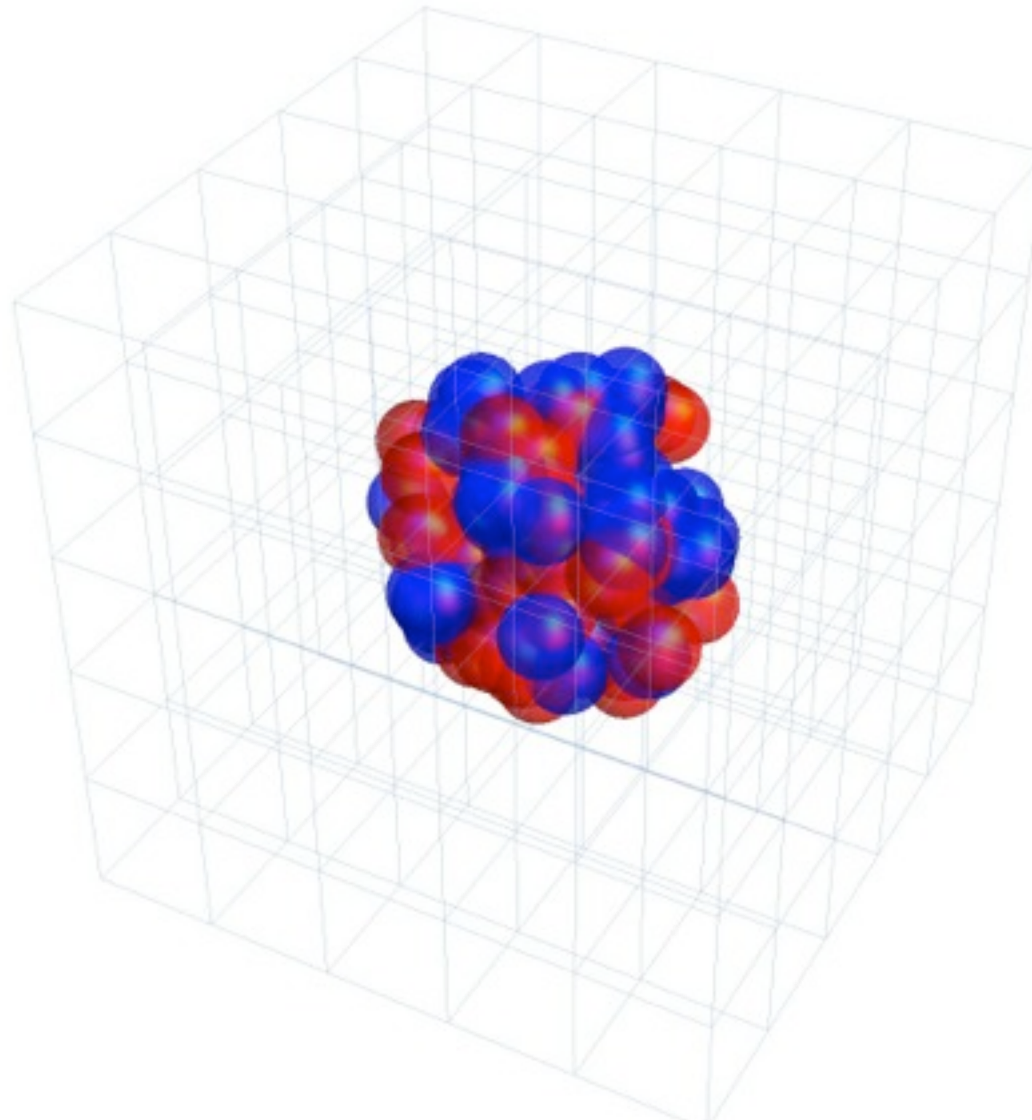


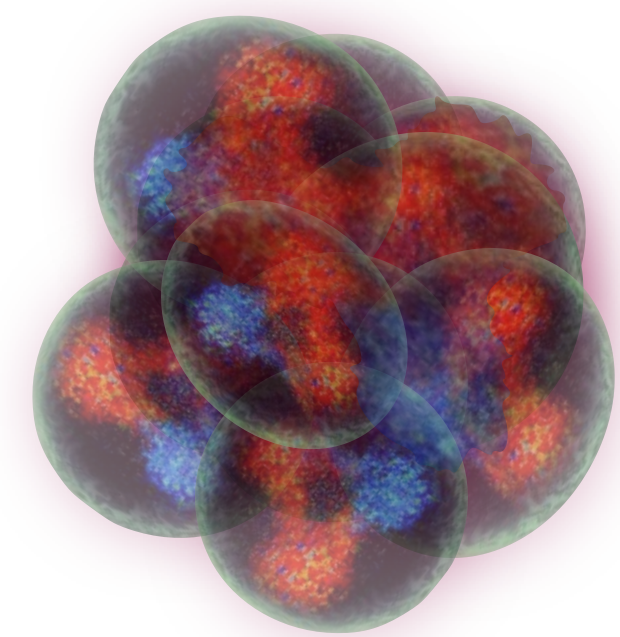
Few body systems in lattice QCD

William Detmold

MIT Lattice Club, September 26th 2012

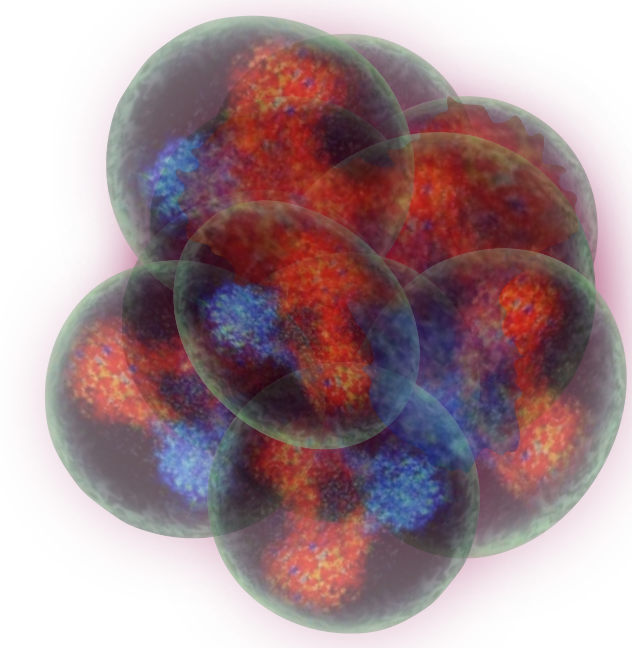
Massachusetts Institute of Technology

From quarks to nuclei



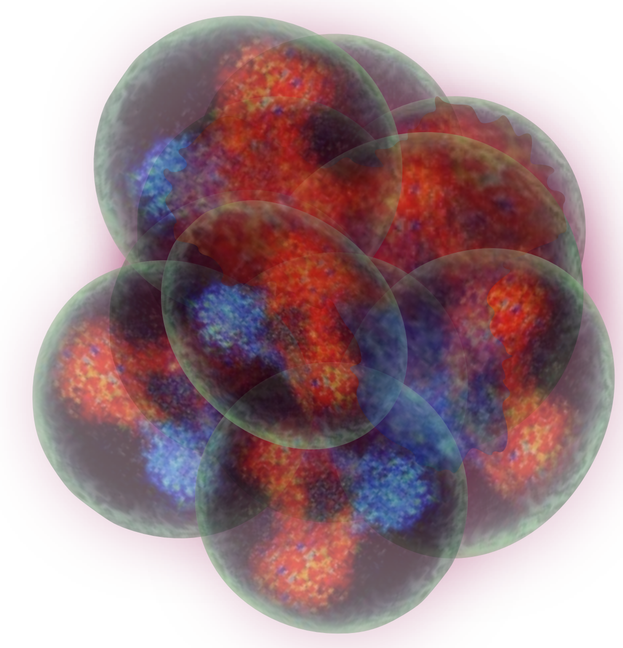
From quarks to nuclei

- Nuclear physics: an emergent phenomenon of the Standard Model



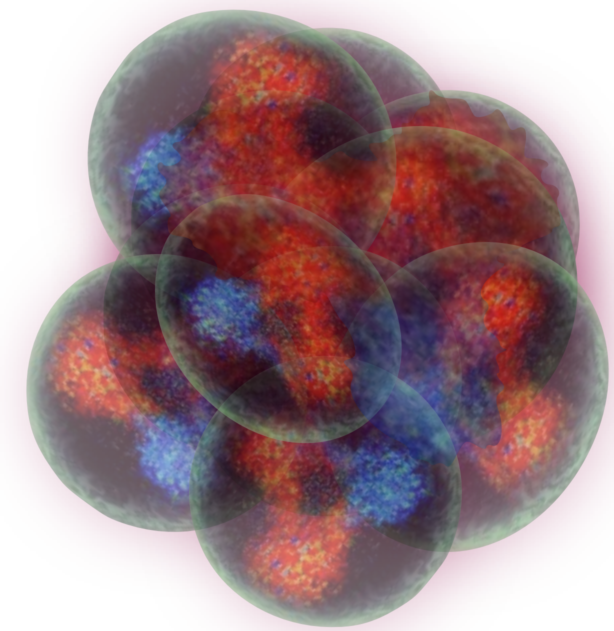
From quarks to nuclei

- Nuclear physics: an emergent phenomenon of the Standard Model
- *How do nuclei emerge from QCD?*



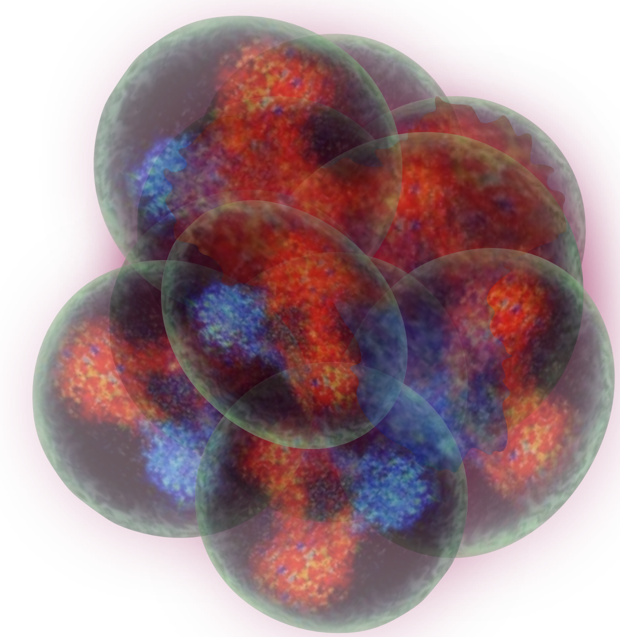
From quarks to nuclei

- Nuclear physics: an emergent phenomenon of the Standard Model
- *How do nuclei emerge from QCD?*
    - Issues



From quarks to nuclei

- Nuclear physics: an emergent phenomenon of the Standard Model
- *How do nuclei emerge from QCD?*
    - Issues
    - Recent progress



Nuclear physics from LQCD

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$$\langle 0|T q_1(t) \ldots q_{624}(t) \overline{q}_1(0) \ldots \overline{q}_{624}(0)|0\rangle$$

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$$\stackrel{t\to\infty}{\longrightarrow} \# \exp(-M_{Pb} t)$$

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- But...

An (exponentially hard)^2 problem?

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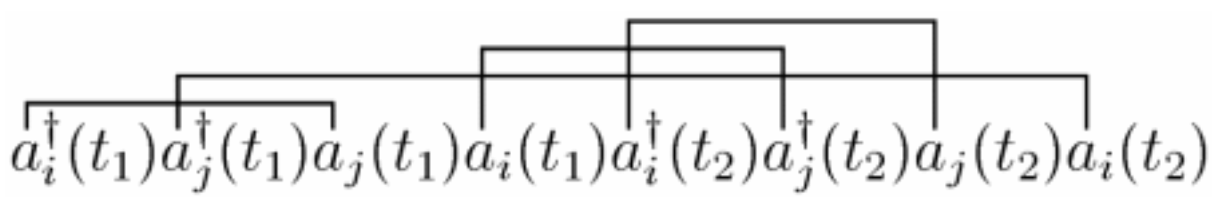
- Complexity: number of Wick contractions = (*A*+*Z*)!(2*A*-*Z*)!

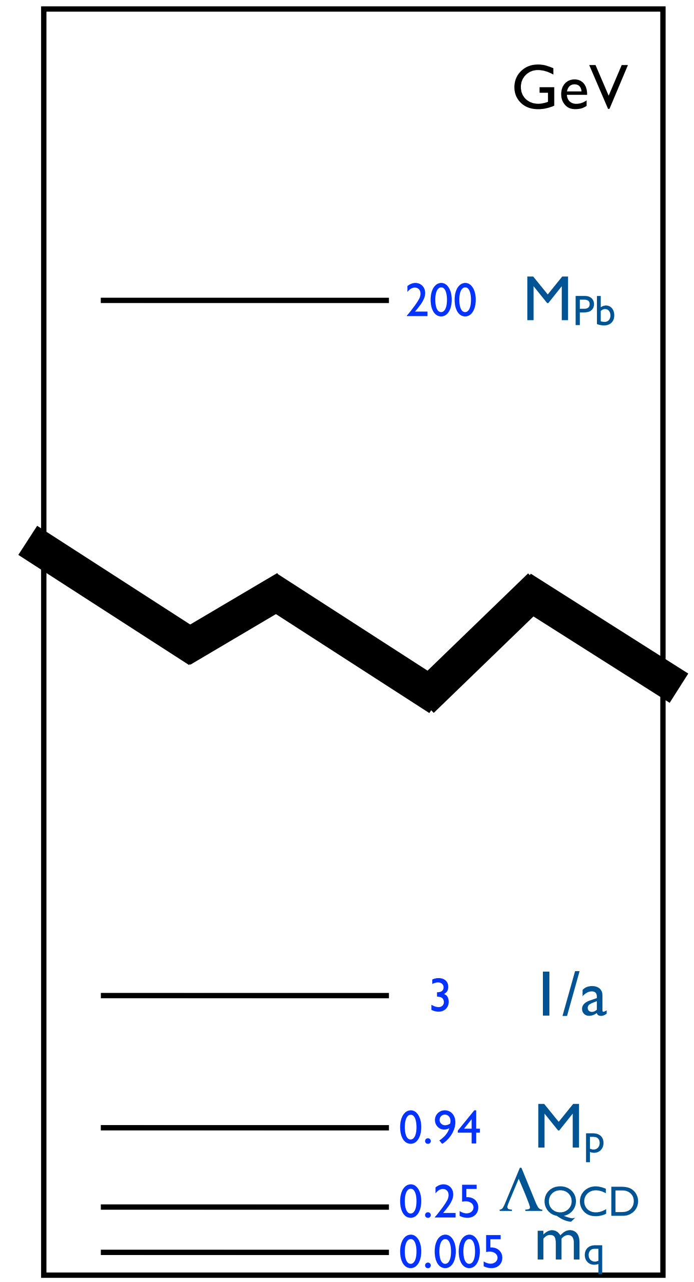
$$a_i^\dagger(t_1)a_j^\dagger(t_1)a_j(t_1)a_i(t_1)a_i^\dagger(t_2)a_j^\dagger(t_2)a_j(t_2)a_i(t_2)$$

An (exponentially hard)^2 problem?

- Complexity: number of Wick contractions = $(A+Z)!(2A-Z)!$



- Dynamical range of scales (numerical precision)



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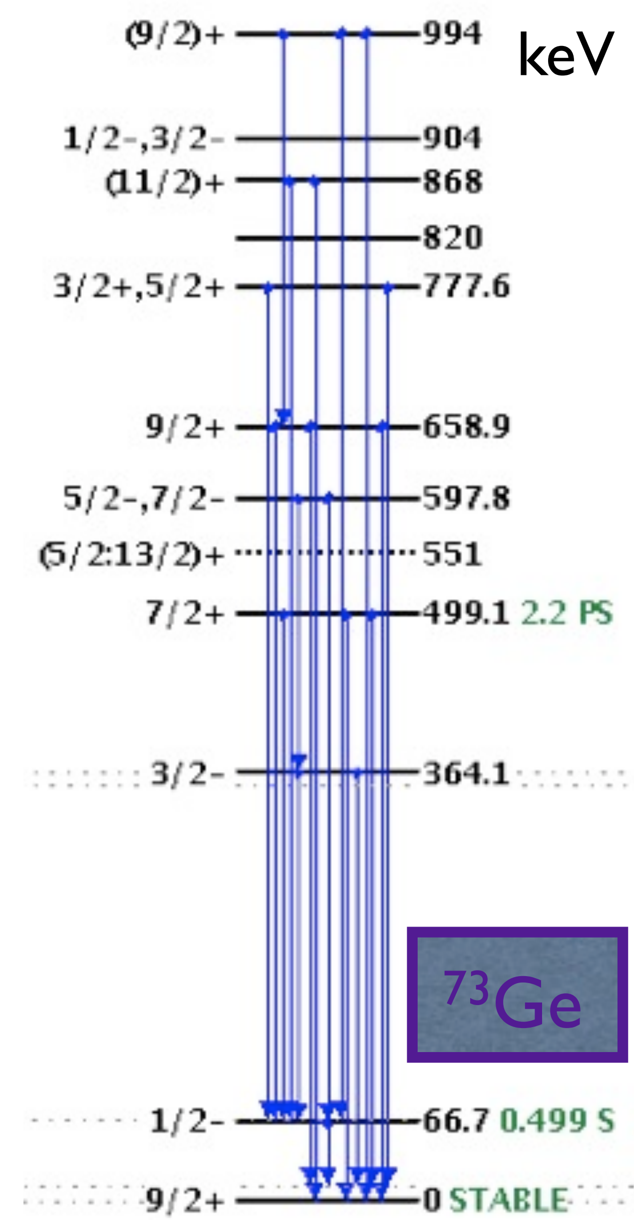
- Dynamical range of scales (numerical precision)
- Small energy splittings



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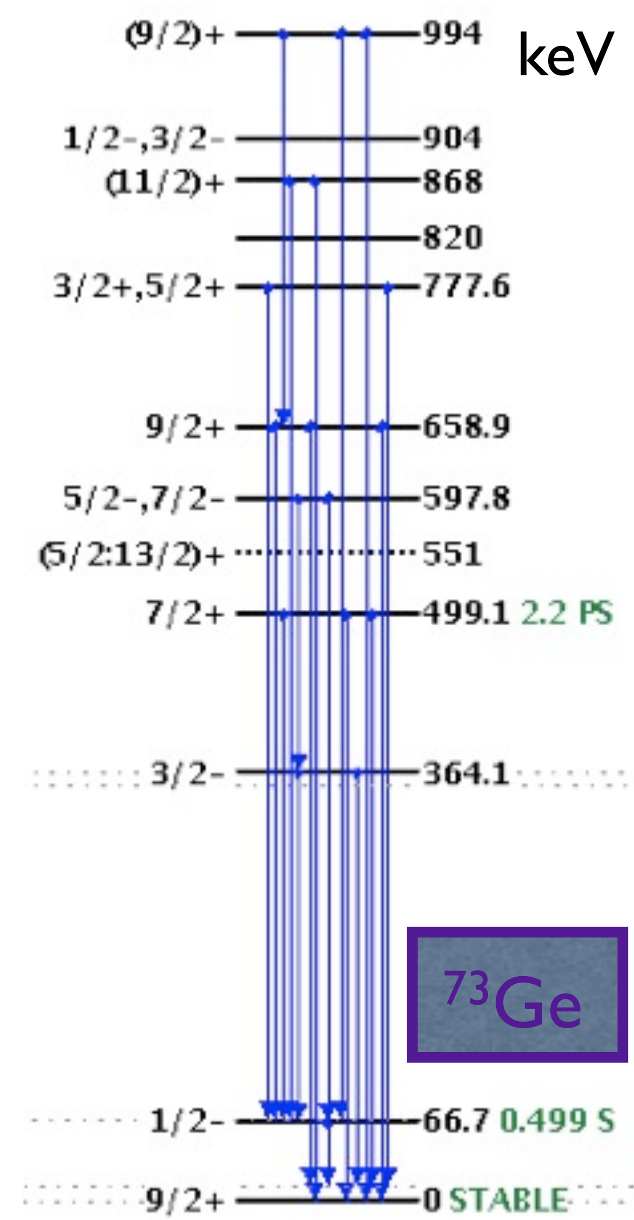
- Dynamical range of scales (numerical precision)

- Small energy splittings

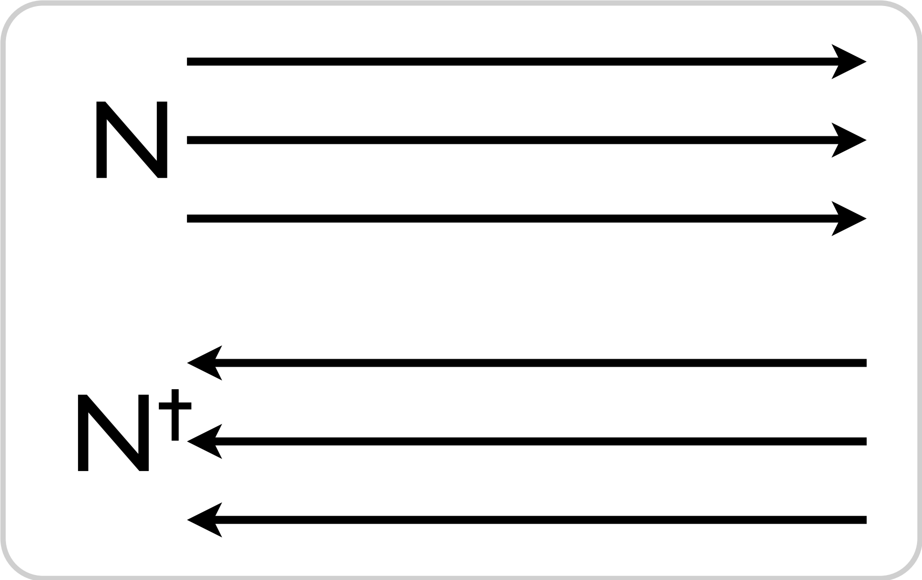
- Importance sampling: statistical noise exponentially increases with A



The trouble with baryons

- Importance sampling of QCD functional integrals
  ➤ correlators determined stochastically
- Variance in single nucleon correlator (C) determined by

$$\sigma^2(C) = \langle C C^\dagger \rangle - |\langle C \rangle|^2$$



- For nucleon:

$$\frac{\text{signal}}{\text{noise}} \sim \exp\left[-(M_N - 3/2 m_\pi)t\right]$$

- For nucleus A:

$$\frac{\text{signal}}{\text{noise}} \sim \exp\left[-A(M_N - 3/2 m_\pi)t\right]$$

[Lepage '89]

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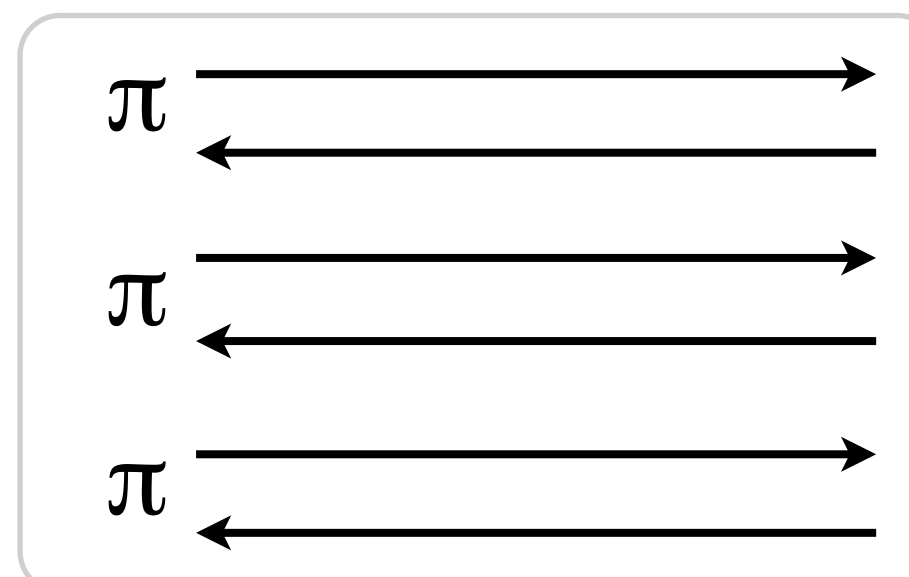

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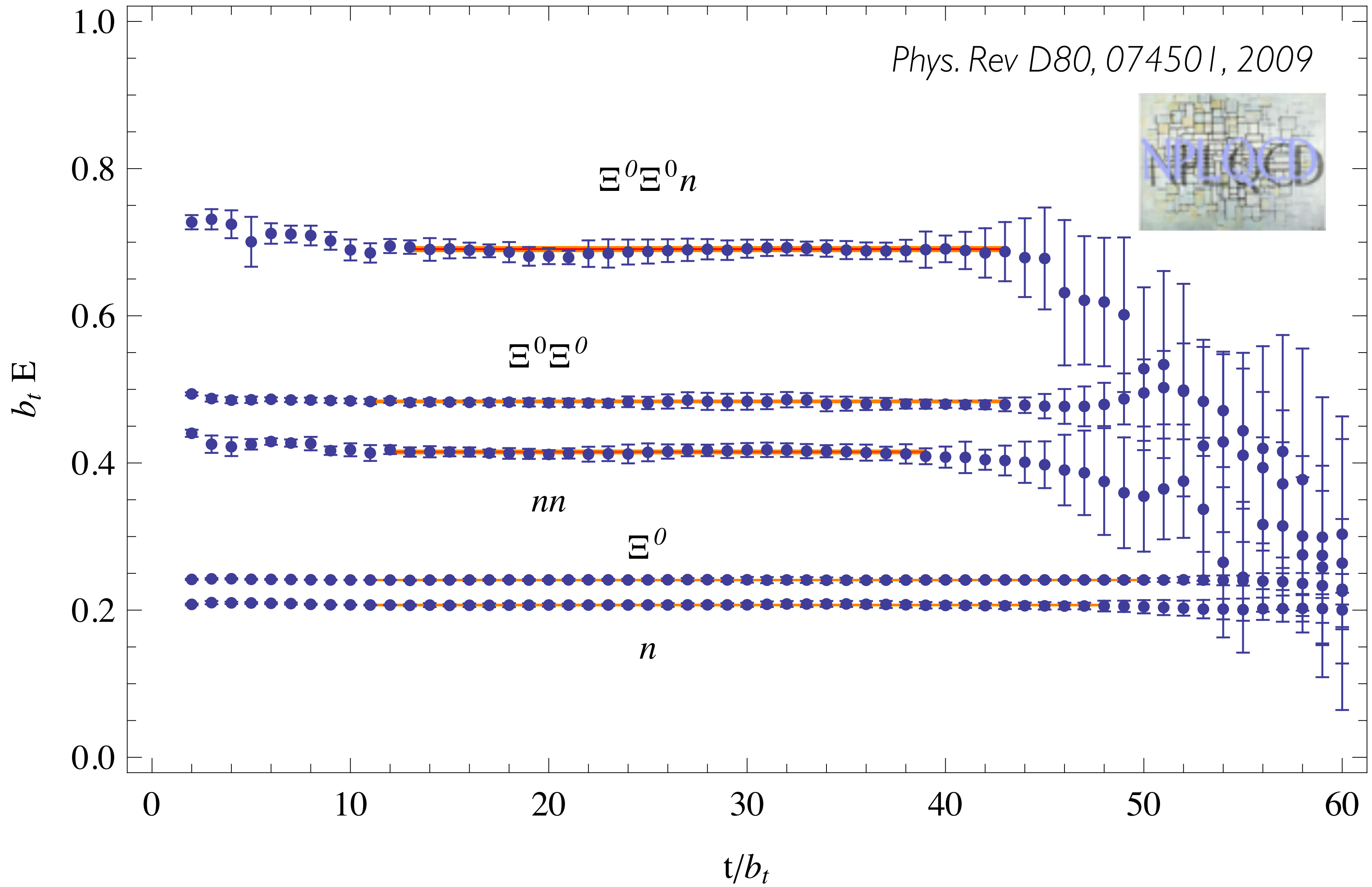
The trouble with baryons

High statistics study using anisotropic lattices (fine temporal resolution)



@ m_π = 390 MeV

The trouble with baryons

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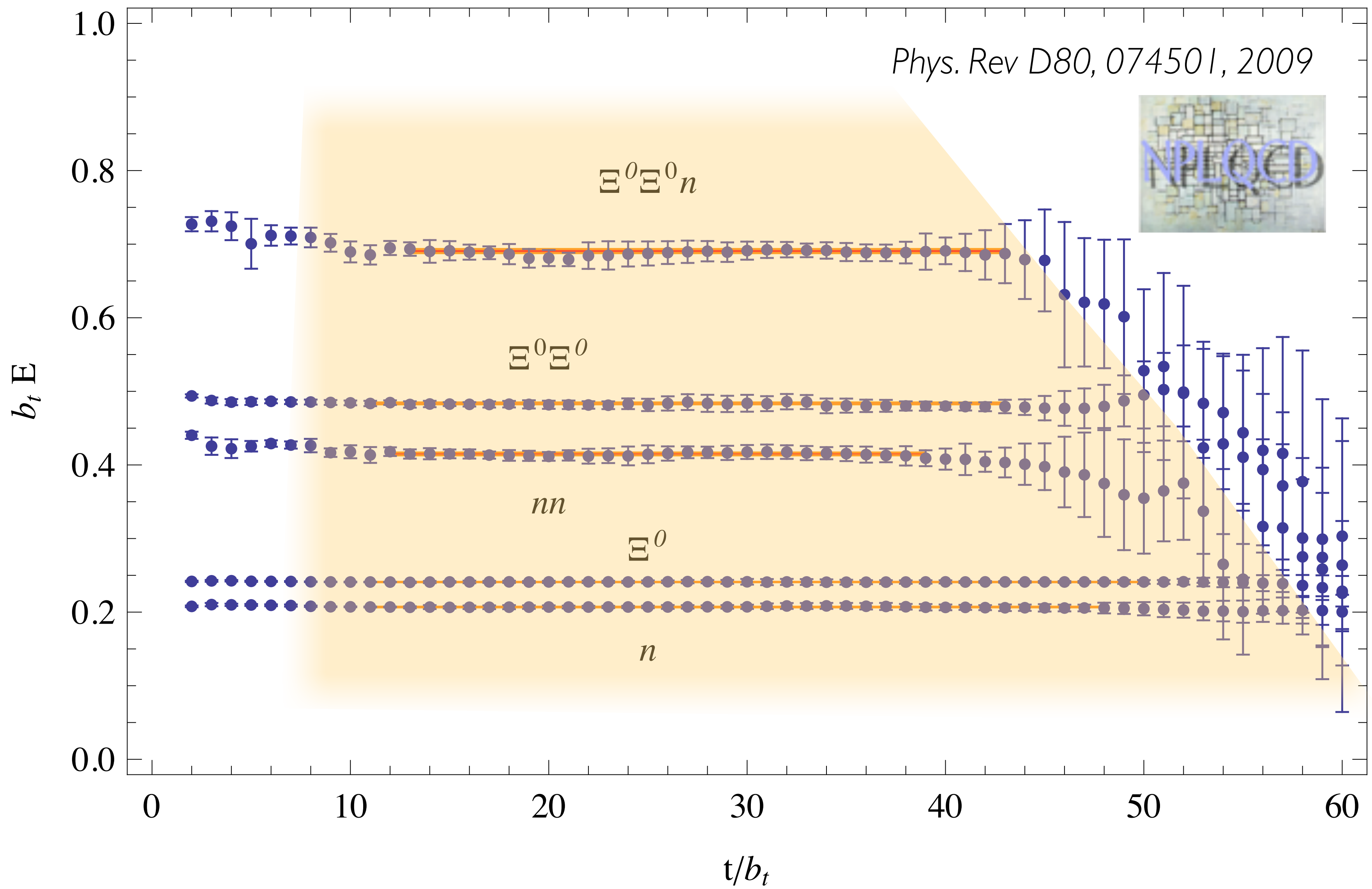

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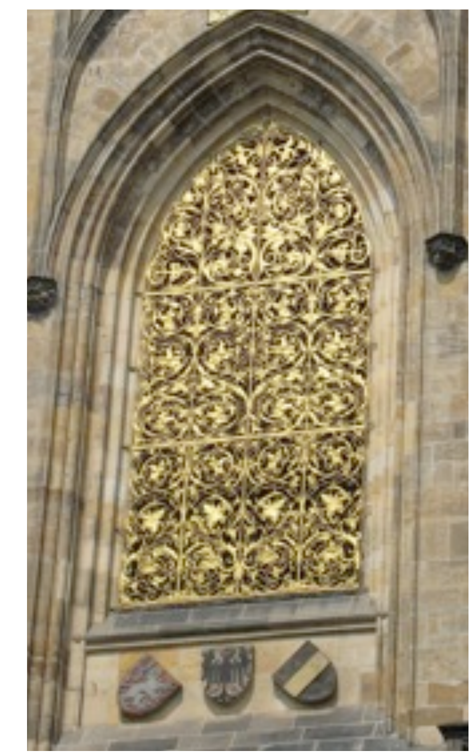
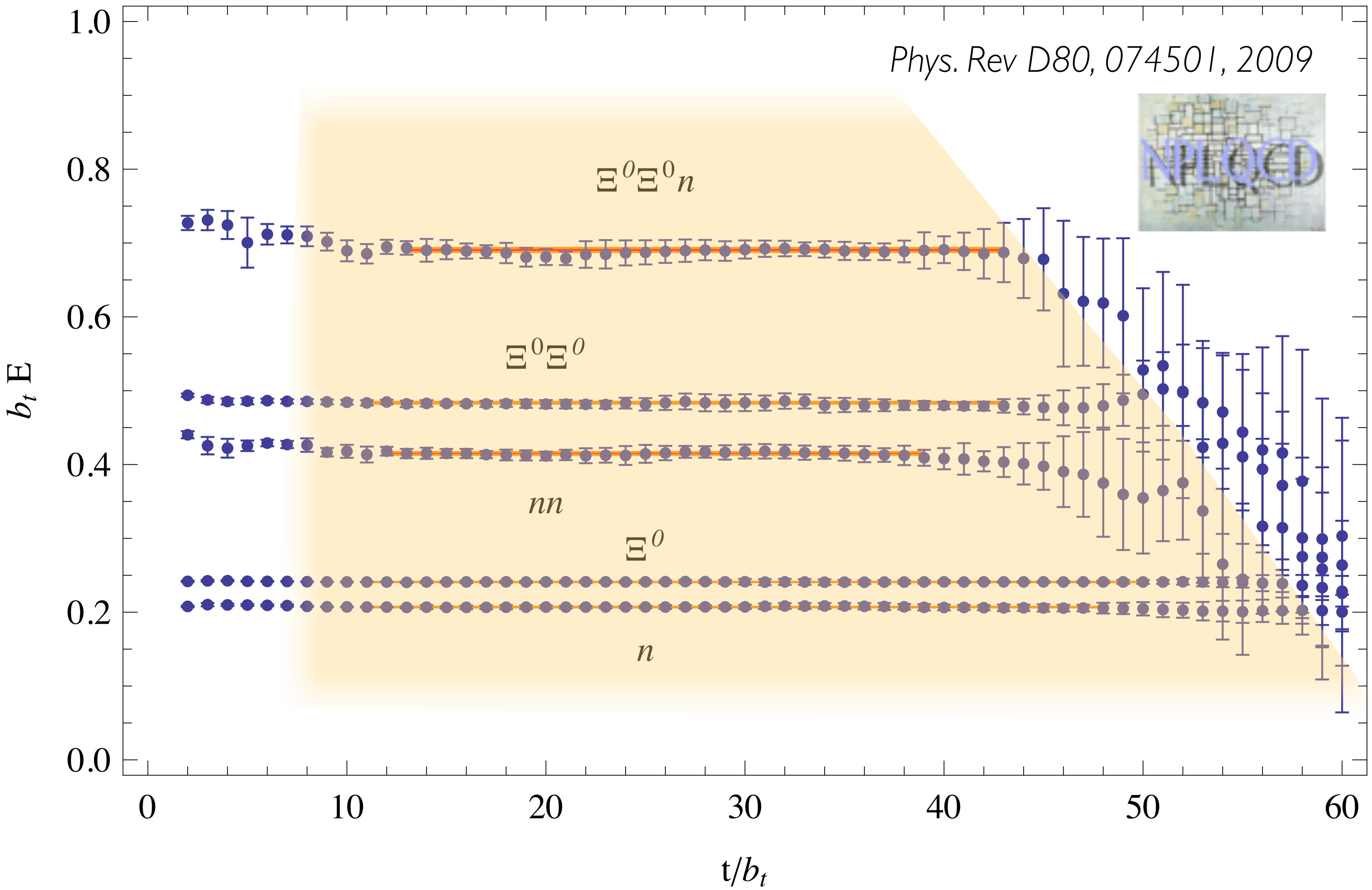


Golden window of time-slices where signal/noise const

No? trouble with baryons

High statistics study using anisotropic lattices (fine temporal resolution)



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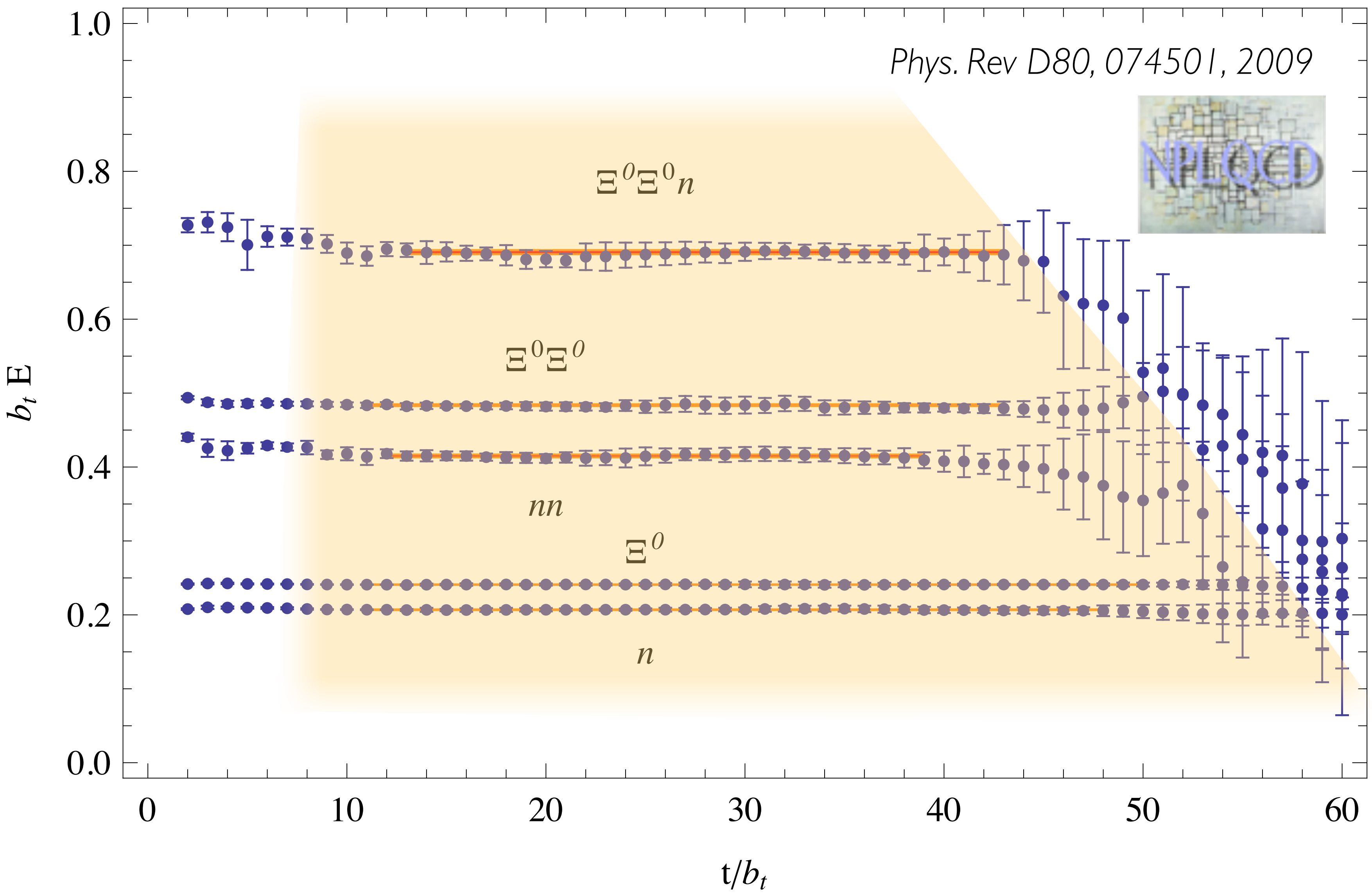

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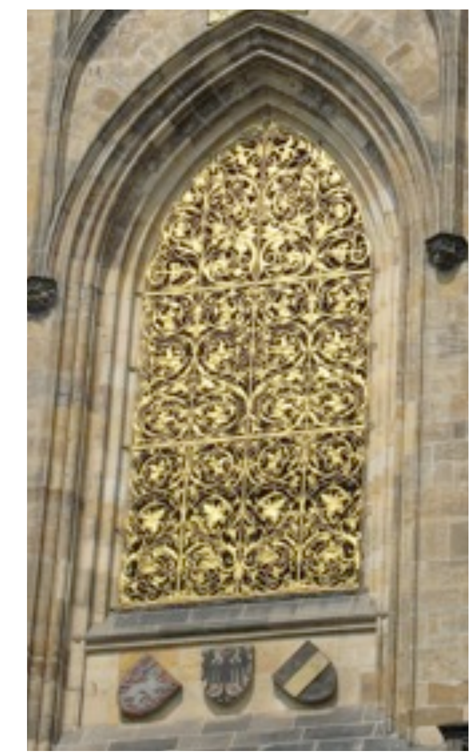
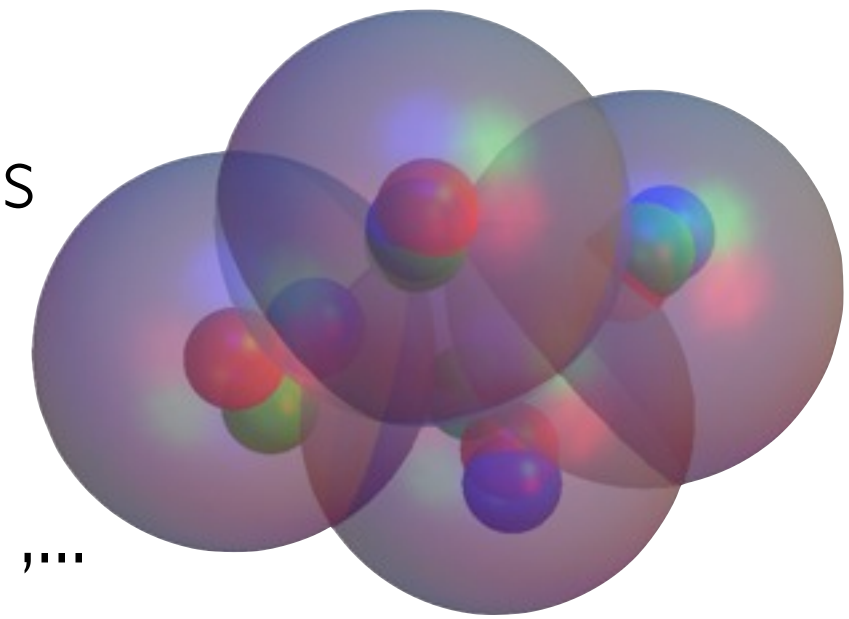


Golden window of time-slices where signal/noise const

Interpolator choice can be used to suppress noise

Multi-baryon systems

- Scattering and bound states
  - NB: Strong interaction bound states
- Dibaryons : H, deuteron, $\Xi\Xi$
- ^3H, ^4He and more exotic: $^4He_\Lambda$, $^4He_{\Lambda\Lambda}$,...
- Correlators for significantly larger A
- Caveat: at unphysical quark masses no electroweak interactions



Bound states at finite volume

- Two particle scattering amplitude in infinite volume

$$\mathcal{A}(p) = \frac{8\pi}{M} \frac{1}{p \cot \delta(p) - ip}$$

scattering phase shift

bound state at $p^2 = -\gamma^2$ when $\cot \delta(i\gamma) = i$

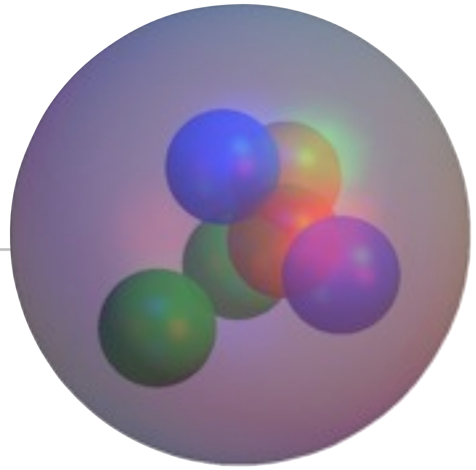
- Scattering amplitude in finite volume (Lüscher method)

$$\cot \delta(i\kappa) = i - i \sum_{\vec{m} \neq 0} \frac{e^{-|\vec{m}|\kappa L}}{|\vec{m}|\kappa L} \qquad \kappa \xrightarrow{L \to \infty} \gamma$$

  - Need multiple volumes

- More complicated for n>2 body bound states

H-dibaryon



- Jaffe [1977]: chromo-magnetic interaction

$$\langle H_m \rangle \sim \frac{1}{4}N(N-10) + \frac{1}{3}S(S+1) + \frac{1}{2}C_c^2 + C_f^2$$

most attractive for spin, colour, flavour singlet

  - H-dibaryon (uuddss) J=I=0, s=-2 most stable

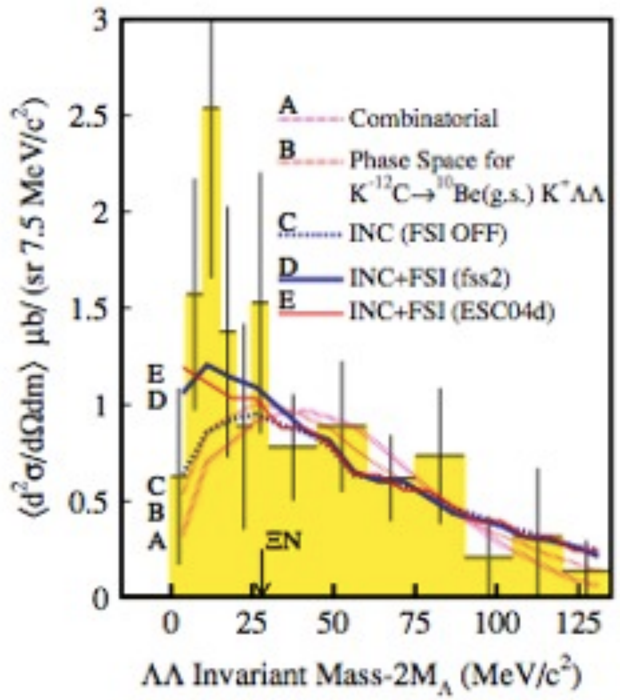
$$\Psi_H = \frac{1}{\sqrt{8}}\left(\Lambda\Lambda + \sqrt{3}\Sigma\Sigma + 2\Xi N\right)$$

  - Bound in a many hadronic models

- Experimental searches
  - Emulsion expts, heavy-ion, stopped kaons
  - No conclusive evidence for or against

KEK-ps (2007)

$K^- \, {}^{12}C \rightarrow K^+ \Lambda\Lambda X$



H dibaryon in QCD

- Early quenched studies on small lattices: mixed results
  [Mackenzie et al. 85, Iwasaki et al. 89, Pochinsky et al. 99, Wetzorke & Karsch 03, Luo et al. 07, Loan 11]

- Semi-realistic calculations

  - *"Evidence for a bound H dibaryon from lattice QCD"*
    PRL 106, 162001 (2011)
    N_f=2+1, a_s=0.12 fm, m_π=390 MeV, L=2.0, 2.5, 3.0, 3.9 fm

  - *"Bound H dibaryon in flavor SU(3) limit of lattice QCD"* *
    PRL 106, 162002 (2011)
    N_f=3, a_s=0.12 fm, m_π=670, 830, 1015 MeV, L=2.0, 3.0, 3.9 fm

- NB: Quark masses unphysical, single lattice spacing

* use a somewhat different method

H dibaryon in QCD

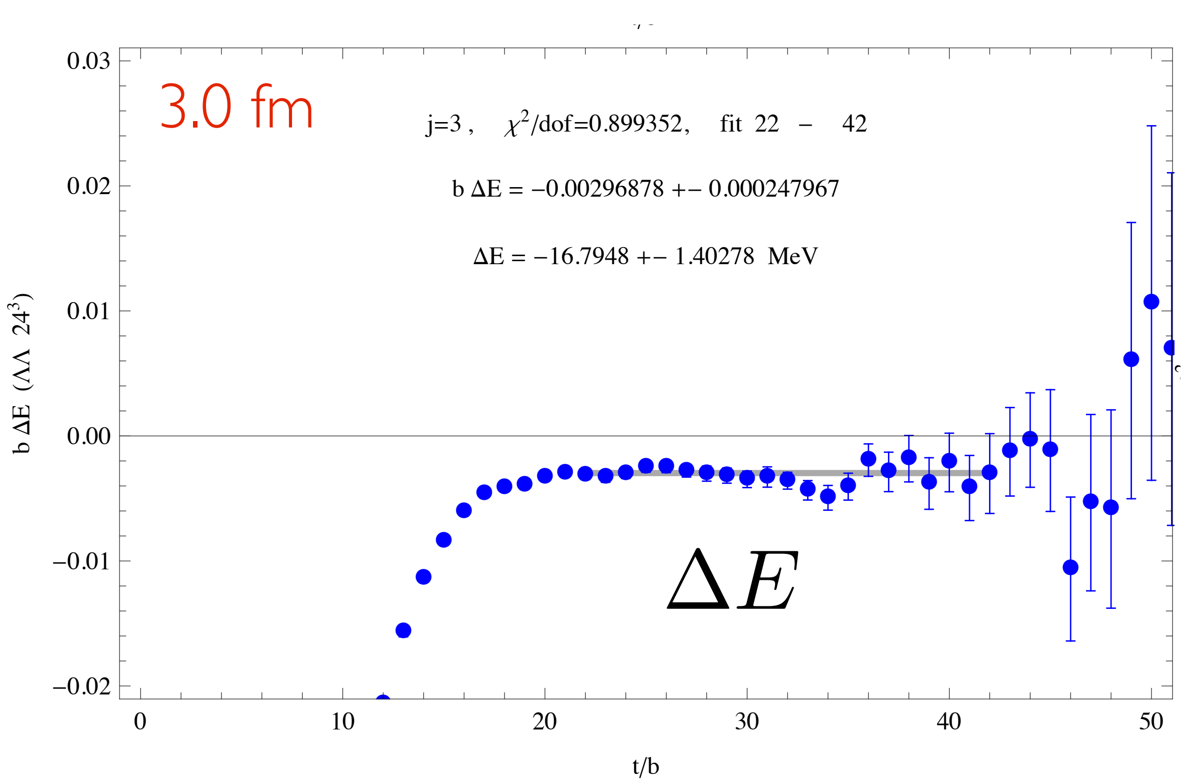
- Extract energy eigenstates from large Euclidean time behaviour of two-point correlators

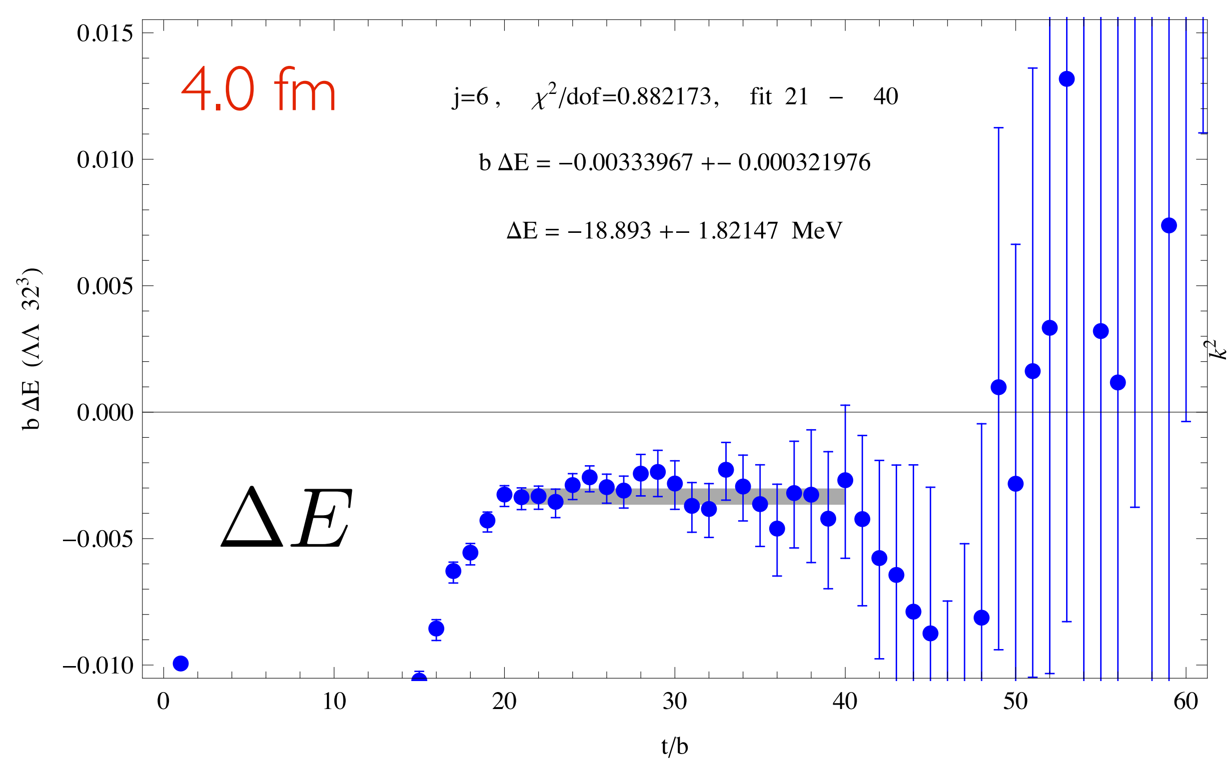
$$C_\Lambda(t) = \sum_{\mathbf{x}} \langle 0|\chi(\mathbf{x},t)\overline{\chi}(0)|0\rangle \stackrel{t\to\infty}{\longrightarrow} Z_\Lambda e^{-M_\Lambda t}$$

$$C_{\Lambda\Lambda}(t) = \sum_{\mathbf{x}} \langle 0|\phi(\mathbf{x},t)\overline{\phi}(0)|0\rangle \stackrel{t\to\infty}{\longrightarrow} Z_{\Lambda\Lambda} e^{-E_{\Lambda\Lambda} t}$$

$$R(t) = \frac{C_{\Lambda\Lambda}(t)}{C_\Lambda^2(t)} \stackrel{t\to\infty}{\longrightarrow} \widetilde{Z} e^{-\Delta E_{\Lambda\Lambda} t}$$

- Correlator ratio allows direct access to energy shift





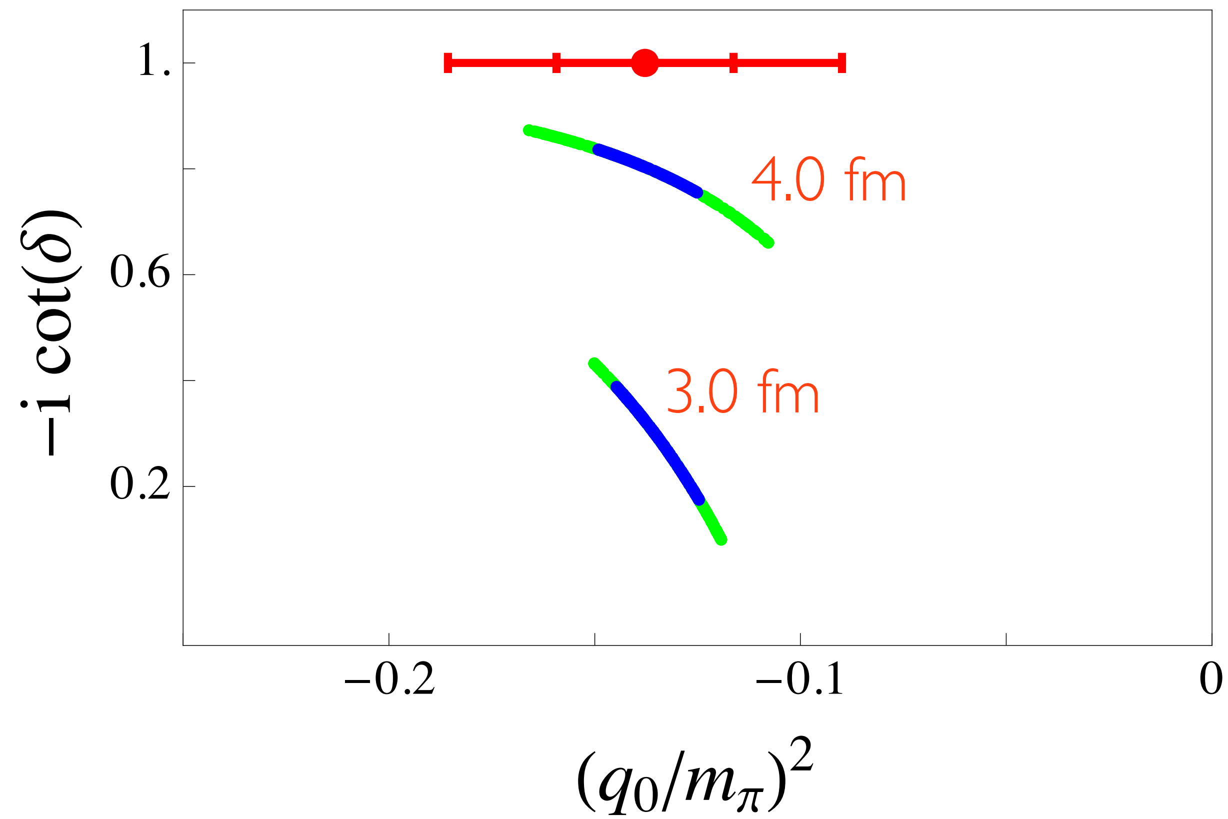
Simple extrapolations

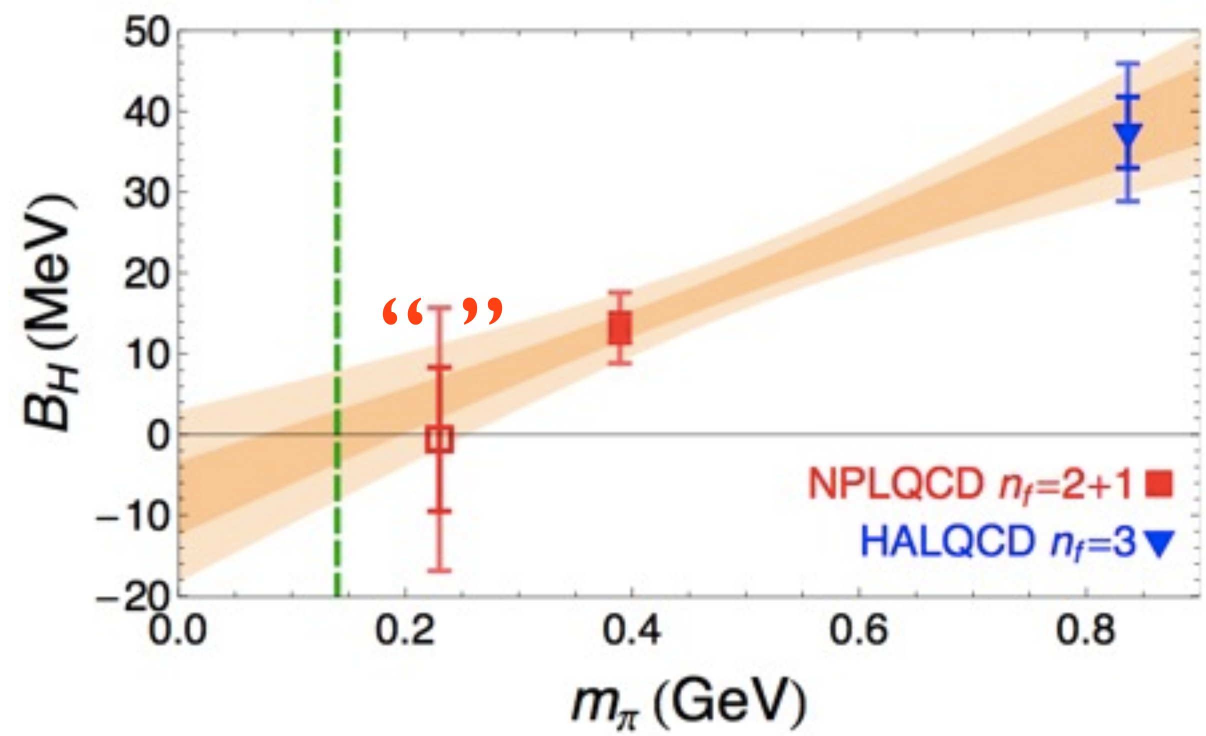
- After volume extrapolation H bound at unphysical quark masses
- Quark mass extrapolation is uncertain and unconstrained

$$B_H^{\text{quad}} = +11.5 \pm 2.8 \pm 6.0 \text{ MeV}$$

$$B_H^{\text{lin}} = +4.9 \pm 4.0 \pm 8.3 \text{ MeV}$$

- Other extrapolations, see
  [Shanahan, Thomas & Young PRL. 107 (2011) 092004, Haidenbauer & Meissner 1109.3590]
- Suggests H is weakly bound or just unbound

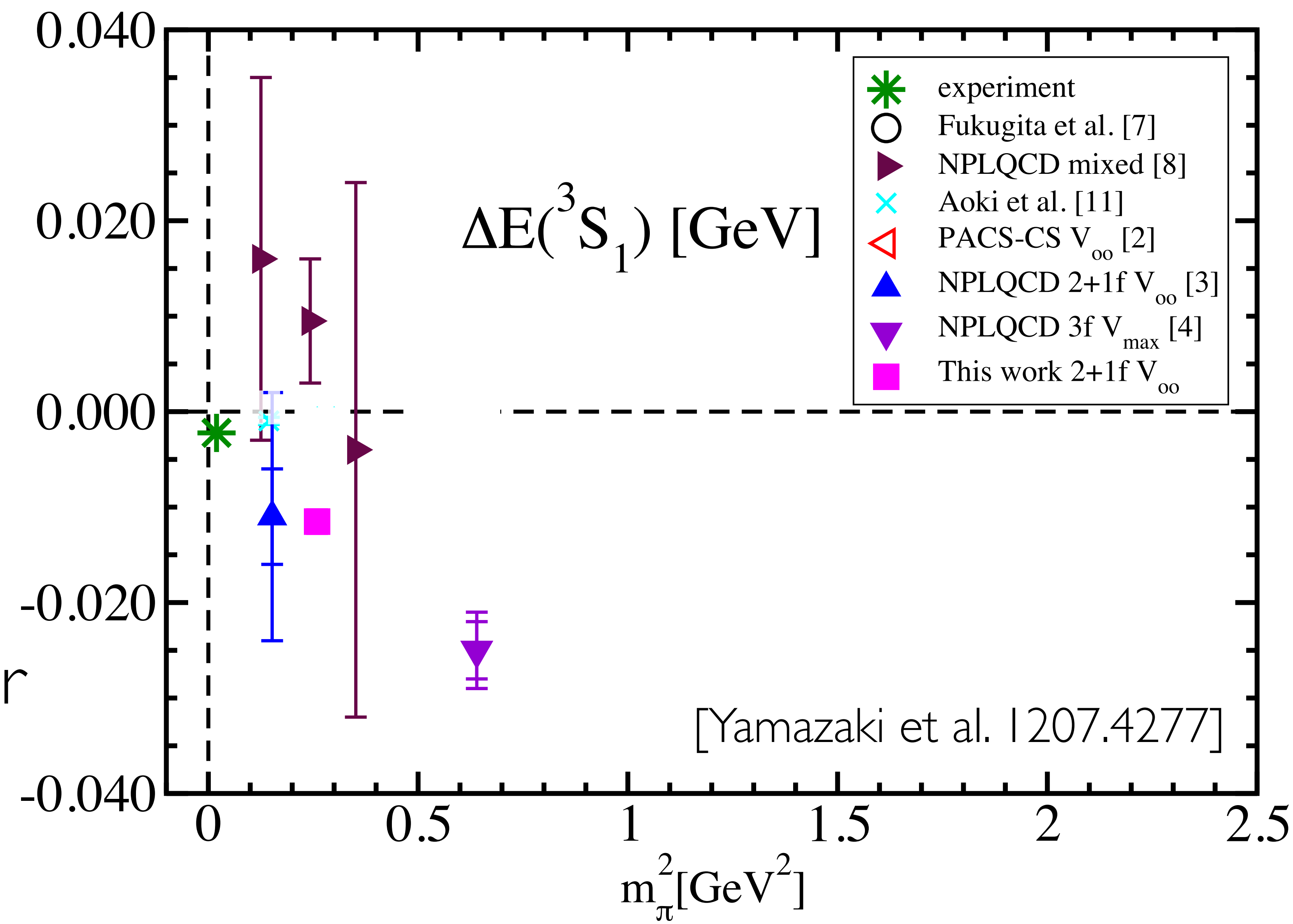




* 230 MeV point preliminary (one volume)

Deuteron

- Deuteron also investigated
  - NPLQCD
  - PACS-CS
- More work needed at lighter masses



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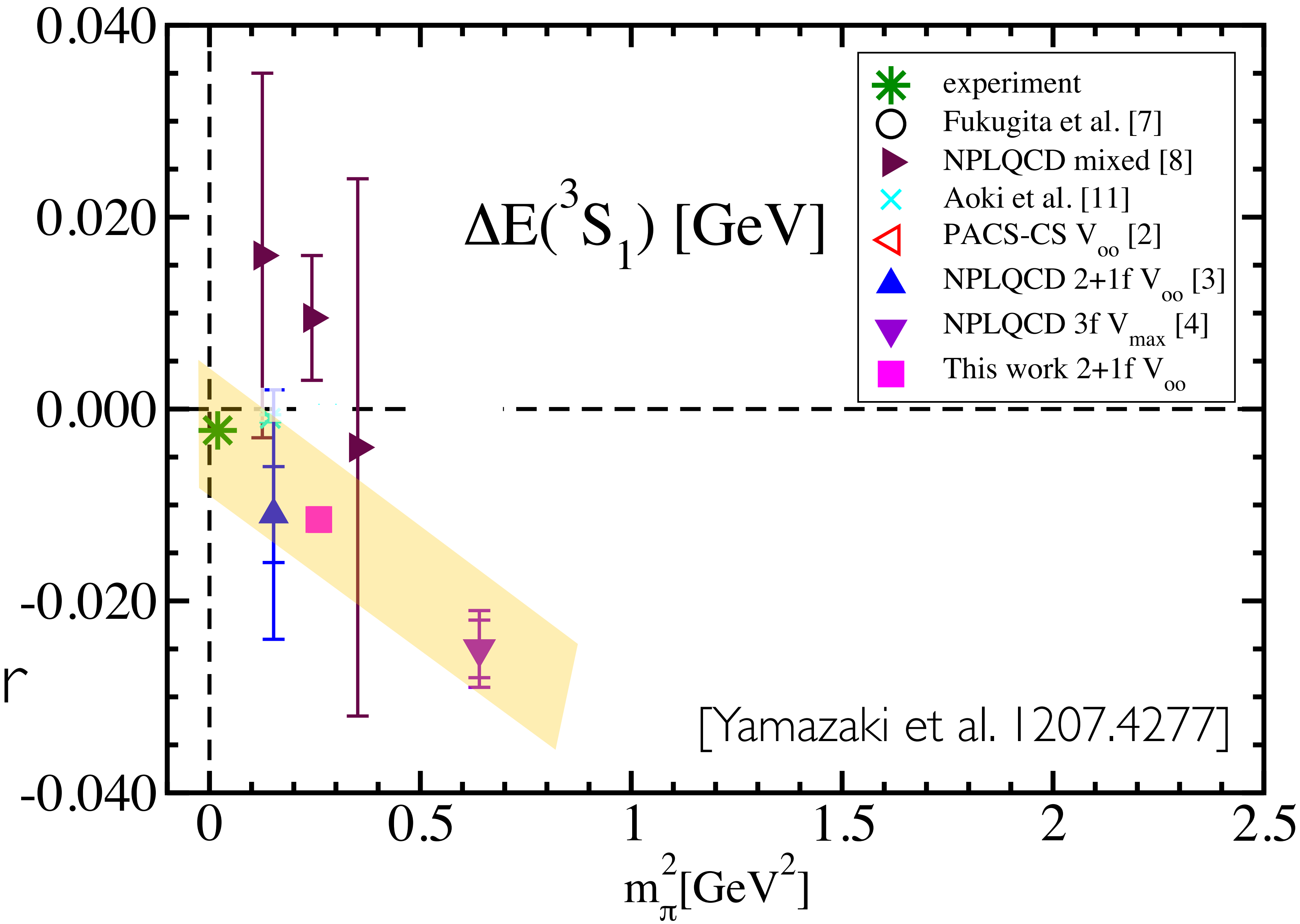


Many baryon systems

- Many baryon correlator construction is somewhat messy
- Interpolating fields – minimal expression as weighted sums

color/spin/flavour/spatial indices

$$\bar{\mathcal{N}}^h = \sum_{k=1}^{N_w} \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \sum_{\mathbf{i}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

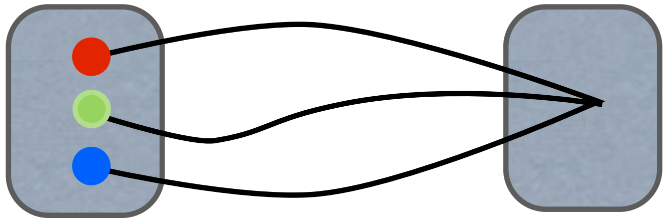
- Generation of weights can be automated (symbolic c++ code) for given quantum numbers
  - Specify final quantum numbers (spin, isospin, strangeness etc)
  - Build up from states of smaller quantum numbers just by using rules of eg angular momentum addition
- Similar ideas by Doi and Endres [1205.0585]
- Contraction just reads in weights and can be implemented independent of the particular process being considered

[WD, K Orginos, 1207.1452 *]*

Many baryon systems

- Given a complex many baryon system to perform contractions for, always possible to group colour singlets at one end (sink)
- Contractions can be written in terms of baryon blocks (objects that are contracted at sink)
- A particular set of quantum numbers b for the block is select by a weighted sum of components of quark propagators

$$\mathcal{B}_b^{a_1,a_2,a_3}(\mathbf{p},t;x_0) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \sum_{k=1}^{N_{B(b)}} \tilde{w}_b^{(c_1,c_2,c_3),k} \sum_{\mathbf{i}} \epsilon^{i_1,i_2,i_3}$$
$$\times S(c_{i_1},x;a_1,x_0)S(c_{i_2},x;a_2,x_0)S(c_{i_3},x;a_3,x_0)$$



- Can be generalised to multi-baryon blocks if desired although storage requirements rapidly increase

Many baryon systems

$$\left[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)\right]_U = \int \mathcal{D}q\mathcal{D}\bar{q}\; e^{-S_{QCD}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h'^{(a'_1,a'_2\cdots a'_{n_q}),k'}\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}}\epsilon^{i_1,i_2,\cdots,i_{n_q}} q(a'_{j_{n_q}})\cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

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$$= e^{-\mathcal{S}_{eff}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\;\tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k}\times$$

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- Make a particular choice of correlation function (momentum projection at sink) and express in terms of blocks (quark-hadron level contraction)

Many baryon systems

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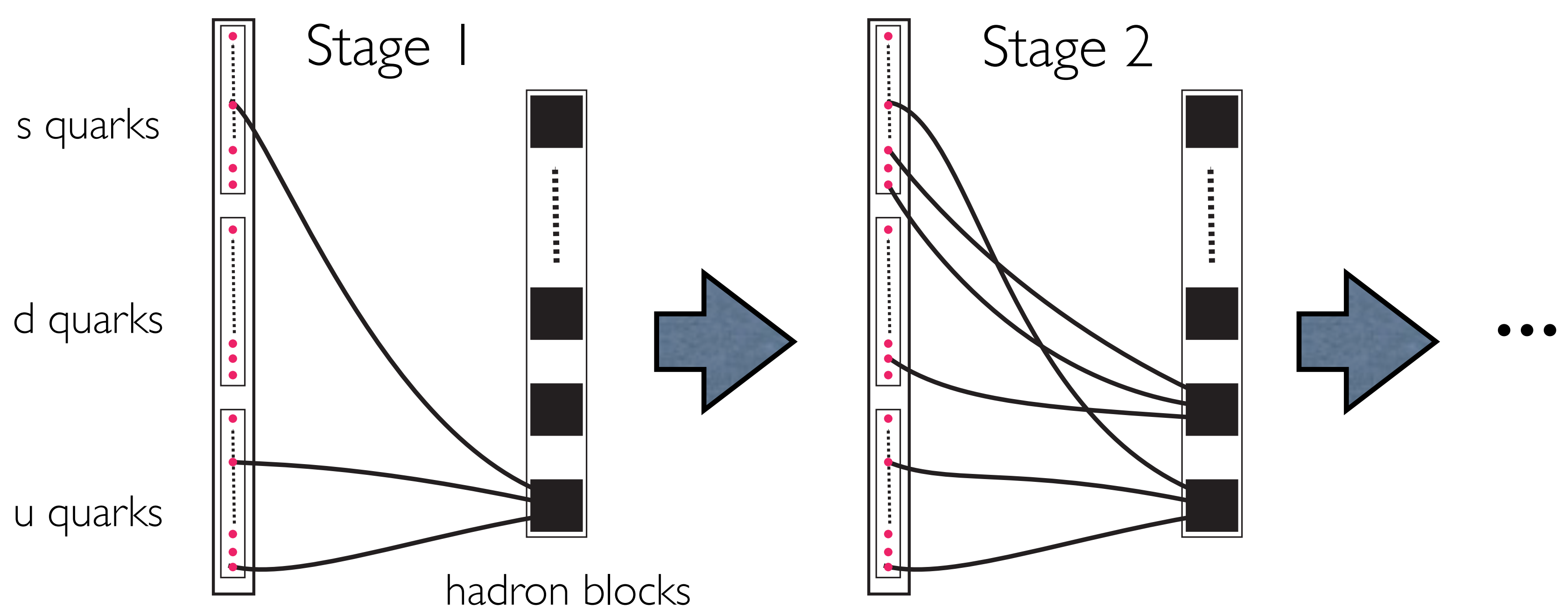

Many baryon systems

$$\left[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)\right]_U = \int \mathcal{D}q\mathcal{D}\bar{q}\; e^{-S_{QCD}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}}\epsilon^{i_1,i_2,\cdots,i_{n_q}} q(a'_{j_{n_q}})\cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

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- Or write as determinant (quark-quark level contraction)

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where

$$G(\mathbf{a}';\mathbf{a})_{j,i} = \begin{cases} S(a'_j;a_i) & a'_j\in\mathbf{a}' \text{ and } a_i\in\mathbf{a} \\ \delta_{a'_j,a_i} & \text{otherwise} \end{cases}$$

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- Determinant can be evaluated in polynomial number of operations (LU decomposition)

Nuclei

- Recent studies at SU(3) point (physical m_s)
  - Isotropic clover lattices
  - Single lattice spacing: 0.145 fm
  - Multiple volumes: 3.4, 4.5, 6.7 fm
  - High statistics

Label	L/b	T/b	β	$b\ m_q$	b [fm]	L [fm]	T [fm]	m_π [MeV]	$m_\pi\ L$	$m_\pi\ T$	$N_{\rm cfg}$	$N_{\rm src}$
A	24	48	6.1	-0.2450	0.145	3.4	6.7	806.5(0.3)(0)(8.9)	14.3	28.5	3822	48
B	32	48	6.1	-0.2450	0.145	4.5	6.7	806.9(0.3)(0.5)(8.9)	19.0	28.5	3050	24
C	48	64	6.1	-0.2450	0.145	6.7	9.0	806.7(0.3)(0)(8.9)	28.5	38.0	1212	32

SU(3) symmetric world

- In flavour SU(3) symmetric case, multi-baryon states come in multiplets

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{27} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{1}$$

$$\mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} = \mathbf{64} \oplus 2\ \mathbf{35} \oplus 2\ \overline{\mathbf{35}} \oplus 6\ \mathbf{27} \oplus 4\ \mathbf{10} \oplus 4\ \overline{\mathbf{10}} \oplus 8\ \mathbf{8} \oplus 2\ \mathbf{1}$$

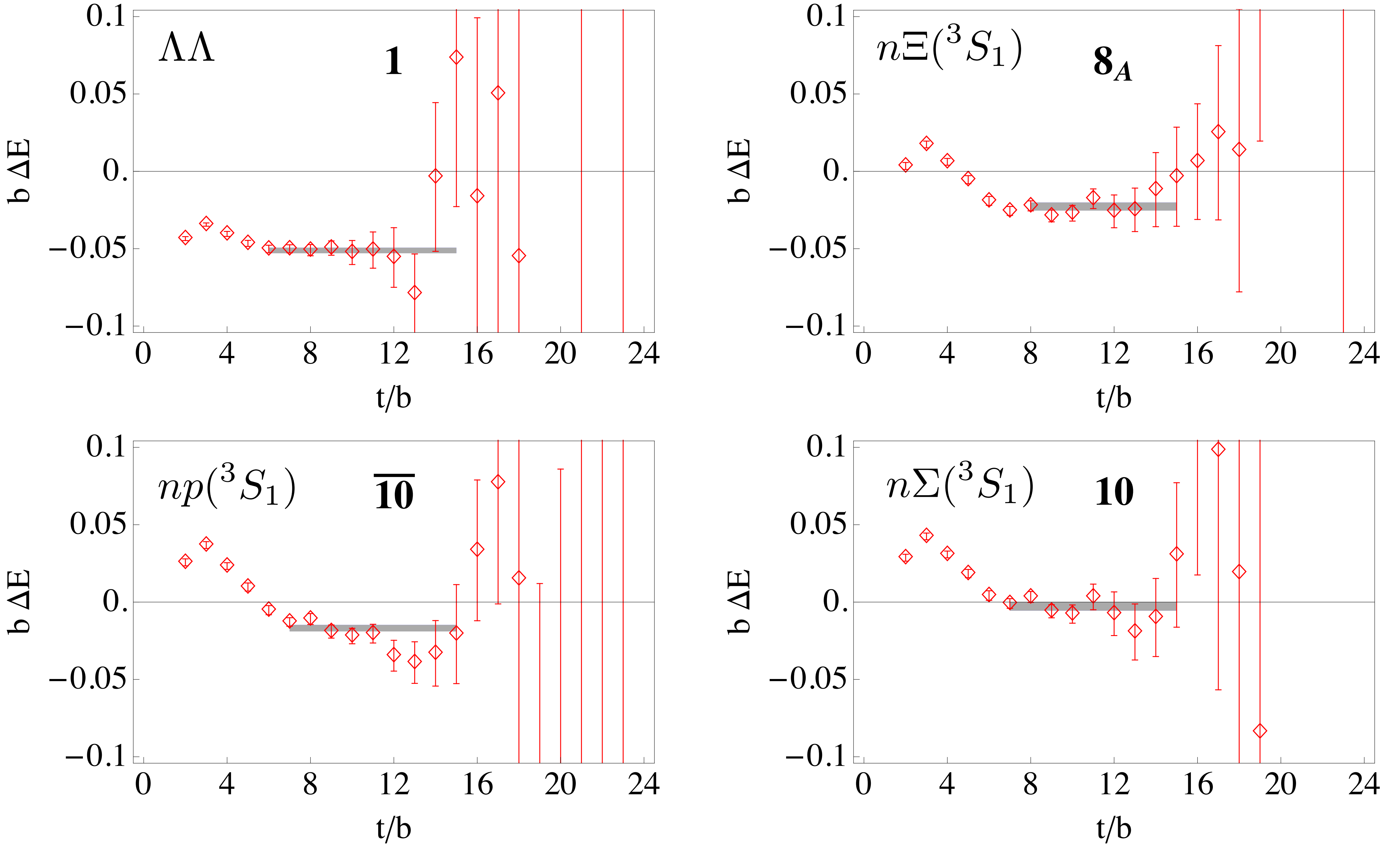
$$\begin{aligned}\mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} = {} & 8\ \mathbf{1} \oplus 32\ \mathbf{8} \oplus 20\ \mathbf{10} \oplus 20\ \overline{\mathbf{10}} \oplus 33\ \mathbf{27} \oplus 2\ \mathbf{28} \oplus 2\ \overline{\mathbf{28}} \oplus 15\ \mathbf{35} \oplus 15\ \overline{\mathbf{35}} \\ & \oplus 12\ \mathbf{64} \oplus 3\ \mathbf{81} \oplus 3\ \overline{\mathbf{81}} \oplus \mathbf{125} \quad , \qquad (1\end{aligned}$$

$$\begin{aligned}\mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} = {} & 32\ \mathbf{1} \oplus 145\ \mathbf{8} \oplus 100\ \mathbf{10} \oplus 100\ \overline{\mathbf{10}} \oplus 180\ \mathbf{27} \oplus 20\ \mathbf{28} \oplus 20\ \overline{\mathbf{28}} \\ & \oplus 100\ \mathbf{35} \oplus 100\ \overline{\mathbf{35}} \oplus 94\ \mathbf{64} \oplus 5\ \mathbf{80} \oplus 5\ \overline{\mathbf{80}} \oplus 36\ \mathbf{81} \oplus 36\ \overline{\mathbf{81}} \\ & \oplus 20\ \mathbf{125} \oplus 4\ \mathbf{154} \oplus 4\ \overline{\mathbf{154}} \oplus \mathbf{216} \quad .\end{aligned}$$

- Unphysical symmetries manifest in spectrum

Nuclei (A=2)

Quark-hadron contraction method



NPLQCD arXiv:1206.5219

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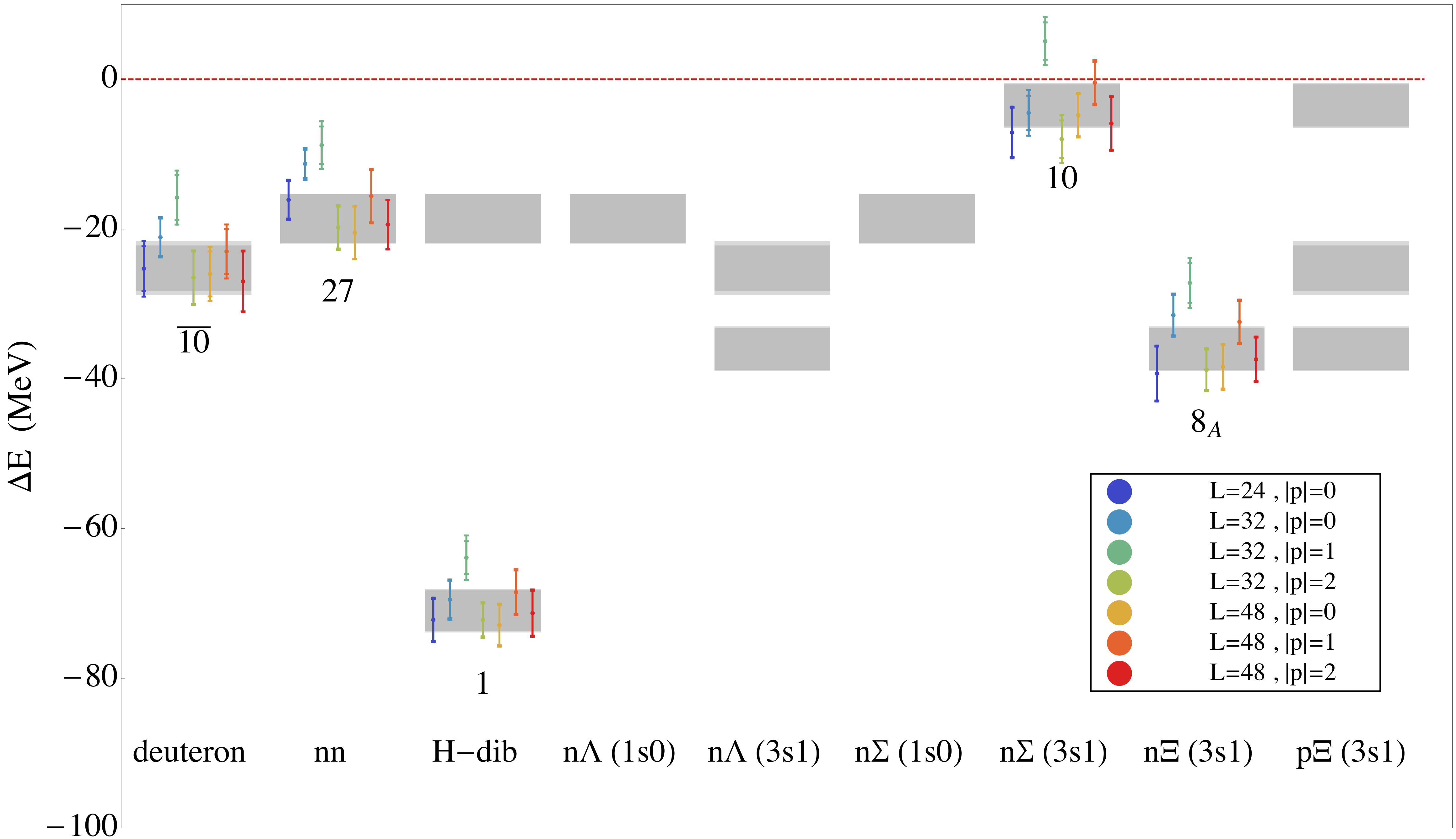

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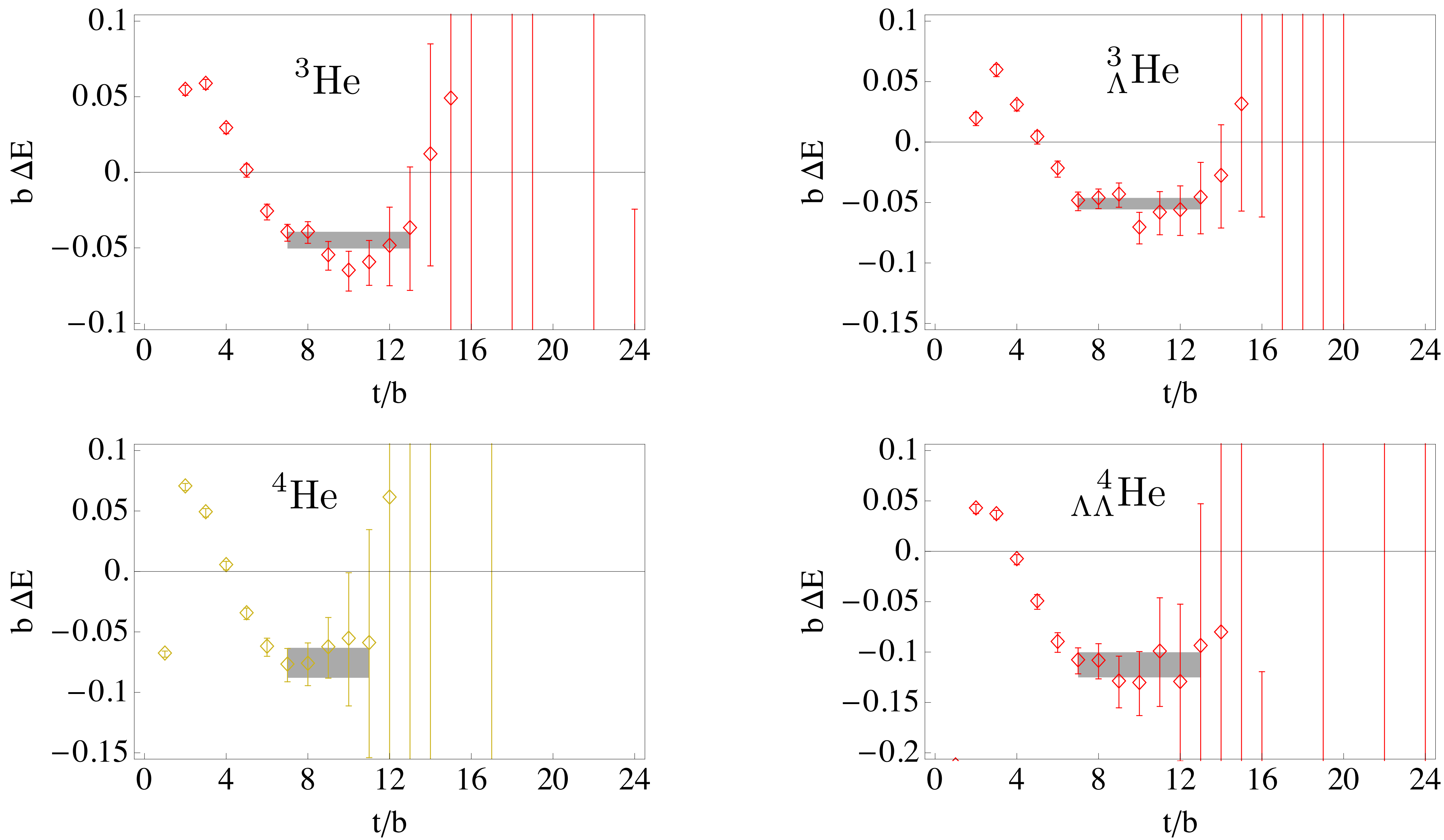
Nuclei (A=2,3,4)

Quark-hadron contraction method



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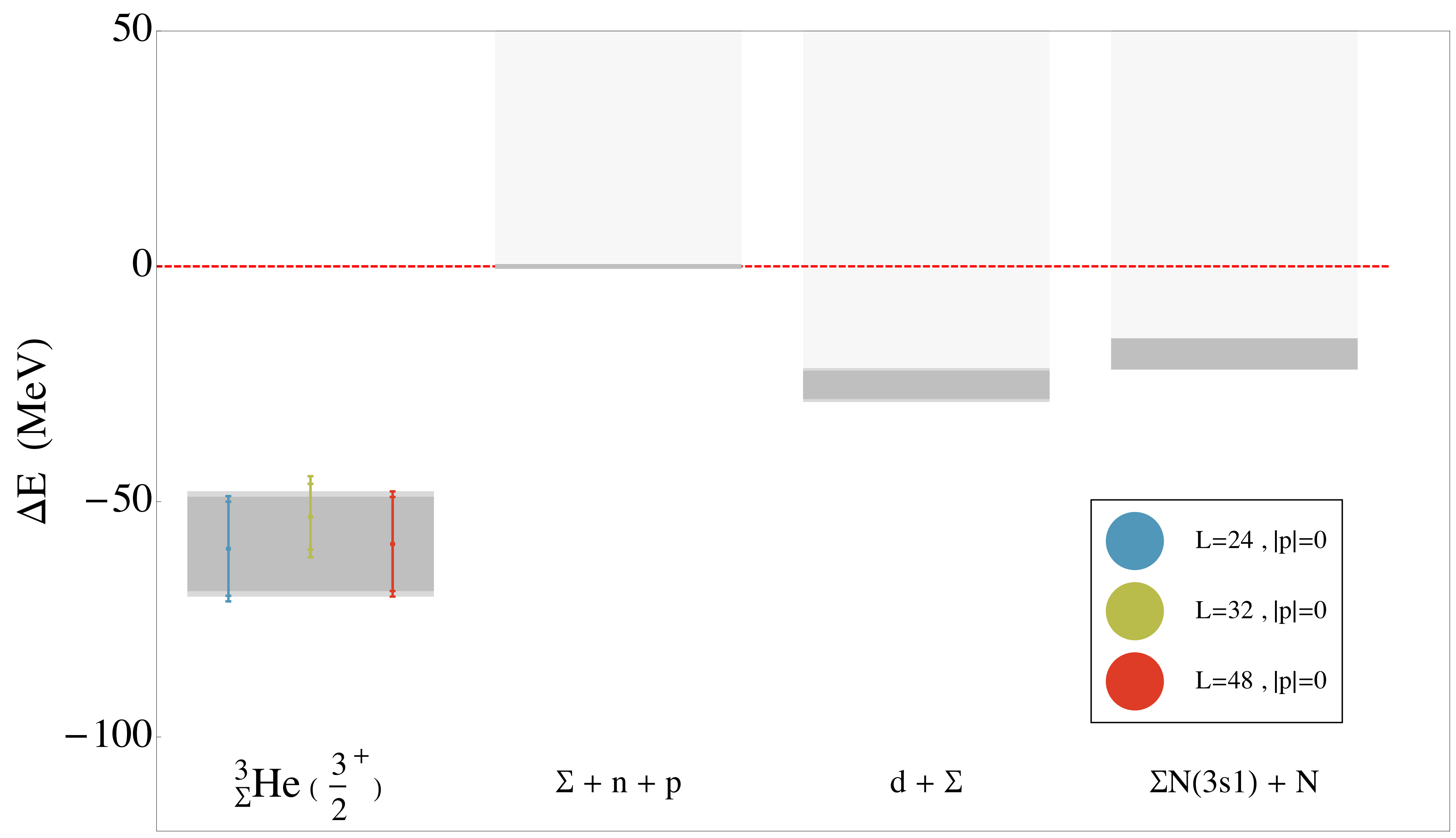
Nuclei (A=3,4)

- Empirically investigate volume dependence
- Need to ask if this is a 2+1 or 3+1 or 2+2 etc scattering state



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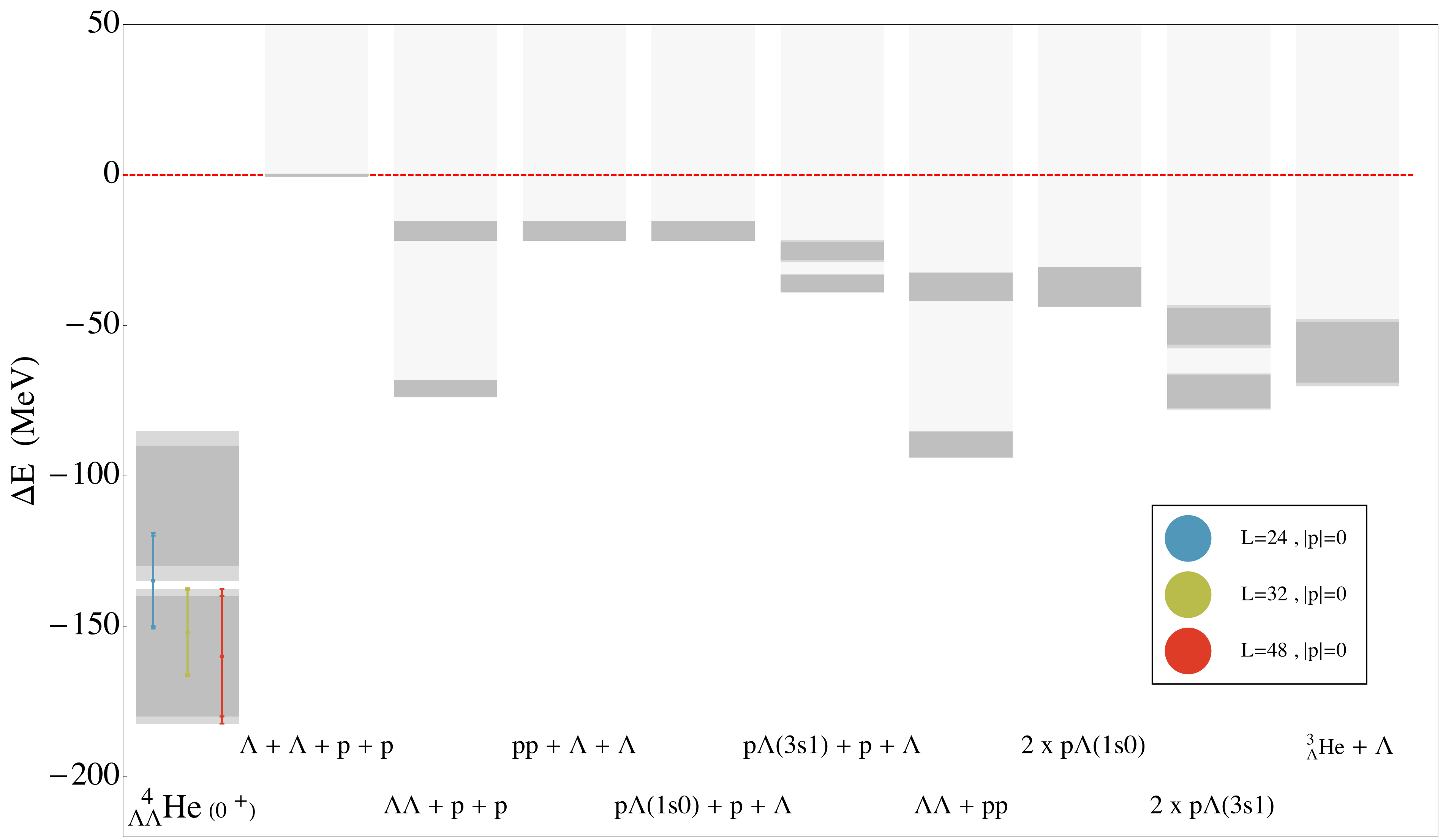

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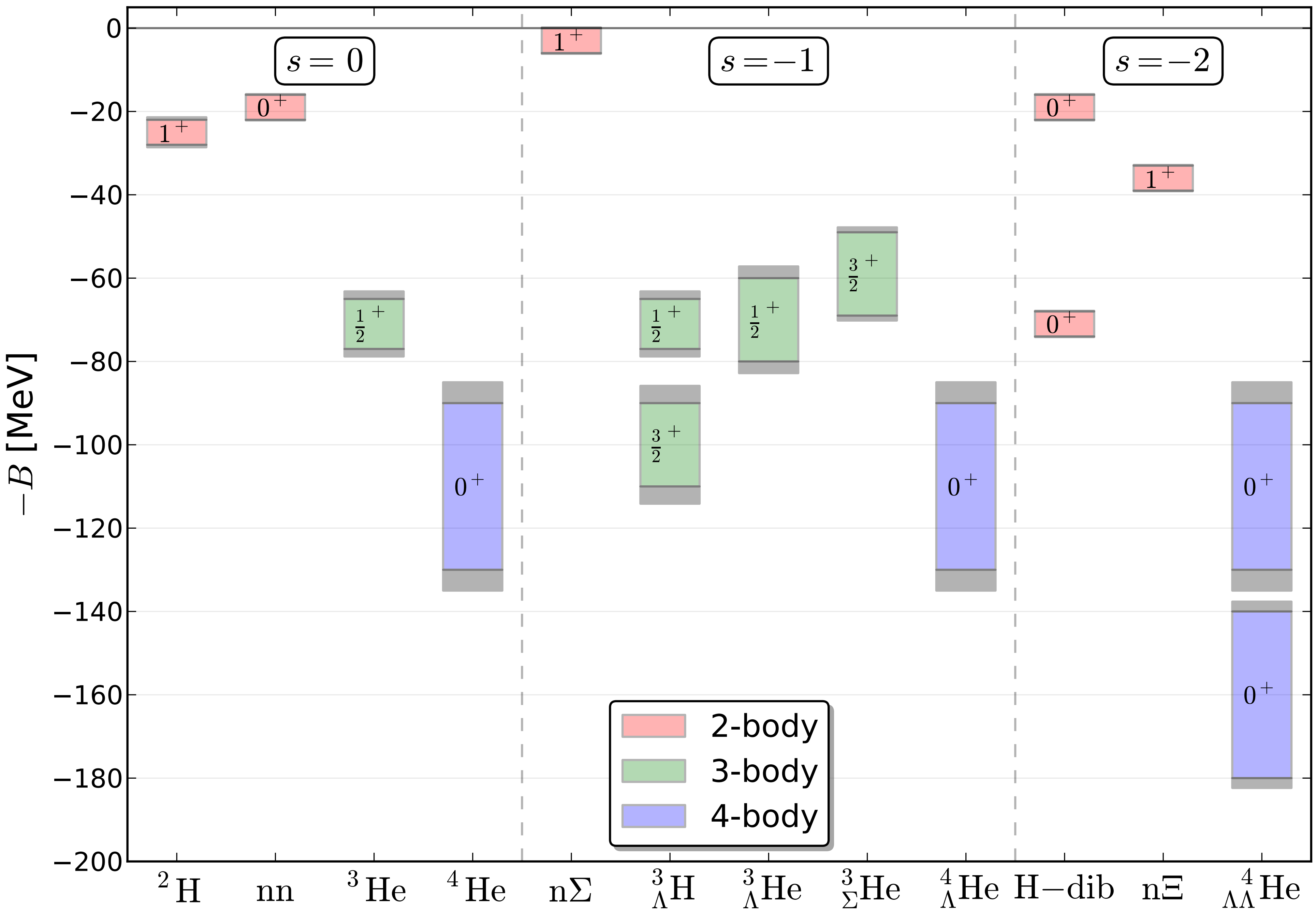
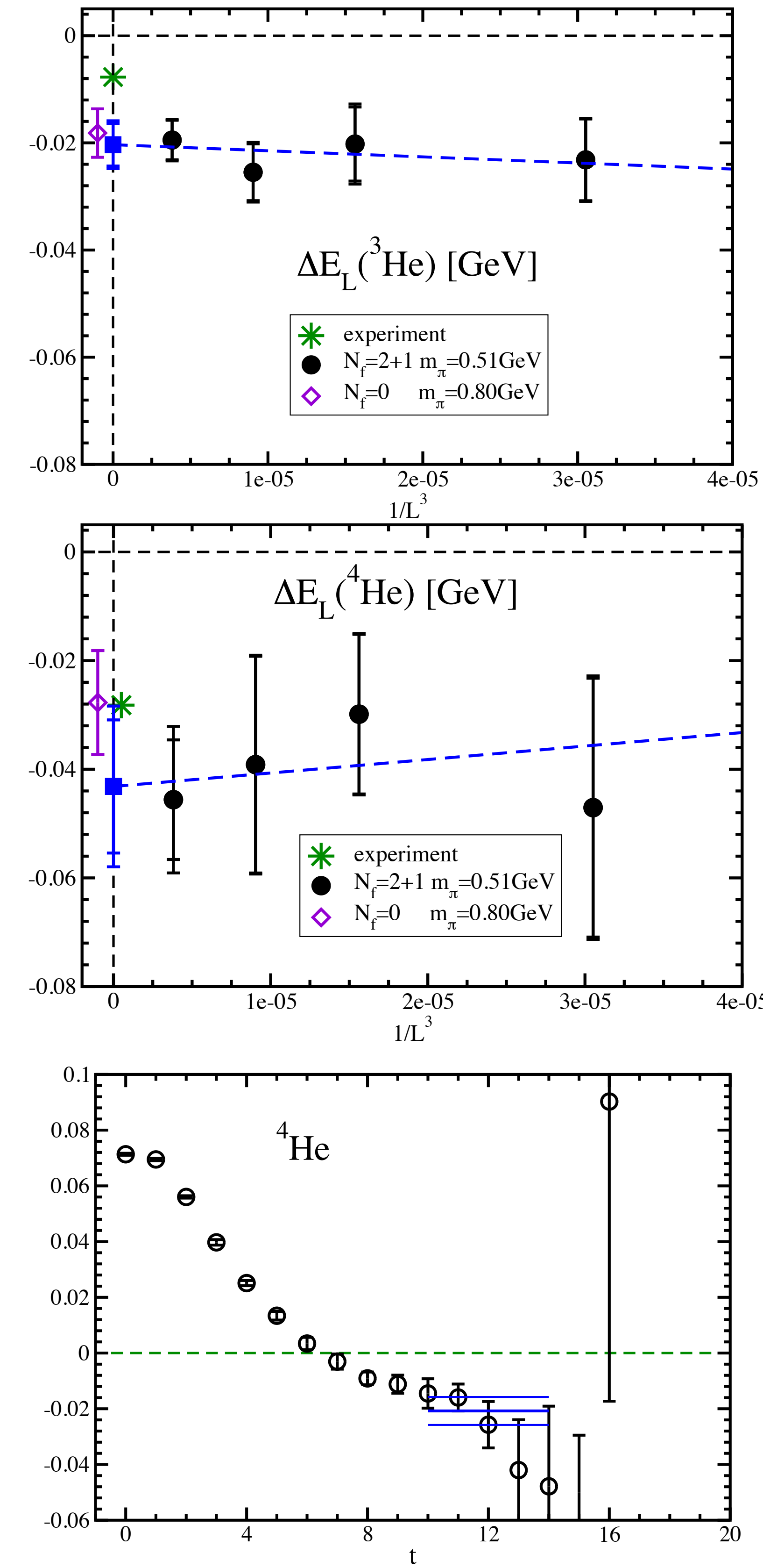


NPLQCD arXiv:1206.5219

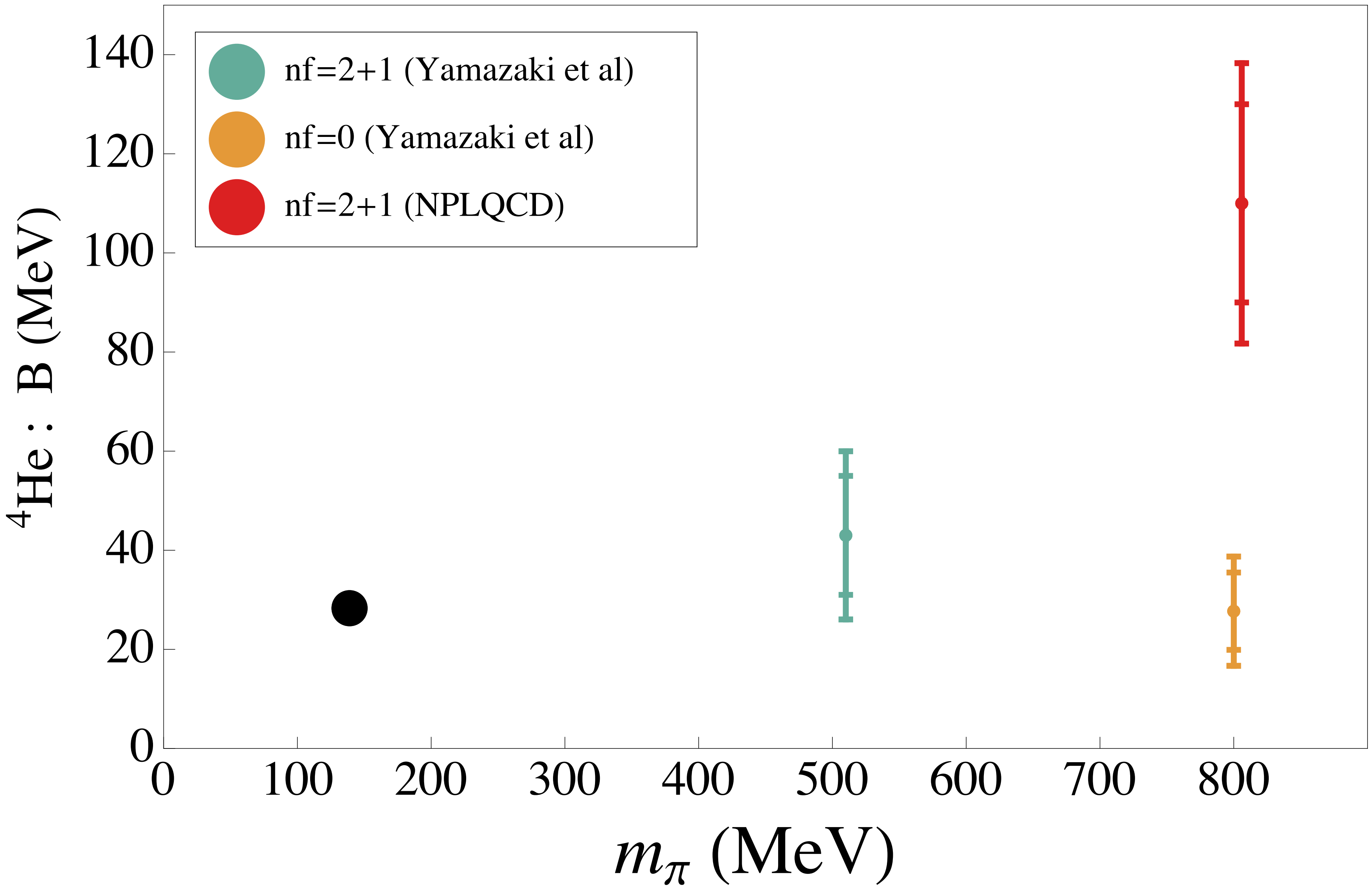
d, nn, ^{3}He, ^{4}He

- PACS-CS: bound d,nn, ^{3}He, ^{4}He
  - Previous quenched work
  - Recent unquenched study at m_π=500 MeV
- HALQCD
  - Extract an NN potential
  - Strong enough to bind H, ^{4}He at m_{PS}=490 MeV SU(3) pt
  - d, nn not bound



[Yamazaki et al. 1207.4277]

^{4}He binding



nf=2+1 (Yamazaki et al)
nf=0 (Yamazaki et al)
nf=2+1 (NPLQCD)
^{4}He : B (MeV)
m_π (MeV)
0
20
40
60
80
100
120
140
0
100
200
300
400
500
600
700
800

Nuclei (A=4,...)

Quark-quark determinant contraction method

(low statistics, single volume)

WD, Kostas Orginos, 1207.1452

Nuclei (A=4,...)

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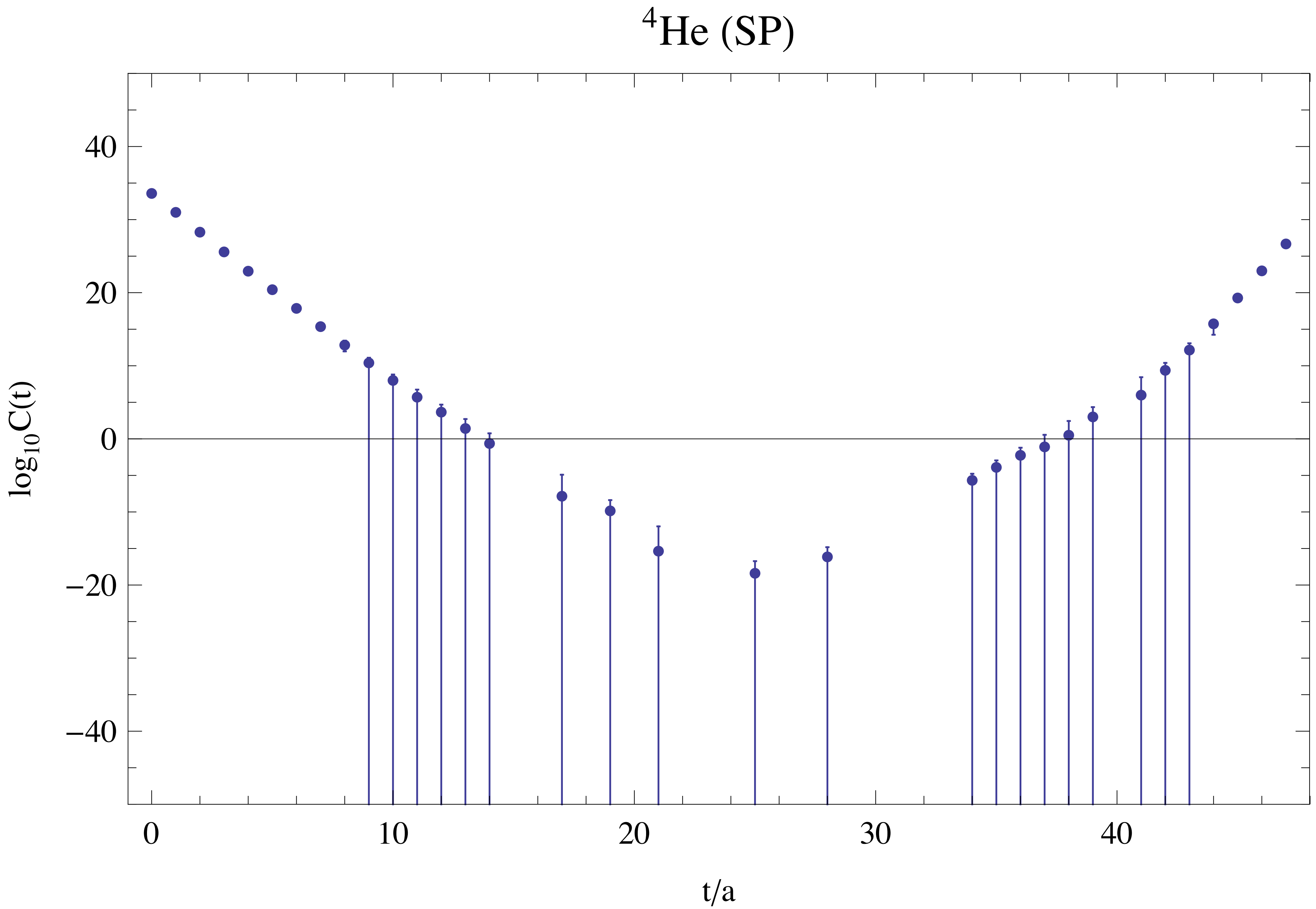

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WD, Kostas Orginos, 1207.1452

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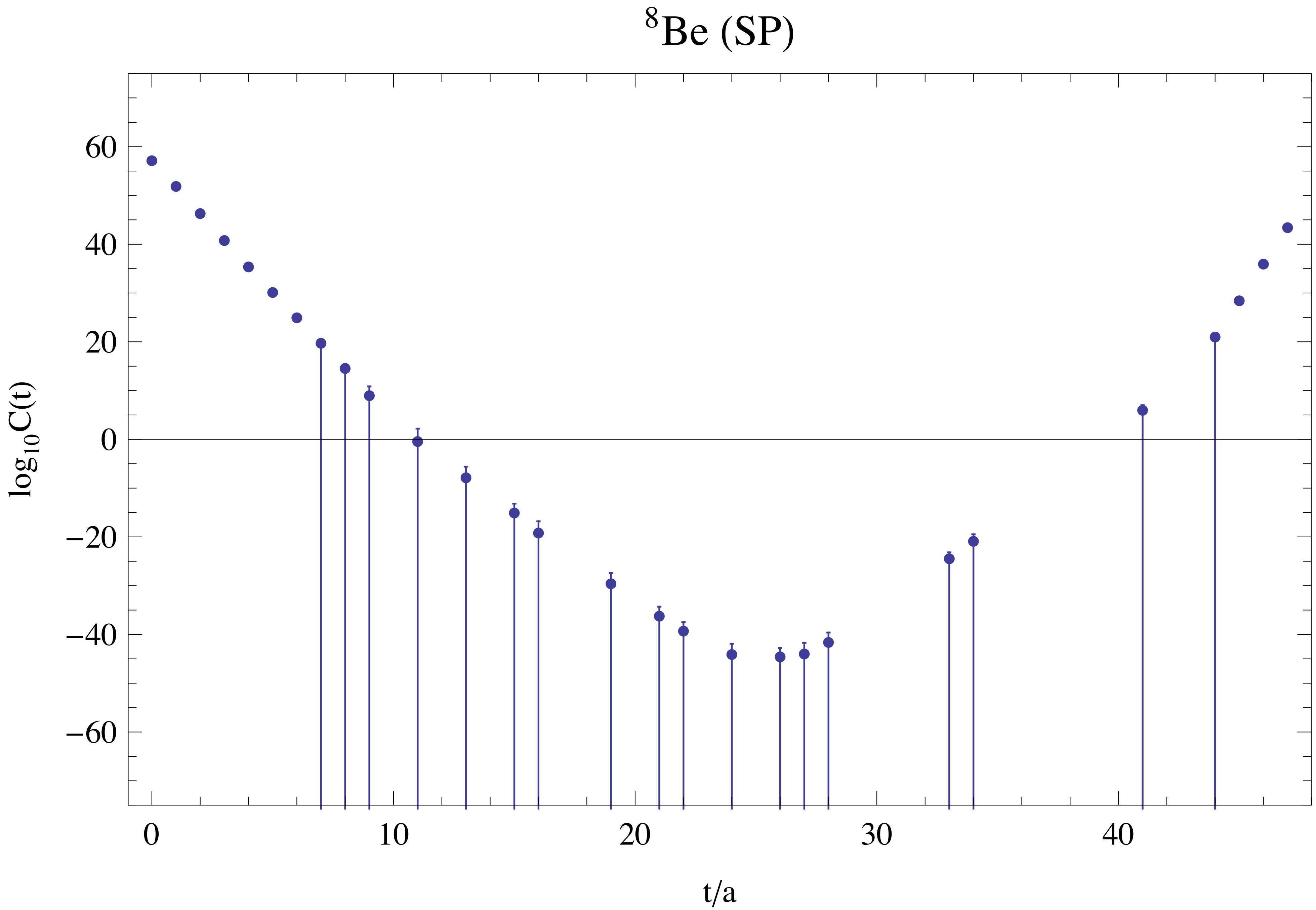

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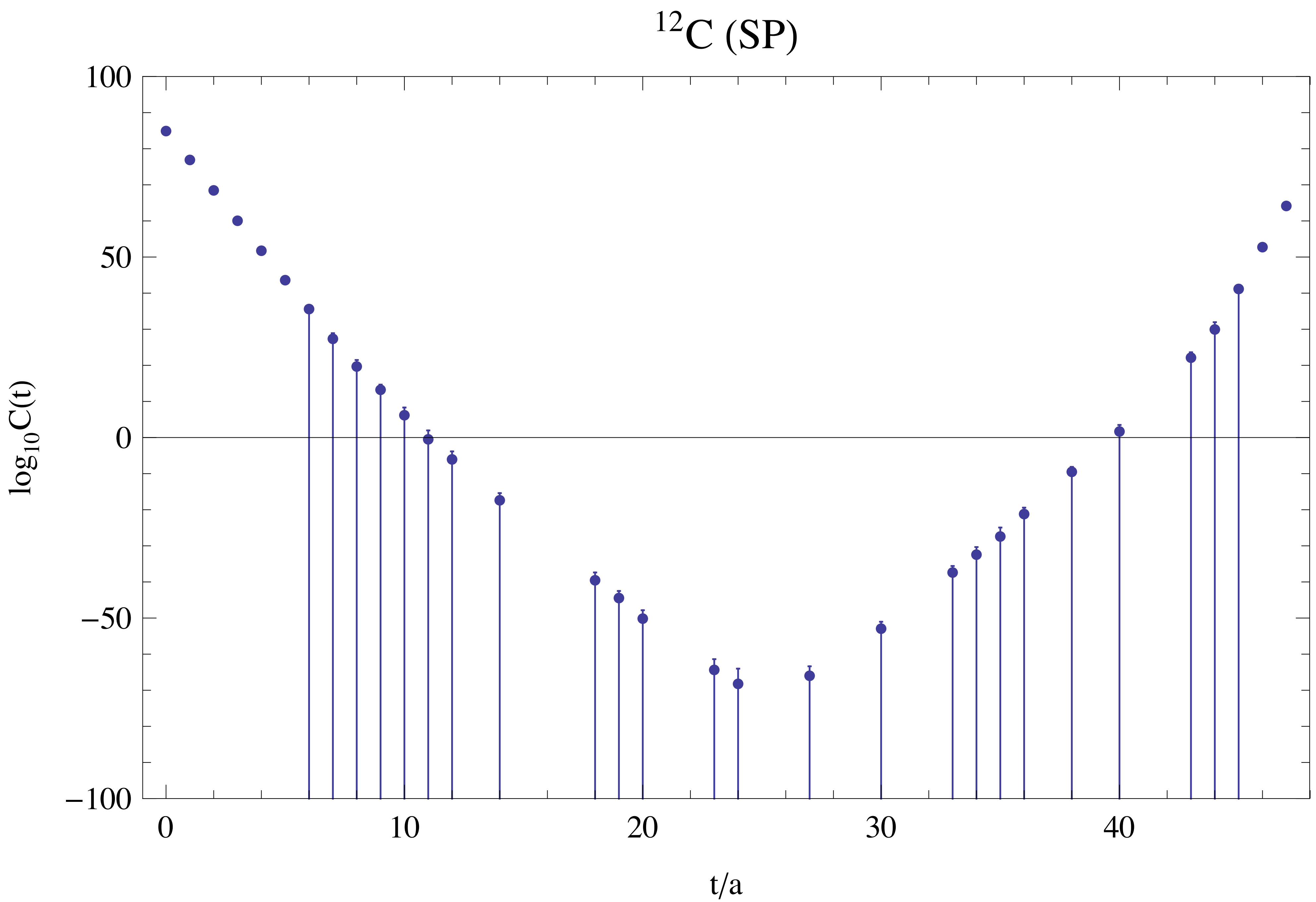

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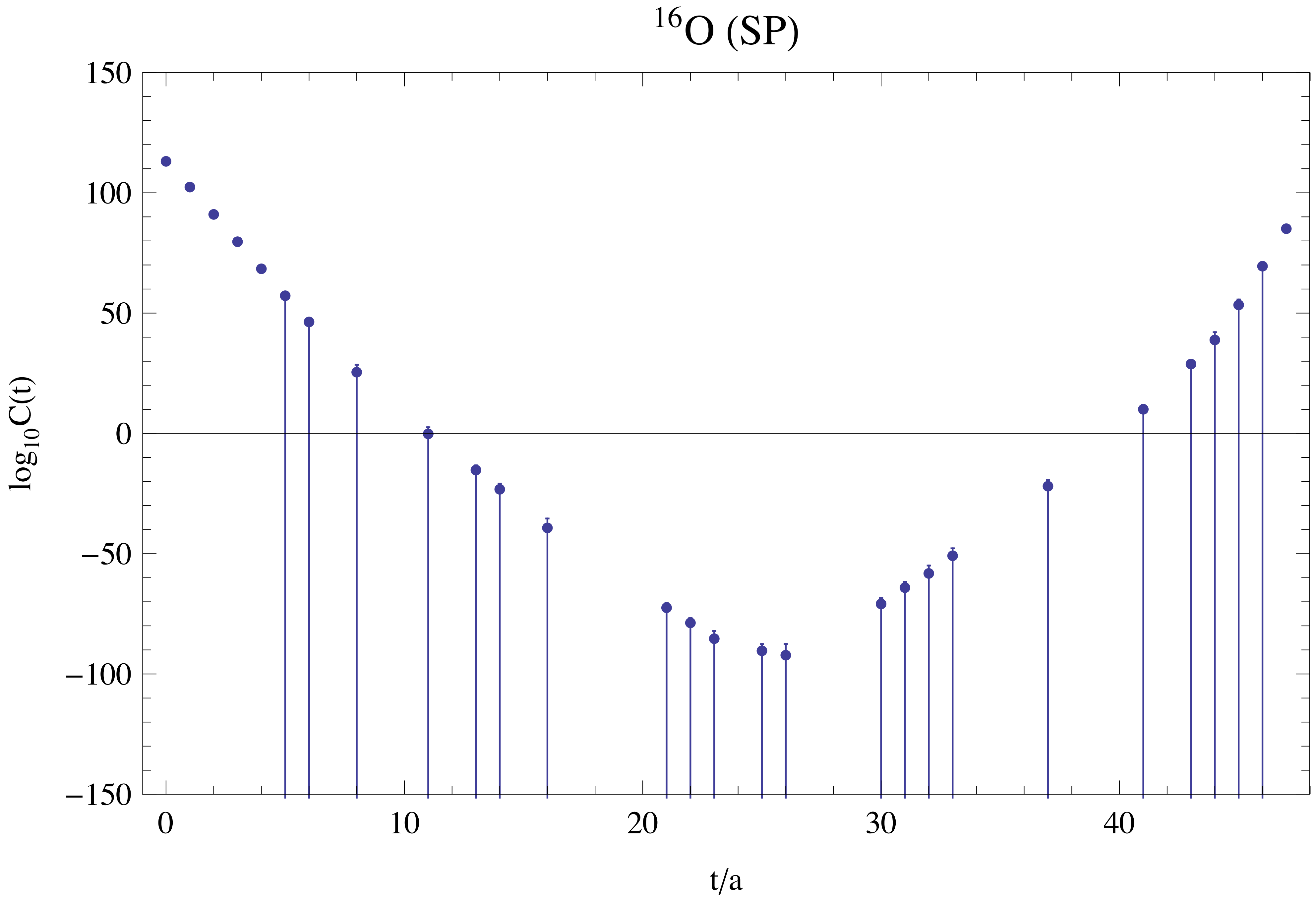

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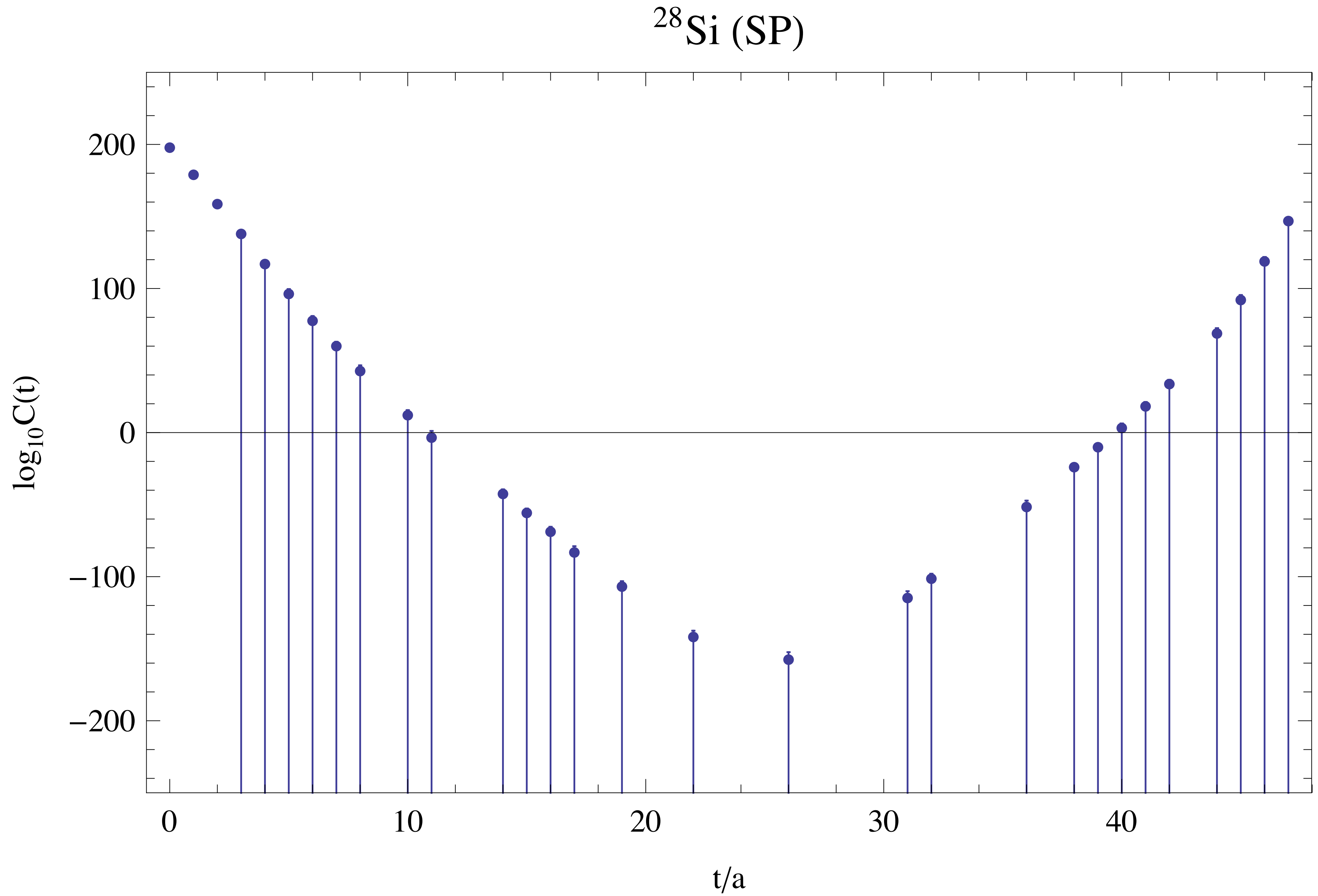

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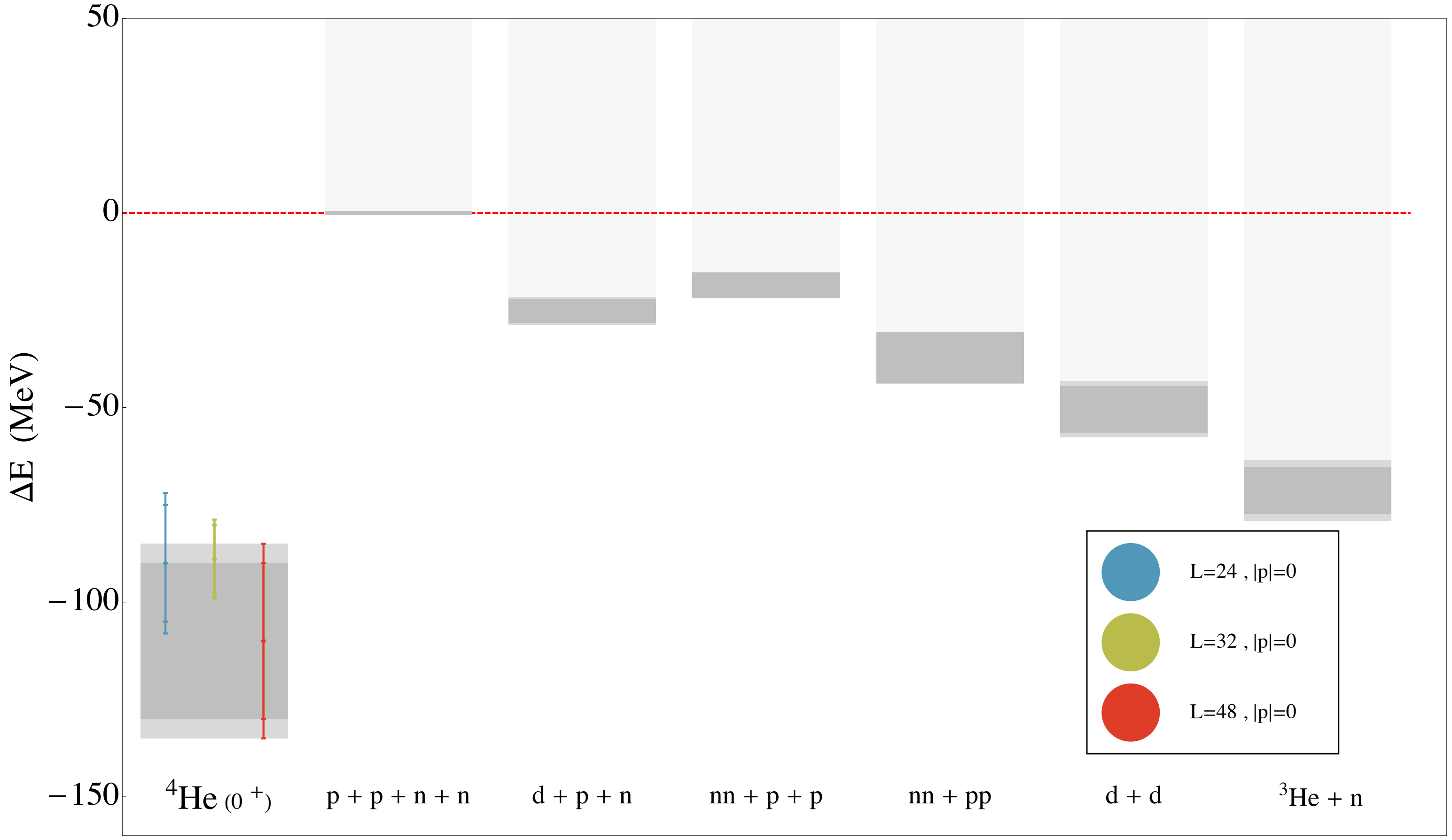
Density of states ...arrrrgh

- Current challenge is the density of scattering states in multi-hadron systems



- States far below thresholds are OK, but how do we learn about d–d scattering?

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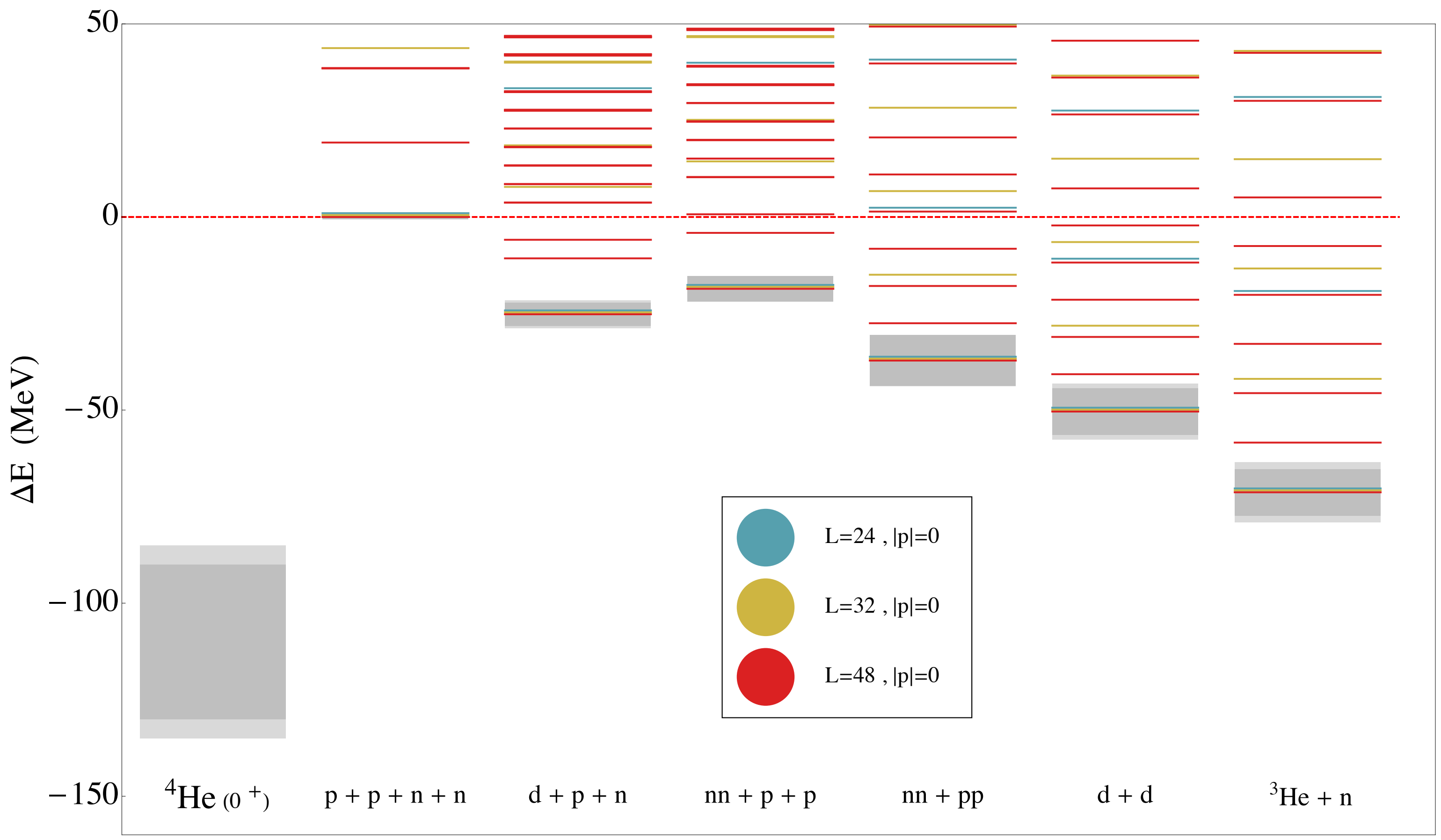

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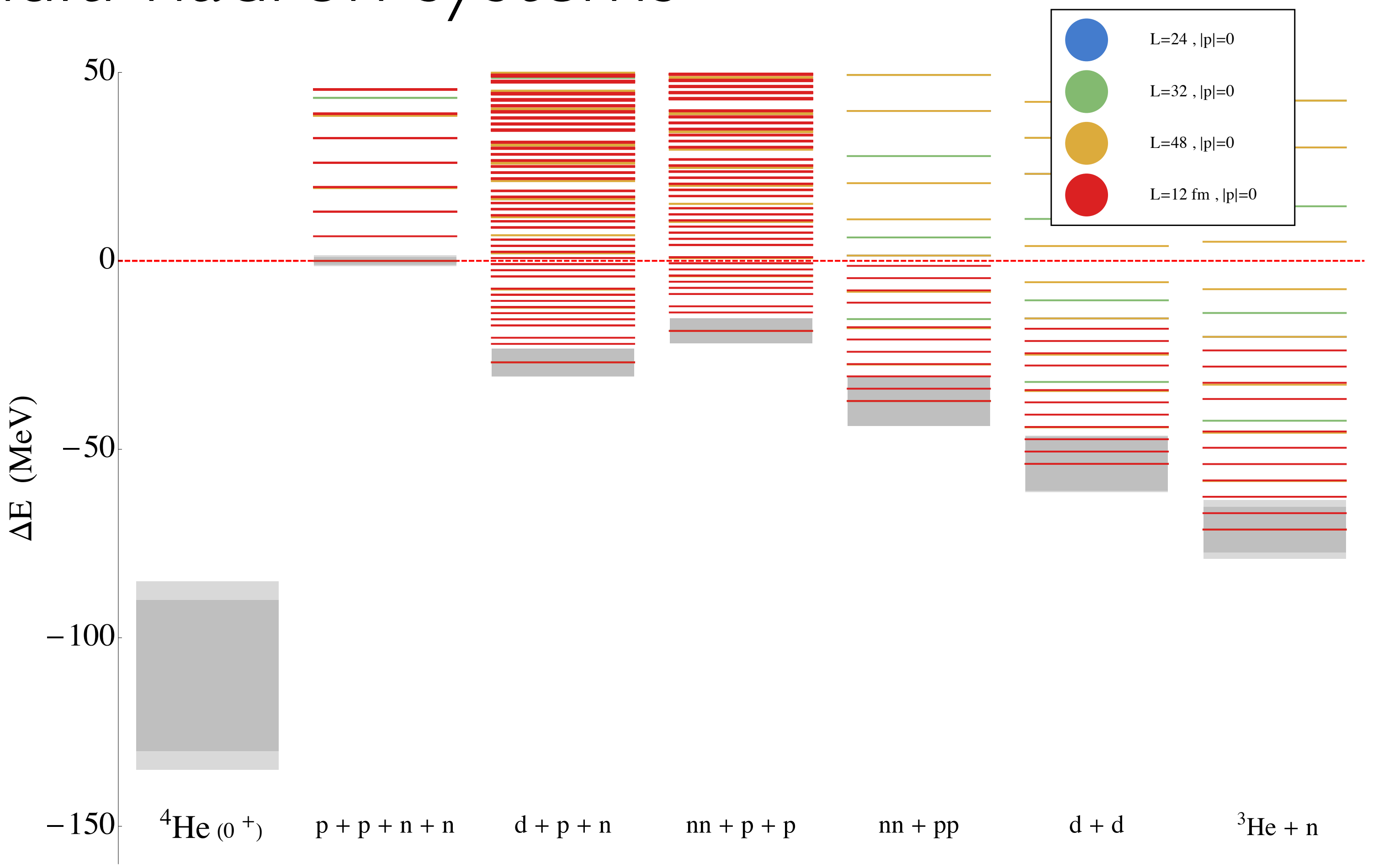

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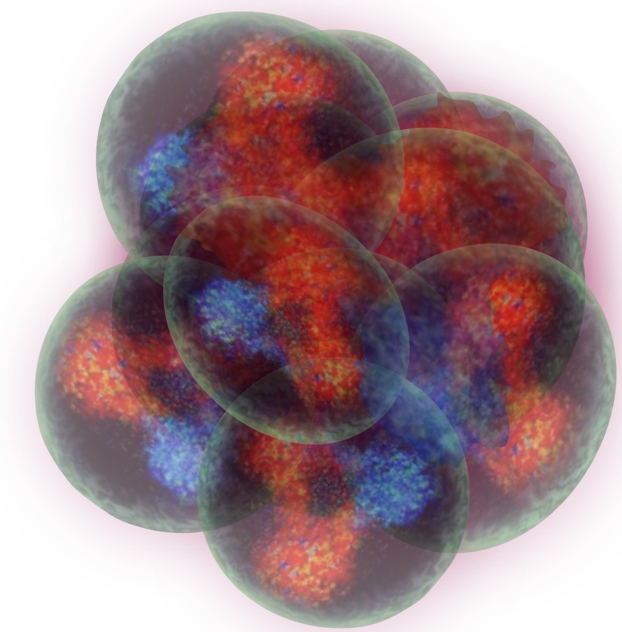
Issues

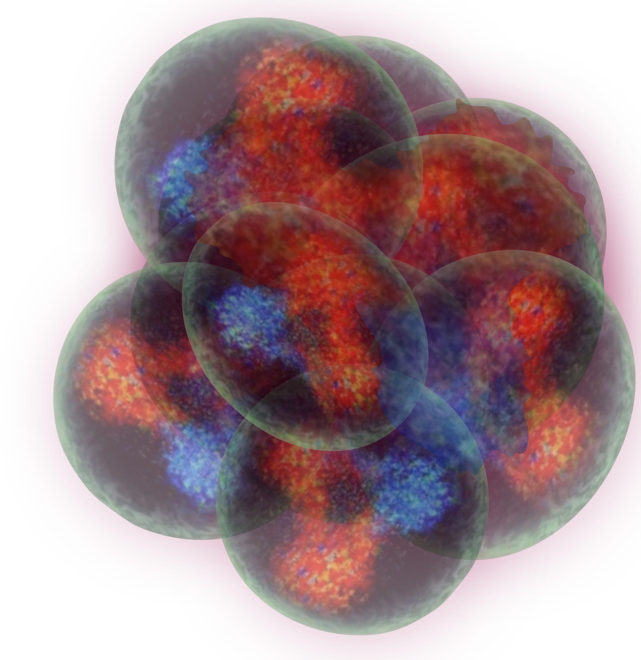
- Can we optimise noise suppression systematically
- For large A systems, how do we control the volume, lattice spacing, unphysical quark mass artefacts?
    - EFT probably loses control/breaks down for $A>4$
    - Maybe just empirically?
- What other kinds of observables can we calculate?
    - Structure of bound nuclei

From quarks to nuclei



From quarks to nuclei

- *QCD calculations of nuclei are possible*



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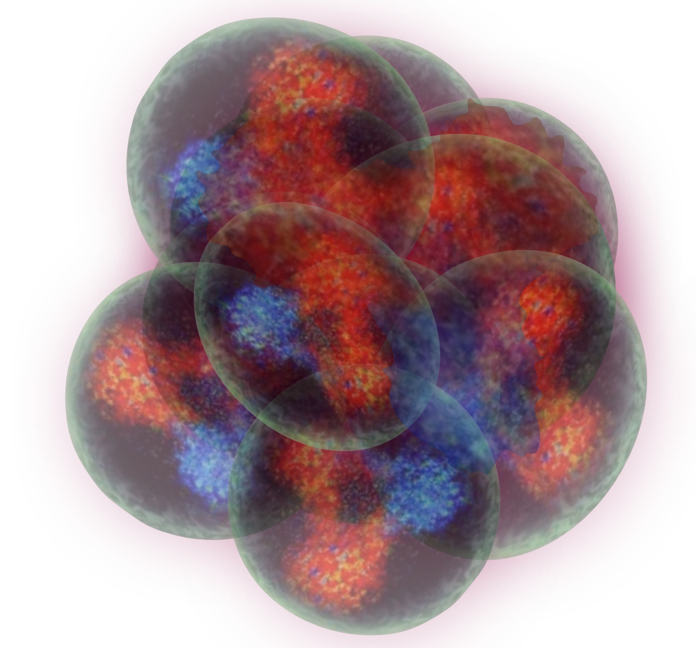

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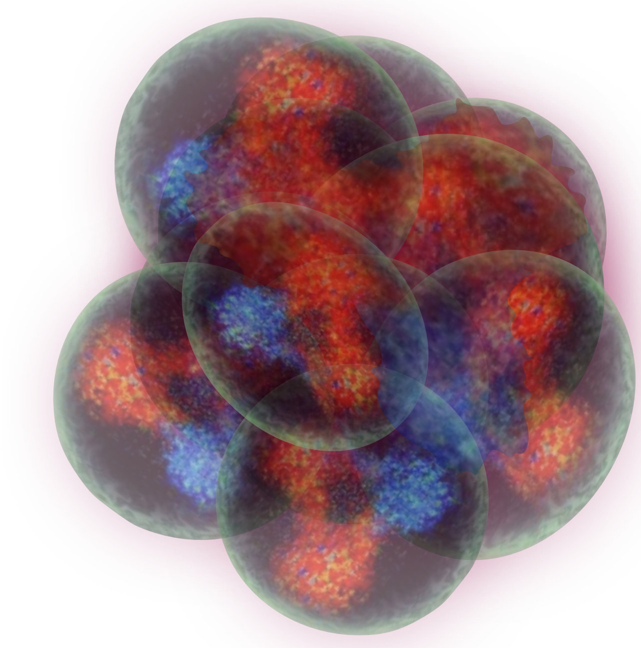

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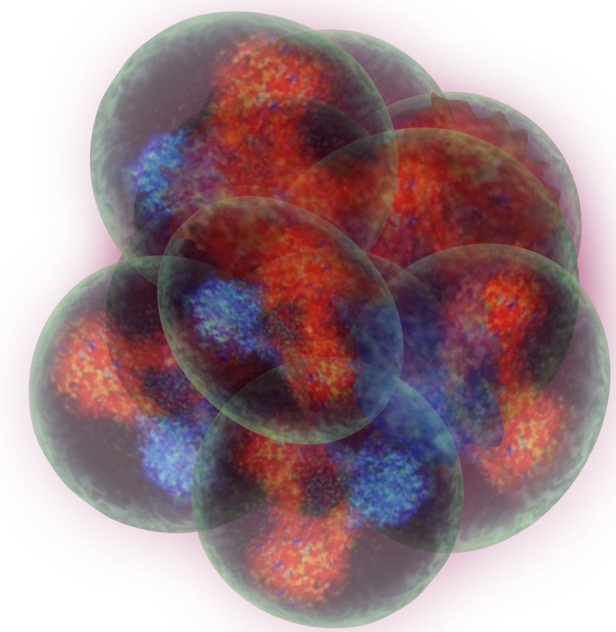

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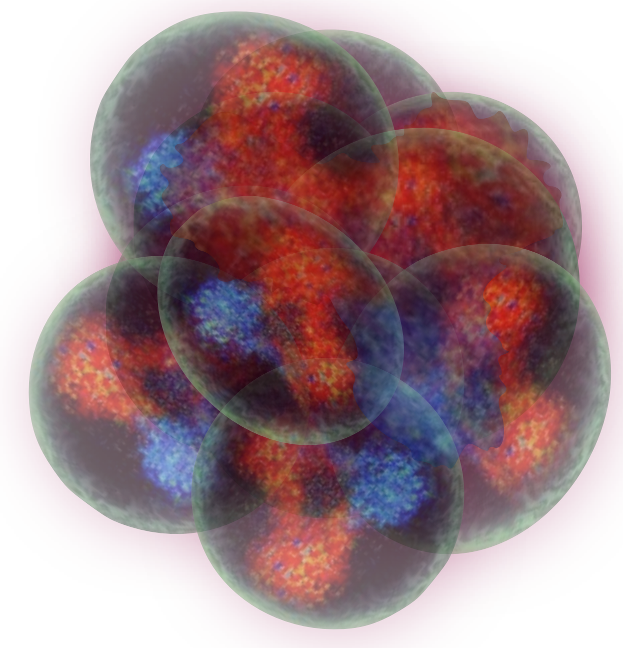

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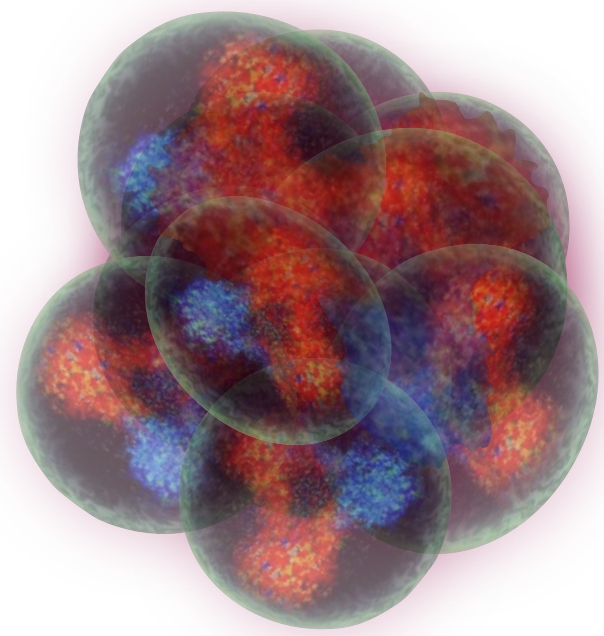

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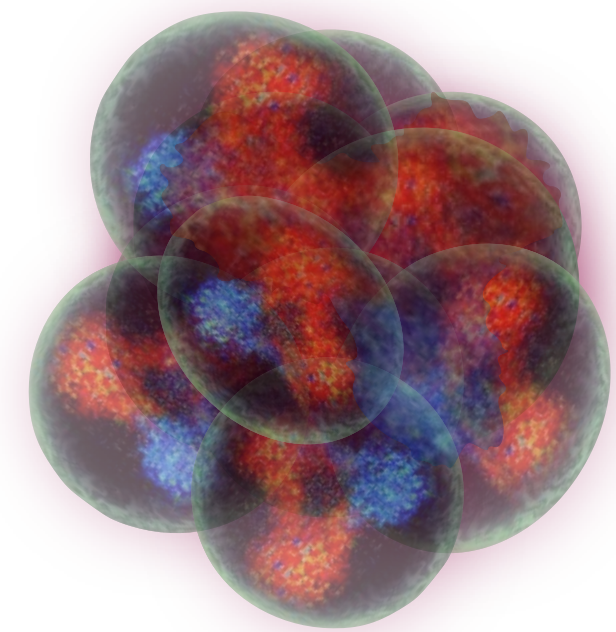

- *Where is the field going?*
  - Strong connections to experimental programs: hypernuclear spectroscopy at JLab, JPARC, FAIR
  - Answer questions that experiments have not and cannot: nnn, quark mass dependence

[FIN]

thanks to

NPLQCD