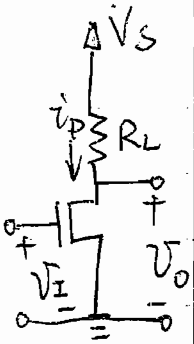
SOLUTION OF PS 6

6.002 Fall 2007

Exercise 6.1: a) $\because V_T = 0 \quad \therefore i_{DS} = \frac{k}{2}(v_{GS} - V_T)^2 = \frac{k}{2} v_{GS}^2$



$$v_O = V_S - i_{DS} \cdot R_L = V_S - \frac{k}{2} v_{GS}^2 \cdot R_L = V_S - \frac{k}{2} v_I^2 \cdot R_L$$

$$\therefore \boxed{v_O = f(v_I) = V_S - \frac{k R_L}{2} \cdot v_I^2}$$

b) To keep the amplifier operating in saturation region, we need:

$$v_{DS} \geq v_{GS} - V_T \geq 0$$

$$\because V_T = 0$$

$$\therefore \quad v_{DS} \geq v_{GS} \geq 0 \implies v_{DS} \geq v_I \geq 0$$

At the boundary of saturation and triode regions, we still have

$$v_{DS} = V_S - \frac{kR_L}{2} \cdot v_I^2$$

Therefore,

$$V_S - \frac{kR_L}{2} \cdot v_I^2 \geq v_I$$

$$\implies \frac{kR_L}{2} \cdot v_I^2 + v_I - V_S \leq 0$$

$$\implies v_I \leq \frac{-1 + \sqrt{1 + 2kR_L V_S}}{kR_L}$$

The negtive solution is eliminated because it will put MOSFE in cutoff region.

Combining all the information we have, we obtain that

$$\boxed{0 \leq v_I \leq \frac{-1 + \sqrt{1 + 2kR_L V_S}}{kR_L}}$$

Becaust output current $i_D = i_{DS} = \frac{K}{2} V_I^2$

$$\therefore \boxed{0 \le i_D \le \frac{1+KR_LV_s - \sqrt{1+2KR_LV_s}}{KR_L^2}}$$

$$\because V_o = V_{DS} = V_s - i_D \cdot R_L$$

$$\therefore \boxed{\frac{\sqrt{1+2kR_LV_s}-1}{KR_L} \le V_o \le V_s}$$

Note that it makes sense when it's on the boundary of saturation and triode, $V_o = V_I = \frac{\sqrt{1+2kR_LV_s}-1}{KR_L}$

c) We want to set bias V_I such that V_o is half way between its valid range deried in part b), therefore,

$$\boxed{V_o = \frac{-1+kR_LV_s+\sqrt{1+2kR_LV_s}}{2kR_L}}$$

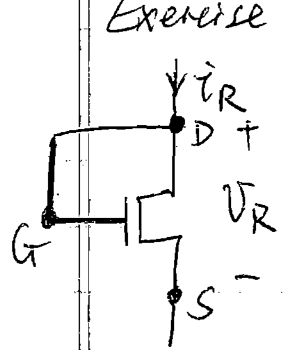
$$\therefore I_D = \frac{V_s - V_o}{R_L}$$

$$\therefore \boxed{I_D = \frac{1+kR_LV_s - \sqrt{1+2kR_LV_s}}{2kR_L^2}}$$

$$\because i_D = \frac{k}{2} V_I^2 \Rightarrow V_I = \sqrt{\frac{2i_D}{k}}$$

$$\therefore \boxed{V_I = \sqrt{\frac{2I_D}{K}} = \sqrt{\frac{1+KR_LV_s - \sqrt{1+2KR_LV_s}}{KR_L}}}$$

Exercise 6.2:



a) For a MosFet operation in saturation region,

$$i_D = \frac{K(v_{GS} - V_T)^2}{2}$$

In this case: $v_{GS} = v_{DS} = v_R$

$$i_D = i_R$$

$$\therefore \boxed{i_R = \frac{K(v_R - V_T)^2}{2}}$$

b) From part a), we have $i_R = \dfrac{K(v_R - V_T)^2}{2}$

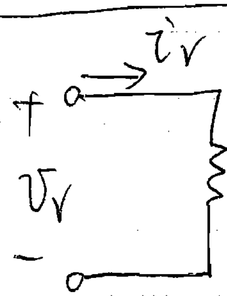
$$\therefore \left.\frac{di_R}{dv_R}\right|_{v_R = V_R} = \left.\frac{2 \cdot K(v_R - V_T)}{2}\right|_{v_R = V_R} = K(V_R - V_T)$$

i.e. When it's operating at dc bias $v_R = V_R > V_T$, the small-signal model will be,

$$\frac{i_r}{v_r} = \left.\frac{di_R}{dv_R}\right|_{v_R = V_R} = \boxed{K(V_R - V_T)}$$

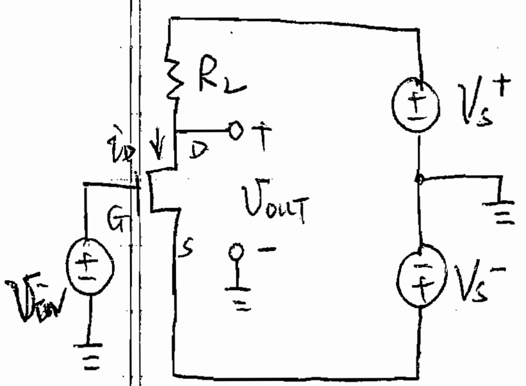
$$\Rightarrow \boxed{\frac{v_r}{i_r} = \frac{1}{K(V_R - V_T)}}$$

$\Rightarrow$



$R = \dfrac{1}{K(V_R - V_T)}$ / $G = K(V_R - V_T)$

Problem 6.1:



a) · From circuit, we know when $V_{IN} = 0V$

$$V_{GS} = V_{IN} - V_S^- = 1.5V + V_{IN} = 1.5V$$

$$V_{OUT} = -i_D \cdot R_L + V_S^+$$

$$= -\frac{K(V_{GS}-V_T)^2}{2} \cdot R_L + V_S^+ = 0V$$

$$\Rightarrow \boxed{R_L = \frac{V_S^+}{\frac{K(V_{GS}-V_T)^2}{2}} = 3(k\Omega)} = \frac{1.5}{\frac{1\times10^{-3}(1.5-0.5)^2}{2}}$$

b) To keep MOSFET in saturation range, we need

$$V_{DS} \geq V_{GS} - V_T$$

Take the boundary point between saturation & triode, we have:

$$V_{DS} = V_{GS} - V_T$$

~~from part a), we have,~~

from circuit, we have, $V_{DS} = V_S^+ - V_S^- - i_D R_L = V_S^+ - V_S^- - \frac{K(V_{GS}-V_T)}{2}$

from part a), we have, $V_T = 0.5V$, $R_L = 3k\Omega$, $K = 1mA/V^2$; $V_S^+ = 1.5V$, $V_S^- = -1.5V$

$$\therefore 3 - \frac{3}{2}(V_{GS}-0.5)^2 = V_{GS} - 0.5$$

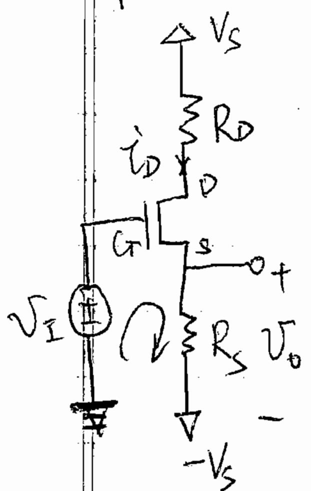
$$\Rightarrow 3 - \frac{3}{2}V_{DS}^2 = V_{DS}$$

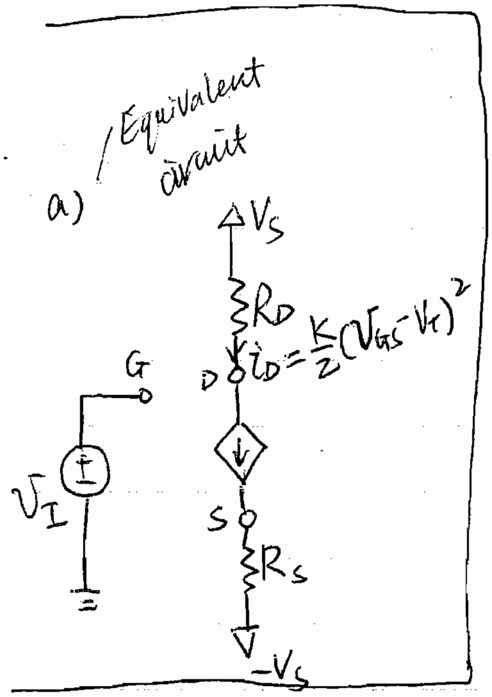
$$\Rightarrow V_{DS}^2 + \frac{2}{3}V_{DS} - 2 = 0$$

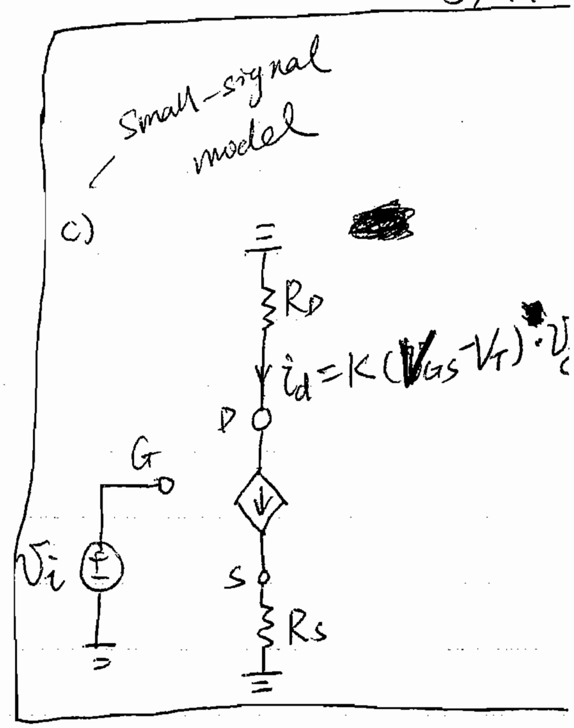
$$\Rightarrow V_{DS} = \frac{-\frac{2}{3} \pm \sqrt{(\frac{2}{3})^2 + 8}}{2} \longrightarrow V_{DS} = 1.12V$$ (only positive solution applies)

$$\therefore V_{OUT} = V_{DS} + V_S^- = -0.38V \therefore \boxed{V_{OUT\,min} = -0.38V}$$

Problem 6.2.







b) Write KVL loop equation around the bottom portion of the circuit and we have,

$$V_I - V_{GS} - I_D R_S = -V_S \Rightarrow V_{GS} = V_I + V_S - I_D R_S$$

So under saturation discipline, we have operating point current

$$I_D = \frac{K}{2}(V_{GS}-V_T)^2 = \frac{K}{2}(V_I+V_S-I_DR_S-V_T)^2$$

Solve the quadratic equation about I_D and notice the negtive term wouldn't apply, we have,

$$\boxed{I_D = \frac{1}{KR_S^2} + \frac{V_I-V_T+V_S}{R_S} - \frac{\sqrt{2KR_S(V_I-V_T+V_S)+1}}{KR_S^2}}$$

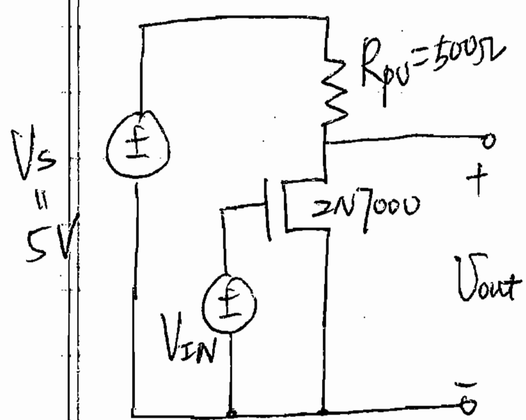
Then, $\boxed{V_0} = I_D R_S + (-V_S)$

$$\boxed{= \frac{1}{KR_S} + V_I - V_T - \sqrt{\frac{2}{KR_S}(V_I-V_T+V_S)+\frac{1}{K^2R_S^2}}}$$

c) See up-right ~~for~~ small-signal model. To find small-signal gain, we take $dv_o/dv_i\big|_{v_i=V_I}$; This will give us

$$\boxed{A_v = \frac{dv_o}{dv_i}\bigg|_{v_i=V_I} = 1-[2KR_S(V_I-V_T+V_S)+1]^{-1/2}}$$

Problem 6.3.



a). · Please see Page 7

b) Please see Page 8

c) Please see Page 9

d) Please see Page 10

e) Because different people could get slightly different data, so it's possible to get slightly different fitted values for K and V_T. The fitting approach should be similiar, though, as following.

① From part a), b) & c), we can pick a good estimation of V_T. From my data, I pick $\boxed{V_T = 2V}$

② From part b), pick a pair of (V_{GS}, V_{DS}), which is in saturation mode. For example, I pick $(V_{GS}=2.2V,\ V_{DS}=4.5V)$, therefore, $i_D = \frac{V_S - V_{DS}}{R_{PU}} = \frac{5-4.5}{500} = 1mA$, which is

consistent with the measured data shown in part a) & b).

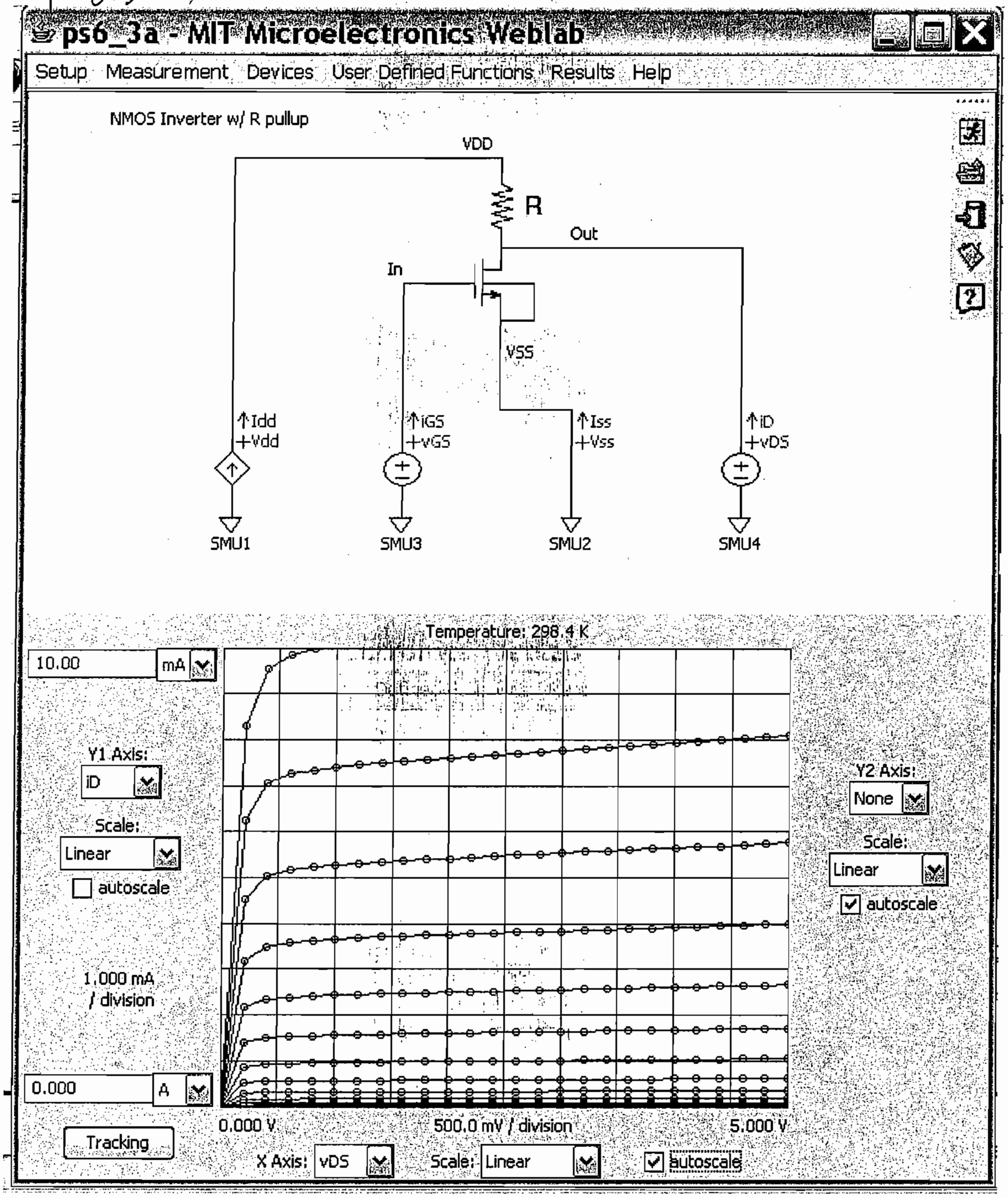
On the other hand, because it's in saturation mode, so we have, $i_D = \frac{K}{2}(V_{GS}-V_T)^2 \Longrightarrow K = \frac{2 i_D}{(V_{GS}-V_T)^2}$

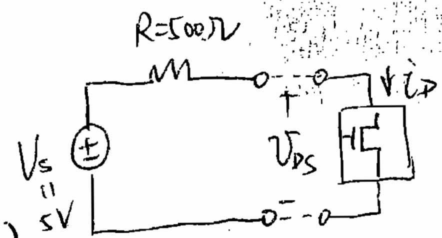
$$= \frac{2 \times 1 \times 10^{-3}}{(2.2-2.0)^2} = 0.05\ A/V^2$$

∴ The fitted values of V_T & K are: $\boxed{V_T = 2.0\ V \quad K = 0.05\ A/V^2}$

Please see page 11 for graphical comparison.

P.6.3.a)


ps6_3a - MIT Microelectronics Weblab
Setup Measurement Devices User Defined Functions Results Help
NMOS Inverter w/ R pullup
VDD
R
Out
In
VSS
Idd
+Vdd
iGS
+vGS
Iss
+Vss
iD
+vDS
SMU1
SMU3
SMU2
SMU4
Temperature: 298.4 K
10.00
mA
Y1 Axis:
iD
Scale:
Linear
autoscale
1.000 mA / division
0.000
A
Tracking
0.000 V
500.0 mV / division
5.000 V
X Axis:
vDS
Scale:
Linear
autoscale
Y2 Axis:
None
Scale:
Linear
autoscale



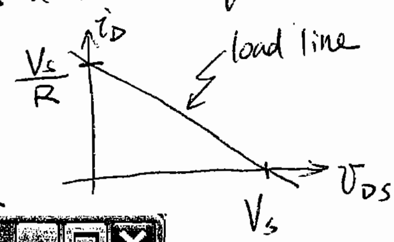
P.6.3. b)

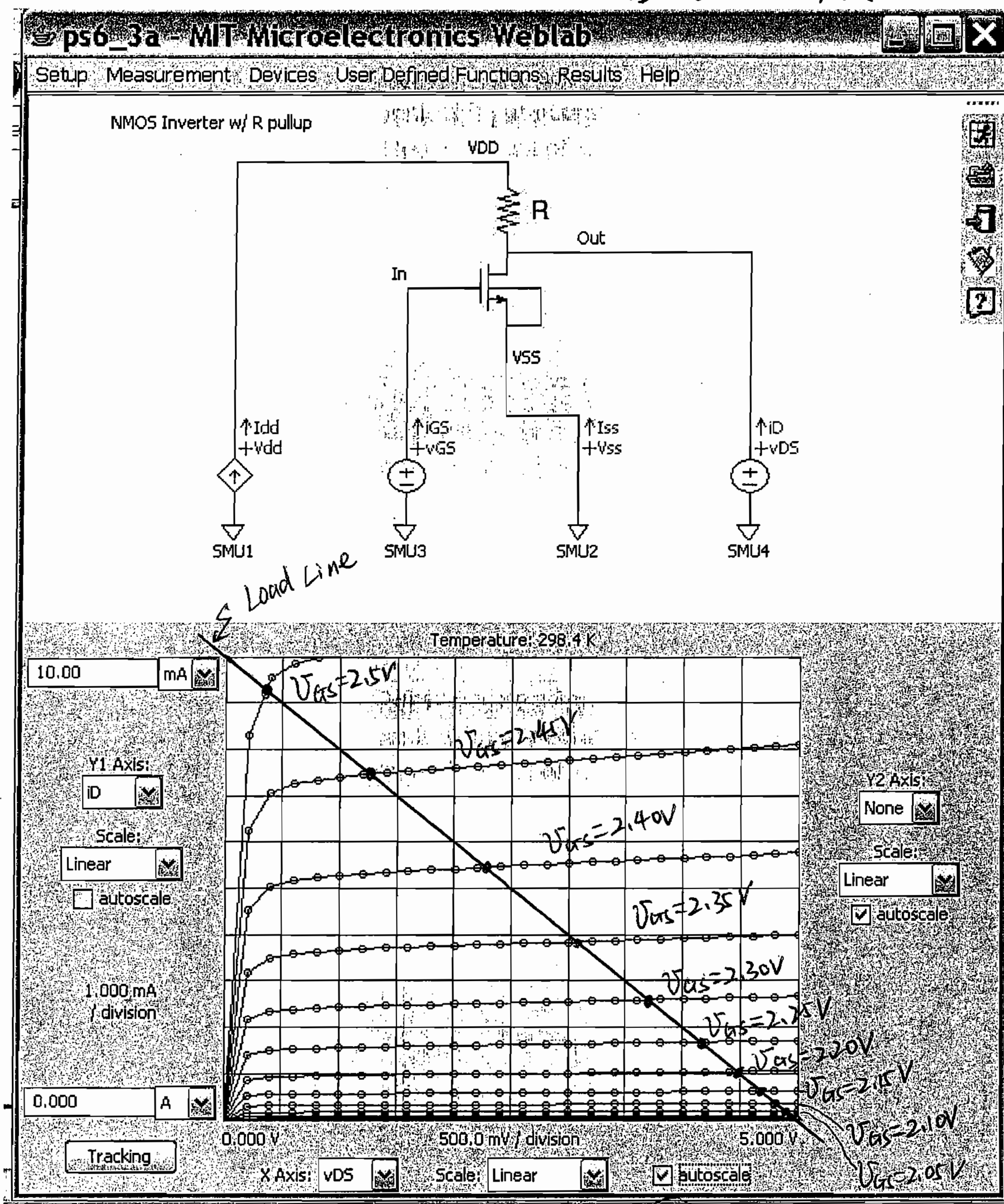
∴ for left side, it is a thevenin equivalent.

$V_{TH} = V_s = 5V$

$R_{TH} = R = 500\Omega$

∴ $i_D = (V_s - v_{DS})/R$





$v_{IN}=v_{GS}$	2.5	2.45	2.4	2.35	2.3	2.25	2.2	2.15	2.10	2.05	1.9
$v_{OUT}=v_{DS}$	0.4	1.25	2.3	3.1	3.7	4.2	4.5	4.7	4.8	4.9	5.0

⇓ part(e)

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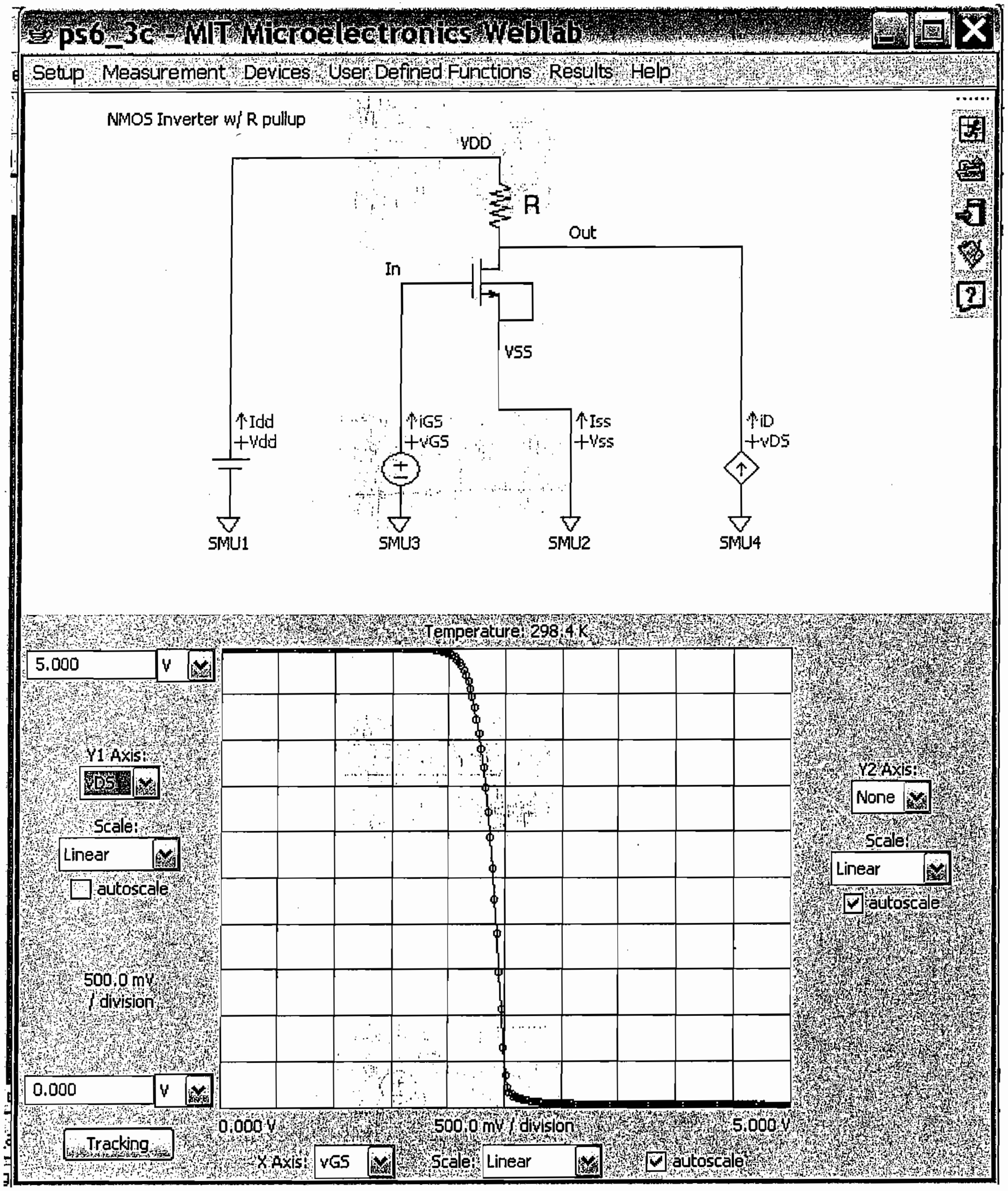
P.6.3. c)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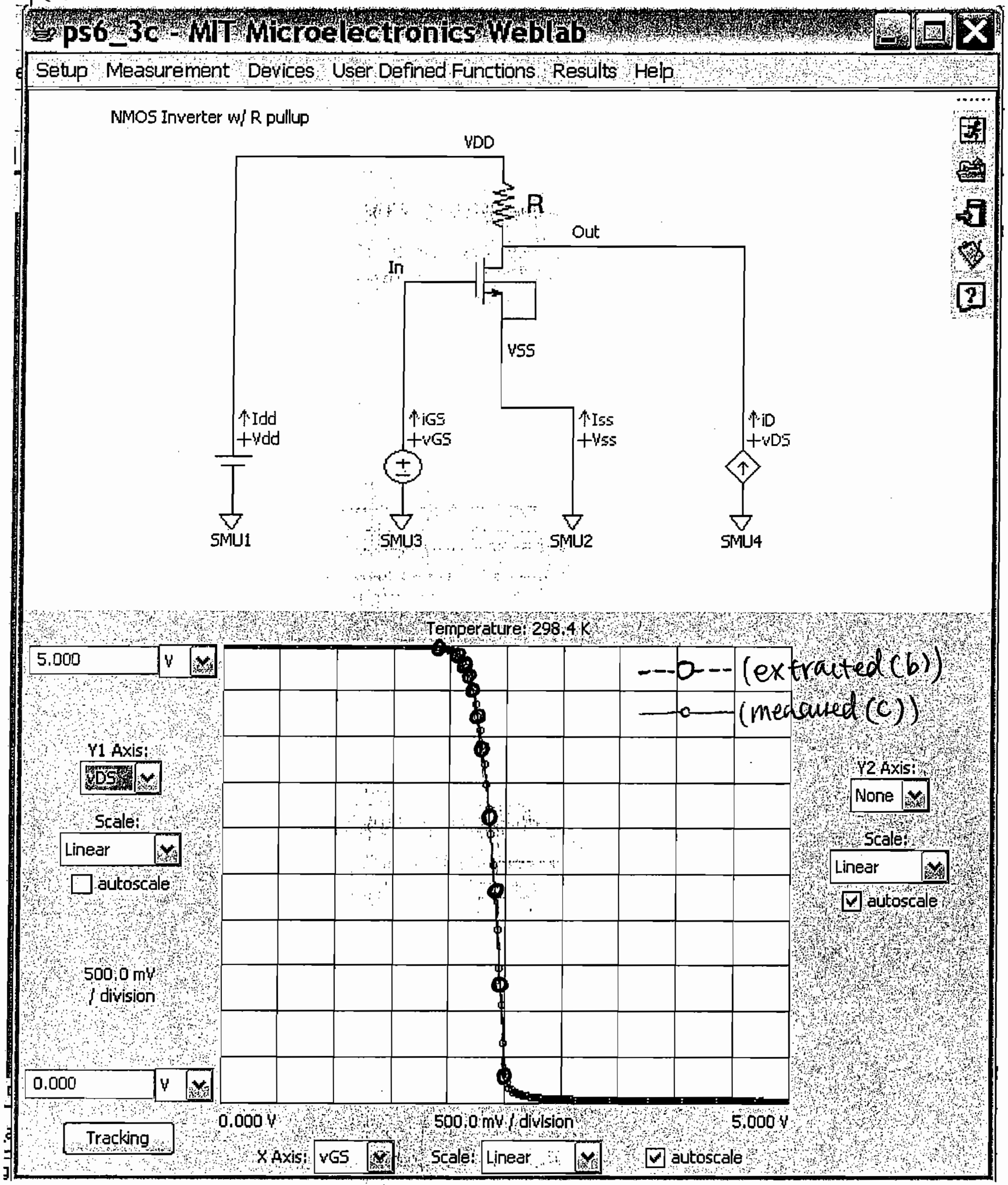
Scale: Linear
autoscale
500.0 mV / division
0.000 V
Tracking
0.000 V
500.0 mV / division
5.000 V
X Axis: vGS
Scale: Linear
autoscale
Y2 Axis: None
Scale: Linear
autoscale

P.6.3 d)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SMU2
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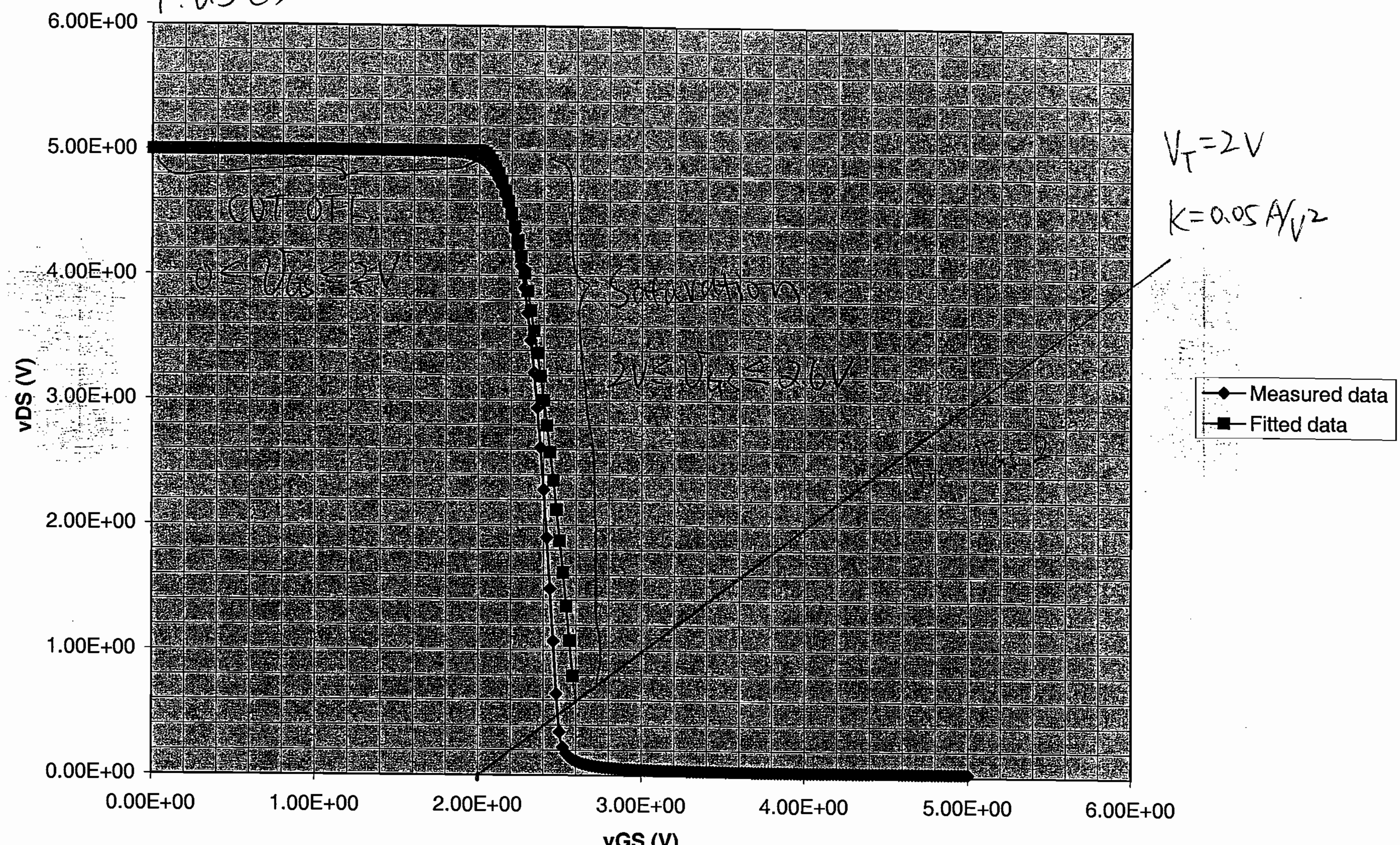
Temperature: 298.4 K
5.000 V
Y1 Axis: VDS
Scale: Linear
autoscale
500.0 mV / division
0.000 V
Tracking
0.000 V
500.0 mV / division
5.000 V
X Axis: vGS
Scale: Linear
autoscale
Y2 Axis: None
Scale: Linear
autoscale
(extracted (b))
(measured (c))

P.6.3 e)

Comparison of measured and fitted data



The fitted data is pretty close to the measured data.
The minor difference is due to the change/transfer from one region to ~~the~~ another is more gradual in the measured data.