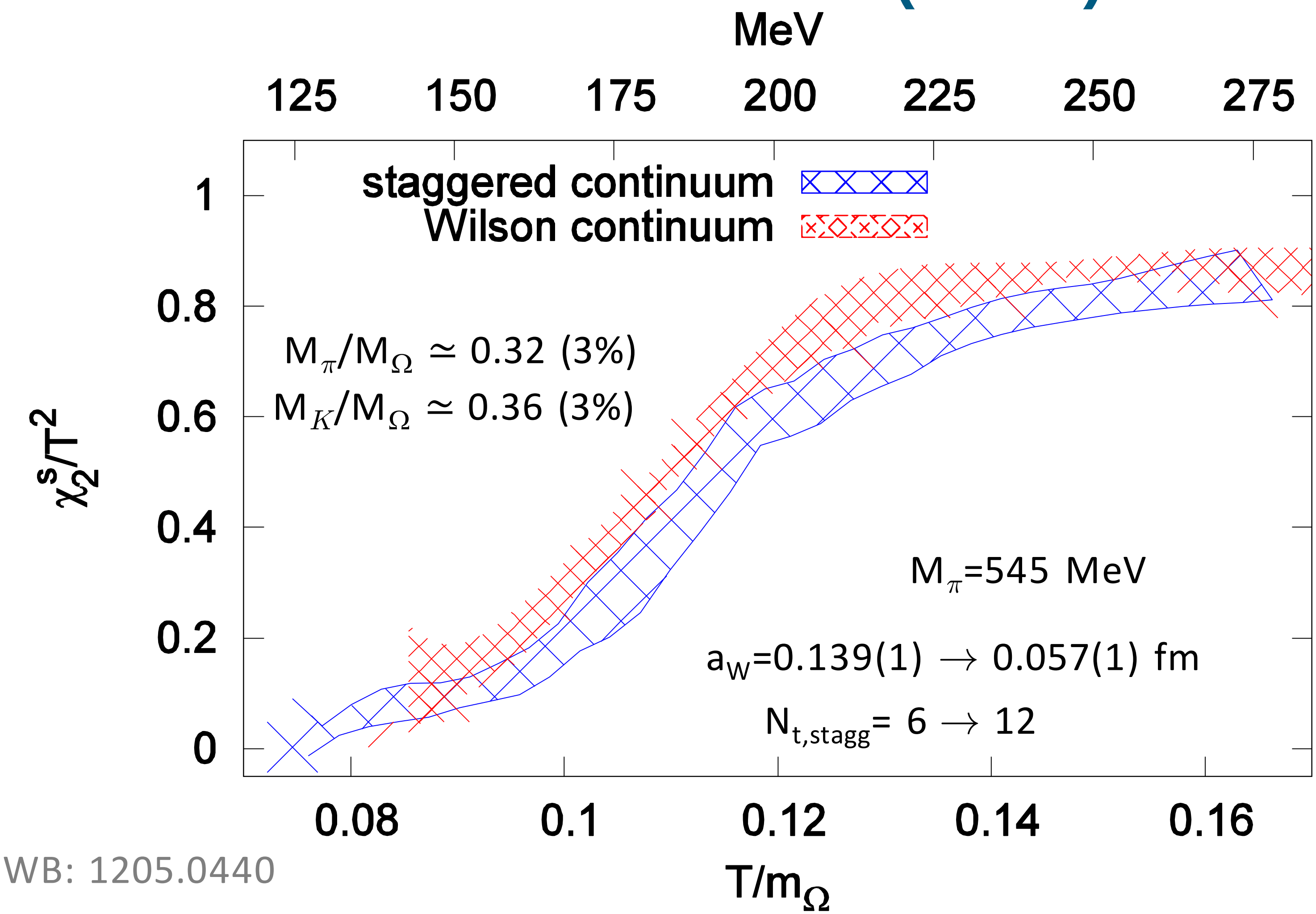
hotQCD: 1205.3535

Crosschecks: Wilson Clover (stout)



WB: 1205.0440

Crosschecks: Wilson Clover (stout)



WB: 1205.0440

Crosschecks: summary

Crosschecks with different lattice actions are available

- Comparison to Wilson (Clover) simulations available at $M_\pi \approx$ 545MeV, smaller masses in production
- Comparison to Overlap simulations available at $M_\pi \approx$ 350MeV
  - Some systematics effects will be analyzed more closely in the future
  - How far down in M_π can you go? (locate 1^{st} order phase-transition?)
- Comparison with Domain Wall not yet at similar M_π (spectrum distortions aside)

Fluctuations: introduction

- Net yields are given by $\langle N_X \rangle = -T \frac{\partial \log Z}{\partial \mu_X}$ $\langle N_X^2 \rangle - \langle N_X \rangle^2 = \frac{\partial^2 \log Z}{(\partial \mu_X / T)^2}$
- Defining: $\hat{\chi}_2^X = \frac{1}{VT^3} \frac{\partial^2 \log Z}{(\partial \mu_X / T)^2}$ $\hat{\chi}_4^X = \frac{1}{VT^3} \frac{\partial^4 \log Z}{(\partial \mu_X / T)^4}$
- Computable at finite μ through:

$$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(T, \mu_q) = \sum_{n=0}^{\infty} c_n(T, m_q) \left(\frac{\mu_q}{T} \right)^n \qquad c_n(T, m_q) = \frac{1}{n!} \frac{1}{VT^3} \frac{\partial^n \ln Z(T, \mu_q)}{\partial(\mu_q / T)^n} \bigg|_{\mu_q = 0}$$

- χ_2 requires (finite μ_B): $\hat{\chi}_{22}^{XB} = \frac{1}{VT^3} \frac{\partial^4 \log Z}{(\partial \mu_X / T)^2 (\partial \mu_B / T)^2}$
- Lattice: large volume and equilibrium.

Challenges:

- **Derivatives come with volume penalty in the statistics**
- **Electric charge is pion-dominated $\rightarrow$ fine lattices and taste-improvement.**
- **Baryon number noisy $\rightarrow$ large statistics.**

Fluctuations: Introduction

Fluctuations of conserved charges

- signal the transition between QGP and hadronic “phase”
- from HRG may be incorrect at T_f
- can be compared directly to experiment (μ_B allowing)

$$P(N_X) = \mathrm{e}^{-N_X^2/(2V_f T_f^2 \chi_2^X)}$$

- (ratio B/Q) can be used to relate fluctuations in proton number (experiment) to fluctuations in baryon number
- Can be used to extract the freeze-out parameters

Fluctuations: Introduction

- Freeze-out parameters directly from QCD:
  - Ratios of baryon number cumulants (Karsch 1202.4173)

$$\frac{\sigma_B^2}{M_B} = \frac{\chi_{2,\mu}^B}{\chi_{1,\mu}^B}, \qquad S_B\sigma_B = \frac{\chi_{3,\mu}^B}{\chi_{2,\mu}^B}, \qquad \kappa_B\sigma_B^2 = \frac{\chi_{4,\mu}^B}{\chi_{2,\mu}^B}$$

$$\frac{\sigma_B^2}{M_B} \equiv \frac{\chi_{2,\mu}^B}{\chi_{1,\mu}^B} = \frac{T}{\mu_B}\left[\frac{1+\frac{1}{2}\frac{\chi_4^B}{\chi_2^B}(\mu_B/T)^2+\ldots}{1+\frac{1}{6}\frac{\chi_4^B}{\chi_2^B}(\mu_B/T)^2+\ldots}\right]$$

$$\kappa_B\sigma_B^2 \equiv \frac{\chi_{4,\mu}^B}{\chi_{2,\mu}^B} = \frac{\chi_4^B(T)}{\chi_2^B(T)}\left[\frac{1+\frac{1}{2}\frac{\chi_6^B(T)}{\chi_4^B(T)}(\mu_B/T)^2+\ldots}{1+\frac{1}{2}\frac{\chi_4^B(T)}{\chi_2^B(T)}(\mu_B/T)^2+\ldots}\right]$$

  - Alternative for T_f: use charge fluctuations instead

Fluctuations: Introduction

- Freeze-out parameters directly from QCD:
  - "Generalized" Ratios of cumulants (Bazavov *et al.*, 1208.1220)

$$M_S \equiv 0 \,, \quad M_Q = r M_B$$

$$\hat{\mu}_Q = q_1 \, \hat{\mu}_B + q_3 \, \hat{\mu}_B^3 \,, \quad \hat{\mu}_S = s_1 \, \hat{\mu}_B + s_3 \, \hat{\mu}_B^3 \,, \quad \hat{\mu}_X = \mu_X / T$$

$$q_1 = \frac{r\left(\chi_2^B \chi_2^S - \chi_{11}^{BS} \chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS}\right)}{\left(\chi_2^Q \chi_2^S - \chi_{11}^{QS} \chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS}\right)} \,, \quad s_1 = -\frac{\chi_{11}^{BS}}{\chi_2^S} - \frac{\chi_{11}^{QS}}{\chi_2^S} \, q_1$$

$$R_{12}^X \equiv \frac{M_X}{\sigma_X^2} = \hat{\mu}_B \left(R_{12}^{X,1} + R_{12}^{X,3} \, \hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)\right)$$

$$R_{12}^{X,1} = \frac{\chi_{11}^{BX}}{\chi_2^X} + q_1 \frac{\chi_{11}^{XQ}}{\chi_2^X} + s_1 \frac{\chi_{11}^{XS}}{\chi_2^X}$$

Fluctuations: Introduction

Correlations

- show a strong temperature dependence and characteristic behavior around T_c
- vanish in an ideal non-interacting QGP
- HTL corrections predict a non-zero value even at large T
- can signal bound-state survival above T_c
- check the applicability of the HRG for low temperatures

Fluctuations: quark mass basis

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q$$
$$\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q$$
$$\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$$

- LQCD action is given in 'quark mass basis'
- express the chemical potentials in terms of flavor chemical potentials

$$\frac{d}{d\mu_B} = \frac{1}{3}\partial_u + \frac{1}{3}\partial_d + \frac{1}{3}\partial_s$$
$$\frac{d}{d\mu_Q} = \frac{2}{3}\partial_u - \frac{1}{3}\partial_d - \frac{1}{3}\partial_s$$
$$\frac{d}{d\mu_I} = \frac{1}{2}\partial_u - \frac{1}{2}\partial_d$$
$$\frac{d}{d\mu_S} = -\partial_s$$

- The interesting fluctuations require derivatives w.r.t. 'rotated' basis
- use above map to 'rotate' derivatives appropriately

Fluctuations: derivatives

$$\chi^{us}_{i,j} = \frac{T}{V}\frac{\partial^{i+j}\log Z}{(\partial\mu_u)^i(\partial\mu_s)^j}$$

$$\partial_i \log Z = \frac{1}{Z}\int \mathcal{D}U\ \partial_i e^{-S_{\rm eff}} = \langle A_i\rangle \qquad (=0 \text{ at } \mu = 0)$$

$$\begin{aligned}\partial_j \langle X\rangle &= -\langle X\rangle(\partial_j \log Z) + \left\langle X\partial_j e^{-S_{\rm eff}}\right\rangle + \langle\partial_j X\rangle \\ &= \langle X A_j\rangle - \langle X\rangle\langle A_j\rangle + \langle\partial_j X\rangle\end{aligned}$$

$$\begin{aligned}\partial_i\partial_j \log Z &= \langle A_i A_j\rangle - \langle A_i\rangle\langle A_j\rangle + \delta_{ij}\langle B_i\rangle \\ \partial_i^4 \log Z &= \left\langle A_i^4\right\rangle - 3\left\langle A_i^2\right\rangle^2 + 3\left(\left\langle B_i^2\right\rangle - \langle B_i\rangle^2\right) \\ &\quad +6\left(\left\langle A_i^2 B_i\right\rangle - \left\langle A_i^2\right\rangle\langle B_i\rangle\right) + 4\langle A_i C_i\rangle + \langle D_i\rangle\end{aligned}$$

Fluctuations: lattice techniques

- Use SET to estimate traces (Gottlieb et al. PRL 59 (**1987**) 2247)

$$\mathtt{tr}(A) \approx \frac{1}{s}\sum_{k=1}^{s} v_k^\top A v_k$$

- Disconnected diagrams $\rightarrow$ use different sets of random vectors ($c_4 \rightarrow 4\times$ inversions)
- diagrams (e.g. $\langle A_i^2 \rangle$ and $\langle A_i^2 \cdot B \rangle$) allow reuse of inversions $\rightarrow$ increase statistics
- We use up to 1000 sources to estimate the trace or O(10k) inversions

Fluctuations: error estimation

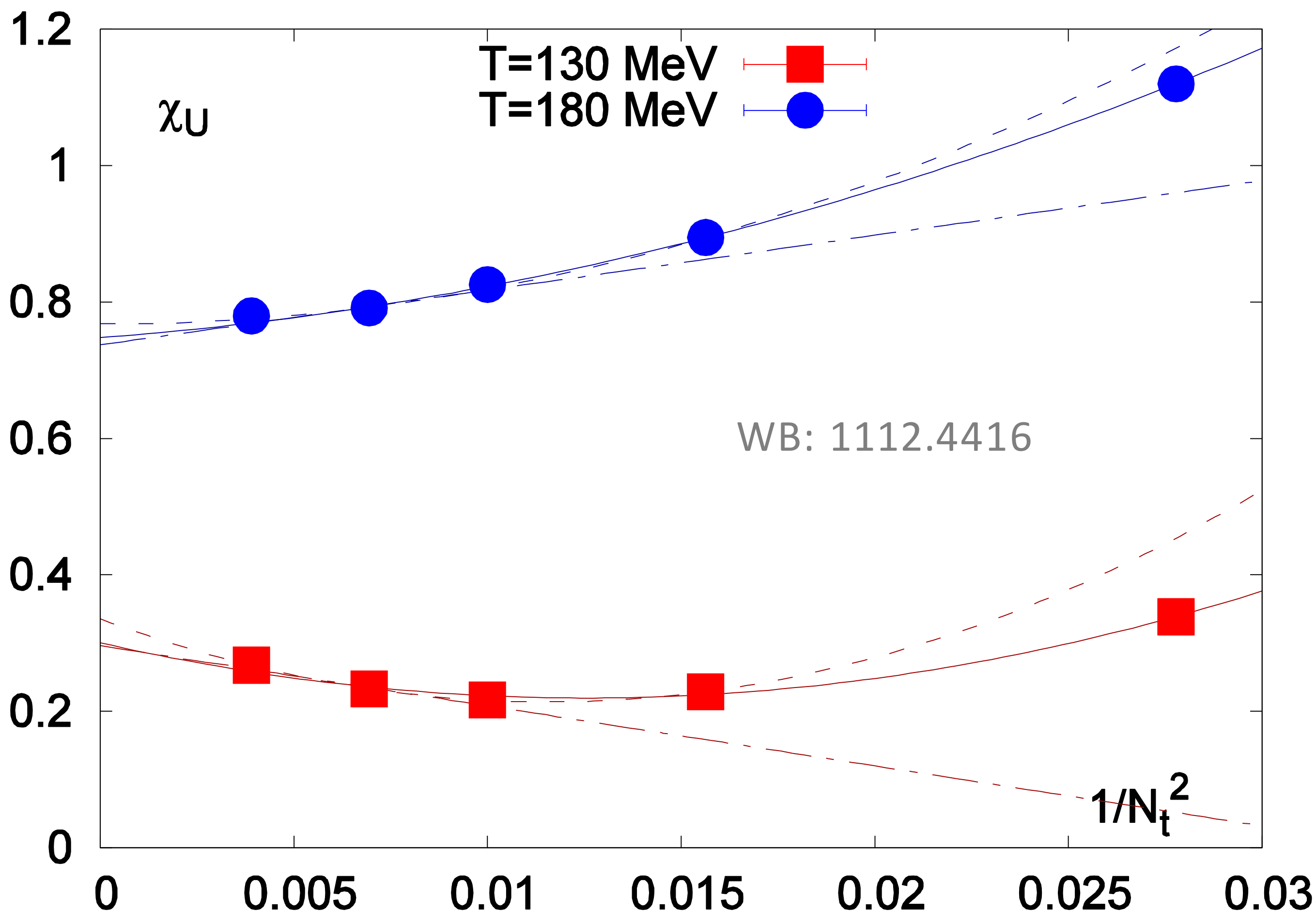
Aim: reliable systematic and statistical errors

- We work at M_π = phys $\rightarrow$ no chiral extrapolation
- Continuum extrapolation:
  - Use 2 sets of nodepoints for spline interpolation at fixed N_t
  - Extrapolate χ and $1/\chi$
  - Use 4 different extrapolation formulae

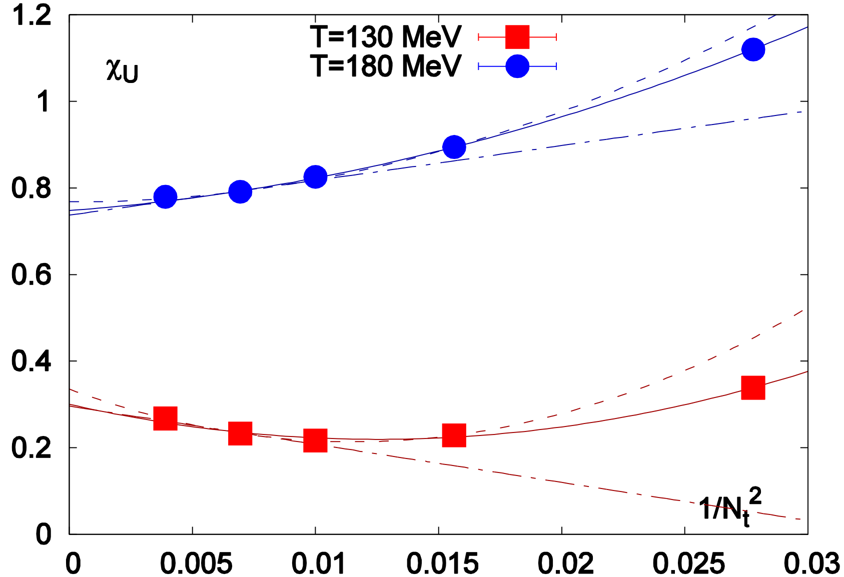
  $\rightarrow$ difference gives estimate of systematic uncertainty
- Use jacknife estimate of deviation form mean to estimate statistical uncertainty

Systematic uncertainty is dominant

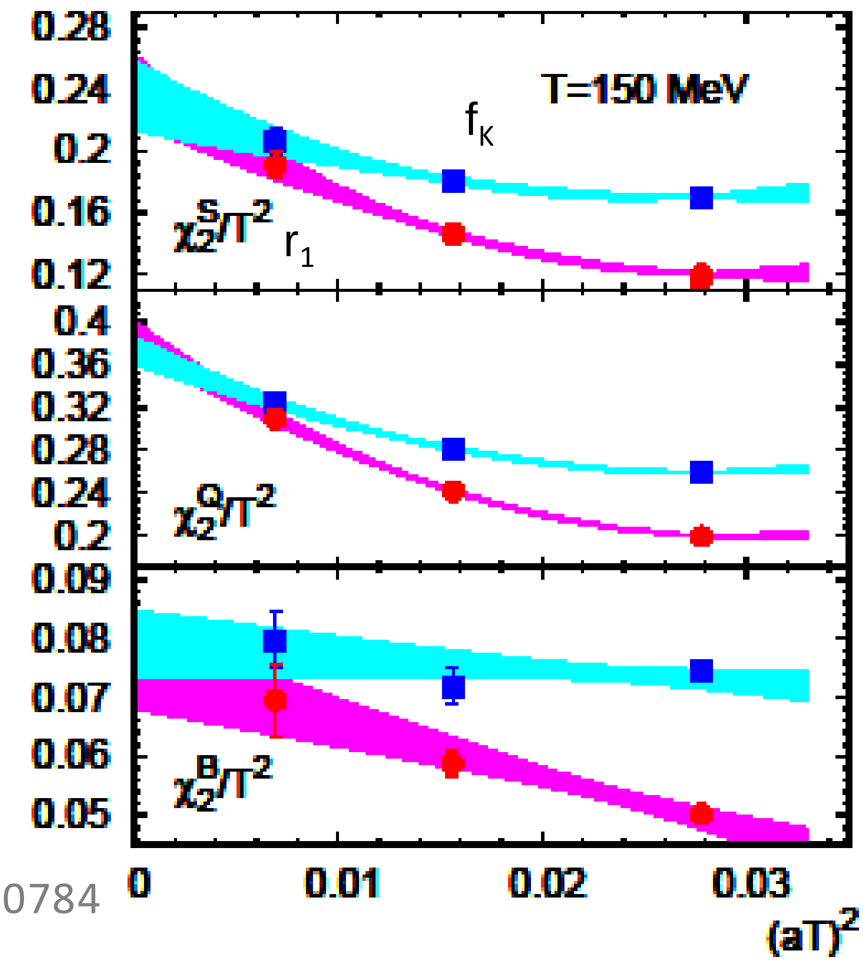
Fluctuations: continuum extrapolation



Fluctuations: continuum extrapolation

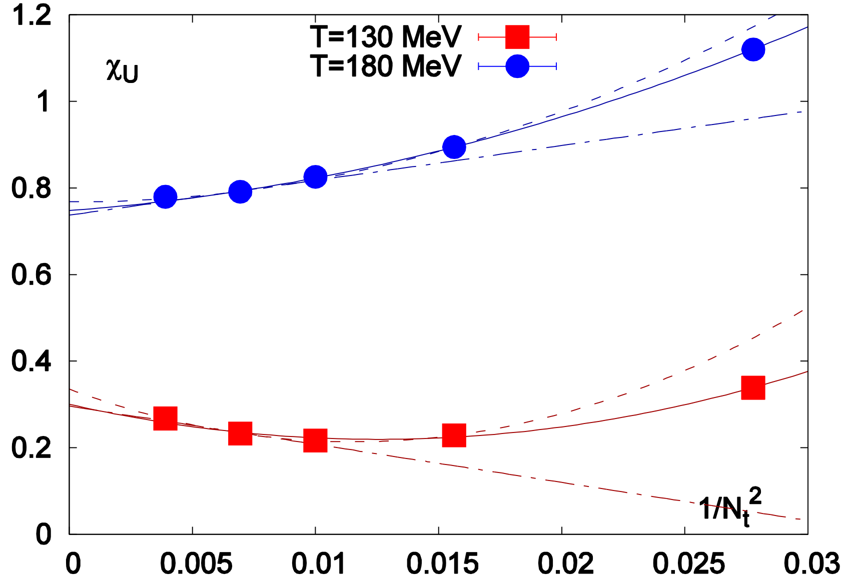


WB: 1112.4416

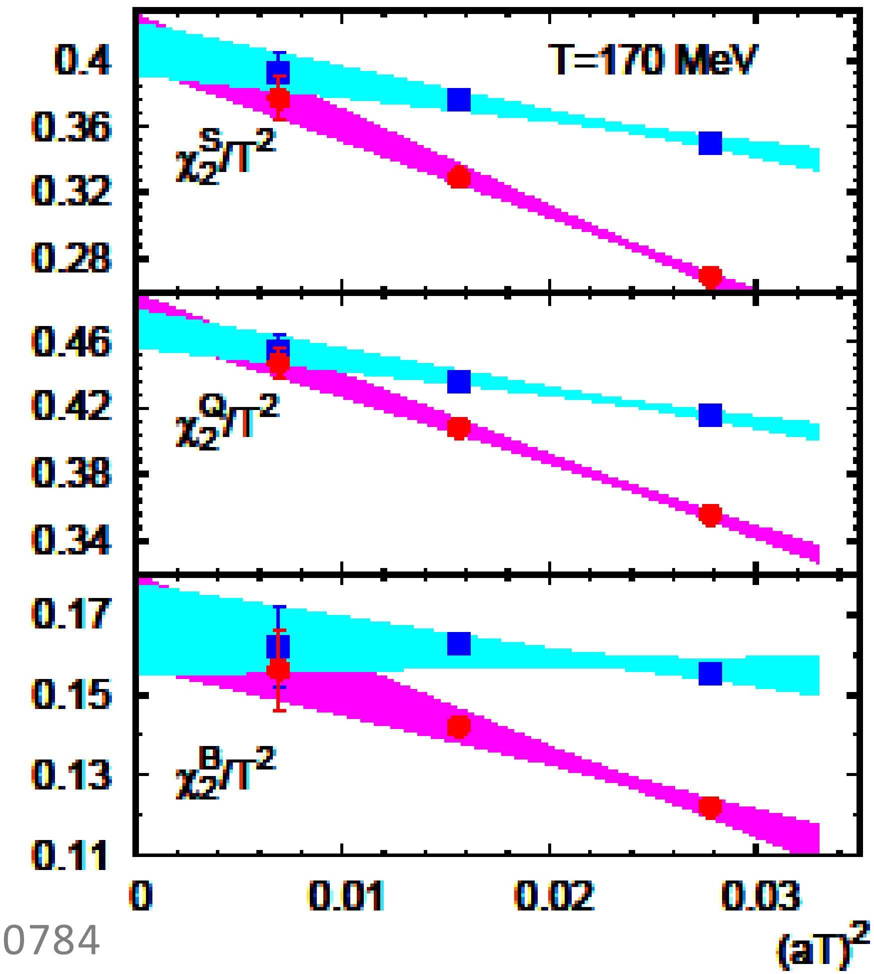


hotQCD: 1203.0784

Fluctuations: continuum extrapolation



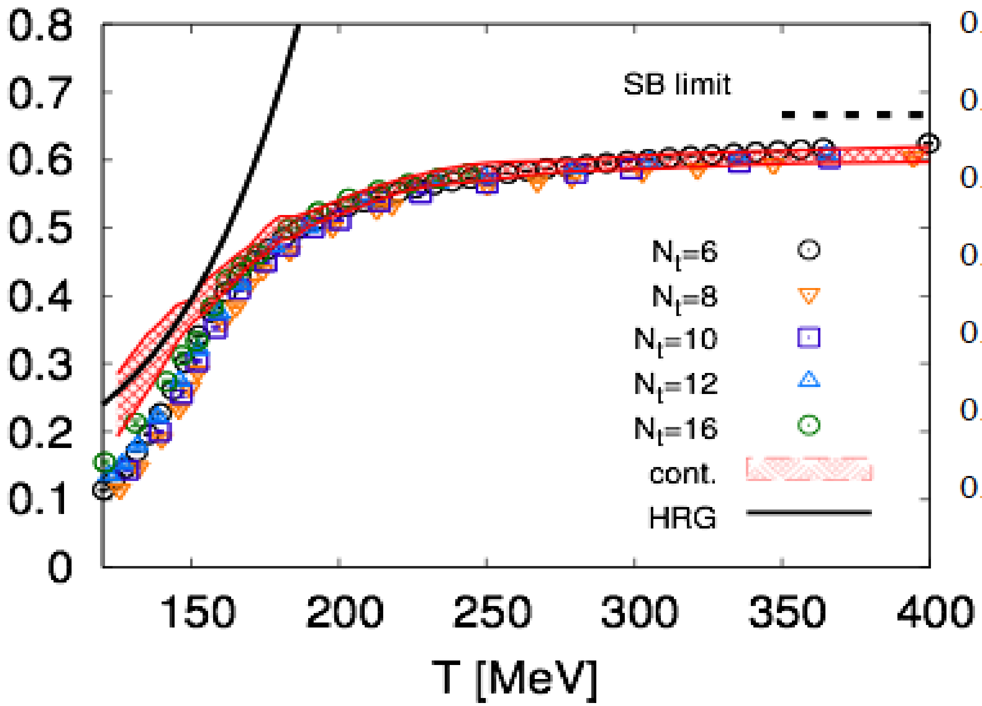
WB: 1112.4416



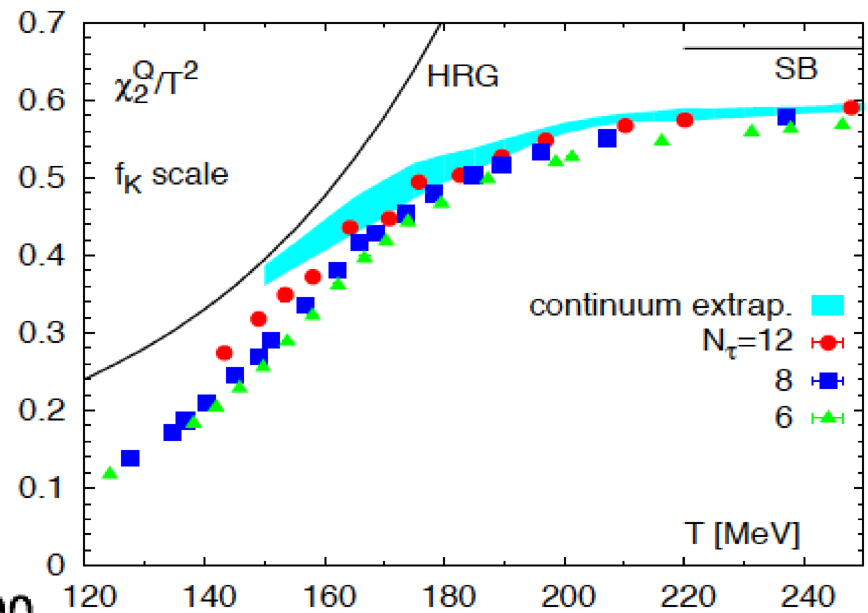
hotQCD: 1203.0784

Fluctuations: c_2 diagonal elements: $\chi_2{}^Q$

WB: 1112.4416



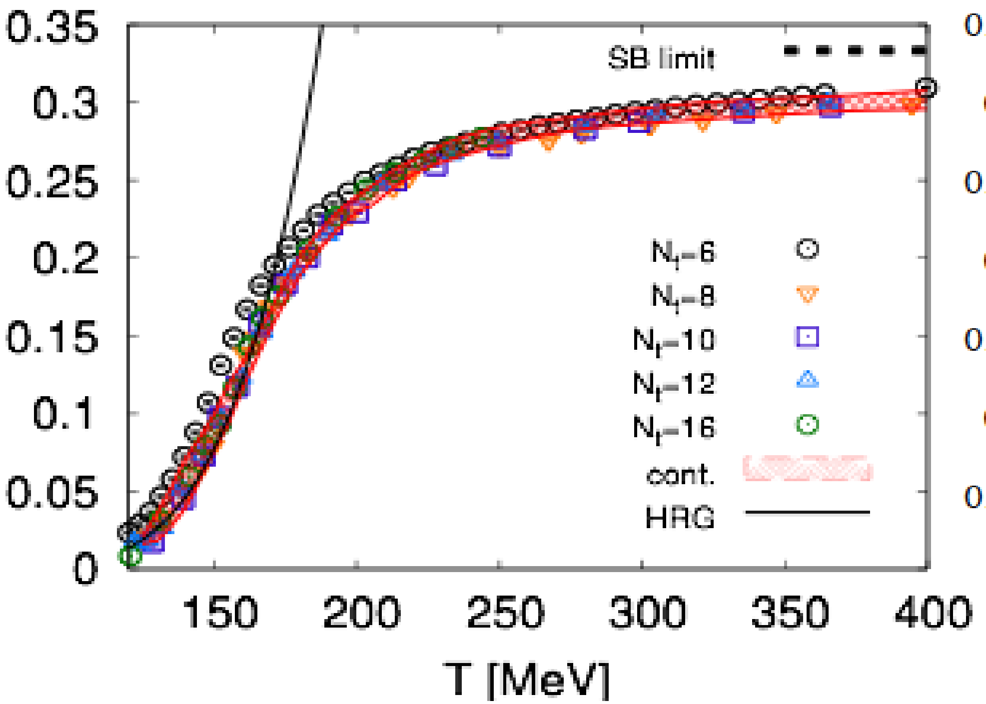
hotQCD: 1203.0784



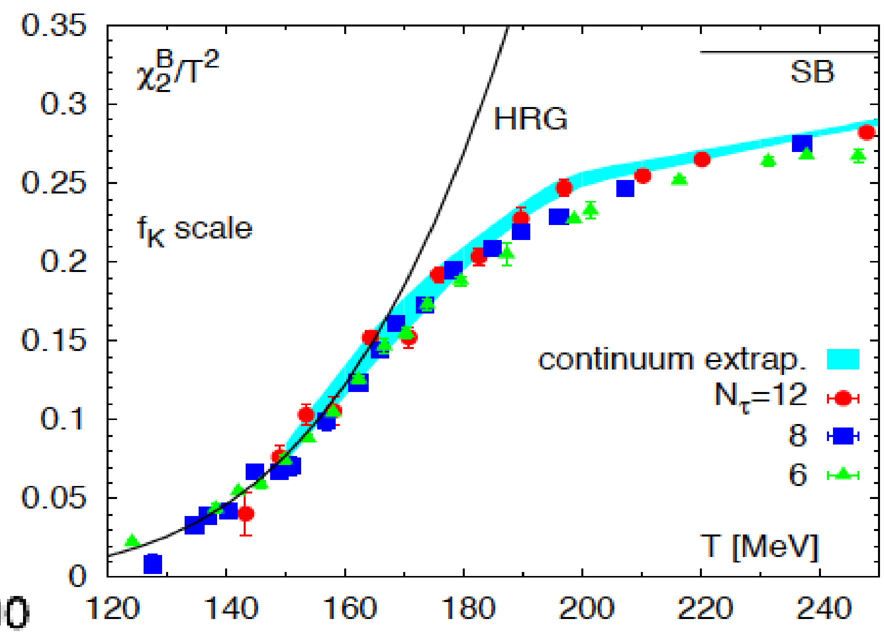
Agreement with HRG visible at small T

Fluctuations: c_2 diagonal elements: $\chi_2{}^B$

WB: 1112.4416

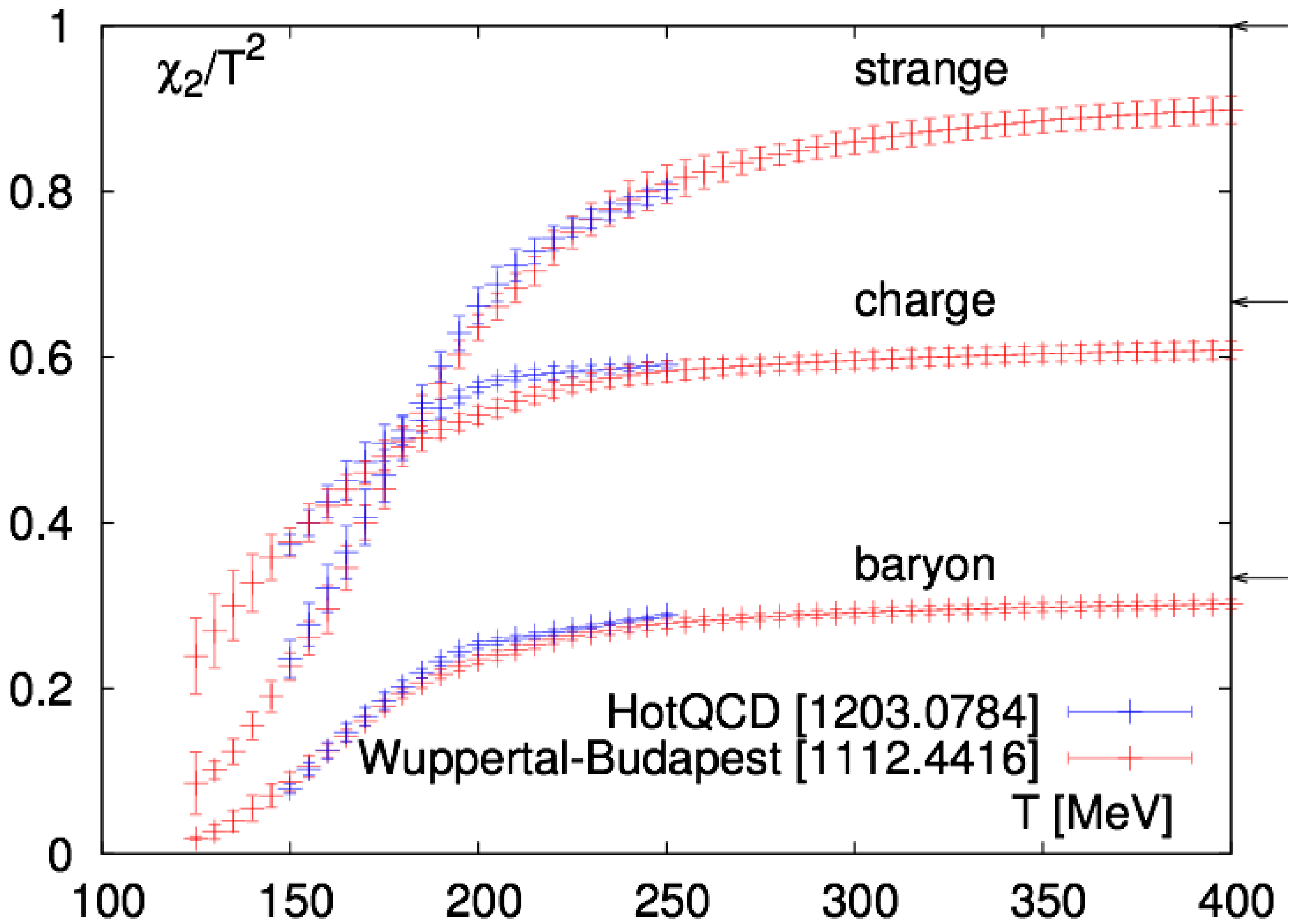


hotQCD: 1203.0784

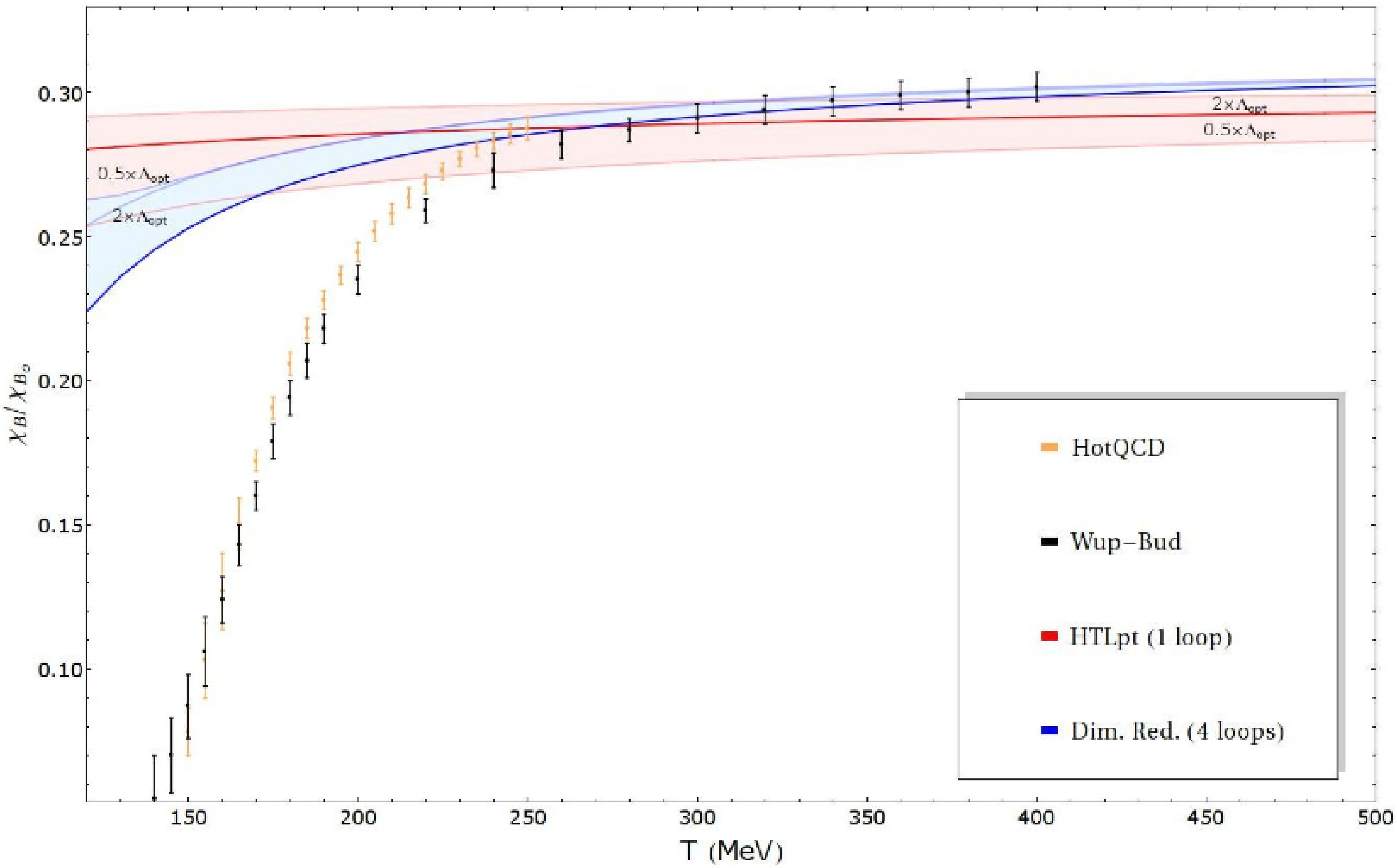


Agreement with HRG visible at small T

Fluctuations: crosscheck for c_2

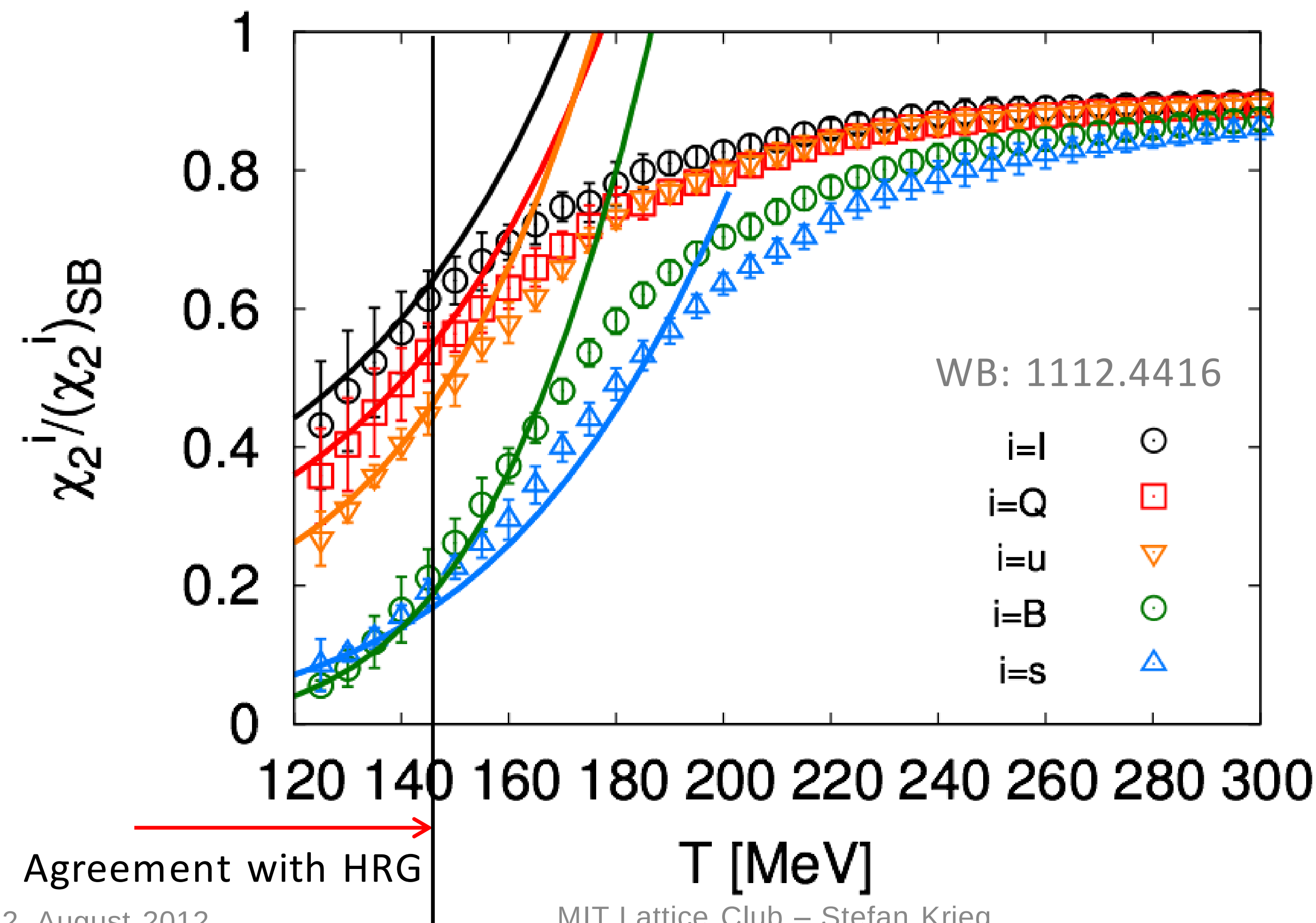


Fluctuations: lattice vs. analytical results



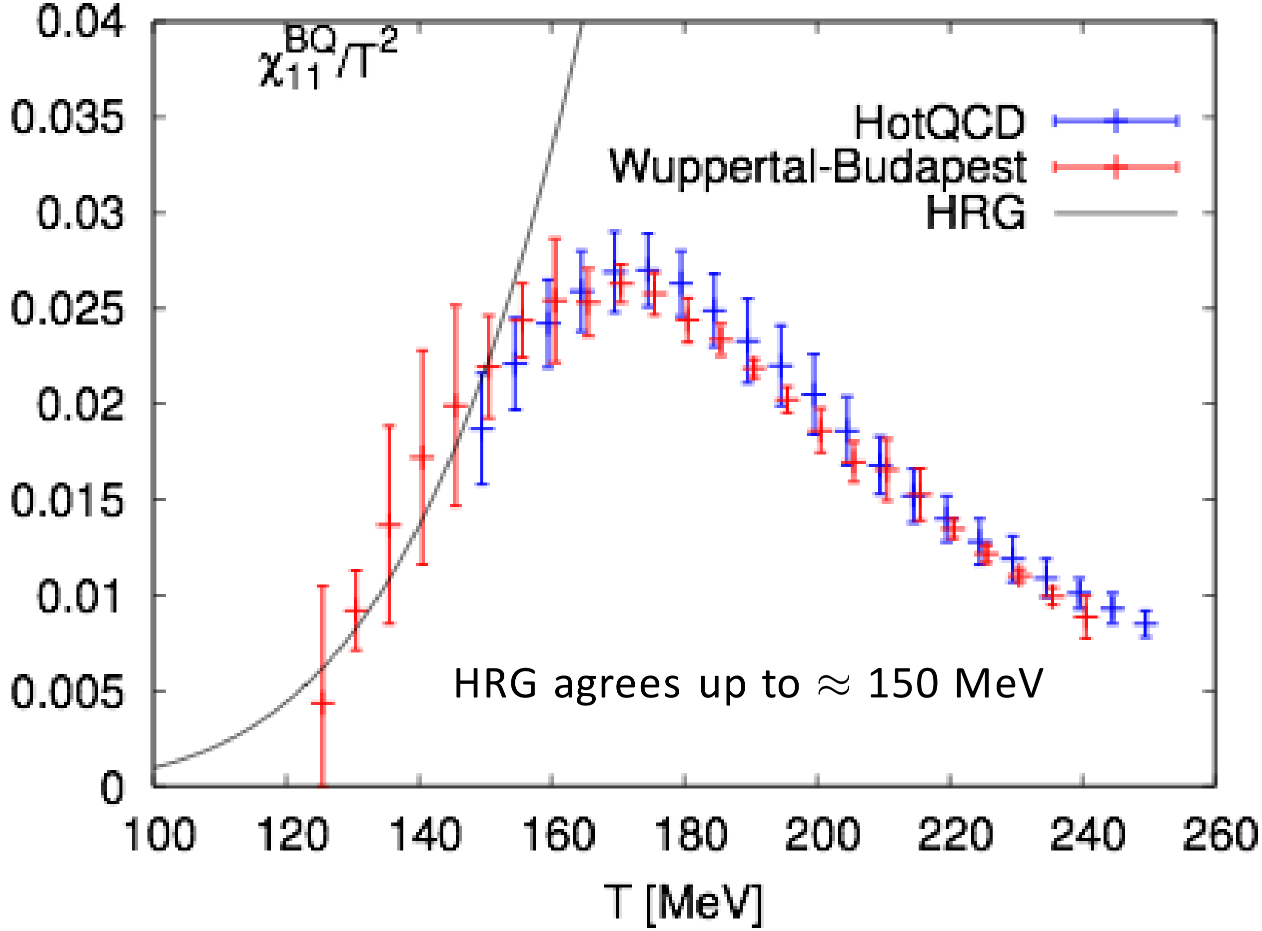
[Andersen, Mogliacci, Su, Vuorinen 2012]

Fluctuations: comparison with HRG

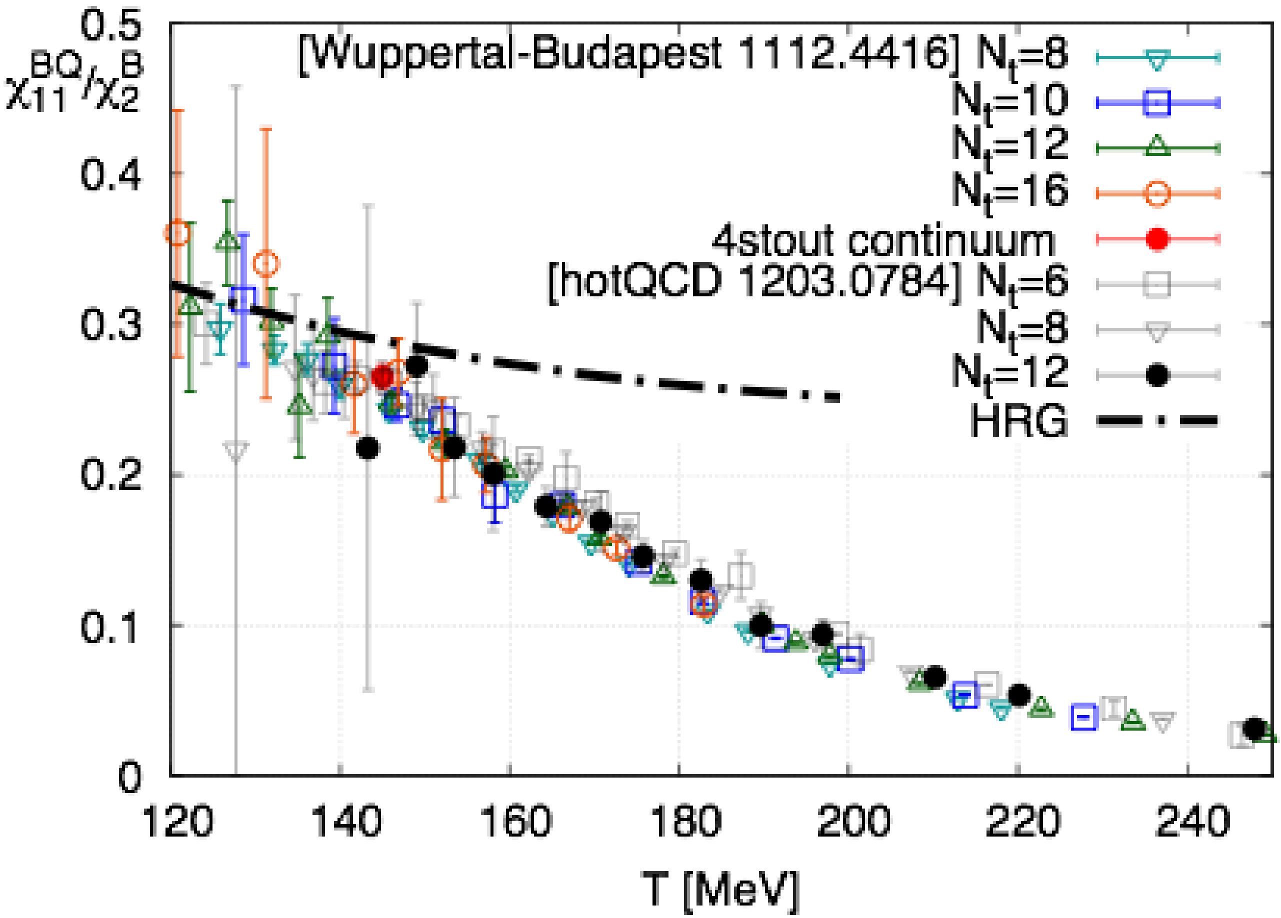


Agreement with HRG

Fluctuations: correlators

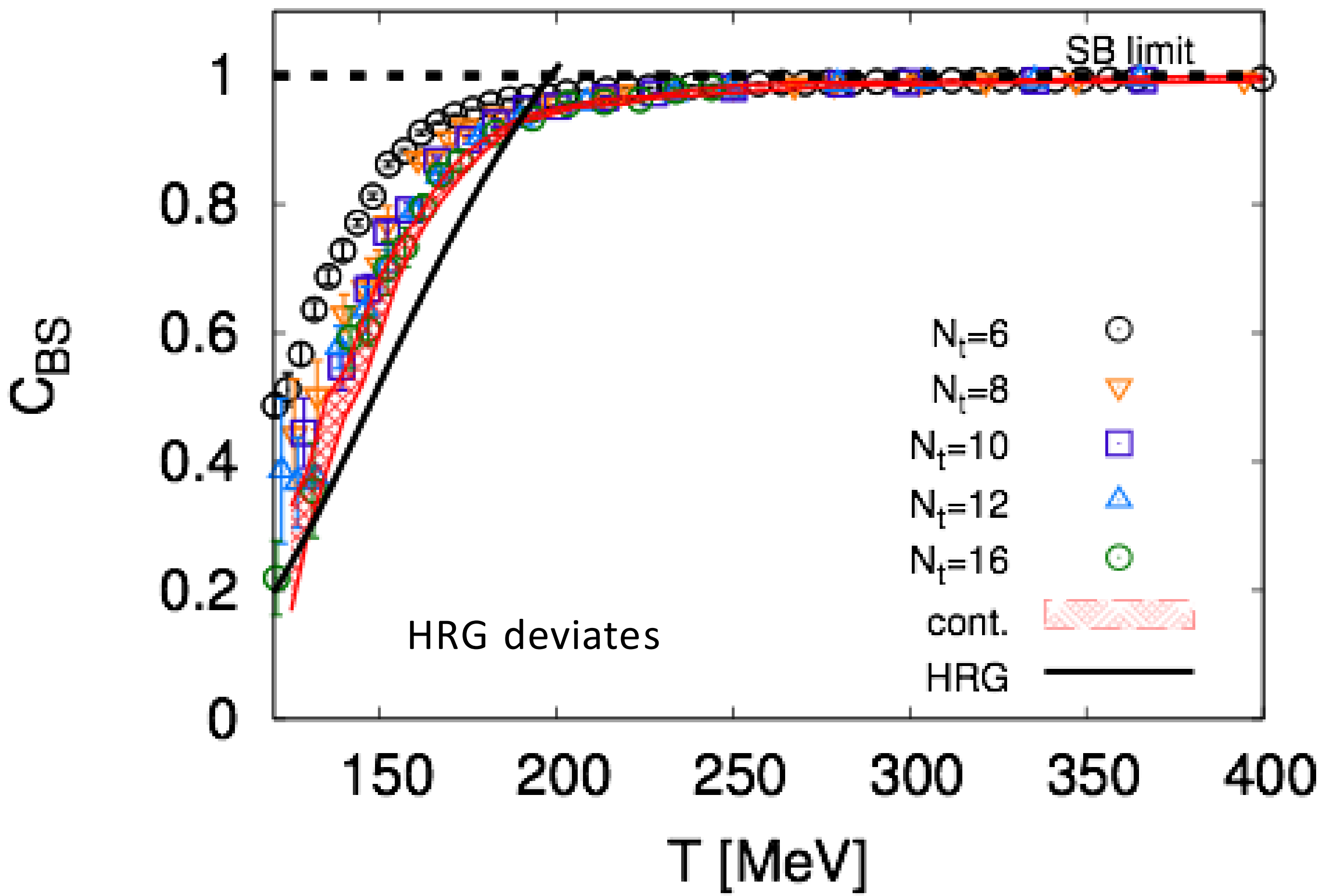


Fluctuations: correlators

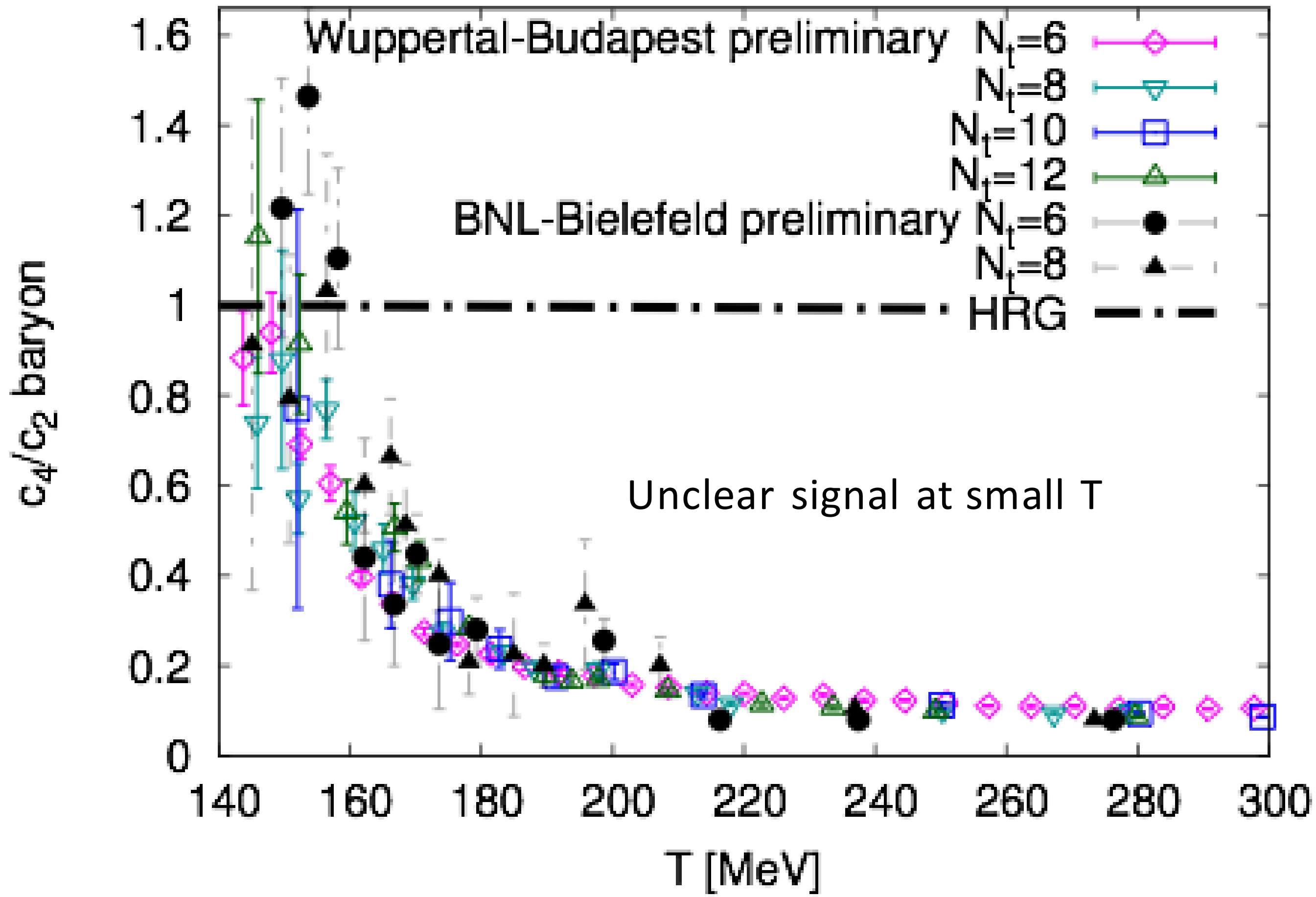


Fluctuations: correlators

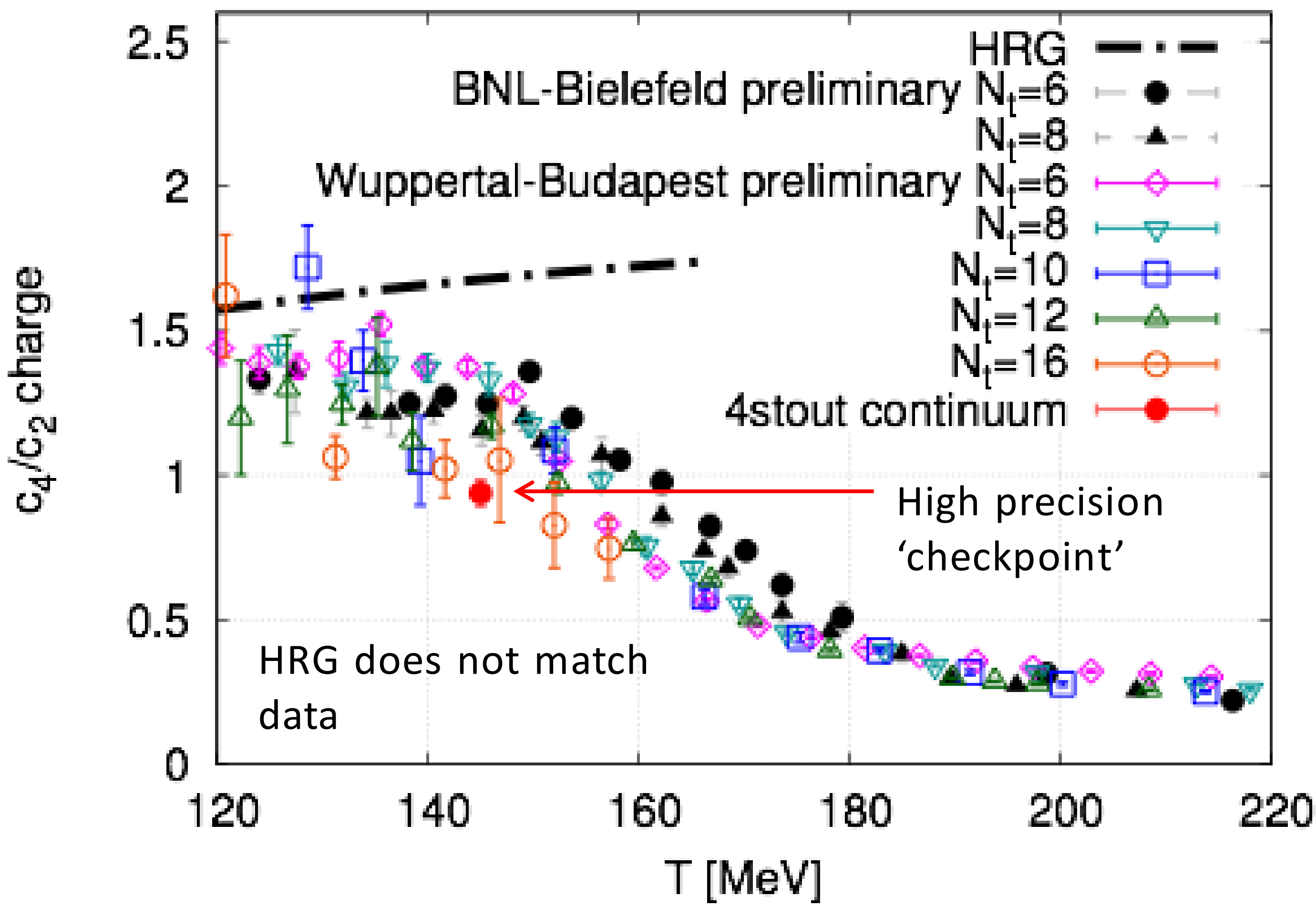
$$C_{BS} = -3\frac{\langle N_B N_S \rangle}{\langle N_S^2 \rangle} = 1 + \frac{\chi_{11}^{us} + \chi_{11}^{ds}}{\chi_2^s}$$



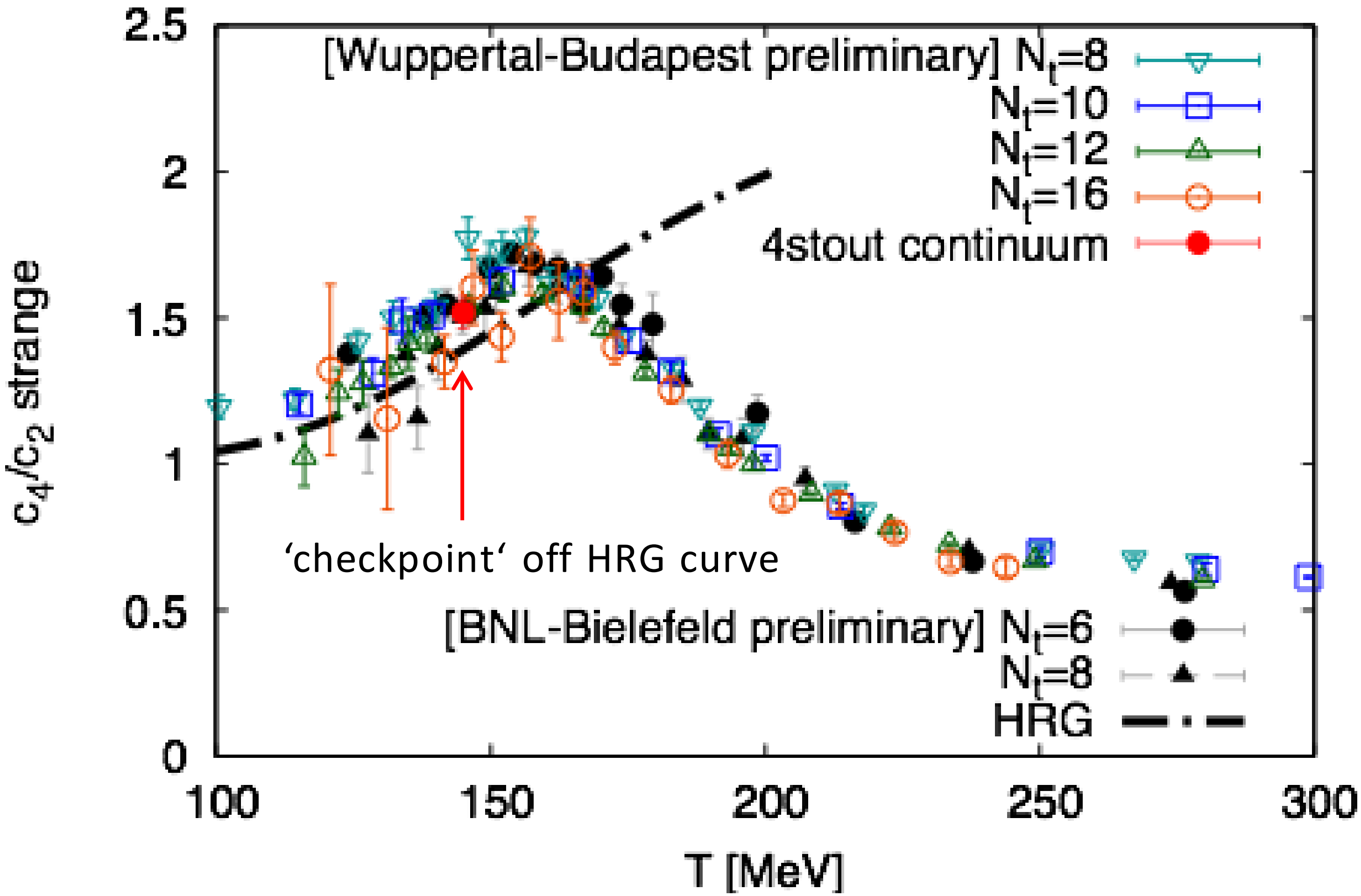
Fluctuations: kurtosis c_4/c_2



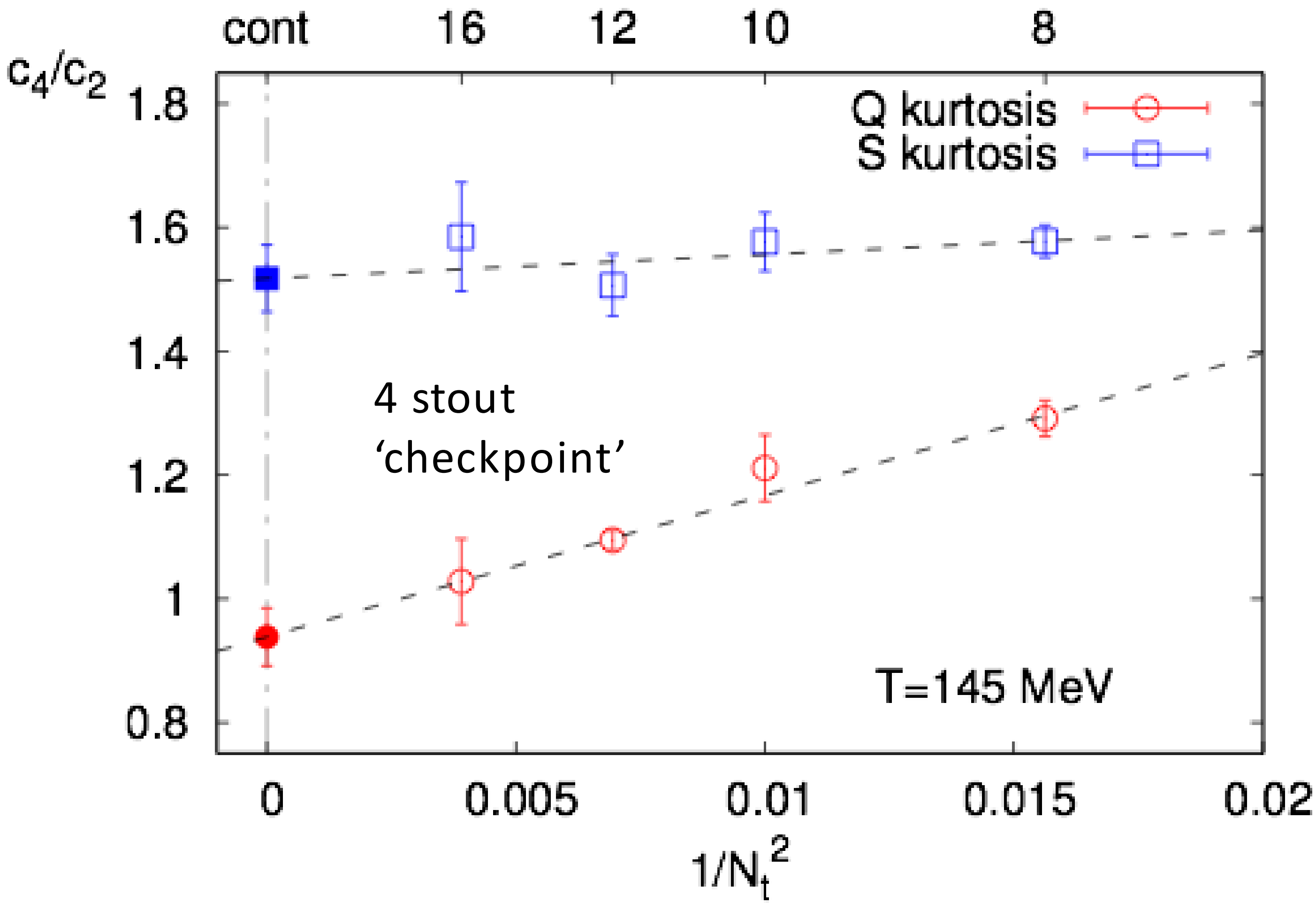
Fluctuations: kurtosis c_4/c_2



Fluctuations: kurtosis c_4/c_2



Fluctuations: kurtosis c_4/c_2: zoom in



Conclusions and outlook

- Lattice results are getting increasingly precise
  - Different collaborations agree on T_c
  - The 'gap' in the EoS is closing
  - Crosschecks
    - provide a handle on systematic uncertainties
    - Agree well with available staggered results so far
- Fluctuations computed on the lattice
  - Deviate from HRG at temperatures as low as $T \approx 130$ MeV
  - Are consistent between the collaborations

Thank you!