Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Circuits & Electronics
Fall 2006

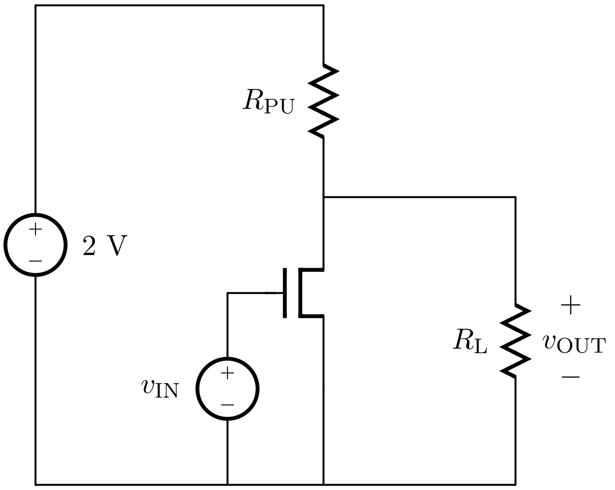
Problem Set #4

Issued 10/6/06 – Due 10/13/06

Projected reading for 10/11 – 10/13 from A&L: Ch 8, Chs 9-10 & Chs 9-10.

Exercise 4.1 (1 Point): Consider the logic function $\mathrm{OUT} = \overline{\mathrm{IN1} + \overline{(\overline{\mathrm{IN2}} \cdot \mathrm{IN3})}}$. Express OUT as a function of IN1, IN2 and IN3 in the form of a truth table. Also, implement this logic function using logic symbols, and using a small number of n-channel MOSFETs and pull-up resistors.

Exercise 4.2 (1 Point): In the circuit shown below, an inverter is loaded with a resistor. The load resistor models possible leakage along the data path from the inverter to subsequent gates. Its resistance is known to satisfy $R_{\mathrm{L}} > 1\ \mathrm{M\Omega}$. In the presence of this partially known load, the inverter must satisfy the static discipline defined by $v_{\mathrm{OL}} = 0.4$ V, $v_{\mathrm{IL}} = 0.8$ V, $v_{\mathrm{IH}} = 1.2$ V, and $v_{\mathrm{OH}} = 1.6$ V. Assume that the MOSFET behaves like a switch having threshold voltage V_{T} and on-state resistance R_{ON}. Over what voltage range must V_{T} be designed, and at what values must R_{PU} and R_{ON} be designed, so that the inverter dissipates minimum power while obeying the desired static discipline in the presence of uncertainty in R_{L}?

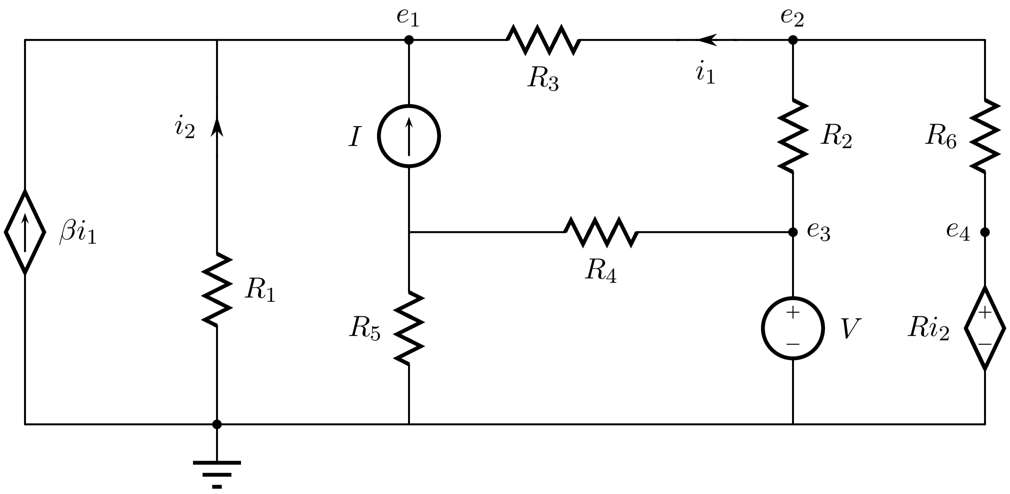


Problem 4.1 (2 Points): Following the node method, develop a set of simultaneous equations for the network shown below that can be solved to determine the unknown node voltages e_1, e_2 and e_3. Express the set of equations in the form

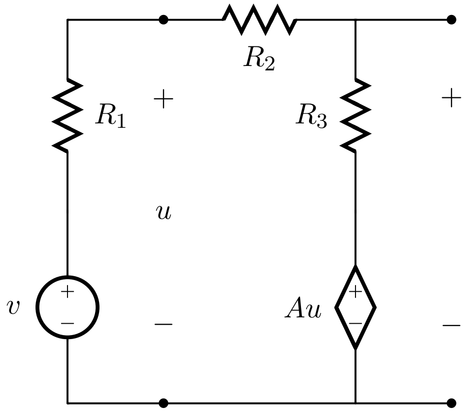
$$G \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = S$$

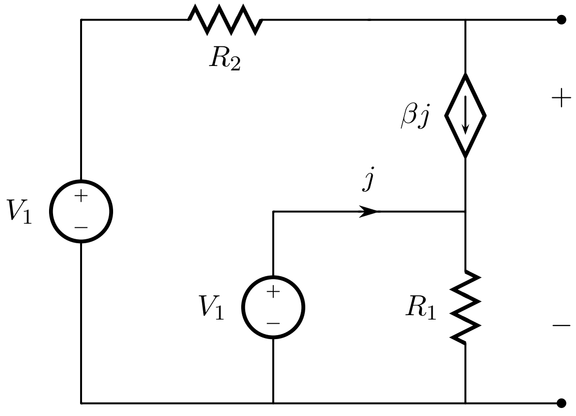
where G is a 3×3 matrix of conductance terms and S is a 3×1 vector of terms involving the *independent* sources. You need not solve the set of equations for the node voltages.

Note that e_4 is not included in the analysis because it is directly sourced by the dependent voltage source from ground, and the source voltage can be expressed in terms of the first three node voltages. In this sense, it is treated like an independent voltage source. Nonetheless, state how e_4 can be determined from e_1, e_2 and e_3 once the latter node voltages are determined.

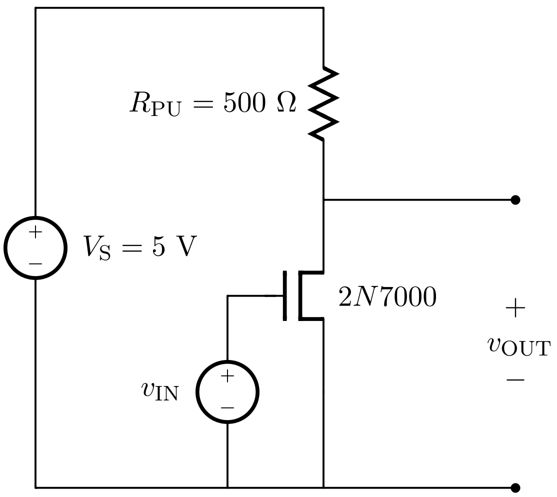


Problem 4.2 (2 Points): Determine the Thevenin equivalent of each network shown below. The coefficients describing the dependent sources satisfy $A > 0$ and $\beta > 0$.





Problem 4.3 (2 Points): This problem uses WebLab to predict and then verify the input-output relation of the circuit shown below. The circuit can function either as a digital inverter or an analog amplifier. To study the circuit, this problem is divided into four parts. The first part uses WebLab to measure the MOSFET characteristics, that is, the relation between v_{DS} and i_{D} for various values of v_{GS}. The second part applies a load-line analysis to the measurements to predict the input-output relation of the circuit. The third part uses WebLab to verify the predicted input-output relation through direct measurement. The fourth part compares the input-output relation predicted with the theoretical MOSFET model to that measured by WebLab over the cutoff and saturation ranges.



(A) Use the following procedure to measure and plot the MOSFET characteristics via WebLab. Feel free to modify the procedure as desired.

- Launch WebLab using the procedure given in Problem 1.4.
- Select Device 6 or Device 7 from the drop-down menu of devices. These two devices are built to be the same. However, because they are physically different devices, they will be slightly different electrically.
- For all measurements, SMU3 should be grounded. To do so, select it and set MODE = COMM. Name the variables, but do not select them for downloading.
- To measure the MOSFET characteristics, SMU1 must be set to act as an open circuit. To do so, select it and set: MODE = I; FUNCTION = CONS; VALUE = 0 A; and COMPLIANCE = 5 V. Name the variables, but do not select them for downloading.
- SMU4 must be set to sweep v_{DS} over an inner loop. To do so, select it and set: MODE = V; FUNCTION = VAR1; and COMPLIANCE = 100 mA. Set VAR1 to sweep from 0 V to 5 V in 200-mV steps. Finally, name both variables and select both for downloading. These variables are v_{DS} and i_{D}.
- SMU2 must be set to sweep v_{GS} over an outer loop. To do so, select it and set: MODE = V; FUNCTION = VAR2; and COMPLIANCE = 100 mA. Set VAR2 to sweep from 1.9 V to 2.5 V in 50-mV steps. Finally, name both variables, but do not select them for

downloading.

- Run WebLab to measure the MOSFET characteristics.

- Plot the measured characteristics with v_{DS} on the horizontal axis over the range 0 V to 5 V, and i_{D} on the vertical axis over the range 0 mA to 10 mA. You can do so directly in WebLab, and then capture a screen shot for printing. Alternatively, you can download the measurements, and then plot and print them using another program such as MatLab or Excel. Note that the measured data will not all fit within the specified graphing range.

(B) On the graph produced in Part (A), plot the v_{DS}-i_{D} load line that is imposed by the 5-V power supply and the 500-Ω resistor in the circuit below. Using the intersections of the load line with the MOSFET characteristics, determine the values of $v_{\mathrm{OUT}} = v_{\mathrm{DS}}$ that correspond to the values of $v_{\mathrm{IN}} = v_{\mathrm{GS}}$. Make a table of the resulting input-output relation.

(C) Use the following procedure to measure and plot the input-output relation of the circuit via WebLab. Feel free to modify the procedure as desired.

- To measure the input-output relation, SMU1 must be set to the power supply voltage of 5 V. To do so, select it and set: MODE = V; FUNCTION = CONS; VALUE = 5 V; and COMPLIANCE = 100 mA.

- SMU2 must be set to sweep $v_{\mathrm{IN}} = v_{\mathrm{GS}}$. To do so, select it, and set: MODE = V; FUNCTION = VAR1; and COMPLIANCE = 100 mA. Set VAR1 to sweep from 0 V to 5 V in 20-mV steps. Select the voltage for downloading. This variable is the circuit input.

- SMU4 must be set to act as an open circuit while it measures the output v_{OUT}. To do so, select it and set MODE = I; FUNCTION = CONS; VALUE = 0 A; and COMPLIANCE = 5 V. Select the voltage for downloading. This variable is the output of the circuit.

- SMU3 should remain grounded, as it was during the measurement of the MOSFET characteristics.

- Run WebLab to measure the input-output relation of the circuit.

- Plot the input-output relation of the circuit, with both variables ranging from 0 V to 5 V. You can do so directly in WebLab, and then capture a screen shot for printing. Alternatively, you can download the measurements, and then plot and print them using another program such as MatLab or Excel.

(D) On the graph produced in Part (C), plot the input-output relation found in Part (B). The input-output relations found in Parts (B) and (C) should be identical.

(E) Following Lectures 7 and 8, $v_{\mathrm{OUT}} = V_{\mathrm{S}}$ for $v_{\mathrm{IN}} < V_{\mathrm{T}}$ (Cutoff Operation) and $v_{\mathrm{OUT}} = V_{\mathrm{S}} - 0.5KR_{\mathrm{PU}}(v_{\mathrm{IN}} - V_{\mathrm{T}})^2$ for $V_{\mathrm{T}} \leq v_{\mathrm{IN}} < V_{\mathrm{T}} + (\sqrt{1 + 2V_{\mathrm{S}}KR_{\mathrm{PU}}} - 1)/(KR_{\mathrm{PU}})$ (Saturation Operation). By comparing these theoretical expressions to the input-output relation measured in Part (C), determine fitted values for K and V_{T}. Given these values, plot the theoretical input-output relation on the graph from Part (D) over the range of v_{IN} that corresponds to cut off and saturation operation.

Problem 4.4 (2 Points): This problem studies the two-stage n-channel MOSFET amplifier shown below. The two stages are built with identical MOSFETs and pull-up resistors. A simplified model for the MOSFET is also given below. The simplification is that the triode region of operation is compressed onto the curve $i_{\rm D} = Kv_{\rm DS}^2/2$, which becomes a common curve of operation for $v_{\rm GS} - V_{\rm T} > v_{\rm DS}$. Hint a load-line analysis may help solve this problem.

(A) Determine the range of $v_{\rm IN}$ over which the first MOSFET operates in cutoff. Also, determine $v_{\rm MID}$ for this operating range.

(B) Assuming that the first MOSFET operates in its saturation region, determine $v_{\rm MID}$ as a function of $v_{\rm IN}$. Also, determine the range of $v_{\rm MID}$ and the range of $v_{\rm IN}$ that correspond to the saturated operation of the first MOSFET.

(C) For values of $v_{\rm IN}$ that are above the range found in Part (B), the first MOSFET operates in its triode region, which in the model below is compressed onto the curve $i_{\rm D} = Kv_{\rm DS}^2/2$. Determine $v_{\rm MID}$ for $v_{\rm IN}$ in this range of operation.

(D) Using the results of Parts (A), (B) and (C), determine $v_{\rm OUT}$ as a function of $v_{\rm MID}$ for the cutoff, saturation and triode regions of operation of the second MOSFET. For each region, state the corresponding operating range of $v_{\rm MID}$ and $v_{\rm OUT}$.

(E) Assuming that both MOSFETs operate in their saturation regions, determine $v_{\rm OUT}$ as a function of $v_{\rm IN}$. Also, determine the range of $v_{\rm MID}$ and then the corresponding range of $v_{\rm IN}$ over which both MOSFETs operate in their saturation regions.

(F) Determine the small-signal gain of the amplifier as a function of the operating-point input $V_{\rm IN}$ assuming that this operating point falls within the range found in Part (E). That is, determine $dv_{\rm OUT}/dv_{\rm IN}$ evaluated at $V_{\rm IN}$.

(G) Let $K = 0.02$ A/V^2, $R = 1$ kΩ, $V_{\rm S} = 10$ V and $V_{\rm T} = 1$ V. Plot $v_{\rm MID}$ as a function of $v_{\rm IN}$ for $0 \le v_{\rm IN} \le 3$ V. On the same graph, plot $v_{\rm OUT}$ as a function of $v_{\rm IN}$ over the same range of $v_{\rm IN}$. Hint: this is particularly simple if you use a program such as MatLab or Excel. Observe the differences of the two plots.

Save a copy of your solutions to help with Problem Set #5.