Lattice Perturbation Theory and *B* Physics

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Outline

- Lattice QCD and the hunt for new physics
- What is Lattice Perturbation Theory (LPT)?
- Why use LPT?
- How is LPT currently used?
    - in different lattice formulations
    - B physics applications
        - an example: heavy-light current matching
- Summary

B physics, CKM unitarity and lattice QCD

$|V_{ub}|$ from

- Semileptonic decays, $B \to \pi \ell \nu$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} \propto |V_{ub}|^2 |f_+(q^2)|^2$$

- Leptonic decays, $B \to \ell \nu$

$$\Gamma \propto |V_{ub}|^2 f_B^2$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & \begin{array}{c} |V_{ub}| \\ B\to\tau\nu \\ B\to\pi\ell\nu \end{array} \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix}$$

nonperturbative parameters

$$\langle \pi | V_\mu | B \rangle \sim f_+ \times \text{kin.} + f_0 \times \text{kin.}$$

$$\langle 0 | A_\mu | B(p) \rangle = i f_B p_\mu$$

overconstrain parameters → tensions ⇒ new physics?

B physics, CKM unitarity and lattice QCD

$|V_{cb}|$ from

- Semileptonic decays,
  $B \to D^{(*)}\ell\nu$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} \propto |V_{cb}|^2 |f_+(q^2)|^2$$

- Leptonic decays, $B_c \to \tau\nu$, ?

$$V_{\text{CKM}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & \underset{B\to D^{(*)}\ell\nu}{|V_{cb}|} \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix}$$

nonperturbative parameters

$$\langle D^{(*)}|V_\mu|B\rangle \sim f_+ \times \text{kin.} + f_0 \times \text{kin.}$$

overconstrain parameters → tensions ⇒ new physics?

B physics, CKM unitarity and lattice QCD

$|V_{td}|$ from

- Neutral $B_d - \overline{B}_d$ mixing,

$$\Delta M_d \propto |V_{td}^* V_{tb}|^2 f_{B_d}^2 \widehat{B}_d$$

$|V_{ts}|$ from

- Neutral $B_s - \overline{B}_s$ mixing,

$$\Delta M_s \propto |V_{ts}^* V_{tb}|^2 f_{B_s}^2 \widehat{B}_s$$

$$V_{\text{CKM}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \\ \langle B_d|\overline{B}_d\rangle & \langle B_s|\overline{B}_s\rangle & \end{pmatrix}$$

nonperturbative parameters

$$\left\langle \overline{B}_q^0 \right| (V-A)_\mu \, (V-A)_\mu \left| B_q^0 \right\rangle = \tfrac{8}{3} M_{B_q}^2 f_{B_q}^2 B_{B_q}$$

overconstrain parameters → tensions ⇒ new physics?

B physics, rare decays and lattice QCD

Flavor-changing Neutral Currents (FCNC) loop suppressed

- Leptonic decays, $B_{(s)} \to \mu^+\mu^-$

$$\mathcal{B}(B_{(s)} \to \mu^+\mu^-) \propto |V_{tb}V_{ts}^*| f_{B_{(s)}}^2$$

$$\mathcal{B}(B_{(s)} \to \mu^+\mu^-) \propto \Delta M_{(s)} \widehat{B}_{B_{(s)}}^{-1}$$

- Semileptonic decays, $B \to K^{(*)}\mu^+\mu^-$

$$\mathcal{B}(B \to K^{(*)}\ell^+\ell^-) \sim f_0(q^2), f_+(q^2), f_T(q^2)$$

nonperturbative parameter

$$\langle K^{(*)}|T_{\mu\nu}|B\rangle \propto f_T(q^2)$$

What is (and Why) Lattice Perturbation Theory (LPT)?

Not an oxymoron. . .!

“perturbation theory for (or with) lattice actions”

Motivations:

- calculating renormalisation parameters
- matching regularisation schemes
- extracting continuum results
- improving lattice actions

account for scales above cutoff ⇒ renormalisation tool

Why can we use LPT?

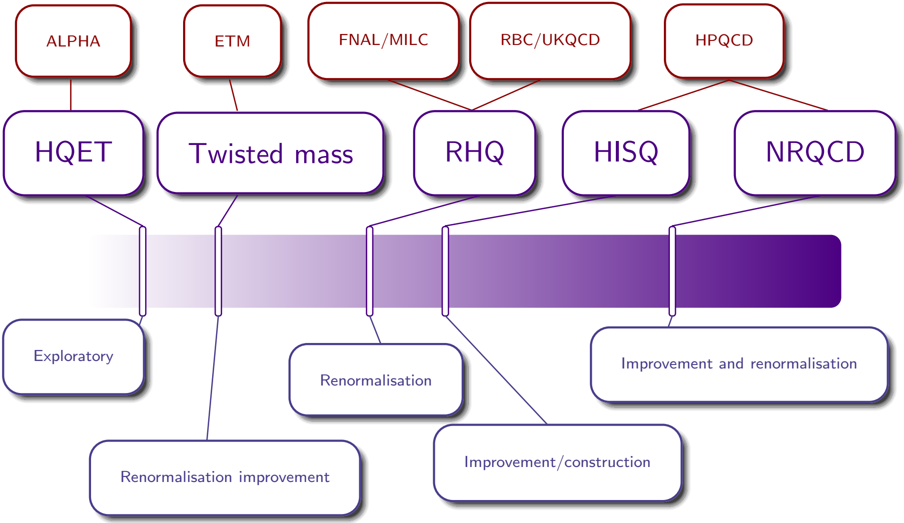
Motivation:

account for scales above cutoff ⇒ renormalisation tool

Justification [G.P. Lepage, 1996]

- lattice = regularisation scheme
  ⇒ require renormalisation to account for high energy effects
- lattice cutoff $= \pi/a \sim$ 5-6 GeV
- at these scales $\alpha_s(\pi/a) \sim 0.2$
  ⇒ perturbation theory should be valid

LPT in B physics on the lattice



ALPHA

ETM

FNAL/MILC

RBC/UKQCD

HPQCD

HQET

Twisted mass

RHQ

HISQ

NRQCD

Exploratory

Renormalisation improvement

Renormalisation

Improvement/construction

Improvement and renormalisation

LPT in *B* physics on the lattice: HQET

HQET (ALPHA Collaboration):

- suitable for heavy-light systems
- leading order

$$\mathcal{L}^{0}_{\mathrm{HQET}} = \overline{\psi}_h(x) D_0 \psi_h(x) \qquad (1)$$

- higher order terms included as operators

$$\mathcal{L}_{\mathrm{HQET}} = \mathcal{L}^{0}_{\mathrm{HQET}} - \omega_{\mathrm{kin}}\mathcal{O}_{\mathrm{kin}} - \omega_{\mathrm{spin}}\mathcal{O}_{\mathrm{spin}} \qquad (2)$$

- renormalisable
  ⇒ nonperturbative renormalisation required
- not much room for LPT . . .
  . . . but enough that automated LPT recently developed

`pastor`: automated LPT for HQET in the Schrödinger functional

[D. Hesse, 2012]

LPT in B physics on the lattice: Twisted mass

Twisted mass (ETM Collaboration):

- Relativistic quark action

$$\mathcal{L}_{\mathrm{TM}} = \overline{\psi}(x)\left(\not{D} + m_0 + i\mu_q\gamma_5\tau^3\right)\psi(x) \tag{3}$$

- Extrapolation up to physics b quark mass
- Nonperturbative renormalisation
- LPT used to improved nonperturbative determinations

LPT in *B* physics on the lattice: RHQ

Relativistic heavy quarks:

- Based on Symanzik effective theory approach
- Includes small $m_q a$ and large $m_q/\Lambda_{\mathrm{QCD}}$ interactions
  - $m_q a \to 0$: $\mathcal{O}(a)$-improved clover action
  - $m_q \gg \Lambda_{\mathrm{QCD}}$: (anisotropic) nonrelativistic action
- 3 parameters in the action

Fermilab Lattice/MILC Collaboration

- fix 2 parameters, tune 3^{rd} nonperturbatively
- renormalisation parameteres calculated with mixed nonperturbative/LPT approach

LPT in B physics on the lattice: RHQ

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- 3 parameters in the action

RBC/UKQCD Collaboration

- tune 3 parameters nonperturbatively
- nonperturbative and LPT renormalisation methods
- automated LPT recently developed

new `Computer Algebra System` for RHQ in "Columbia" formulation

[C. Lehner, 2012]

LPT in *B* physics on the lattice: HISQ

HISQ (HPQCD Collaboration):

- Relativistic staggered quark action
- Taste-breaking interactions removed by 2 levels of smearing
- Certain operators absolutely normalised (V_μ, A_μ currents) $\Rightarrow$ no matching calculations required . . .
- . . . but LPT still used:
    - (in construction to remove taste-breaking errors at one loop)
    - certain matching parameters (4q operators, tensor current)

LPT in B physics on the lattice: NRQCD

NRQCD (HPQCD Collaboration):

- suitable for heavy-light and heavy-heavy systems
- nonrenormalisable effective theory
  $\Rightarrow$ requires $1/m_b < a < 1/\Lambda_{\mathrm{QCD}}$
- automated LPT used extensively (1 and 2 loop calculations):
  - action improvement
  - renormalisation parameters
  - matching calculations

`HiPPy/HPsrc`: for AsqTad, HISQ, NRQCD, Wilson, clover ...

[Hart *et al.*, 2006,2009]

LPT and BSM searches

Process	B physics	Lattice parameter	Role of LPT
$B \to \pi \ell \nu$	V_{ub}	FF	heavy-light V_μ
$B \to D^* \ell \nu$	V_{cb}	FF	heavy-heavy V_μ
$B_s \to \mu^+ \mu^-$	rare	f_{B_s}	heavy-light A_μ
$B_q^0 - \overline{B_q^0}$	V_{td}/V_{ts}	ξ	$Q\overline{Q}q\overline{q}$ matching
$B \to \tau \nu$	V_{ub}	f_B	heavy-light A_μ
$B \to D \ell \nu$	V_{cb}	FF	heavy-heavy V_μ
$B \to K \ell^+ \ell^-$	rare	FF	heavy-light V_μ
$B \to K^* \gamma$	rare	FF	heavy-light T_μ
$B_c \to \tau \nu$	V_{cb}	f_{B_c}	heavy-heavy A_μ

LPT and BSM searches: heavy-light currents

Semileptonic decays: $B \to \pi \ell \nu$

- $\langle \pi | V_\mu | B \rangle$ parameterised by form factors f_+, f_0
- $m_\ell \to 0$:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} \propto |V_{ub}|^2 |f_+(q^2)|^2$$

Leptonic decays: $B_{(s)} \to \ell \nu$

- $\langle 0 | A_\mu | B_{(s)} \rangle$ parameterised by decay constant $f_{B_{(s)}}$

$$\Gamma \propto |V_{ub}|^2 f_B^2$$

Both require heavy-light current renormalisation

LPT and BSM searches: heavy-heavy currents

Semileptonic decays: $B \to D^{(*)}\ell\nu$

- $\langle D^{(*)}|V_\mu|B\rangle$ parameterised by form factors f_+, f_0
- $m_\ell \to 0$:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} \propto |V_{cb}|^2|f_+(q^2)|^2$$

Leptonic decays: $B_c \to \ell\nu$

- $\langle 0|A_\mu|B_c\rangle$ parameterised by decay constant f_{B_c}

$$\Gamma \propto |V_{cb}|^2 f_{B_c}^2$$

Both require heavy-heavy current renormalisation

LPT example: heavy-light currents

Simulations carried out with

- NRQCD *b* quarks
- HISQ light quarks
- effective lattice NRQCD operators

$$J_0^{(0)} = \overline{\Psi}_q \Gamma_0 \Psi_Q \qquad J_0^{(1)}(x) = -\frac{1}{2M_b}\overline{\Psi}_q \Gamma_0 \gamma \cdot \overrightarrow{\nabla} \Psi_Q$$

$$J_0^{(2)}(x) = -\frac{1}{2M_b}\overline{\Psi}_q \gamma \cdot \overleftarrow{\nabla} \gamma_0 \Gamma_0 \Psi_Q$$

Matching relation:

$$\langle A_0 \rangle_{QCD} = (1 + \alpha_s \rho_0) \langle J_0^{(0)} \rangle + (1 + \alpha_s \rho_1) \langle \widetilde{J_0}^{(1)} \rangle + \alpha_s \rho_2 \langle \widetilde{J_0}^{(2)} \rangle$$

Use improved currents with better power law behaviour.

$$\widetilde{J_0}^{(i)} = J_0^{(i)} - \alpha_s \zeta_{10} J_0^{(0)}$$

Matching coefficients:

$$\rho_0 = B_0 - \frac{1}{2} \underbrace{(Z_H + Z_q)}_{\text{wavefn. renorm.}} - \zeta_{00}$$

mixing matrix elements

$$\rho_1 = B_1 - \frac{1}{2} \overbrace{(Z_H + Z_q)} - Z_M - \overbrace{\zeta_{01} - \zeta_{11}}$$

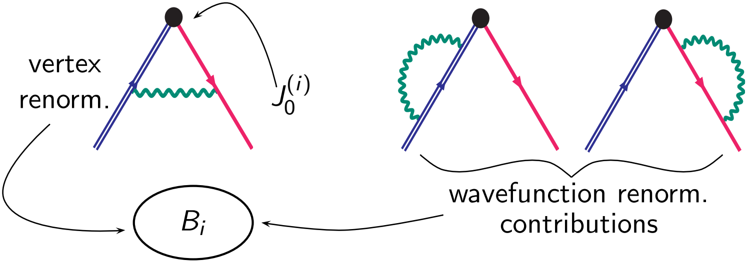
cont. contributions

mass. renorm.

$$\rho_2 = B_2 - \underbrace{\zeta_{02} - \zeta_{12}}$$

Calculating matching coefficients

Diagrams for continuum $\langle A_0 \rangle_{QCD}$:



vertex renorm.

$J_0^{(i)}$

wavefunction renorm. contributions

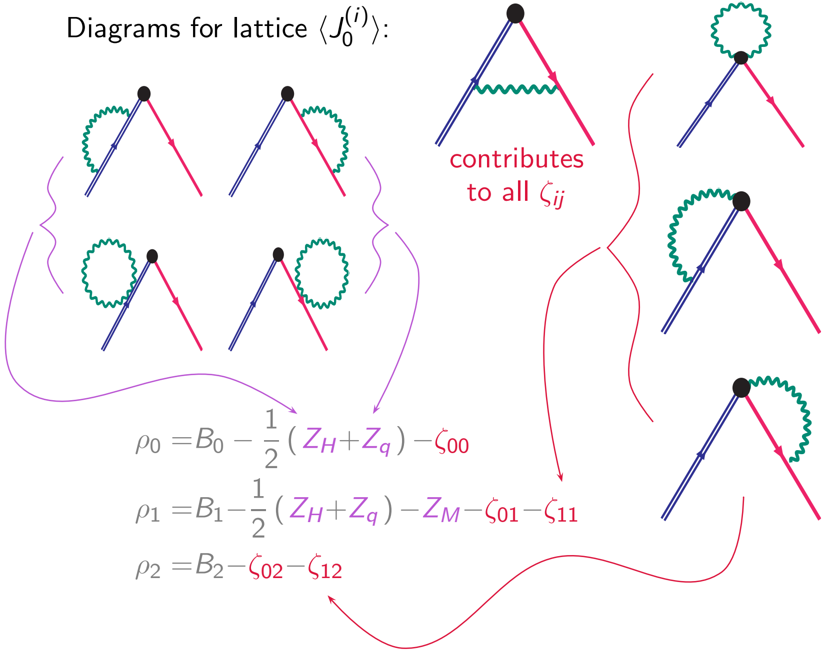
B_i

$$\rho_0 = B_0 - \frac{1}{2}\left(Z_H + Z_q\right) - \zeta_{00}$$

$$\rho_1 = B_1 - \frac{1}{2}\left(Z_H + Z_q\right) - Z_M - \zeta_{01} - \zeta_{11}$$

$$\rho_2 = B_2 - \zeta_{02} - \zeta_{12}$$

Diagrams for lattice $\langle J_0^{(i)} \rangle$:



contributes to all ζ_{ij}

$$\rho_0 = B_0 - \frac{1}{2}\left(Z_H + Z_q \right) - \zeta_{00}$$

$$\rho_1 = B_1 - \frac{1}{2}\left(Z_H + Z_q \right) - Z_M - \zeta_{01} - \zeta_{11}$$

$$\rho_2 = B_2 - \zeta_{02} - \zeta_{12}$$

Results

aM_b	ζ_{00}	ζ_{10}	ρ_0	ρ_1	ρ_2
2.688	0.721(1)	-0.1144(3)	-0.109(1)	0.014(2)	-0.712(5)
2.650	0.725(1)	-0.1157(3)	-0.113(1)	0.013(2)	-0.699(4)
1.832	0.802(1)	-0.1595(3)	-0.164(1)	-0.039(2)	-0.314(4)
1.826	0.804(1)	-0.1595(3)	-0.166(1)	-0.040(2)	-0.312(4)

(More interesting) results

We find

Na, 2012

$$f_B = 0.191(9)\,\mathrm{GeV} \qquad \text{and} \qquad f_{B_s} = 0.228(10)\,\mathrm{GeV}$$

so

$$\frac{f_{B_s}}{f_B} = 1.188(18)$$

Agreement with previous HPQCD HISQ result a non-trivial consistency check: $f_{B_s}^{(HISQ)} = 0.225(4)\,\mathrm{GeV}$

Combining NRQCD-HISQ ratio with HISQ $f_{B_s}^{(HISQ)}$

$$\boxed{f_B = \frac{f_B}{f_{B_s}} \times f_{B_s}^{(HISQ)} = 0.189(4)\,\mathrm{GeV}}$$

Summary

- B physics important for BSM physics searches:
  - rare decays
  - CKM unitarity
- Lattice QCD vital in the hunt for precision
- Spectrum of applications of lattice perturbation theory by major collaborations in precision B physics

Thank you!

Fits and correlators

- Delta function and Gaussian smearing used at both source and sink for meson correlators
- Random wall sources in operator-meson correlators
- Correlators fitted between
    - $t_{\rm min} = 2 \sim 4$ and $t_{\rm max} = 16$ on coarse ensembles
    - $t_{\rm min} = 4 \sim 8$ and $t_{\rm max} = 24$ on fine ensembles
- Bayesian multiexponential fits with $t_{\rm min}$, $t_{\rm max}$ fixed and no. exponentials increased until saturation in results

Chiral and lattice spacing fits

Fit to lattice spacing dependence as described by Rachel Dowdall, but include chiral fit.

- Fit to

$$\Phi_q = f_{B_q}\sqrt{M_{B_q}} = \Phi_0\left(1 + \delta f_q + [\text{analytic}]\right)\left(1 + [\text{disc.}]\right)$$

- δf_q includes chiral logs using one-loop χPT and lowest order in $1/M$
- [analytic] - powers of m_{val}/m_c and m_{sea}/m_c, with m_c scale chosen for convenience
- [disc.] - powers of $(a/r_1)^2$ with expansion coefficient functions of aM_b or am_q

ASQTad action correct to $\mathcal{O}(a^2)$, strongly reduced $\mathcal{O}(\alpha_S a^2)$ errors:

$$S_{\mathrm{ASQTad}} = \sum_x \overline{\psi}(x) \left(\gamma^\mu \Delta_\mu^{\mathrm{ASQTad}} + m\right) \psi(x)$$

where

$$\Delta_\mu^{\mathrm{ASQTad}} = \Delta_\mu^F - \frac{1}{6}(\Delta_\mu)^3.$$

F indicates

$$U_\mu \to \mathcal{F}_\mu \widetilde{U}_\mu = u_0^{-1} \left[\prod_{\nu\neq\mu}\left(1 + \frac{\Delta_\nu^{(2)}}{4}\right)_{\mathrm{symm}} - \sum_{\nu\neq\mu}\frac{(\Delta_\nu)^2}{4}\right] U_\mu$$

HISQ action correct to $\mathcal{O}(a^4)$, $\mathcal{O}(\alpha_S a^2)$ with reduced taste=changing:

$$S_{\mathrm{HISQ}} = \sum_x \overline{\psi}(x) \left(\gamma^\mu \Delta_\mu^{\mathrm{HISQ}} + m\right) \psi(x)$$

where

$$\Delta_\mu^{\mathrm{HISQ}} = \Delta_\mu \left[\mathcal{F}_\mu^{\mathrm{HISQ}} U_\mu(x)\right] - \frac{1+\epsilon}{6}(\Delta_\mu)^3 \left[U\mathcal{F}_\mu^{\mathrm{HISQ}} U_\mu(x)\right].$$

and

$$\mathcal{F}_\mu^{\mathrm{HISQ}} = \mathcal{F}_\mu^{\mathrm{ASQTad}} U_\mu \mathcal{F}_\mu^{\mathrm{ASQTad}}$$

Lattice NRQCD action

$$S_{\mathrm{NRQCD}} = \sum_{\mathbf{x},\tau} \psi^{+}(\mathbf{x},\tau)\left[\psi(\mathbf{x},\tau) - \kappa(\tau)\psi(\mathbf{x},\tau-1)\right]$$

with

$$\kappa(\tau) = \left(1 - \frac{\delta H}{2}\right)\left(1 - \frac{H_0}{2n}\right)^{n} U_4^{\dagger}\left(1 - \frac{H_0}{2n}\right)^{n}\left(1 - \frac{\delta H}{2}\right)$$

- Link variable in temporal direction: $U_4^{\dagger}$
- Leading nonrelativistic kinetic energy: $H_0 = -\Delta^{(2)}/2M$
- Higher order terms in δH:
  - Chromoelectric and chromomagnetic interactions
  - Leading relativistic kinetic energy correction
  - Discretisation error corrections

Automated LPT: HiPPy

`HiPPy` generates Feynman rules, encoded as “vertex files”
To generate vertex files:

- Expand link variables

Lüscher and Weisz, NPB 266 (1986) 309

$$U_{\mu>0}(x) = \exp\left(gA_\mu\left(x+\frac{\hat{\mu}}{2}\right)\right) = \sum_{r=0}^{\infty}\frac{1}{r!}\left(gA_\mu\left(x+\frac{\hat{\mu}}{2}\right)\right)^r$$

with $U_{-\mu} \equiv U_\mu^\dagger(x-\hat{\mu})$

- Actions built from products of link variables - Wilson lines

$$L(x,y;U) = \sum_r \left(\frac{g^r}{r!}\right) \sum_{k_1,\mu_1,a_1} \cdots \sum_{k_r,\mu_r,a_r} \widetilde{A}_{\mu_1}^{a_1}(k_1)\cdots\widetilde{A}_{\mu_r}^{a_r}(k_r) \\ \times V_r(k_1,\mu_1,a_1;\ldots;k_r,\mu_r,a_r)$$

where the V_r are “vertex functions”

- Vertex functions decomposed into colour structure matrix, C_r and "reduced vertex", Y_r

$$V_r(k_1,\mu_1,a_1;\ldots;k_r,\mu_r,a_r) = C_r(a_1;\ldots;a_r)Y_r(k_1,\mu_1;\ldots;k_r,\mu_r)$$

- Reduced vertices are products of exponentials

$$Y_r(k_1,\mu_1;\ldots;k_r,\mu_r) = \sum_{n=1}^{n_r} f_n \exp\left(\frac{i}{2}\left(k_1\cdot v_1^{(n)} + \cdots + k_r\cdot v_r^{(n)}\right)\right)$$

where the f_n are amplitudes and the $v^{(n)}$ the locations of each of the r factors of the gauge potential

- Feynman rules encoded as ordered lists

$$E = (\mu_1,\cdots,\mu_r;x,y;v_1,\cdots,v_r;f)$$

For example, the product of two links, $L(0, 2x, U) = U_x(0)U_x(x)$, is

$$\begin{aligned}
U_x(0)U_x(x) = & \left[\sum_{r_1=0}^{\infty} \frac{1}{r_1!} \left(gA_x\left(\frac{x}{2}\right)\right)^{r_1}\right] \left[\sum_{r_2=0}^{\infty} \frac{1}{r_2!} \left(gA_x\left(\frac{3x}{2}\right)\right)^{r_2}\right] \\
&= 1 + g\sum_{k_1} \widetilde{A}_x(k_1)e^{ik_1 \cdot x/2} + g\sum_{k_2} \widetilde{A}_x(k_2)e^{i2_1 \cdot 3x/2} + \dots \\
&= 1 + g\sum_{k_1}\sum_{a_1} \widetilde{A}_x^{a_1}(k_1)T^{a_1}\left(e^{ik_1 \cdot x/2} + e^{ik_1 \cdot 3x/2}\right)
\end{aligned}$$

Vertex function

$$V_1(k_1, x, a_1) \equiv C_1(a_1)Y_1(k_1, x) = T^{a_1}\left(e^{ik_1 \cdot x/2} + e^{ik_1 \cdot 3x/2}\right)$$

Reduced vertex

$$Y_1(k_1, x) = \left(e^{ik_1 \cdot x/2} + e^{ik_1 \cdot 3x/2}\right)$$

Reduced vertex

$$Y_1(k_1, x) = \sum_{n=1}^{n_1=2} f_n \exp\left(\frac{i}{2}\left(k_1 \cdot v_1^{(n)}\right)\right)$$

So in this case

$$f_1 = f_2 = 1\ ;\ v_1^{(1)} = (1,0,0,0)\ ,\ v_1^{(2)} = (3,0,0,0)$$

We store this information as the list

$$\begin{aligned} E &= (\mu_1; x, y; v_1^{(1)}, v_1^{(2)}; f) \\ &= (x; (0,0,0,0), (2,0,0,0); (1,0,0,0), (3,0,0,0); (1,1)) \end{aligned}$$