Quark and Glue Components of Proton Spin and Meson Masses

- Status of nucleon spin components
- Momentum and angular momentum sum rules
- Lattice results
- Quark spin from anomalous Ward identity
- Meson masses

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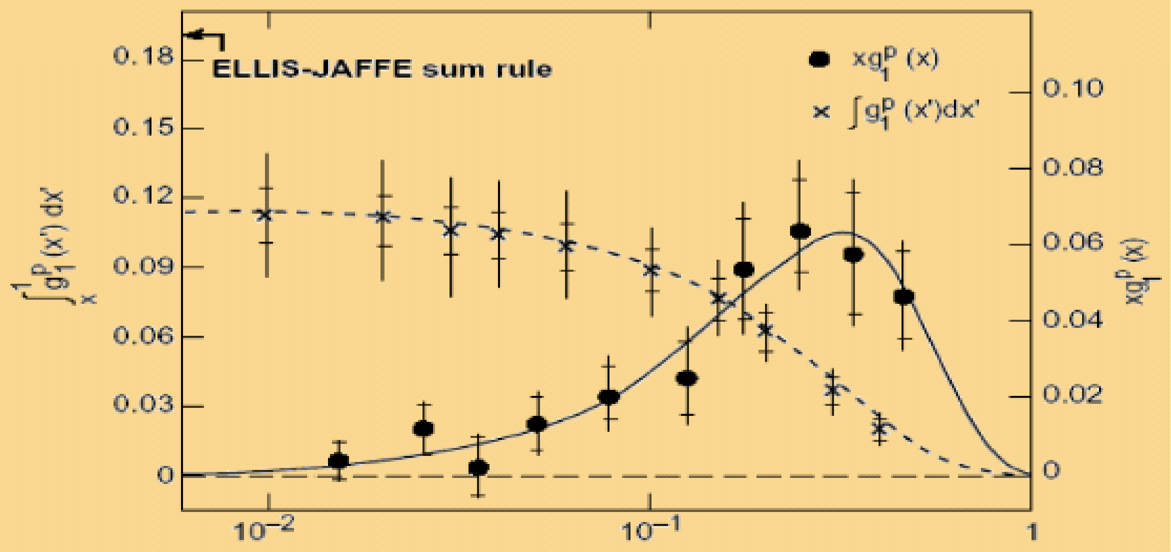
MIT, Feb. 21, 2014

Where does the spin of the proton come from?

Twenty years since the "spin crisis"

5 ^ (handwritten correction: "Twenty" → "Twenty-five" implied)

- EMC experiment in 1988/1989 – "the plot":



$$g_1(x) = \frac{1}{2}\sum_q e_q^2\,[\Delta q(x) + \Delta\bar{q}(x)] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

$$\Delta q = \int_0^1 dx\,\Delta q(x) = \langle P, s_{||}|\overline{\psi}_q(0)\gamma^+\gamma_5\psi_q(0)|P, s_{||}\rangle$$

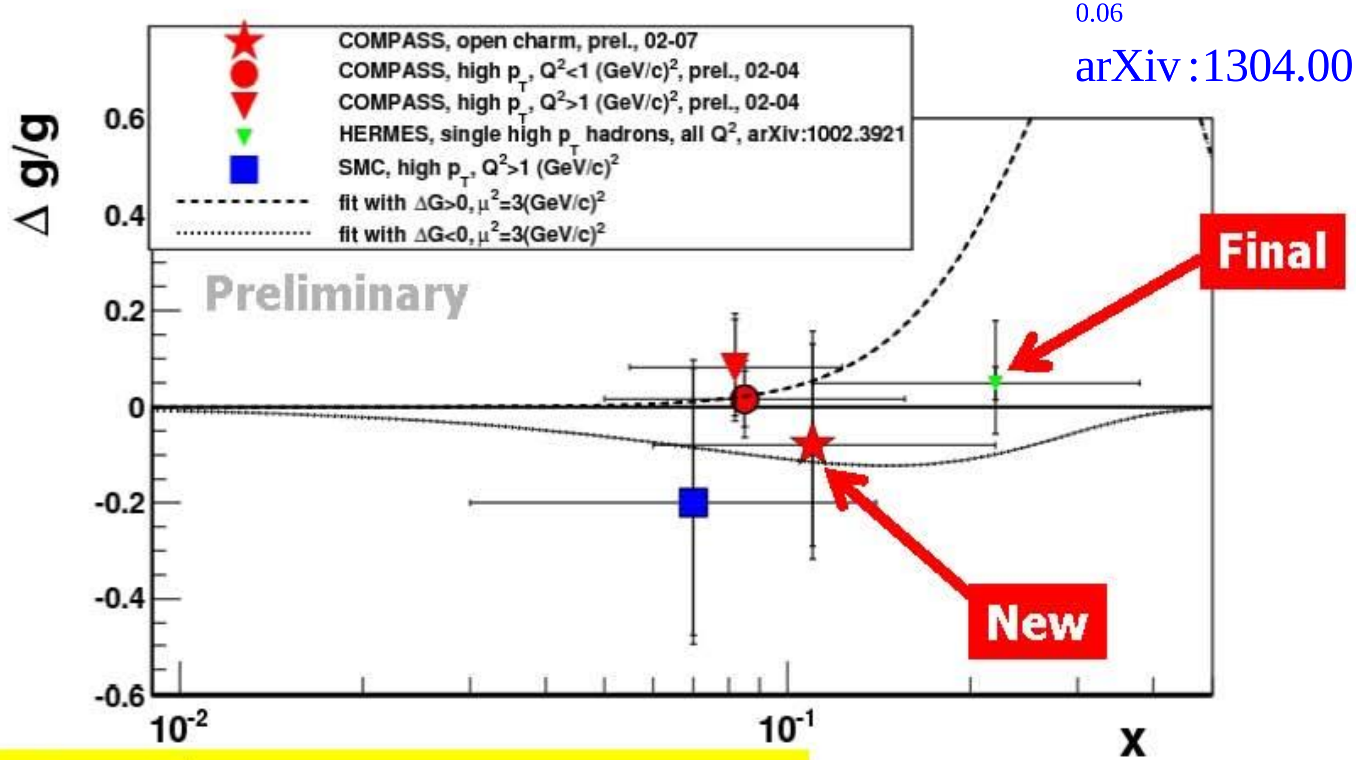
- "Spin crisis" or puzzle: $\Delta\Sigma = \sum_q \Delta q + \Delta\bar{q} = 0.2 - 0.3$

Summary Gluon Polarization

Presently all Analysis in LO only

$$\int_{0.06}^{0.2} \Delta g(x)\,dx = 0.1 \pm^{0.06}_{0.07}$$

arXiv:1304.0079



COMPASS Open Charm:

$\Delta G/G$=-0.08+- 0.21(stat) +- 0.11(sys.)
(Systematic error still under investigations)

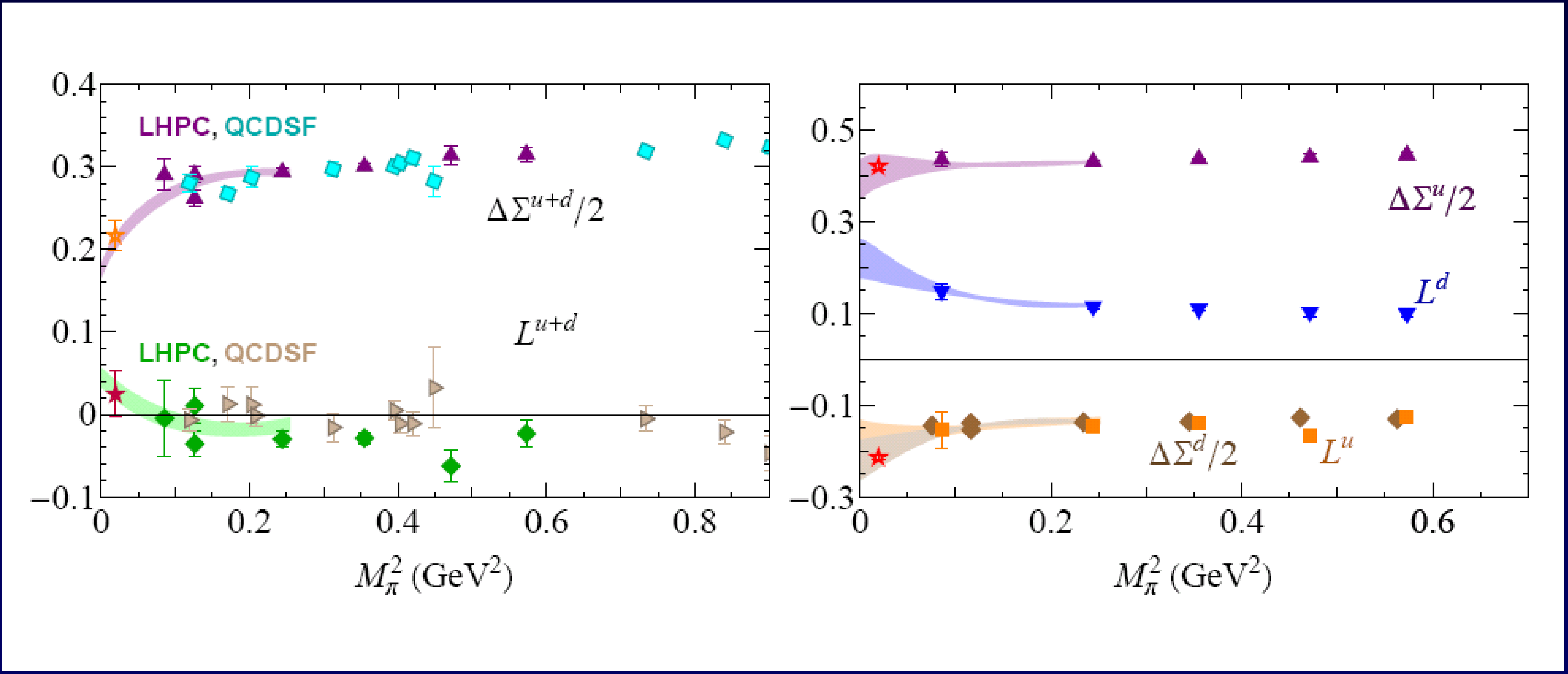
(Value supersedes previous publication)

C.Franco

See Talk 1193 by F. Kunne

Horst Fischer DIS2010

Quark Orbital Angular Momentum
(connected insertion)



Status of Proton Spin

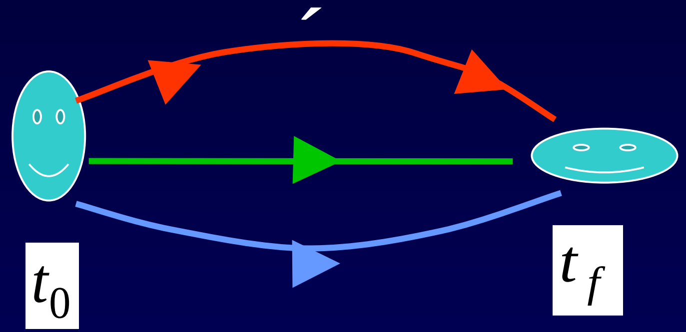
- Quark spin $\Delta\Sigma \sim 20 - 30\%$ of proton spin (DIS, Lattice)
- Quark orbital angular momentum? (lattice calculation (LHPC,QCDSF)→ ~ 0)
- Glue spin $\Delta G/G$ small (COMPASS, STAR) ?
- Glue orbital angular momentum is zero (Brodsky and Gardner) ?

→ Dark Spin ?

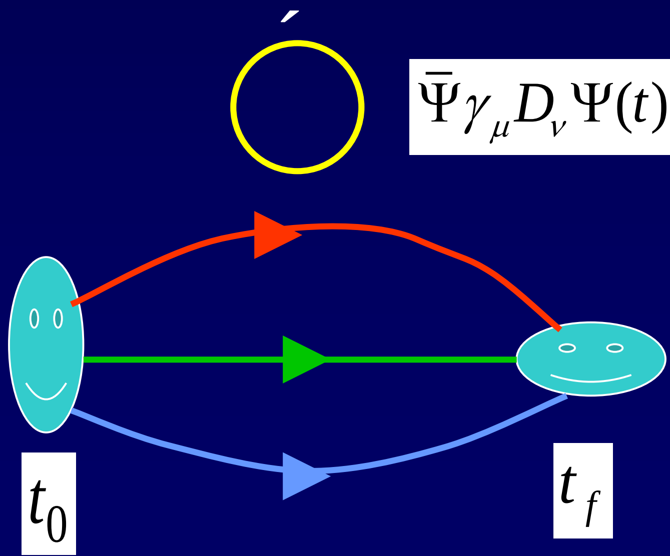
Hadron Structure with Quarks and Glue

- Quark and Glue Momentum and Angular Momentum in the Nucleon

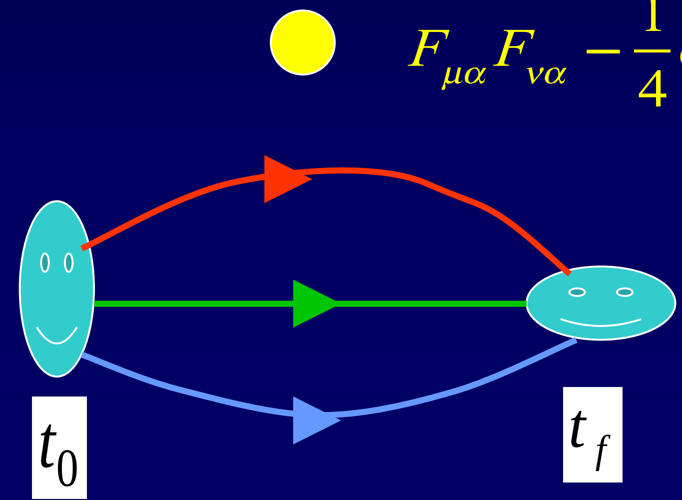
$$(\bar{u}\gamma_\mu D_\nu u + \bar{d}\gamma_\mu D_\nu d)(t)$$



$$\bar{\Psi}\gamma_\mu D_\nu \Psi(t)(u,d,s)$$



$$F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$$



Momenta and Angular Momenta of Quarks and Glue

- Energy momentum tensor operators decomposed in quark and glue parts gauge invariantly --- Xiangdong Ji (1997)

$$T^q_{\mu\nu} = \frac{i}{4}\left[\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi + (\mu \leftrightarrow \nu)\right] \rightarrow \vec{J}_q = \int d^3x \left[\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi + \vec{x}\times\bar{\psi}\gamma_4(-i\vec{D})\psi\right]$$

$$T^g_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} - \frac{1}{4}\delta_{\mu\nu}F^2 \rightarrow \vec{J}_g = \int d^3x\left[\vec{x}\times(\vec{E}\times\vec{B})\right]$$

- Nucleon form factors

$$\langle p,s\,|\,T_{\mu\nu}\,|\,p's'\rangle = \bar{u}(p,s)[T_1(q^2)\gamma_\mu\bar{p}_\nu - T_2(q^2)\bar{p}_\mu\sigma_{\nu\alpha}q_\alpha/2m$$

$$-iT_3(q^2)(q_\mu q_\nu - \delta_{\mu\nu}q^2)/m + T_4(q^2)\delta_{\mu\nu}m/2]u(p's')$$

- Momentum and Angular Momentum

$$Z_{q,g}T_1(0)_{q,g}\ [\text{OPE}] \rightarrow \langle x\rangle_{q/g}(\mu,\overline{\text{MS}}),\quad Z_{q,g}\left[\frac{T_1(0)+T_2(0)}{2}\right]_{q,g} \rightarrow J_{q/g}(\mu,\overline{\text{MS}})$$

$T_1\,(q^2)$ and $T_2\,(q^2)$

- 3-pt to 2-pt function ratios

$$G_m^{3pt}(\vec{p},t_2;\vec{q},t_1) = \sum_{\vec{x}_1,\vec{x}_2} e^{-i\vec{p}\cdot\vec{x}_2 + i\vec{q}\cdot\vec{x}_1} \left\langle 0 \,|\, T\left[c_N(\vec{x}_2,t_2) T_m(t_1) \bar{c}_N(0) \right] \right\rangle;$$

$$\mathrm{Tr}\left[\Gamma_m G_m^{3pt}(\vec{p}=0,t_2;\vec{q},t_1) \right] = We^{-m(t_2-t_1)} e^{-Et_1} \left[T_1(q^2) + T_2(q^2) \right]$$

- Need both polarized and unpolarized nucleon and different kinematics (p_i, q_j, s) to separate out $T_1\,(q^2)$, $T_2\,(q^2)$ and $T_3\,(q^2)$

Renormalization and Quark-Glue Mixing

Momentum and Angular Momentum Sum Rules

$$\langle x\rangle_q^R = Z_q \langle x\rangle_q^L, \quad \langle x\rangle_g^R = Z_g \langle x\rangle_g^L,$$

$$J_q^R = Z_q J_q^L, \quad J_g^R = Z_g J_g^L,$$

$$Z_q \langle x\rangle_q^L + Z_g \langle x\rangle_g^L = 1,$$

$$Z_q J_q^L + Z_g J_g^L = \frac{1}{2} \Rightarrow \begin{cases} Z_q T_1^q(0) + Z_g T_1^g(0) = 1, \\ Z_q (T_1^q + T_2^q)(0) + Z_g (T_1^g + T_2^g)(0) = 1, \\ Z_q T_2^q(0) + Z_g T_2^g(0) = 0 \end{cases}$$

Mixing

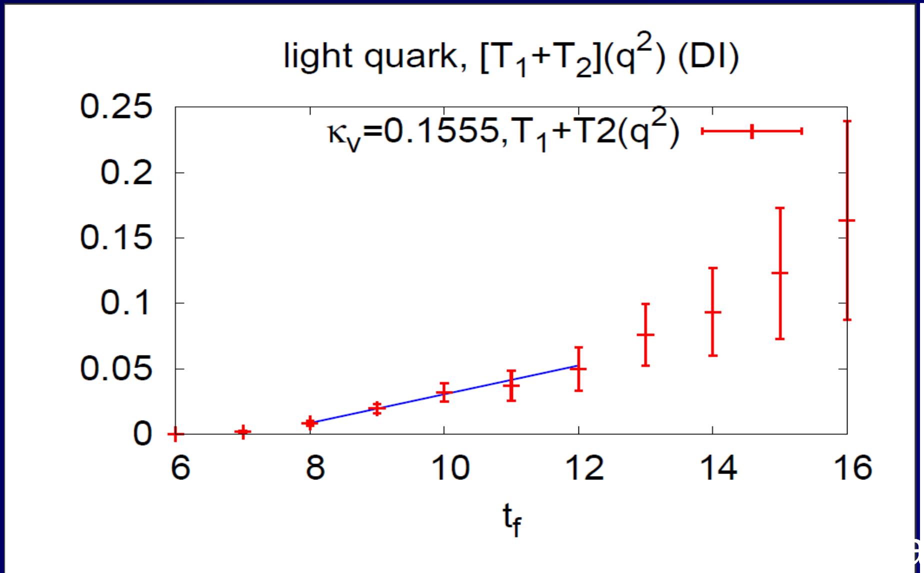
$$\begin{bmatrix} \langle x\rangle_q^{\overline{MS}}(\mu) \\ \langle x\rangle_g^{\overline{MS}}(\mu) \end{bmatrix} = \begin{bmatrix} C_{qq}(\mu) & C_{qg}(\mu) \\ C_{gq}(\mu) & C_{gg}(\mu) \end{bmatrix} \begin{bmatrix} \langle x\rangle_q^R \\ \langle x\rangle_g^R \end{bmatrix}$$

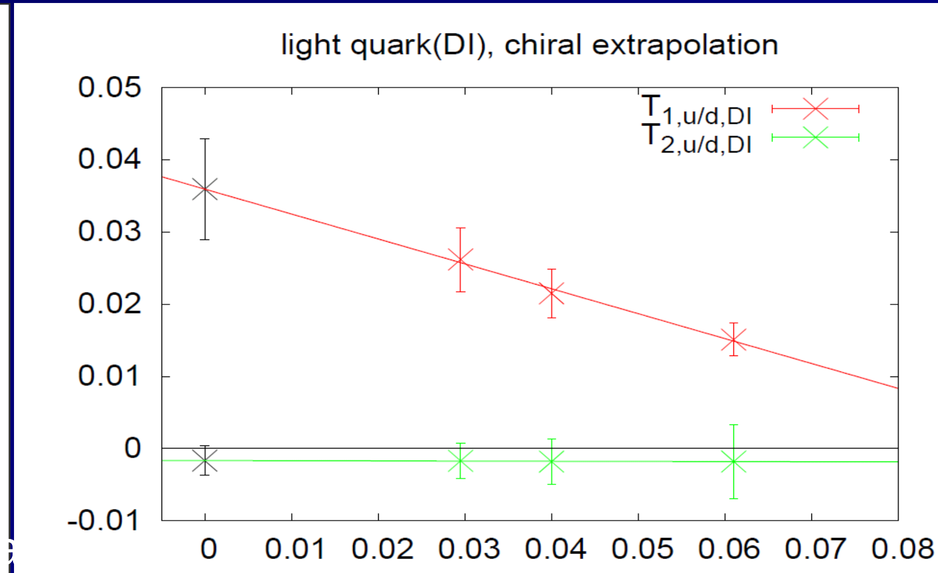
M. Glatzmaier

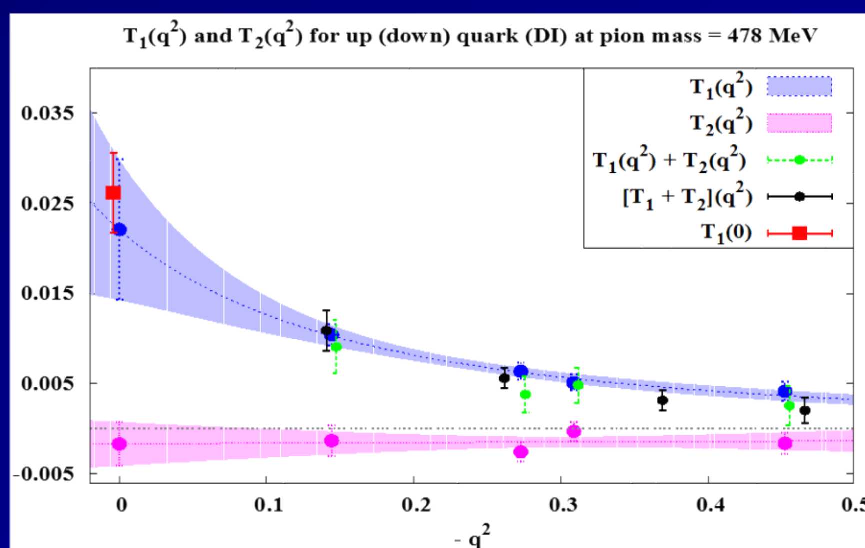
Lattice Parameters

- Quenched 16^3 x 24 lattice with Wilson fermion
- Quark spin and <x> were calculated before for both the C.I. and D.I.
- κ = 0.154, 0.155, 0.1555 (m_π = 650, 538, 478 MeV)
- 500 gauge configurations
- 400 noises (Optimal Z_4 noise with unbiased subtraction) for DI
- 16 nucleon sources

Disconnected Insertions of T_1 (q^2) and T_2 (q^2) for u/d Quarks







Gauge Operators from the Overlap Dirac Operator

- Overlap operator

$$D_{ov} = 1 + \gamma_5 \varepsilon(H); \quad H = \gamma_5 D_W(m_0)$$

- Index theorem on the lattice (Hasenfratz, Laliena, Niedermayer, Lüscher)

$$\text{index}\, D_{ov} = -Tr\gamma_5 (1 - \frac{a}{2} D_{ov})$$

- Local version (Kikukawa & Yamada, Adams, Fujikawa, Suzuki)

$$q_L(x) = -tr\gamma_5 (1 - \frac{a}{2} D_{ov}(x,x)) \xrightarrow[a \to 0]{} a^4 q(x) + O(a^6)$$

- Study of topological structure of the vacuum
  - Sub-dimensional long range order of coherent charges (Horvàth et al; Thacker talk in Lattice 2006)
  - Negativity of the local topological charge correlator (Horvàth et al)

- We obtain the following result

$$\mathbf{tr}_s \sigma_{\mu\nu} a D_{ov}(x,x) = c^T a^2 F_{\mu\nu}(x) + O(a^3),$$

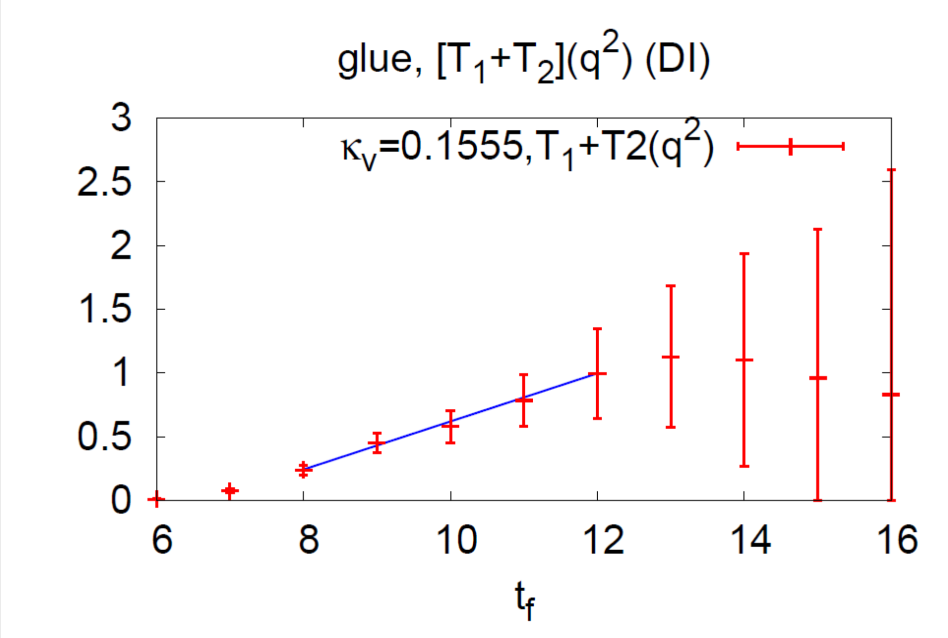
$$c^T = \rho \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{2\left[(\rho + r\sum_{\lambda}(c_\lambda - 1))c_\mu c_\nu + 2rc_\mu s_\nu^2\right]}{(\sum_\mu s_\mu^2 + [\rho + \sum_\nu (c_\nu - 1)]^2)^{3/2}}$$

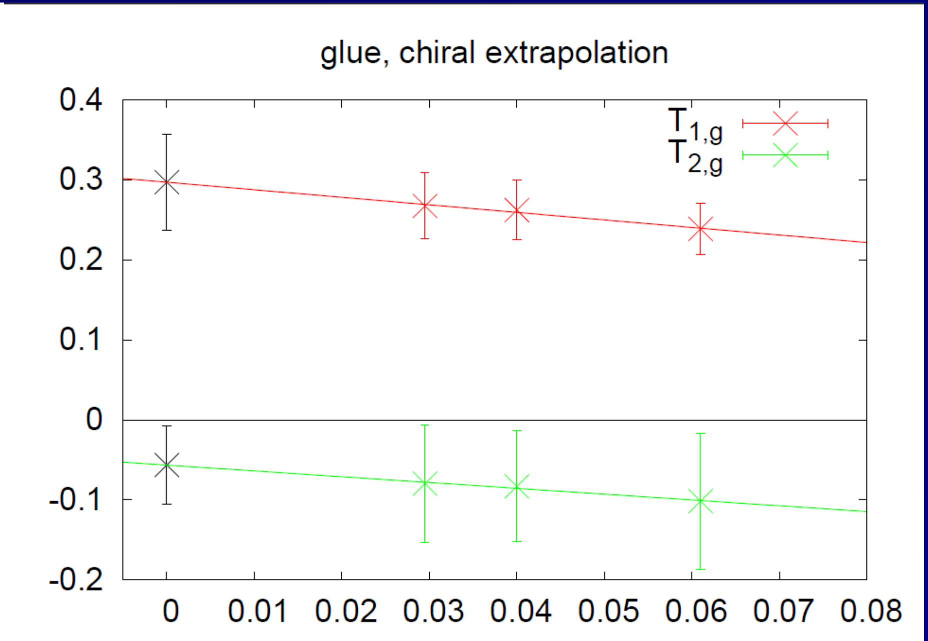
where, $r = 1, \;\; \rho = 1.368, \quad c^T = 0.11157$

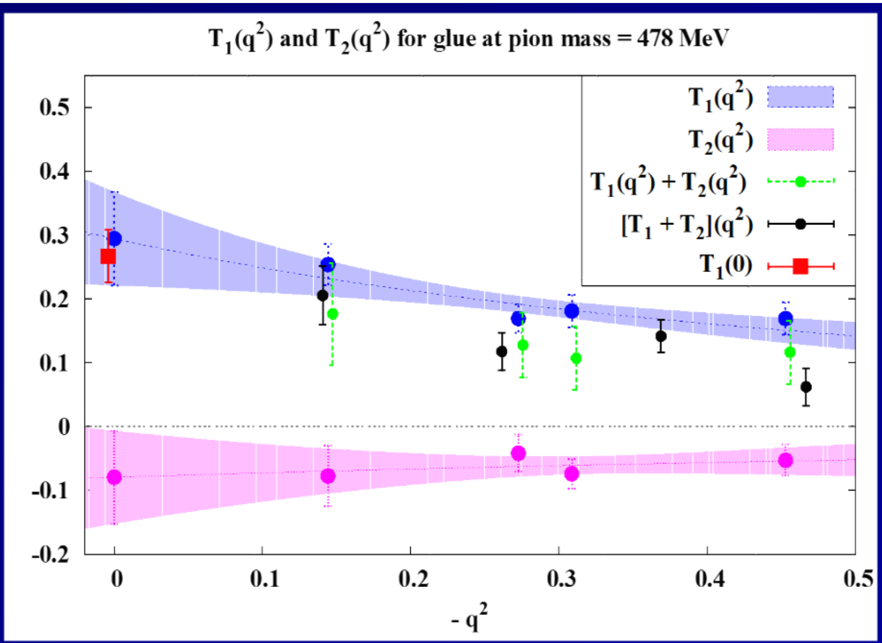
Liu, Alexandru, Horvath – PLB 659, 773 (2007)

- Noise estimation $D_{ov}(x,x) \rightarrow \left\langle \eta_x^\dagger (D_{ov}\eta)_x \right\rangle$
  with Z_4 noise with color-spin dilution and some dilution in space-time as well.

Glue $T_1(q^2)$ and $T_2(q^2)$







Renormalized results: $Z_q = 1.05$, $Z_g = 1.05$

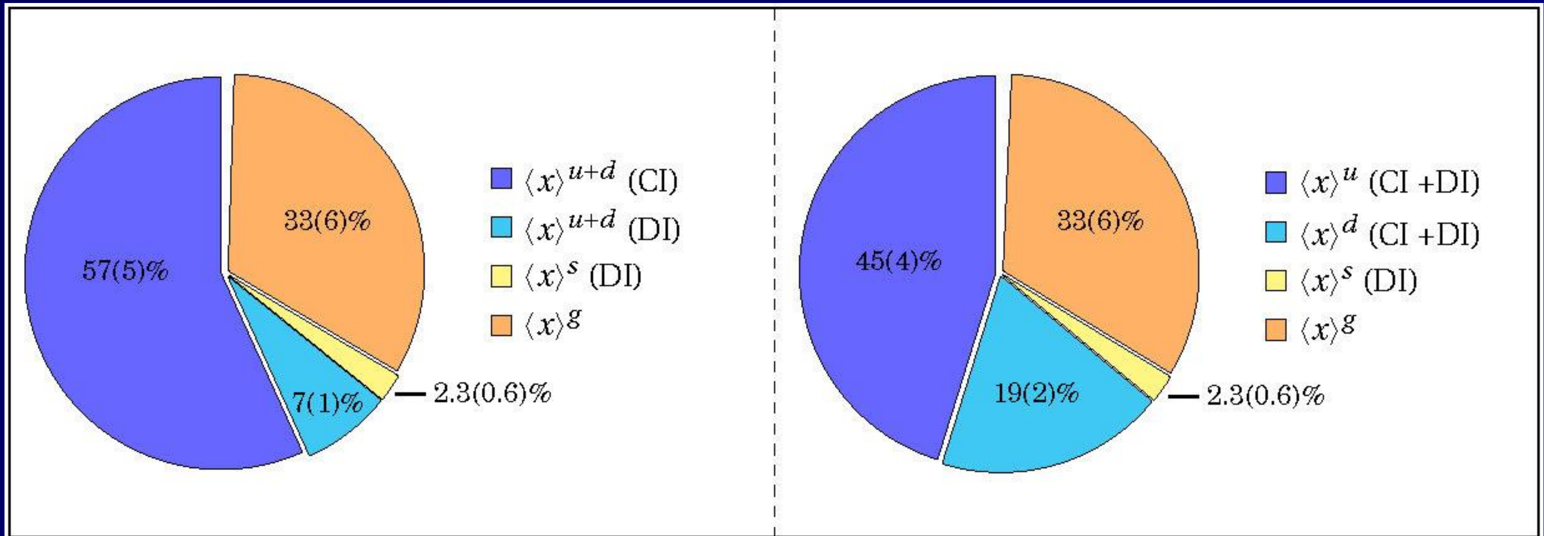
	CI(u)	CI(d)	CI(u+d)	DI(u/d)	DI(s)	Glue
<x>	0.416 (40)	0.151 (20)	0.567 (45)	0.037 (7)	0.023 (6)	0.334 (56)
$T_2(0)$	0.283 (112)	-.217 (80)	0.061 (22)	-0.002 (2)	-.001 (3)	-.056 (52)
2J	0.704 (118)	-.070 (82)	0.629 (51)	0.035 (7)	0.022 (7)	0.278 (76)

$$T_2(0)^R_{CI} + T_2(0)^R_{DI} + T_2(0)^R_g = 0$$

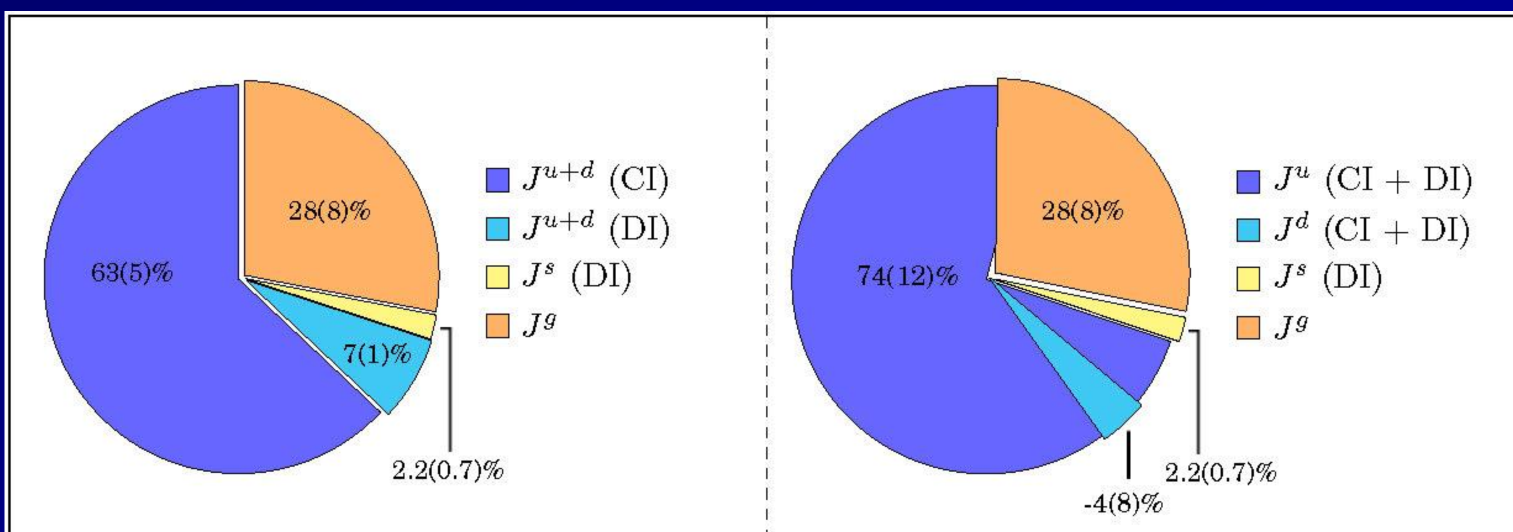
I.Yu. Kobzarev, L.B. Okun, Zh. Eksp. Teor. Fiz. 43, 1904 (1962) [Sov. Phys. JETP 16, 1343 (1963);

S. Brodsky et al. NPB 593, 311(2001) → no anomalous gravitomagnetic moment

Momentum fractions <x>^{q}, <x>^{g}



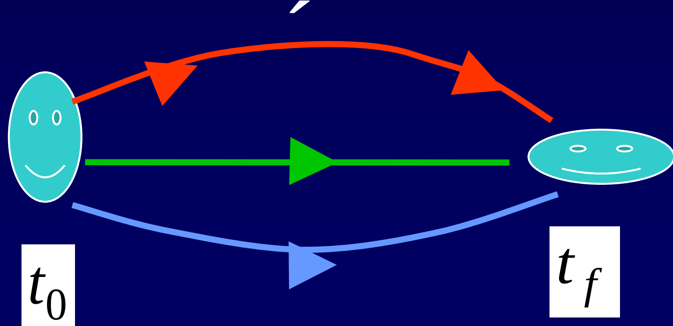
Angular Momentum fractions J^q, J^g



Flavor-singlet g_A

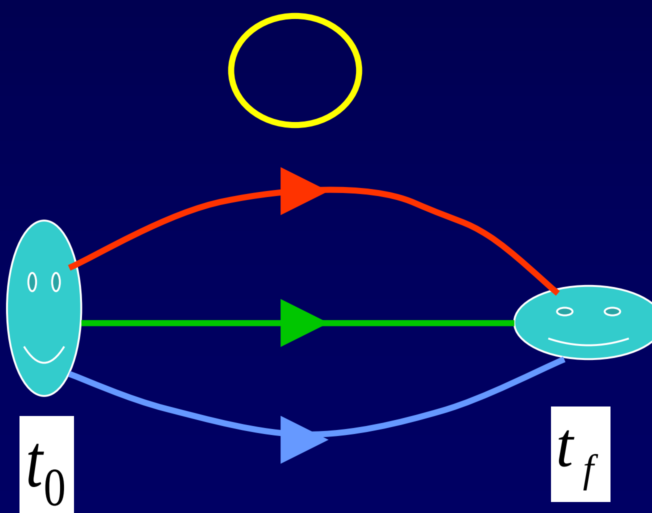
- Quark spin puzzle (dubbed `proton spin crisis')
  - $$g_A^0 = \Delta u + \Delta d + \Delta s = \begin{cases} 1 & \text{NRQM} \\ 0.75 & \text{RQM} \end{cases}$$
  - Experimentally (EMC, SMC, … $\Delta\Sigma = g_A^0 \sim 0.2 - 0.3$

$$(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)(t)$$



$$g^0_{A,con} = (\Delta u + \Delta d)_{con}$$

$$\overline{\Psi}\gamma_\mu\gamma_5\Psi(t)(u,d,s)$$



$$g^0_{A,dis} = (\Delta u + \Delta d + \Delta s)_{dis}$$

S.J. Dong, J.-F. Lagae, and KFL, PRL 75, 2096 (1995)

$$g_A^0 = (\Delta u + \Delta d)_{con} + (\Delta u + \Delta d + \Delta s)_{dis} = 0.62(9) + 3(-0.12(1)) = 0.25(12)$$

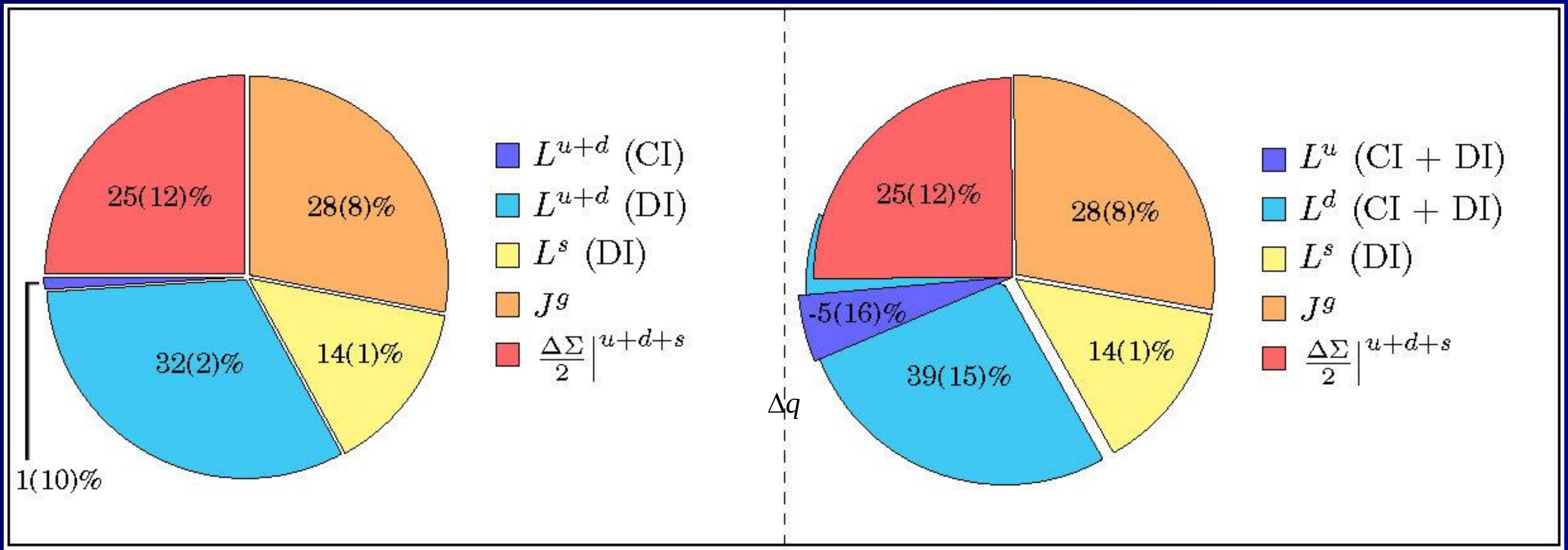
	Lattice	Expt. (SMC)	NRQM	RQM
$g_A^0 = \Delta u + \Delta d + \Delta s$	0.25(12)	0.22(10)	1	0.75
$g_A^3 = \Delta u - \Delta d$	1.20(10)	1.2573(28)	5/3	1.25
$g_A^8 = \Delta u + \Delta d - 2\Delta s$	0.61(13)	0.579(25)	1	0.75
Δu	0.79(11)	0.80(6)	1.33	1
Δd	-.42(11)	-0.46(6)	-0.33	-0.25
Δs	-.12(1)	-0.12(4)	0	0
F_A	0.45(6)	0.459(8)	0.67	0.5
D_A	0.75(11)	0.798(8)	1	0.75
F_A / D_A	0.60(2)	0.575(16)	0.67	0.67

$$F_A = (\Delta u - \Delta s)/2; \quad D_A = (\Delta u - 2\Delta d + \Delta s)/2$$

Renormalized results:

	CI(u)	CI(d)	CI(u+d)	DI(u/d)	DI(s)	Glue
2J	0.704 (118)	-.070 (82)	0.629 (51)	0.035 (7)	0.022 (7)	0.278 (76)
g_A	0.91 (11)	-0.30 (12)	0.62 (9)	-0.12 (1)	-0.12 (1)	
2 L	-0.21 (16)	0.23 (15)	0.01 (10)	0.16 (1)	0.14 (1)	

Quark Spin, Orbital Angular Momentum, and Gule Angular Momentum



$\Delta q \approx 0.25;$

$2\ L_q \approx 0.47\ (0.01(\mathrm{CI})+0.46(\mathrm{DI}));$

$2\ \mathrm{J}_g \approx 0.28$

Summary of Quenched Lattice Calculations

- Complete calculation of momentum fractions of quarks (both valence and sea) and glue have been carried out for a quenched lattice:
  - Glue momentum fraction is ~ 33%.
  - g_A^0 ~ 0.25 in agreement with expt.
  - Glue angular momentum is ~ 28%.
  - Quark orbital angular momentum is large for the sea quarks (~ 47%).
- These are quenched results so far.

Overlap fermion on 2+1 flavor DWF configurations (RBC-UKQCD-LHPC)

La ~ 2.8 fm
m_π ~ 330 MeV

24^3 x 32, a =0.115 fm

↓ ($O(a^2)$ extrapolation)

La ~ 2.7 fm
m_π ~ 295 MeV

32^3 x 64, a =0.085 fm

DSDR

La ~ 4.5 fm
m_π ~ 180 MeV

32^3 x 64, a =0.12 fm

Some Desirable Features of Overlap

- Calculating eigenmodes is relatively easy

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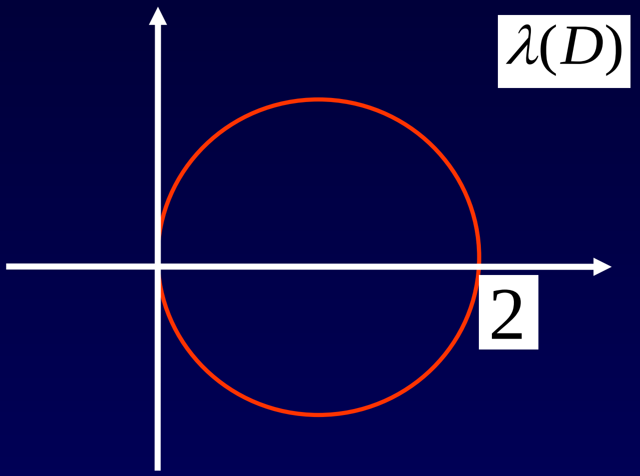


- Normality ($D^\dagger D = D D^\dagger$) and GW relation

→ Eigenvalues are on a unit circle and
λ = 0, 2 are chiral modes.
The rest are complex pairs.

- Normality and

$$[\gamma_5, D^\dagger D] = 0$$

$$\Rightarrow \; D^\dagger D \varphi_{L,R} = |\lambda|^2 \varphi_{L,R};$$

$$\text{Diagonalize } \langle \varphi_{L,R} | D | \varphi_{L,R} \rangle \;\Rightarrow\; D\psi = \lambda\psi$$

Overlap with Deflation (multimass with same eigenvectors)

$$D(m,\rho)X^{H}_{L,R} = \eta_{L,R} - \sum_{i=1}^{n}(1 \mp \gamma_5)|i\rangle\langle i|\eta_{L,R}\rangle$$

where,

$$D(0,\rho)|i\rangle = \lambda_i |i\rangle; \quad D(0,\rho)\gamma_5|i\rangle = \lambda_i^{*}\gamma_5|i\rangle$$

Therefore,

$$X^{H}_{L,R} = D^{-1}(m,\rho)\eta_{L,R} - X^{L}_{L,R}$$

where,

$$X^{L}_{L,R} = \sum_{i=1}^{n}\left[\frac{|i\rangle\langle i|\eta_{L,R}\rangle}{\rho\lambda_i + m(1-\lambda_i/2)} \mp \frac{\gamma_5|i\rangle\langle i|\eta_{L,R}\rangle}{\rho\lambda_i^{*} + m(1-\lambda_i^{*}/2)}\right]$$

and

$$\mathrm{X} = (\mathrm{X}^{H}_{L} + X^{H}_{R}) + (\mathrm{X}^{L}_{L} + X^{L}_{R}) \quad \text{except for the zero modes.}$$

Speedup with deflation and HYP smearing

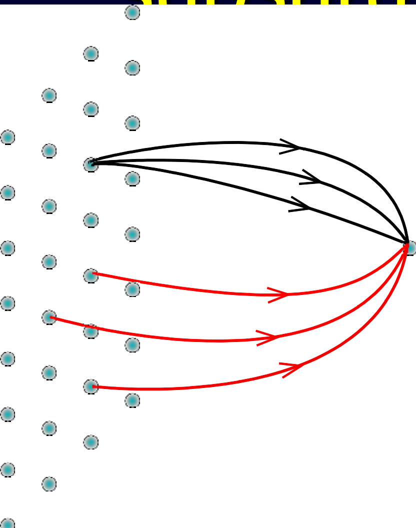
		16^3 x 32			24^3 x 64			32^3 x 64		
	res	w/o D	D	D+S	w/o D	D	D+S	w/o D	D	D+S
Low mode	10^{-8}	0	200	200	0	200	200	0	400	400
Inner iter	10^{-11}	340	321	108	344	341	107	309	281	101
Outer iter	10^{-8}	627	72	85	2931	147	184	4028	132	156
Speedup				23			51			79
Overhead				5 prop			5 prop			8 prop

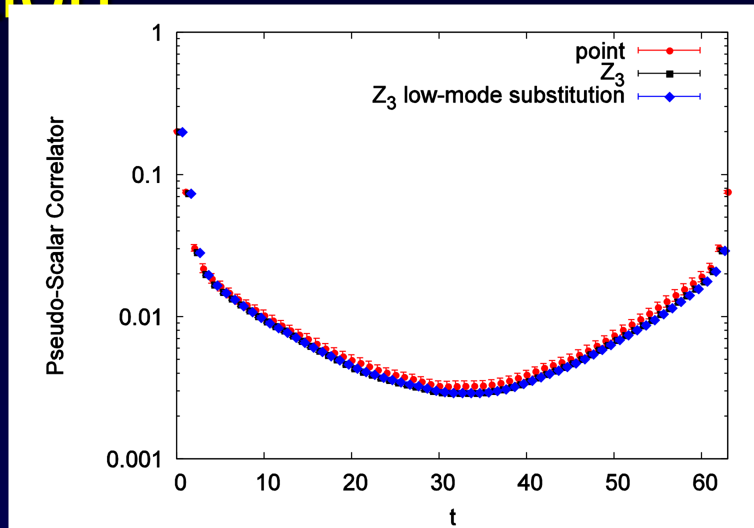
$$D(0,\rho) = 1 + \gamma_5 \varepsilon = 1 + \gamma_5 \frac{H_W(\rho)}{\sqrt{H_W{}^2(\rho)}} \approx 1 + \gamma_5 H_W \sum_{i=1}^{n} \frac{b_i}{H_W^2 + c_i}$$

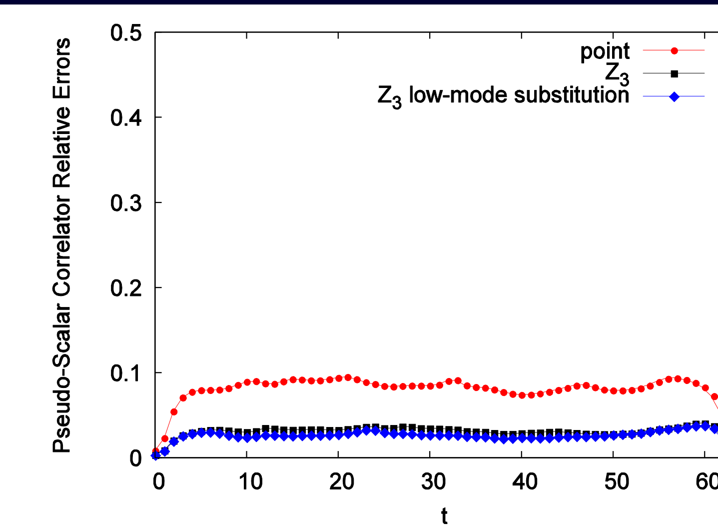
- No critical slowing down
- Multii-mass inversion (10-20 masses)

A. Li, et al, PRD 82, 114501 (2010)

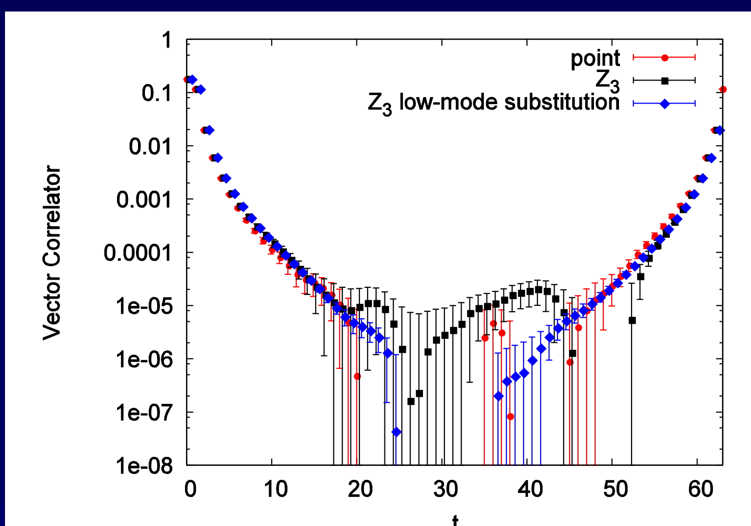
Z_3 grid (64) source with low-mode substitution

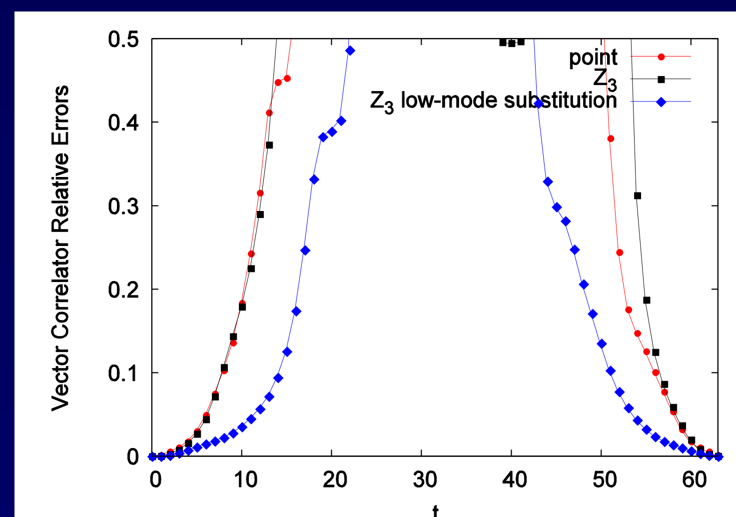




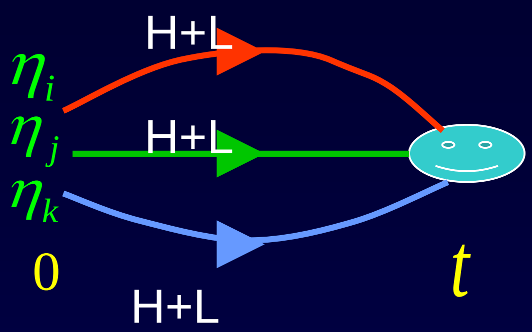


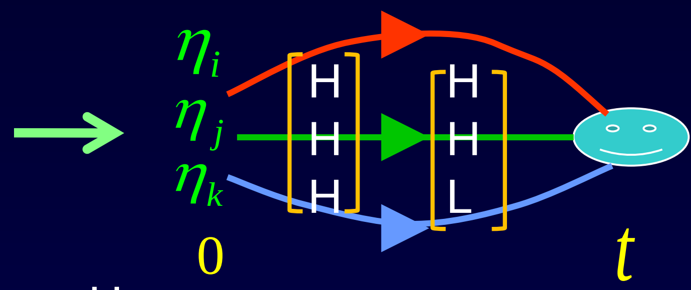
32^3 x 64 lattice, m_l (sea)= 0.004 at m_π ~ 200 MeV, 50 conf.





Nucleon with LLL and HLL substitution

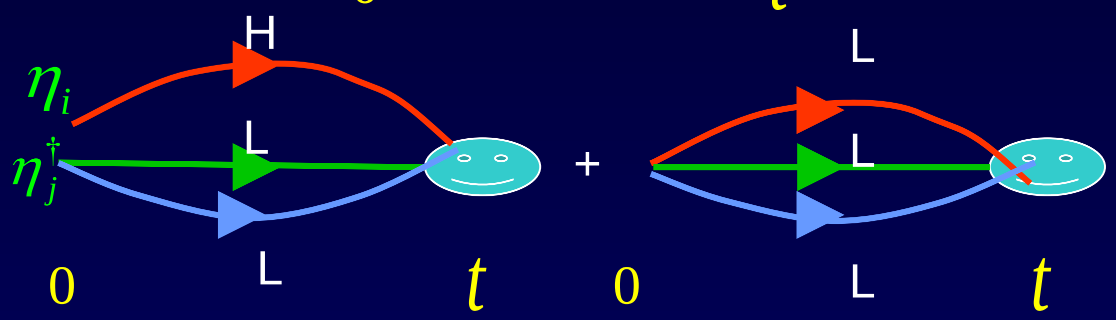




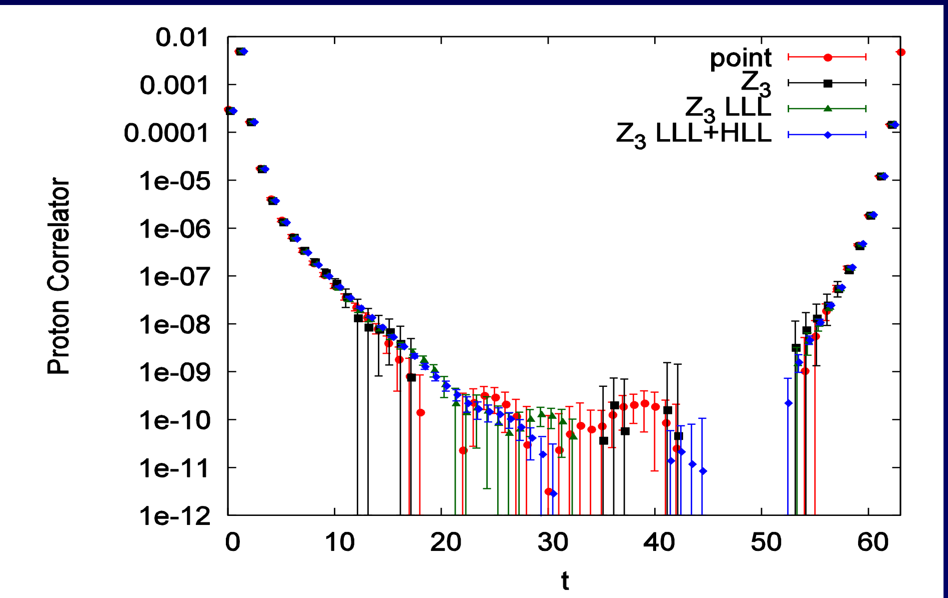
$$\frac{C_N(t)}{\sigma_N(t)} \approx \sqrt{\frac{n}{V_3}}\, e^{-(m_N-3/2m_\pi)t}$$

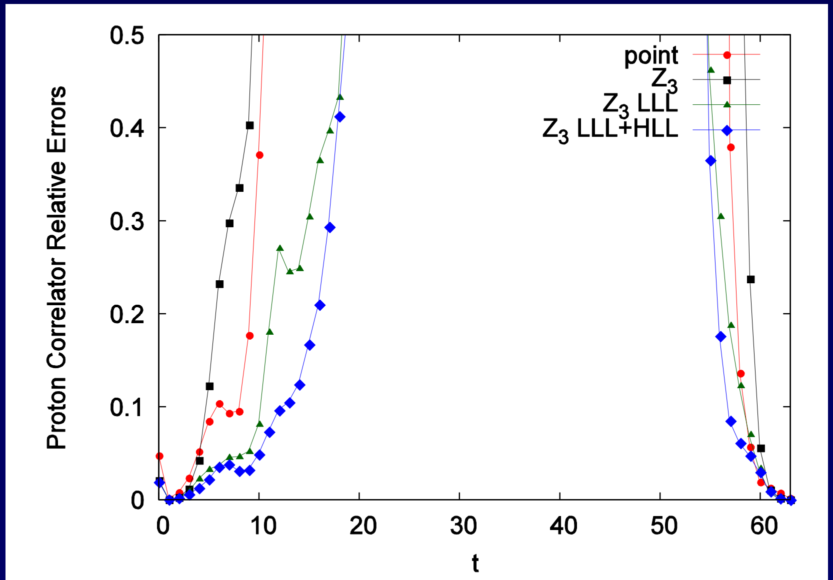
$$\langle \eta_i \eta_j \eta_k \rangle = \delta_{ij}\delta_{jk}$$

$$\langle \eta_i^\dagger \eta_j \rangle = \delta_{ij}$$



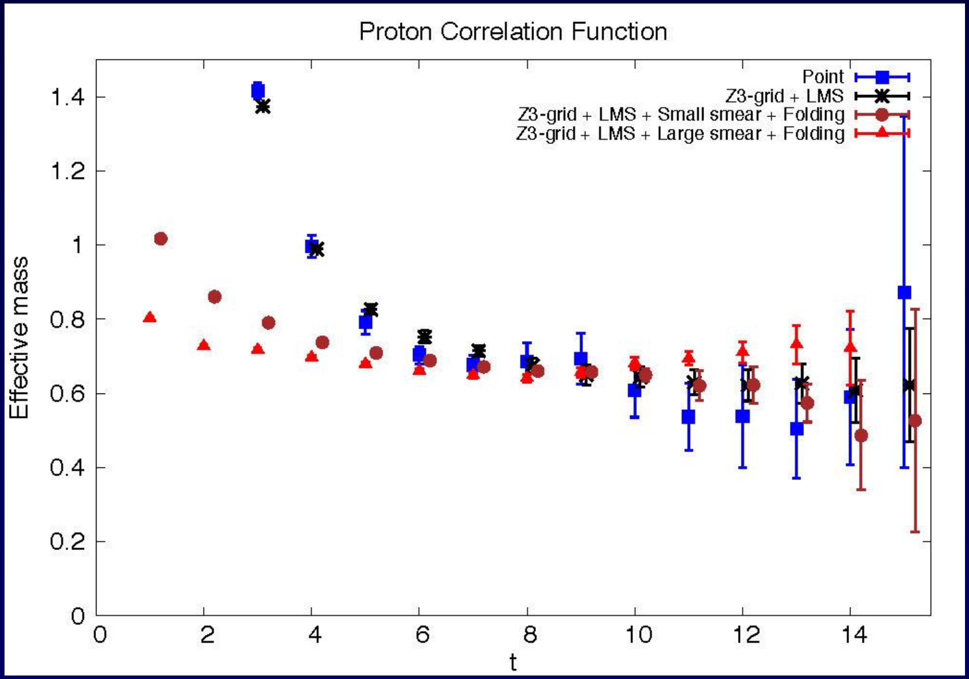
m_π ~ 300 MeV

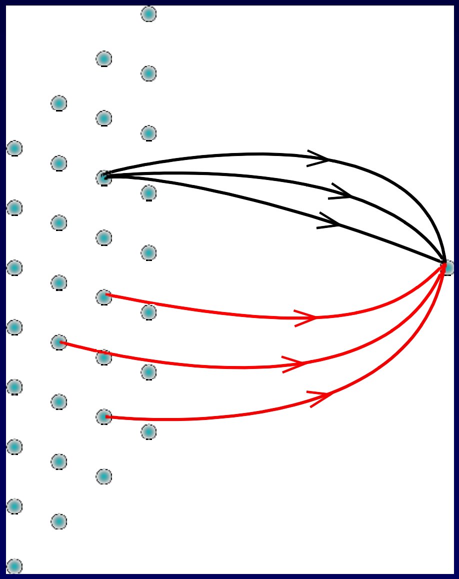




Nucleon Mass with Low Mode Substitution

- Improvement of nucleon correlator with low-mode substitution





$24^3 \times 64$ lattice with $m_\pi = 331$ MeV, a = 1.73 GeV^{-1}

47 configurations

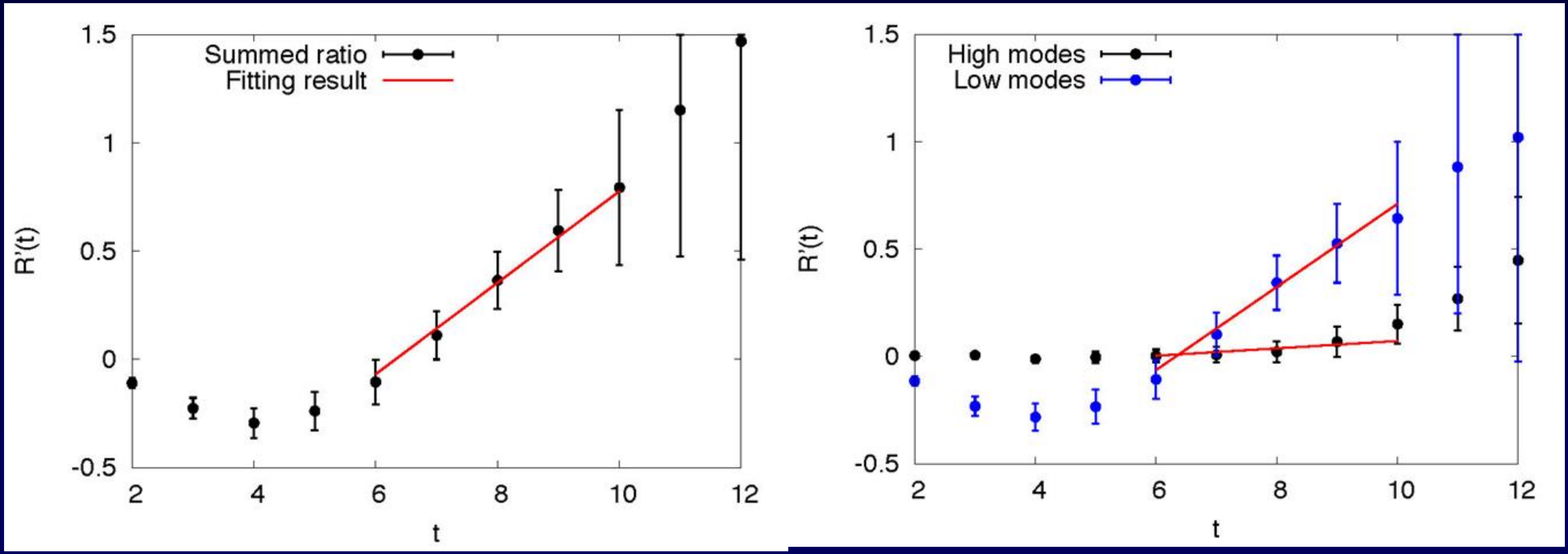
Point source: $m_N = 1.13(14)$ GeV;

Z_3 grid source: $m_N = 1.08(5)$ GeV;

Z_3 small smeared grid: $m_N = 1.14(2)$ GeV;

Z_3 large smeared grid: $m_N = 1.13(1)$ GeV

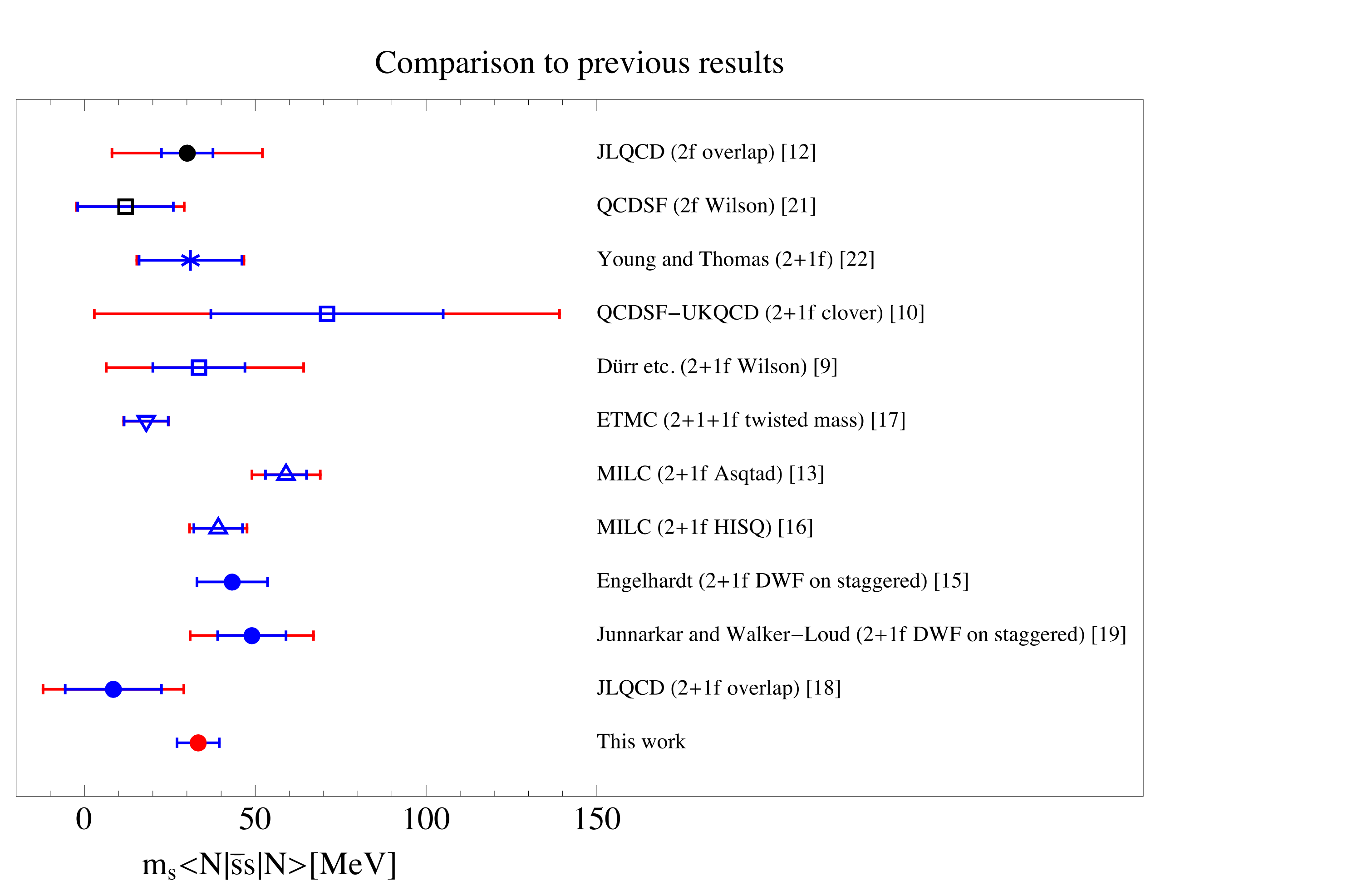
Quark loop with low-mode averaging and Z_4 noise estimate of high modes with grids and time dilution



M. Gong, et al.,
PRD 88, 014503 (2013)

$$\text{constant} + m_s < N | \bar{s}s | N > t$$

24^3 x 64, $m_l = 0.005$, $m_s = 0.04$, 176 conf. → 5 sigma signal



$$f_{T_s} = m_s \langle N | \bar{s}s | N \rangle / m_N = 0.0334(62)$$

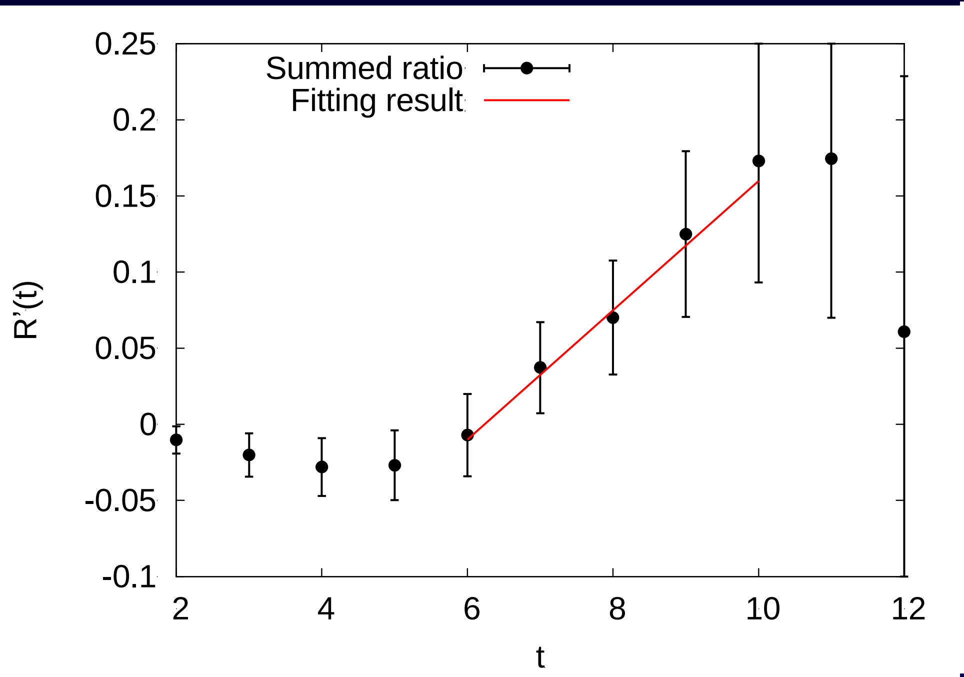
Comparison of statistics:

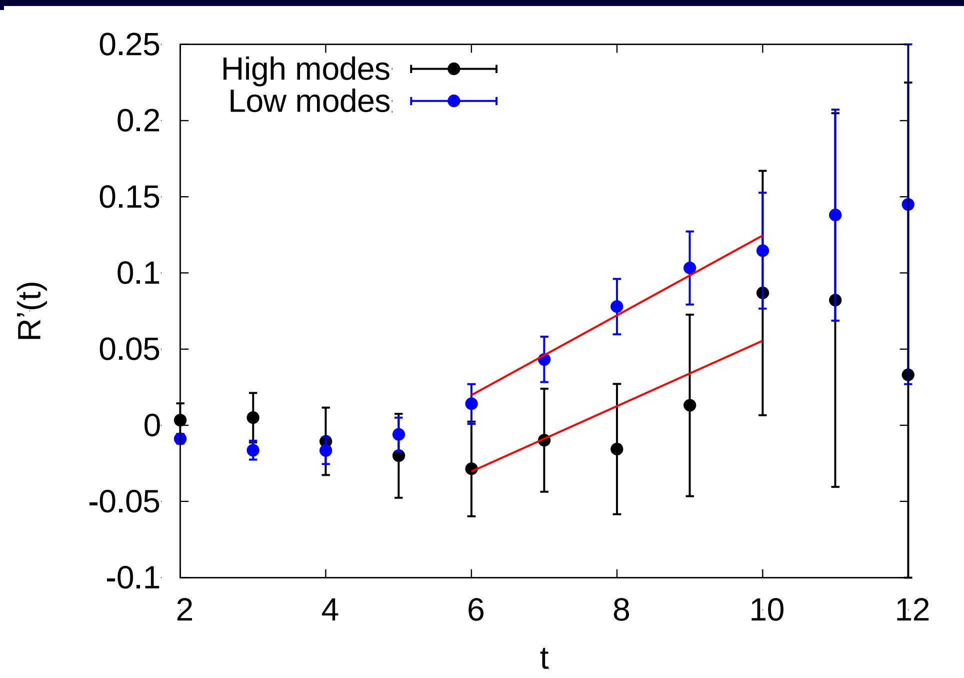
1) This work -- 176 conf. 48 noises each

2) Engelhardt -- 468 conf. 1200 noises each

3) JLQCD -- 50 conf. 288 noises each (4 times error bar)

Charmness



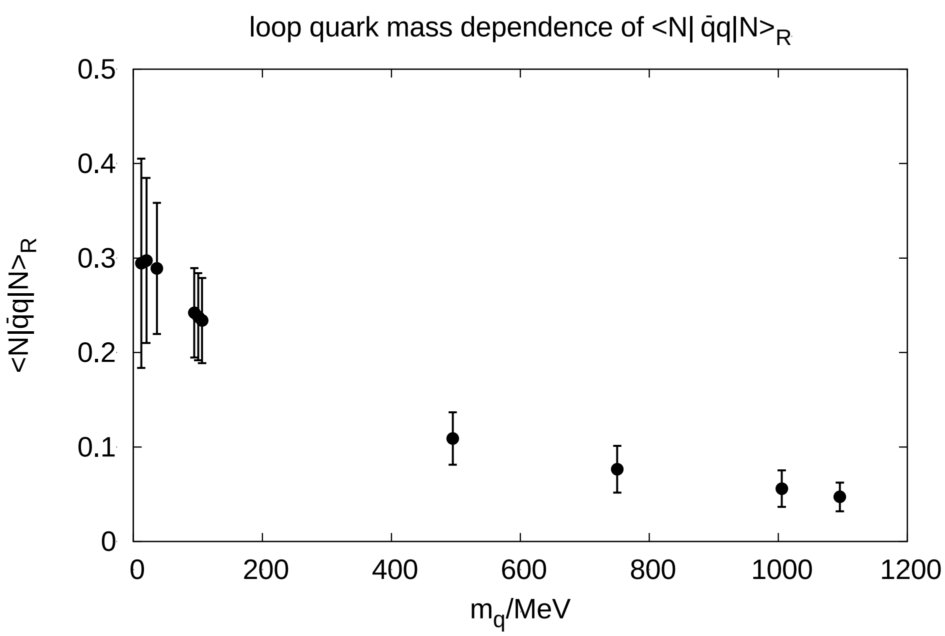


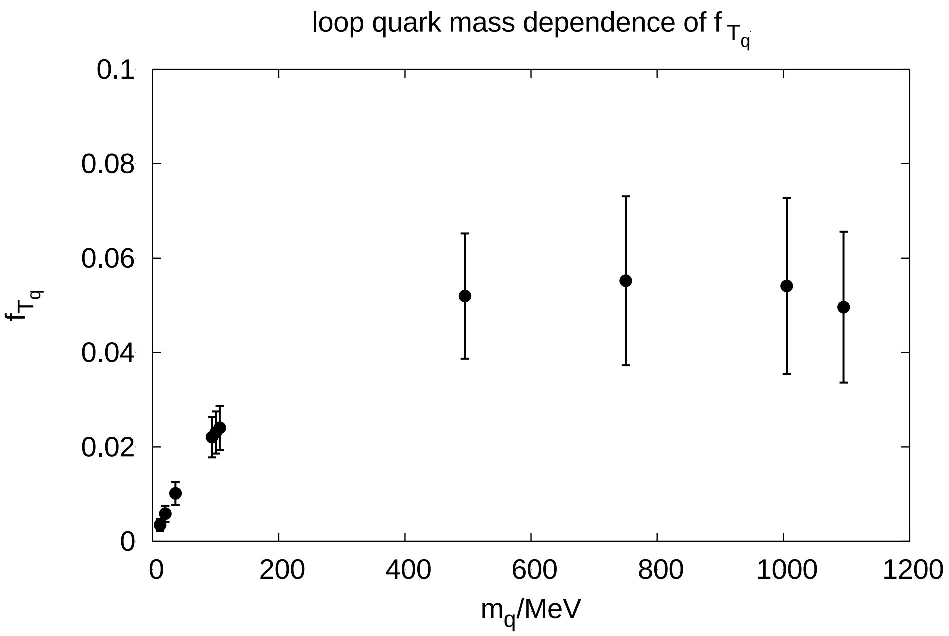
$$m_c < N|\bar{c}c|N > = 94(31)\ \text{MeV}$$

M. Gong, et al., PRD 88, 014503 (2013)

Consistent with the estimate of ~ 70 MeV from heavy quark expansion and trace anomaly for the nucleon mass (Shifman, …)

- In the case of heavy quarks, the scalar matrix elements seem to fall as 1/m and f_T level off for m > 500 MeV.
- We use ma = 0.67 for the charm mass.





Uncertainty of Quark Spin Calculation

- Recent calculation of strange quark spin with dynamical fermions
  - R. Babich et al. (1012.0562)

    $$\Delta s = -0.019(11)$$

  - QCDSF (G. Bali et al. 1206.4205) gives

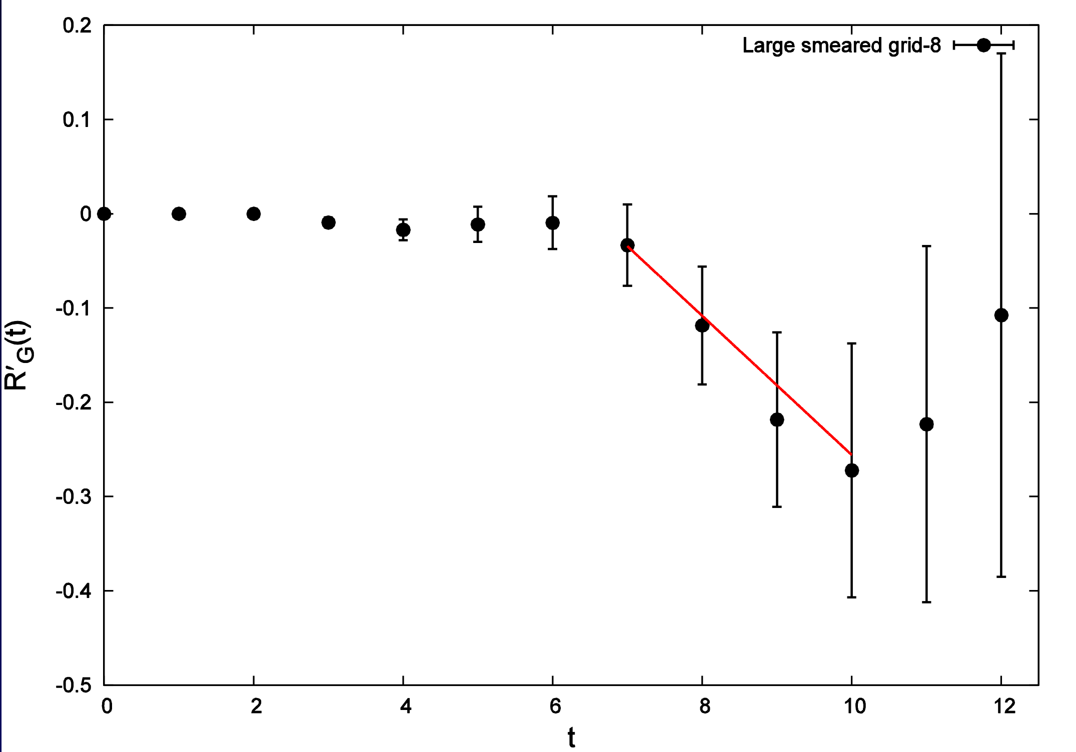
    $$Ds = -\ 0.020(10)(4)$$

  much smaller than that of of quenched result.
  - C. Alexandrou et al. (arXiv:1310.6339)

    $$\Delta s \sim -0.0227(34)$$

Quark Spin from Anomalous Ward Identify

- Calculation of the axial-vector in the DI is very noisy
- Instead, try AWI $\partial_\mu A_\mu^0 = 2mP + \frac{N_f}{8\pi^2} G_{\mu\nu}\tilde{G}_{\mu\nu}$
  - Overlap fermion --> mP is RGI.
  - Overlap operator for $q(x) = -1/2\,\mathrm{Tr}\,\gamma_5 D_{ov}(x,x)$ is RGI.
  - P is totally dominated by small eigenmodes.
  - q(x) from overlap is exponentially local and is dominated by high eigenmodes.
  - Direct check the origin of \`proton spin crisis'.



The anomaly contribution to the quark spin per flavor

Slope = - 0.074(27)

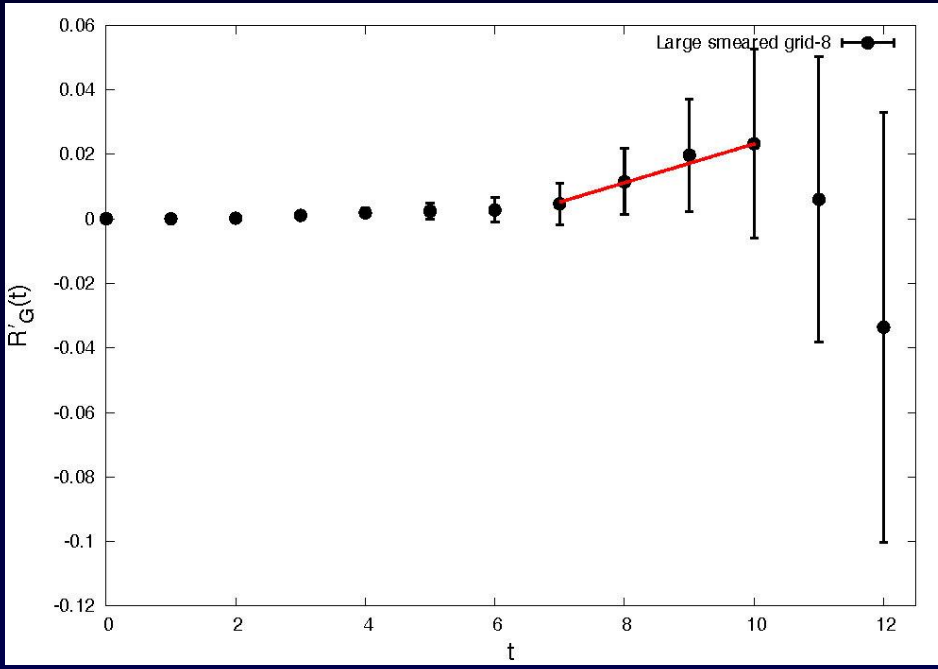
at $|\vec{q}| = 2\pi / La$

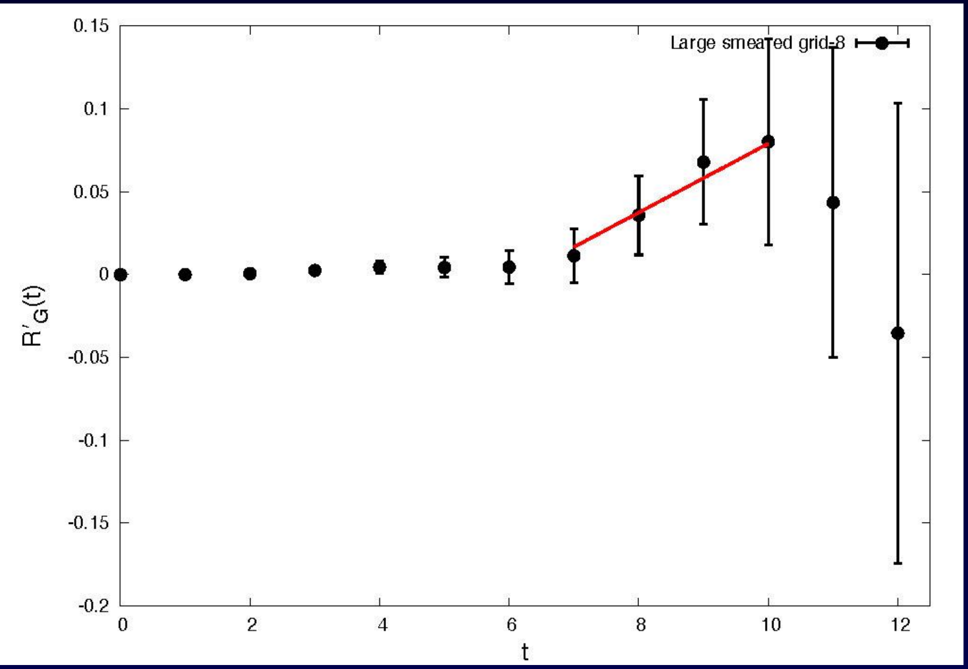
24 x 64 DWF config

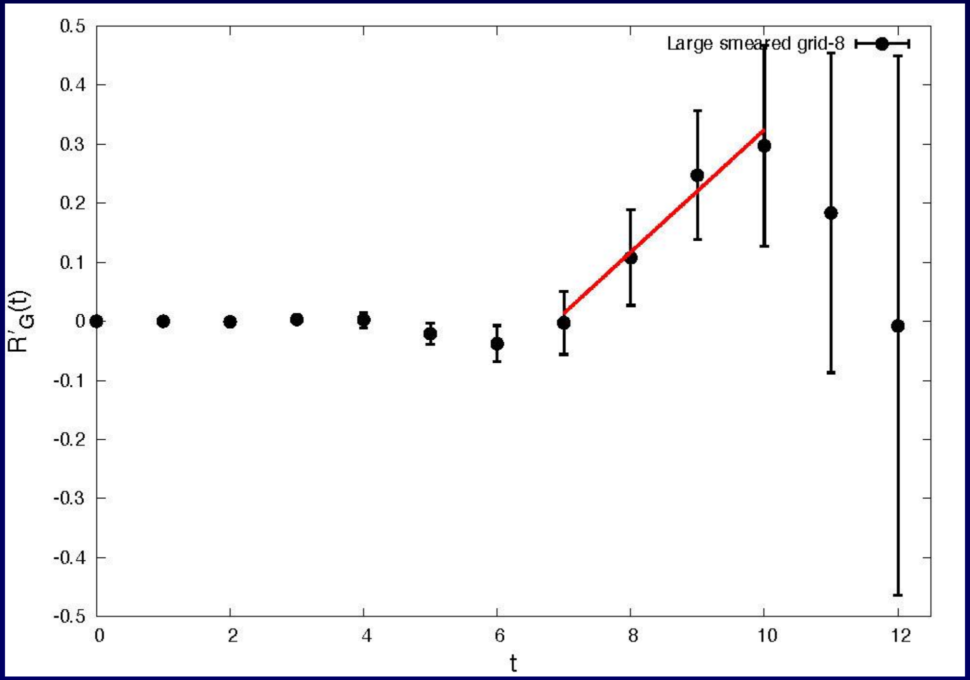
Overlap valence

m ~ 330 MeV

79 configurations







Δu (DI) = 0.0060(49)
Δs = 0.021(11)
Δc = 0.104(34)

$$|\vec{q}| = 2\pi / La$$

Status of Lattice Calculation

- The quark orbital angular momentum is ~ 50% of the nucleon spin (quenched).
- The quark OAM is small for the CI in both dynamical and quenched calculations.
- Quark spin from dynamical fermion calculation is not settled.
- AWI to address `proton spin crisis'.

Quark and Glue Components of Hadron Mass

- Energy momentum tensor

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2 \qquad \langle P | T_{\mu\nu} | P \rangle = P_\mu P_\nu / M$$

- Trace anomaly

$$T_{\mu\mu} = -(1+\gamma_m)\bar{\psi}\psi + \frac{\beta(g)}{2g}G^2$$

- Separate into traceless part $\bar{T}_{\mu\nu}$ and trace part $\hat{T}_{\mu\nu}$

$$\langle P | \bar{T}^{q,g}_{\mu\nu} | P \rangle = \langle x \rangle_{q,g}(\mu^2)(P_\mu P_\nu - \frac{1}{4}\delta_{\mu\nu}P^2)/M, \quad \langle x \rangle_q(\mu^2) + \langle x \rangle_g(\mu^2) = 1$$

$$\langle \bar{T}_{44} \rangle = -3/4M; \qquad \langle \hat{T}_{44} \rangle = -1/4M$$

- Decomposition of hadron mass

X.-D. Ji, PRL 74, 1071 (1995)

$$M=-\langle T_{44}\rangle=\langle H_q\rangle+\langle H_g\rangle+\langle H_a\rangle=\langle H_k\rangle+\langle H_m\rangle+\langle H_g\rangle+\langle H_a\rangle;$$

$$\frac{1}{4}M=-\langle \hat{T}_{44}\rangle=\frac{1}{4}\langle H_m\rangle+\langle H_a\rangle;$$

where

$$H_q=\sum_{u,d,s\ldots}\int d^3x\, -\bar{\psi}(\gamma_4 D_4)\psi;\ \ H_k=\sum_{u,d,s\ldots}\int d^3x\ \bar{\psi}(\vec{\gamma}\cdot\vec{D})\psi;\ \ H_m=\sum_{u,d,s\ldots} m_f\int d^3x\ \bar{\psi}\psi;$$

$$H_g=\int d^3x\ (B^2-E^2);\ \ H_a=\int d^3x\ \frac{-\beta(g)}{2g}(B^2+E^2)$$

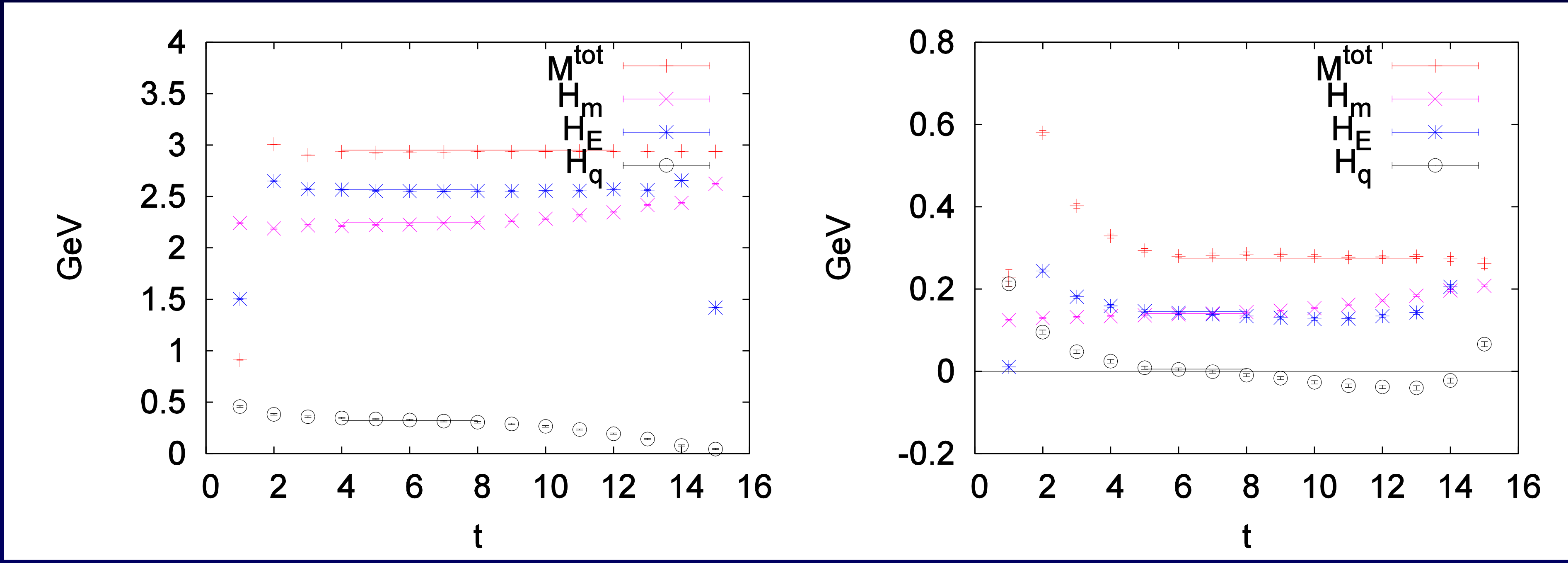
  - Equation of motion

$$\sum_z (D_c+m)(x,z)\frac{1}{D_c+m}(z,y)=\delta_{x,y}\Rightarrow\begin{cases}0 \text{ for CI}\\ \text{cont for DI}\end{cases}\qquad D_c=\frac{\rho D_{ov}}{1-D_{ov}/2}$$

Therefore,

$$\langle H_q\rangle-\langle H_k\rangle=\langle H_m\rangle+O(a^2)$$

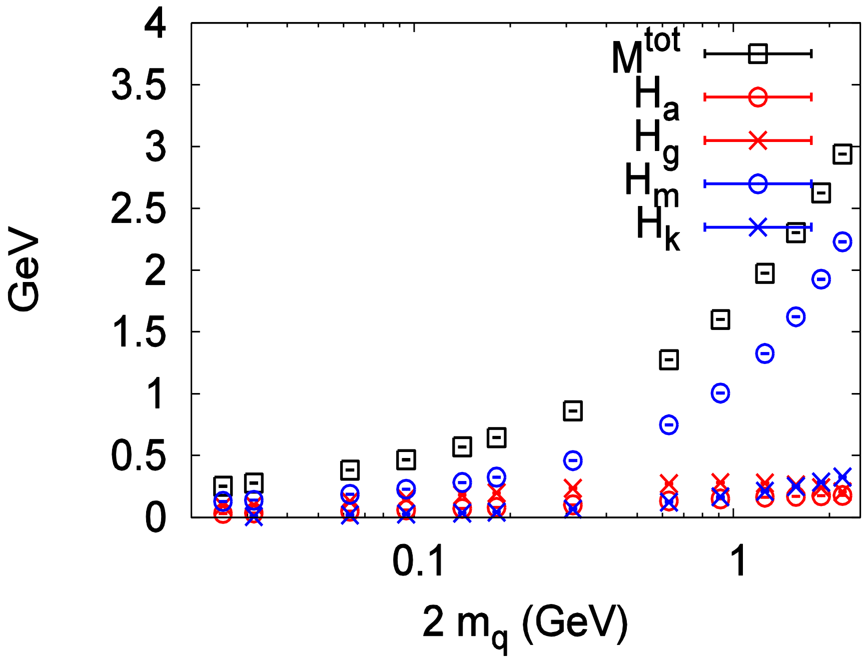
Ratios of three-to-two point functios

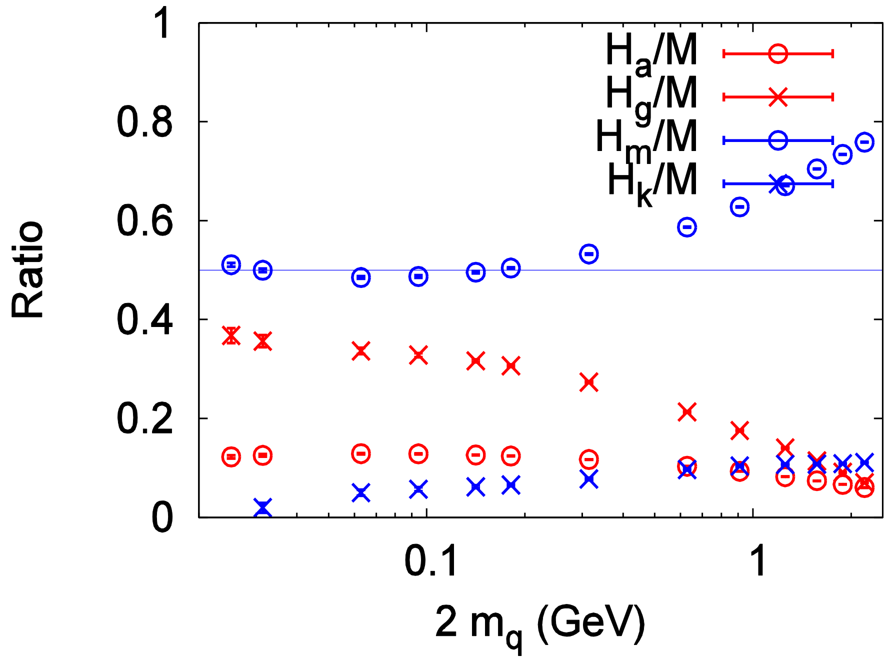


η_c

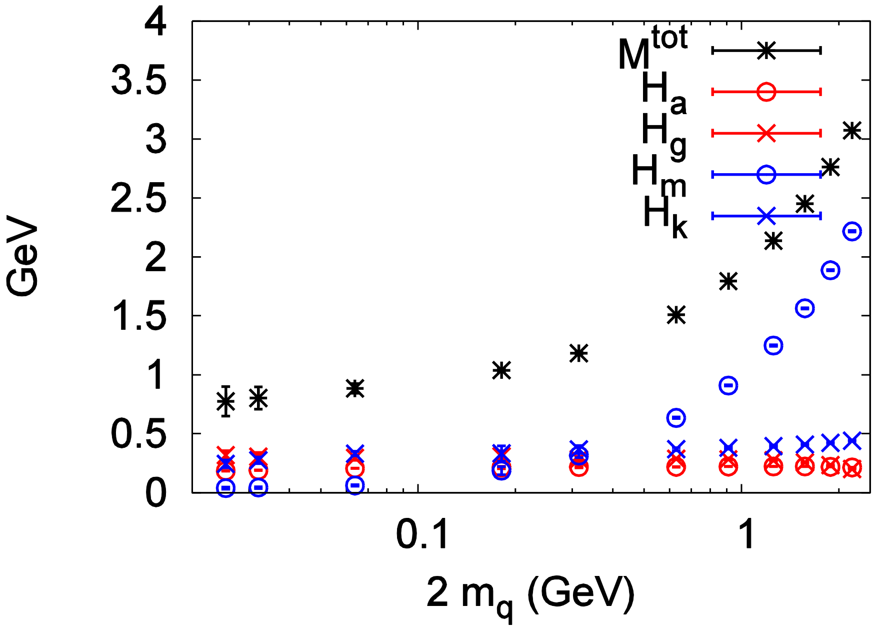
m_π ~ 300 MeV

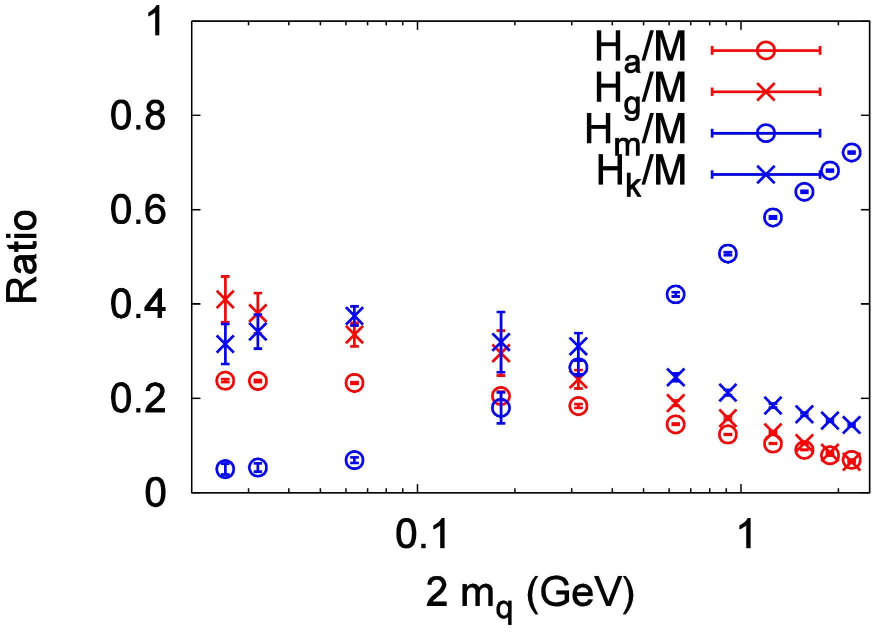
Pseudoscalar meson masses from m_π ~ 200 MeV to η_c ~3 GeV





Vector meson masses from m_ρ ~ 800 MeV to J/ψ ~3 GeV





Challenges ahead

- Continuum limit and physical pion extrapolations with 3 smaller lattices
- 48^3 x 96 and 60^3 x 128 lattices with large number of eigenvectors (~ 2000)
- Decomposition of glue angular momentum into glue helicity and glue orbital angular momentum