Problem Set # 6

1. Since $V_{BS} = -1$ First we need to Find the new threshold voltage (V_{Tn})

$$V_{Tn} = V_{Ton} + \gamma_n \left(\sqrt{-2\Phi_p - V_{BS}} - \sqrt{-2\Phi_p}\right)$$

$$= 1\,v + 0.6\left(\sqrt{0.84+1} - \sqrt{0.84}\right) = 1.26\ v$$

Since $V_{GS} = 3\,v$ is bigger than $V_{Tn} = 1.26\ v$ and $V_{DS} = 5\,v$ is bigger than $V_{GS} - V_{Tn} = 1.74\ v$, the nMOS is in saturation, as expected.

Note that channel length modulation is ignored to make the calculations simpler

$$I_D = \frac{W}{2L}\mu_n C_{ox}(V_{GS} - V_{Tn})^2 = \frac{22}{2\times 1.8}\ 50\,\frac{\mu A}{V^2}\,(3v - 1.26v)^2 = \boxed{925\,\mu A}$$

note that $L = L_{gate} - 2L_D = 2.0\ \mu m - 0.2\ \mu m = 1.8\ \mu m$

a)

$$g_m = \sqrt{2\frac{W}{L}\mu_n C_{ox} I_D} = \sqrt{2\ \frac{22}{1.8}\ 50\times10^{-6} \times 925\times10^{-6}} = \boxed{1063\ \mu S}$$

$$g_{mb} = \frac{\gamma_n g_m}{2\sqrt{-2\Phi_p - V_{BS}}} = \frac{0.6\times 1063\times10^{-6}}{2\sqrt{0.84+1}} = \boxed{235\ \mu S}$$

$$g_o = \frac{1}{r_o} = \lambda_n I_D = \frac{0.1}{1.8}\times 925\times10^{-6} = \boxed{51\ \mu S} \Rightarrow r_o = \boxed{19.5\ k\Omega}$$

b)

$$C_{gs} = \frac{2}{3}WLC_{ox} + WC_{ov} \simeq \frac{2}{3}WLC_{ox} + WL_DC_{ox} = \left(\frac{2}{3}L + L_D\right)WC_{ox}$$

$$= \left(\frac{2}{3}\,1.8\,\mu m + 0.1\,\mu m\right) 22\,\mu m \times 1.42\ FF/\mu m^2 = \boxed{40.6\ FF}$$

$$C_{gd} = WC_{ov} \simeq WL_DC_{ox} = 22\,\mu m \times 0.1\,\mu m \times 1.42\ FF/\mu m^2 = \boxed{3.1\ FF}$$

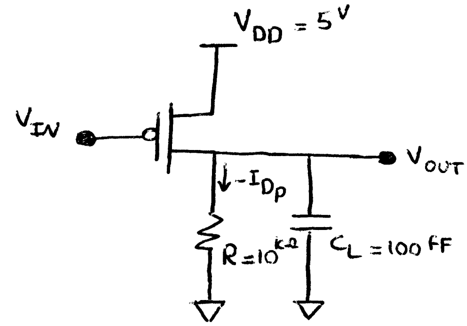
$$C_{sb} = WL_{diff}\sqrt{\frac{q\varepsilon_s N_a}{2(\Phi_B - V_{BS})}} = 22\,\mu m \times 6\,\mu m\sqrt{\frac{1.6\times10^{-19}\times 11.7\times 8.85\times10^{-14}\times10^{17}}{2(0.97+1)}}$$

$$= \boxed{85.6\ FF}$$

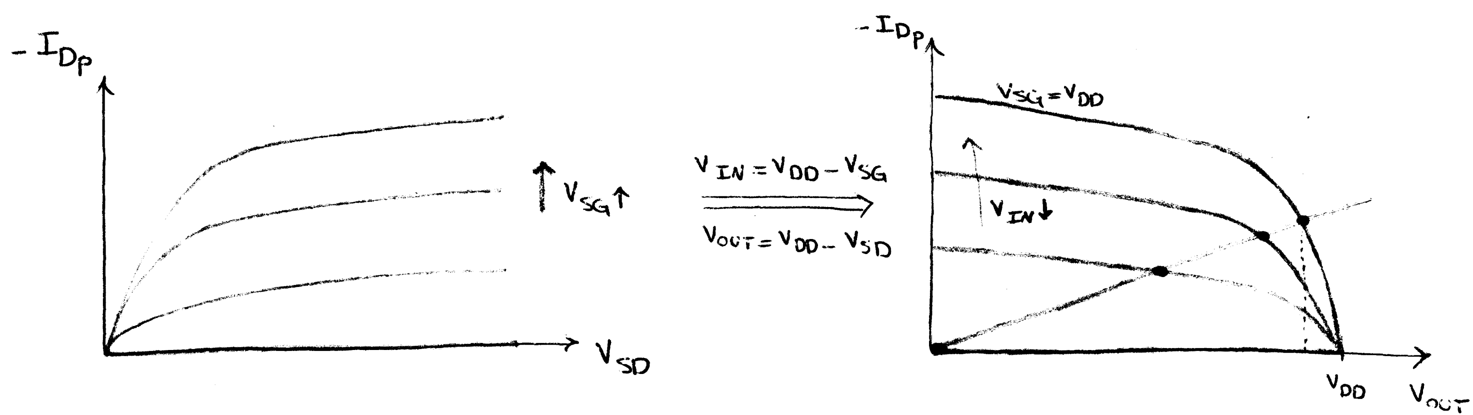
$$C_{db} = WL_{diff}\sqrt{\frac{q\varepsilon_s N_a}{2(\Phi_B - V_{BD})}} = 22\,\mu m\times 6\,\mu m\sqrt{\frac{1.6\times10^{-19}\times 11.7\times 8.85\times10^{-14}\times10^{17}}{2(0.97+6)}}$$

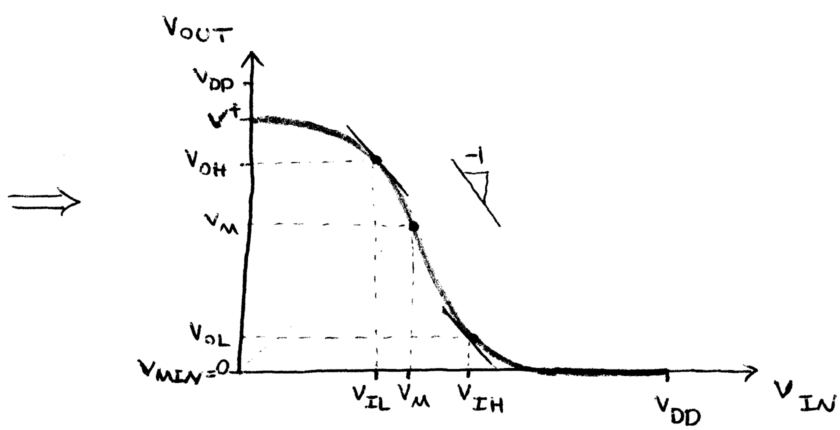
$$= 45.5\ FF$$

2.



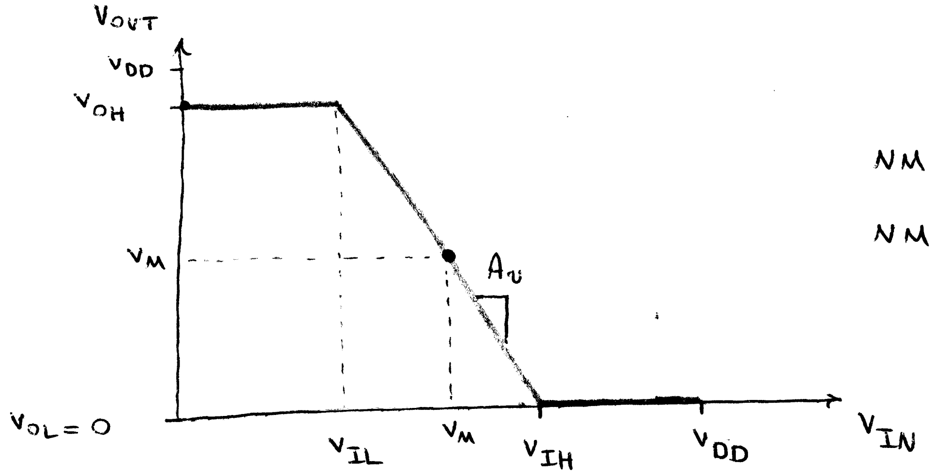
a) The voltage transfer charcteristic can be calculated from drain characteristics of the pMOS :





notice $V^+ < V_{DD}$ and $V_{MIN} = 0^V$

b) In the simplified model the transfer characteritics is approximated by a piecewise linear function :



$$NM_H = V_{OH} - V_{IH}$$

$$NM_L = V_{IL} - V_{OL} = V_{IL}$$

at $V_{IN} = V_M$, where $V_{OUT} = V_M$, the pMOS is in saturation, there :

$$-I_{Dp} = \frac{W}{2L} \mu_p C_{ox} (V_{SG} + V_{Tp})^2 = I_R = \frac{V_{OUT}}{R}$$

$$\Rightarrow \frac{W}{2L} \mu_p C_{ox} (V_{DD} - V_M + V_{Tp})^2 = \frac{V_M}{R}$$

$$\Rightarrow \frac{20}{2 \times 1} 25 \times 10^{-6} (5 - V_M - 1)^2 = \frac{V_M}{10^4} \Rightarrow V_M^2 - 8.4 V_M + 16 = 0$$

$$\Rightarrow \boxed{V_M = 2.9 \text{ V}}$$

Also $-I_{Dp} = I_R = \frac{V_M}{R} = \frac{2.9 \text{ V}}{10 \text{ k}\Omega} = 290 \ \mu A$

therefore $g_m = \sqrt{2 \frac{W}{L} \mu_p C_{ox} (-I_D)} = \sqrt{2 \frac{20}{1} 25 \times 10^{-6} \times 290 \times 10^{-6}}$

$$= 539 \ \mu S$$

so $A_v = -g_m (r_o \| R) \simeq -g_m R = \boxed{-5.39}$

By knowing V_M and A_v we can calculate V_{IH} :

$$V_{IH} = V_M - \frac{V_M}{A_v} = 2.9^V + \frac{2.9^V}{5.39} = \boxed{3.4^V}$$

To figure out V_{OH} we need to write down the equation for drain current at $V_{IN} = 0$ ($V_{OUT} = V_{OH}$). since the pMOS is in triode regime at this point:

$$\frac{W}{L} \mu_p C_{ox} \left(V_{SG} + V_{Tp} - \frac{V_{SD}}{2} \right) V_{SD} = \frac{V_{OUT}}{R}$$

$$\Rightarrow \frac{W}{L} \mu_p C_{ox} \left(V_{DD} + V_{Tp} - \frac{V_{DD} - V_{OH}}{2} \right) (V_{DD} - V_{OH}) = \frac{V_{OH}}{R}$$

$$\Rightarrow \frac{W}{2L} \mu_p C_{ox} (V_{DD} + 2V_{Tp} + V_{OH})(V_{DD} - V_{OH}) = \frac{V_{OH}}{R}$$

$$\Rightarrow \frac{20}{2 \times 1} 25 \times 10^{-6} (5 - 2 + V_{OH}) (5 - V_{OH}) = \frac{V_{OH}}{10^4}$$

$$\Rightarrow V_{OH}^2 - 1.6 V_{OH} - 15 = 0 \Rightarrow \boxed{V_{OH} = 4.8^V}$$

therefore

$$V_{IL} = \frac{1 - A_v}{-A_v} V_M - \frac{V_{OH}}{-A_v} = \frac{1 + 5.39}{5.39} 2.9 - \frac{4.8}{5.39} = \boxed{2.5^V}$$

Finally

$$NM_H = V_{OH} - V_{IH} = 4.8 - 3.4 = 1.4^V$$

$$NM_L = V_{IL} - V_{OL} = 2.5 - 0 = 2.5^V$$

c)

$$C_{DB} = W L_{diff} C_{jp} + (W + 2L_{diff}) C_{JSWp}$$

$$= 22\,\mu m \times 6\,\mu m \times 0.3\ FF/\mu m^2 + (22\,\mu m + 12\,\mu m) \times 0.35\ FF/\mu m$$

$$= \boxed{51.5\ FF}$$

$$C_{Total} = (C_L \parallel C_{DB}) = C_L + C_{DB} = 100 + 51 = \boxed{151\ FF}$$

$$t_{PLH} = R C_{Total} \ln(2) = 10^4 \times 151 \times 10^{-15} \ln(2) = \boxed{1.05\ n\,sec}$$

$$t_{PHL} = R C_{Total} \ln\left(\frac{-2 I_D R}{-2 I_D R - V_{DD}}\right) \quad \text{where } -I_D = 290\,\mu A$$

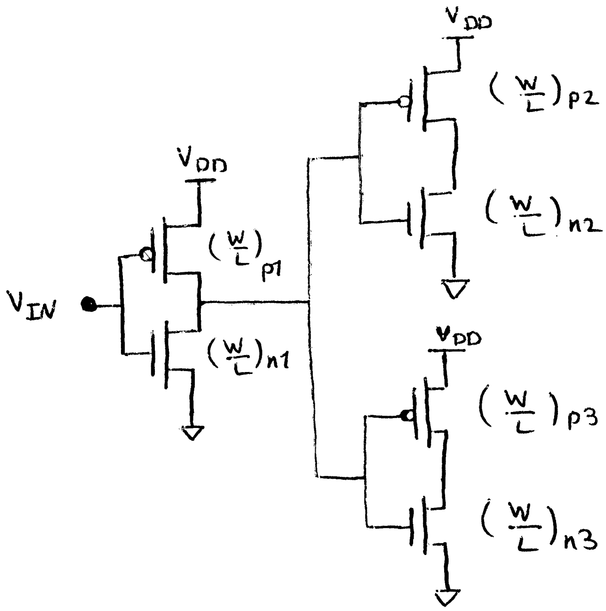
(From part (b))

$$\Rightarrow t_{PHL} = 10^4 \times 151 \times 10^{-15} \ln\left(\frac{2 \times 290 \times 10^{-6} \times 10^4}{2 \times 290 \times 10^{-6} \times 10^4 - 5}\right) = \boxed{2.99\ n\,sec}$$

d) Static power dissipation is due to the power consumed by the pull down resistor when V_{OUT} is high (V_{OH})

$$P_{diss} = \frac{V_{OH}^2}{R} = \frac{(4.8)^2}{10^4} = 2.3 \times 10^{-3}\ J/s$$

3.



$t_{ox} = 20$ nm

$\Rightarrow C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = 0.17\ \mu F/cm^2$

a) $$V_M = \frac{V_{Tn} + \sqrt{\frac{k_p}{k_n}}\,(V_{DD} + V_{Tp})}{1 + \sqrt{\frac{k_p}{k_n}}}$$

$$\Rightarrow \frac{k_p}{k_n} = \left(\frac{V_M - V_{Tn}}{V_{DD} + V_{Tp} - V_M}\right)^2 = \left(\frac{2.5 - 0.7}{5 - 0.9 - 2.5}\right)^2 = 1.26$$

$$\frac{W_n}{W_p} = \frac{\mu_p}{\mu_n}\,\frac{k_n}{k_p} = \frac{200}{500}\,\frac{1}{1.26} = \boxed{0.32}$$

b) $$I_{Dn} = \left(\frac{W}{2L}\right)_n \mu_n C_{ox}\,(V_M - V_{Tn})^2 \Rightarrow$$

$$1\,mA = \frac{W_n}{2 \times 2\,\mu m}\,500 \times 0.17 \times 10^{-6}\,(2.5 - 0.7)^2 \Rightarrow \boxed{W_n = 14.5\,\mu m}$$

$$-I_{Dp} = \left(\frac{W}{2L}\right)_p \mu_p C_{ox}\,(V_{DD} - V_M + V_{Tp})^2 \Rightarrow$$

$$1\,mA = \frac{W_p}{2 \times 2\,\mu m}\,200 \times 0.17 \times 10^{-6}\,(5 - 2.5 - 0.9)^2 \Rightarrow \boxed{W_p = 46.0\,\mu m}$$

c) $C_{DB} = W_n L_{diff_n} C_{jn} + (W_n + 2L_{diff_n}) C_{JSW_n} + W_p L_{diff_p} C_{jp} + (W_p + 2L_{diff_p}) C_{JSW_p}$

$= 14.5\,\mu m \times 6\,\mu m \times 0.1\ fF/\mu m^2 + (14.5\,\mu m + 12\,\mu m)\ 0.5\ fF/\mu m$

$+ 46.0\,\mu m \times 6\,\mu m \times 0.3\ fF/\mu m^2 + (46.0\,\mu m + 12\,\mu m)\ 0.35\ fF/\mu m$

$= \boxed{125\ fF}$

d) Since the source acting as ground for the four nMOS and pMOS in the second stage :

$$C_L = 2 C_{gs_n} + 2 C_{gs_p} \simeq 2\left(\tfrac{2}{3} W_n L C_{ox} + \tfrac{2}{3} W_p L C_{ox}\right)$$

$$= 2\left(\tfrac{2}{3} \times 14.5\,\mu m \times 2\,\mu m \times 0.17\ \tfrac{\mu F}{cm^2} + \tfrac{2}{3} \times 46.0\,\mu m \times 2\,\mu m \times 0.17\ \tfrac{\mu F}{cm^2}\right)$$

$$= \boxed{274\ fF}$$

e) $$t_{PLH} \simeq \frac{C_{total}\ V_{DD}}{\frac{W_p}{L_p} \mu_p C_{ox} (V_{DD} + V_{Tp})^2} = \frac{(125\ fF + 274\ fF)\ 5v}{\frac{46}{2}\ 200 \times 0.17 \times 10^{-6}\ (5 - 0.9)^2}$$

$$= \boxed{152\ p\ sec}$$

$$t_{PHL} \simeq \frac{C_{total}\ V_{DD}}{\frac{W_n}{L_n} \mu_n C_{ox} (V_{DD} - V_{Tn})^2} = \frac{(125\ fF + 274\ fF)\ 5v}{\frac{14.5}{2}\ 500 \times 0.17 \times 10^{-6}\ (5 - 0.7)^2}$$

$$= \boxed{175\ p sec}$$

4.

a) The graph for transistor output characteristics is on next page.

b) For small V_{DS} and $V_{BS} = 0$:

$$I_D = \frac{W}{L} \mu_n C_{ox} \left(V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} \simeq \frac{W}{L} KP \left(V_{GS} - V_{Ton} \right) V_{DS}$$

Therefore KP can be calculated from the slope of the graph in part (a) for the small V_{DS} region.

For high V_{DS} and $V_{BS} = 0$:

$$I_D = \frac{W}{2L} \mu_n C_{ox} \left(V_{GS} - V_T \right)^2 \left(1 + \lambda V_{DS} \right)$$

$$= \left[\frac{W}{2L} KP(V_{GS} - V_{To})^2 \right] + \left[\frac{W}{2L} KP(V_{GS} - V_{To})^2 \lambda \right] V_{DS}$$

Therefore LAMBDA can be calculated from the slope of the graph in part (a) for high V_{DS} region.

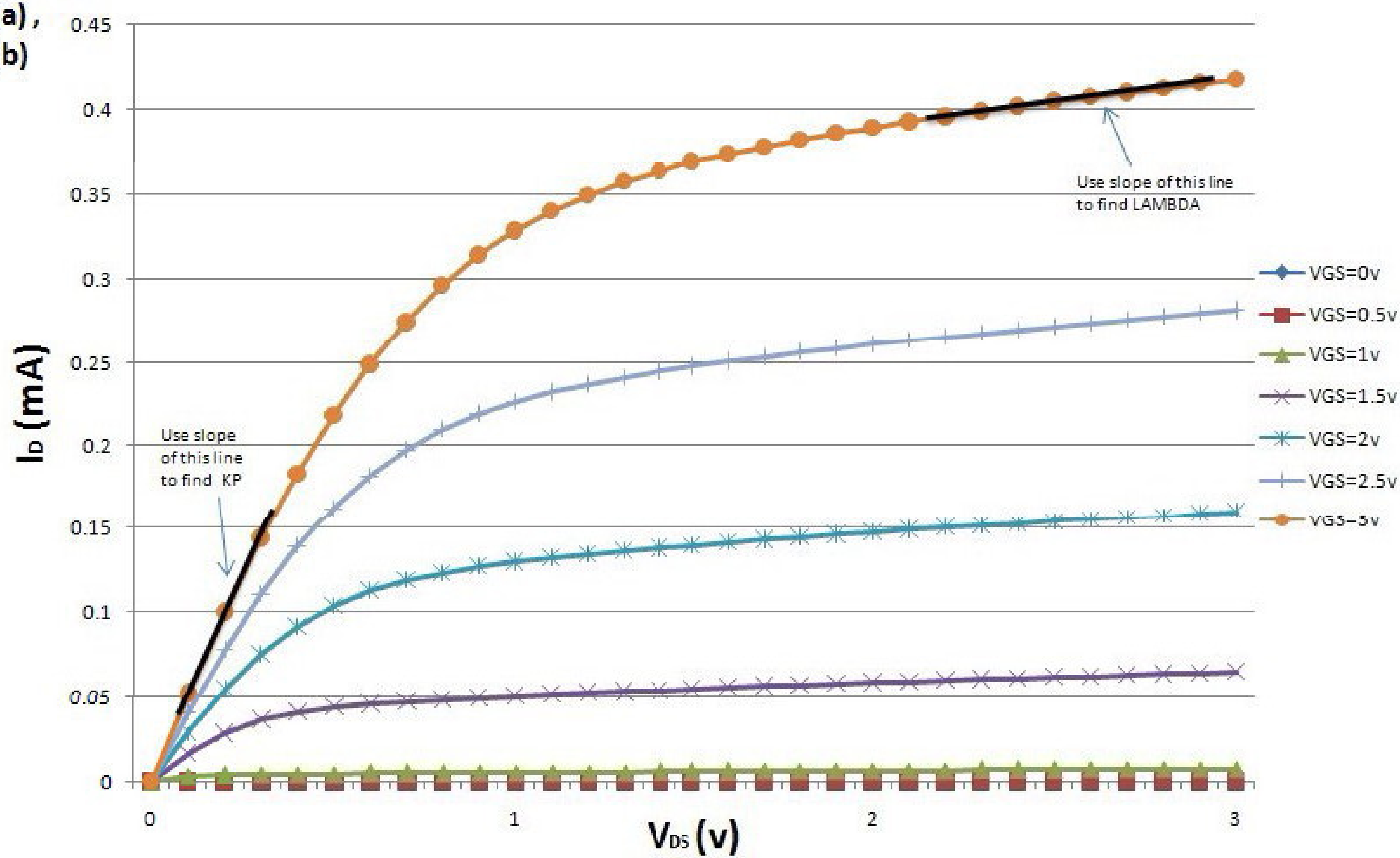
summary of results:

KP = $33.5 \frac{\mu A}{V^2}$, LAMBDA = $0.058\ V^{-1}$

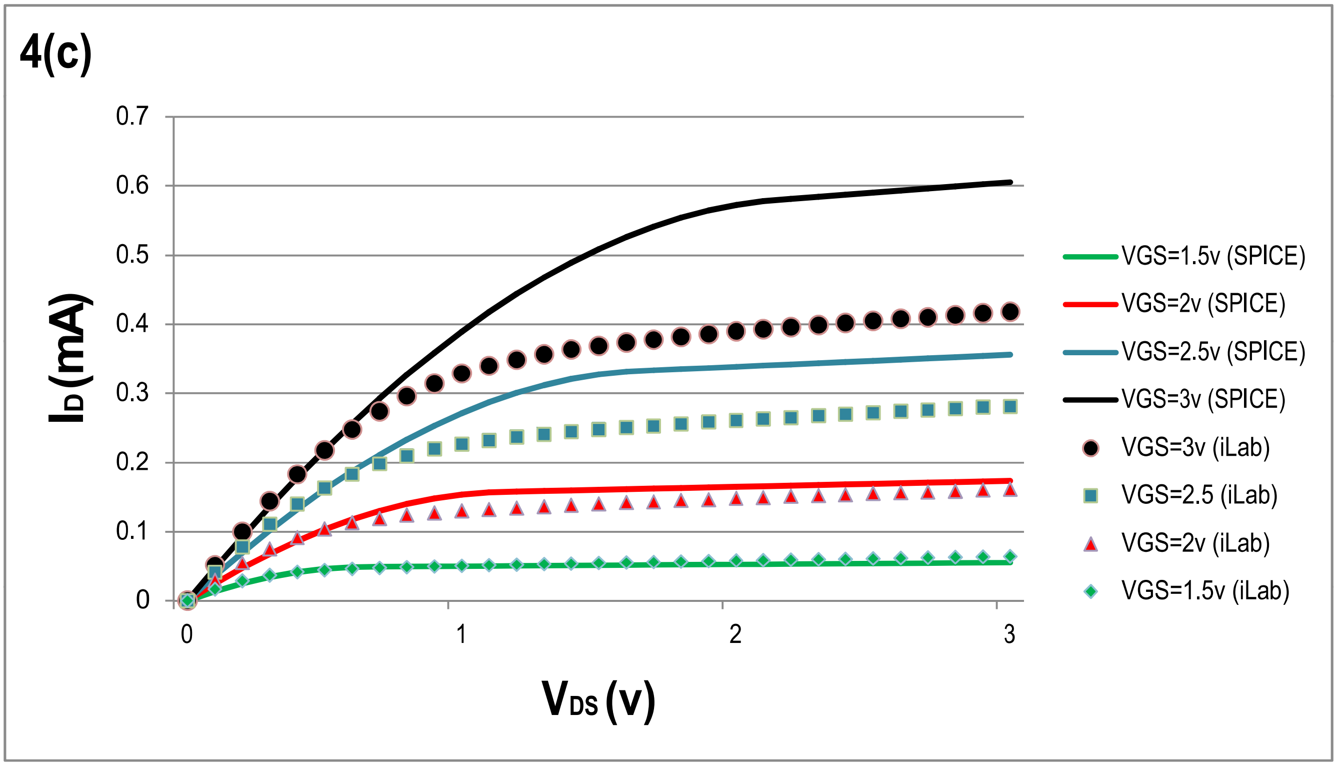
From pset #5 → VTO = 0.85 v , GAMMA = $1.6\ V^{1/2}$, PHI = 0.8 v

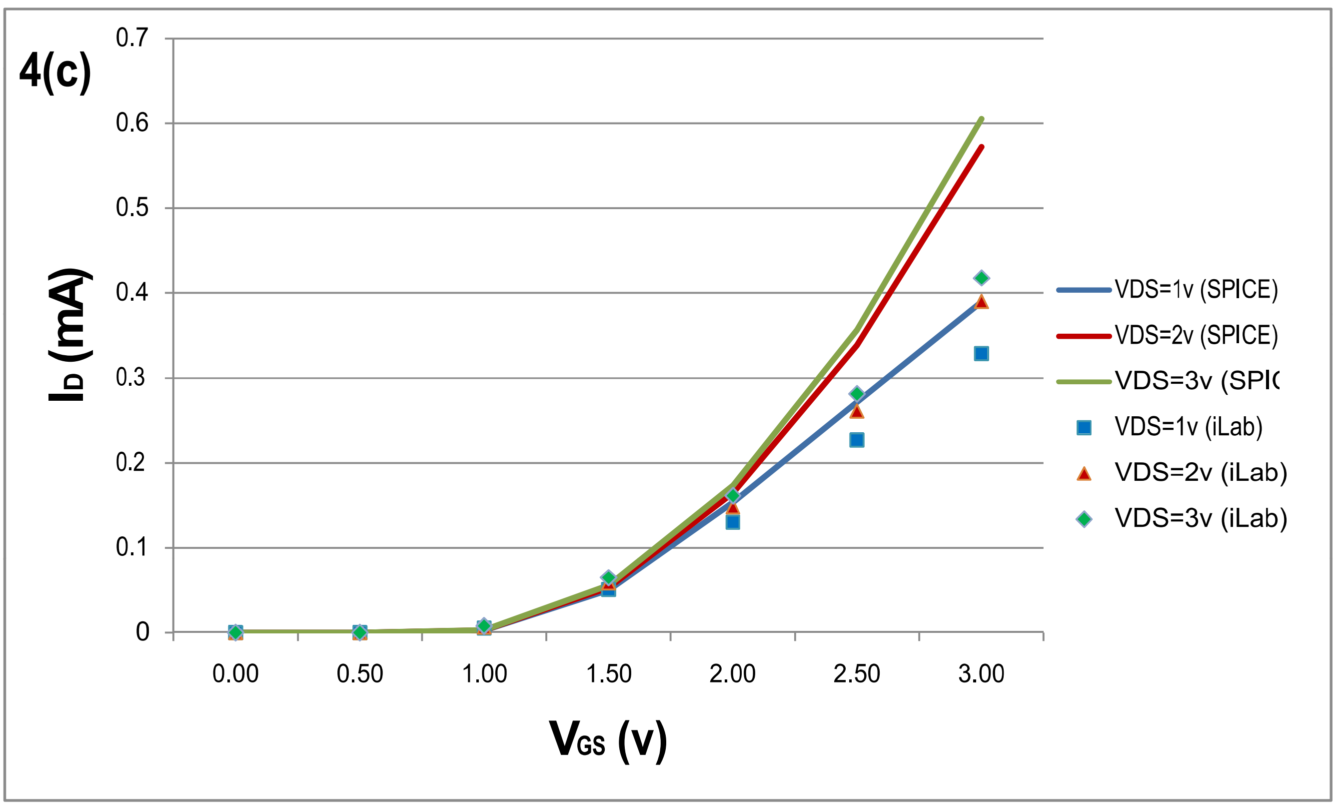
c) Output characteristics (I_D vs V_{DS}) and transfer characteristics (I_D vs V_{GS}) is on next pages.

4(a) ,
4(b)



0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
I_D (mA)
Use slope of this line to find KP
Use slope of this line to find LAMBDA
0
1
2
3
V_DS (v)
VGS=0v
VGS=0.5v
VGS=1v
VGS=1.5v
VGS=2v
VGS=2.5v


4(c)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
VDS (v)
ID (mA)
VGS=1.5v (SPICE)
VGS=2v (SPICE)
VGS=2.5v (SPICE)
VGS=3v (SPICE)
VGS=3v (iLab)
VGS=2.5 (iLab)
VGS=2v (iLab)
VGS=1.5v (iLab)


4(c)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.00
0.50
1.00
1.50
2.00
2.50
3.00
VGS (v)
ID (mA)
VDS=1v (SPICE)
VDS=2v (SPICE)
VDS=3v (SPI(
VDS=1v (iLab)
VDS=2v (iLab)
VDS=3v (iLab)