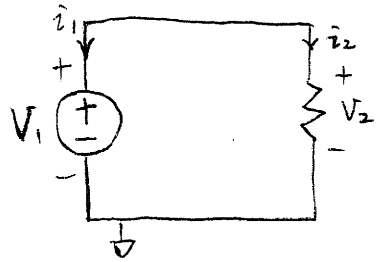
6.002 PSet 1 Solution

1. A&L P1.3

(1)



(2) Current in the bulb.

$$i_2 = \frac{V_2}{R} = \frac{7.2}{100} = 0.072 \text{ (A)}$$

Power dissipation in the bulb:

$$P_2 = V_2 i_2 = 7.2 \times 0.072 = 0.52 \text{ W}$$

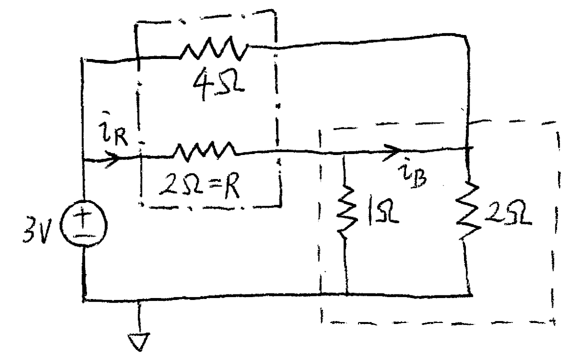
(3) Power into the battery

$$P_1 = V_1 i_1 = V_1 \cdot (-i_2) = -0.52 \text{ W}$$

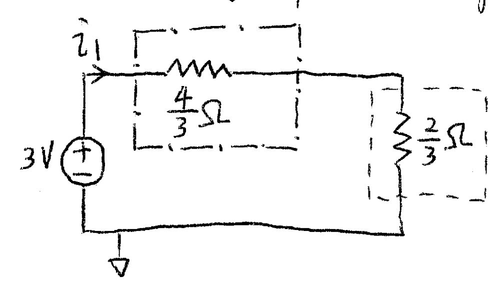
(4) $P_1 + P_2 = 0.52 - 0.52 = 0$

(5) $t = \frac{W}{P_2} = \frac{10000}{0.52} = 1.9 \times 10^5 \text{ s}$

2. A&L P2.9



⇓ parallel simplification



Total current from the battery (see above)

$$i_1 = \frac{3}{\frac{4}{3} + \frac{2}{3}} = 1.5 \text{ A}$$

Current through R (see above)

$$i_R = \frac{4}{2+4} \times 1.5 = 1 \text{ A}$$

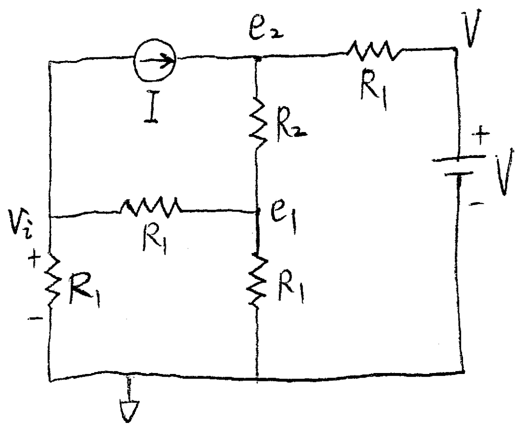
Power dissipation in R:

$$P_R = i_R^2 \cdot R = 2 \text{ W}$$

(Note: It's a balanced bridge. So $i_B = 0$.

$$i_R = \frac{3}{2+1} = 1 \text{ A})$$

3. A&L P 3.6



Node method:

@ v_i :

$$\frac{V_i}{R_1} + \frac{V_i - e_1}{R_1} + I = 0$$

@ e_1:

$$\frac{e_1 - V_i}{R_1} + \frac{e_1 - e_2}{R_2} + \frac{e_1}{R_1} = 0$$

@ e_2:

$$\frac{e_2 - e_1}{R_2} + \frac{e_2 - V}{R_1} - I = 0$$

$$\begin{bmatrix} \frac{2}{R_1} & -\frac{1}{R_1} & 0 \\ -\frac{1}{R_1} & \frac{2}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} \\ 0 & -\frac{1}{R_2} & \frac{1}{R_1}+\frac{1}{R_2} \end{bmatrix} \begin{bmatrix} V_i \\ e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} -I \\ 0 \\ I + \frac{V}{R_1} \end{bmatrix}$$

$$\therefore V_i = \frac{V - 2I(R_1+R_2)}{5 + \frac{3R_2}{R_1}} = \frac{2 - 2\times3\times5}{5+\frac{9}{2}}$$

$$= -2.95\ V$$

$$\left(e_1 = 2V_i + IR_1 = 0.105\ V \right.$$

$$\left. e_2 = V + IR_1 + V_i - 2e_1 = 4.84\ V \right)$$

4. i1ab

(1) Measurements: i_1, V_2

We also know V_1, as a known voltage source

i_2 is always zero.

A possible measurement is:

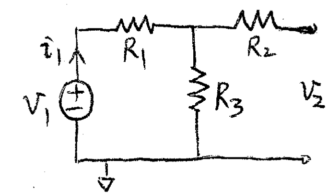
$V_1 = 1\ V$, $V_2 = 0.66\ V$, $i_1 = 34\ \mu A$.

Therefore,

$$\frac{V_2}{V_1} = \frac{R_3}{R_1+R_3} = 0.66$$

$$\frac{V_1}{i_1} = R_1 + R_3 = 29\ k\Omega$$

$$\begin{cases} R_3 = 19\ k\Omega \\ R_1 = 10\ k\Omega \end{cases}$$



(2) measurement(s): i_1, i_2

We also know V_1.

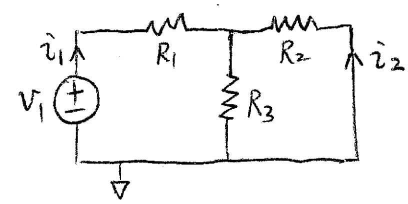
V_2 is always zero.

A possible measurement is.

$V_1 = 1\ V$, $i_1 = 42\ \mu A$, $i_2 = -12\ \mu A$

Therefore

$$\frac{V_1}{i_1} = R_1 + \frac{R_2 R_3}{R_2+R_3} = 24\ k\Omega$$



$$\left|\frac{i_2}{i_1}\right| = \frac{R_3}{R_2+R_3} = 0.29$$

(3) measurements : v_1, v_2.

We also know i_1.

i_2 is always zero.

A possible measurement is,

$i_1 = 100\mu A$, $v_1 = 2.95 V$, $v_2 = 1.96 V$

Therefore, similar to (1).

$$\frac{v_2}{v_1} = \frac{R_3}{R_1+R_3} = 0.66$$

$$\frac{v_1}{i_1} = R_1 + R_3 = 30\ k\Omega.$$

$$\therefore \begin{cases} R_3 = 20\ k\Omega \\ R_1 = 10\ k\Omega \end{cases}$$

(4) measurements: v_1, i_2.

We also know i_1.

v_2 is always zero.

A possible measurement is,

$i_1 = 100\mu A$, $v_1 = 2.39 V$, $i_2 = -28.5\mu A$.

There fore, similar to (2)

$$\frac{v_1}{i_1} = R_1 + \frac{R_2 R_3}{R_2+R_3} = 24\ k\Omega$$

$$\left|\frac{i_2}{i_1}\right| = \frac{R_3}{R_2+R_3} = 0.29$$

(5) ① Three measurements are required to determine three unknown resistor values.

② However, we can do multiple measurement in one experiment. E.g. in (1), we can determine both R_1 and R_3.

③ Within experiments (1) ~ (4), we need at least two of them to determine R_1, R_2 and R_3.

④ However, if we are allowed to design any experiment, it's possible to do all the three measurements in one experiment:

$$\begin{cases} v_1 = 1\ V & \text{constant voltage source} \\ i_2 = 0 \sim 10\mu A & \text{variable current source} \end{cases}$$

Record v_1, i_1, v_2, i_2 for all steps.

(6) From experiment (1) & (2), we know

$R_1 = 10\ k\Omega$, $R_3 = 19\ k\Omega$.

$$\frac{R_3}{R_2+R_3} = 0.29$$

Therefore,

$$\begin{cases} R_1 = 10\ k\Omega, \\ R_2 = 47\ k\Omega \\ R_3 = 19\ k\Omega \end{cases}$$