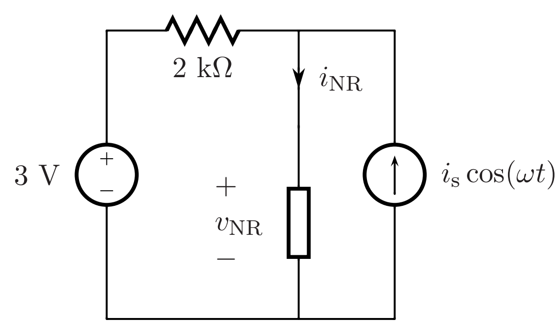
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Circuits & Electronics
Spring 2008

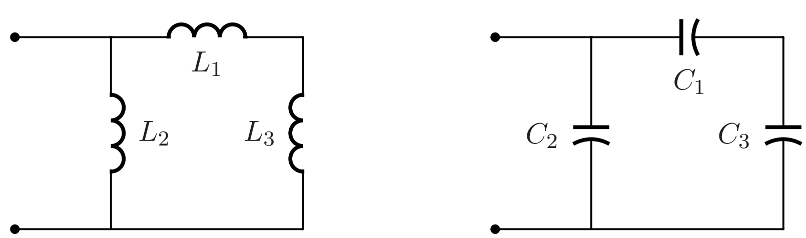
Problem Set #6

Issued 3/12/08 – Due 3/19/08

Exercise 6.1 (1 Point): Consider the network shown below, which contains a Thevenin equivalent, a nonlinear resistor and a small-signal current source, all in parallel. The current through, and the voltage across, the nonlinear resistor are related according to $v_{\mathrm{NR}} = (1\ \mathrm{V/mA/mA}) \cdot i_{\mathrm{NR}}^2$. First, assume that the small-signal current amplitude i_s is zero. Show that V_{NR}, the bias component of v_{NR} is 1 V. Second, assume that $i_{\mathrm{s}} = 1\ \mu\mathrm{A}$ and use a small signal model to determine v_{nr}, the small-signal component of v_{NR}.



Exercise 6.2 (1 Point): Find the inductance of the all-inductor network, and the capacitance of the all-capacitor network, shown below.



Problem 6.1 (2 Points): This problem studies the propagation delay of digital signals through the inverter shown below. Assume that the MOSFET in the inverter acts as a switch with on-state resistance R_{ON}. The inverter is loaded by a capacitor, having capacitance C_{G}, that models the combined input capacitance of the logic gates connected to its output. Assume that the inverter obeys the static discipline defined in part by V_{OL} and V_{OH}.

(A) Assume that the MOSFET has been off for a very long time. At $t = 0$, v_{IN} turns the MOSFET on. Determine $v_{\text{G}}(t)$ for $t \geq 0$.

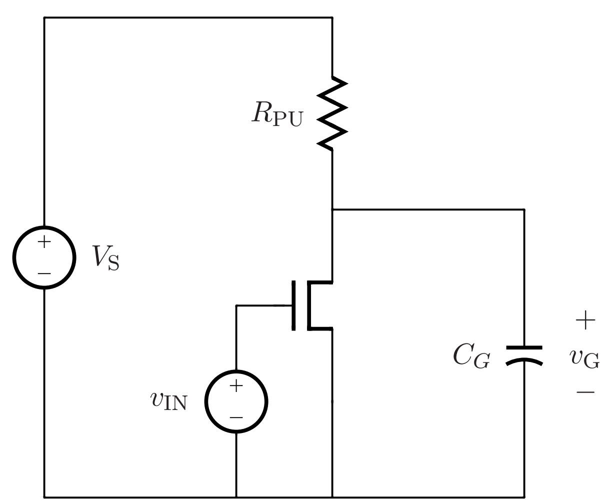
(B) How long does it take $v_{\text{G}}(t)$ to pass by V_{OL}? This delay is the fall time of the inverter.

(C) Assume that the MOSFET has been on for a very long time. At $t = 0$, v_{IN} turns the MOSFET off. Determine $v_{\text{G}}(t)$ for $t \geq 0$.

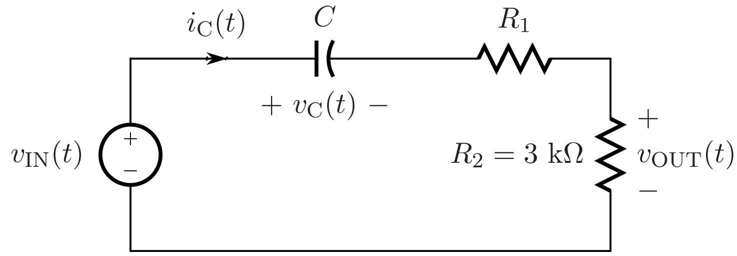
(D) How long does it take $v_{\text{G}}(t)$ to pass by V_{OH}? This delay is the rise time of the inverter.

(E) If more gates are connected to the output of the inverter will the delays found in Parts (B) and (D) become shorter or longer? Why?

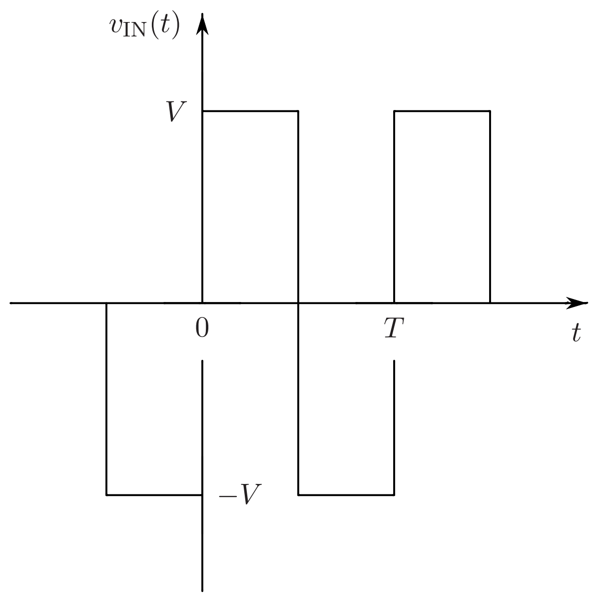
(F) How can the fall and rise times be shortened via the design of R_{PU}? What limits the extent to which this design path may be followed?



Problem 6.2 (2 Points): This problem studies the response of a series RC network, both theoretically and experimentally. The experiments will be performed using the ELVIS iLab. The circuit to be studied is shown below. It comprises a capacitor, two resistors and a voltage source all in series. The voltage $v_{\mathrm{OUT}}(t)$ across R_2 can be measured and used to determine the current through the series network.



Consider first a theoretical study of the network. Let the voltage $v_{\mathrm{IN}}(t)$ be a periodic square wave with amplitude V and period T as shown below. The period T is much larger that the RC time constant of the network. Assume that $v_{\mathrm{IN}}(t)$ has been applied long before $t = 0$, while any measurements start at $t = 0$. Thus, the network has reached its periodic steady state before any measurements are taken.



(A) Derive an expression for $v_C(t)$, the voltage across the capacitor. Your answer should include separate expressions for the time period over which $v_{\mathrm{IN}}(t) = V$, and the time period over which $v_{\mathrm{IN}}(t) = -V$. Hint: consider the consequences of T being much longer than the RC time constant of the network, and use reasonable engineering judgement.

(B) Derive an expression for $i_C(t)$, the current flowing through network.

(C) Derive an expression for $v_{\mathrm{OUT}}(t)$, the voltage across R_2.

Now consider an experimental study of the network. First, log in to the ELVIS iLab as in previous homeworks. After launching the iLab, you should see a network that is equivalent to the one shown above.

First, select the voltage source, or FGEN signal generator, and set its parameters to WaveForm = SQUARE, Frequency = 200 Hz, Amplitude = 1 V, and Offset = 0 V. Second, select the SCOPE output measurement unit and program it with a suitable sampling rate that will allow you to see at least one full cycle of $v_{\text{OUT}}(t)$ with enough resolution. Note that the system will only allow you to take a maximum of 201 data samples at the output. Third, run the experiment. Finally, select $v_{\text{IN}}(t)$ for the Y1 axis and $v_{\text{OUT}}(t)$ for the Y2 axis, and use linear axes for both. When the figure resembles what you expect, capture a screen shot for subsequent analysis.

(D) From the experimental data, extract the RC time constant of the network. You can see the actual numerical values of the data that you have obtained by looking into *View Data* under the *Results* menu. You can also download the data to Excel using the *Results* menu.

(E) From the experimental data, extract the value of the resistor R_1. When you do this, note that even though you selected 1 V as the amplitude, the signal generator does not impose this voltage very accurately; the actual amplitude is is measured as v_{IN}.

(F) From the experimental data, extract the value of C.

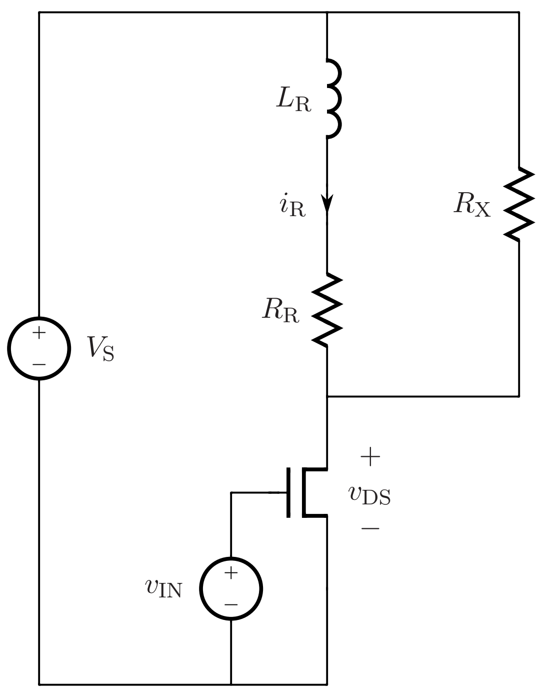
Problem 6.3 (2 Points): In the circuit shown below, a MOSFET and an external resistor having resistance R_X are used to control the current i_R in the winding of a relay. Here, the relay is modeled as a series inductor and resistor having inductance L_R and resistance R_R, respectively. The MOSFET may be modeled as an ideal switch.

(A) At $t = 0$, v_{IN} turns the MOSFET on so that $v_{DS} = 0$. Determine $i_R(t)$ for $t \geq 0$ given that $i_R(t = 0) = 0$.

(B) Next, at $t = T$, v_{IN} turns the MOSFET off. Determine both $i_R(t)$ and $v_{DS}(t)$ for $t \geq T$. Hint: $i_R(t)$ is continuous at $t = T$.

(C) Sketch and clearly label graphs of both $i_R(t)$ and $v_{DS}(t)$ for $t \geq 0$ assuming that $T \approx 5L_R/R_R$ and $R_X = R_R$.

(D) The relay control circuit would be less expensive without the external resistor, which may be "removed" from the circuit by considering the limit $R_X \to \infty$. Why might such a cost reduction be unwise?



Problem 6.4 (2 Points): At $t = 0^-$, the two networks shown below both have zero initial state. That is, the inductor current $i(t)$ and the capacitor voltage $v(t)$ are both zero at $t = 0^-$. At $t = 0$, the current source produces an impulse of area Q, and the voltage source produces an impulse of area Λ.

(A) Derive the differential equations that relate $i(t)$ to $I(t)$ and $v(t)$ to $V(t)$. Hint: consider using Thevenin and/or Norton equivalents to simplify the work.

(B) Find the inductor current $i(t)$ and capacitor voltage $v(t)$ at both $t = 0^+$ and $t = \infty$. One way to find the state at $t = 0^+$ is to integrate the corresponding differential equation from $t = 0^-$ to $t = 0^+$ under the assumption that the state remains finite during that time; you should justify this assumption. Then, substitute the initial condition at $t = 0^-$ into the result to determine the state at $t = 0^+$. Try to determine the states at $t = \infty$ through physical, rather than mathematical, reasoning.

(C) Next, find the time constant by which each state goes from its initial value at $t = 0^+$ to its final value at $t = \infty$.

(D) Using the previous results, and without necessarily solving the differential equations directly, construct $i(t)$ and $v(t)$ for $t \geq 0$.

(E) Verify that the solutions to Part (D) are correct by substituting them into the differential equation found in Part (A).