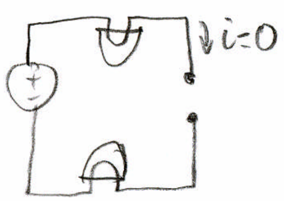
6.002 - Spring 2008

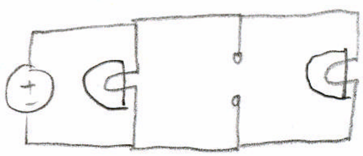
Problem Set #2 Solutions

Exercise 2.1: Assuming that the intensity of light is proportional to the power dissipated in the bulb:

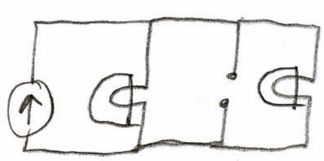
1st Circuit) Dimmer - and go out. Opening any of the 3 bulbs in series will open the entire circuit. Current and power will be zero for all 3 bulbs.



2nd Circuit) No Change - with all 3 bulbs in parallel, the voltage and power, as $P = \frac{V^2}{R}$, will remain constant on the 2 remaining bulbs

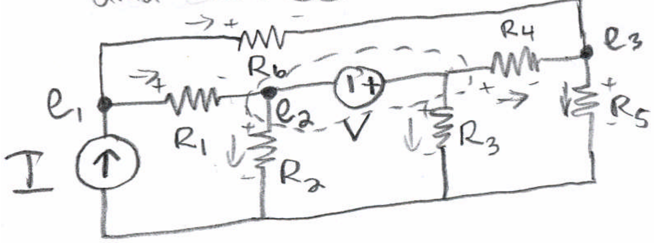


3rd Circuit) Brighter - the current and power, as $P = I^2R$, increase as the same total current is now split between the 2 remaining bulbs.



Exercise 2.2

Using the node method: $G_n = \frac{1}{R_n}$, need to find the terms for the **G** and **S** conductance matrix and source vector.



at e_1: $I = (e_1 - e_3)G_6 + (e_1 - e_2)G_1$

$$I = e_1(G_1 + G_6) - e_2 G_1 - e_3 G_6$$

at e_2 "super node":

$$(e_1 - e_2)G_1 = e_2 G_2 + (e_2 + V)G_3 + (e_2 + V - e_3)G_4$$

$$-V(G_3 + G_4) = -e_1 G_1 + e_2(G_1 + G_2 + G_3 + G_4) - e_3 G_4$$

at e_3: $(e_1 - e_3)G_6 + (e_2 + V - e_3)G_4 = e_3 \cdot G_5$

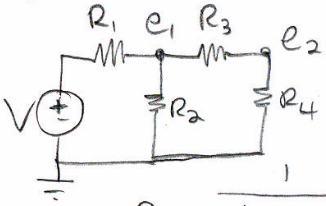
$$V \cdot G_4 = -e_1 \cdot G_6 - e_2 \cdot G_4 + e_3(G_4 + G_5 + G_6)$$

and as $\mathbf{G}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \mathbf{S}$

$$\begin{bmatrix} (G_1 + G_6) & -G_1 & -G_6 \\ -G_1 & G_1 + G_2 + G_3 + G_4 & -G_4 \\ -G_6 & -G_4 & G_4 + G_5 + G_6 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} I \\ -V(G_3 + G_4 \\ V \cdot G_4 \end{bmatrix}$$

Problem 2.1

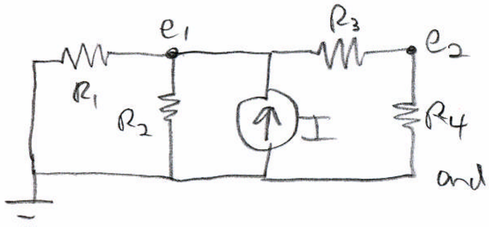
(A) First including the voltage source and opening the current source:



Using 2 voltage dividers: $e_2 = e_1\left(\frac{R_4}{R_3+R_4}\right)$ and $e_1 = V\left(\frac{R_{eq}}{R_1+R_{eq}}\right)$ where

$$R_{eq} = \frac{1}{\frac{1}{R_2}+\frac{1}{R_3+R_4}} = \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}$$

Including the current source and shorting the voltage source:



e_1 is the total voltage across the current source. $e_1 = I\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3+R_4}}\right)$

and again $e_2 = e_1\left(\frac{R_4}{R_3+R_4}\right)$

Combining both parts gives:

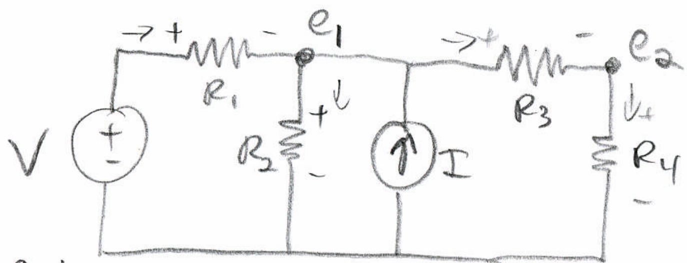
$$\boxed{e_1 = \frac{V\cdot R_2(R_3+R_4) + I\cdot R_1R_2(R_3+R_4)}{R_1R_2+R_1R_3+R_1R_4+R_2R_3+R_2R_4}}$$

and $e_{2_{tot}} = e_{1_{tot}}\left(\frac{R_4}{R_3+R_4}\right)$

$$\boxed{e_2 = \frac{(V\cdot R_2 + I\cdot R_1R_2)R_4}{R_1R_2+R_1R_3+R_1R_4+R_2R_3+R_2R_4}}$$

Problem 2.1 (continued)

(B) Using the Node Method:



at e_1:

$$\frac{V-e_1}{R_1} + I = \frac{e_1}{R_2} + \frac{e_1-e_2}{R_3}$$

at e_2:

$$\frac{e_1-e_2}{R_3} = \frac{e_2}{R_4}$$

$$e_1 \cdot R_4 = e_2(R_3+R_4)$$

$$e_2 = e_1 \cdot \left(\frac{R_4}{R_3+R_4}\right) \text{ again}$$

$$\frac{V}{R_1} + I = e_1\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} - \frac{R_4}{R_3(R_3+R_4)}\right] = e_1\left[\frac{(R_3+R_4)(R_2R_3+R_1R_3+R_1R_2) - R_4R_1R_2}{R_1R_2R_3(R_3+R_4)}\right]$$

$$\Rightarrow e_1 = \left(\frac{V}{R_1}+I\right)\left(\frac{R_1R_2\cancel{R_3}(R_3+R_4)}{R_2R_3\cancel{R_3} + R_2\cancel{R_3}R_4 + R_1R_3\cancel{R_3} + R_1\cancel{R_3}R_4 + R_1R_2\cancel{R_3} + \cancel{R_1R_2R_4} - \cancel{R_1R_2R_4}}\right)$$

$$\boxed{\begin{aligned} e_1 &= \frac{(V\cdot R_2 + I\cdot R_1R_2)(R_3+R_4)}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4} \\ e_2 &= \frac{R_4(V\cdot R_2 + I\cdot R_1R_2)}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4} \end{aligned}}$$

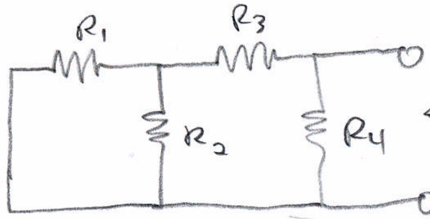
(C) The solutions to parts (A) and (B) are identical.

Problem 2.1 (continued)

(D) $$\boxed{V_{TH} = V_{oc} = e_2 = \frac{R_2 R_4 (V + I R_1)}{R_1 R_2 + (R_1 + R_2)(R_3 + R_4)}}$$

from either part (A) or (B)

The equivalent resistance, R_{TH}, looking into the terminals, with the voltage source shorted and the current source open is:

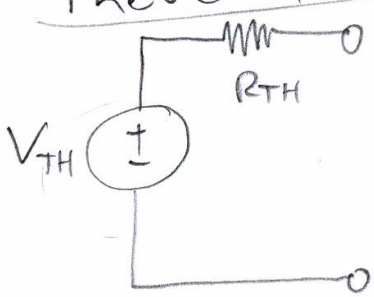


$$\Leftarrow R_{TH} = \left(\frac{1}{\frac{1}{R_4} + \frac{1}{R_3 + \frac{R_1 R_2}{R_1 + R_2}}} \right)$$

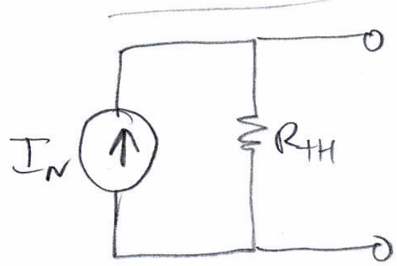
R_4 is in parallel with the series combination of R_3 with the parallel combination of R_1 and R_2.

$$\boxed{R_{TH} = \frac{R_4 (R_1 R_2 + R_1 R_3 + R_2 R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3 + R_1 R_4 + R_2 R_4}}$$

Thevenin OR Norton



OR

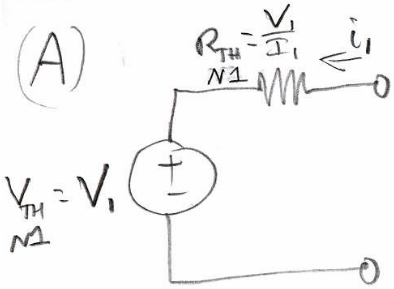


Where $$\boxed{I_N = \frac{V_{TH}}{R_{TH}}}$$ the short circuit current

$$\boxed{I_N = \frac{V_{TH}}{R_{TH}} = \frac{R_2 (V + I R_1)}{R_1 R_2 + R_1 R_3 + R_2 R_3}}$$

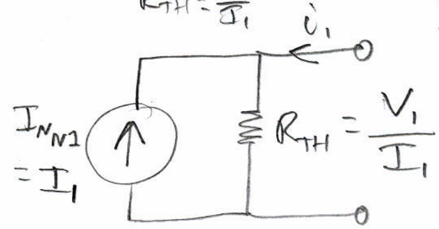
Problem 2.2

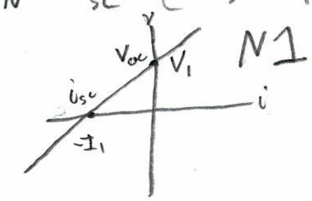
(A)



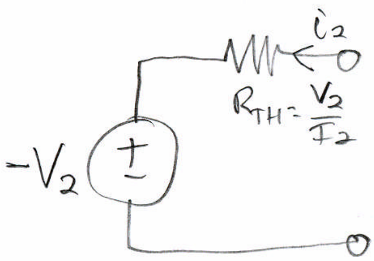
For N1: $V_{TH} = V_{oc} = V_1$, $R_{TH} = \frac{V_1}{I_1}$

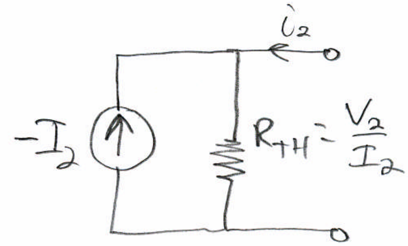
$I_N = -i_{sc} = -(-I_1) = I_1$

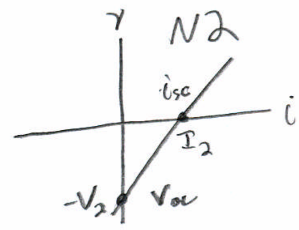




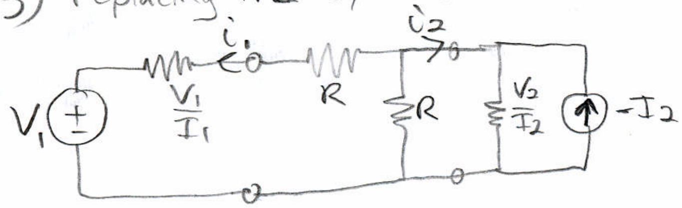
For N2: $V_{TH} = V_{oc} = -V_2$; $I_N = -i_{sc} = -I_2$; $R_{TH} = \frac{V_{TH}}{I_N} = \frac{-V_2}{-I_2} = \frac{V_2}{I_2}$







(B) Replacing N1 by Thevenin and N2 by Norton equivalents:



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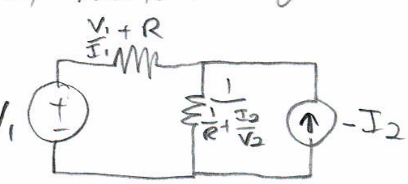


i_1 is still the total current out of N1, using superposition and voltage and current dividers:

$$i_1 = \left(\frac{-V_1}{\left(\frac{V_1}{I_1} + R\right) + \left(\frac{V_2 R}{V_2 + I_2 R}\right)}\right) - I_2\left(\frac{\frac{V_2 R}{V_2 + I_2 R}}{\left(\frac{V_1}{I_1} + R\right) + \frac{V_2 R}{V_2 + I_2 R}}\right)$$

$$i_2 = -i_{1\,\text{due to } V_1}\left(\frac{R}{R + \frac{V_2}{I_2}}\right) + I_2\left(\frac{V_2/I_2}{R_{eq} + \frac{V_2}{I_2}}\right)$$

where $R_{eq} = \dfrac{1}{\frac{1}{R} + \frac{1}{R + \frac{V_1}{I_1}}}$

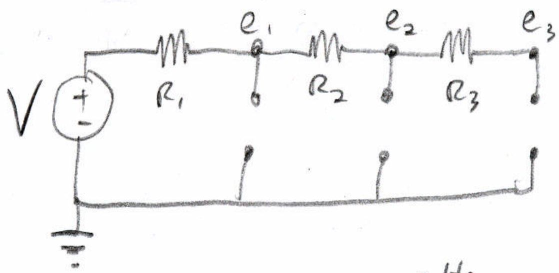
Simplify to:

$$i_1 = \frac{-I_1\left[V_1 V_2 + I_2 R (V_1 + V_2)\right]}{(V_1 + I_1 R)(V_2 + I_2 R) + I_1 V_2 R} \; ; \quad i_2 = \frac{I_2\left[V_1 V_2 + I_1 R (V_1 + 2V_2)\right]}{(V_1 + I_1 R)(V_2 + I_2 R) + I_1 V_2 R}$$

Problem 2.3

(A) $\boxed{\max e_2 = V}$ volts

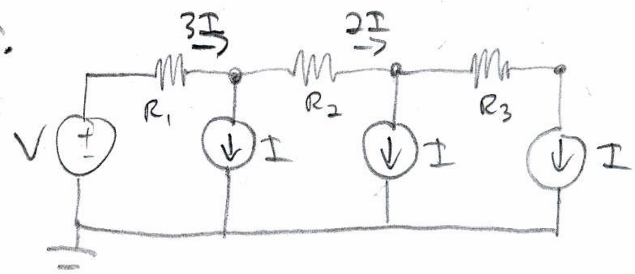
When all loads are "off," all currents are zero, the current sources act as open circuits and there is no voltage drop across R_1 and R_2.



$V = e_1 = e_2 = e_3$

$\boxed{\min e_2 = V - 3IR_1 - 2IR_2}$ volts

For minimum e_2, need maximum voltage drop and current thru R_1 and R_2. This occurs when all 3 current sources are "ON" and carry I amps. KCL at each node gives the current in each resistor.



(B) $\boxed{3IV}$ Watts

The maximum power will be supplied to the maximum load, when all 3 current sources, representing processor loads, are "ON." This will also give maximum current and power loss in the resistors. All 3 current sources will sink, dissipate, power since all 3 node voltages and all 3 current sources are all positive. I (i_n, v_n) $P_n = i_n v_n > 0$ So the voltage source is the only power source and it must output a maximum $3I$ A. The same result could be found by adding up the power dissipated in all 3 resistors and all 3 current sources.

Problem 2.3 (continued)

(C) $\boxed{V_{min} = e_{min} + 3IR_1 + 2IR_2 + IR_3 \text{ Volts}}$

The lowest possible voltage, after maximum voltage drop from V across all 3 resistors, occurs at e_3, when all 3 current sources are "ON" for maximum current in all resistors. In this case $e_3 = V - (3I)R_1 - (2I)R_2 - (I)R_3 \geq e_{min}$

and for $V \geq e_{min} + (3I)R_1 + (2I)R_2 + IR_3$,

$$V_{min} = e_{min} + (3I)R_1 + (2I)R_2 + (I)R_3$$

(D) $\boxed{V_{max} = e_{max}}$ Similar to part (A) the maximum voltages occur with minimum current and voltage drop across the resistors. With all of the current sources open, at 0 A, no loads are connected, no current flows thru any of the resistors and $e_1 = e_2 = e_3 = V$.

so in this case for $e_1 = e_2 = e_3 \geq e_{max}$

$$V_{max} = e_{max}$$

Problem 2.4

(A) $\boxed{V_T \approx 2.58 \cdot 10^{-2} \text{ volts}}$ using $T = 299.4$ Kelvin in $V_T = \frac{kT}{q}$

(B) $i_D \approx -I_s$ for around $V_D \leq -0.1$ volts

Using a linear scale for current is deceptive in this case, making all of the small currents below about 0.4 V appear constant around 0 A. The log plot agrees with the theoretical expectation in that $e^{-3} \approx 0.05$, so the exponential part of the relation will have less than a 5% impact on i_D for $V_D < -3V_T$ or $V_D < -0.08$ volts

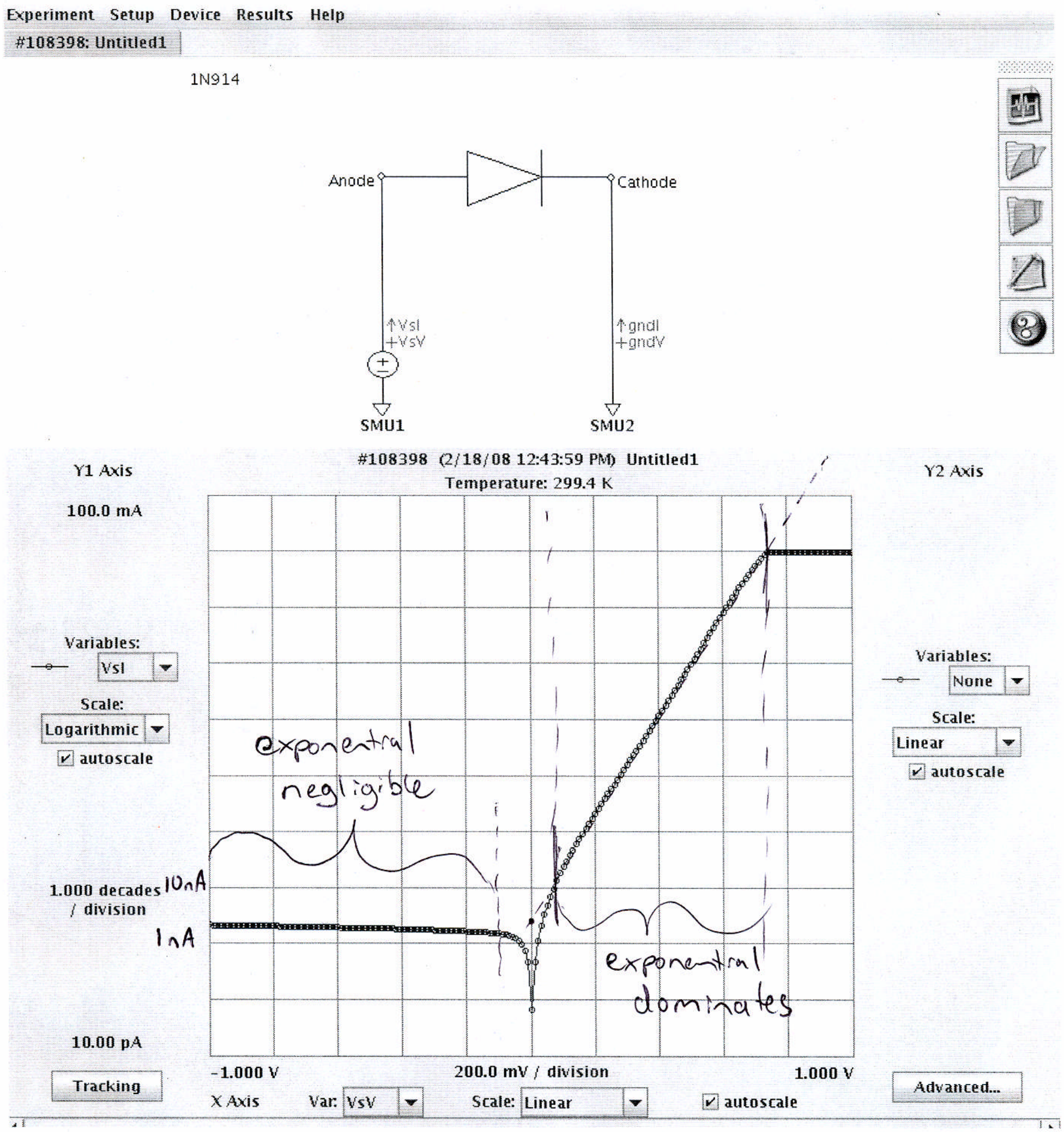
(C) With a graphical solution it is again more difficult to determine where the exponential behavior begins on the plot using a linear current scale. Using the log scale, the plot appears linear with the exponential dominating for about

$\boxed{V_D > 0.1 V}$ This matches theoretical expectation that for above $\frac{V_D}{V_T} \approx 3$ or 4 at $V_D \approx 0.8V$ or $V_D \approx 0.1V$, the exponential will be contributing about 95% or 98% of i_D.

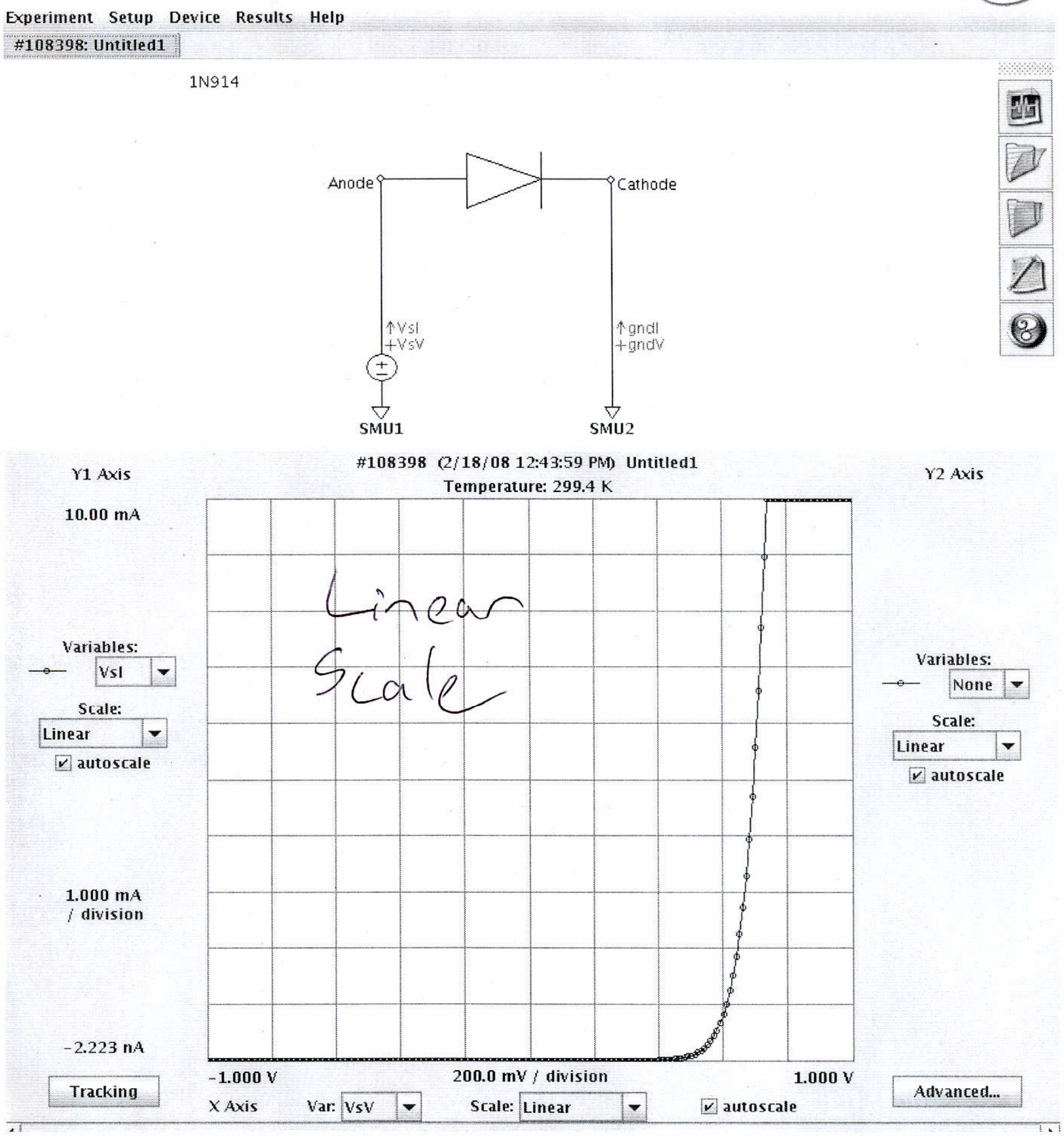
On the exponential branch of the data, I_s would be the y-intercept of the extended linear-looking plot of the exponential. From $i_D = I_s(e^{V_D/V_T} - 1)$ and neglecting the -1 gives $i_D \approx I_s e^{V_D/V_T}$ and at $V_D = 0$
$i_D \approx I_s$

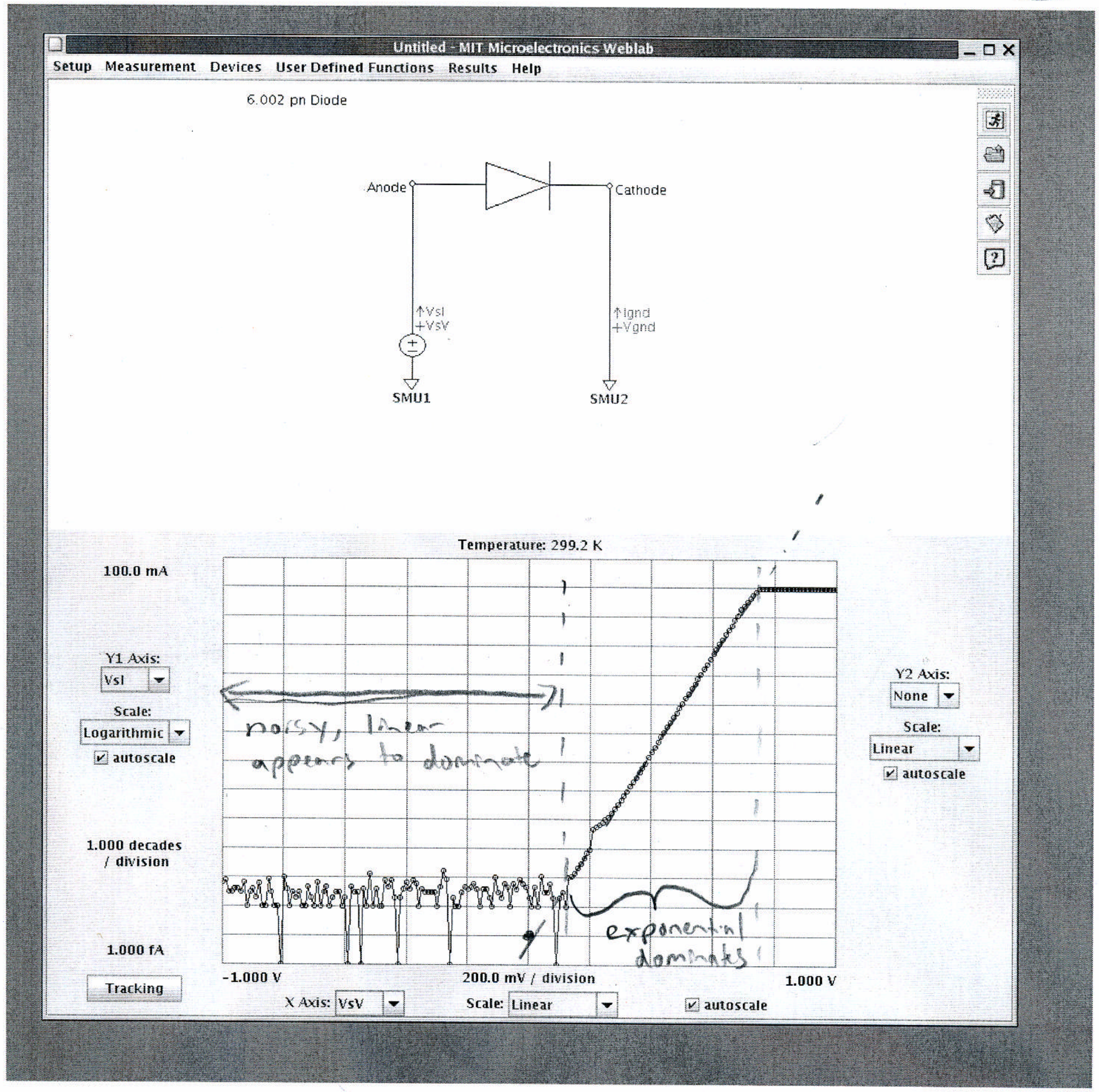
Here $\boxed{I_s \approx 2 nA}$

This roughly agrees with $-I_s$ at the extreme negative V_D.



Experiment Setup Device Results Help
#108398: Untitled1
1N914
Anode
Cathode
↑VsI
+VsV
↑gndI
+gndV
SMU1
SMU2
#108398 (2/18/08 12:43:59 PM) Untitled1
Temperature: 299.4 K
Y1 Axis
Y2 Axis
100.0 mA
Variables:
VsI
Scale:
Logarithmic
autoscale
1.000 decades / division
10.00 pA
10nA
1nA
exponential negligible
exponential dominates
Variables:
None
Scale:
Linear
autoscale
-1.000 V
200.0 mV / division
1.000 V
Tracking
X Axis
Var: VsV
Scale: Linear
autoscale
Advanced...


Experiment Setup Device Results Help
#108398: Untitled1
1N914
Anode
Cathode
↑VsI
+VsV
↑gndI
+gndV
SMU1
SMU2
#108398 (2/18/08 12:43:59 PM) Untitled1
Temperature: 299.4 K
Y1 Axis
10.00 mA
Variables:
VsI
Scale:
Linear
autoscale
1.000 mA
/ division
-2.223 nA
Tracking
Linear
Scale
Y2 Axis
Variables:
None
Scale:
Linear
autoscale
-1.000 V
200.0 mV / division
1.000 V
X Axis
Var: VsV
Scale: Linear
autoscale
Advanced...



Using the 6.002 pn Diode, the exponential still dominates over about $V_D > 0.1$ V, roughly agreeing with some theoretical expectations. But here the lowest current signal is noisy and it appears that the linear portion dominates for $V_D < 0.1$V, when theoretical calculations would expect around $V_D < -0.1$V. Graphically finding I_S from the exponential portion, the y-axis intercept of the extended linear-looking plot gives $I_S \approx 10$fA. This is lower than the ≈ 300 fA average from the lowest voltage portion.