Bounded Model Checking with SAT/SMT

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Outline – Something Old, Something New

- Bounded Model Checking Using SAT
- Bounded Model Checking for Hybrid Systems
  - How to use numerical methods safely.

Method used by most "industrial strength" model checkers:

- uses Boolean encoding for state machine and sets of states.
- can handle much larger designs – hundreds of state variables.
- BDDs traditionally used to represent Boolean functions.

Problems with BDDs

- BDDs are a canonical representation. Often become too large.
- Variable ordering must be uniform along paths.
- Selecting right variable ordering very important for obtaining small BDDs.
  - Often time consuming or needs manual intervention.
  - Sometimes, no space efficient variable ordering exists.

BMC is an alternative approach to symbolic model checking that uses SAT procedures.

Advantages of SAT Procedures

- SAT procedures also operate on Boolean expressions but do not use canonical forms.
- Do not suffer from the potential space explosion of BDDs.
- Different split orderings possible on different branches.
- Very efficient implementations available.

Bounded Model Checking (Clarke, Biere, Cimatti, Zhu)

- Bounded model checking uses a SAT procedure instead of BDDs.
- We construct Boolean formula that is satisfiable iff there is a counterexample of length k.
- We look for longer and longer counterexamples by incrementing the bound k.
- After some number of iterations, we may conclude no counterexample exists and specification holds.
- For example, to verify safety properties, number of iterations is bounded by diameter of finite state machine.

Main Advantages of Our Approach

- Bounded model checking finds counterexamples fast. This is due to depth first nature of SAT search procedures.
- It finds counterexamples of minimal length. This feature helps user understand counterexample more easily.
- It uses much less space than BDD based approaches.
- Does not need manually selected variable order or costly reordering. Default splitting heuristics usually sufficient.
- Bounded model checking of LTL formulas does not require a tableau or automaton construction.

Implementation

- Implemented a tool BMC in 1999.
- It accepts a subset of the SMV language.
- Given k, BMC outputs a formula that is satisfiable iff counterexample exists of length k.
- If counterexample exists, a standard SAT solver generates a truth assignment for the formula.

- There are many examples where BMC significantly outperforms BDD based model checking.
- In some cases BMC detects errors instantly, while SMV fails to construct BDD for initial state.

Armin's example: Circuit with 9510 latches, 9499 inputs.
BMC formula has 4×10^6 variables, 1.2×10^7 clauses.
Shortest bug of length 37 found in 69 seconds.

Basic Definitions and Notation

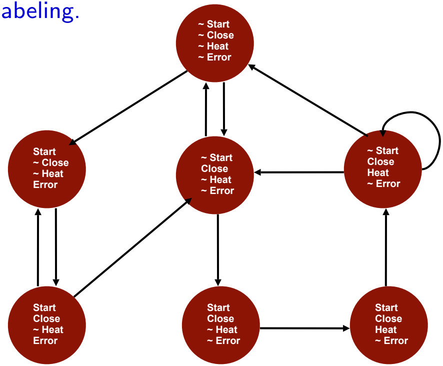
- We use linear temporal logic (LTL) for specifications.
- Basic LTL operators:

next time	'$\mathbf{X}$'	*eventuality*	'$\mathbf{F}$'
globally	'$\mathbf{G}$'	*until*	'$\mathbf{U}$'
release	'$\mathbf{R}$'		

- Only consider existential LTL formulas $\mathbf{E}f$, where
  - $\mathbf{E}$ is the existential path quantifier, and
  - f is a temporal formula with no path quantifiers.
- Recall that $\mathbf{E}$ is the dual of the universal path quantifier $\mathbf{A}$.
- Finding a witness for $\mathbf{E}f$ is equivalent to finding a counterexample for $\mathbf{A}\neg f$.

- System described as a Kripke structure $M = (S, I, T, \ell)$, where
  - S is a finite set of states and I a set of initial states,
  - $T \subseteq S \times S$ is the transition relation,
    (We assume every state has a successor state.)
  - $\ell\colon S \to \mathcal{P}(\mathcal{A})$ is the state labeling.
- The Microwave Oven Example:

$\mathbf{AG}(\mathit{start} \to (\neg \mathit{heat}\ \mathbf{U}\ \mathit{close}))$



Definitions and Notation (Cont.)

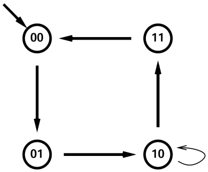
- In symbolic model checking, a state is represented by a vector of state variables $s = (s(1), \ldots, s(n))$.
- We define propositional formulas $f_I(s)$, $f_T(s,t)$ and $f_p(s)$ as follows:
  - $f_I(s)$ iff $s \in I$,
  - $f_T(s,t)$ iff $(s,t) \in T$, and
  - $f_p(s)$ iff $p \in \ell(s)$.
- We write $T(s,t)$ instead of $f_T(s,t)$, etc.

- If $\pi = (s_0, s_1, \ldots)$, then $\pi(i) = s_i$ and $\pi^i = (s_i, s_{i+1}, \ldots)$.
- π is a path if $\pi(i) \rightarrow \pi(i+1)$ for all i.
- $\mathbf{E}f$ is true in M ($M \models \mathbf{E}f$) iff there is a path π in M with $\pi \models f$ and $\pi(0) \in I$.
- Model checking is the problem of determining the truth of an LTL formula in a Kripke structure. Equivalently,

  Does a witness exist for the LTL formula?

Example To Illustrate New Technique

Two-bit counter with an erroneous transition:



- Each state s is represented by two state variables $s[1]$ and $s[0]$.
- In initial state, value of the counter is 0. Thus, $I(s) = \neg s[1] \wedge \neg s[0]$.
- Let $inc(s, s') = (s'[0] \leftrightarrow \neg s[0]) \wedge (s'[1] \leftrightarrow (s[0] \oplus s[1]))$
- Define $T(s, s') = inc(s, s') \vee (s[1] \wedge \neg s[0] \wedge s'[1] \wedge \neg s'[0])$
- Have deliberately added erroneous transition!!

- Suppose we want to know if counter will eventually reach state (11).
- Can specify the property by $\mathbf{AF}q$, where $q(s) = s[1] \wedge s[0]$.

  On all execution paths, there is a state where $q(s)$ holds.
- Equivalently, we can check if there is a path on which counter never reaches state (11).
- This is expressed by $\mathbf{EG}p$, where $p(s) = \neg s[1] \vee \neg s[0]$.

  There exists a path such that $p(s)$ holds globally along it.

- In bounded model checking, we consider paths of length k.
- We start with $k = 0$ and increment k until a witness is found.
- Assume k equals 2. Call the states s_0, s_1, s_2.
- We formulate constraints on s_0, s_1, and s_2 in propositional logic.
- Constraints guarantee that (s_0, s_1, s_2) is a witness for $\mathbf{EG}p$ and, hence, a counterexample for $\mathbf{AF}q$.

Example (Cont.)

- First, we constrain (s_0, s_1, s_2) to be a valid path starting from the initial state.
- Obtain a propositional formula

$$[\![M]\!] = I(s_0) \wedge T(s_0, s_1) \wedge T(s_1, s_2).$$

Example (Cont.)

- Second, we constrain the shape of the path.
- The sequence of states s_0, s_1, s_2 can be a loop or lasso.
- If so, there is a transition from s_2 to the initial state s_0, s_1 or itself.
- We write $L_l = T(s_2, s_l)$ to denote the transition from s_2 to a state s_l where $l \in [0, 2]$.
- We define L as $\bigvee_{l=0}^{2} L_l$. Thus $\neg L$ denotes the case where no loop exists.

Example (Cont.)

- The temporal property $\mathbf{G}p$ must hold on (s_0, s_1, s_2).
- If no loop exists, $\mathbf{G}p$ does not hold and $[\![\ \mathbf{G}p\ ]\!]$ is $false$.
- To be a witness for $\mathbf{G}p$, the path must contain a loop (condition L, given previously).
- Finally, p must hold at every state on the path

$$[\![\ \mathbf{G}p\ ]\!] = p(s_0) \wedge p(s_1) \wedge p(s_2).$$

- We combine all the constraints to obtain the propositional formula

$$[\![\ M\ ]\!] \wedge ((\neg L \wedge false) \vee \bigvee_{l=0}^{2} (L_l \wedge [\![\ \mathbf{G}p\ ]\!])).$$

- In this example, the formula is satisfiable.
- Truth assignment corresponds to counterexample path (00), (01), (10) followed by self-loop at (10).
- If self-loop at (10) is removed, then formula is unsatisfiable.

- Diameter d: Least number of steps to reach all reachable states. If the property holds for $k \geq d$, the property holds for all reachable states.
- Finding d is computationally hard:
  - State s is reachable in j steps:

$$R_j(s) := \exists s_0, \ldots, s_j : s = s_j \wedge I(s_0) \wedge \bigwedge_{i=0}^{j-1} T(s_i, s_{i+1})$$

  - Thus, k is greater or equal than the diameter d if

$$\forall s : R_{k+1}(s) \Longrightarrow \exists j \leq k : R_j(s)$$

This requires an efficient QBF checker!

Work in Progress: Bounded Model Checking Hybrid Systems (with S. Gao and J. Avigad)

Hybrid systems combine finite automata with continuous dynamical systems.

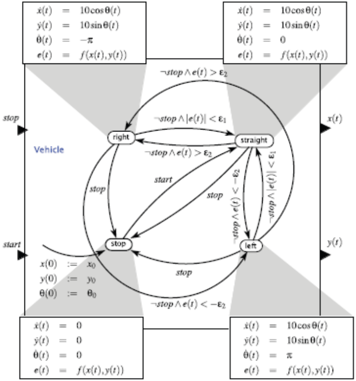
- They are widely used to model cyber-physical systems. (e.g., aerospace, automotive, and biological systems)
- They pose a grand challenge to formal verification.
  - Reachability for simple systems is undecidable.
  - Existing tools do not scale on realistic systems.
    - Less than ten variables and mostly constant dynamics.

$\mathcal{H} = \langle X, Q, \text{Init}, \text{Flow}, \text{Jump} \rangle$

- A state space $X \subseteq \mathbb{R}^k$ and a finite set of modes Q.
- Init $\subseteq Q \times X$: initial configurations
- Flow: continuous flows
  - Each mode q is equipped with differential equations $\dfrac{d\vec{x}}{dt} = \vec{f}_q(\vec{x}, t)$.
- Jump: discrete jumps
  - The system can be switched from $(q, \vec{x})$ to $(q', \vec{x}')$, resetting modes and variables.

Continuous flows are interleaved with discrete jumps.

Controller of an automated guided vehicle [Lee and Seshia, 2011]



Encoding Continuous Dynamics

Logical encoding is not limited to discrete systems.

- Continuous Dynamics: $\dfrac{d\vec{x}(t)}{dt} = \vec{f}(\vec{x}(t), t)$
  - The solution curve:
    $\alpha : \mathbb{R} \to X,\ \ \alpha(t) = \alpha(0) + \int_0^t \vec{f}(\alpha(s), s)ds.$
  - Define the predicate
    $[\![\mathsf{Flow}_f(\vec{x}_0, t, \vec{x})]\!]^{\mathcal{M}} = \{(\vec{x}_0, t, \vec{x}) : \alpha(0) = \vec{x}_0, \alpha(t) = \vec{x}\}$

Reachability:

$$\exists \vec{x}_0, \vec{x}, t.\ (\mathsf{Init}(\vec{x}_0) \wedge \mathsf{Flow}_f(\vec{x}_0, t, \vec{x}) \wedge \mathsf{Unsafe}(\vec{x}))\ ?$$

Combining continuous and discrete behaviors, we can encode bounded reachability for hybrid systems:

- "$\vec{x}$ is reachable after after 0 discrete jumps" is definable as:

$$\mathsf{Reach}^0(\vec{x}) := \quad \exists \vec{x}_0, t.\ [\mathsf{Init}(\vec{x}_0) \wedge \mathsf{Flow}(\vec{x}_0, t, \vec{x})]$$

- Inductively, "$\vec{x}$ is reachable after $k+1$ discrete jumps" is definable as:

$$\mathsf{Reach}^{k+1}(\vec{x}) := \exists \vec{x}_k, \vec{x}'_k, t.\ [\mathsf{Reach}^k(\vec{x}_k) \wedge \mathsf{Jump}(\vec{x}_k, \vec{x}'_k) \wedge \mathsf{Flow}(\vec{x}'_k, t, \vec{x})]$$

Reachability within n discrete jumps:

$$\exists \vec{x}.\ (\bigvee_{i=0}^{n} \mathsf{Reach}^i(\vec{x}) \wedge \mathsf{Unsafe}(\vec{x}))\ ?$$

The formulas that we have shown are first-order formulas over reals. Because of the dynamical systems involved, they usually contain a rich set of nonlinear functions:

- polynomials
- exponentiation and trigonometric functions
- solutions of ODEs, mostly no analytic forms

Difficulty with Symbolic Decision Procedures

Symbolic decision procedures are unlikely to scale on realistic problems.

- The arithmetic theory $(\times/+)$ is decidable but already highly complex.
  - Double-exponential (PSPACE for SMT, theoretically).
  - Very active research in the past twenty years. (Cylindrical Decomposition, Gröbner Bases, Postivstellensatz,...)
  - Available solvers: Hard to scale to more than ten variables.
- The general first-order theory over exp, sin, ODEs, ...
  - Wildly undecidable.

Scaling up: Use Numerical Methods?

However, large systems of real equalities/inequalities/ODEs are routinely solved **numerically**.

- They are perfect for simulation, but usually regarded inappropriate for verification because of their inevitable numerical errors.
  - (Platzer and Clarke, HSCC 2008)
- Is there a way of using them still?
- We need to start with a good formalization of "numerical algorithms".

What does it mean to say a function f over reals is "numerically computable"?

- There exists an algorithm M_f, such that given a good approximation of x, M_f can find a good approximation of $f(x)$.
  - "A real function is computable if we can draw it faithfully on a computer!"
- This leads to the well-developed framework of Computable Analysis (a.k.a. Type-II Computability) over real numbers. [A. Turing, A. Grzegorczyk, K. Weihrauch, S. Cook]

- Any real number a is encoded by a name $\gamma_a : \mathbb{N} \to \mathbb{Q}$ satisfying

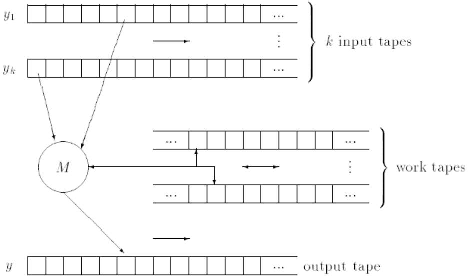
$$\forall i,\ |a - \gamma_a(i)| < 2^{-i}$$

- A Type-II Turing machine extends the ordinary by allowing input and output tapes to be both infinite. The working tape remains finite.
- Note that each symbol on the output tape of a Type-II machine needs to be written down after finitely many operations in the machine.

Type-II Computable Functions

- A function f is Type-II computable, if there exists a Type-II Turing machine $\mathcal{M}_f$, such that given any name of $\gamma_{\vec{x}}$ of $\vec{x} \in dom(f)$,

$$\mathcal{M}_f \text{ outputs a name of } \gamma_{f(\vec{x})} \text{ of } f(\vec{x}).$$



$f_M(y_1, \ldots, y_k) = y$

First-order Formulas with Computable Functions

- Let $\mathcal{F}$ be the set of all Type-II computable functions.
  - This is a very general framework: $\mathcal{F}$ contains polynomials, $\exp$, $\sin$, and solutions of Lipschitz-continuous ODEs.
- Consider the first-order the structure $\mathbb{R}_{\mathcal{F}} = \langle \mathbb{R}, 0, 1, \mathcal{F}, < \rangle$ and the corresponding language $\mathcal{L}_{\mathcal{F}}$.
- Can we solve SMT problems in $\mathcal{L}_{\mathcal{F}}$ over $\mathbb{R}_{\mathcal{F}}$?
  - This would allow us to solve formulas that arise in bounded model checking of hybrid systems.

Suppose we want to decide a formula in $\mathcal{L}_{\mathcal{F}}$:

$$\exists x \in I.(f(x) = 0 \wedge g(x) = 0).$$

($I \subseteq \mathbb{R}$ is a bounded interval where f and g are defined).

- Numerical algorithms can never compute $f(x)$ and $g(x)$ precisely for all x.
- But how about fixing some error bound δ, and relaxing the formula to:

$$\exists x \in I.(|f(x)| < \delta \wedge |g(x)| < \delta)?$$

We can consider formulas whose satisfiability is invariant under numerical perturbations. Formally:

- Consider any formula $\varphi := \bigwedge_i (\bigvee_j f_{ij}(\vec{x}) = 0)$.
  - Inequalities are turned into interval bounds on slack variables.
- A δ-perturbation on φ is a constant vector $\vec{c}$ satisfying $||\vec{c}|| < \delta$ ($|| \cdot ||$ denotes the maximum norm)

$$\varphi^{\vec{c}} := \bigwedge_i (\bigvee_j f_{ij}(\vec{x}) = c_{ij})$$

- We say φ is δ-robust, if its satisfiability is invariant under δ-perturbations:

For any δ-perturbation $\vec{c}$, $\exists \vec{x}.\varphi \leftrightarrow \exists \vec{x}.\varphi^{\vec{c}}$.

Computational Benefits

As it turns out, robust formulas in $\mathcal{L}_{\mathcal{F}}$ have nice computational properties.

- Theorem:
  Satisfiability of robust bounded SMT problems over $\mathbb{R}_{\mathcal{F}}$ is decidable.
  - This is significant given the richness of $\mathcal{F}$: exp, sin, ODEs...
- Decidability can be extended to quantified formulas.
- (Reasonably low) complexity results are in progress.

For general formulas, we can produce decision procedures using numerical oracles (with an error bound δ) that guarantee:

- If φ is decided as "unsatisfiable", then it is indeed unsatisfiable.
- If φ is decided as "satisfiable", then:

Under some δ-perturbation $\vec{c}$, $\varphi^{\vec{c}}$ is satisfiable.

If a decision procedure satisfies this property, we say it is "δ-complete".

Delta-Complete Bounded Model Checking

Recall that when bounded model checking a hybrid system $\mathcal{H}$, we ask if

$$\varphi : \mathsf{Reach}_{\mathcal{H}}^{\leq n}(\vec{x}) \wedge \mathsf{Unsafe}(\vec{x})$$

is satisfiable.

- If φ is unsatisfiable, then $\mathcal{H}$ is safe up to depth n.
- If φ is satisfiable, then $\mathcal{H}$ is unsafe.

Consequently, using a δ-complete decision procedure we can guarantee:

- If φ is "unsatisfiable", then $\mathcal{H}$ is safe up to depth n.
- If φ is "satisfiable", then

$\mathcal{H}$ is unsafe under some δ-perturbation!

Consequently, if a system can become unsafe under some δ-perturbation, we will be able to detect such unsafety.

- This can not be achieved using precise symbolic algorithms.

We are developing the practical SMT solver `dReal`.

- DPLL(T) + Interval Constraint Propagation (ICP).
  - ICP = Interval Arithmetic + Constraint Propagation
    - Floating-point arithmetic (no need for precise arithmetic)
    - ICP can handle highly complex nonlinear constraint systems with thousands of variables.
  - The DPLL(T) framework: SAT solver + ICP solver.
- Currently solvable signature: $+/\times$ exp, sin. [Gao et al. FMCAD 2010]
- In progress: Numerically stable ODEs.