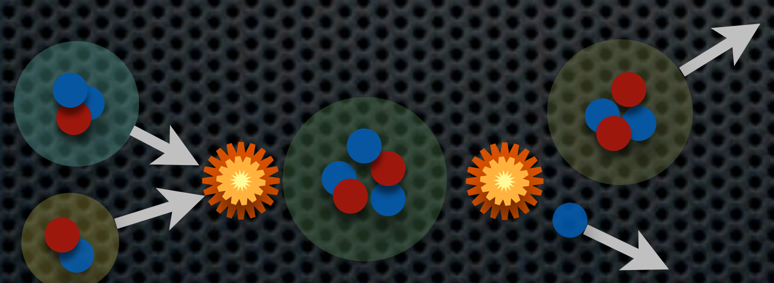
MIT Lattice Club
Nov. 21, 2012

Towards Nuclear Reactions from Lattice QCD

Raúl Briceño

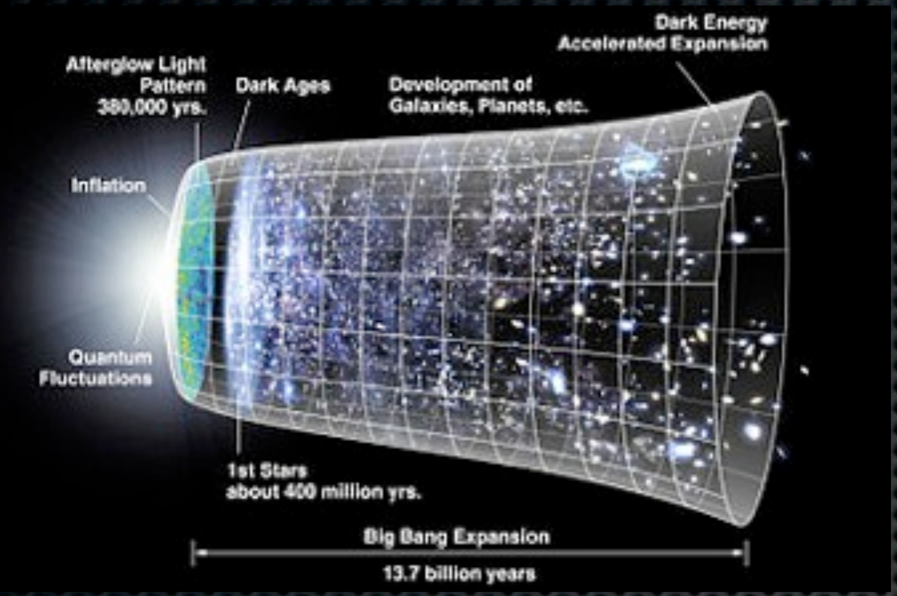
(in collaboration with Zohreh Davoudi)



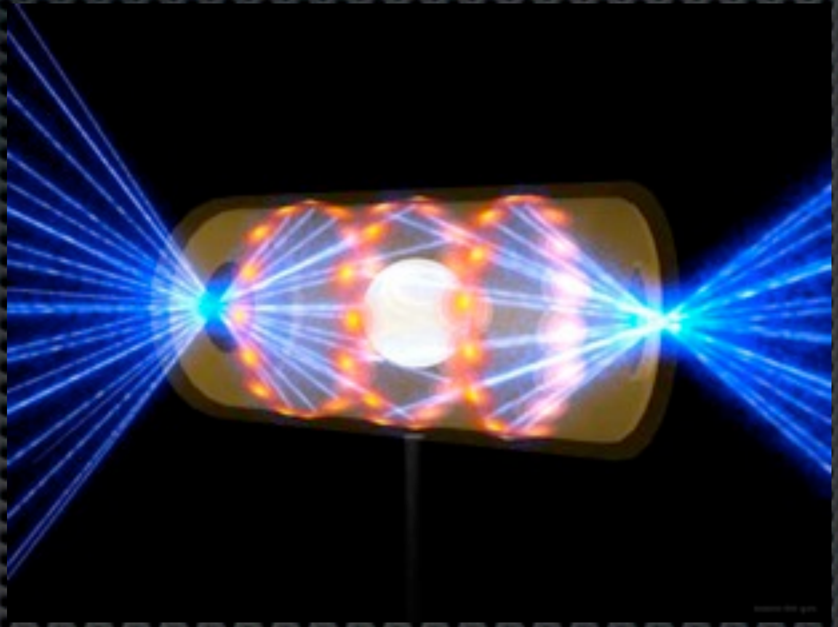
$$ {}^2H +{}^3 H \to {}^5He^* \to {}^4He + n $$

Paving the Road From QCD to Nuclear Reactions

Big Bang Nucleosynthesis

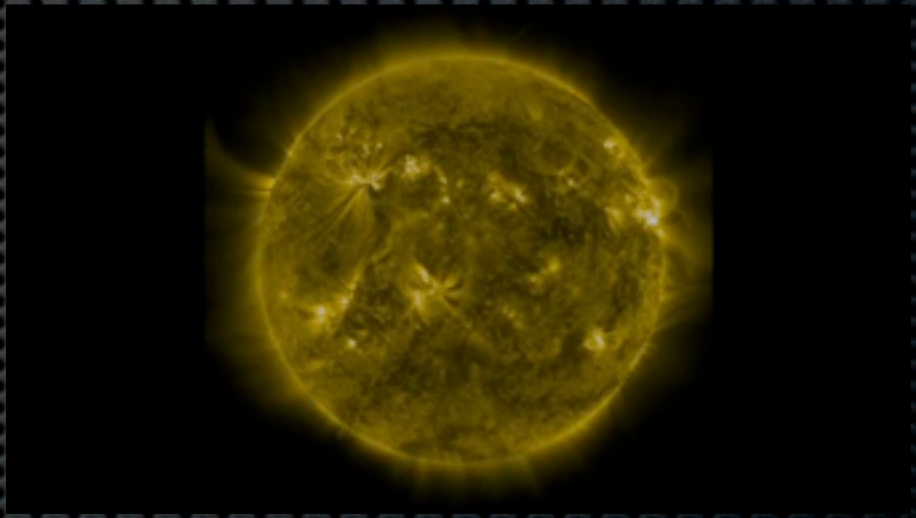


Fusion



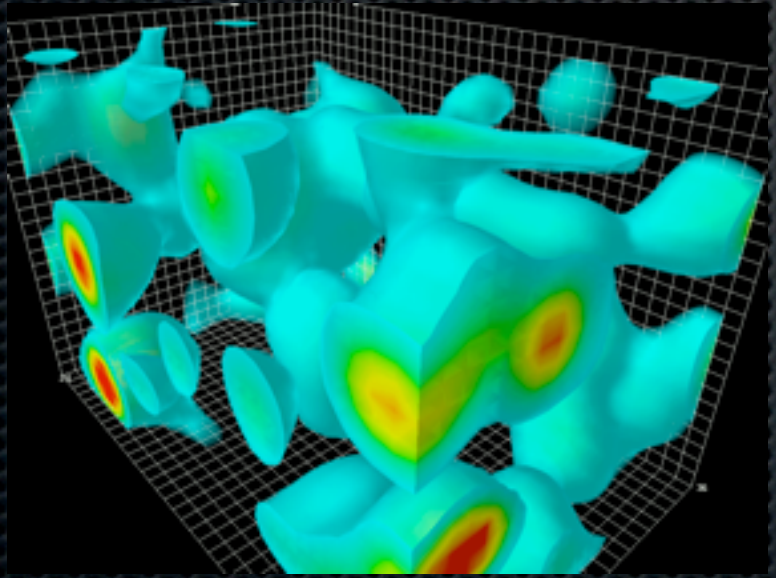
Inertial Confinement Fusion Target (NIF)

Stars, Supernovae, Neutron Stars



Solar Dynamics Observatory (NASA)

QCD (Vacuum)

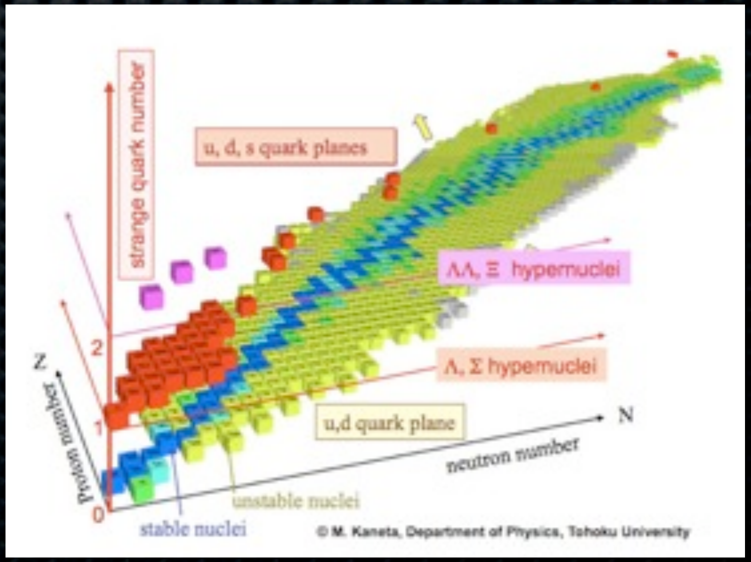


Leinweber

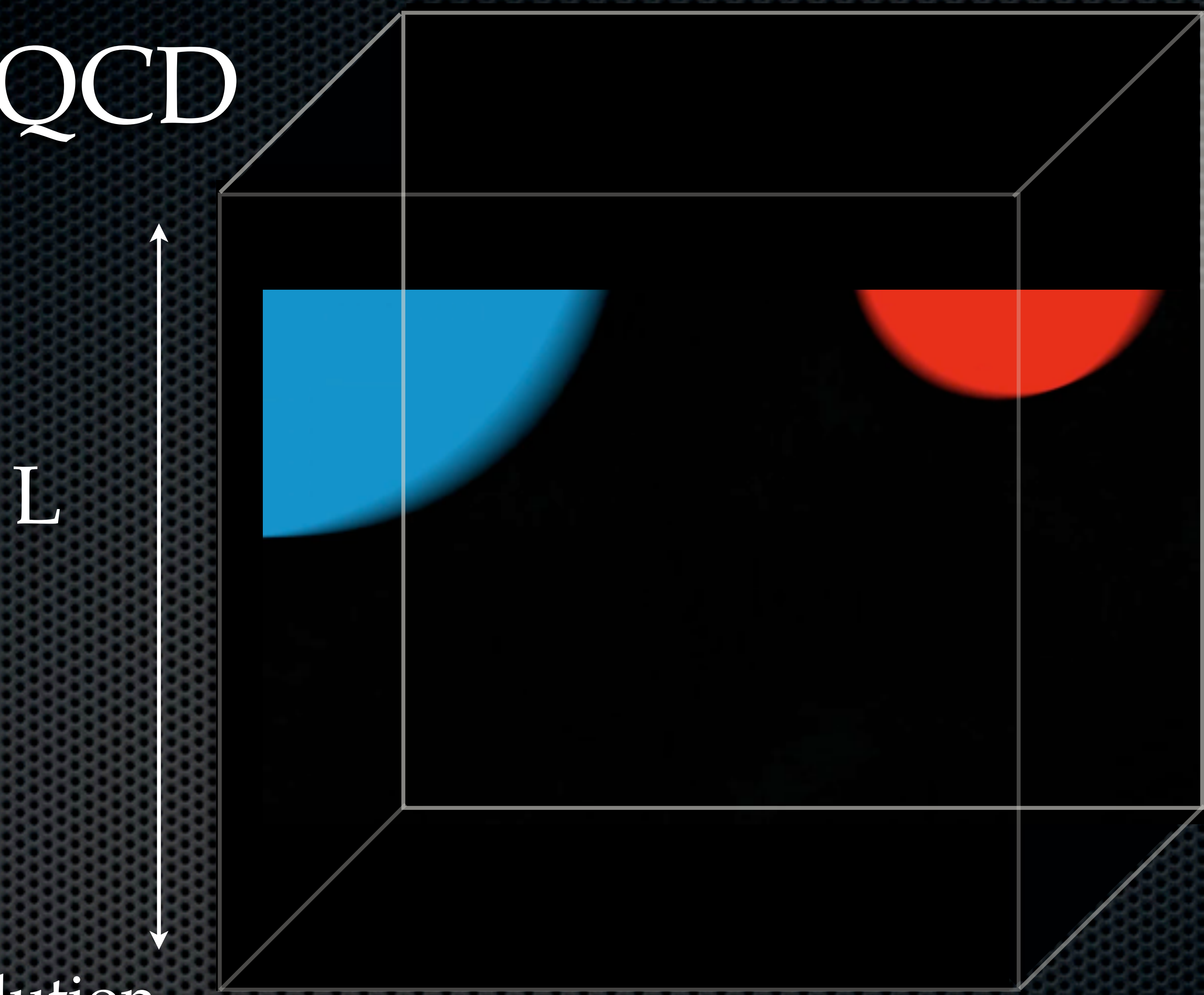
LQCD

Sequoia (LLNL)

Spectrum



Lattice QCD



$a \to 0$

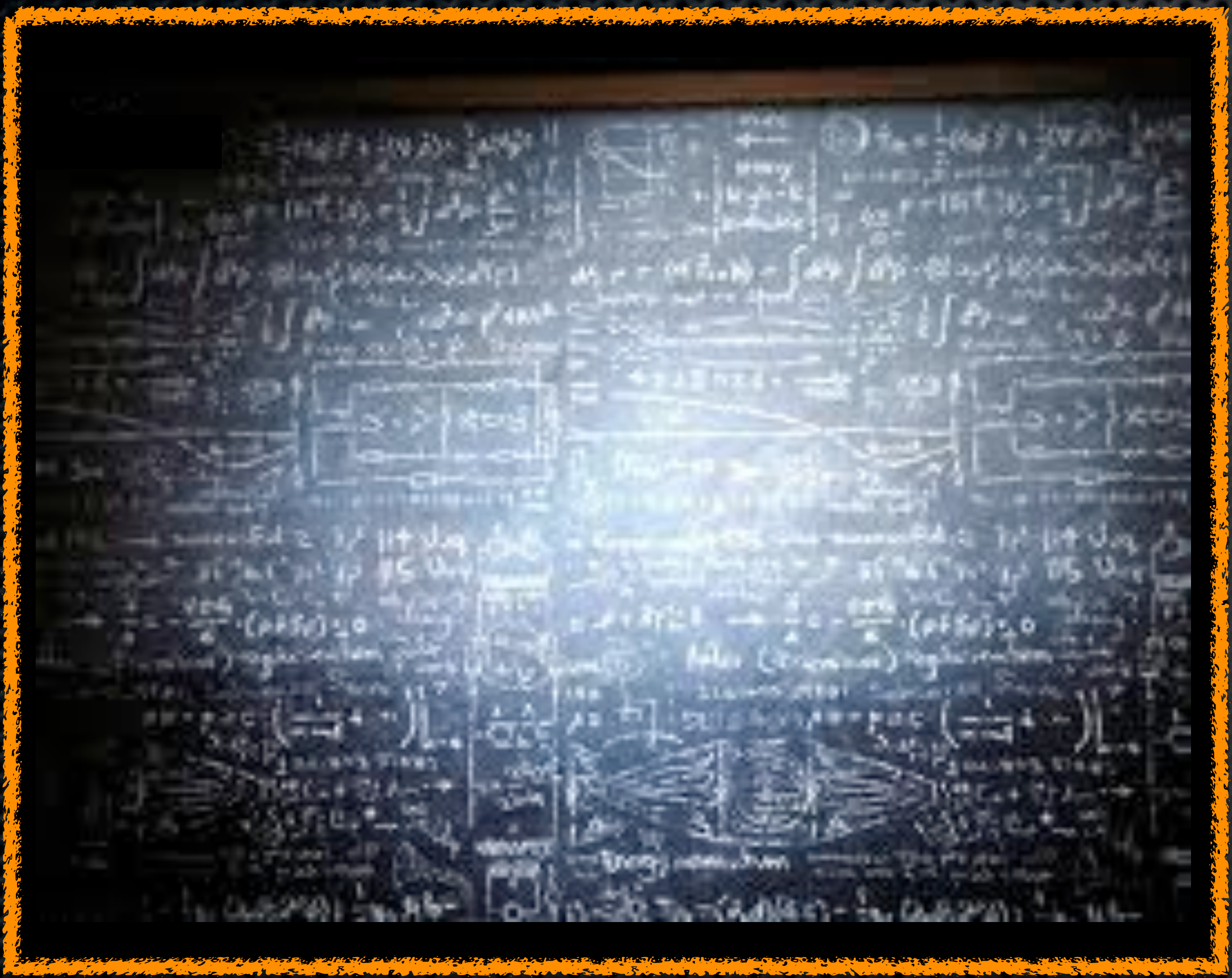
Movie by
José Rodríguez

- Numerical solution
- Finite periodic Euclidean spacetime
- Consider continuum limit
- Maiani-Testa theorem (1990)
- Lüscher (1991) : $E_L, m \to \delta(q^*)$

Community effort!

Analytical

Numerics

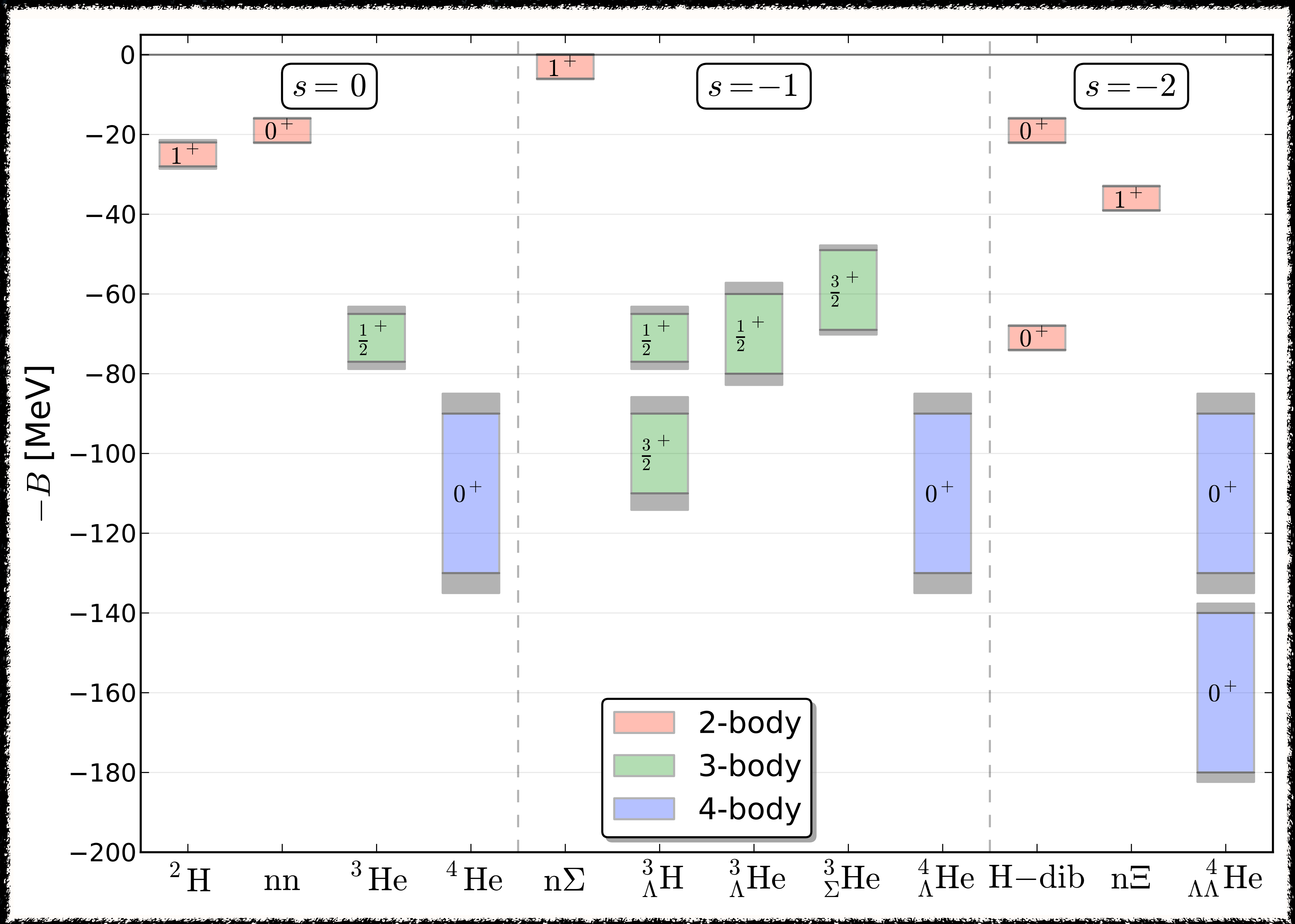


Actions, observables, systematics,...

Inversion algorithms, code development, production,...

Status Report: Spectrum

$m_\pi = m_K \sim 800\text{MeV}$ $\qquad L = 3.5 - 6.7 \text{ fm}$

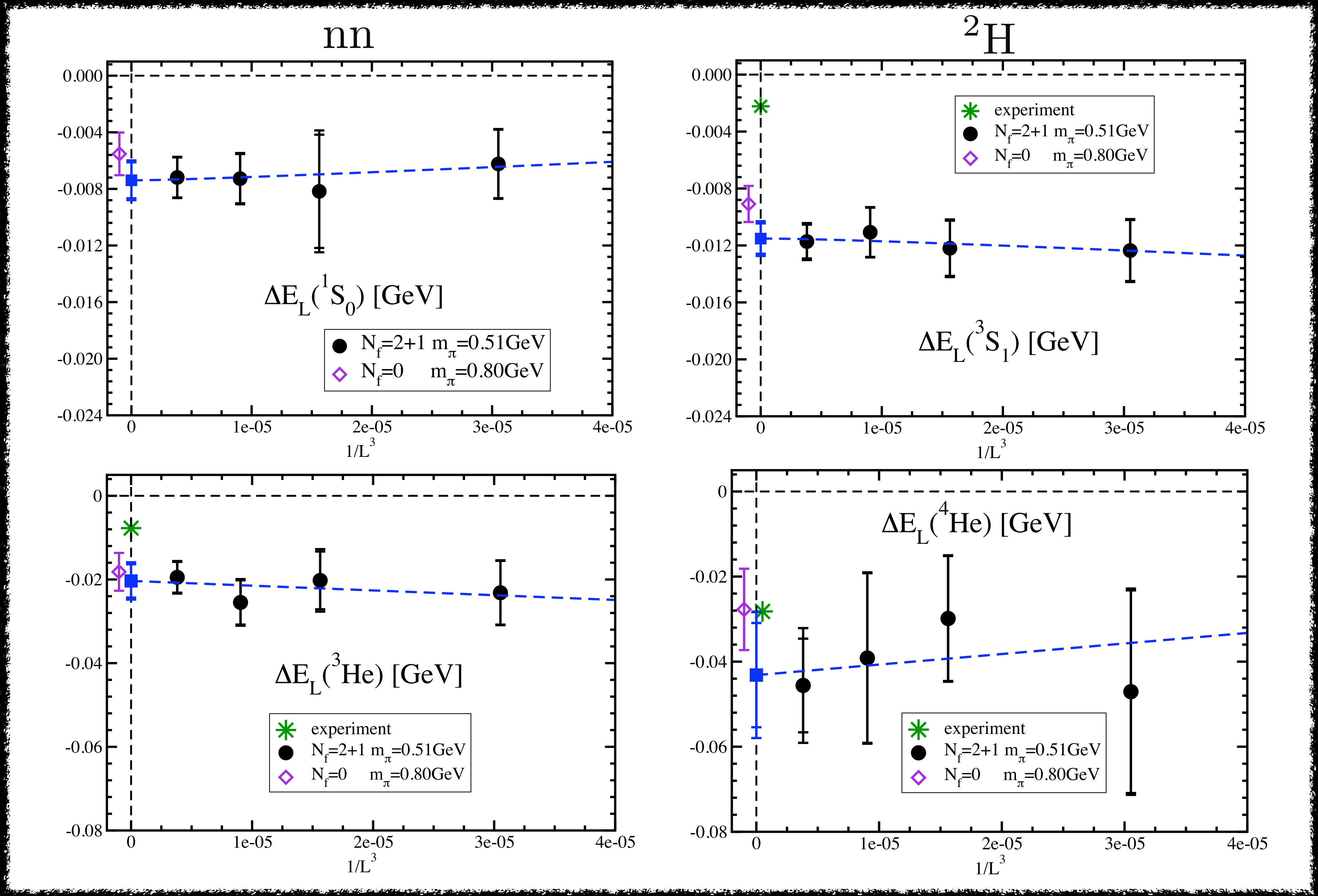


high statistics

NPLQCD (2012): Beane *et al.*

Status Report: Spectrum

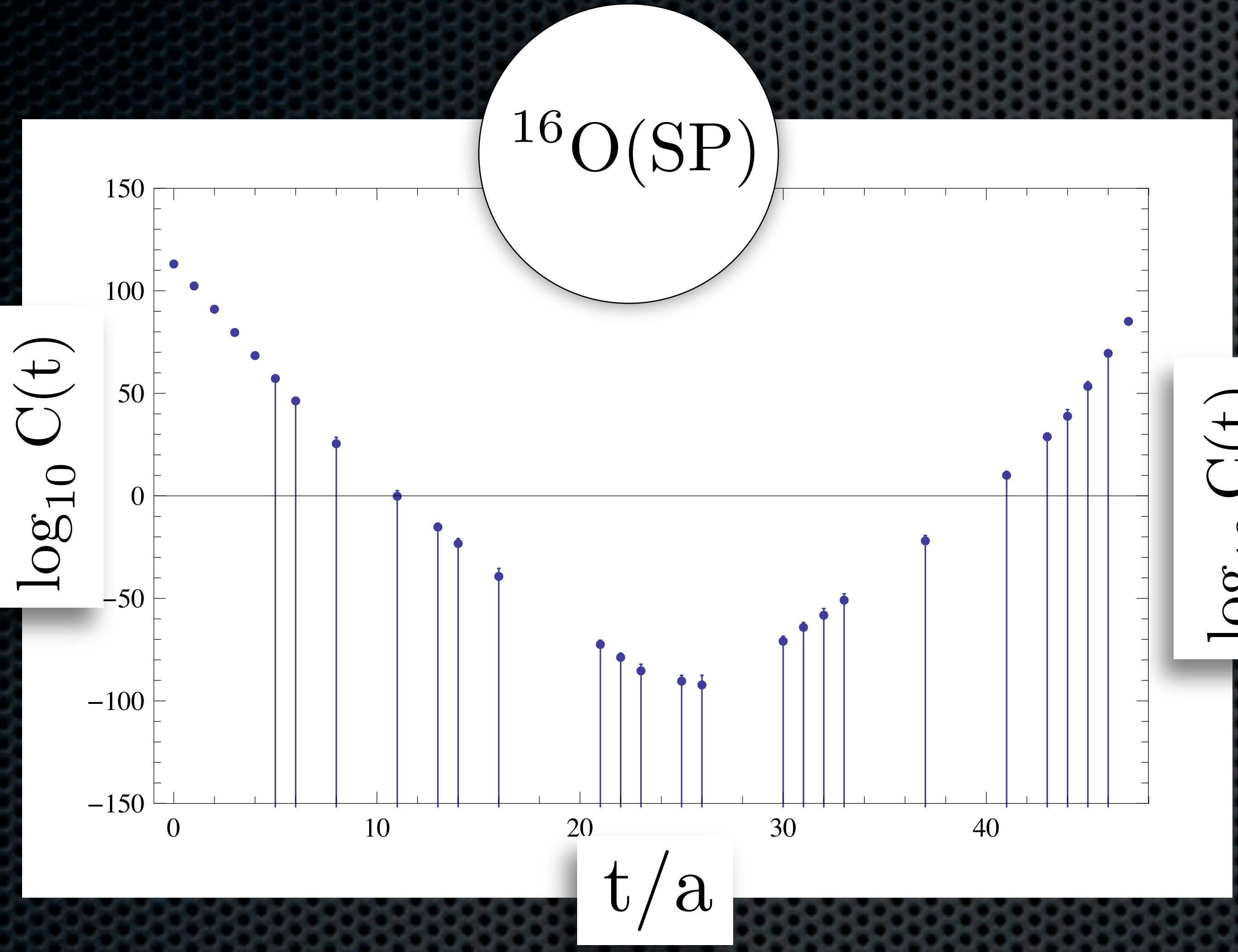
$m_\pi \sim 510\text{MeV}$ $\quad L = 2.9 - 5.8$ fm

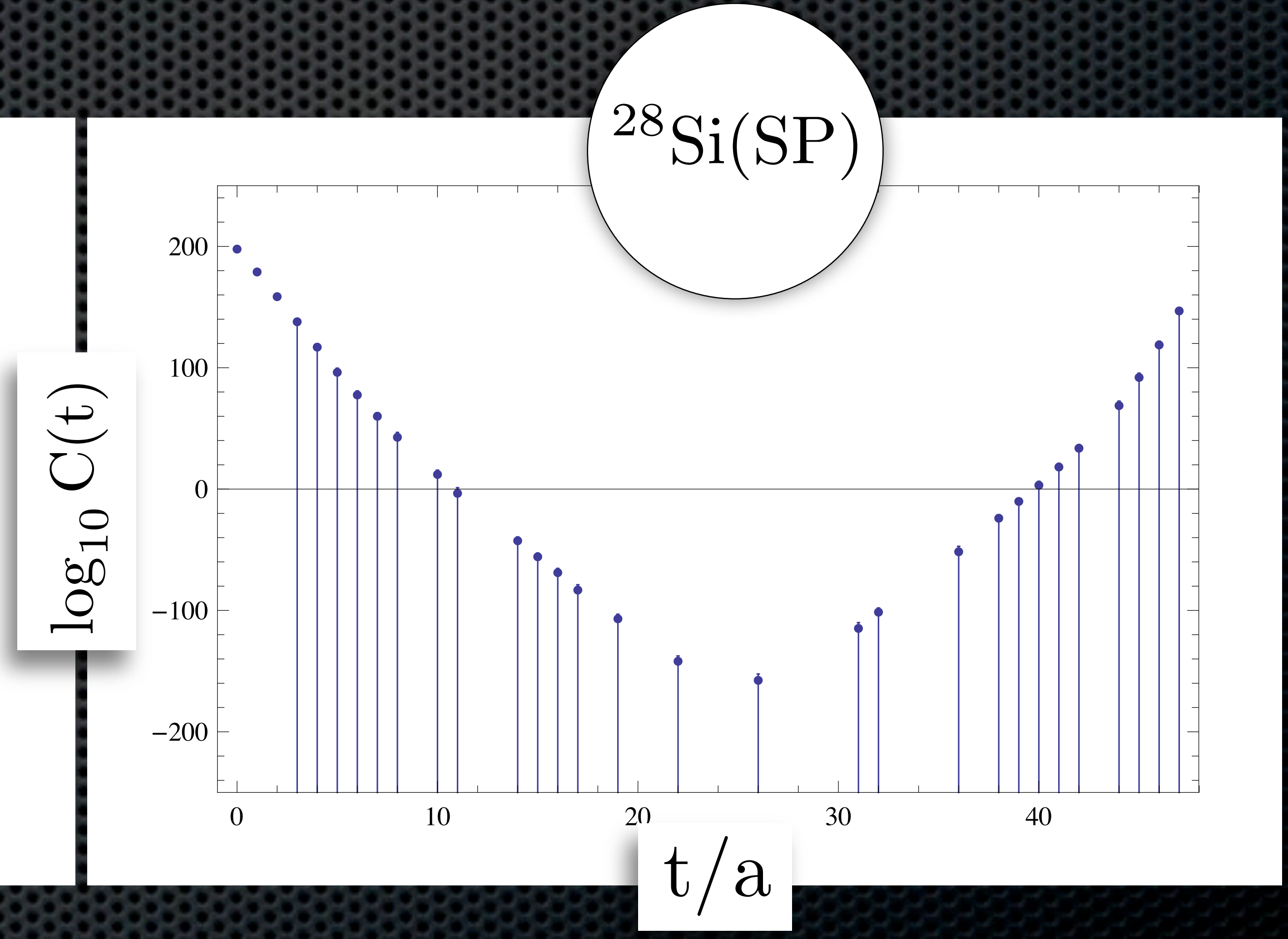


Yamazaki *et al.* (2012)

Status Report:
A glimpse into the future

$$C(t) = Z_0 e^{-tE_0} + \cdots$$





84 quarks!

Wick contractions: **naive scaling ~ $n_u!n_d!n_s!$**

improved scaling ~ $(n_u n_d n_s)^3$

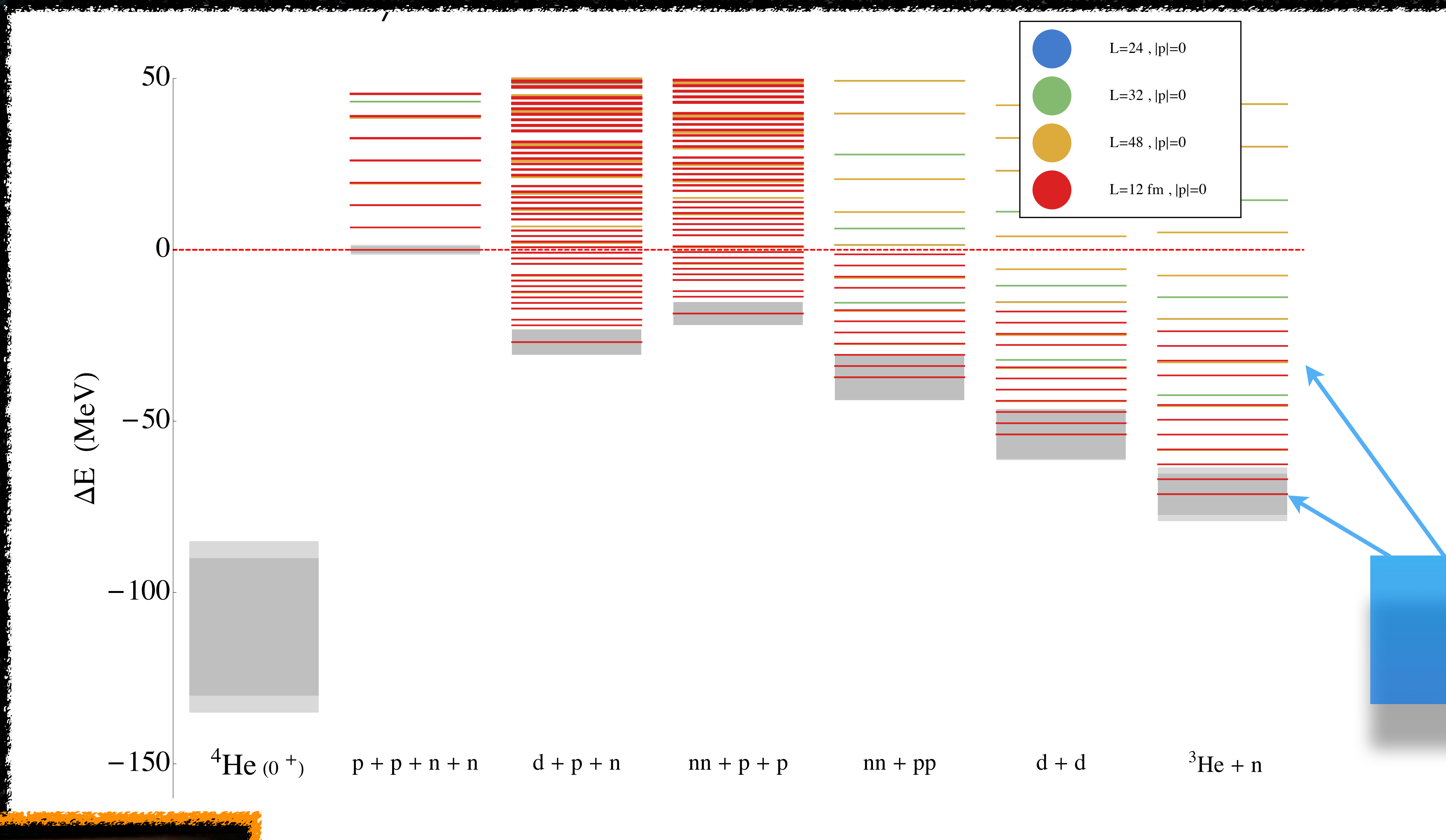
low statistics ~ 250 configurations

Detmold and Orginos (2012)

Bound to unbound?

NPLQCD

Expected spectrum for the $J^\pi = 0^+$ ^{4}He sector.



Neglecting interactions

If we did obtain this spectrum, what would it mean?

NPLQCD (2012): Beane *et al.*

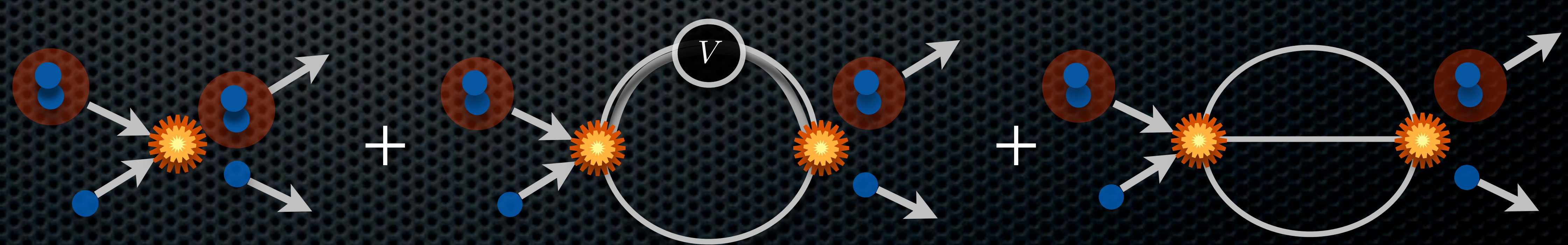
Optimistic yet Cautious!

Numerical Observation:

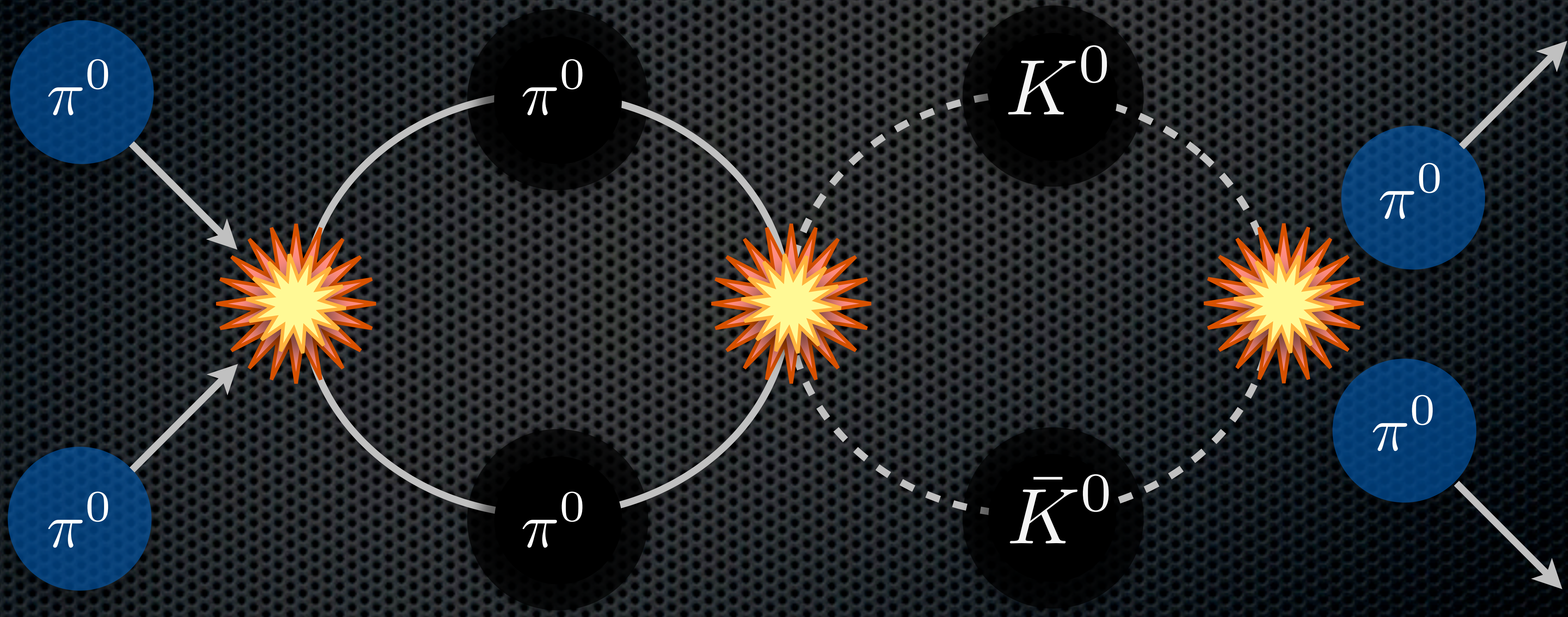
- Below break-up: 3-Body ➞ Lüscher [Bour, Hammer, Lee, Meißner (2012)]

Questions/Issues:

- Proof?
- Validity of Lüscher?
- Compact two-particle states? [Guo, Dudek, Edwards, Szczepaniak (2012)]
- FV effects from off-shell states?
- Partial wave mixing? [Kreuzer, Hammer, Greißhammer (2009-2012)]
- *Model independent* break-up/recombination?

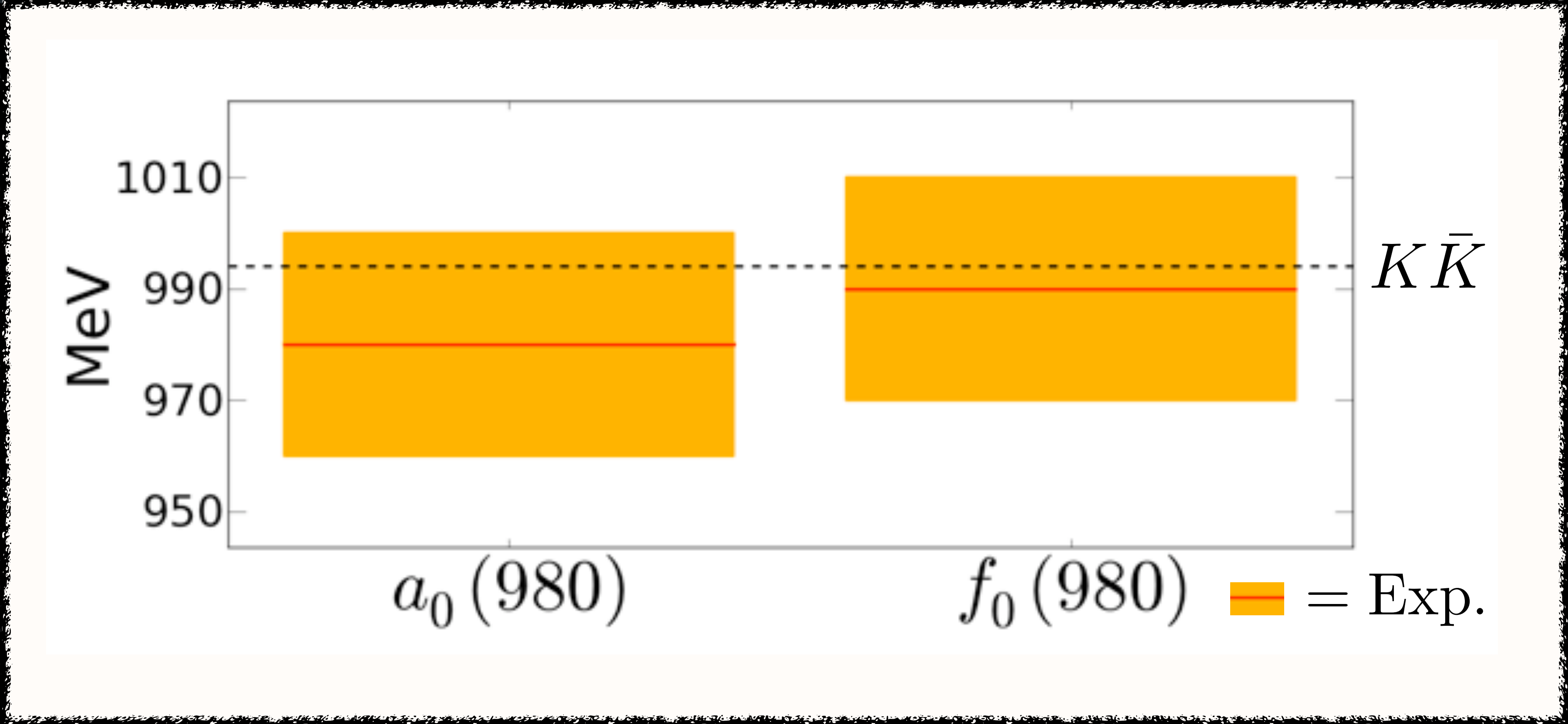


Coupled Channels: 2-Body



Coupled Channels: 2-Body

$$f_0(980), a_0(980) : \{\pi\pi, 4\pi, 6\pi, K\bar{K}, \ldots\}$$



- Two-channels: 2 phases + 1 mixing angle
- Boosted systems*

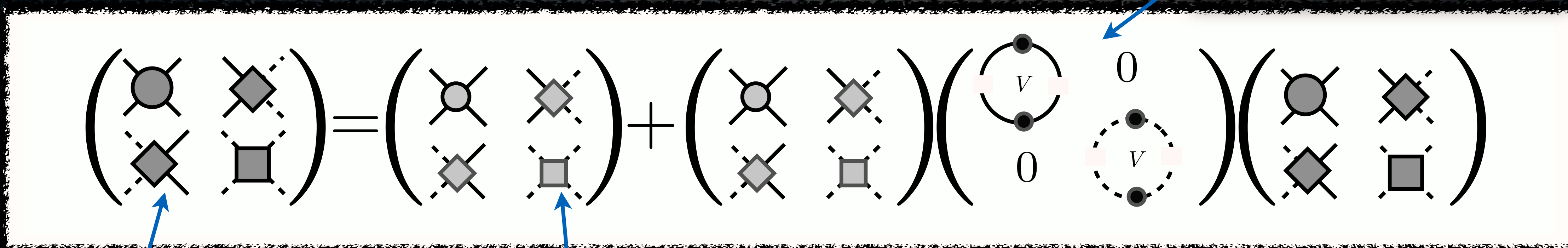
*Rummukainen and Gottlieb (1995) / Kim, Sachrajda, and Sharpe (2005)

$\pi\pi - K\bar{K}$

$[m_\pi \sim \ 310\mathrm{MeV}, \mathrm{m_K} \sim 530\mathrm{MeV}]$

- Bellow 4 pion threshold
- Spectrum from poles of:

relativistic two particle propagator



FV Scattering Amplitude

s-channel 2PI

- Quantization condition (RB & Zohreh Davoudi arXiv:1204.1110):

$$\det\left(\mathcal{M}^{-1} + \delta\mathcal{G}^{V}\right) = 0$$

Holds for arbitrary numbers of channels

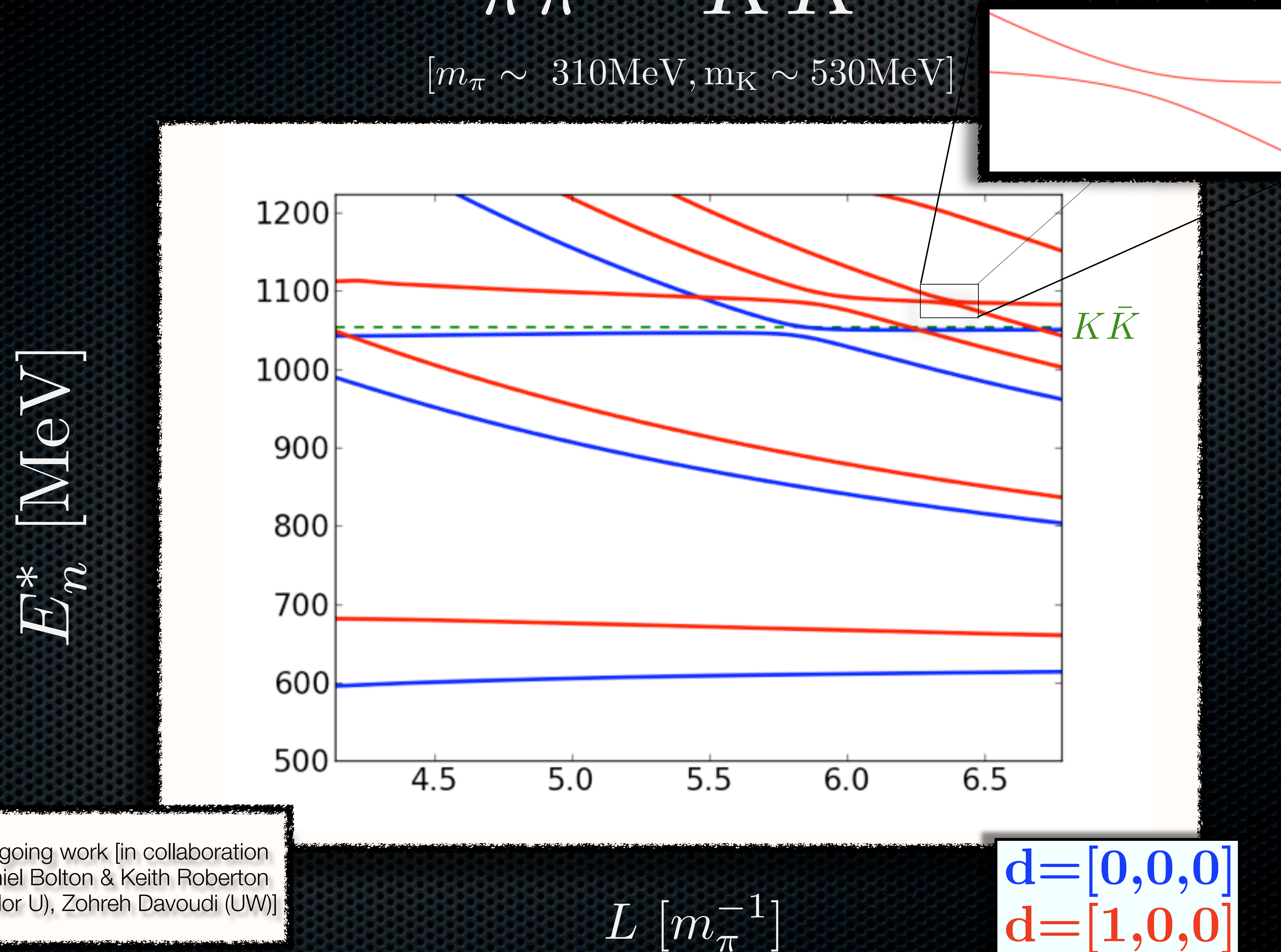
IV Scattering Amplitude

Kinematic function (L, E*,m)

- Derived independently by Max Hansen and Steve Sharpe (2012)

$\pi\pi - K\bar{K}$

$[m_\pi \sim\ 310\text{MeV}, \text{m}_\text{K} \sim 530\text{MeV}]$



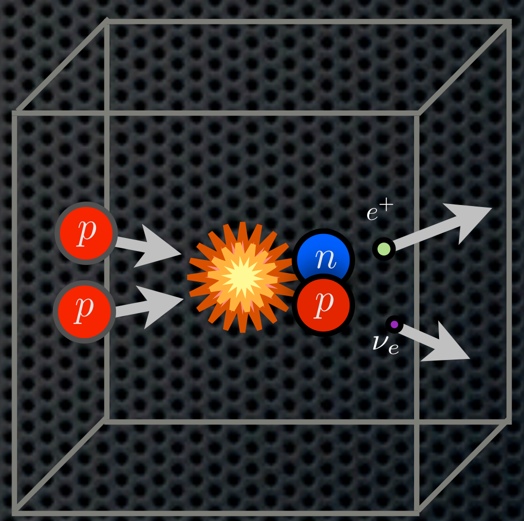
d=[0,0,0]
d=[1,0,0]

On going work [in collaboration Daniel Bolton & Keith Roberton (Baylor U), Zohreh Davoudi (UW)]

NN Weak Matrix Elements

- Axial vector current: $A^{\mu=3} = \frac{1}{2}\left(\bar{u}\gamma^3\gamma^5 u - \bar{d}\gamma^3\gamma^5 d\right)$
  - 2-Body ~ dominant uncertainty in deuteron break-up
- Detmold & Savage (2004): background field
  - ${}^1S_0 - {}^3S_1$ coupled channels
- 5-point correlation functions

reaction in FV



IV Weak Matrix Element

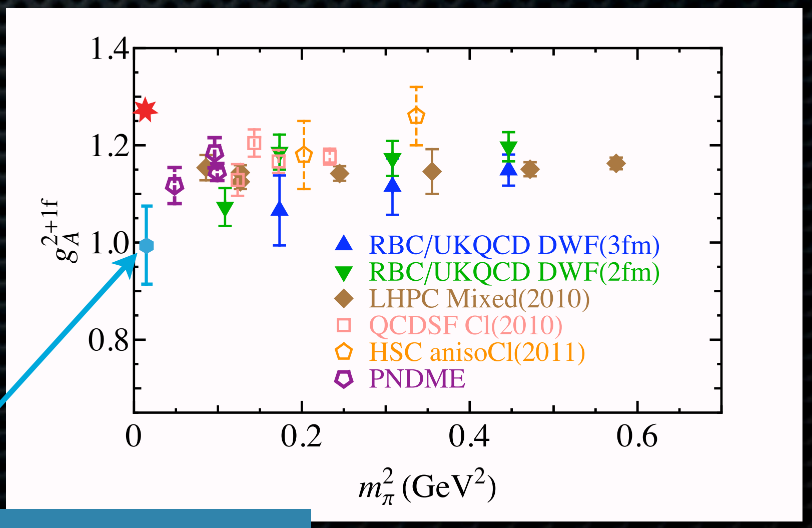
FV Weak Matrix Element

$$\left(\left|\mathcal{M}^{\infty}_{{}^1S_0-{}^3S_1}\right| - g_A W_3 \frac{\delta J_0^V e^{i2\phi}}{\left(\delta I_0^V\right)^2}\right)^2 = \left(\frac{2\pi V}{q_0^{*2}}\right)^2 \left(\phi' + \delta'_{{}^3S_1}\right)\left(\phi' + \delta'_{{}^1S_0}\right)\left|\mathcal{M}^V_{{}^1S_0-{}^3S_1}\right|^2$$

Kinematic function (L, E*)

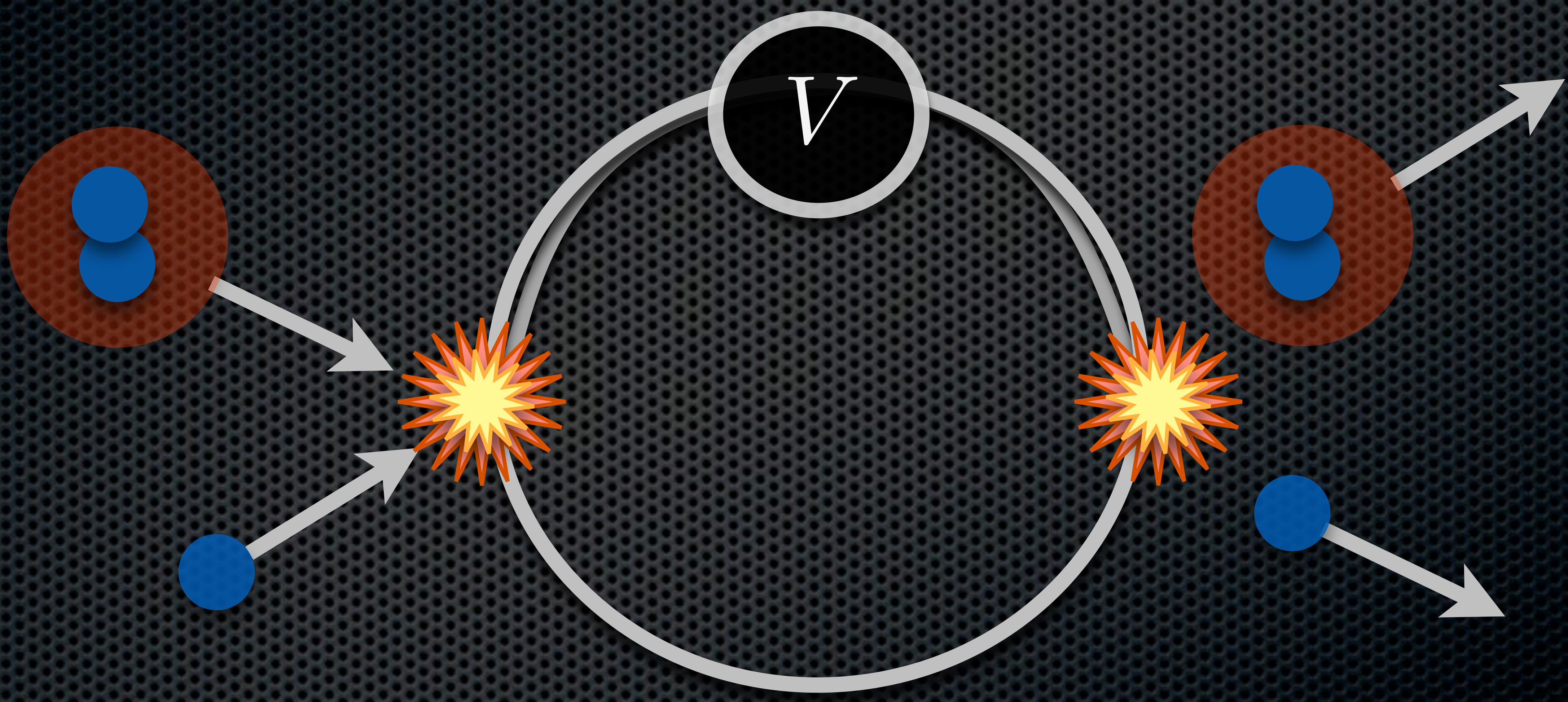
Future LQCD work

Summary plot by H-W Lin (2011)



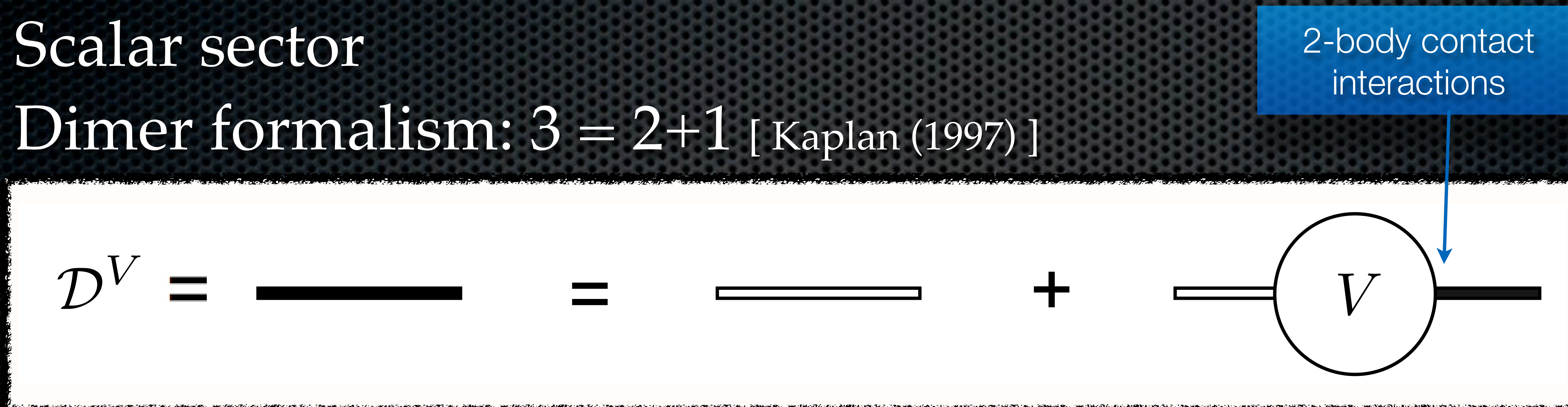
Green et al. (2012) [extrapolated with lightest m_π=149(1) MeV, and $m_\pi L$ ~ 4]

Unto the 3-Body Problem

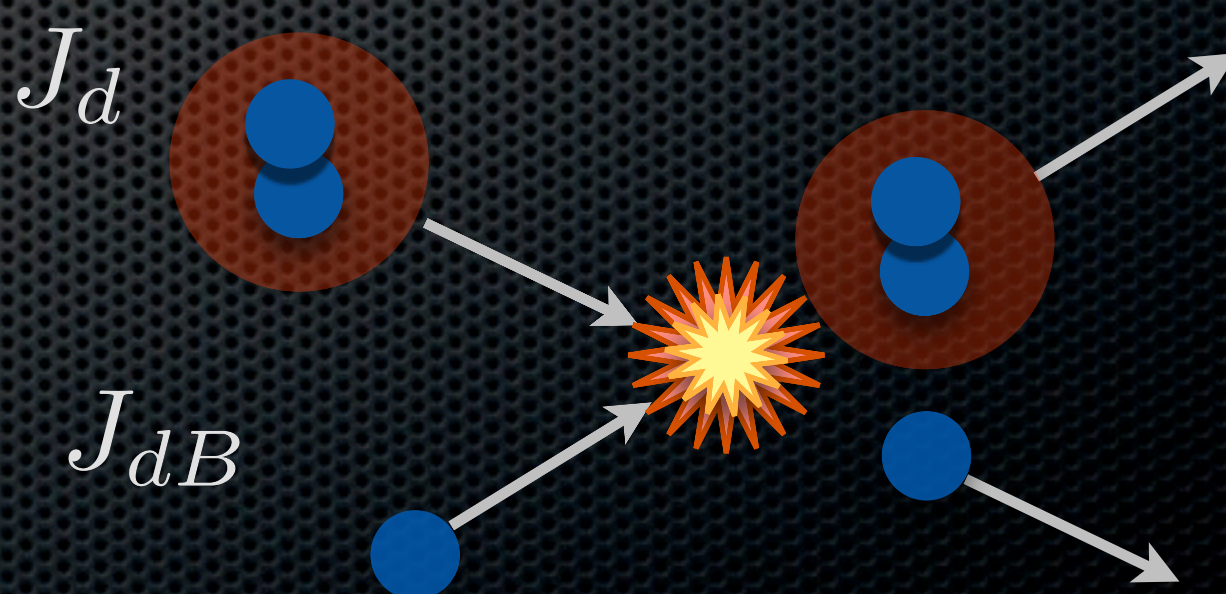


Unto the 3-Body Problem

- Scalar sector
- Dimer formalism: 3 = 2+1 [Kaplan (1997)]



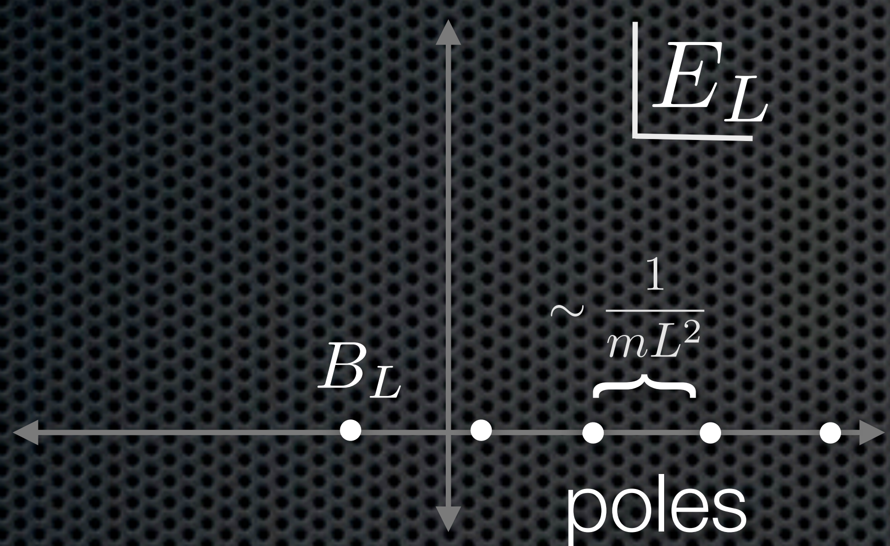
- Simplification comes at a cost:
  - $J_d = 0$

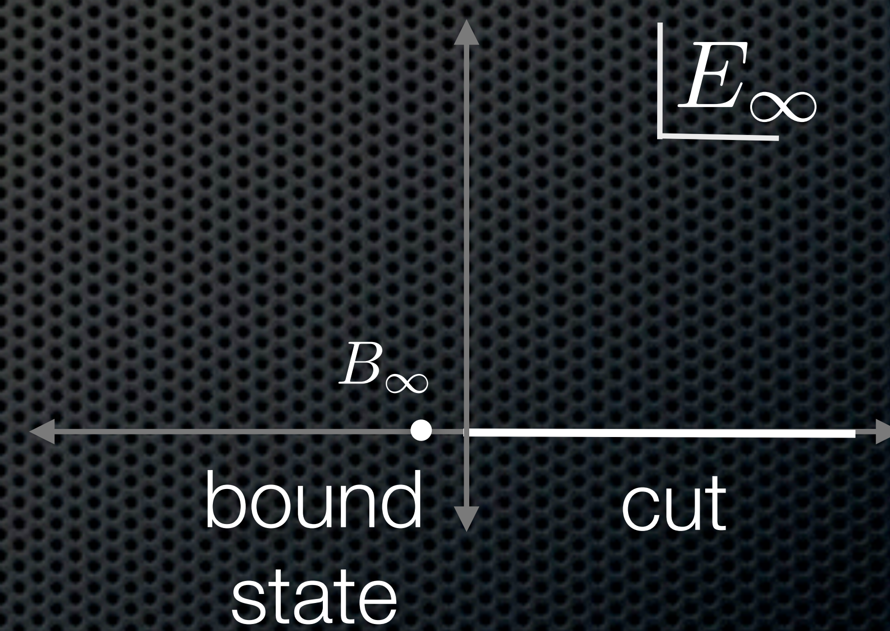


Infinite vs. Finite Volume Spectrum

$$\boxed{(\mathcal{D}^V)^{-1} = 0}$$

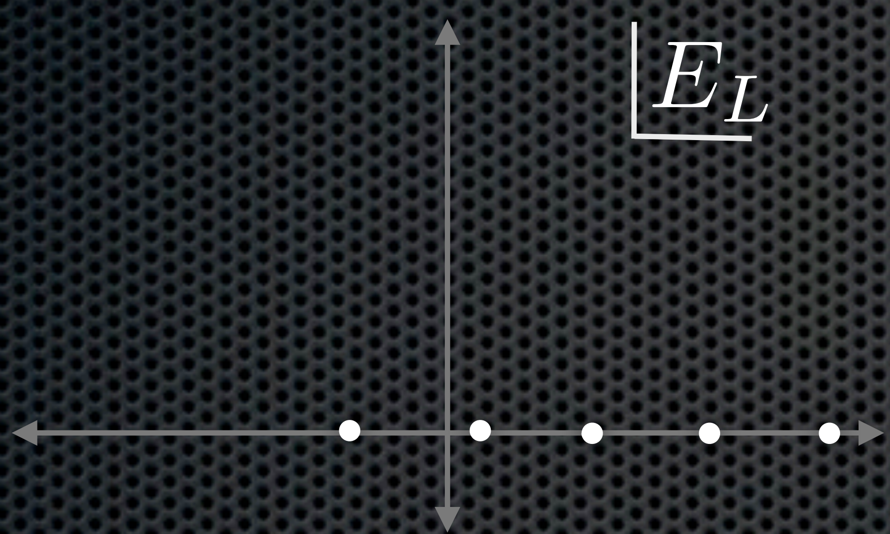
$$\boxed{(\mathcal{D}^\infty)^{-1} = 0}$$





Infinite vs. Finite Volume Spectrum

$$\left(\mathcal{D}^V\right)^{-1} = 0$$

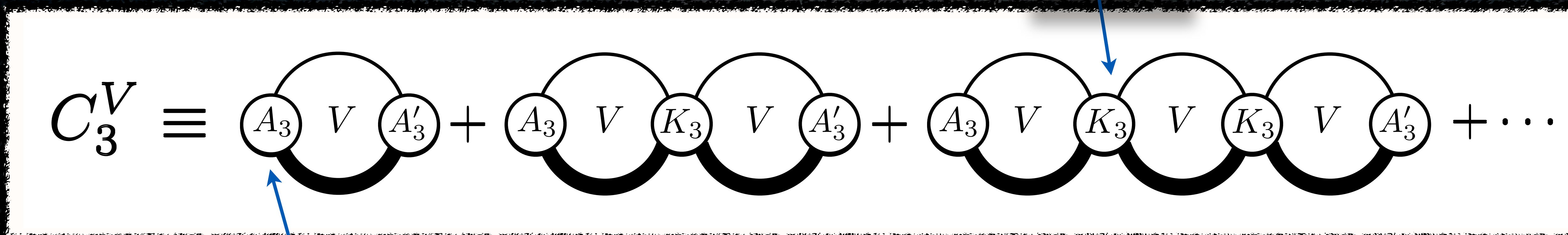


$\mathcal{D}^V \to$

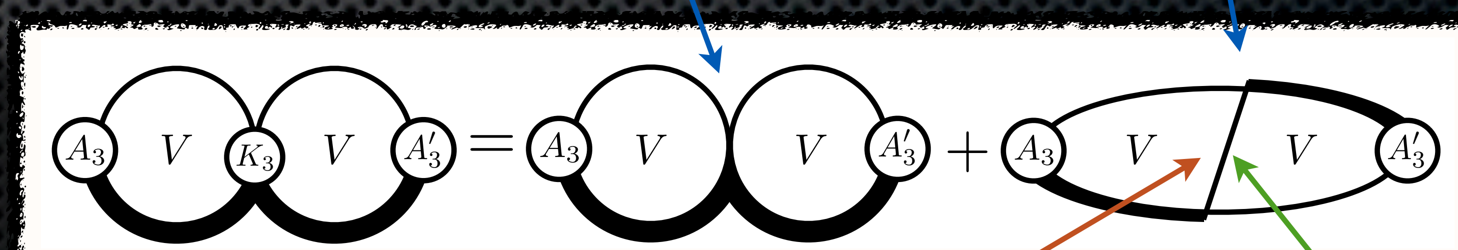
FV spectrum is ALWAYS discretized
No cuts/integrals
Only poles/sums

Some Technicalities

- Spectrum from poles of correlation function



- Two loop diagrams:

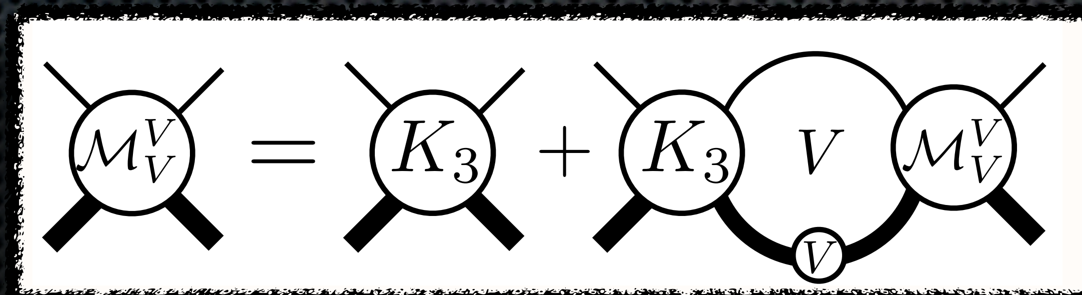


- Only dimer poles contribute!

- **Loops decouple!**

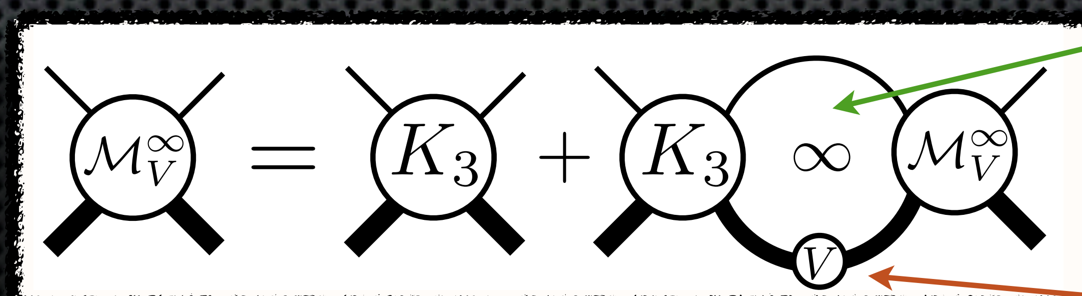
Some Technicalities

Finite volume "scattering amplitude"



Poles determine spectrum in finite volume

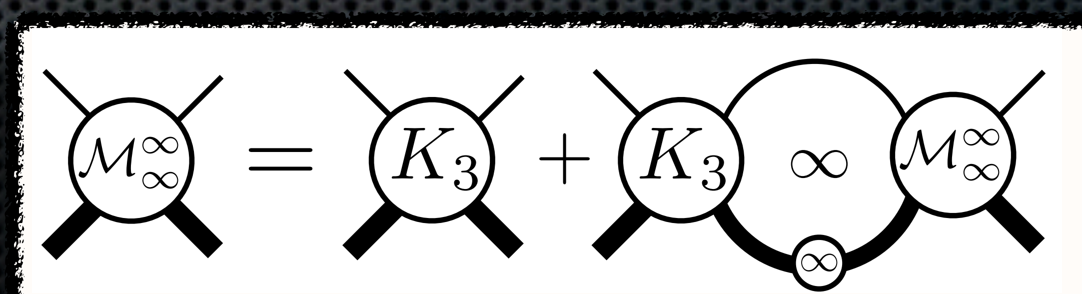
"Scattering amplitude" between boson and finite volume dimer



Continuous boson-dimer relative momenta

Finite volume dimer

Infinite volume scattering amplitude



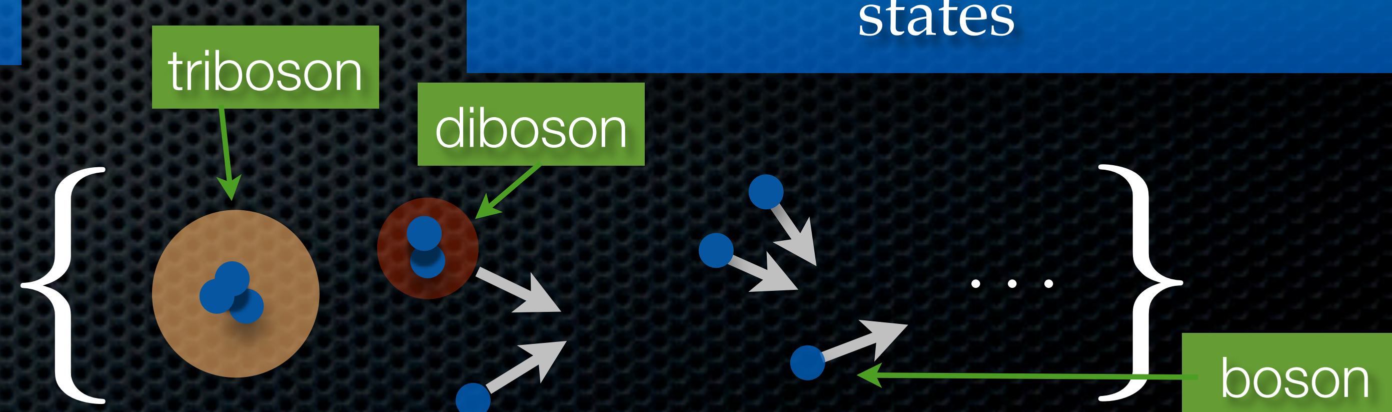
Three-Body Result

Disagreement with Guo et al. (2012)

$$\det\left({\mathcal{M}_V^{\infty}}^{-1} + \delta\mathcal{G}^V\right) = 0$$

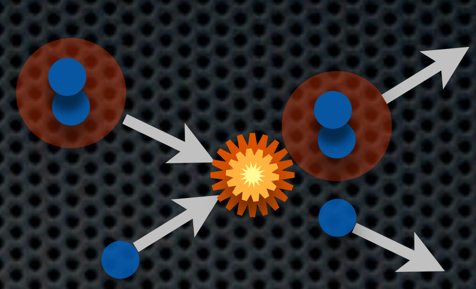
$\mathcal{M}_V^{\infty}$	$\delta\mathcal{G}^V$
"Scattering amplitude" between boson and finite volume dimer	Kinematic function of (L, E_L)
Diagonal in angular momentum	Mixes angular momentum
Mixed the three particle states (coupled-channels)	Diagonal in the three particle states

Three-particle states:



Recovering Lüscher

(Negative energies, deeply bound diboson)



- Below break-up:

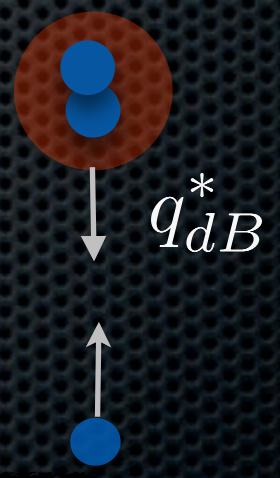
$$\mathcal{M}_{dB} = \frac{3\pi}{m}\frac{1}{q^*_{dB}\cot\delta_{dB} - iq^*_{dB}}$$

- CM momentum:

$$q^{*2}_{dB} \equiv \frac{4m}{3}\left(E^* + \frac{\gamma^{*2}_d}{m}\right)$$

diboson binding energy in the moving frame

Consistent with Bour et al. (2012)



- Quantization condition:

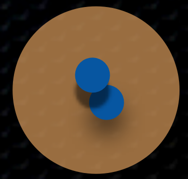
$$q^*_{dB}\cot\delta_{dB} = \frac{1}{\pi L}S^P\left(\left(q^*_{dB}L/2\pi\right)^2\right)$$

~ Boosted Zeta function for two particles with $m_2=2m_1$

$$S^P\left(\tilde{p}^2\right) = \sum_{\mathbf{n}}^{\Lambda_n}\frac{1}{(\mathbf{n} - L\mathbf{P}/6\pi)^2 - \tilde{p}^2} - 4\pi\Lambda_n$$

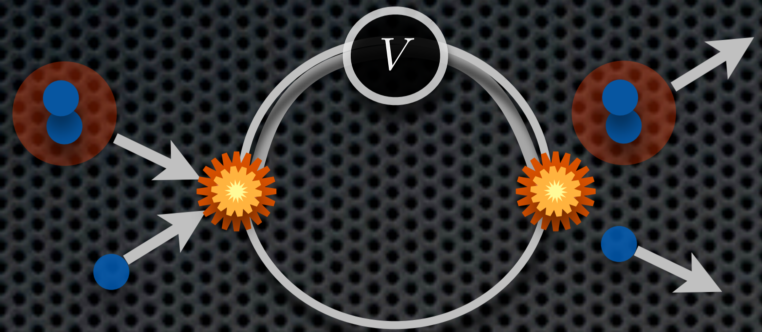
- Bound states:

$$\gamma_{dB} + q^*_{dB}\cot\delta_{dB}\big|_{q^{*2}_{dB}=-\gamma^2_{dB}} = \mathcal{O}(e^{-\gamma_{dB}L})$$



Exponential Corrections

- Finite volume dimer:



Obtained from 3-particle spectrum

$$\delta^L_{dB} = \delta^\infty_{dB} + \mathcal{O}(e^{-\gamma_d^* L})$$

Extrapolate to infinite volume!

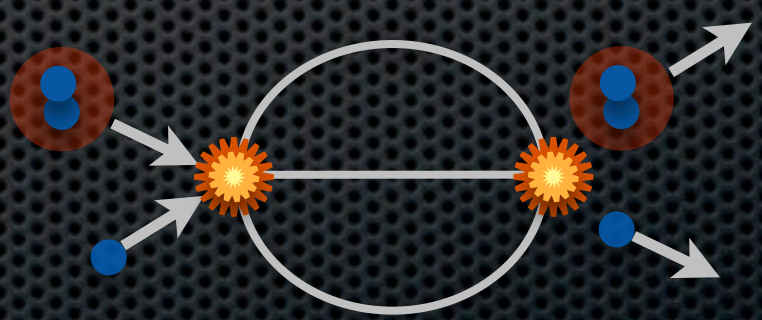
Dimer is NOT compact
[Guo et al. (2012)]

- Excited state: $q^{*2}_{E^*,1} \equiv \frac{4}{3}(E^* m - q_1^{*2}) < 0$

off-shell

$$\text{Corrections} \sim e^{-2L\sqrt{(q_1^{*2} - E^* m)/3}}$$

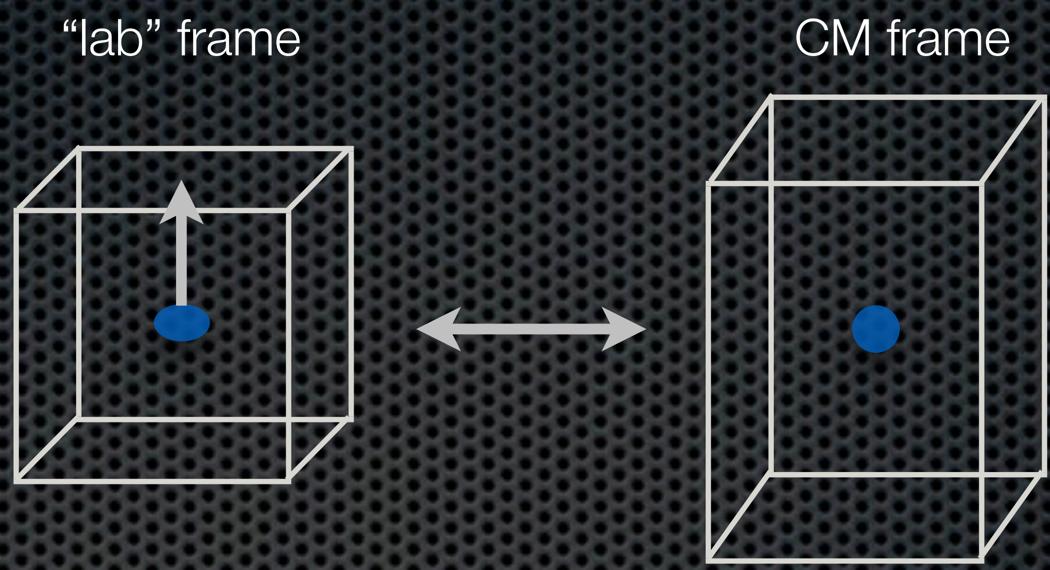
Included in quantization condition



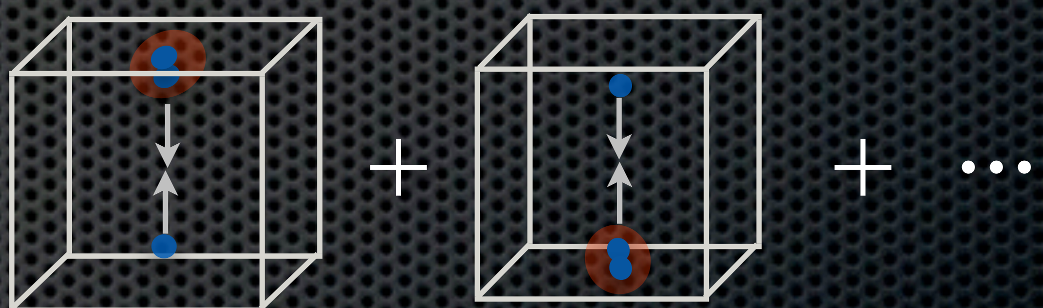
- For positive energies: FV effects are power-law

Boosts

Symmetry is reduced:

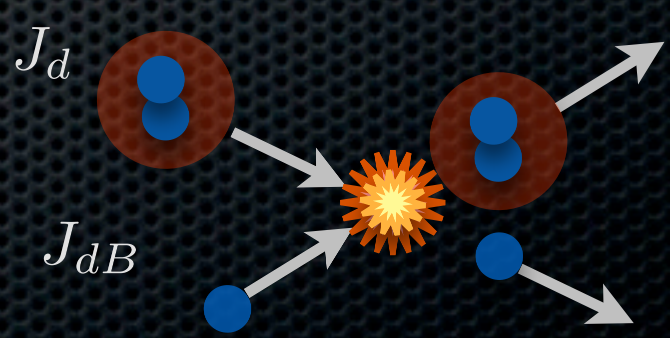


Boson-diboson CM:



- diboson is boosted: $J_d = \{0, 2, 4, \ldots\}$
- dB is unboosted: $J_{dB} = \{0, 4, 6, \ldots\}$

Boson-diboson Boosted:

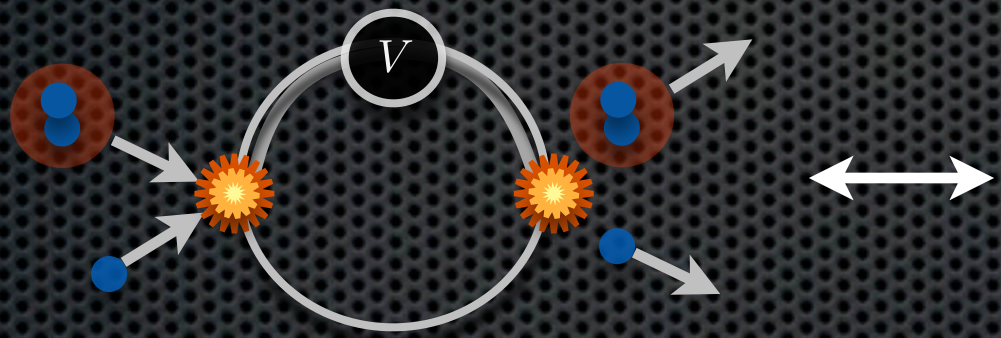


- $J_{dB} = \{0, 1, 2, \ldots\}$

Bour et al. (2011), Davoudi & Savage (2011), Fu (2012):
Boosted two-particle system with different masses

Take-Home Message

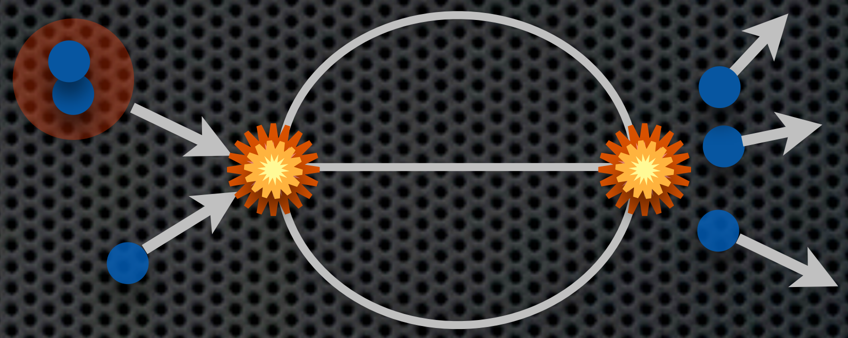
- FV spectrum is ALWAYS discretized
- 3-Body quantization condition reduces to Luscher-like equation
- $\{E_L^*, a_d, r_d\} \to \{q_{dB}^*, q_d^*, \delta_{dB}^L\}$
- Boson-diboson phase shift has *large* FV effects



$$\delta_{dB}^L = \delta_{dB}^\infty + \mathcal{O}(e^{-\gamma_d^* L})$$

- Requires extrapolation
- Partial wave mixing: J = 2 (unboosted), J = 1 (boosted)
- **Three-body problem requires caution!**

In progress...



- Above threshold!
- Nuclear sector
- Partial wave mixing due to boost
- Cubic dimer propagator

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Quan
May 6
A. An

Advan
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F. Pe

Nucle
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R. Briceno, Z. Davoudi, T. Luu

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(Last modified 9/13/2012)

Nuclear Reactions from Lattice QCD
March 11-12, 2013
R. Briceno, Z. Davoudi, T. Luu

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