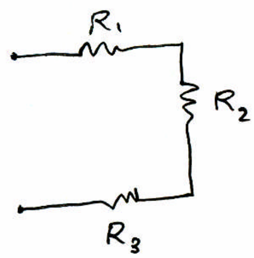
6.002 – Circuits & Electronics

Problem Set #1 – Solutions

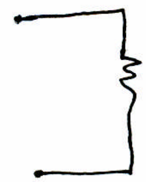
Exercise 1.1

A)



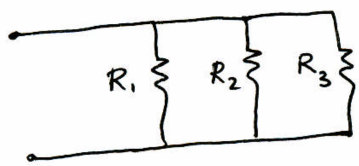
$\Longrightarrow$

series
resistances
add



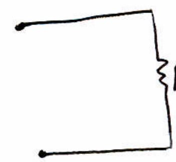
$R_A = R_1 + R_2 + R_3$

B)



$\Longrightarrow$

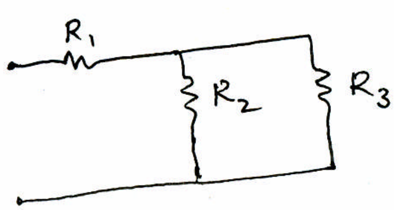
parallel
conductances
add



$$R_B = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$G_B = \frac{1}{R_B} = G_1 + G_2 + G_3 = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

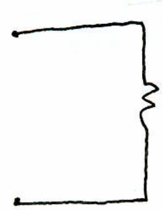
C)



$\Longrightarrow$

parallel-series
combination

$$R_{par} = \frac{R_2 R_3}{R_2 + R_3}$$



$$R_C = R_1 + R_{par} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

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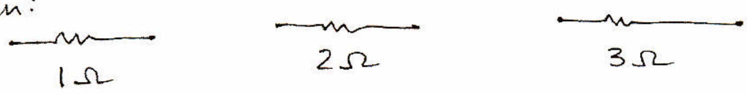
6.002 Problem Set 1 - Solutions

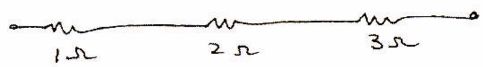
Exercise 1.2

Given:

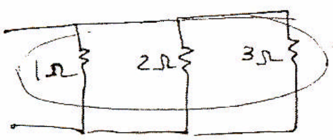


the largest possible resistor fabricated by combining the above resistors is the series combination of all three:

$\Rightarrow R_1 = 1\Omega + 2\Omega + 3\Omega = \boxed{6\Omega}$



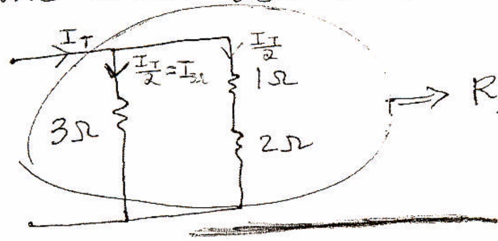
The smallest possible resistor fabricated by combining the above resistors is the parallel combination of all three:



$$\Rightarrow R = \frac{1\Omega(2\Omega)(3\Omega)}{1\Omega(2\Omega) + 1\Omega(3\Omega) + 2\Omega(3\Omega)}$$

$$R_2 = \frac{6}{11}\,\Omega \simeq \boxed{0.54\,\Omega}$$

To fabricate a 1.5Ω resistor using the above three resistors, one could use the following configuration:



$$\Rightarrow R_3 = \frac{3(1+2)}{3+1+2} = 1.5\Omega$$

The max power that each resistor can dissipate is $1W \Rightarrow$

$$P = 1W = I^2R$$

The same current flows through all resistors $\left(\frac{I_T}{2}\right)$ so the 3Ω resistor will dissipate the most power.

$$P_{3\Omega} = 1W = I_{3\Omega}^2(3\Omega)$$

$$\Rightarrow I_{3\Omega}^2 = \frac{1}{3}$$

$$\Rightarrow I_{3\Omega} = \frac{1}{\sqrt{3}}$$

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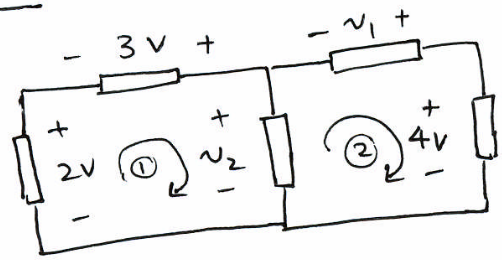
The total max power dissipation $= P_T = \left(\frac{1}{\sqrt{3}}\right)^2(3) + \left(\frac{1}{\sqrt{3}}\right)^2(2) + \left(\frac{1}{\sqrt{3}}\right)^2(1)$

$$P_T = 2W$$

6.002 Problem Set 1 - Solutions

Problem 1.1

A)



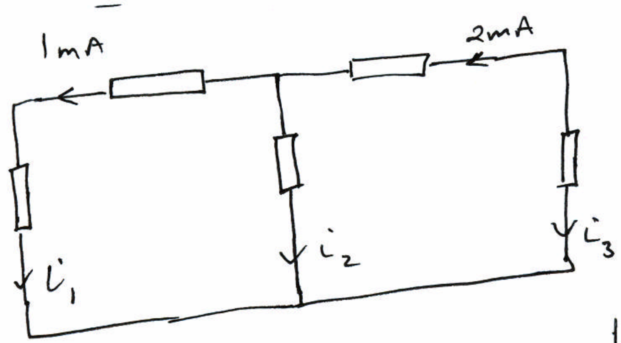
KVL in loop 1

$2V + 3V = \boxed{v_2 = 5V}$

KVL in loop 2

$v_2 + v_1 = 4V$ $\boxed{\therefore v_1 = -1V}$

B)



KCL @ corner nodes

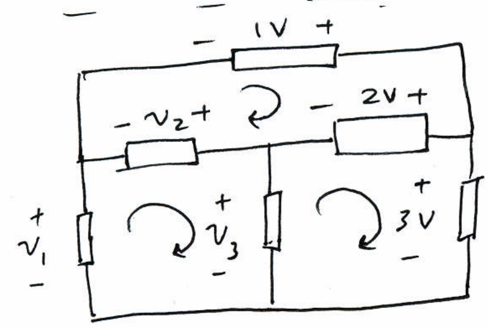
$\boxed{i_1 = 1mA}$

$\boxed{i_3 = -2mA}$

KCL @ middle node

$1mA + i_2 = 2mA$ $\boxed{\therefore i_2 = 1mA}$

C)



KVL top loop

$1V - 2V - v_2 = 0$ $\boxed{\therefore v_2 = -1V}$

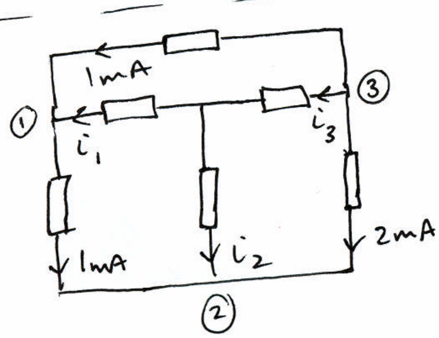
KVL bottom right loop

$v_3 + 2V = 3V$ $\boxed{\therefore v_3 = 1V}$

KVL bottom left loop

$v_1 + v_2 = v_3$ $\boxed{\therefore v_1 = 2V}$

D)



KCL @ node 1 $1mA + i_1 = 1mA$ $\boxed{\therefore i_1 = 0A}$

KCL @ node 2 $1mA + i_2 + 2mA = 0$

$\boxed{\therefore i_2 = -3mA}$

KCL @ node 3 $1mA + i_3 + 2mA = 0$

$\boxed{\therefore i_3 = -3mA}$

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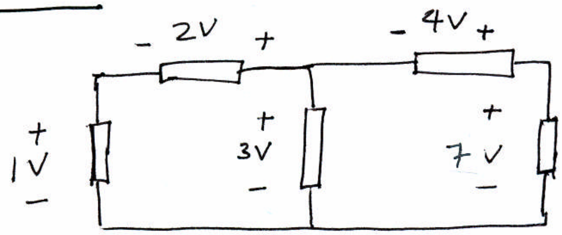
6.002 Problem Set 1 - Solutions

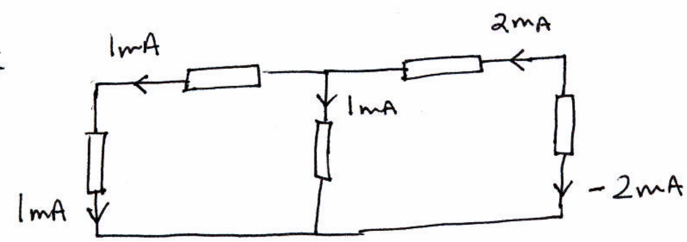
Problem 1.1 cont'd.

LH Network



N.B. Values are arbitrary but satisfy KVL.

RH Network



Total power ⇒ $P = 1V(1mA) + 2V(1mA) + 3V(1mA) + 4V(2mA) +$
$+ 7V(-2mA)$

$= 1mW + 2mW + 3mW + 8mW - 14mW$

$= 14mW - 14mW$

$\boxed{P = 0.}$

∴ The total power dissipated is equal to the total power generated as predicted by Tellegen's Theorem.

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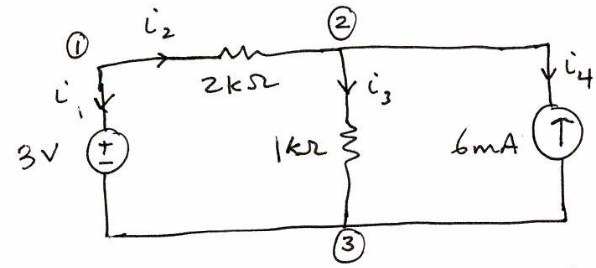
6.002 Problem Set 1 – Solutions

Problem 1.2

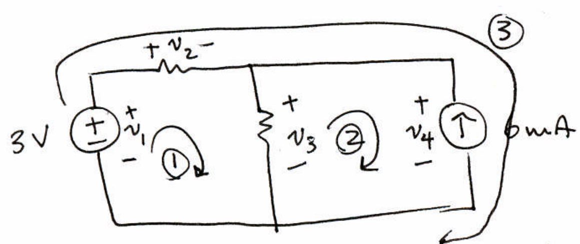


(A) There are <u>three</u> nodes in the network.

by KCL:
$i_1 + i_2 = 0$ Node ①
$i_3 + i_4 = i_2$ Node ②
$i_1 + i_3 + i_4 = 0$ Node ③

Only <u>two</u> of the KCL equations are independent. In any network, the number of independent KCL equations is always <u>one less</u> than the number of nodes.

(B)



There are <u>three</u> loops in the network as indicated.

By KVL:
$v_1 = v_2 + v_3$ Loop ①
$v_3 = v_4$ Loop ②
$v_1 = v_2 + v_4$ Loop ③

Again, only <u>two</u> of the KVL equations are independent. The third is simply the sum of the first two.

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S.F.

Problem 1.2 cont'd.

(c) There are four elements.

i) Ideal voltage source: $v_1 = 3V$ independent of i_1

ii) $2k\Omega$ resistor: Ohm's Law $v_2 = i_2(2k\Omega)$

iii) $1k\Omega$ resistor: Ohm's Law $v_3 = i_3(1k\Omega)$

iv) Ideal current source: $i_4 = -6mA$ independent of v_4.

(D) Solving equations in (A), (B), and (C) using substitution of equations or matrix algebra, we get.

Table 1

Currents	Voltages	Branch Power
$i_1 = 1mA$	$v_1 = 3V$	$P_1 = v_1 i_1 = 3mW$
$i_2 = -1mA$	$v_2 = -2V$	$P_2 = v_2 i_2 = 2mW$
$i_3 = 5mA$	$v_3 = 5V$	$P_3 = 25mW$
$i_4 = -6mA$	$v_4 = 5V$	$P_4 = -30mW$

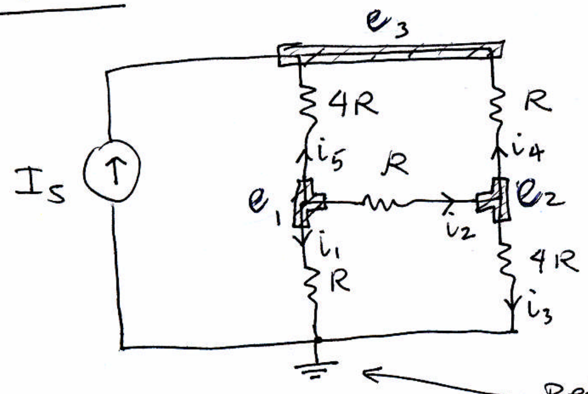
(see Appendix I for sample algebra)

$$\sum_{n=1}^{4} P_n = (3+2+25-30)mW = 0\ mW$$

(E) Refer to above Table for branch powers. We see that branches 2 & 3 (passive resistors) dissipate power. The Ideal Voltage Source (branch 1) also dissipates power since the current and voltage follow the same convention as the resistors. That leaves the final element, the Ideal Current Supply, which must provide or source power.

6.002 Problem Set 1 - Solutions

Problem 1.3



Reference Node (GND)

Nodal Analysis:

(1) Reference Node (given)

(2) Labeled remaining node voltages (e_1, e_2, e_3).

(3) KCL for 3 nodes:

$$\left.\begin{aligned} \frac{e_1}{R} + \frac{e_1 - e_2}{R} + \frac{e_1 - e_3}{4R} &= 0. \\ \frac{e_2}{4R} + \frac{e_2 - e_1}{R} + \frac{e_2 - e_3}{R} &= 0 \\ \frac{e_3 - e_1}{4R} + \frac{e_3 - e_2}{R} &= I_s \end{aligned}\right\} \Rightarrow \begin{aligned} 9e_1 - 4e_2 - e_3 &= 0 \\ -4e_1 + 9e_2 - 4e_3 &= 0 \\ -e_1 - 4e_2 + 5e_3 &= 4RI_s \end{aligned}$$

(4) Solving the equations using matrix algebra, we get

$$\boxed{e_1 = \frac{5}{7} RI_s, \quad e_2 = \frac{8}{7} RI_s, \quad e_3 = \frac{13}{7} RI_s.}$$

(5) Back solving for the branch currents, we get

$$i_1 = \frac{e_1}{R} = \frac{5}{7} I_s, \quad i_2 = \frac{e_1 - e_2}{R} = \frac{-3}{7} I_s, \quad i_3 = \frac{e_2}{4R} = \frac{2}{7} I_s,$$

$$i_4 = \frac{e_2 - e_3}{R} = \frac{-5}{7} I_s, \quad i_5 = \frac{e_1 - e_3}{4R} = \frac{-2}{7} I_s$$

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6.002 Problem Set 1 - Solutions

Problem 1.4 - WebLab

(A) With SMU1 as a voltage source and SMU2 as an open circuit, we can read the open circuit voltage v_2. Since there should ideally be no current through R_2 in this case, this yields the following resistor-divider ratio:

$$v_2 = \frac{R_3}{R_1 + R_3} v_1 \Rightarrow \boxed{\frac{v_2}{v_1} = 0.66} \quad \text{(from below data)}$$

v_1	v_2	i_1
1 V	0.6648	33.88 μA
4 V	2.656	135.5 μA
7 V	4.646	237.0 μA
10 V	6.642	338.6 μA

* include screen shot

(B) With SMU1 as a voltage source and SMU2 as a short circuit to ground, we can read the input current i_1 which is equal to the equivalent resistor ratio

$$\frac{v_1}{i_1} = R_1 + R_2 \| R_3 = R_1 + \frac{R_2 R_3}{R_2 + R_3} \Rightarrow \boxed{\frac{v_1}{i_1} = 23.9\,k\Omega} \quad \text{(from below data)}$$

v_1	i_1
1 V	41.79 μA
4 V	167.2 μA
7 V	292.2 μA
10 V	417.7 μA

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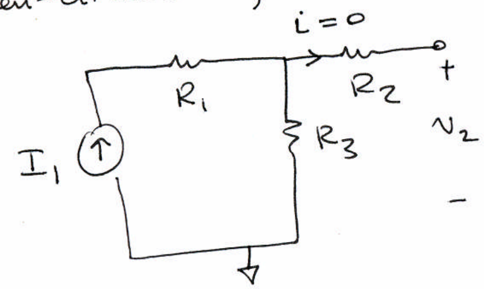
S. F.

Problem 1.4 cont'd.

(C) With SMU1 as a current source and SMU2 open-circuited, we have

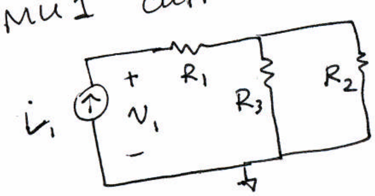


Since SMU2 open-circuit, the current through R_2 is 0 A, so v_2 is the voltage across R_3. From Ohm's Law

$$v_2 = R_3 i_1 \Rightarrow \frac{v_2}{i_1} = R_3 \Rightarrow \boxed{R_3 = 19.6\,k\Omega}$$ (from below data)

i_1	v_2
1 μA	20.2 mV
201 μA	3.94 V
301 μA	5.906 V

(D) SMU1 current source, SMU2 GND, we can measure v_1



$$\frac{v_1}{i_1} = R_1 + \frac{R_2 R_3}{R_2 + R_3} \Rightarrow \boxed{\frac{v_1}{i_1} = 23.9\,k\Omega}$$

i_1	v_1
51 μA	1.22 V
201 μA	4.814 V
351 μA	8.404 V

N.B. The value obtained here agrees with the value from part (B) as expected.

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S.F.

Problem 1.4 cont'd.

(E) Only three experiments are required to determine R_1, R_2, and R_3. Since each experiment yields one relationship between the three resistors, we require three experiments to determine three unknowns.

(F) Choosing the experiments outlined in parts (A), (B), and (C), we obtain the following information

$$\frac{R_3}{R_1+R_3} = 0.66 \Rightarrow R_1 = 0.515 R_3 \qquad \text{from (A)}$$

$$R_1 + \frac{R_2 R_3}{R_2+R_3} = 23.9\,k\Omega \qquad \text{from (B) \& (D)}$$

$$R_3 = 19.6\,k\Omega \qquad \text{from (C)}$$

Solving the above equations yields

$$\boxed{\begin{aligned} R_1 &= 10\,k\Omega \\ R_2 &= 47.8\,k\Omega \\ R_3 &= 19.6\,k\Omega. \end{aligned}}$$

Note that the values are given to within $100\,\Omega$ since that was the approximate accuracy of the measurements.

S.F.

6.002 Problem Set 1 - Solutions

Appendix I

From KCL,

$$i_4 = -6 \text{ mA} \qquad \text{(Current Source)}$$

$$i_1 = -i_2 \qquad \text{(Node 1)}$$

Combining the above in part A) Node 3,

$$-i_2 + i_3 - 6 \text{ mA} = 0 \qquad (*)$$

From KVL,

$$v_1 = 3\text{V} \qquad \text{(Voltage Source)}$$

$$v_3 = v_4 \qquad \text{(Loop 2)}$$

Using the above in equations for Loop 1 or Loop 3,

$$3\text{V} = v_2 + v_3 \qquad (**)$$

From Ohm's Law,

$$v_2 = i_2 (2k\Omega) \Rightarrow i_2 = \frac{v_2}{2k\Omega}$$

$$v_3 = i_3 (1k\Omega) \Rightarrow i_3 = \frac{v_3}{1k\Omega}$$

Substituting i_2 and i_3 into equation $(*)$, we get

$$-\frac{v_2}{2k\Omega} + \frac{v_3}{1k\Omega} - 6 \text{ mA} = 0$$

or $$-v_2 + 2v_3 = 12\text{V} \qquad (***)$$

Now combining $(**)$ and $(***)$, we solve for v_2 and v_3 then backsolve to get the values in Table I.