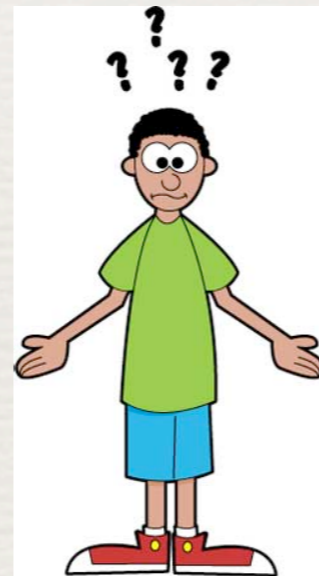


Neutron-Antineutron Oscillations on the Lattice



Michael I. Buchoff

Lawrence Livermore National Laboratory

In collaboration with Chris Schroeder and Joe Wasem

Why should we care?

- ◆ Other particle/antiparticle mixings occur

$$K^0 \leftrightarrow \overline{K}^0$$

$$B^0 \leftrightarrow \overline{B}^0$$

- ◆ Expect baryon number number to be broken
 - Baryon-antibaryon asymmetry

$$\Delta B = 1 \text{ (Proton Decay)}$$

$$\Delta B = 2 \text{ (} N\overline{N} \text{ Oscillations)}$$

- ◆ Natural in GUT theories with Majorana neutrinos
 - Usual sphaleron process: $\Delta(B - L) = 0$

1980

$$\nu = \bar{\nu} \Rightarrow \Delta L = 2$$

$$B - L = 0 \Rightarrow \Delta B = 2$$

Basic Idea

- ♦ BSM physics leads to off-diagonal mixing

$$H = \begin{pmatrix} E_n & \delta m \\ \delta m & E_{\bar{n}} \end{pmatrix} = \begin{pmatrix} E + V & \delta m \\ \delta m & E - V \end{pmatrix}$$

$$V = 0 \Rightarrow \text{Free System}$$

- ♦ Transition Probability

$$P_{n \rightarrow \bar{n}}(t) = \frac{\delta m^2}{\delta m^2 + V^2} \sin^2 \left[\sqrt{\delta m^2 + V^2} t \right] \quad \tau_{n\bar{n}} = \frac{1}{\delta m}$$

Restricting GUTs

Examples:

TeV-scale seesaw mechanism
for neutrino masses in
 $SU(2)_L \times SU(2)_R \times SU(4)_c$

$$10^{10} - 10^{11} \text{ sec}$$

Babu,
Bhupal Dev,
Mohapatra
(2009)

SO(10) seesaw mechanism
with adequate baryogenesis

$$10^9 - 10^{12} \text{ sec}$$

Babu,
Mohapatra
(2012)

Extra-dimensional particles
above 45 TeV

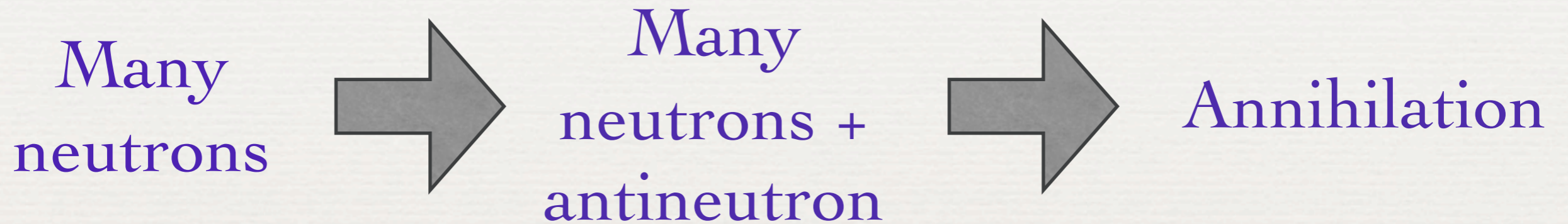
$$> 10^8 \text{ sec}$$

Nussinov,
Shrock
(2002)

♦ **Estimates** for ruling out large classes of GUTs

$$\tau_{n\bar{n}} > 10^{10} - 10^{11} \text{ sec}$$

Experimental prospects



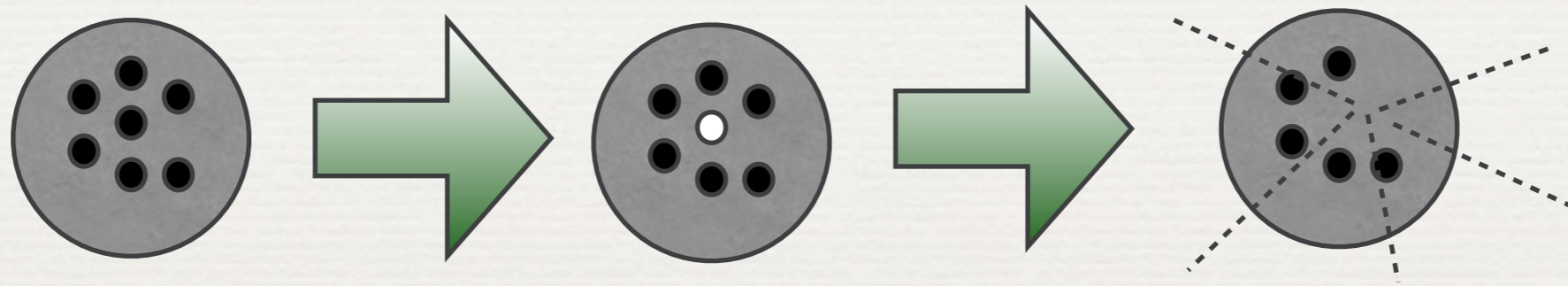
- ♦ Neutron-antineutron annihilation signals

Primary channel $n\bar{n} \rightarrow 5\pi$ (“Zero background” signal)

- ♦ Two Types of experimental searches

Experimental progress

1. Neutron-antineutron annihilation in nuclei



Straight-forward question:
Why have we not annihilated yet?

Crude Answer: Limited time
neutron is “free” in nuclei

$$P \sim \frac{1}{\tau_{Nuc}} \sim \frac{1}{\Delta t} \left(\frac{\Delta t}{\tau_{n\bar{n}}} \right)^2$$

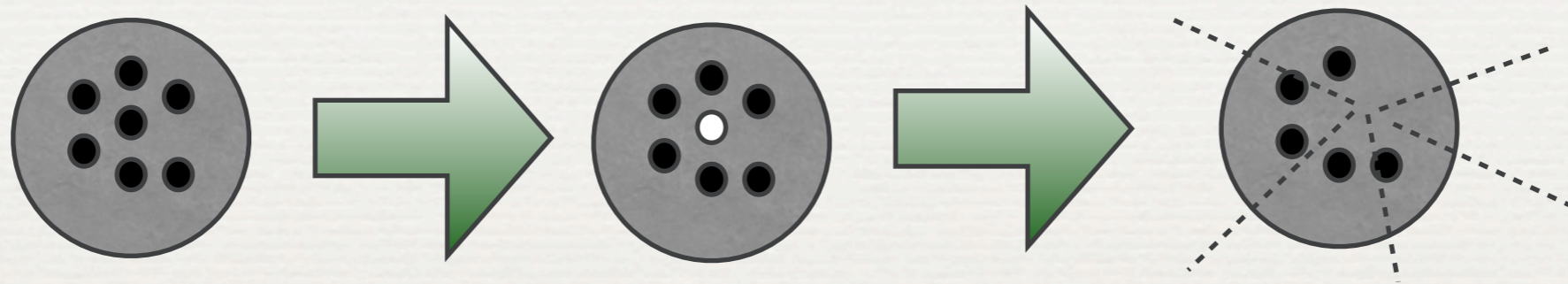
Friedman,
Gal
2008

-Nuclear suppression: $\tau_{Nuc} = (3 \times 10^{22}) \frac{\tau_{n\bar{n}}^2}{\text{sec}}$

Super-K bounds (2011) $\tau_{n\bar{n}} > 3.5 \times 10^8 \text{ sec}$

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Friedman,
Gal
2008

-Nuclear suppression: $\tau_{Nucl} = (3 \times 10^{22}) \frac{\tau_{n\bar{n}}^2}{\text{sec}}$ Model estimate

Super-K bounds (2011) $\tau_{n\bar{n}} > 3.5 \times 10^8 \text{ sec}$

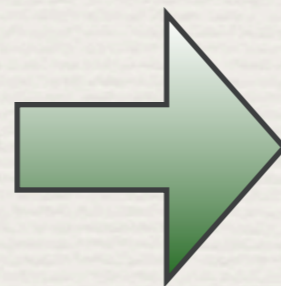
Experimental progress

2. Free, Cold neutron annihilation with target

Designed to:

1. Maximize number of neutrons
2. Minimize energy of neutrons
3. Maximize time of flight
4. Minimize External Magnetic Field

Minimize external
potential



Most model-independent
measurement

ILL bound (1993)

$$\tau_{n\bar{n}} > 0.86 \times 10^8 \text{ sec}$$

Experimental prospects

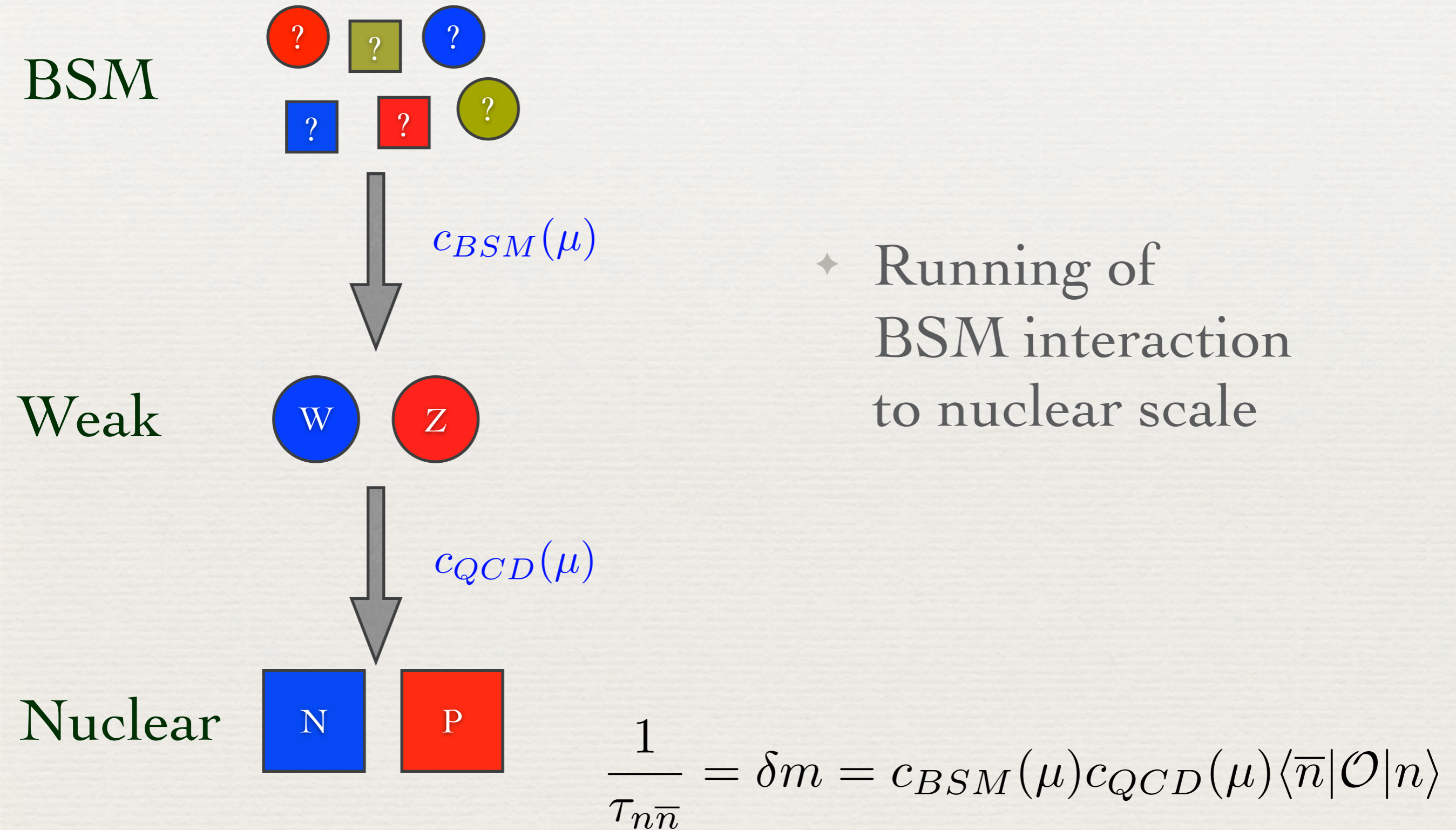
- ♦ Cost Estimates (Project X meeting, June 2012)

$$\tau_{n\bar{n}} \gtrsim 3 \times 10^9 \text{ sec:} \quad \sim \$10 \text{ million}$$

$$\tau_{n\bar{n}} \gtrsim 1 \times 10^{11} \text{ sec:} \quad \gtrsim \$200 \text{ million}$$

Bottom line: Lattice allows for rigorous, first-principle understanding of QCD input

Origin of Oscillations



Where Lattice Can Help

- ♦ Is BSM running non-perturbative?
 - Model-dependent (assume pert. models for now)
- ♦ Is QCD running non-perturbative?
 - Should be calculated (pert. running reasonable)
- ♦ What is neutron-antineutron matrix element?
 - Inherently non-perturbative question
- ♦ What is effect in nuclei?
 - Very interesting, **VERY** hard question

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 - Very interesting, VERY hard question

Six-quark Operators

Rao, Shrock (1982)

Three pairs of quarks:

1. $u^T C u$ or $u^T C d$ or $d^T C d$

2. $u_L^T C d_L$ or $u_R^T C d_R$

3. $\Gamma_{ijklmn}^s = \epsilon_{mik} \epsilon_{njl} + \epsilon_{nik} \epsilon_{mjl} + \epsilon_{mjk} \epsilon_{nil} + \epsilon_{njk} \epsilon_{mil}$
 $\Gamma_{ijklmn}^a = \epsilon_{mij} \epsilon_{nkl} + \epsilon_{nij} \epsilon_{mkl}$

Six-quark Operators

Rao, Shrock (1982)

$$\chi_i = L, R$$

1. $\mathcal{O}_{\chi_1\chi_2\chi_3}^1 = (u_{i\chi_1}^T C u_{j\chi_1})(d_{k\chi_2}^T C d_{l\chi_2})(d_{m\chi_3}^T C d_{n\chi_3}) \Gamma_{ijklmn}^s$

2. $\mathcal{O}_{\chi_1\chi_2\chi_3}^2 = (u_{i\chi_1}^T C d_{j\chi_1})(u_{k\chi_2}^T C d_{l\chi_2})(d_{m\chi_3}^T C d_{n\chi_3}) \Gamma_{ijklmn}^s$

3. $\mathcal{O}_{\chi_1\chi_2\chi_3}^3 = (u_{i\chi_1}^T C d_{j\chi_1})(u_{k\chi_2}^T C d_{l\chi_2})(d_{m\chi_3}^T C d_{n\chi_3}) \Gamma_{ijklmn}^a$

$$\Gamma_{ijklmn}^s = \epsilon_{mik} \epsilon_{njl} + \epsilon_{nik} \epsilon_{mjl} + \epsilon_{mjk} \epsilon_{nil} + \epsilon_{nj k} \epsilon_{mil}$$

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Six-quark Operators

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$$\mathcal{O}_{\chi_1 LR}^1 = \mathcal{O}_{\chi_1 RL}^1$$

$$\chi_i = L, R \quad \mathcal{O}_{LR\chi_3}^{2,3} = \mathcal{O}_{RL\chi_3}^{2,3}$$

1.
$$\mathcal{O}_{\chi_1\chi_2\chi_3}^1 = (u_{i\chi_1}^T C u_{j\chi_1})(d_{k\chi_2}^T C d_{l\chi_2})(d_{m\chi_3}^T C d_{n\chi_3}) \Gamma_{ijklmn}^s$$

2.
$$\mathcal{O}_{\chi_1\chi_2\chi_3}^2 = (u_{i\chi_1}^T C d_{j\chi_1})(u_{k\chi_2}^T C d_{l\chi_2})(d_{m\chi_3}^T C d_{n\chi_3}) \Gamma_{ijklmn}^s$$

3.
$$\mathcal{O}_{\chi_1\chi_2\chi_3}^3 = (u_{i\chi_1}^T C d_{j\chi_1})(u_{k\chi_2}^T C d_{l\chi_2})(d_{m\chi_3}^T C d_{n\chi_3}) \Gamma_{ijklmn}^a$$

$$\Gamma_{ijklmn}^s = \epsilon_{mik} \epsilon_{njl} + \epsilon_{nik} \epsilon_{mjl} + \epsilon_{mjk} \epsilon_{nil} + \epsilon_{nj k} \epsilon_{mil}$$

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18 Independent Operators

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$$\mathcal{O}_{\chi_1\chi_2\chi_3}^2 = (u_{i\chi_1}^T C d_{j\chi_1})(u_{k\chi_2}^T C d_{l\chi_2})(d_{m\chi_3}^T C d_{n\chi_3}) \Gamma_{ijklmn}^s$$

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Caswell, Milutinovic,
Sejanovic (1983)

18 Independent Operators  14 Indep. Operators

Six-quark Operators

Rao, Shrock (1982)

If invariant under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$:

Restricts operators to:

-All combination of right-handed singlets

$$\mathcal{P}_1 = (u_{iR}^T C u_{jR})(d_{kR}^T C d_{lR})(d_{mR}^T C d_{nR}) \Gamma_{ijklmn}^s = \mathcal{O}_{RRR}^1$$

$$\mathcal{P}_2 = (u_{iR}^T C d_{jR})(u_{kR}^T C d_{lR})(d_{mR}^T C d_{nR}) \Gamma_{ijklmn}^s = \mathcal{O}_{RRR}^2$$

$$\mathcal{P}_3 = (u_{iR}^T C d_{jR})(u_{kR}^T C d_{lR})(d_{mR}^T C d_{nR}) \Gamma_{ijklmn}^a = \mathcal{O}_{RRR}^3$$

Six-quark Operators

Rao, Shrock (1982)

If invariant under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$:

Restricts operators to:

- All combination of right-handed singlets
- Flavor anti-symmetric combinations of left-handed doublets

$$\mathcal{P}_4 = ([q_{iL}^T]^w C [q_{jL}]^x) (u_{kR}^T C d_{lR}) (d_{mR}^T C d_{nR}) \Gamma_{ijklmn}^a \epsilon^{wx} = 2\mathcal{O}_{LRR}^3$$

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$$\mathcal{P}_5 = ([q_{iL}^T]^w C [q_{jL}]^x) ([q_{kL}^T]^y C [q_{lR}]^z) (d_{mR}^T C d_{nR}) \Gamma_{ijklmn}^a \epsilon^{wx} \epsilon^{yz} = 4\mathcal{O}_{LLR}^3$$

Six-quark Operators

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$$\begin{aligned} \mathcal{P}_6 &= ([q_{iL}^T]^w C [q_{jL}]^x) ([q_{kL}^T]^y C [q_{lR}]^z) (d_{mR}^T C d_{nR}) \Gamma_{ijklmn}^s (\epsilon^{wy} \epsilon^{xz} + \epsilon^{xy} \epsilon^{wz}) \\ &= 4(\mathcal{O}_{LLR}^1 - \mathcal{O}_{LLR}^2) \end{aligned}$$

Six-quark Operators

Rao, Shrock (1982)

If invariant under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$:

(Six Operators)

$$\mathcal{P}_1 = \mathcal{O}_{RRR}^1$$

$$\mathcal{P}_2 = \mathcal{O}_{RRR}^2$$

$$\mathcal{P}_3 = \mathcal{O}_{RRR}^3$$

$$\mathcal{P}_4 = 2\mathcal{O}_{LRR}^3$$

$$\mathcal{P}_5 = 4\mathcal{O}_{LLR}^3$$

$$\mathcal{P}_6 = 4(\mathcal{O}_{LLR}^1 - \mathcal{O}_{LLR}^2)$$

- ♦ Matrix elements cannot be calculated perturbatively

Six-quark Operators

Rao, Shrock (1982)

If invariant under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$:

(Six Operators)

$$\mathcal{P}_1 = \mathcal{O}_{RRR}^1$$

(Four Operators)

$$\mathcal{P}_2 = \mathcal{O}_{RRR}^2$$

Caswell, Milutinovic,
Sejanovic (1983)

$$\mathcal{P}_3 = \mathcal{O}_{RRR}^3$$

$$\mathcal{P}_4 = 2\mathcal{O}_{LRR}^3$$

$$\mathcal{P}_6 = -3\mathcal{P}_5$$

$$\mathcal{P}_5 = 4\mathcal{O}_{LLR}^3$$

$$\mathcal{P}_2 - \mathcal{P}_1 = 3\mathcal{P}_3$$

$$\mathcal{P}_6 = 4(\mathcal{O}_{LLR}^1 - \mathcal{O}_{LLR}^2)$$

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(Four Operators)

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$$\mathcal{P}_6 = 4(\mathcal{O}_{LLR}^1 - \mathcal{O}_{LLR}^2)$$

- Matrix elements cannot be calculated perturbatively

Current Understanding of Matrix Elements

MIT bag Model: (Rao, Shrock 1982)

- Model dependent estimation
- Results roughly consistent with DA
- No QCD input

Lattice Motivation:

- Numerical QCD calculation
- Quantification of uncertainties
- Pinpoint target sensitivity for experiment
- Large enhancements/suppressions?

Lattice Calculation

Correlation Functions via path integral:

$$C_{\mathcal{O}} = \langle \mathcal{O} \rangle = \int d[U] \mathcal{O} \det(D_{lat}(U)) e^{-S_G(U)}$$

$$C_{NN}(t) = \langle \bar{N}(t) N(0) \rangle \rightarrow |\langle N|n \rangle|^2 e^{-m_n t}$$

$$C_{\bar{N}\bar{N}}(t) = \langle N(t) \bar{N}(0) \rangle \rightarrow |\langle \bar{N}|\bar{n} \rangle|^2 e^{-m_n t}$$

$$C_{\bar{N}\mathcal{O}N}(t_1, t_2) = \langle N(t_1) \mathcal{O}(0) \bar{N}(t_2) \rangle \rightarrow \langle \bar{N}|\bar{n} \rangle \langle N|n \rangle e^{-m_n(t_1+t_2)} \langle n|\mathcal{O}|\bar{n} \rangle$$

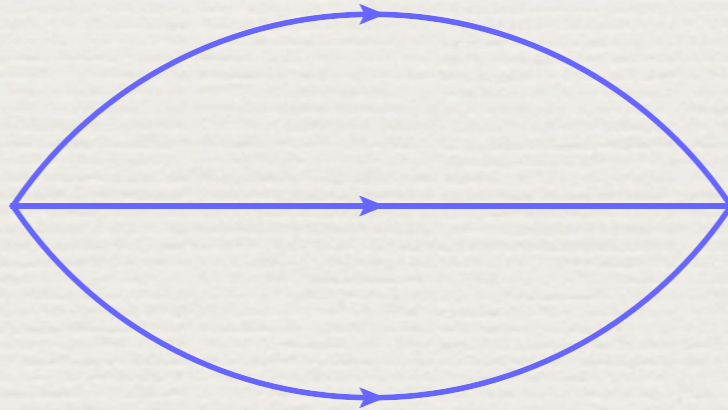
$$\mathcal{R} = \frac{C_{\bar{N}\mathcal{O}N}(t_1, t_2)}{C_{\bar{N}\bar{N}}(t_1 + t_2)} \left[\frac{C_{NN}(t_1) C_{\bar{N}\bar{N}}(t_2) C_{\bar{N}\bar{N}}(t_1 + t_2)}{C_{\bar{N}\bar{N}}(t_1) C_{NN}(t_2) C_{NN}(t_1 + t_2)} \right]^{\frac{1}{2}} \rightarrow \langle \bar{n}|\mathcal{O}|n \rangle$$

Lattice Contractions

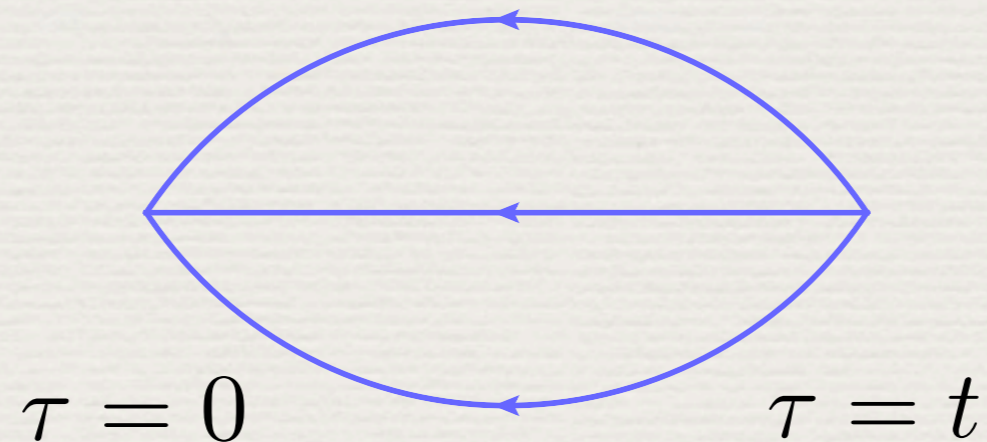
Propagator Contractions:

$$\underbrace{\bar{q}_{i'}^{\alpha'}(y) q_i^\alpha(x)} = S_{i'i}^{\alpha'\alpha}(y, x) \quad S^\dagger = \gamma_5 S \gamma_5$$

$C_{NN}(t)$



$C_{\overline{NN}}(t)$



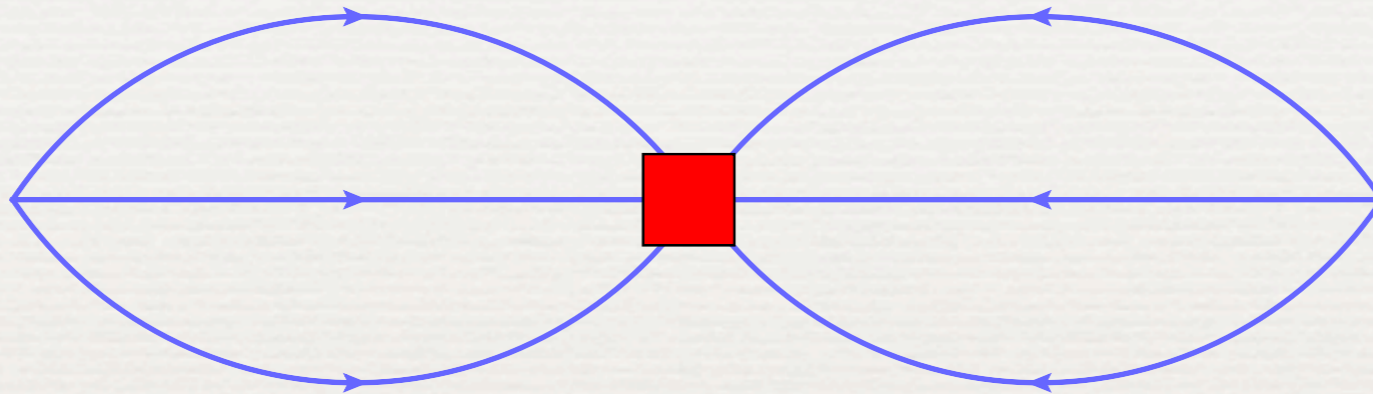
Lattice Contractions

$$\tau = -t_1$$

$$\tau = 0$$

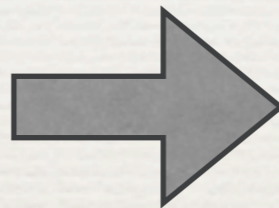
$$\tau = t_2$$

$$C_{\overline{N}ON}(t_1, t_2)$$



No
Dis.
Diagrams

1 Propagator



Two measurement
ALL time insertion

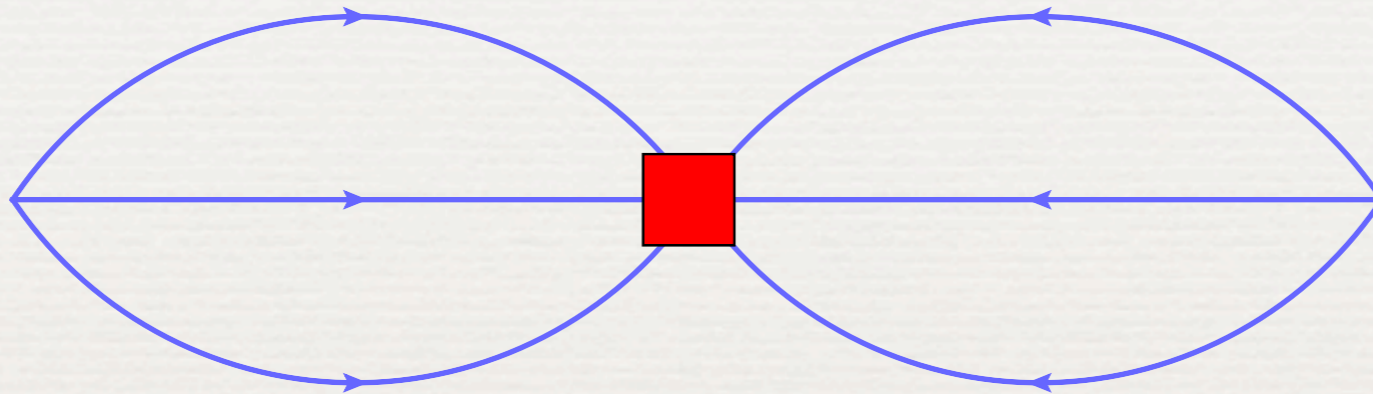
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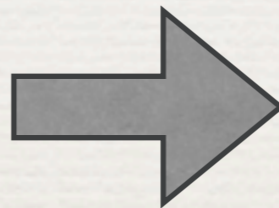
$$\tau = t_2$$

$$C_{\overline{N}ON}(t_1, t_2)$$



No
Dis.
Diagrams

1 Propagator



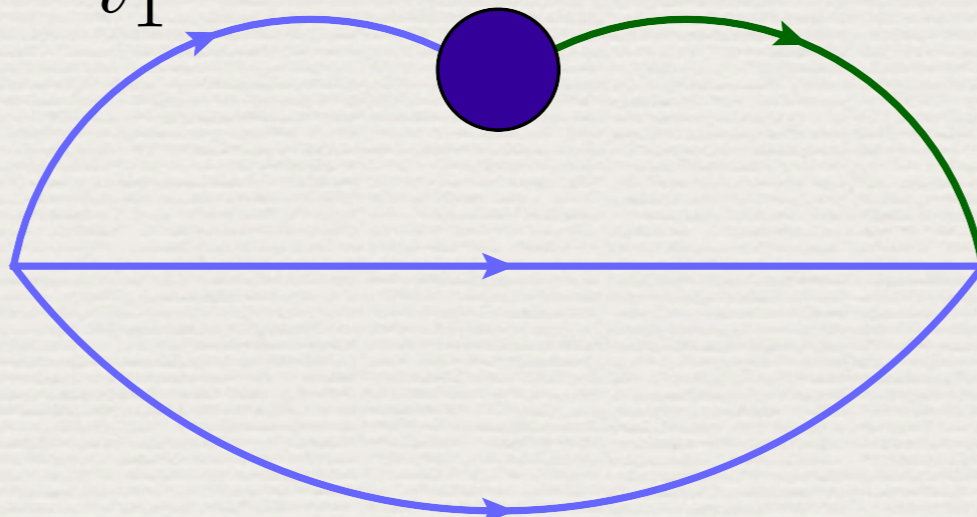
Two measurement
ALL time insertion

$$\tau = -t_1$$

$$\tau = 0$$

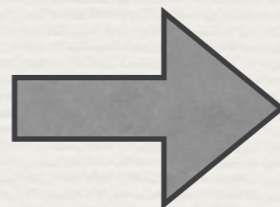
$$\tau = t_2$$

Typical
3-point



Often
Dis.
Diagrams

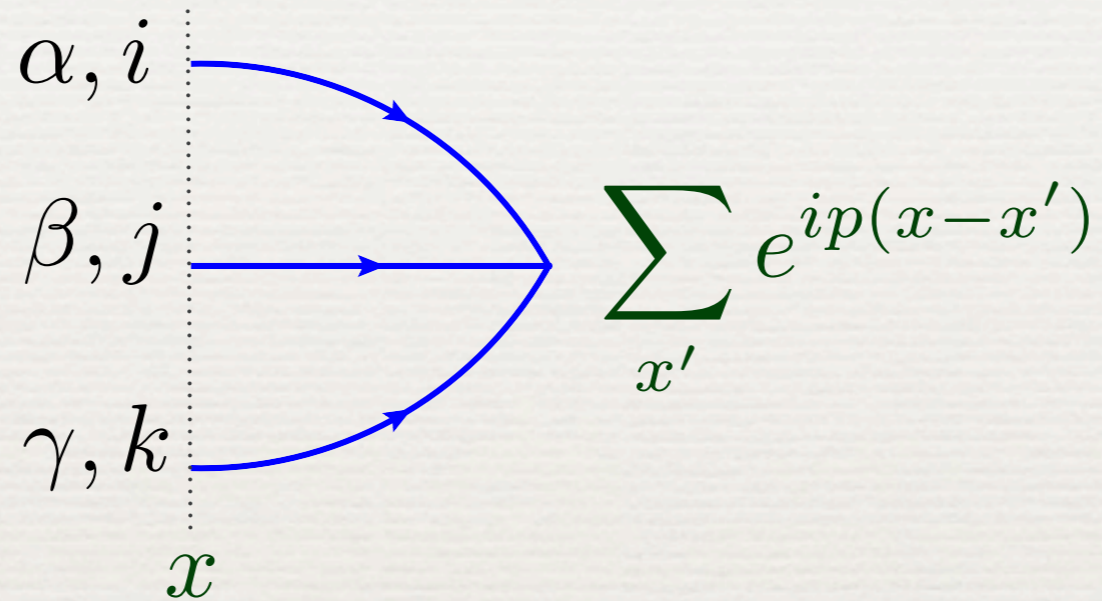
2 Propagators



One measurement
One time insertion

Neutron Blocks

Construct sink-contracted neutron blocks:

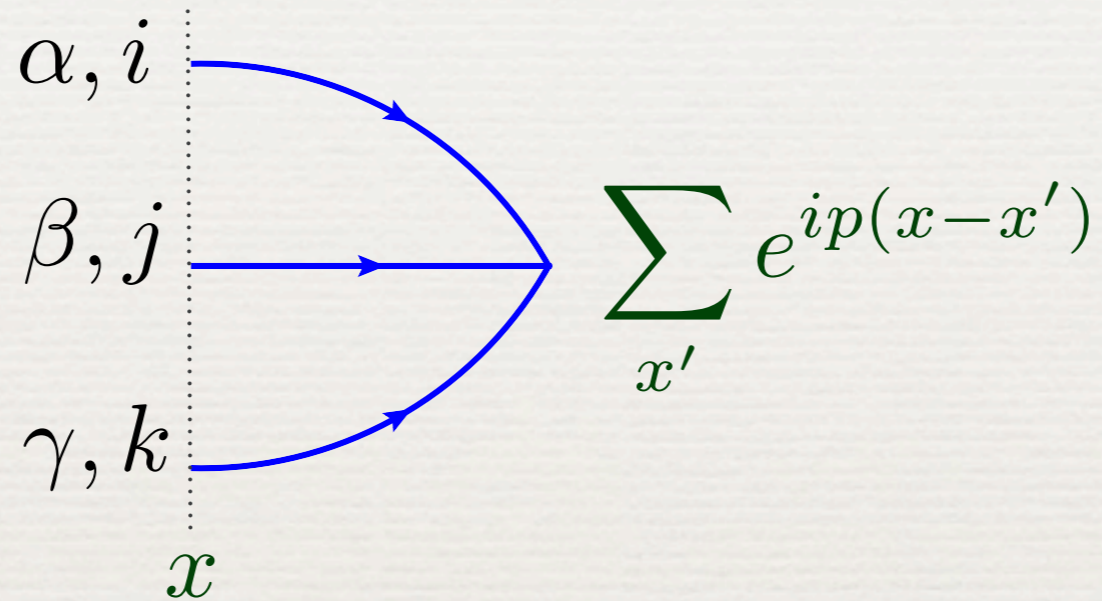


Project onto
lattice irrep
 G_1^+

$12 \times 12 \times N_t$ Object  N_s^3 times smaller than prop

Neutron Blocks

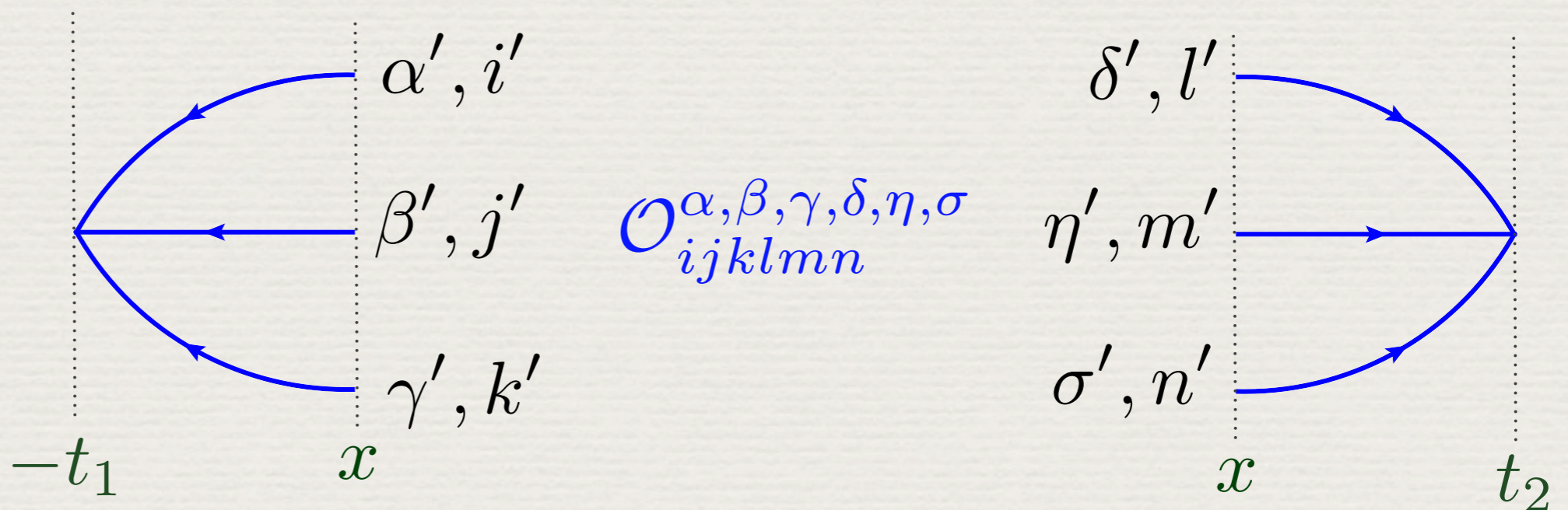
Construct sink-contracted neutron blocks:



Project onto
lattice irrep
 G_1^+

$12 \times 12 \times N_t$ Object  N_s^3 times smaller than prop

12
Indep.
Contractions



Executive Summary

- ♦ Advantages of Neutron-Antineutron calculations

For same cost:

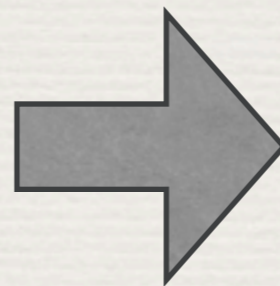
More Statistics

All Operator Insertions

No Quark Loop or Disconnected Diagrams

- ♦ Disadvantages of Neutron-Antineutron calculations

More Propagator
Multiplications



Potentially Worse
Signal

Lattice Details

Original Calculation

Problems:

Operator Smearing
Insufficient statistics

- $32^3 \times 256$ anisotropic clover-Wilson lattices
- $m_\pi \sim 390$ MeV
- $a_t \sim 0.04$ fm, $a_s \sim 0.125$ fm
- $L \sim 4$ fm
- 159 configurations (every 4th trajectory)
- 7268 propagators (Gaussian smeared sources)

-
- $20^3 \times 256$ anisotropic clover-Wilson lattices
 - $m_\pi \sim 390$ MeV
 - $a_t \sim 0.04$ fm, $a_s \sim 0.125$ fm
 - $L \sim 4$ fm
 - 875 configurations (every 5th trajectory)
 - 62,548 propagators

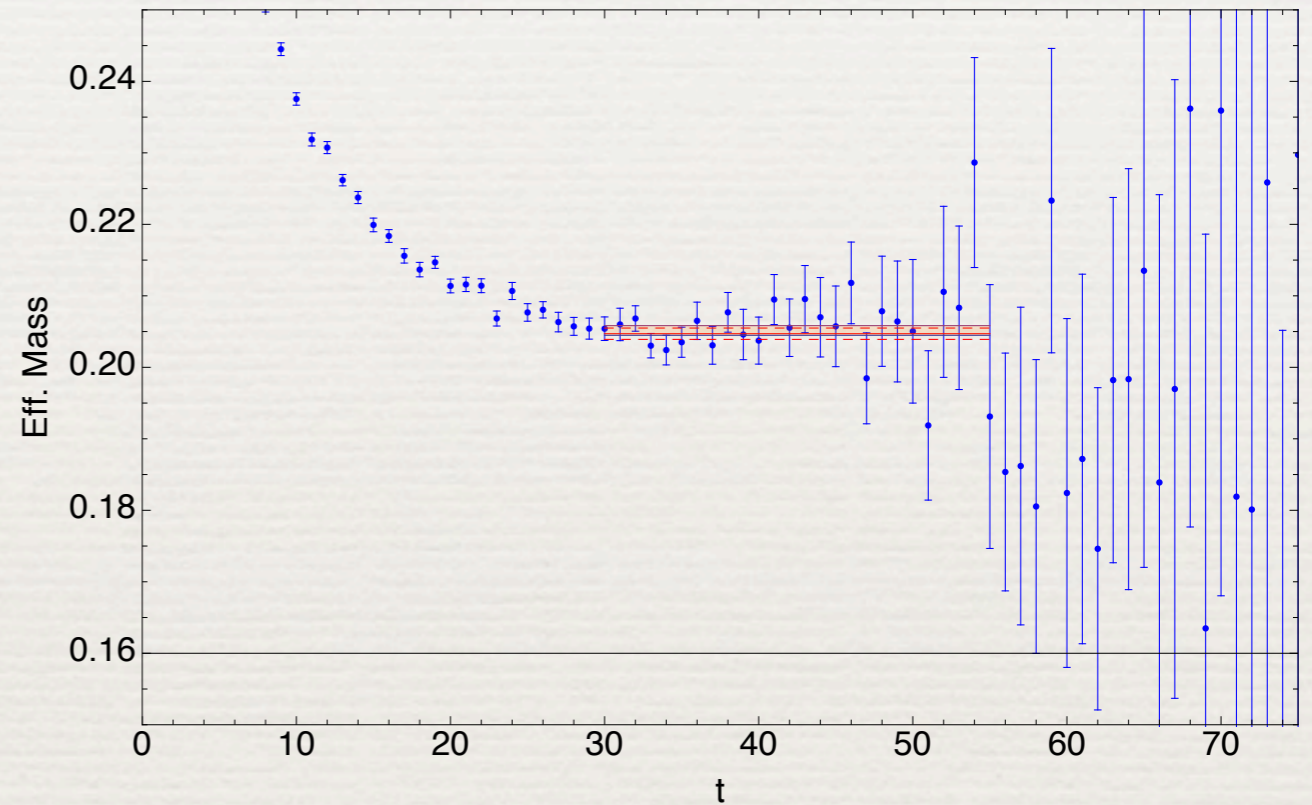
Aim:
1149

Aim:
100,000

Nucleon Effective Mass

Eff. Mass

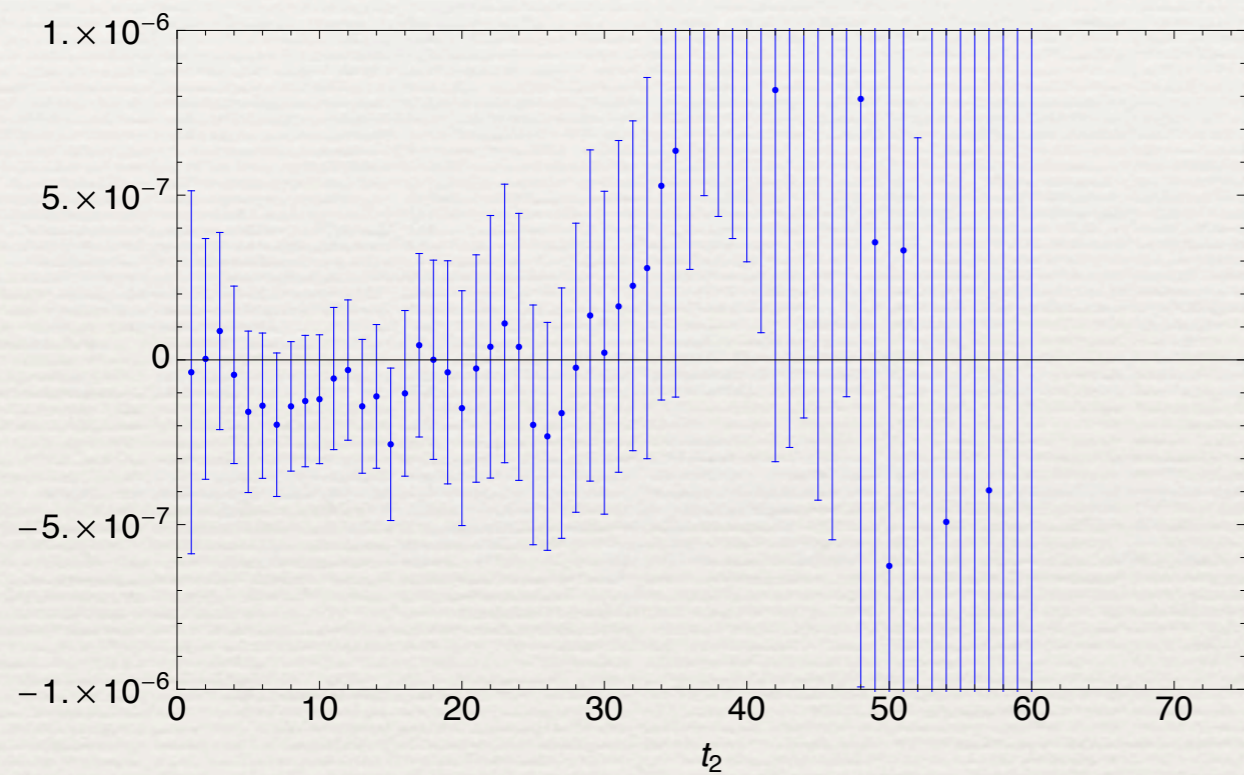
$$\frac{C_{NN}(t+1)}{C_{NN}(t)} \rightarrow m_n$$



$$a_t M_N = 0.20470(\pm 0.00079)(+0.00113)(-0.00022)$$

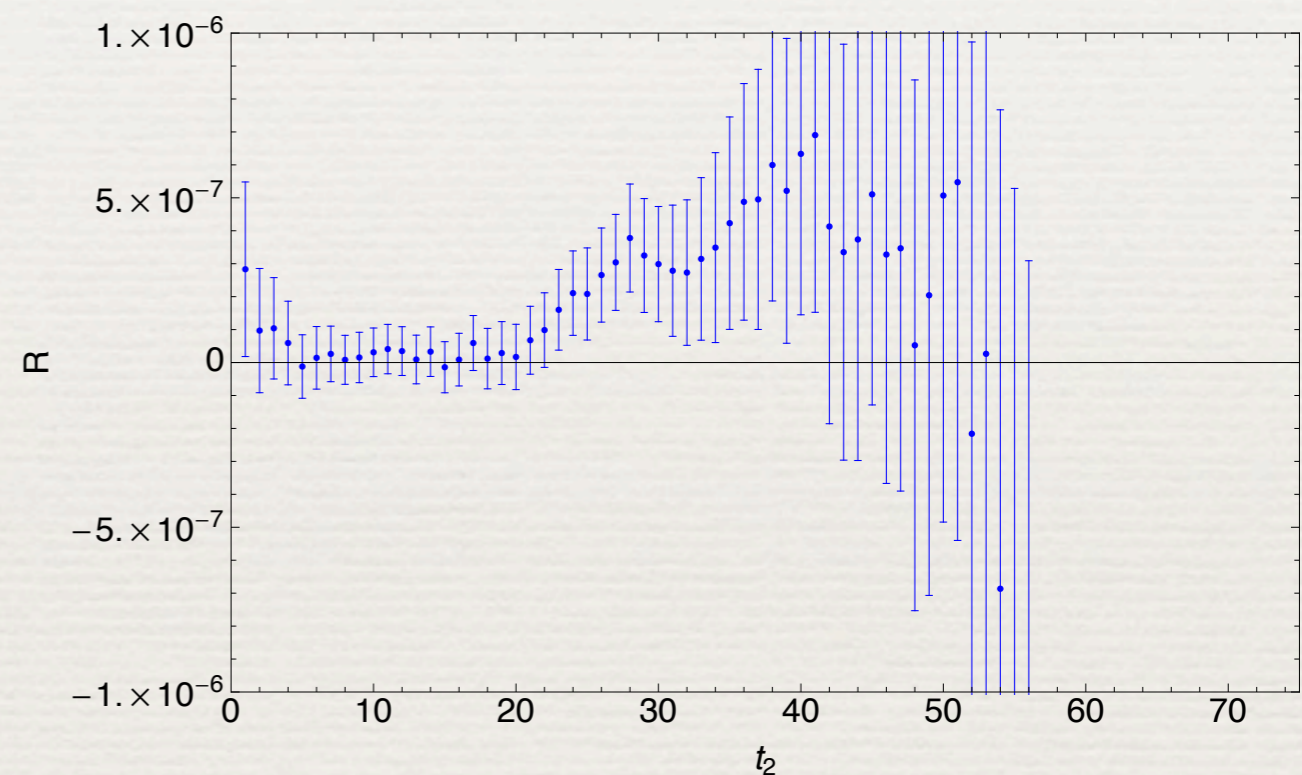
N-NBar Matrix Element

$$t_1 = 15$$



$\sim 10,000$
Measurements

$$t_1 = 15$$

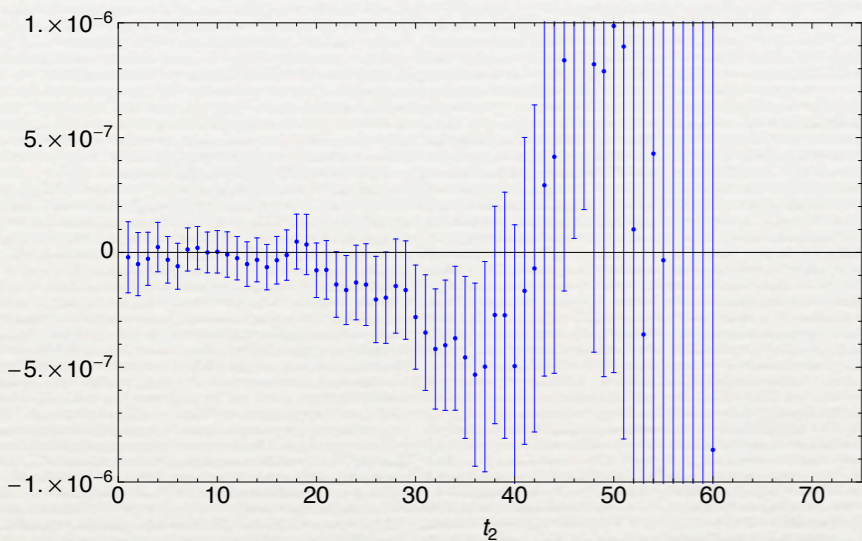


$\sim 60,000$
Measurements

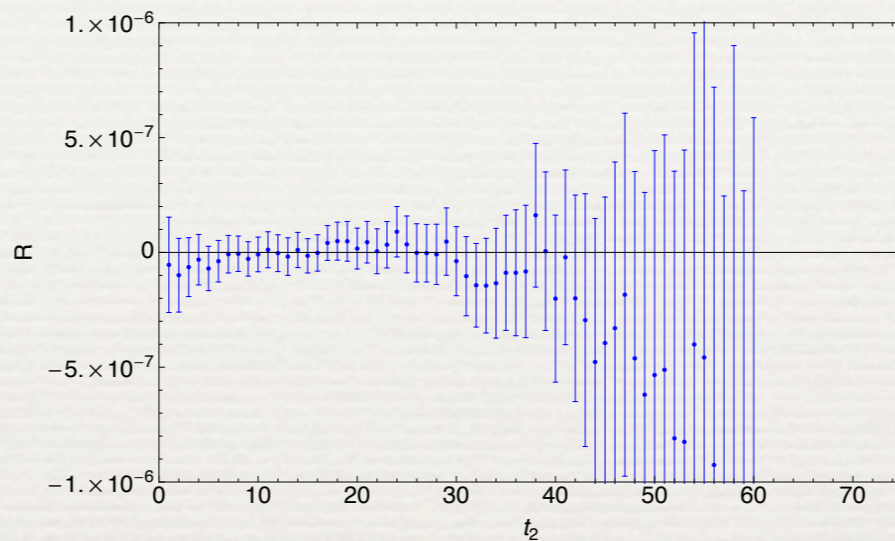
$$\mathcal{R} \rightarrow \langle \bar{n} | \mathcal{O} | n \rangle$$

N-NBar Matrix Element

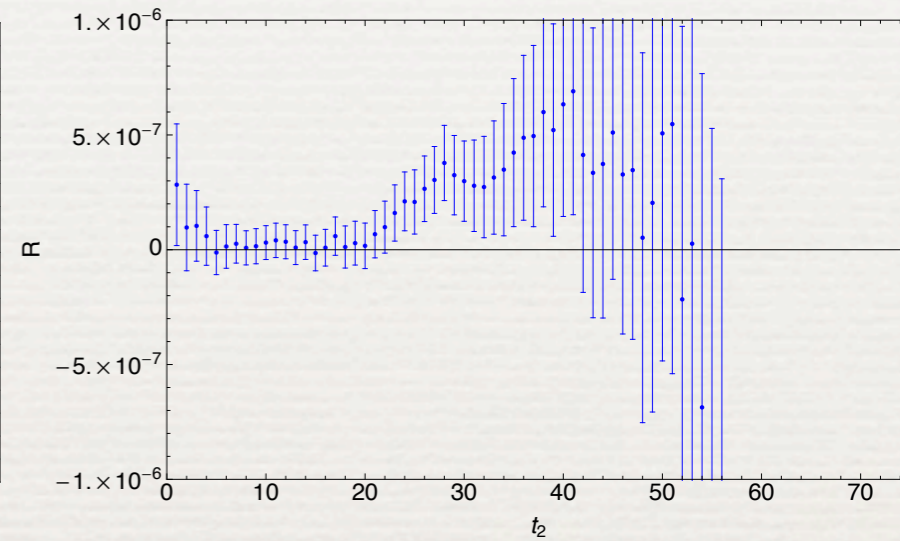
$$t_1 = 5$$



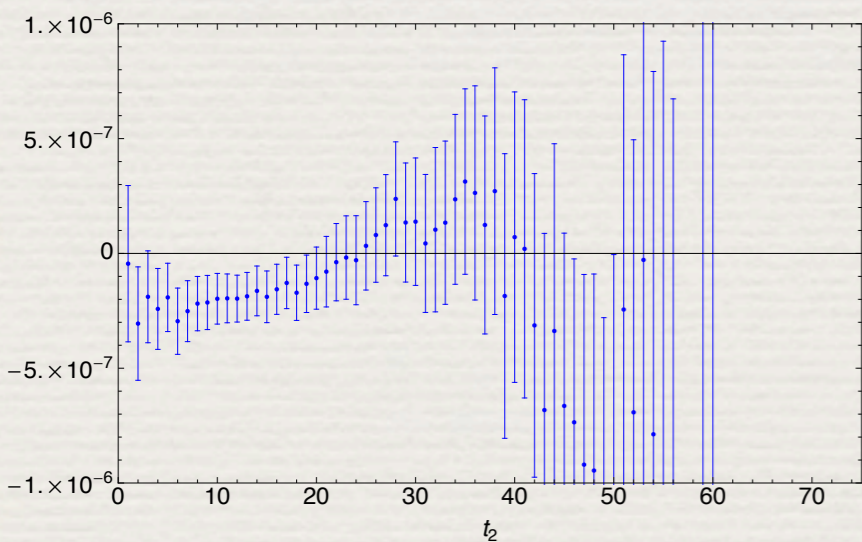
$$t_1 = 10$$



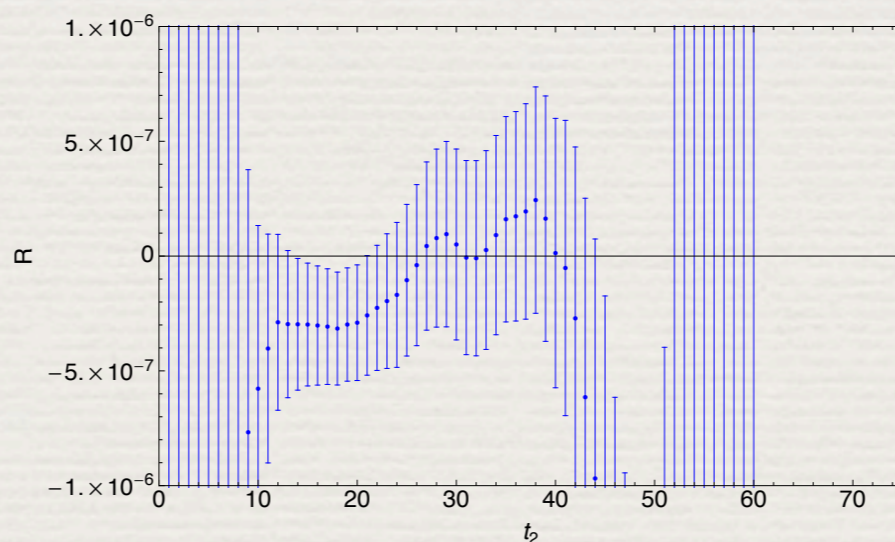
$$t_1 = 15$$



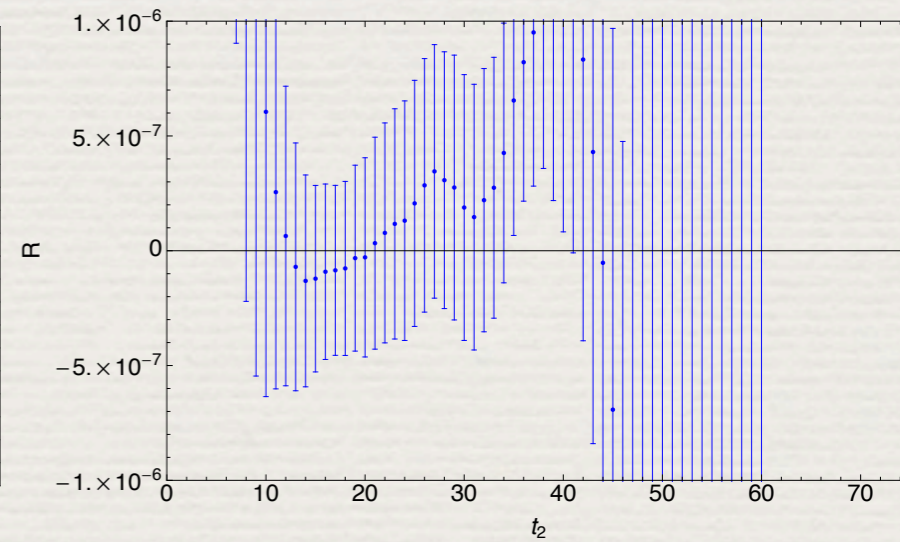
$$t_1 = 20$$



$$t_1 = 25$$



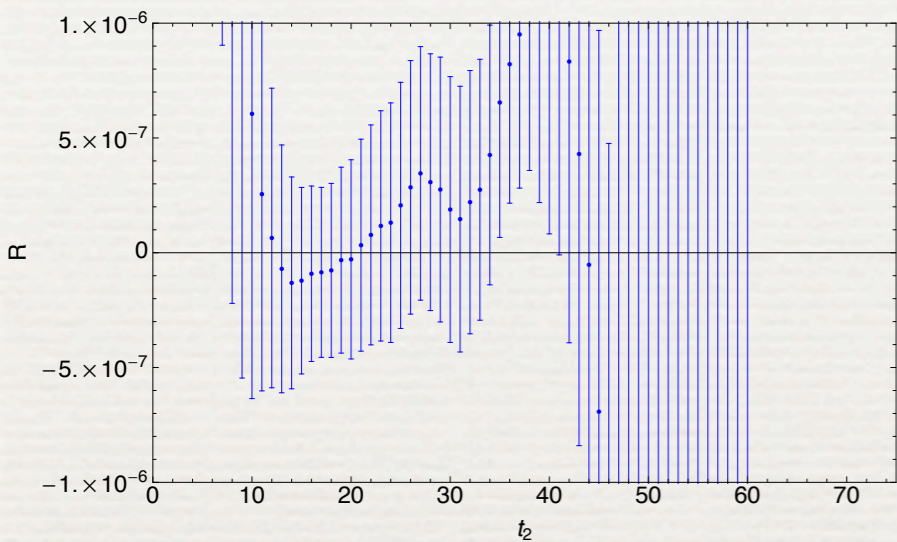
$$t_1 = 30$$



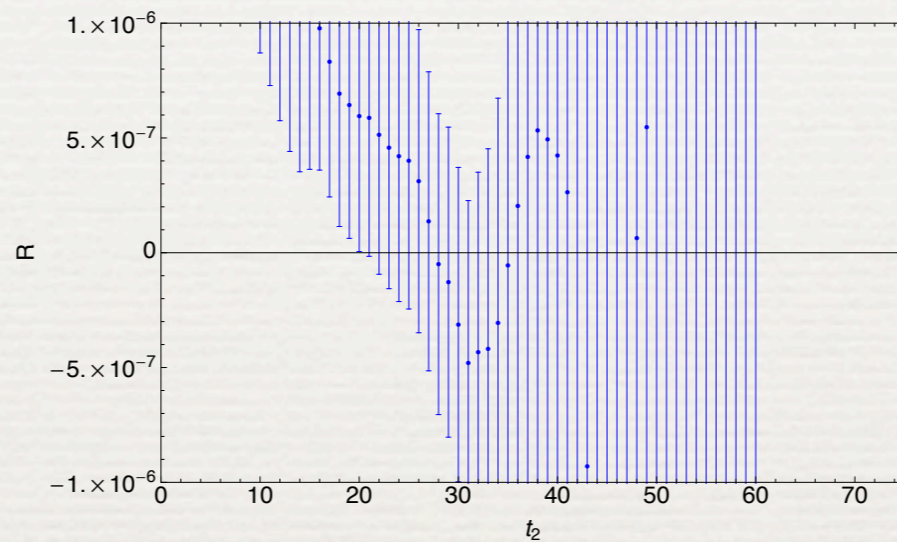
$$\mathcal{R} \rightarrow \langle \bar{n} | \mathcal{O} | n \rangle$$

N-NBar Matrix Element

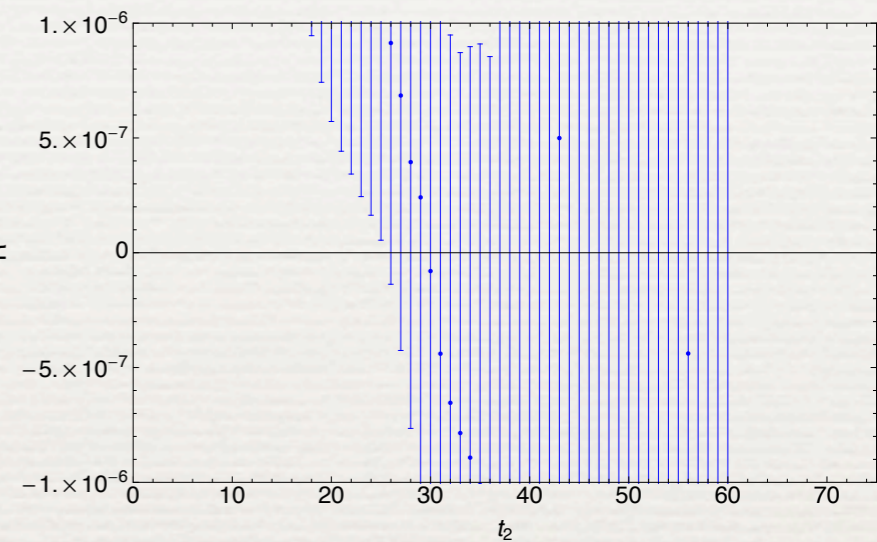
$$t_1 = 30$$



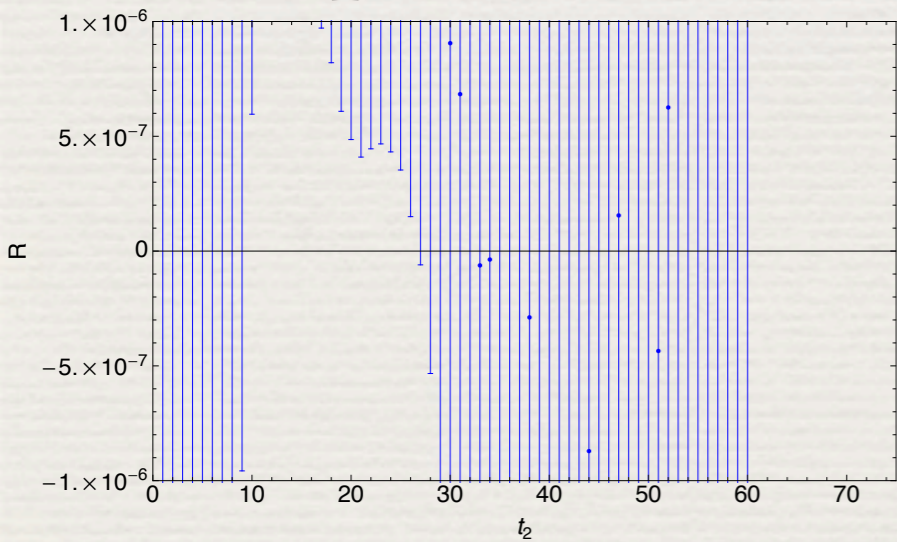
$$t_1 = 35$$



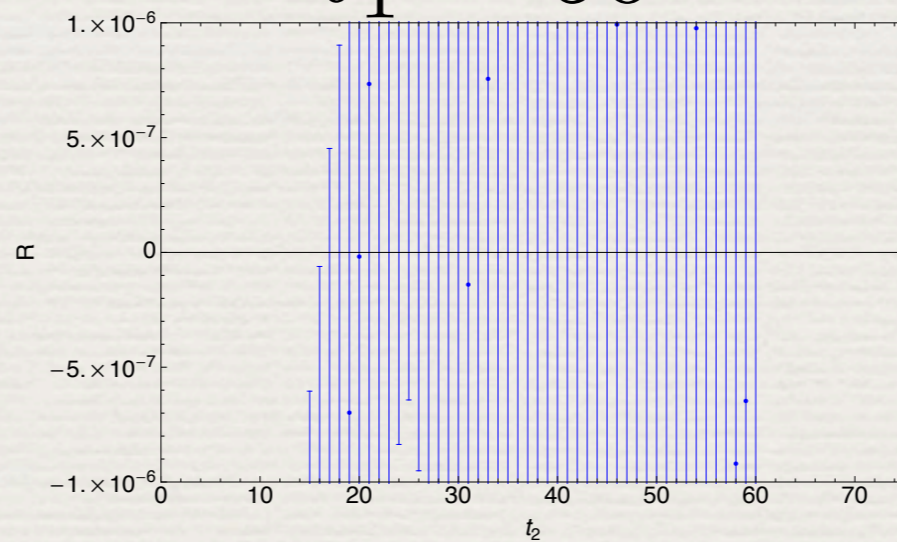
$$t_1 = 40$$



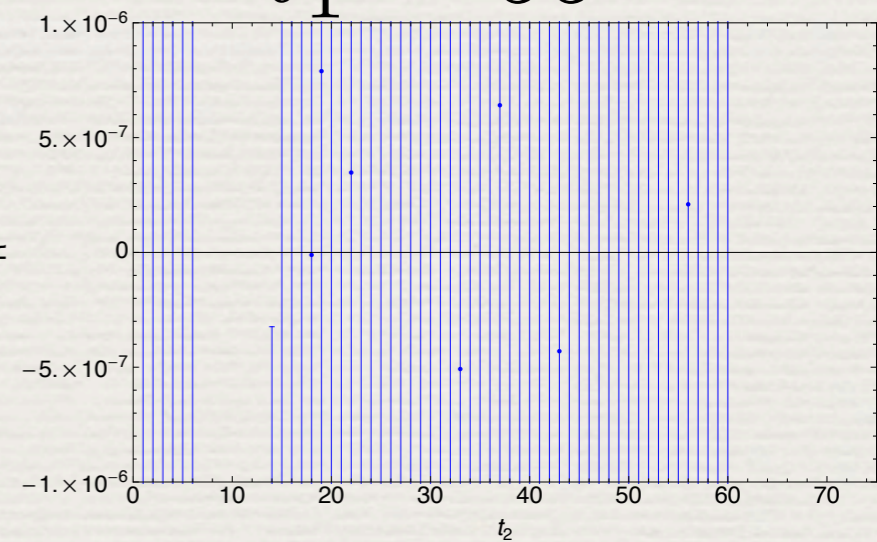
$$t_1 = 45$$



$$t_1 = 50$$



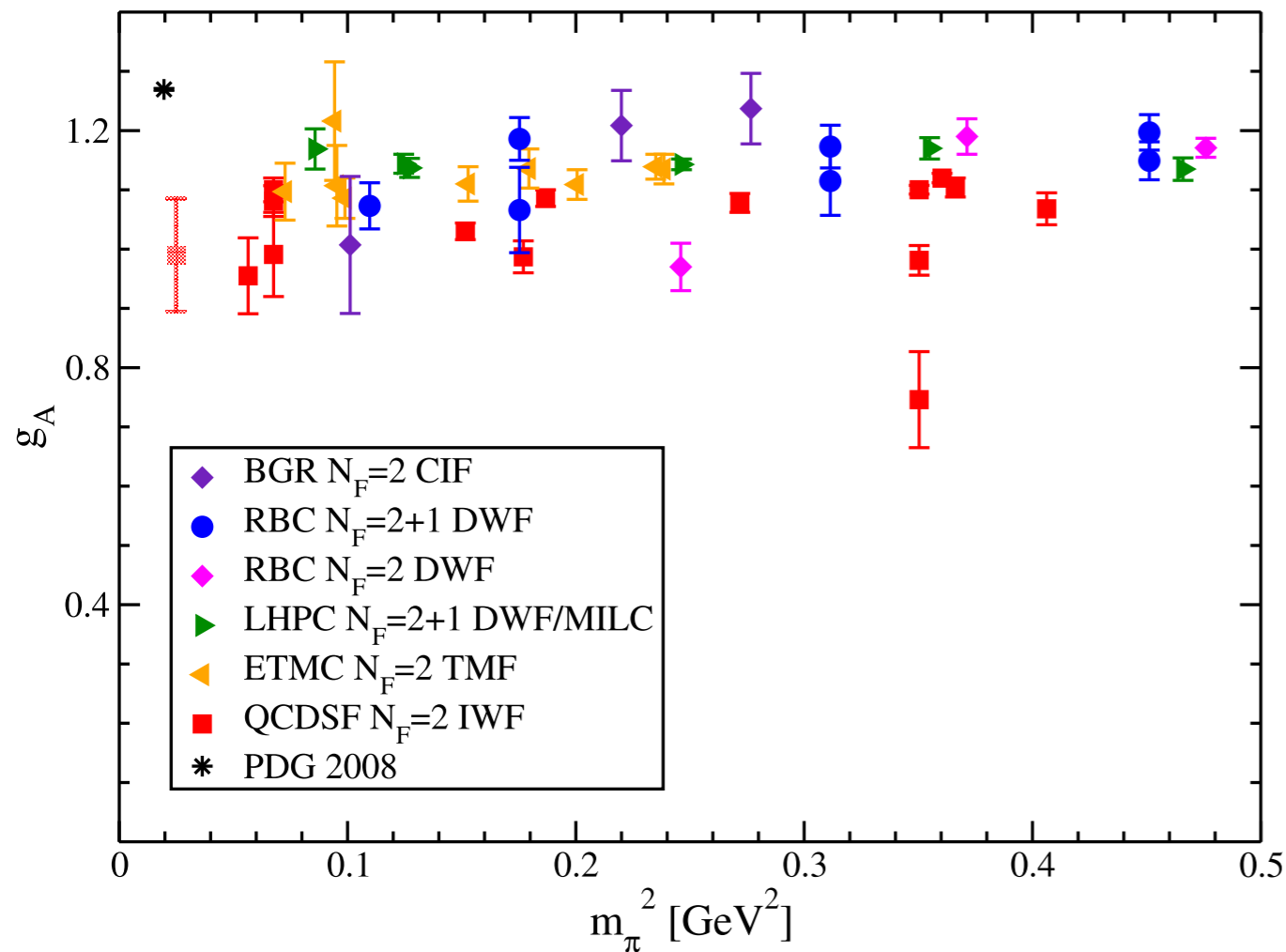
$$t_1 = 55$$



$$\mathcal{R} \rightarrow \langle \bar{n} | \mathcal{O} | n \rangle$$

Possible Systematics Studies

Hosts of systematics can plague nucleon three-point functions



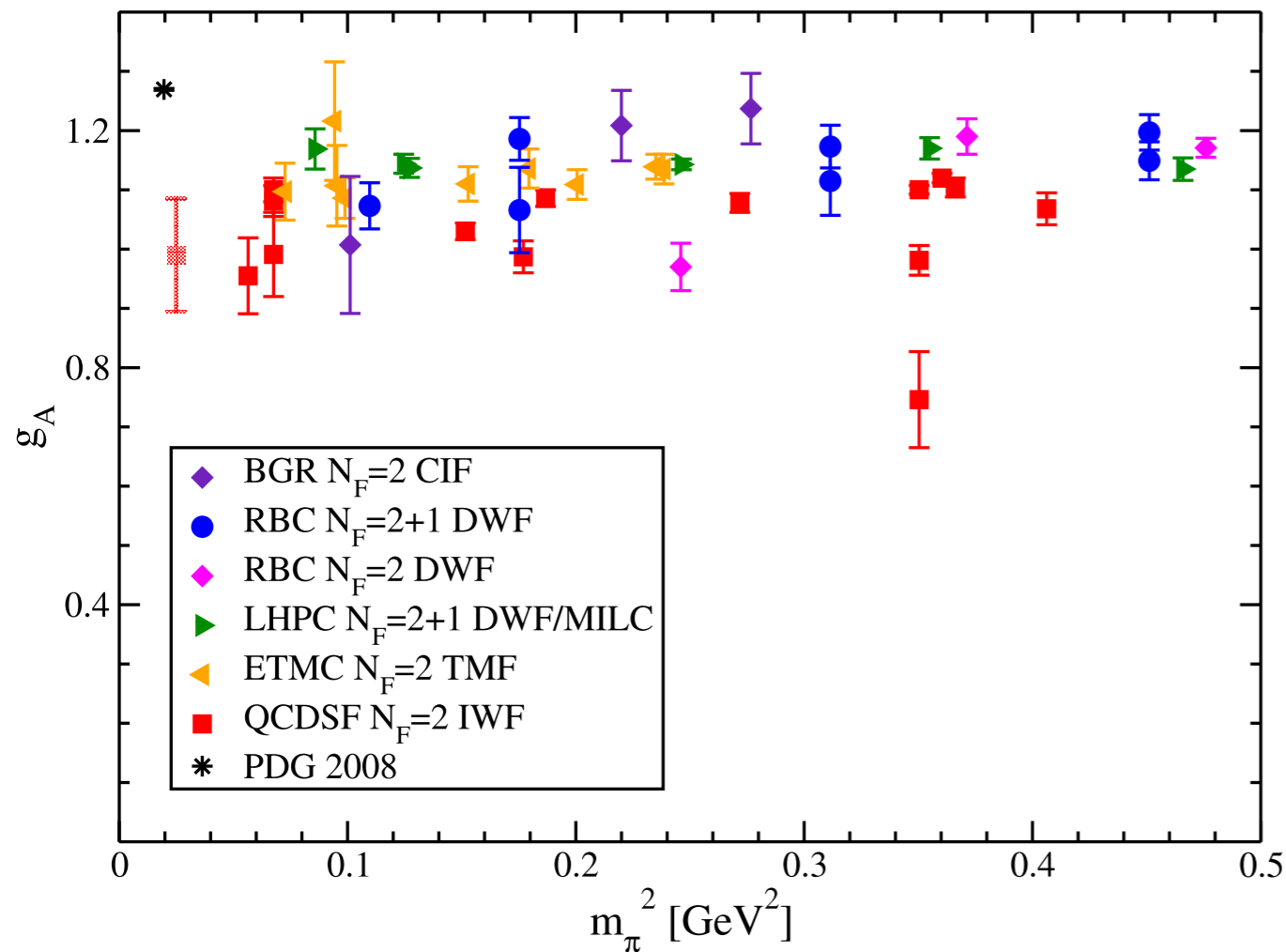
- 1) Volume Effects
- 2) Excited State Effects
- 3) Renormalization
- 4) Signal-to-noise
- 5) Statistics

Renner,
arXiv:1002.0925

Neutron-antineutron calculations
can be a testing ground!!

Possible Systematics Studies

Hosts of systematics can plague nucleon three-point functions



1) Volume Effects

2) Excited State Effects

3) Renormalization

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Renner,
arXiv:1002.0925

Neutron-antineutron calculations
can be a testing ground!!

Variety of Three-point analyses

Examined here:

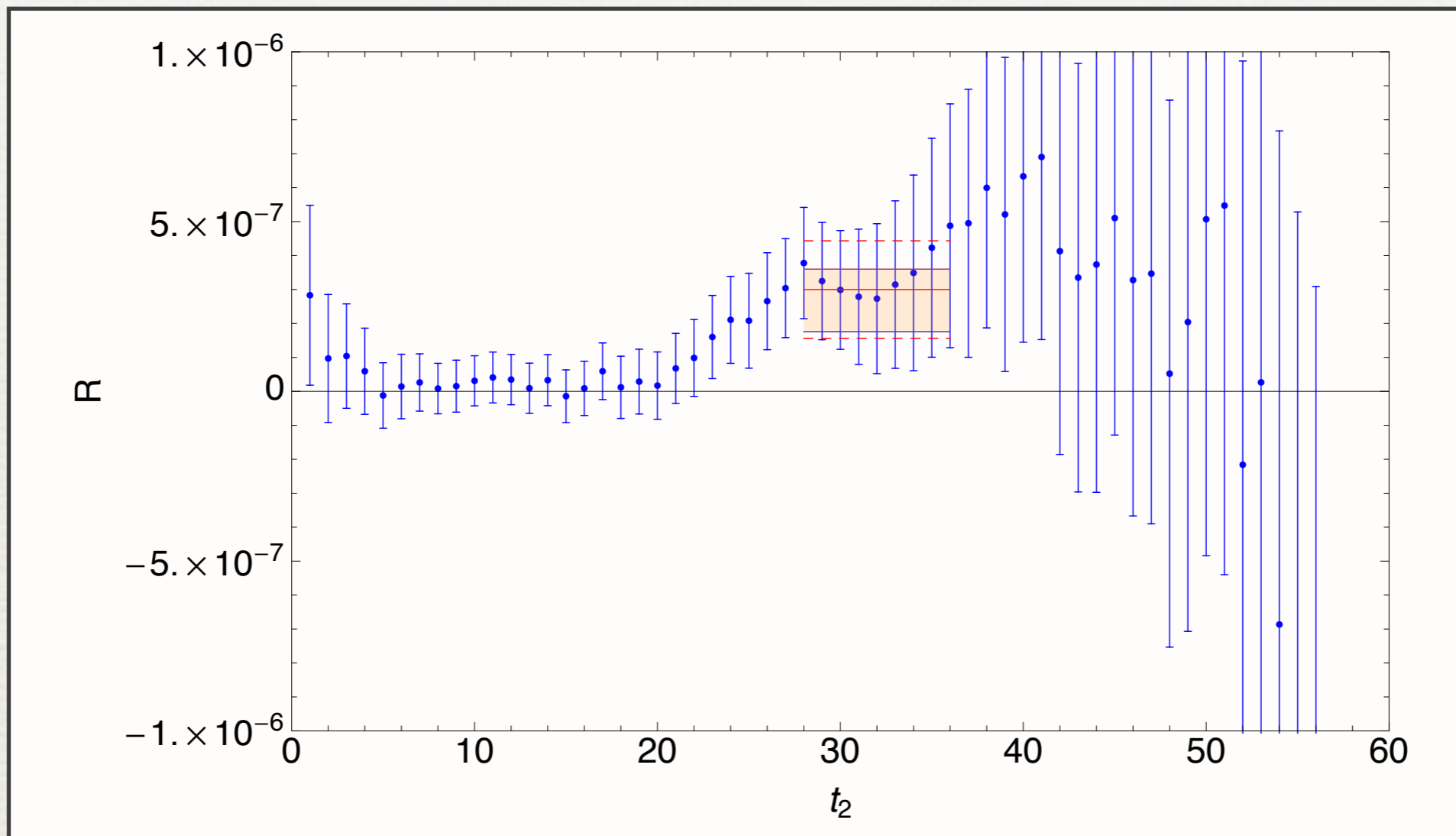
- 1) Single source-operator separation
- 2) Center-line Fit
- 3) 2D Center-line Fit (source-op & op-sink)
- 4) Summation Method

To be explored:

- 1) Three-point matrix-Prony
- 2) Folding
- 3) Other Suggestions?

One Separation Fit

$$t_1 = 15$$

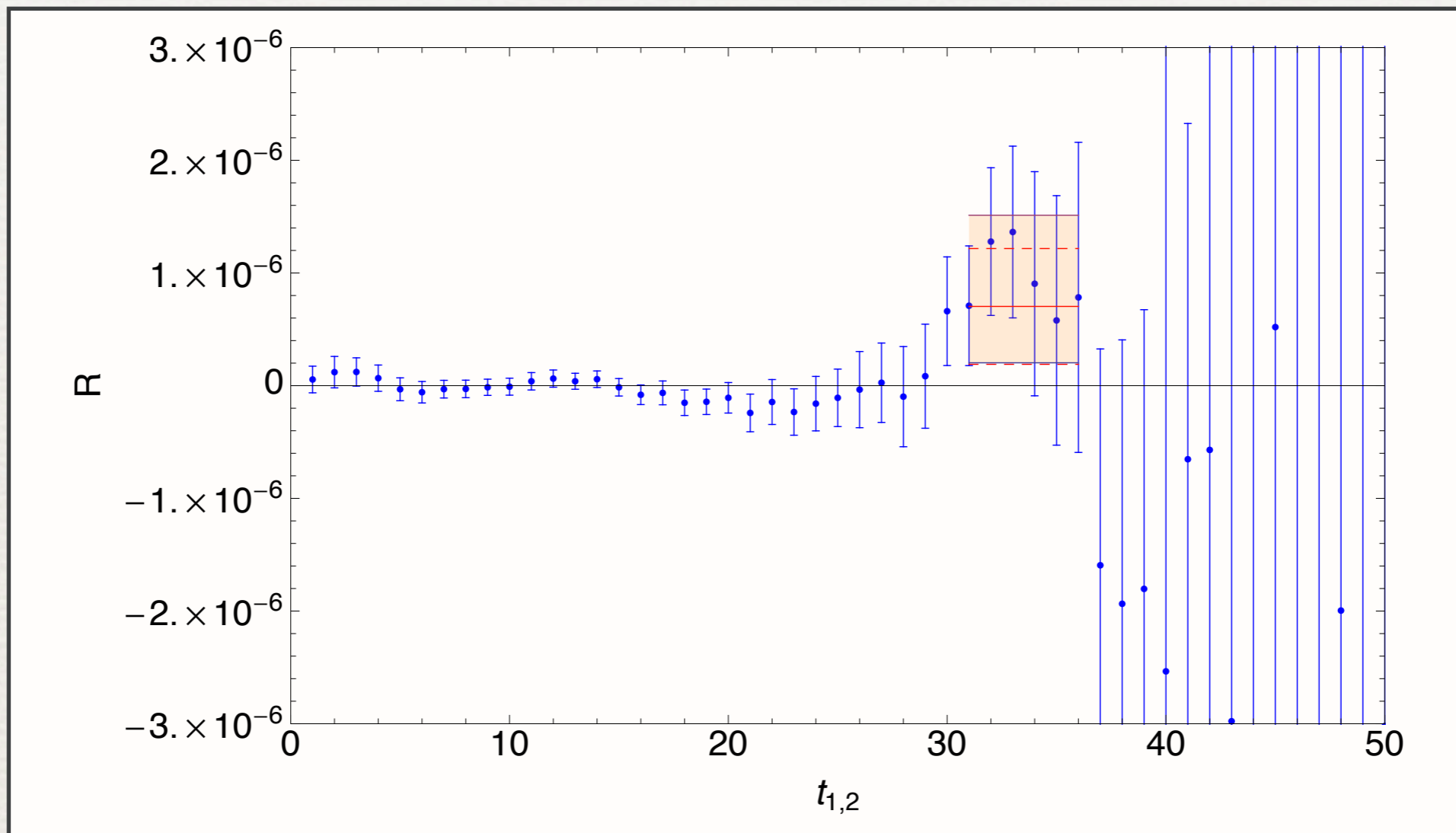


$$a_s^6 \langle N | \mathcal{O}_{RRR}^1 | \bar{N} \rangle = (3.00 \pm 1.44_{-1.24}^{+0.60}) \times 10^{-7}$$

$$\chi^2/\text{dof} = 0.859$$

Center-line Fit

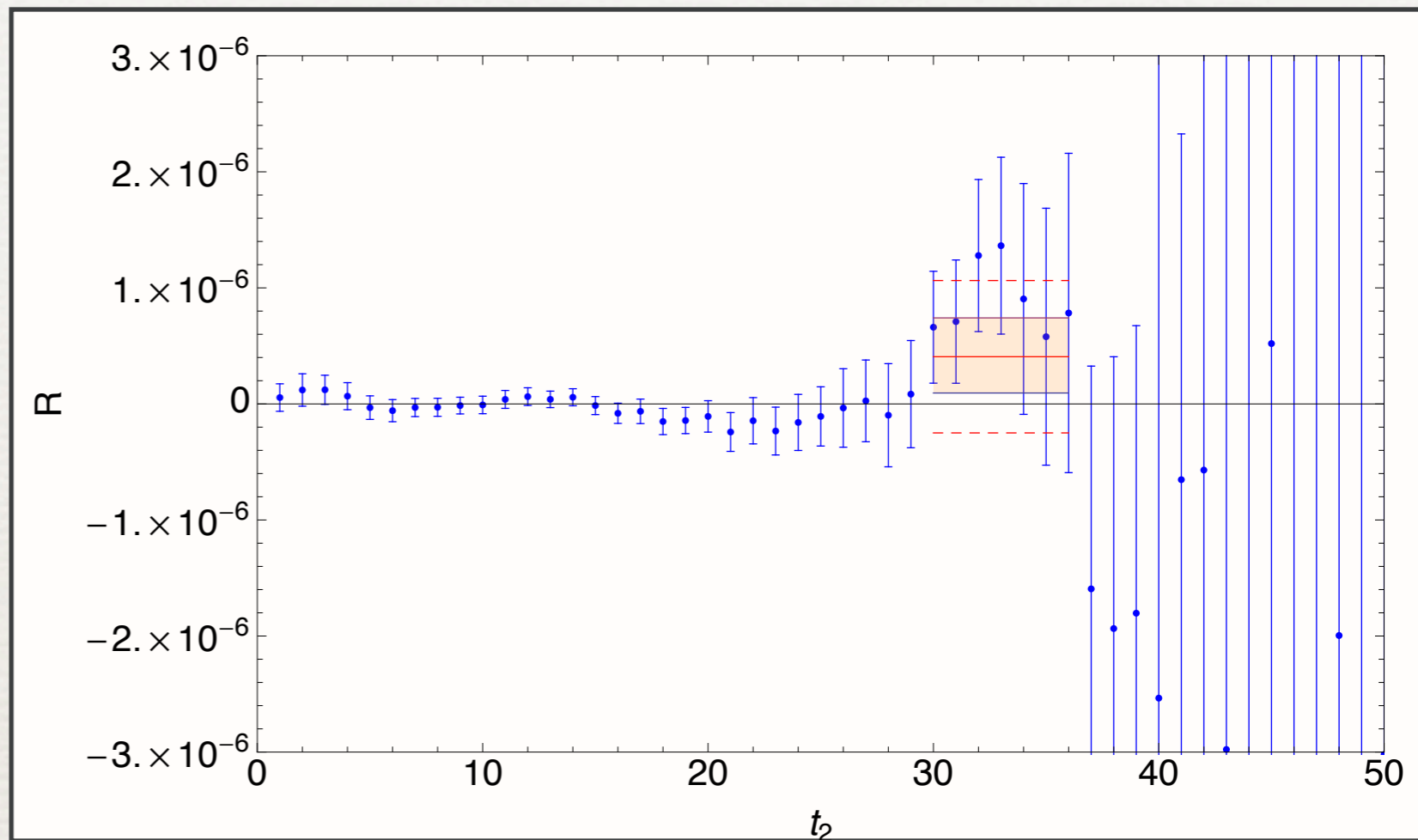
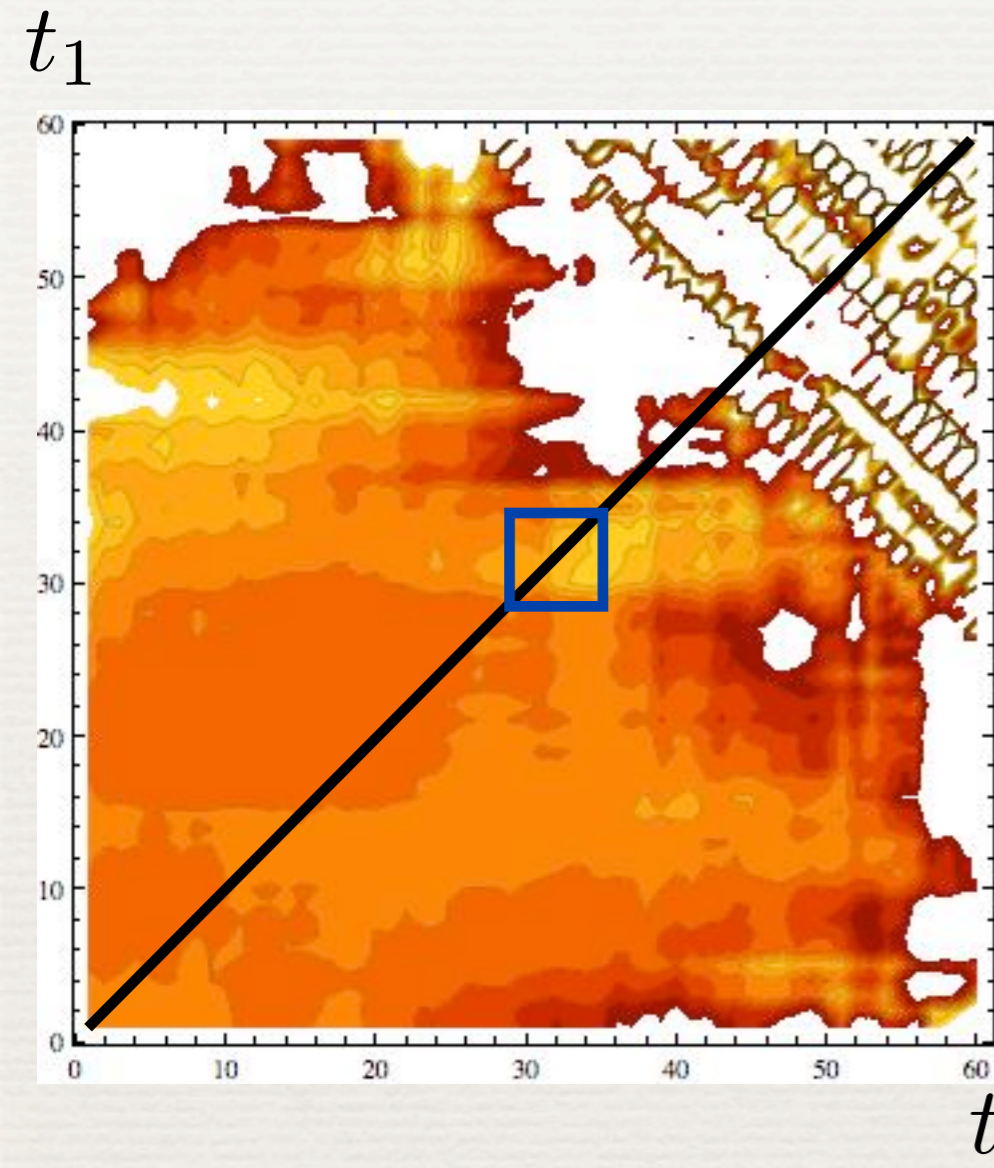
$$t_{1,2} = t_1 = t_2$$



$$a_s^6 \langle N | \mathcal{O}_{RRR}^1 | \bar{N} \rangle = (7.03 \pm 5.13_{-5.00}^{+8.09}) \times 10^{-7}$$

$$\chi^2/\text{dof} = 0.834$$

2D Center Line Fit



$$30 \leq t_1 \leq 36$$

$$30 \leq t_2 \leq 36$$

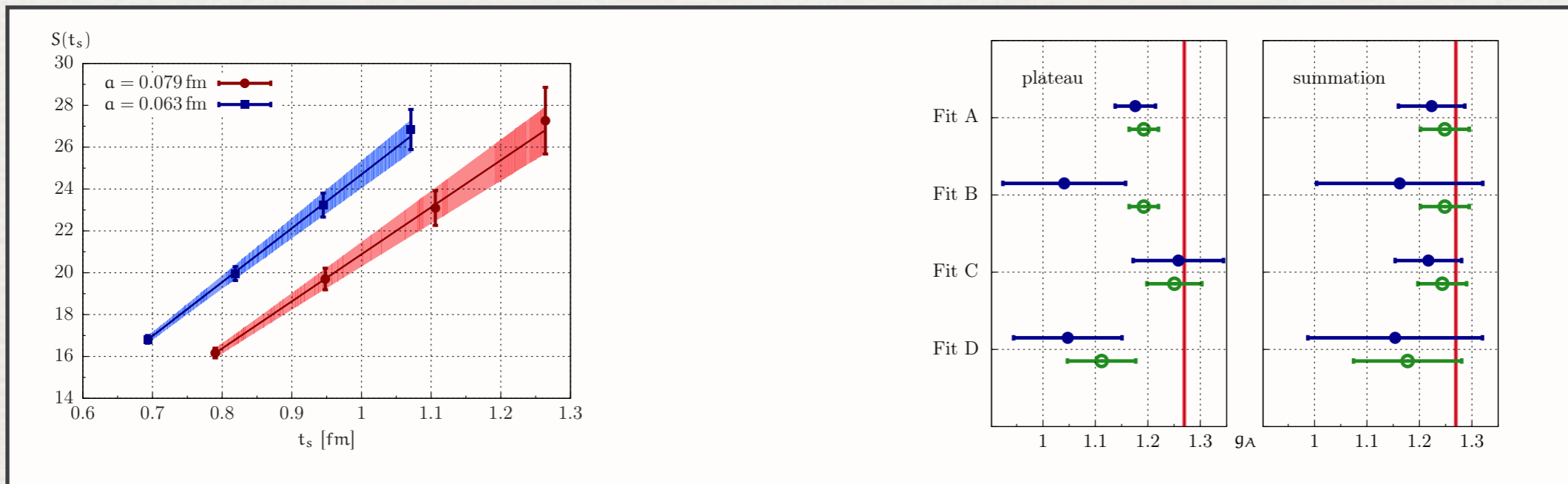
$$a_s^6 \langle N | \mathcal{O}_{RRR}^1 | \bar{N} \rangle = (4.07 \pm 6.57_{-3.13}^{+3.34}) \times 10^{-7}$$

$$\chi^2/\text{dof} = 1.32$$

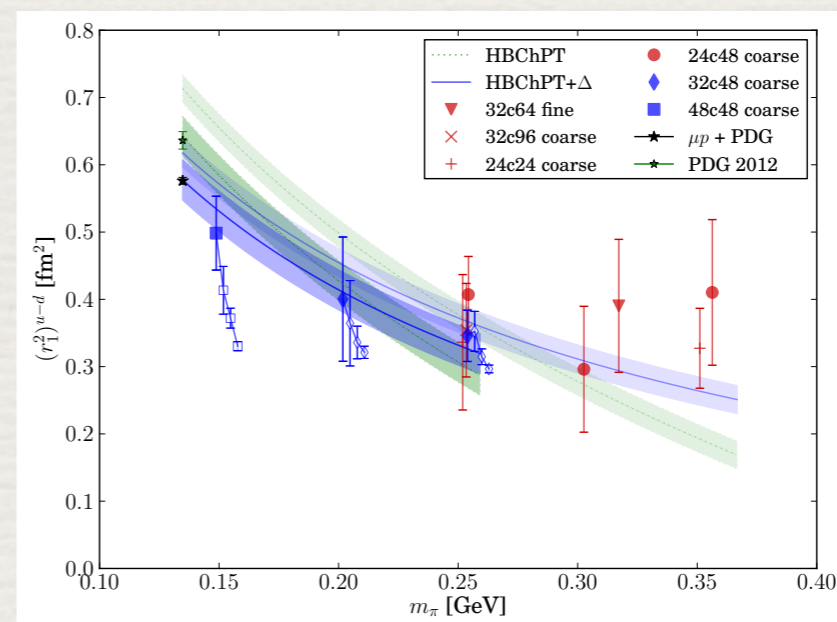
Summation Method

$$S(t_s) = \sum_{t=0}^{t_s} \mathcal{R}(t, t_s - t) \xrightarrow{t_s \gg 0} c + t_s \langle N | \mathcal{O} | \bar{N} \rangle$$

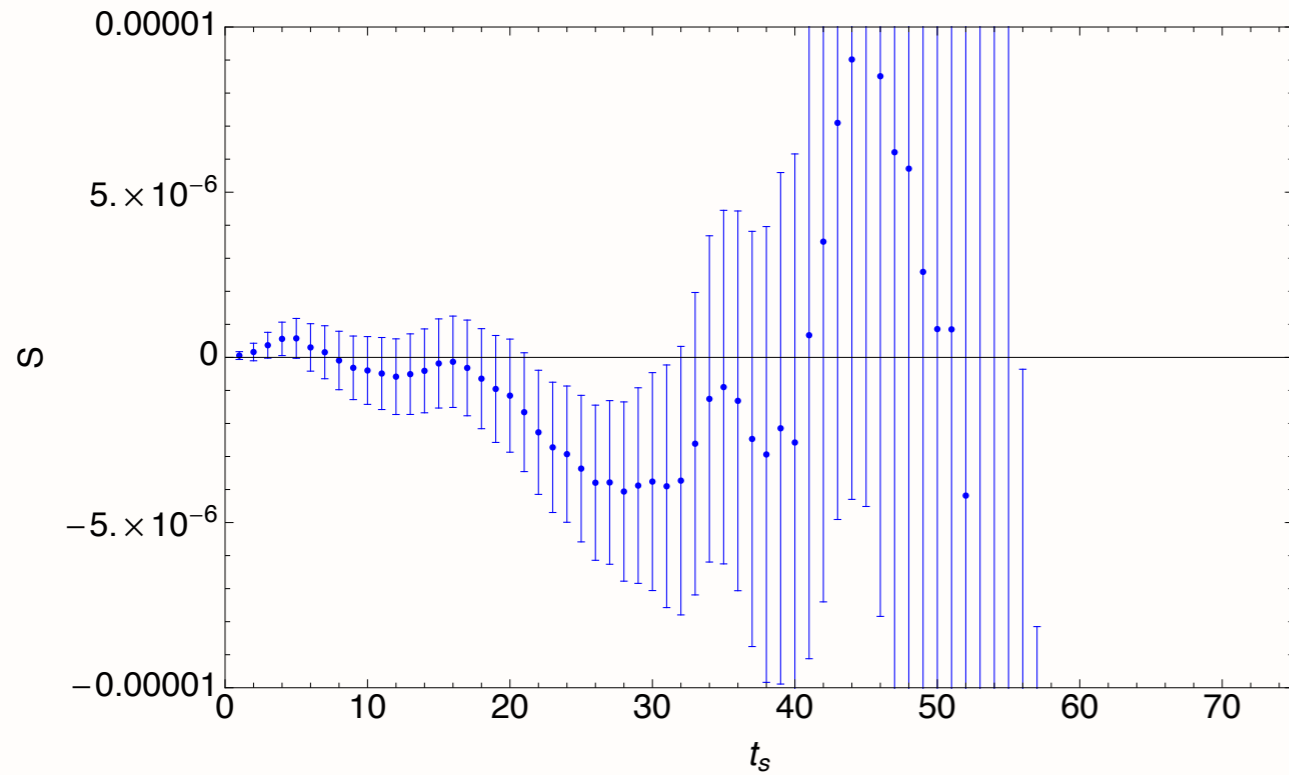
Capitani et al., 2012, arXiv:1205.0180



Green et al., 2012,
arXiv:1205.0180



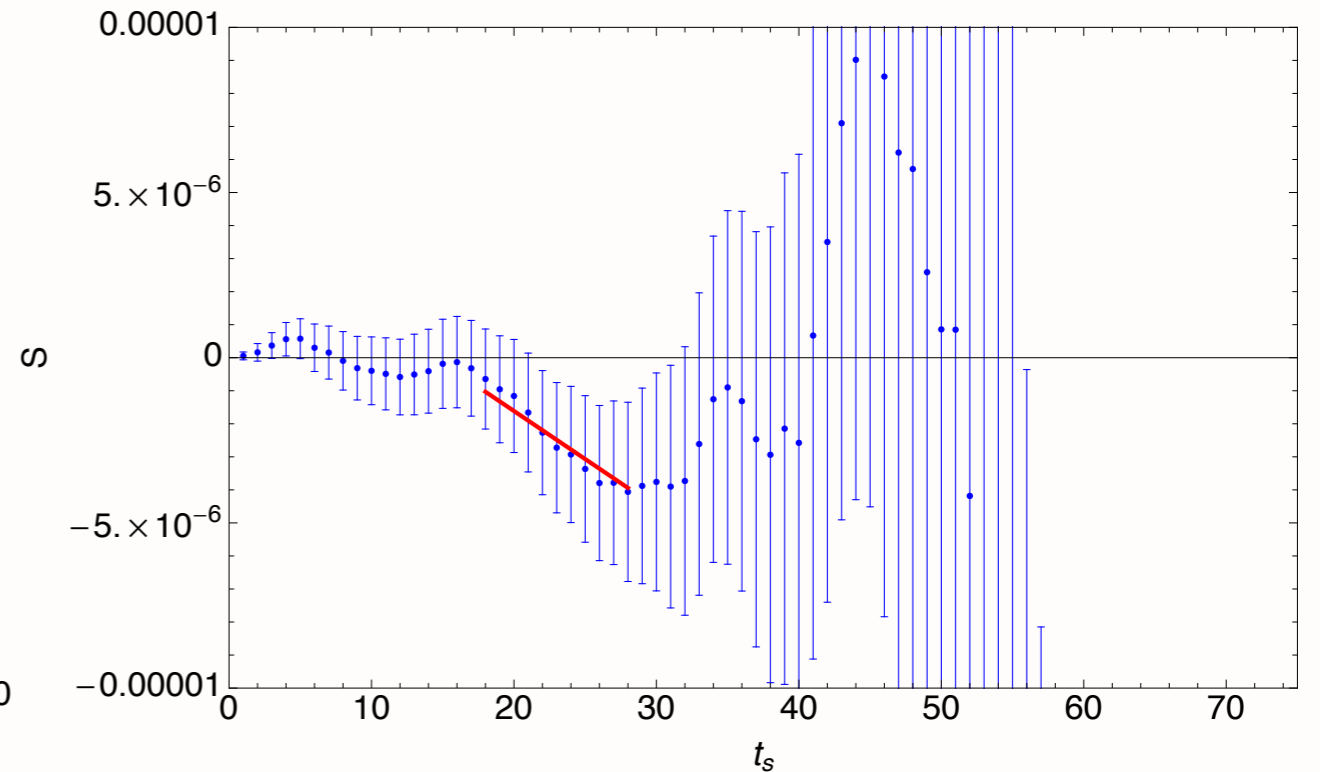
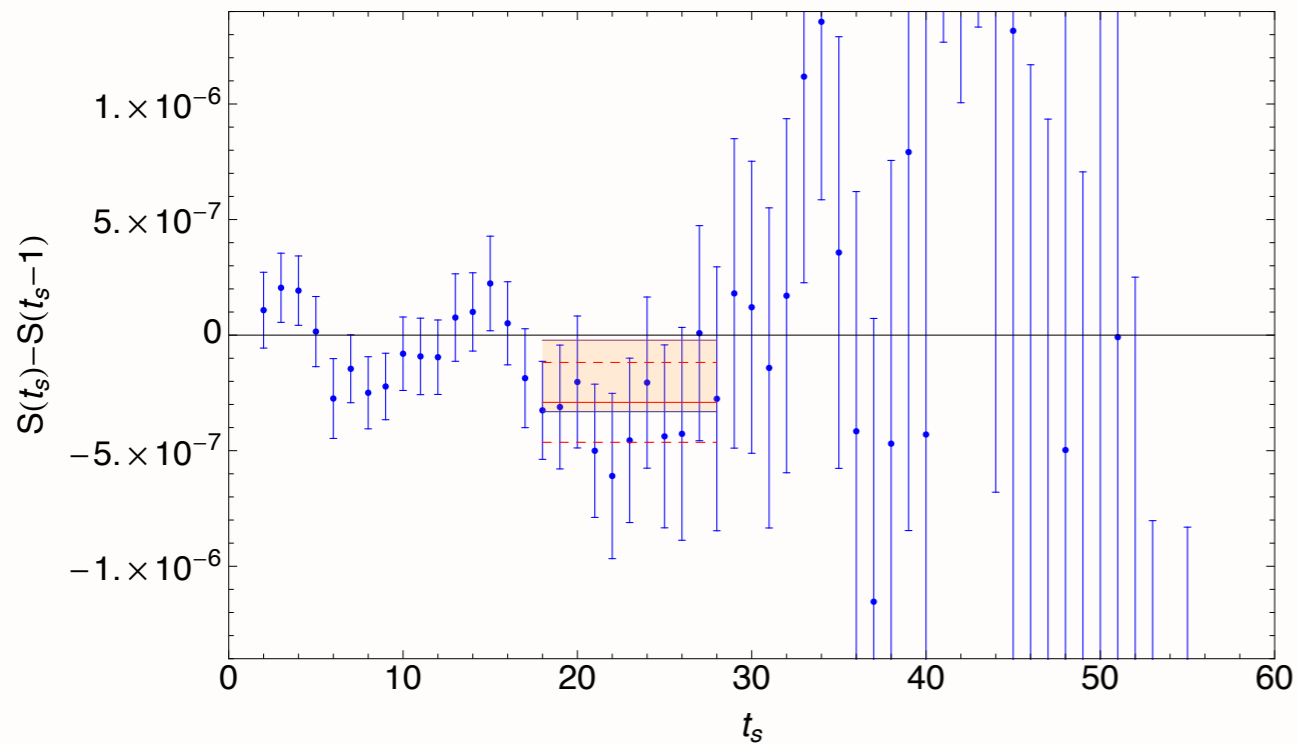
Summation Method



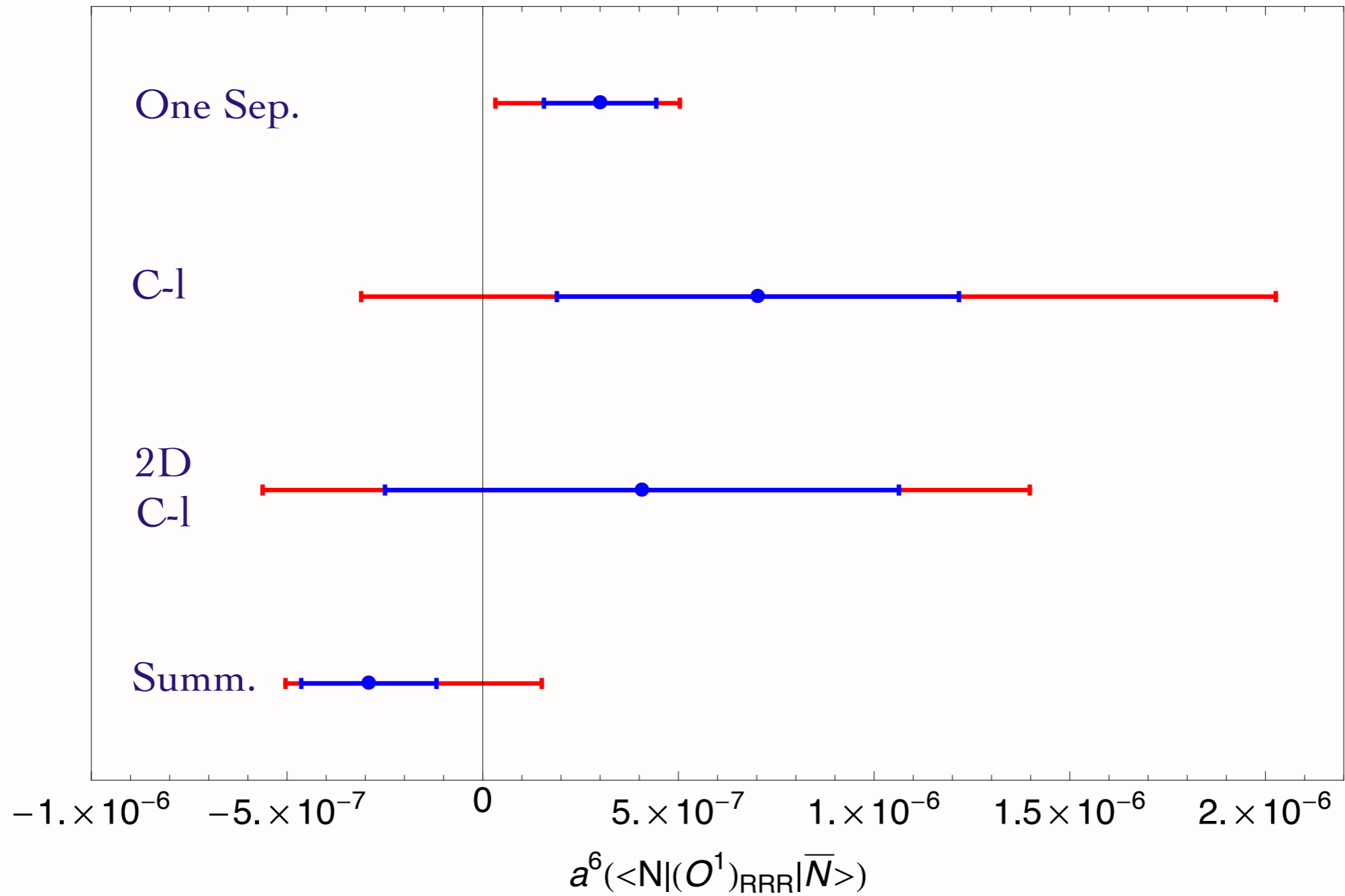
$$a_s^6 \langle N | \mathcal{O}_{RRR}^1 | \bar{N} \rangle =$$

$$(-2.91 \pm 1.73^{+2.69}_{-0.40}) \times 10^{-7}$$

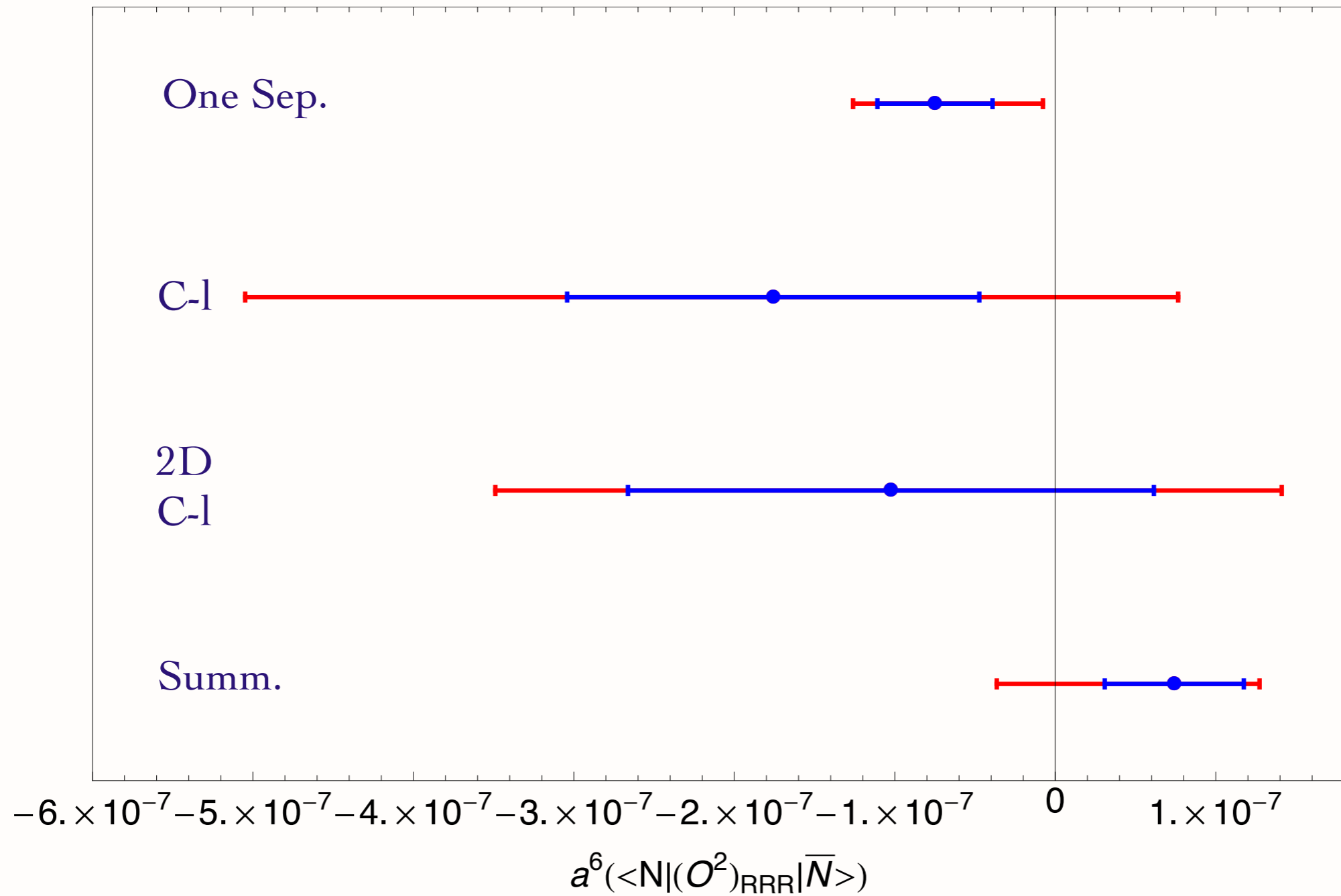
$$\chi^2/\text{dof} = 0.792$$



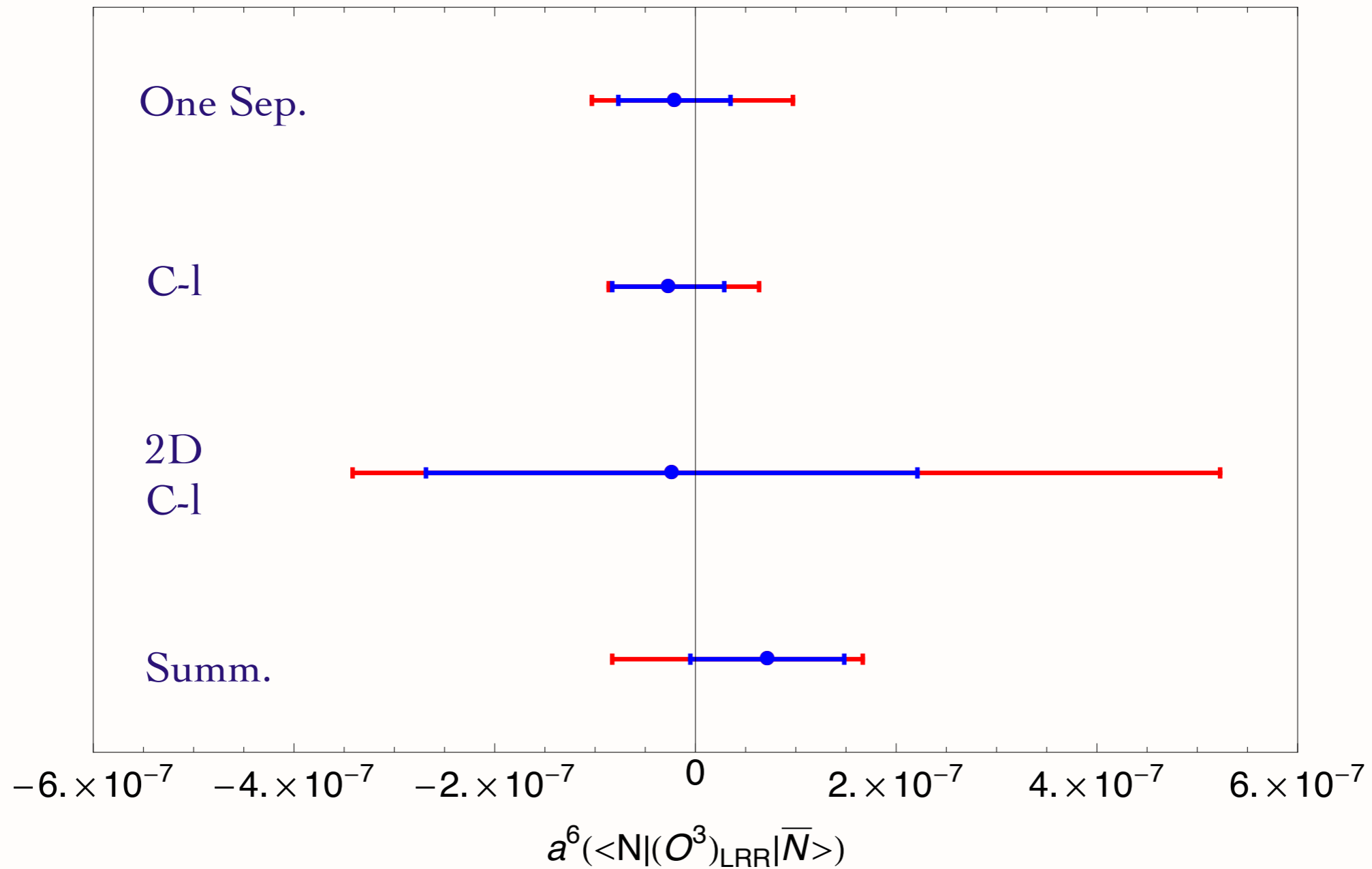
All results for \mathcal{O}_{RRR}^1



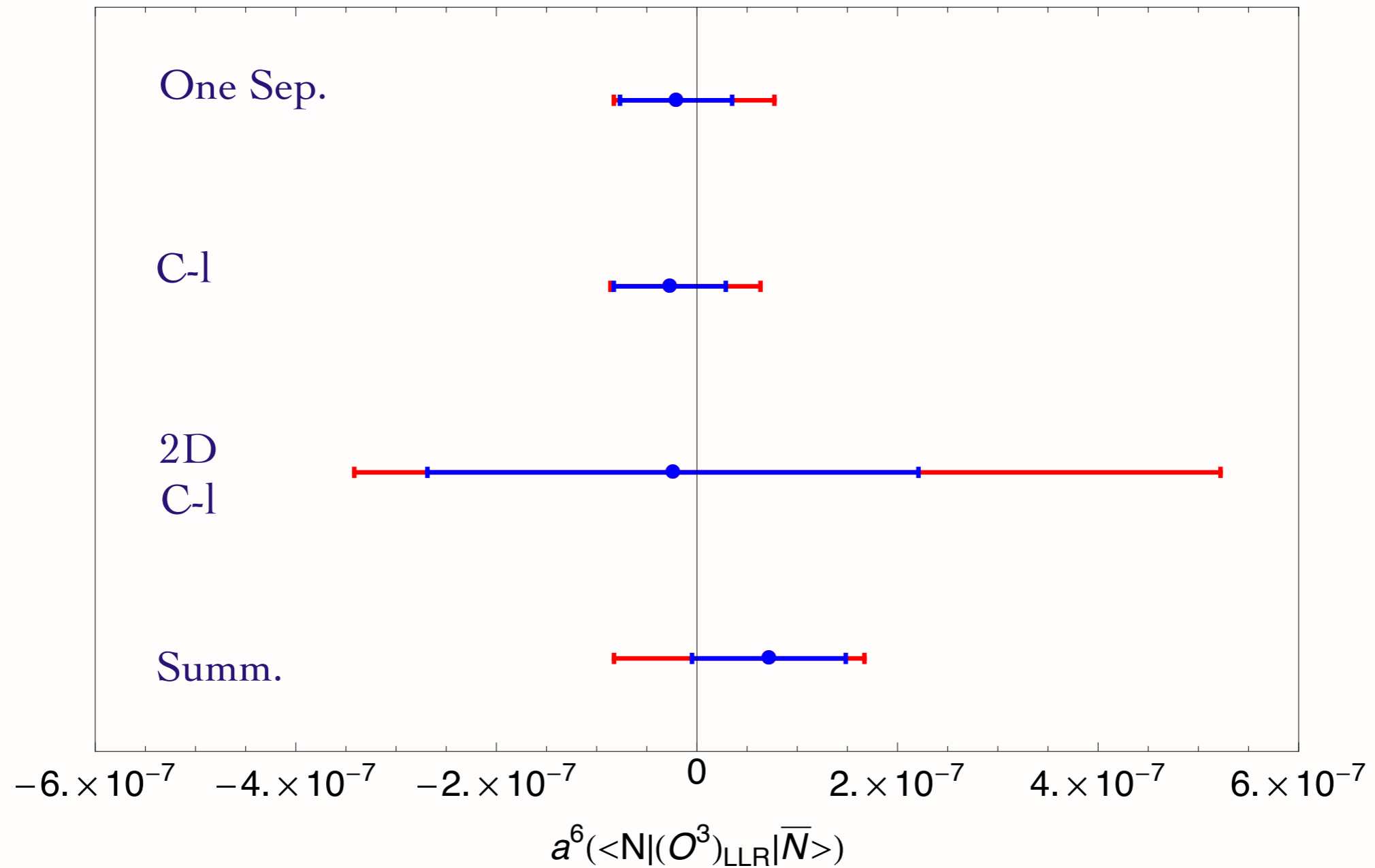
All results for \mathcal{O}_{RRR}^2



All results for \mathcal{O}_{LRR}^3



All results for \mathcal{O}_{LLR}^3



Preliminary Results

	Lattice (Bare)	MIT Bag Model
$\langle \bar{n} \mathcal{P}_1 n \rangle$	$0.81 \pm 1.30^{+0.66}_{-0.62}$	-6.56
$\langle \bar{n} \mathcal{P}_2 n \rangle$	$-0.21 \pm 0.32^{+0.15}_{-0.16}$	1.64
$\langle \bar{n} \mathcal{P}_4 n \rangle$	$-0.10 \pm 0.96^{+1.20}_{-0.28}$	-6.36
$\langle \bar{n} \mathcal{P}_5 n \rangle$	$-0.20 \pm 1.92^{+2.40}_{-0.54}$	9.64
	$\times 10^{-5} \text{ GeV}^6$	$\times 10^{-5} \text{ GeV}^6$

Still multiple important lattice systematics

Systematic Effects

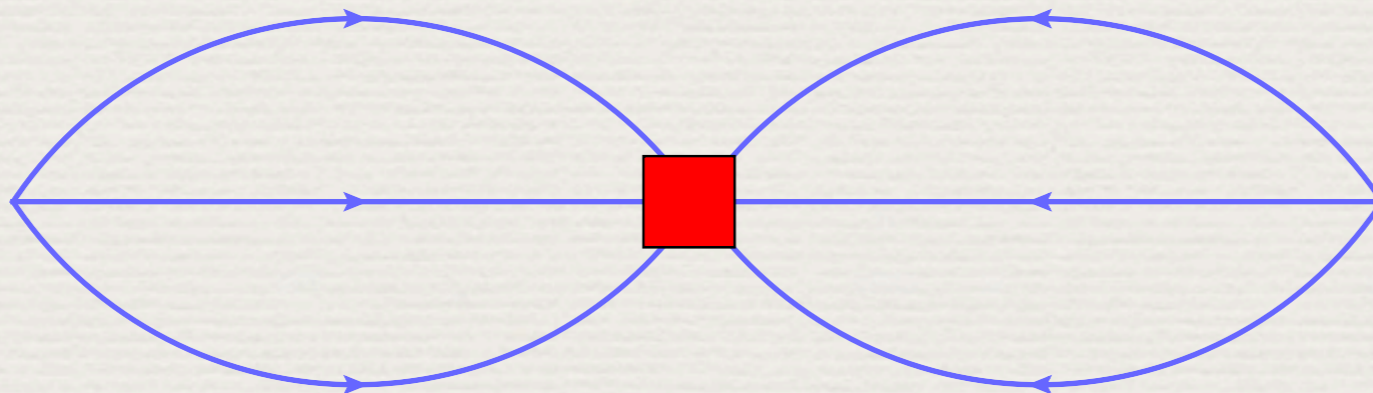
- ◆ Unphysical Pion Mass
 - No chiral extrapolation (yet...)
 - Near physical point enhancements?

$n\bar{n}$:

$$\tau = -t_1$$

$$\tau = 0$$

$$\tau = t_2$$



Systematic Effects

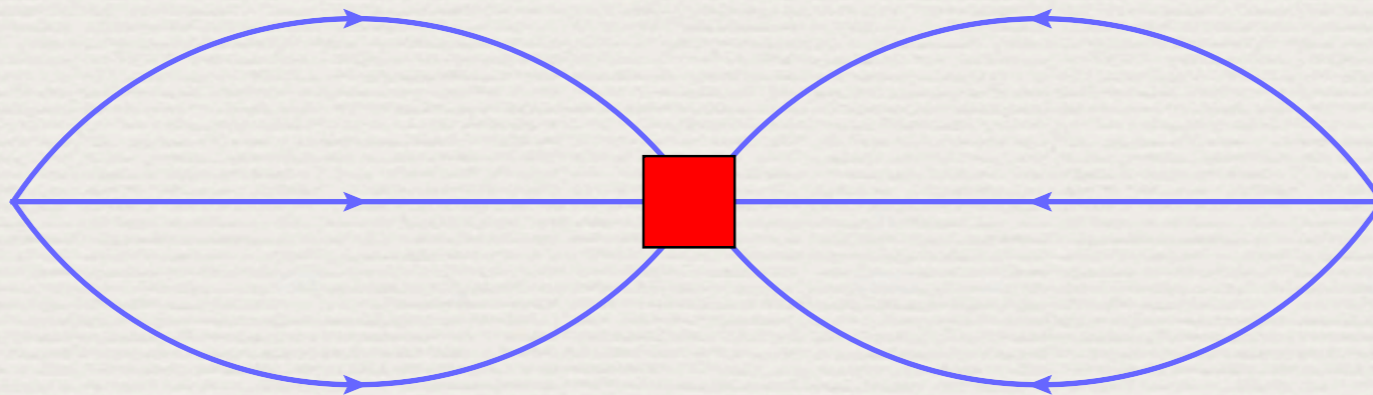
- ◆ Unphysical Pion Mass
 - No chiral extrapolation (yet...)
 - Near physical point enhancements?

$n\bar{n}$:

$$\tau = -t_1$$

$$\tau = 0$$

$$\tau = t_2$$



nn scattering:

$$\tau = t_2$$

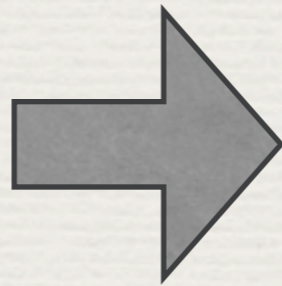
$$\tau = 0$$

$$\tau = t_2$$

Other Systematic Effects

- ♦ Renormalization/discretization effects
 - Most violent case should not occur
No lower dimensional $\Delta B = 2$ operator
 - Perturbative and non-perturbative renormalization needed

Six-quark operator



Potentially larger
Z-factor

Future Outlook

Currently in progress:

- ◆ Independent analysis checks
- ◆ Lattice Renormalization
- ◆ Systematic way to separate excited states & noise from ground state signal

Near Future:

- ◆ More Statistics
- ◆ Chiral Extrapolation

Future Outlook

Feasible in the next few years:

- ◆ Physical Point Calculation
- ◆ Chiral Fermion Calculation
- ◆ Construct Variational Basis
- ◆ Low-mode/all-mode averaging
- ◆ Separate wall sources?



Final word

Intriguing prospects for Neutron-antineutron oscillations!

Physically:

- Can unveil new physics or provide stringent constraints
- Proposed experiments can finally probe region of interest

Lattice:

- Can rigorously pinpoint bounds from various GUT theories
 - At the same time, calculations can help address systematic effects for nucleon three-point functions

This research was supported by the LLNL LDRD “Unlocking the Universe with High Performance Computing” 10-ERD-033 and by the LLNL Multiprogrammatic and Institutional Computing program through the Tier 1 Grand Challenge award that has provided us with the large amounts of necessary computing