Cottingham Formula for the Electromagnetic Self-Energy Contribution to M_p - M_n

MIT Lattice Club

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Introduction: M_p - M_n

- Neutron-Proton mass splitting plays an important role in the formation of light nuclei in the early universe

Initial conditions for Big Bang Nucleosynthesis (BBN)

$$\frac{X_n}{X_p} = e^{-(M_n - M_p)/T}$$

- BBN places stringent constraints on time-variation of fundamental constants

- We would like to understand this mass splitting from first principles

Introduction: M_p - M_n

- Given only electro-static forces, one would predict

$$M_p > M_n$$

- Nature: $M_p - M_n = -1.29333217(42)\ \mathrm{MeV}$

- Standard Model of Physics has two sources of isospin breaking $\hat{Q} = \frac{1}{6}\mathbb{1} + \frac{1}{2}\tau_3 \qquad m_q = \hat{m}\mathbb{1} - \delta\tau_3$

- We now know contribution from $m_d - m_u$ is comparable in size, but opposite in sign to the electromagnetic contribution

Introduction: M_p - M_n

- We would like to understand the Neutron-Proton mass splitting from first principles

- $M_p - M_n = \delta M^{\gamma} + \delta M^{m_d - m_u}$ Separation only valid at LO in isospin breaking

- $\delta M^{m_d - m_u}$ Well understood from lattice QCD

  δM^{γ} Disparate scales relevant for QCD and QED make this a very challenging problem to solve with LQCD: large systematic uncertainties

- Alternative means to determine δM^{γ}
  Cottingham Formulation

Introduction: M_p - M_n

What do we know?

- $$\delta M^{m_d - m_u}_{LQCD} = -2.53(40)\ \text{MeV}$$

  Weighted average of 3 independent lattice QCD calculations

NPLQCD	Blum, Izubuchi, et al	RM123
DWF on MILC	DWF on DWF	twisted mass LQCD

- $$\delta M^{\gamma} = 0.76(30)\ \text{MeV}$$

  Gasser & Leutwyler
  Nucl. Phys. B94 (1975)
  Phys. Rept. 87 (1982) "Quark Masses"

  central value from elastic contribution

  uncertainty from estimates of inelastic contributions

- Experiment & lattice QCD

$$\delta M^{phys}_{p-n} - \delta M^{m_d - m_u}_{LQCD} = 1.24(40)\ \text{MeV}$$

Introduction: M_p - M_n

- Desire to improve this determination with modern knowledge of nucleon structure functions

- Updating G&L result uncovered a "technical oversight"
  - The application of the Cottingham Formula requires the use of a subtracted dispersion integral.
  - Gasser & Leutwyler had an argument to evade the unknown subtraction function.
  - The argument was based on incorrect assumptions about scaling violations of the parton model
  - this has gone (mostly) unnoticed since 1982

Introduction: M_p - M_n

Walker-Loud, Carlson, Miller PRL 108 (2012) [arXiv:1203.0254]

$$\delta M^{\gamma} = \delta M^{el} + \delta M^{inel} + \delta M^{sub} + \delta M^{ct}$$

elastic	inelastic	unknown subtraction	counter-term renormalization
↑	↑	↑	↑
precisely determined	newly determined (precisely)	newly determined (imprecisely)	determined by J.C. Collins

$$\delta M^{\gamma}_{p-n} = 1.30 \pm 0.03 \pm 0.47 \text{ MeV}$$

Introduction: M_p - M_n

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$$\delta M^{\gamma} = 0.76(30)\ \mathrm{MeV}$$ Gasser & Leutwyler

$$\delta M^{phys}_{p-n} - \delta M^{m_d - m_u}_{LQCD} = 1.24(40)\ \mathrm{MeV}$$ Experiment & LQCD

Introduction: M_p - M_n

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~~$\delta M^{\gamma} = 0.76(30)\ \mathrm{MeV}$~~ Gasser & Leutwyler

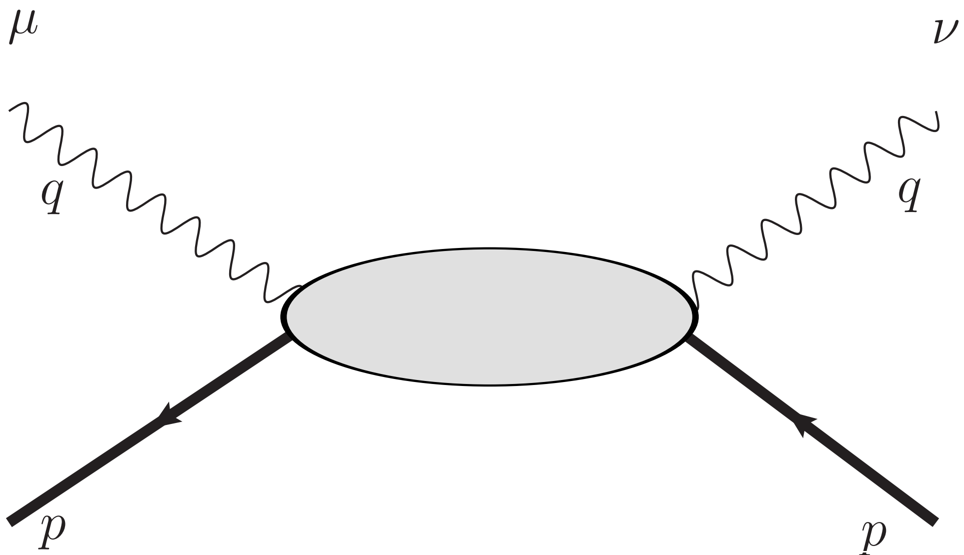
$$\delta M^{phys}_{p-n} - \delta M^{m_d - m_u}_{LQCD} = 1.24(40)\ \mathrm{MeV}$$ Experiment & LQCD

Cottingham's Formula

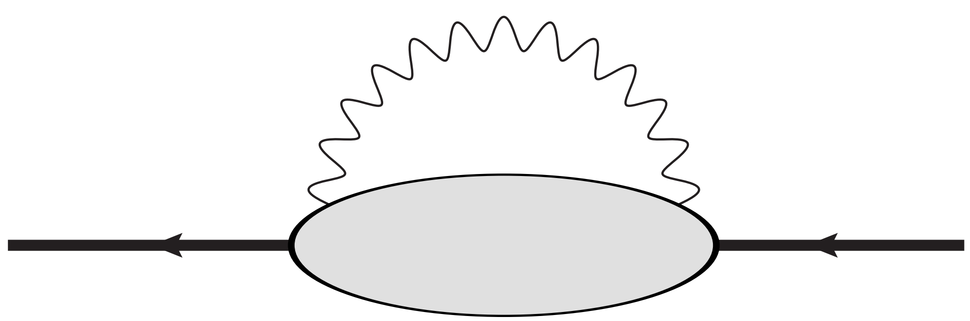
AWL, C.Carlson, G.Miller: PRL 108 (2012)

Cini, Ferrari, Gato: PRL 2 (1959)

Cottingham: Annals Phys 25 (1963)



$$T_{\mu\nu} = \frac{i}{2} \sum_{\sigma} \int d^4\xi \; e^{iq\cdot\xi} \langle p\sigma | T\left\{ J_\mu(\xi) J_\nu(0) \right\} | p\sigma \rangle$$



$$\delta M^{\gamma} = \frac{i}{2M} \frac{\alpha}{(2\pi)^3} \int_R d^4q \frac{T^{\mu}_{\mu}(p,q)}{q^2 + i\epsilon}$$

$$\alpha = \frac{e^2}{4\pi}$$

Integral diverges and must be renormalized

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

$$\delta M^{\gamma} = \frac{i}{2M} \frac{\alpha}{(2\pi)^3} \int_R d^4q \frac{T^{\mu}_{\mu}(p,q)}{q^2 + i\epsilon}$$

- Wick rotate $q^0 \to i\nu$
- variable transform $Q^2 = \mathbf{q}^2 + \nu^2$

$$\delta M^{\gamma} = \frac{\alpha}{8\pi^2} \int_0^{\Lambda^2} dQ^2 \int_{-Q}^{+Q} d\nu \frac{\sqrt{Q^2 - \nu^2}}{Q^2} \frac{T^{\mu}_{\mu}}{M} + \delta M^{ct}(\Lambda)$$

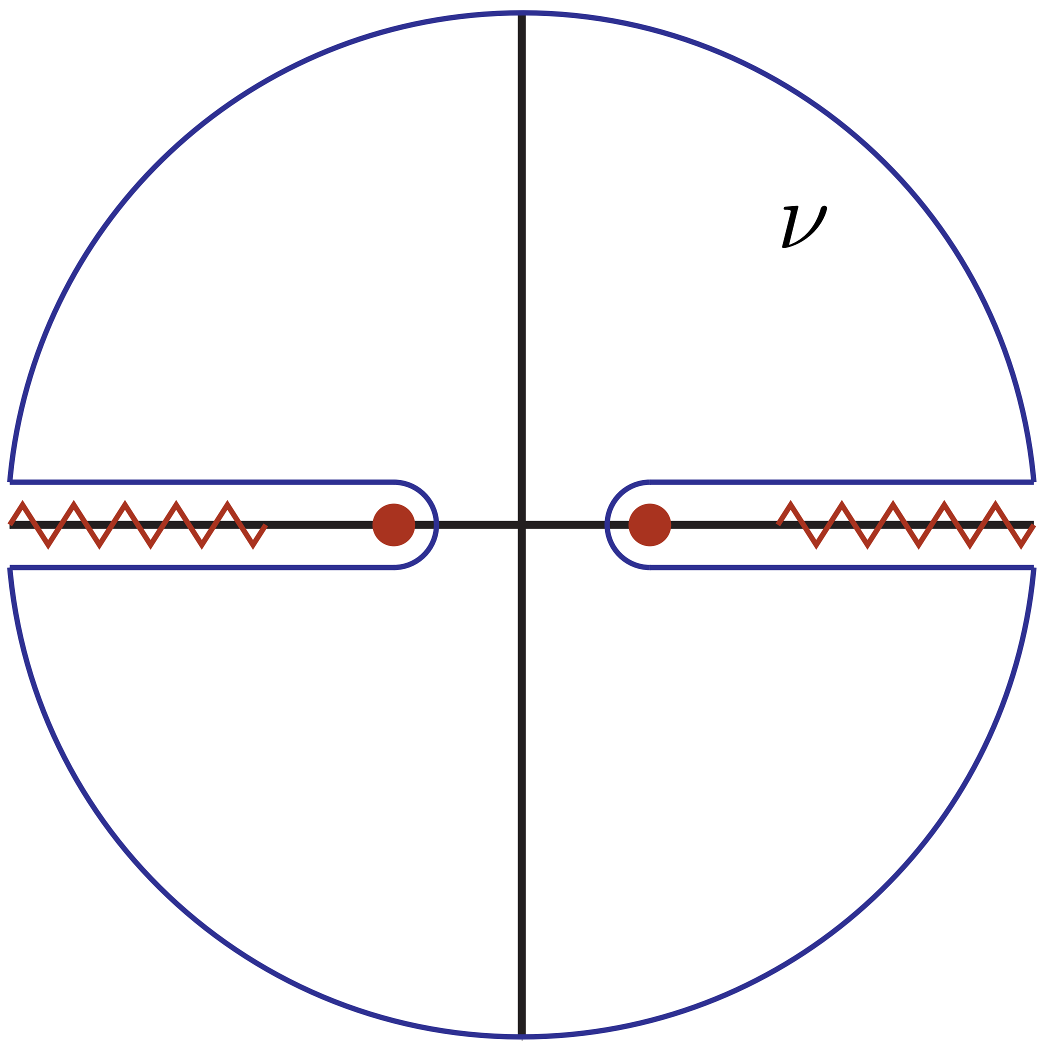
$$T^{\mu}_{\mu} = -3\,T_1(i\nu, Q^2) + \left(1 - \frac{\nu^2}{Q^2}\right) T_2(i\nu, Q^2)\,, \qquad (7a)$$

$$= -3Q^2\,t_1(i\nu, Q^2) + \left(1 + 2\frac{\nu^2}{Q^2}\right) Q^2 t_2(i\nu, Q^2)\,. \qquad (7b)$$

use dispersion integrals to evaluate scalar functions

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)



$$T_i(\nu, Q^2) = \frac{1}{2\pi} \oint d\nu' \frac{T_i(\nu', Q^2)}{\nu' - \nu}$$

Crossing Symmetric

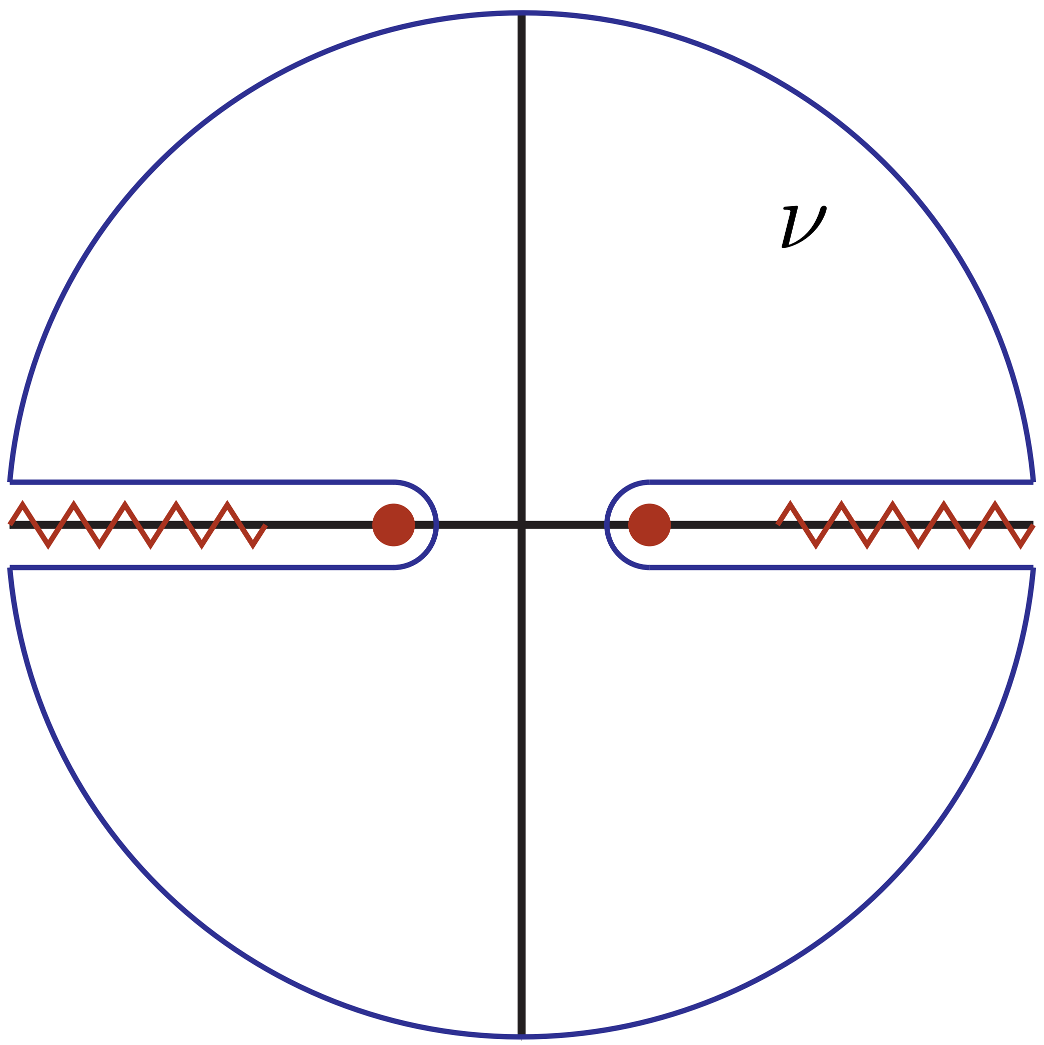
$$T_i(\nu, Q^2) = T_i(-\nu, Q^2)$$

$$T_i(\nu, Q^2) = \frac{1}{2\pi} \int_{\nu_t}^{\infty} d\nu' \frac{2\nu'}{(\nu')^2 - \nu^2} 2\mathrm{Im} T_i(\nu' + i\epsilon, Q^2)$$

(provided contour and infinity vanishes)

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)



if contour at infinity does not vanish

subtracted dispersion integral

$$g(\nu) = \frac{T_i(\nu, Q^2)}{\nu^2}$$

introduces new pole at $\nu = 0$
which you need to subtract

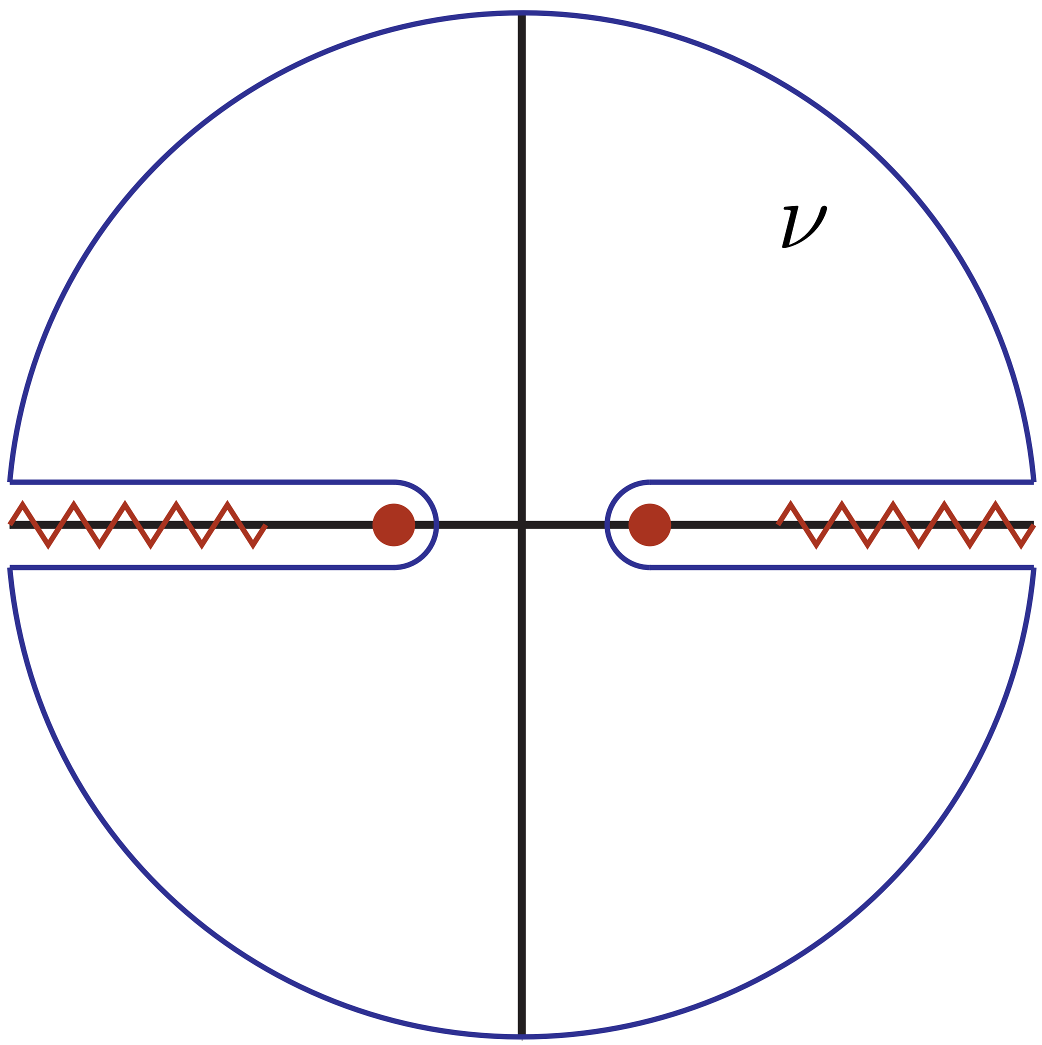
$$T_i(\nu, Q^2) = \frac{\nu^2}{2\pi} \int_{\nu_t}^{\infty} d\nu' \frac{2\nu'}{\nu'^2(\nu'^2 - \nu^2)} 2\mathrm{Im}T_i(\nu' + i\epsilon, Q^2) + T_i(0, Q^2)$$

measured experimentally

unknown function

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)



It is known that

$$T_2(\nu, Q^2) \quad [t_2(\nu, Q^2)]$$

satisfies unsubtracted dispersion integral while

$$T_1(\nu, Q^2) \quad [t_1(\nu, Q^2)]$$

requires a subtraction

Regge behavior

$$\mathrm{Im} t_1 [T_1] \Big|_{p-n} \propto \nu^{1/2}$$

H. Harari: PRL 17 (1966)

H.D. Abarbanel S. Nussinov: Phys.Rev. 158 (1967)

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

at the time, introducing an unknown subtraction function would be disastrous for getting a precise value:

they provided an argument based upon various assumptions to avoid the subtracted dispersive integral

$$\delta M^{\gamma}_{p-n} = 0.76(30)\ \mathrm{MeV}$$

central value: from elastic contribution

uncertainty: estimates of inelastic structure contributions

however, one can show their arguments are incorrect:

one must face the subtraction function

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

$$\delta M^{\gamma} = \frac{\alpha}{8\pi^2} \int_0^{\Lambda^2} dQ^2 \int_{-Q}^{+Q} d\nu \frac{\sqrt{Q^2-\nu^2}}{Q^2} \frac{T_\mu^\mu}{M} + \delta M^{ct}(\Lambda)$$

$$T_\mu^\mu = -3\,T_1(i\nu, Q^2) + \left(1 - \frac{\nu^2}{Q^2}\right) T_2(i\nu, Q^2)\,, \qquad (7a)$$

$$= -3Q^2\, t_1(i\nu, Q^2) + \left(1 + 2\frac{\nu^2}{Q^2}\right) Q^2 t_2(i\nu, Q^2)\,. \quad (7b)$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

$$T_{\mu\nu} = \frac{i}{2}\sum_\sigma \int d^4\xi\; e^{iq\cdot\xi}\langle p\sigma|T\left\{J_\mu(\xi)J_\nu(0)\right\}|p\sigma\rangle$$

Insert complete set of states:
isolate elastic contributions

$$1 = \sum_\Gamma |\Gamma\rangle\langle\Gamma|$$

$$\delta M^{el}_{unsub,a} = \frac{\alpha}{\pi}\int_0^{\Lambda^2} dQ \left\{ \left[G_E^2(Q^2) - 2\tau_{el} G_M^2(Q^2)\right] \frac{(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2} - \frac{3}{2}\sqrt{\tau_{el}}}{1+\tau_{el}} - \frac{3}{2} G_M^2(Q^2)\frac{{\tau_{el}}^{3/2}}{1+\tau_{el}} \right\}, \qquad (8a)$$

$$\delta M^{el}_{unsub,b} = \frac{\alpha}{\pi}\int_0^{\Lambda^2} dQ \left\{ \left[G_E^2(Q^2) - 2\tau_{el}\, G_M^2(Q^2)\right] \frac{(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2}}{1+\tau_{el}} + 3 G_M^2(Q^2)\frac{{\tau_{el}}^{3/2}}{1+\tau_{el}} \right\}, \qquad (8b)$$

typically quoted as elastic Cottingham

$$\delta M^\gamma = \frac{\alpha}{8\pi^2}\int_0^{\Lambda^2} dQ^2 \int_{-Q}^{+Q} d\nu \frac{\sqrt{Q^2-\nu^2}}{Q^2}\frac{T^\mu_\mu}{M} + \delta M^{ct}(\Lambda)$$

$$T^\mu_\mu = -3\,T_1(i\nu, Q^2) + \left(1 - \frac{\nu^2}{Q^2}\right) T_2(i\nu, Q^2)\,, \qquad (7a)$$

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One must use a subtracted dispersive integral even for elastic terms

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

perform once subtracted dispersion integral for $T_1(t_1)$
and unsubtracted dispersion integral for $T_2(t_2)$

$$\delta M^{\gamma} = \delta M^{el} + \delta M^{inel} + \delta M^{sub} + \delta \tilde{M}^{ct}$$

$$\delta M^{el} = \frac{\alpha}{\pi} \int_0^{\Lambda_0^2} dQ \left\{ \frac{3\sqrt{\tau_{el}} G_M^2}{2(1+\tau_{el})} + \frac{\left[G_E^2 - 2\tau_{el}\, G_M^2\right]}{1+\tau_{el}} \left[(1+\tau_{el})^{3/2} - \tau_{el}{}^{3/2} - \frac{3}{2}\sqrt{\tau_{el}} \right] \right\}$$

$$\delta M^{inel} = \frac{\alpha}{\pi} \int_0^{\Lambda_0^2} \frac{dQ^2}{2Q} \int_{\nu_{th}}^{\infty} d\nu \left\{ \frac{3F_1(\nu, Q^2)}{M} \left[\frac{\tau^{3/2} - \tau\sqrt{1+\tau} + \sqrt{\tau}/2}{\tau} \right] + \frac{F_2(\nu, Q^2)}{\nu} \left[(1+\tau)^{3/2} - \tau^{3/2} - \frac{3}{2}\sqrt{\tau} \right] \right\},$$

$$\tau_{el} = \frac{Q^2}{4M^2} \qquad \tau = \frac{\nu^2}{Q^2}$$

$$\delta M^{sub} = -\frac{3\alpha}{16\pi M} \int_0^{\Lambda_0^2} dQ^2 \; T_1(0, Q^2)\,,$$

$$\delta \tilde{M}^{ct} = -\frac{3\alpha}{16\pi M} \int_{\Lambda_0^2}^{\Lambda_1^2} dQ^2 \sum_i C_{1,i} \langle \mathcal{O}^{i,0} \rangle\,,$$

OPE: operators and Wilson coeffic.
J.C. Collins: Nucl. Phys. B149 (1979)

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

what is the flaw in the argument?

$$\delta M^\gamma = \frac{\alpha}{8\pi^2} \int_0^{\Lambda^2} dQ^2 \int_{-Q}^{+Q} d\nu \frac{\sqrt{Q^2 - \nu^2}}{Q^2} \frac{T_\mu^\mu}{M} + \delta M^{ct}(\Lambda)$$

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$$= -3Q^2\, t_1(i\nu, Q^2) + \left(1 + 2\frac{\nu^2}{Q^2}\right) Q^2 t_2(i\nu, Q^2)\,. \qquad (7b)$$

is there some motivation to pick t_i vs T_i ?

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

what is the flaw in the argument?

in the point limit (electron) $t_1(\nu, Q^2) = 0!$

for the nucleon (with motivated resummations) the elastic contribution is

$$t_1(\nu, Q^2) = \frac{2}{Q^2}\left[\frac{Q^4 \frac{G_M^2 - G_E^2}{1+\tau}}{(Q^2 - i\epsilon)^2 - 4M^2\nu^2} - \underbrace{\left(F_1^2 - \frac{G_E^2 + \tau G_M^2}{1+\tau}\right)}\right]$$

"Fixed-Pole" missed by unsubtracted dispersion relation

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

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numerically, this term is negligible

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

what is the flaw in the argument?

in the point limit (electron) $t_1(\nu, Q^2) = 0!$

real problem comes in the Regge limit: Q^2 fixed, $\nu \to \infty$

$$\mathrm{Im}t_1(\nu, Q^2) = \frac{\pi M \nu}{Q^4}\left[2xF_1(x, Q^2) - F_2(x, Q^2)\right] \qquad x = \frac{Q^2}{2M\nu}$$

in the strict DIS limit: Callan-Gross relation

$$2xF_1(x) - F_2(x) = 0$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

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Gasser and Leutwyler assumed

$$2xF_1(x, Q^2) - F_2(x, Q^2) = \frac{H_1(x)}{\nu}$$

if this were true, their argument would go through, however...

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

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Zee, Wilczek and Treiman Phys.Rev. D10 (1974)

$$2xF_1(x) - F_2(x) = \frac{-32}{9}\frac{\alpha_s(Q^2)}{4\pi}F_2(x)$$

Both IR and UV safe

This criticism first given by J.C. Collins: Nucl. Phys. B149 (1979)

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

what is the flaw in the argument?

in the point limit (electron) $t_1(\nu, Q^2) = 0!$

real problem comes in the Regge limit: Q^2 fixed, $\nu \to \infty$

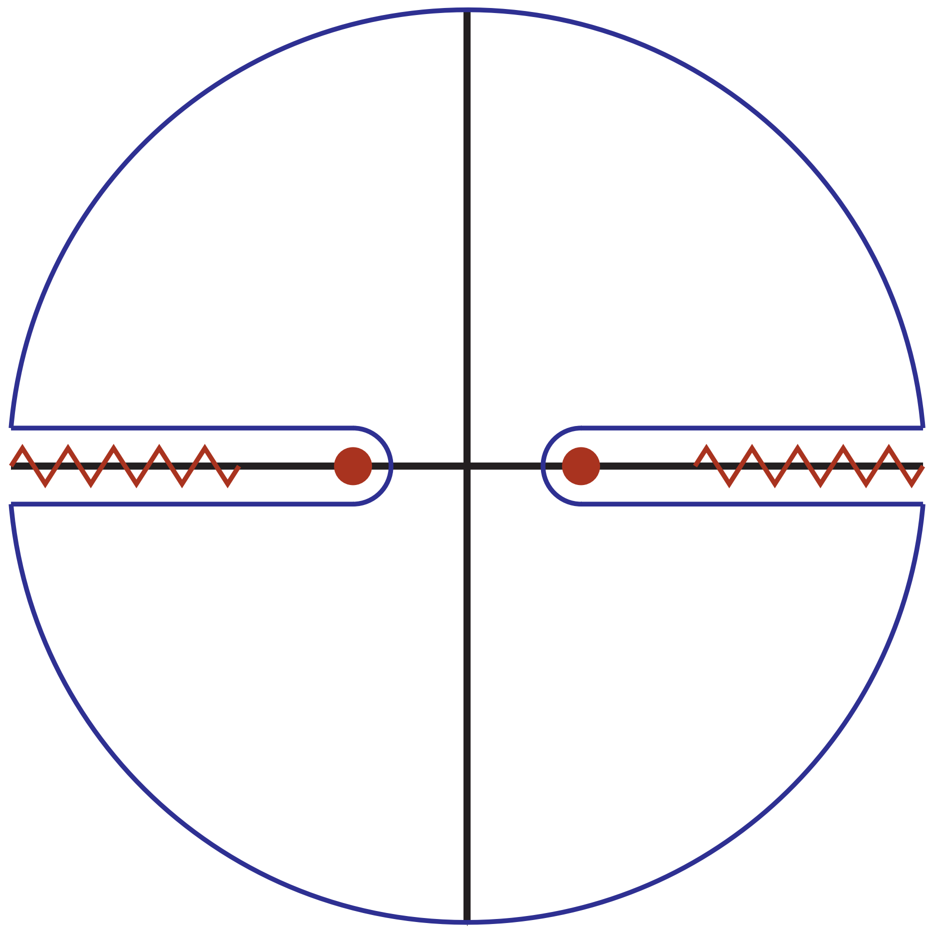
$$\lim_{x \to 0} F_2^{p-n}(x) \propto x^{1/2} \qquad x = \frac{Q^2}{2M\nu}$$

$$\mathrm{Im} t_1^{p-n}(\nu, Q^2) \propto \alpha_s(Q^2) \frac{\sqrt{M\nu}}{Q^3}$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

$$t_1(\nu, Q^2) = \frac{1}{2\pi}\int_{\nu_t}^{\infty} 2\nu' d\nu' \frac{2\mathrm{Im} t_1(\nu' + i\epsilon, Q^2)}{(\nu')^2 - \nu^2}$$



$$\mathrm{Im} t_1^{p-n}(\nu, Q^2) \propto \alpha_s(Q^2)\frac{\sqrt{M\nu}}{Q^3}$$

→ subtracted dispersion integral is unavoidable evaluation of various contributions

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

elastic contribution: use well measured form factors

$$\delta M^{el} = \frac{\alpha}{\pi}\int_0^{\Lambda_0^2} dQ \left\{ \frac{3\sqrt{\tau_{el}}G_M^2}{2(1+\tau_{el})} + \frac{\left[G_E^2 - 2\tau_{el}\, G_M^2\right]}{1+\tau_{el}}\left[(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2} - \frac{3}{2}\sqrt{\tau_{el}}\right]\right\}$$

$$\delta M^{el}\Big|_{p-n} = 1.39(02)\ \mathrm{MeV}$$

- insensitive to value of Λ_0 since form factors fall as $1/Q^4$
- uncertainty from Monte Carlo evaluation of parameters describing form factors

central values: $\Lambda_0^2 = 2\ \mathrm{GeV}^2$

uncertainties: $1.5\ \mathrm{GeV}^2 \leq \Lambda_0^2 \leq 2.5\ \mathrm{GeV}^2$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

inelastic terms: use modern knowledge of structure functions to improve determination of inelastic contributions

$$\delta M^{inel} = \frac{\alpha}{\pi}\int_0^{\Lambda_0^2}\frac{dQ^2}{2Q}\int_{\nu_{th}}^{\infty} d\nu \left\{ \frac{3F_1(\nu,Q^2)}{M}\left[\frac{\tau^{3/2}-\tau\sqrt{1+\tau}+\sqrt{\tau}/2}{\tau}\right] + \frac{F_2(\nu,Q^2)}{\nu}\left[(1+\tau)^{3/2}-\tau^{3/2}-\frac{3}{2}\sqrt{\tau}\right]\right\},$$

$$\delta M^{inel}\Big|_{p-n} = 0.057(16)\ \mathrm{MeV}$$

- contributions from two regions:
  - resonance region — Bosted and Christy: Phys.Rev. C77, C81
  - scaling region — Capella et al: PLB 337; Sibirtsev et al: Phys. Rev. D82
- uncertainty dominated by choice of transition between two regions

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

renormalization: [J.C. Collins Nucl. Phys. B149 (1979)] one can show the contribution from the operator is numerically second order in isospin breaking with Naive Dimensional Analysis and suitable renormalization (dim. reg.)

quark mass operator renormalizes EM self-energy: can not cleanly separate these two contributions (but mixing is higher order in isospin breaking)

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

renormalization: [J.C. Collins Nucl. Phys. B149 (1979)]

$$\delta M^{\gamma}_{UV} \sim \frac{3\alpha_{f.s.}}{16\pi M} \int_{\Lambda^2}^{\infty} \left[\frac{M^2}{Q^2} \int_0^1 dx \Big(2xF_1(x) + F_2(x) \Big) - T_1(0, Q^2) \right]$$

subtraction function

- use OPE to connect to perturbative QCD
- log divergence arising from $2xF_1(x) + F_2(x)$ exactly cancels against log divergence from $T_1(0, Q^2)$
- counter term comes entirely from subtraction function

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

renormalization: [J.C. Collins Nucl. Phys. B149 (1979)]

$$\delta M^{\gamma} = \frac{3\alpha_{f.s.}}{16\pi M}\left\{ \int_0^{\mu^2} \frac{dQ^2}{Q^2} f(Q^2) + \lim_{\Lambda^2\to\infty}\left[\int_{\mu^2}^{\Lambda^2} \frac{dQ^2}{Q^2}\left(f(Q^2) + \sum_i C^0_{1,i}\langle \mathcal{O}^{i,0}\rangle\right)\right]\right\}$$

$$\langle N|\sum_i C^0_{1,i}\mathcal{O}^{i,0}|N\rangle_{p-n} = \frac{2}{Q^2}(e_u^2 m_u - e_d^2 m_d)\langle p|\bar{u}u - \bar{d}d|p\rangle$$

- $\ln(\Lambda^2)$ divergence exactly cancels
- residual dependence on scale μ
- use Naive Dimensional Analysis to estimate size

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

renormalization: [J.C. Collins Nucl. Phys. B149 (1979)]

$$\delta \tilde{M}^{ct} = -\frac{3\alpha}{4\pi}\sigma_{\pi N} \ln\left(\frac{\Lambda_1^2}{\Lambda_0^2}\right) \frac{3\hat{m} - 5\delta}{9\hat{m}} \frac{\langle p|\bar{u}u - \bar{d}d|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle}$$

$$\sigma_{\pi N} = \frac{1}{2M}\langle p|\hat{m}(\bar{u}u + \bar{d}d)|p\rangle \simeq 45 \text{ MeV}$$

- saturate matrix elements in valence limit $$\frac{\langle p|\bar{u}u - \bar{d}d|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle} \leq \frac{1}{3}$$
- vary arbitrary scales in scaling region $$\Lambda_0^2 = 2 \text{ GeV}^2\,, \quad \Lambda_1^2 = 100 \text{ GeV}^2$$

$$|\delta \tilde{M}^{ct}| \lesssim 0.02 \text{ MeV}$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M^{sub} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2\ T_1(0,Q^2)\,,$$

- low energy: constrained by effective field theory

$$T_1(0,Q^2) = 2\kappa(2+\kappa) - Q^2\left\{\frac{2}{3}\left[(1+\kappa)^2 r_M^2 - r_E^2\right] + \frac{\kappa}{M^2} - 2M\frac{\beta_M}{\alpha}\right\} + \mathcal{O}(Q^4)\,,$$

most of these contributions come from Low Energy Theorems and are "elastic" (arising from a photon striking an on-shell nucleon)

intimately related to the proton size puzzle which suffers from the same subtracted dispersive problem

K. Pachucki: Phys. Rev. A53 (1996); A. Pineda: Phys. Rev. C67 (2003); Phys. Rev. C71 (2005);
R.J. Hill, G. Paz: PRL 107 (2011); C. Carlson, M.Vanderhaeghen: Phys.Rev.A84 (2011); arXiv1109.3779;
M.. Birse, J. McGovern: arXiv:1206.3030

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M^{sub} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2\ T_1(0,Q^2)\,,$$

- high energy: OPE (perturbative QCD) constrains

$$\lim_{Q^2\to\infty} T_1(0,Q^2) \propto \frac{1}{Q^2}$$

$$T_1(0,Q^2) \simeq 2G_M^2(Q^2) - 2F_1^2(Q^2) + Q^2 2M\frac{\beta_M}{\alpha}\left(\frac{m_0^2}{m_0^2+Q^2}\right)^2$$

$\mathcal{O}(Q^4)$ inelastic terms known

Birse and McGovern Eur.Phys.J A48 (2012) [arXiv:1206.3030]

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M^{sub}_{el} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2 \left[2G_M^2 - 2F_1^2\right], \qquad \delta M^{sub}_{el}\Big|_{p-n} = -0.62\ \mathrm{MeV}$$

$$\delta M^{sub}_{inel} = -\frac{3\beta_M}{8\pi}\int_0^{\Lambda_0^2} dQ^2 Q^2 \left(\frac{m_0^2}{m_0^2+Q^2}\right)^2$$

$$\beta_M^{p-n} = -1.0 \pm 1.0 \times 10^{-4}\ \mathrm{fm}^3$$

H.W. Griesshammer, J.A. McGovern, D.R. Phillips, G. Feldman: Prog.Nucl.Part.Phys. (2012)

taking $m_0^2 = 0.71\ \mathrm{GeV}^2$

$$\delta M^{sub}_{inel}\Big|_{p-n} = 0.47 \pm 0.47\ \mathrm{MeV}$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

adding it all up:

$$
\begin{aligned}
\delta M^{\gamma}|_{p-n} =& +1.39(02) \\
& -0.62(02) \\
& +0.057(16) \\
& +0.47(47)\ \text{MeV} \\
\hline
=&\ 1.30(03)(47)\ \text{MeV}
\end{aligned}
$$

$+1.39(02)$ and $-0.62(02)$: $= 0.77(03)\ \text{MeV}$ elastic terms

$+0.057(16)$: inelastic terms

$+0.47(47)\ \text{MeV}$: unknown subtraction term

recall the fixed pole in the elastic contribution makes a negligible contribtion adding it all up:

$$\delta M^\gamma\Big|_{p-n} = 1.30(03)(47)\ \mathrm{MeV}$$

AWL, C.Carlson, G.Miller:
PRL 108 (2012)

$$= 0.76(30)\ \mathrm{MeV}$$

J. Gasser and H. Leutwyler:
Nucl Phys B94 (1975)

We reduced the uncertainty from structure by an order of magnitude! But we uncovered an oversight that dominates the uncertainty :(

adding it all up:

$$\delta M^\gamma \Big|_{p-n} = 1.30(03)(47)\ \text{MeV}$$

AWL, C.Carlson, G.Miller:
PRL 108 (2012)

$$= 0.76(30)\ \text{MeV}$$

J. Gasser and H. Leutwyler:
Nucl Phys B94 (1975)

expectation from experiment + lattice QCD

$$\delta M^\gamma \Big|_{p-n} = -1.29333217(42) + 2.53(40)\ \text{MeV}$$

$$= 1.24(40)\ \text{MeV}$$

average of 3 independent lattice results

Baryons and lattice QCD: Conclusions

- attempt to improve the old determination of nucleon iso-vector EM self-energy uncovered an oversight
  - no avoiding the subtraction (dispersion integral)
  - modeling was necessary to control uncertainty subtraction function
  - a central value was found in much better agreement with expectations from lattice QCD + experiment
  - comparison with independent determinations of iso-vector nucleon magnetic polarizability show the modeling is not crazy
- improvements will come from three areas
  - improved measurement of β_M^{p-n}
  - lattice QCD calculation of β_M^{p-n}
  - including EM effects with lattice QCD:

Fin