6.002 Spring 2008

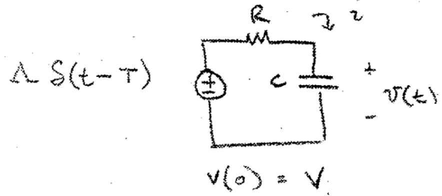
Pset #7 Solutions

3/26
KJ

Updated: 4/17

Exercise 7.1:

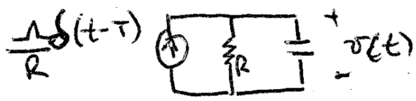
(a)



Impulses will give initial states, ignore in KVL.

$$iR + v(t) = 0$$

$$RC\frac{dv}{dt} + v(t) = 0$$



$$\frac{dv}{dt} + \frac{v}{RC} = 0 \Rightarrow v(t) = A e^{-t/RC}$$

Two Conditions:

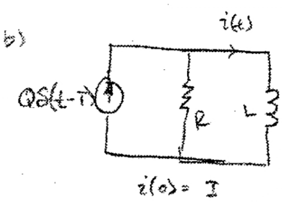
① @ $t=0$; $v(0) = V \Rightarrow v_1(t) = Ve^{-t/RC}$ $t>0$

② @ $t=T$; $v(T) = \frac{\Lambda}{RC} \Rightarrow v_2(t) = \frac{\Lambda}{RC} e^{-(t-T)/RC}$ $t>T$

Using Superposition:

$$\boxed{v(t) = \begin{cases} Ve^{-t/RC} & 0 \le t < T \\ Ve^{-t/RC} + \frac{\Lambda}{RC}e^{-(t-T)/RC} & t \ge T \end{cases}}$$

(b)



KCL: $\frac{v}{R} + i(t) = 0$

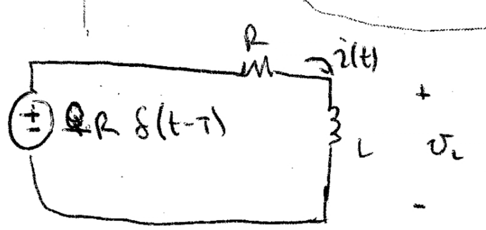
$$\frac{L}{R}\frac{di}{dt} + i = 0 \Rightarrow i(t) = A e^{-t\frac{R}{L}}$$

IC's

① @ $t=0$, $i(0) = I$ $\qquad i_1(t) = Ie^{-t\frac{R}{L}}$

② @ $t=T$, $i(T) = \frac{RQ}{L}$ $\qquad i_2(t) = \frac{RQ}{L}e^{-(t-T)\frac{R}{L}}$

$$\Rightarrow \boxed{i(t) = \begin{cases} Ie^{-t\frac{R}{L}} & 0 \le t < T \\ Ie^{-t\frac{R}{L}} + \frac{RQ}{L}e^{-(t-T)\frac{R}{L}} & t \ge T \end{cases}}$$



Exercise 7.2

$\tau = \frac{L}{R}$

At $t<0$, the inductor is a short

At $t=0^+$ the inductor discharges through the resistor with time constant $\frac{L}{R}$

At $t=0$ $\quad V = 1V$

At $t = 1\mu s$ $\quad V \approx e^{-1}\ V$

$\frac{L}{R} = 1 \times 10^{-6}$

$L = 1 \times 10^{-6} \cdot R$

At $t = 0^+$

$V = 1V = 1 \times 10^{-3} \times R$

$\boxed{R = 1\,k\Omega}$

$\boxed{L = 1\,mH}$

Problem 7.1

(A) In DC, the capacitor C will be an open.
the inductor L in the transformer will be a short.

No voltage from v_{IN} will be seen at the gate of the MOSFET.

$$\boxed{V_{GS} = \frac{R_2}{R_1+R_2} \cdot V_S}$$

$$\boxed{I_D = \frac{K}{2}(V_{GS} - V_T)^2 = \frac{K}{2}\left(\frac{R_2}{R_1+R_2} V_S - V_T\right)^2}$$

Since L is a short; $v_1 = 0$.

$$\therefore \boxed{V_{OUT} = 0}$$

(B) No, a bias at v_{IN} will not be useful since the capacitor will block DC voltage. The MOSFET is biased by R_1 & R_2.
Since $V_{OUT} = 0$, there is no large signal to remove from the output.

(C) In DC:

$V_{DS} = V_S$.

for saturation, $v_{DS} > v_{GS} - V_T$.

This will always be true. V_{GS} needs to be greater than V_T

$$V_{GS} > V_T$$

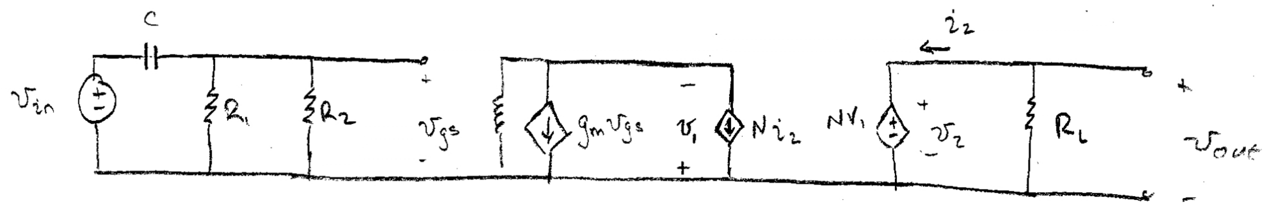
$$\frac{R_2}{R_1+R_2} V_S > V_T$$

$$\boxed{\frac{R_2}{R_1} > \frac{V_T/V_S}{1 - V_T/V_S}}$$

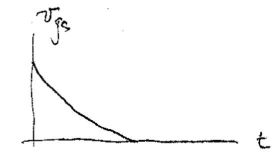
Assume that in small signal:

1) the capacitor is a short

2) the inductor is an open

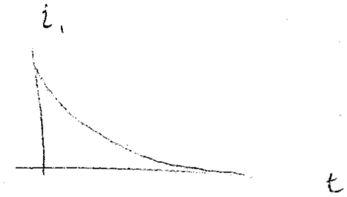


The capacitor C will cause the v_{gs} to decay

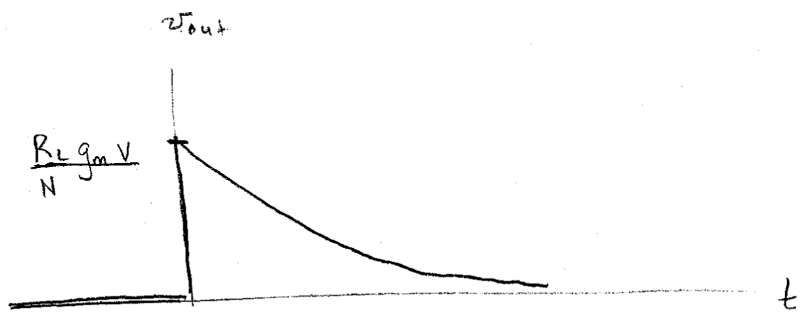


The inductor will cause the current i_1 to decay as well

to a step in current ($g_m v_{gs}$)



Therefore, to a step the output will contain two time constants



To a step input

$$v_{gs} = V e^{-t/\tau} \qquad \tau_1 = (R_1 // R_2)\, C$$

$$i_d = \underbrace{K(V_{GS} - V_T)}_{g_m} \cdot v_{gs}$$

$$i_d = V \cdot g_m \, e^{-t/\tau_1}$$

$$v_1 = \frac{v_{out}}{N} \qquad i_2 = \frac{-v_{out}}{R_L} \qquad i_1 = -\frac{v_{out} \cdot N}{R_L}$$

Using KCL above the MOSFET:

$$-i_1 + i_L - i_d = 0$$

$$+\frac{v_{out}\, N}{R_L} + i_L - V g_m e^{-t/\tau_1} = 0$$

$$\frac{d v_{out}}{dt} + \frac{R_L}{LN} v_{out} + \frac{V \cdot g_m}{\tau_1} e^{-t/\tau_1} = 0$$

$$v_{out,hom} = A e^{-t/\tau_2} \qquad A = \frac{V g_m}{N} \cdot R_L \quad \text{initial jump in } v_{out}$$

$$\tau_2 = \frac{LN}{R_L}$$

$$v_{out,part} = \frac{-V \cdot g_m / \tau_1}{\frac{R_L}{LN} - 1/\tau_1} e^{-t/\tau_1} = \frac{-V g_m LN}{R_L \cdot \tau_1 - LN} e^{-t/\tau_1}$$

$$\boxed{v_{out} = \frac{V \cdot g_m R_L}{N} e^{-t/\tau_2} - \frac{V g_m LN}{R_L \tau_1 - LN} e^{-t/\tau_1} \qquad \tau_1 = (R_1//R_2)\, C \qquad \tau_2 = \frac{LN}{R_L}}$$

(F) Notice that the two solutions have different time constants that will both be large.

N is the number of turns around the transformer. Typically this is very large.

$$\tau_2 > \tau_1$$

The decay will be dominated by τ_1.

Problem 7.2

(a) For the time constant, τ, the initial slope of the rise & fall curves are $\frac{1}{\tau}$.

See attached graphs

Pull-up

$$\frac{1}{\tau_{pu}} = \frac{^{-}1.394 - ^{-}2.298}{51\,ms - 50\,ms}$$

$$\boxed{\tau_{pu} = 1.1\ ms}$$

Pull-down

$$\frac{1}{\tau_{PD}} = \frac{2.138 - ^{-}2.460}{1\,ms}$$

$$\boxed{\tau_{PD} = 0.22\ ms}$$

(b) Offset of -75 mV was given to achieve output offset of -180 mV.

Pullup

$$\frac{1}{\tau_{pu}} = \frac{^{-}1.133 - ^{-}1.806}{1\,ms}$$

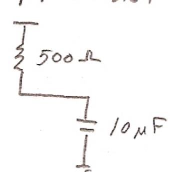
$$\boxed{\tau_{pu} = 1.5\ ms}$$

Pull-down

$$\frac{1}{\tau_{PD}} = \frac{1.636 - 0.8698}{1\,ms}$$

$$\boxed{\tau_{PD} = 1.3\ ms}$$

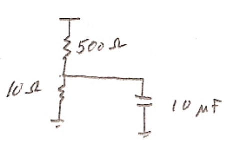
(c) For Pull-up: MOSFET is off



$$\tau = RC = (500\,\Omega)(10\,\mu F)$$

$$\boxed{\tau_{pu} = 5\ ms}$$

For Pull-down: MOSFET is ON

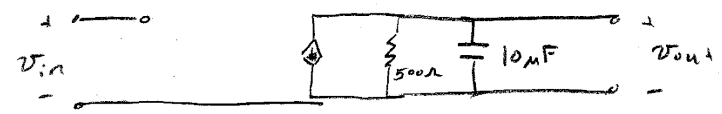


$$\tau = RC = \frac{(500\,\Omega)(10\,\Omega)}{(510\,\Omega)} \cdot 10\,\mu F$$

$$\boxed{\tau_{PD} = 0.1\ ms}$$

Discrepancies can arise from the estimation of the initial slope. Device tolarances and the estimation Ron also have an effect on the measured value.

(D) Small signal model:

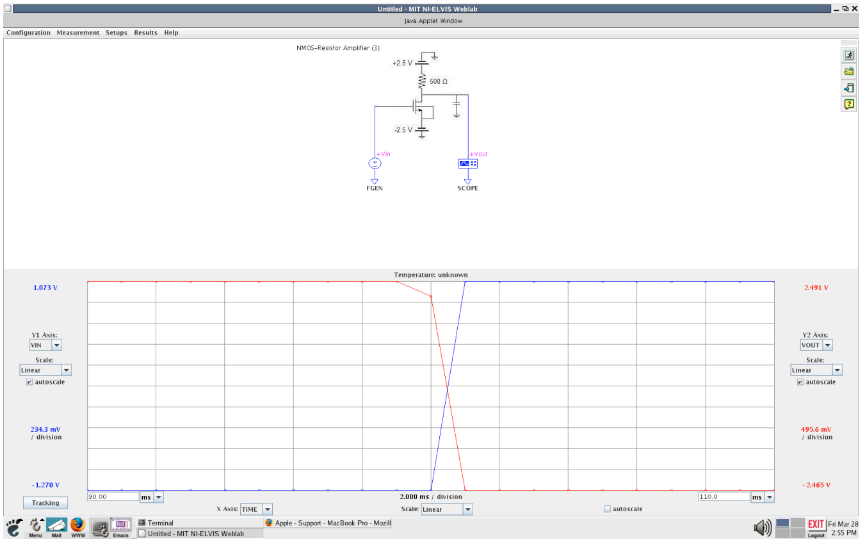


The resistance looking from v_{out} is just 500Ω

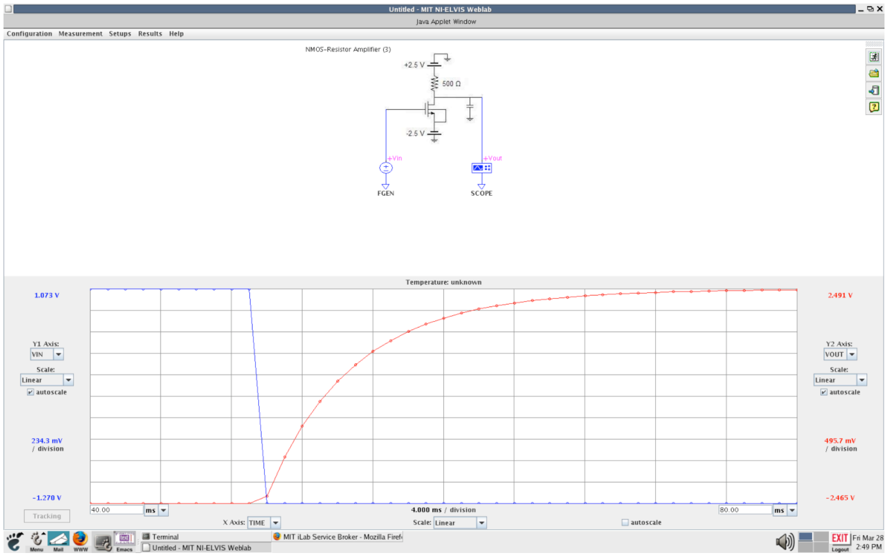
$\tau = (500\Omega)(10\mu F)$

$\boxed{\tau = 5ms}$

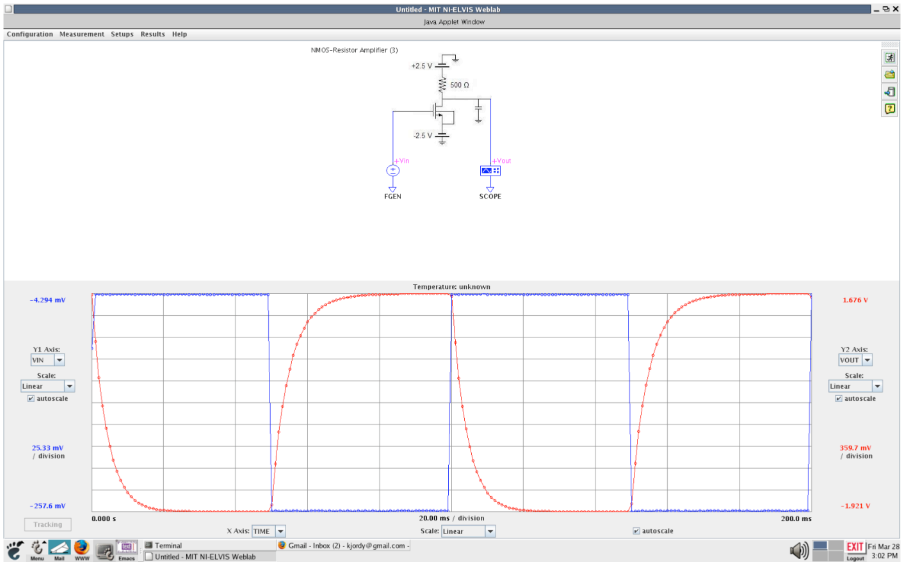
Note: for the small signal there is no difference between pull-up and pull-down. Therefore the time constants should be the same.



Part A: Pull-down



Part A: Pull-up



Part B

Problem 7.3

(a) Calculate power for each combination $P = \frac{V^2}{R}$

00 : $P = \frac{V_s^2}{R_{pu} + R_{ON}} \approx \frac{V_s^2}{R_{pu}}$

Note for:

$R_{pu} + R_{ON} \approx R_{pu}$

for $R_{on} \ll R_{pu}$

01 : $P = \frac{V_s^2}{R_{pu_1} + R_{ON_2}} \approx \frac{V_s^2}{R_{pu}}$

00 : $P = \frac{V_s^2}{R_{pu}}$

10 : $P = \frac{V_s^2}{R_{pu}}$

$$\boxed{\bar{P} = \frac{V_s^2}{R_{pu}}}$$

(b) Make the assumption that V_{g3} goes to zero when either M1 or M2 is on. Thus V_{g3} is either 0 or V_s.

In one cycle V_{g3} goes from L→H and H→L once.

From Lecture 12:

Dissipation of $C_{gs3} = C_{GS} V_s^2 f = \frac{C_{GS} V_s^2}{4T}$.

Dissipation of $C_{out} = \frac{N\, C_{GS} V_s^2}{4T}$

$$\boxed{\text{Total Dynamic Dissipation} = \frac{C_{GS} V_s^2}{4T}(N+1)}$$

(b)

$$\frac{\text{Static Loss}}{\text{Dynamic Loss}} = \frac{V_s^2 / R_{PU}}{\frac{C_{GS} V_s^2}{4T}(N+1)}$$

$$= \frac{5T}{C_{GS} R_{PU} (N+1)} > 0$$

for $T \gg C_{GS} R_{ON}$ & N not much larger than 1

$$\boxed{\text{Static Loss is greater}}$$

Problem 7.4

$\omega = 10^7$ rad/s

$v_{peak} = 100\,mV$

$i_{peak} = 10\,mA$

(A) $\frac{1}{\omega} = \sqrt{LC}$

$i_{peak} = v_{peak}\sqrt{\frac{C}{L}}$

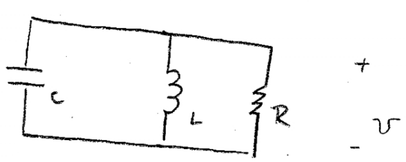
$$\boxed{\begin{array}{l} L = 1\,\mu H \\ C = 100\,\mu F \end{array}}$$

(B) Total energy:

When $i = 0$ no energy in inductor.

$$\boxed{E = \frac{1}{2} C V_{pk}^2}$$

(C)



For Lecture:

$$v(t) = e^{-\alpha t}\left(\cos(\omega_d t + \phi)\right)$$

$\alpha = \frac{1}{2RC}$

$\omega_0 = \sqrt{1/LC}$

Power decays by $e^{-\alpha t}$

Time const: $1/\alpha = 2RC$

$2RC = \tau = 10\,\mu s$

$$\boxed{R = 5\,m\Omega}$$