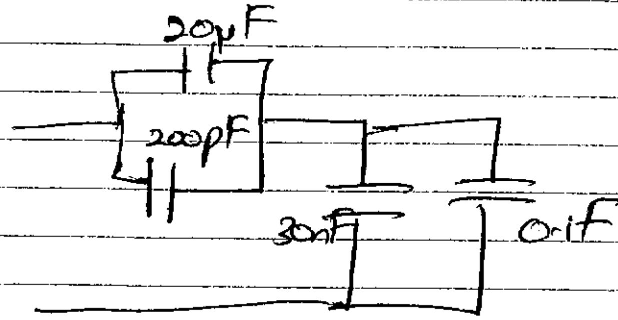
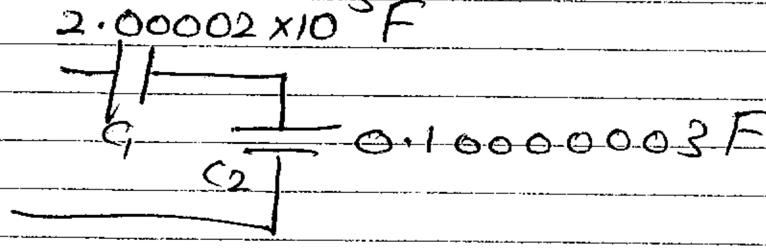
1

a)

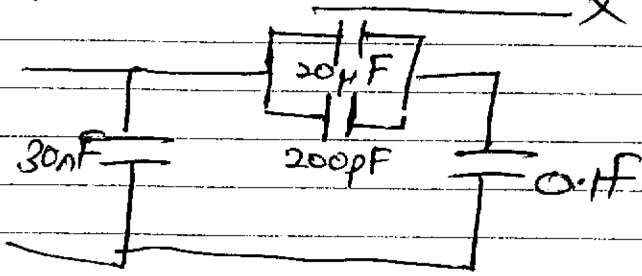


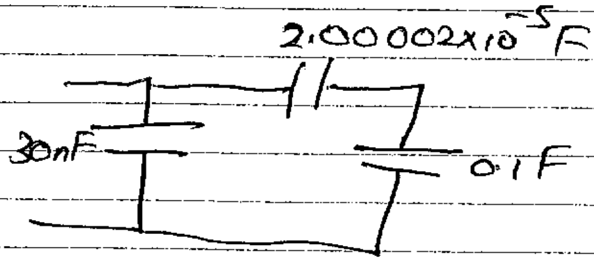


$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_T = 1.99962 \times 10^{-5} F$$

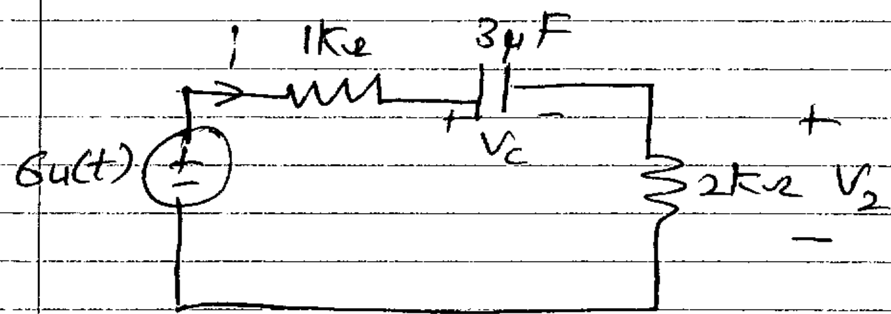
b)





$$C_T = 2.0026 \times 10^{-5} F$$

2.



$t<0 \quad V_c = 0 \quad V_2 = 0$

$t>0 \quad v_c = 6(1 - e^{-t/\tau}) \qquad \tau = 3k \times 3\mu = 9\times10^{-3}\,s$

$i = \frac{6}{3k} e^{-t/\tau}$

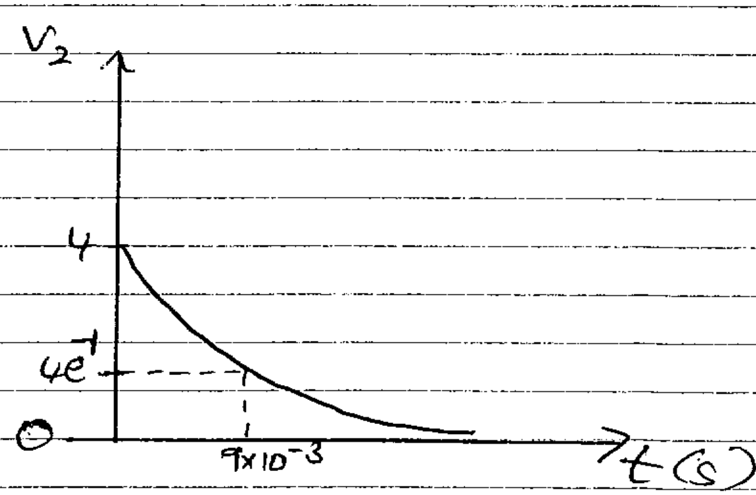
$-6 + i1k + v_c + V_2 = 0$

$V_2 = 6 - v_c - i1k$

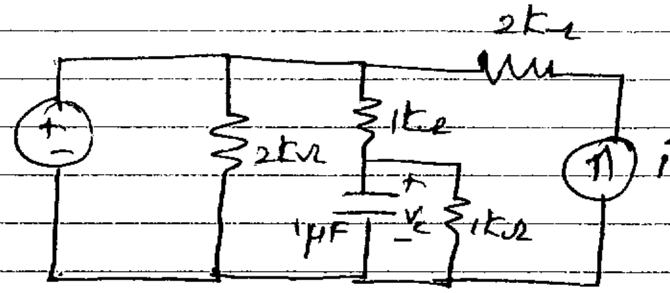
$V_2 = 6 - 6(1 - e^{-t/\tau}) - 2e^{-t/\tau}$

$V_2 = 4e^{-t/\tau} \qquad \tau = 9\times10^{-3}$

— x —

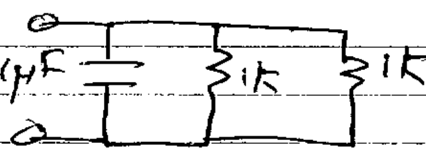


3. $t<0$ (switch closed)



$v_c = 2V$

Thevenin resistance looking from capacitor

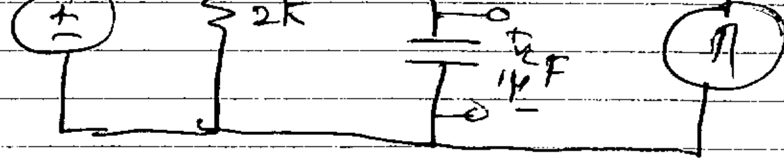


$R = 0.5k$
$C = 1\mu F$

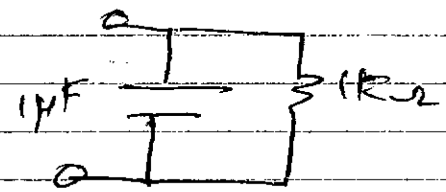
$\tau_2 = RC = 0.5ms$
$= 0.5 \times 10^{-3} s$

$V_1 = 2V$

$t>0$ (switch open)



R_{th}



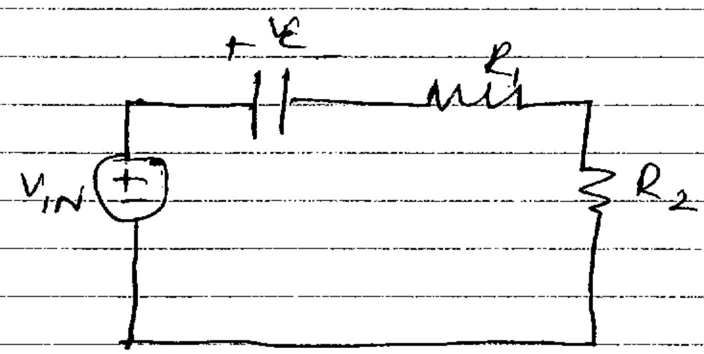
$\tau_1 = RC = 1\times10^{3} \times 1\mu F$
$= 1\times10^{-3}\, s$

$\boxed{\tau_1 = 1\times10^{-3}\, s}$

— x —

4.

a).



$$v_c(t) = V_{c_{initial}} e^{-t/\tau} + V_{c_{final}} \left(1 - e^{-t/\tau}\right)$$

if $V_{IN}(t) = V$

$V_{c_{initial}} = -V$ since $T >> RC$ so capacitor has settled to final voltage

$$v_c(t) = -Ve^{-t/\tau} + V\left(1 - e^{-t/\tau}\right).$$

$$\boxed{v_c(t) = V\left(1 - 2e^{-t/\tau}\right) \quad \text{if } V_{in}(t) = V}$$

if $V_{IN}(t) = -V$

$$v_c(t) = Ve^{-t/\tau} - V\left(1 - e^{-t/\tau}\right)$$

$$\boxed{v_c(t) = -V\left(1 - 2e^{-t/\tau}\right) \quad \text{if } v_{IN}(t) = -V}$$

$$\boxed{\tau = (R_1 + R_2)C}$$

b) if $V_{IN}(t) = V$

$$v_c(t) = V(1 - 2e^{-t/\tau})$$
$$\tau = (R_1 + R_2)C$$

$$i_c = C\frac{dV_c}{dt}$$

$$i_c = CV\frac{2}{\tau}e^{-t/\tau}$$

$V_{IN} = V$ $\boxed{i_c = \frac{2V}{R_1 + R_2}e^{-t/\tau}}$ $\tau = (R_1 + R_2)C$

if $V_{IN}(t) = -V$

$$v_c(t) = -V(1 - 2e^{-t/\tau})$$

$$i_c = C\frac{dV_c}{dt}$$

$$= -C\frac{V2}{\tau}e^{-t/\tau}$$

$V_{IN} = -V$ $\boxed{i_c = -\frac{2V}{R_1 + R_2}e^{-t/\tau}}$ $\tau = (R_1 + R_2)C$

— x —

c) $V_{out} = i_c R_2$

if $V_{IN} = V$ $\quad V_{out} = \frac{2VR_2}{R_1+R_2} e^{-t/\tau}$ $\quad \tau = (R_1+R_2)C$

if $V_{IN} = -V$ $\quad V_{out} = \frac{-2VR_2}{R_1+R_2} e^{-t/\tau}$ $\quad \tau = (R_1+R_2)C$

— x —

d) from plot

$t = 0$ $\quad V_{out} = 203.92 \times 10^{-3} V$

$t = 850 \times 10^{-6}$ $\quad V_{out} = 16.98 \times 10^{-3} V$

$$16.98m = 203.92m \, e^{-\frac{850 \times 10^{-6}}{\tau}}$$

$$\boxed{\tau = 340 \mu S}$$

e) $V_{out} = 174.59 mV$

$V_{IN} = 1.069 V$

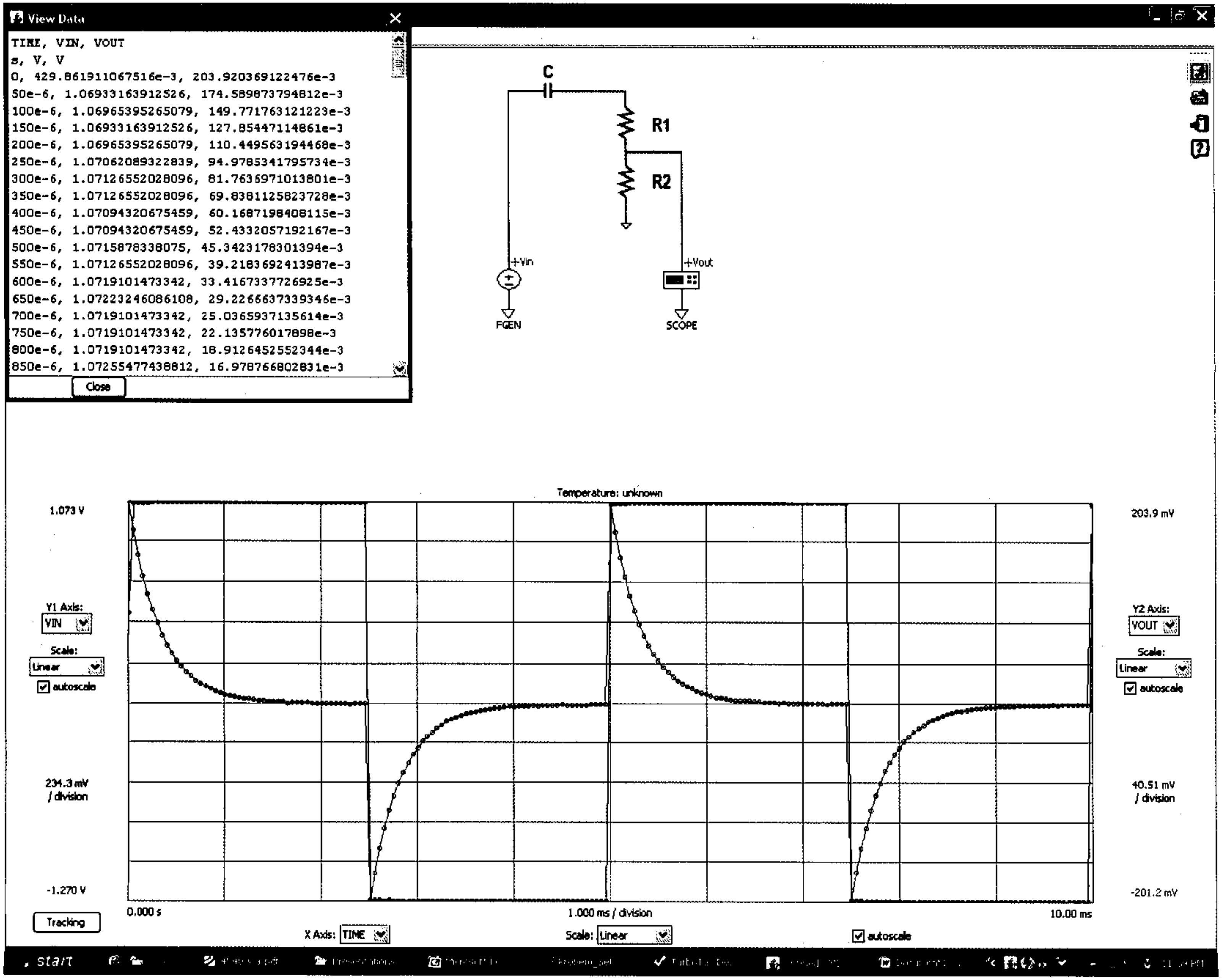
$t = 50 \times 10^{-6}$

$$174.59m = \frac{2(1.069)3k}{R_1 + 3k} e^{-\frac{50 \times 10^{-6}}{340 \times 10^{-6}}}$$

$$\boxed{R_1 = 28.7 k\Omega}$$

f) $\tau = 340 \mu S = (28.7+3)k \, C$

$$\therefore \boxed{C = 10.7 nF}$$


View Data
TIME, VIN, VOUT
s, V, V
0, 429.861911067516e-3, 203.920369122476e-3
50e-6, 1.06933163912526, 174.589873794812e-3
100e-6, 1.06965395265079, 149.771763121223e-3
150e-6, 1.06933163912526, 127.85447114861e-3
200e-6, 1.06965395265079, 110.449563194468e-3
250e-6, 1.07062089322839, 94.9785341795734e-3
300e-6, 1.07126552028096, 81.7636971013801e-3
350e-6, 1.07126552028096, 69.8381125823728e-3
400e-6, 1.07094320675459, 60.1687198408115e-3
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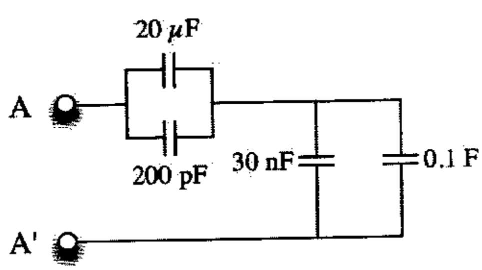
500e-6, 1.0715878338075, 45.3423178301394e-3
550e-6, 1.07126552028096, 39.2183692413987e-3
600e-6, 1.0719101473342, 33.4167337726925e-3
650e-6, 1.07223246086108, 29.2266637339346e-3
700e-6, 1.0719101473342, 25.0365937135614e-3
750e-6, 1.0719101473342, 22.135776017898e-3
800e-6, 1.0719101473342, 18.9126452552344e-3
850e-6, 1.07255477438812, 16.978766802831e-3
Close
C
R1
R2
+Vin
+Vout
FGEN
SCOPE
Temperature: unknown
1.073 V
Y1 Axis:
VIN
Scale:
Linear
autoscale
234.3 mV
/ division
-1.270 V
Tracking
0.000 s
1.000 ms / division
10.00 ms
X Axis: TIME
Scale: Linear
autoscale
203.9 mV
Y2 Axis:
VOUT
Scale:
Linear
autoscale
40.51 mV
/ division
-201.2 mV
start

Homework #6 – March 18, 2009

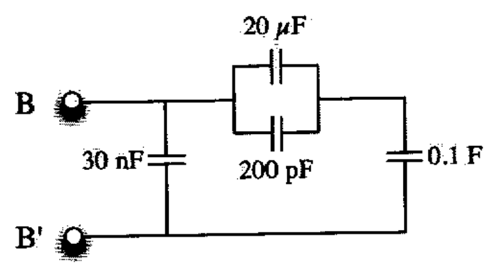
Due: April 1, 2009 at recitation

No late homework accepted

1. *[20 points]* Find the equivalent capacitance of the following two networks, (a) and (b):



(a)



(b)

2. *[20 points]* Exercise 10.16, p572 of Agarwal and Lang. (Note 'Exercise' not 'Problem')

3. *[20 points]* Exercise 10.20, p572 of Agarwal and Lang.

4. *[40 points]* This problem studies the response of a series RC network, both theoretically and experimentally. The experiments will be performed using the ELVIS iLab. The circuit to be studied is shown below. It comprises a capacitor, two resistors and a voltage source all in series, Figure 4.1. The voltage $v_{OUT}(t)$ across R_2 can be measured and used to determine the current through the series network.

b) Derive an expression for $i_C(t)$, the current flowing through network.

c) Derive an expression for $v_{OUT}(t)$, the voltage across R_2.

Now consider an experimental study of the network. First, log in to the ELVIS iLab as in previous homeworks. After launching the iLab, you should see a network that is equivalent to the one shown above.

First, select the voltage source, or FGEN signal generator, and set its parameters to WaveForm = SQUARE, Frequency = 200 Hz, Amplitude = 1 V, and O□set = 0 V. Second, select the SCOPE output measurement unit and program it with a suitable sampling rate that will allow you to see at least one full cycle of $v_{OUT}(t)$ with enough resolution. Note that the system will only allow you to take a maximum of 201 data samples at the output. Third, run the experiment. Finally, select $v_{IN}(t)$ for the Y1 axis and $v_{OUT}(t)$ for the Y2 axis, and use linear axes for both. When the figure resembles what you expect, capture a screen shot for subsequent analysis.

d) From the experimental data, extract the *RC* time constant of the network. You can see the actual numerical values of the data that you have obtained by looking into *View Data* under the *Results* menu. You can also download the data to Excel using the *Results* menu

e) From the experimental data, extract the value of the resistor R_1. When you do this, note that even though you selected 1 V as the amplitude, the signal generator does not impose this voltage very accurately; the actual amplitude is measured as v_{IN}.

f) From the experimental data, extract the value of *C*.