Simulation Based Micro-founded Structural Market Analysis: A Case Study of the Copper Industry

By

Jingshu Zhang

B.S. in Materials Science and Technology, Cornell University, 2012
M.S. in Materials Science and Technology, Massachusetts Institute of Technology, 2014

Submitted to the Institute for Data, Systems, and Society in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Engineering Systems at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Feb 2018

Signature of Author ____
Engineering Systems
Institute for Data, Systems, and Society

Certified by____
Prof. Joel Clark
Professor of Engineering Systems
Thesis Committee Chair
Certified by____
Dr. Randolph Kirchain
Senior Researcher of Engineering Systems

Certified by____
Prof. Roy Welsch
Eastman Kodak Leaders for Global Operations Professor of Management

Certified by____
Prof. Weiqiang Chen
Professor of Industrial & Urban Ecology
Chinese Academy of Science
Accepted by____
Stephen C. Graves
Professor of Mechanical Engineering & Engineering Systems
IDSS Graduate Officer

Simulation Based Micro-founded Structural Market Analysis: A Case Study of the Copper Industry

By Jingshu Zhang

Submitted to the Institute for Data, Systems and Society
In Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

Abstract

This work aims to provide a widely applicable modeling framework that can be used to credibly investigate materials scarcity risks for various types of commodities. Different from existing literature, this work contributes to a better understanding of commodity scarcity risk, specifically copper future consumption on several fronts. Firstly, it introduces an elaborate price mechanism absent in comparable materials flow assessment. It teases out short term and long term substitution, allowing consumers to switch from one type of commodity to another based on price signals and their respective price elasticities of demand. Secondly, the model allows for individual deposit tracking, which allows the modeler to extract ore grade information as a function of consumption and reserve size. Thirdly, it models the supply side on an agent-based basis, allowing for aggregation of granular information, capturing potential emergent phenomena. We believe these three aspects, which are least addressed (none of existing work has addressed the first aspect, and few have addressed the second or the third), are important in assessing scarcity risks. Without them, scarcity assessment is likely to be biased. We hope our work may serve as some sort of foundation upon which more reliable future work on mineral scarcity evaluation can be carried out.

Beyond the Realm of Rationality (In Lieu of Foreword)

Our flight to Urumqi was delayed. Mr. Yang and I were supposed to arrive at 1 PM, but it was already past 9 when we landed. After six long hours of flight, I already felt a trace of fatigue. If I had any idea what was ahead of us, I would probably have bailed out at that moment.

In the next six weeks, Mr. Yang and I would traverse the better part of the entire China, nonstop. We traveled from Five Colored Bay to Holingol, from Qingxiu Mountain to Anshun City. We would suddenly regale in the cooling winds of Tongliao, only to find ourselves bathed in sweat amidst the withering heat of Nanning; we would marvel at the mighty torrents of Huangguoshu waterfall, and in the next moment find ourselves amongst the noble antique constructions where Yimou Chang had his crew raise those red lanterns which would enshrine him in eternal fame.

Our goal was to collect first-hand intelligence on the detailed cost structure and production capacity of a certain type of commodity production plants in the vast land of China.

To be honest, I cannot recall a single production data now.

What left me with the deepest impression across the wild spectrum of six long years of the most precious years of the first thirty years of my life was an afternoon in Jinan, Shandong. Mr. Yang and I finished our interview at the Shandong Provincial Land & Resource Agency quite early in the day, so we decided to have a walk around the Daming Lake. For those who believe in romance, Daming Lake is where Emperor Qianlong met his controversial love, Yuhe Xia, whose offspring wove a most visceral episode of emotional Odyssey recorded in the timeless novel named *My Fair Princess*. Summer prevailed, and an entire lake of lotuses blossomed with such fervency that you'd feel like they were soon going to surround you, swallow you.

A silver haired old man, holding his grandson in his arms, was tottering towards the lake. He was so thin that through the vaguely transparent white t-shirt he was wearing, I could almost see his bare bones. He was not moving fast. As a matter of fact, he was trying hard just to hold his grandson steady. Yet his step was firm, so were his arms. His grandson, still babbling with that type of insouciance so characteristic of fellows without the slightest idea of worries, was pointing his little finger towards the lake, bidding his grandpa to speed up. I fell into a trance, and could not help but following their path, diverging away from where Mr. Yang was heading. As we approached Daming Lake, the innocent chubby angel couldn't wait to see the lotuses. The silver haired old man suddenly mustered enough strength to hold his grandson onto his neck – there, the satisfied young fellow swiftly took the advantage to ride on his grandpa's neck. Wavering, his grandpa muttered: "smell the lotus fragrance, smell that light, light lotus fragrance". I stopped in front of this holy vision of love and harmony as the duet, the lotuses, the shimmering lake, and the setting sun were altogether interwoven into a most sentimental tapestry beyond my furthest imagination. And that grandpa, that old fellow who seemed enervated yet at the same time mysteriously spirited, started chanting with such redoubtably profound emotionality: "Lotus scent fades, mat feels cold; Unlace my robe, step on my boat; Who could bring the note to alight? When geese return the moon shines load."

Alas, how can I forget that this is Qingzhao Lee’s hometown?! A thousand years earlier, when the crushing steeds of the Northern barbarians smashed open the soft womb of the fragile imperial gate of Northern Song Dynasty, when her husband Mingcheng Zhao was forever taken away from her, when Qingzhao, a female so obscure and insignificant in front of the bloody massacres of her era was trying all that she could to make whole of the ancient rubbings which were the crystallization of their celestial love, when she, sitting on her mat, full of reminiscence, was dreaming of a day of reunion in soothe unobtainable until the day she would perish – those beautiful words were born, and they survived countless generations and dynasties; they survived yesterday and survive today and will survive tomorrow; they survive, because of countless seemingly feeble grandpas gently imparting that undying spirit to their babbling grandchildren – such, is the palpable fire of life of this ancient civilization.

Like that sensual Meursault who ultimately understood the full connotation of his life by the end of Camu’s *L'Étranger*, I was struck by something so transparently self-explanatory yet at the same time so unspeakably irrational that I felt fundamentally overwhelmed.

I wept like a child.

I am grateful for the privileged opportunity that Jinjiang Group gave me. I am forever indebted to the benign folks at Jinjiang Group who have not only generously sponsored two years of my PhD career, but has kindly provided me with countless resources that I could only dream of possessing. I am particularly grateful for the help from Mr. Tonghai Yang, CEO Yuanluo Wang, and Chairman Zhengang Dou. I have learned some most valuable lessons from these splendid individuals.

I want to genuinely thank MSL for supporting me. If it were not for Randy's audacity in giving the project a shot, I would not have the chance to witness and meditate on the aforementioned vision that breaks the boundary of rationality. Randy is perhaps one of the closest individuals that I would classify as a saint, harnessing all of the four essential cardinal virtues of humanity. I am sincerely grateful to Randy and Rich for spending hundreds of hours with me in perfecting our model and improving my thesis. I will not forget watching sunset with Rich in that Siberian-like Holingol with all of our four ears ruthlessly ice-bitten in that winter of 2015. I am grateful to Omar and Michele as trusted comrades who fought with me on the same front towards knowledge and truth. I am also immensely indebted to Prof. Joel Clark, Prof. Weiqiang Chen and Prof. Roy Welch for being both sagacious scholars and incredibly virtuous human beings tremendously supportive throughout my intellectual journey.

I want to thank my mum and dad, my grandpa and grandma for supporting me without reservation during the six long years of relentless pursuit. Their patience, encouragement, and understanding warm my heart and give me courage at moments of desperation.

Last but not least, the expressive power of English language defies my capability in exhausting my love and admiration towards my most endearing wife Zhen Dai. She's the most precious jewel of my life. She is, as I always like to say, a "nymphetic being".

No one in the world understands or loves me more than she does. I am forever indebted to her immeasurable sacrifice for our unification. She’s not only a lot wiser than me, but also makes me a better human being through exemplifications with her worthy demeanors. This work is impossible without her.

My work is also dedicated to those countless anonymous individuals who I have met during my journey across the vastness of my motherland. You are not marionettes haplessly manipulated by the merciless hands of fate, nor are you simply rational agents represented in stylistic models. You are a lot more and you encompass far more than what is known in the ivory tower as rationality. You constitute what is known as – Humanity.

Table of Content:

Chapter One: What This Dissertation is About

Minerals scarcity has been on the radar of intellectuals since the industrial revolution. As early as 1798, Thomas R. Malthus argued that the power of population is indefinite and has been expanding exponentially, whereas the power in the earth to produce subsistence for them pales in comparison.[1] Scarcity refers to a gap between limited resources and theoretically limitless wants. The notion of scarcity is that there is never enough of something to satisfy all conceivable human wants, even at advanced states of human technology.[2] Malthus's argument has been echoed ever since. For instance, King Hubbert proposed the Hubbert peak theory, which argues that the rate of petroleum production tends to follow a bell-shaped curve, and the peak is likely to transpire in the 1970s. [3] In 2009, Aaron Regent, president of the Canadian gold giant Barrick Gold said that global output of precious metals has been falling by roughly one million ounces a year since the start of the decade. The total global mine supply has dropped by 10 pc as ore quality erodes. He told the Daily Telegraph at the RBC's annual gold conference in London that "there is a strong case to be made that we are already at "peak gold", and production peaked around 2000 and it has been in decline ever since, and we forecast that decline will continue. It is increasingly difficult to find ore".[4] Scholars such as S. Northey and G.M. Mudd construct bottom-up structural commodity models which indicate we are running out of copper in the next century given a reasonable projection of population growth and intensity of use growth. [5]

All of these studies have tried to explore a clearly important question: are we going to run out of resource X? To quantitatively investigate mineral scarcity necessarily involves constructing a systemic model to track mineral production and short term equilibrium prices so one can answer various relevant questions, change assumptions, and examine outcomes. Because of the anticipatory nature of scarcity investigation work, as we will see later in this dissertation, seemingly unimportant negligence of modeling components can lead to vastly different conclusions.

Existing literature has not addressed the issue of long term price elasticity of demand in the commodities arena and has not addressed a particular aspect of the commodities economics, namely the potential of substitution, has not addressed the issue of ore grade deterioration on a deposit level, and has not sufficiently consider the effect of long term feedback on suppliers. One naturally wonders if leaving out these components might lead to inaccurate estimation of future demand, supply, equilibrium prices, and further conclusions regarding mineral scarcity. As we will see later in the dissertation, leaving out these components may indeed lead to considerably biased results in projecting demand & supply as well as assessing scarcity.

Our goal is to propose a modeling framework that fills the gaps of existing literature in the said field, and provide a useful tool for policy makers, corporate strategists, and researchers alike to investigate mineral scarcity and short term & long term equilibrium mechanism with reasonable degree of confidence. In order to answer the scarcity question, a partial equilibrium model that involves both supply and demand has to be constructed, and the researcher has to make sure that critical components are integrated into the model rather than left out.

We have chosen copper as a case-in-point since scholars have argued that we might run out of copper before the end of this century due to large quantity of consumption (third most consumed metal following steel and aluminum) and limited size of reserve base. We provide some historical background of copper utilization as well as a relevant & interesting story related to copper in the following chapter.

Chapter Two: On the Cusp of a Copper Century

Since my work is fundamentally on the methodology of prediction, and prediction has historically been viewed as a bet against the unpredictable hands of fate, therefore please allow me to start our journey with a story that has everything to do with the whimsical hands of fortune. This story becomes even more intriguing given that it has everything to do with our protagonist – copper.

It was a sultry afternoon on a hot summer day in 1880. A lugubrious George Warren was brooding over the Copper Queen Mine whose orebody ran 23% copper – an ore grade unheard of among anyone born within the last five decades. As a reference, the average ore grade across all operating mines in 2015 is 1.12%. George Warren felt dizzy. He could not recall anything that transpired yesterday. He was lost. One day ago, George Warren still owned 1/9 of the entire mine. Now, he owns next to nothing. [6]

Figure 1: Left: Copper Queen Mine in the early 1900s; Right: George Warren.

Before we dwell on the story of George, let's take a brief look at the history of copper, since it will be of central importance to our dissertation. The earliest usage of copper dates from prehistoric times when copper in native form was collected and beaten into primitive tools by stone-age people in Cyprus (where its name originates), Northern Iran, and the Lake region in Michigan.[7] The use of copper expanded greatly since the invention of smelting around 5000 BP, and copper was mainly used for the development of bronze (an alloy of copper with tin). Iron metallurgy's development around 3000 BP replaced bronze as a widely used material in the production of weapons, tools, pipes, and roofing. Copper conceded its glory to iron until a huge increase of copper production took place with the discovery of brass (an alloy of copper and zinc) in Roman times. In the 150-year period straddling the birth of Christ, copper production reached a peak of 16,000 tonnes per year. The implementation of pumping drainage and basic leaching techniques made copper extraction a lot more efficient. After the Roman Empire fell apart, copper and all metals consumption declined and production was sustained by the use of copper in the manufacturing of bronze cannons for both land and naval use, and as Christianity spreads for roofing and bells in churches. [8]

The industrial revolution in the half eighteenth century marked a new era in mining and usage for all metals, except copper. Copper's fate has been largely tied up with electricity. Even today, more than half of its total consumption is still related to electricity. The full blown of copper usage was enabled by the rapid burgeoning of the electricity and telecommunications industry. In 1866, a telegraph cable made of copper was laid across the Atlantic to connect North America and Europe; in 1876, the first message was transmitted through a copper telephone wire by Alexander G. Bell; 1878, Thomas A. Edison produced an incandescent lamp powered through a copper wire. [9] After hundreds of years of dormancy, copper seemed to have wakened, and it seemed eager to reassert its dominance among its metal peers.

George Warren was born at the right time, and he was at the right place. He moved to New Mexico to join his father after his mother passed away when he was ten years old. While herding horses, the Apaches attacked and killed his father, and abducted George. He was saved after 18 months until prospectors saw the white boy among the Indians and traded 20 pounds of sugar for his freedom. This marked the beginning of a remarkable prospecting life. [6]

On September 27th, 1877, George filed a claim. Over the next six months, his name is mentioned either as the locator or witness in several other claims in the Tombstone Canyon and Mule Mountains and established what became known as the George Mining District. So far, the goddess of Fortune seemed to have favored George – she saved George from hostage, and landed him among a group of people who were on the road to become millionaires of their era.

George was intoxicated in his triumph. He had a reputation as a drunkard. While drinking with some friends in Charleston, he argued with his friend George W. Atkins about the agility and speed of men versus horses. George Warren claimed he could outrun a man on a horse over a distance of 100 yards, or equivalently, 91 meters. Atkins took the bet. George Warren misjudged the odds so badly, that his wager became what's known as "Wager of a Lifetime". If Atkins beat George Warren, Atkins was to receive George Warren's interest in the Copper Queen; if George Warren beat Atkins, he was to receive Atkins's horse. [10]

To put it into perspective, the American Quarter Horse Association estimates the price of a healthy horse to be around $50,000 (including annual costs of owning it), whereas George Warren’s stake in the Copper Queen was valued at $500 million in today’s terms.[10] In other words, assuming that George Warren possessed superhuman running capability that made him as fast as a horse and the game is even odds, the size of the downside versus the size of the upside is a staggering 10,000. This is why you should never take a wager under alcoholic influences. George was, of course, no superman, and he lost out to the horse badly, and with it he lost his 1/9 ownership of the Copper Queen Mine.

The mining company provided him a small pension, and George worked as a blacksmith and tool dresser. He swept floors or cleaned the cuspidors in exchange for a drink of whisky. [11]

George Warren died a pauper in 1892, half insane.

Soon after George Warren died, copper embraced its golden era. In 1913, the International Electro-technical Commission (IEC) established copper as the standard reference for electrical conductivity. Since then, the use of copper has spread to many industrial and service sectors. Copper wires have been used to conduct electricity and telecommunications across long distances as well as inside houses and buildings, cars, aircrafts, and many electric devices. Copper’s corrosion resistance, heat conductivity, and malleability has made it an excellent material for plumbing and heating applications such as car radiators and air conditioners, among others.[8] By 1925, US along produced roughly 50% of copper in the world, and Arizona was one of few states that dominated the production of copper – the Copper Queen Mine was the most productive copper mine in Arizona throughout the early 1900s. Despite of the fact that copper mine profitability varies tremendously, so much that only 51% of new mines opened from 1989 to 2008 carry the average cost of exploration, copper production flew through the roof, going from less than 500,000 tonnes in the beginning of the 20th century to more than 25 million tonnes in 2015. [12] As a matter of fact, archaeological, historical, and modern data show that the amounts of most metals mined and smelted before 1900 were tiny in comparison with the amounts extracted during the 20th century. From human origin to 1900, only 10 million tonnes of copper was mined and smelted, about 30% of 2015's global copper production. [13]

Through George Warren's tortuous fate and the massive explosion of copper consumption over the past century following more than a millennium's silence, we can probably get a good sense of both the vicissitudes of fate and the unpredictability of social evolution. Countless haphazard micro-agent's decision-making processes interact and aggregate into a macroscopic picture unfathomably opaque and oftentimes utterly incomprehensible. This dissertation can do little for those forfeited by the ruthless hands of fate other than referring such individuals to Cavafy's valedictory "Apoleipein o Theos Antonion" ("the God Abandons Antony") and that immortal admonishment: "Just listen while shaken by emotion but not with the coward's imploration and complaints." This dissertation, on the other hand, does share the ambition of understanding macro-level complex systems. The macroscopic unpredictability makes the task of trying to comprehend the behavior of complex systems incredibly fascinating. This dissertation attempts to understand and predict the equilibrium system dynamics of the copper market with a reasonable degree of confidence. We believe existing literature, despite of their tremendous contributions in extending the frontier of understanding of this particular field, tends to overlook several critical aspects we believe should be included in any commodity structural market models in evaluating scarcity. In the next chapter, we discuss the gap in existing literature in detail.

Chapter Three: Filling the Gap

In this chapter, we will first take a closer look at the copper value chain to get a better sense of the copper production process. As we will see in chapter five, we have constructed a model where the suppliers' cost structure is broken down into its components. Different copper production processes are used to process different types of ores, and can lead to very different component cost structure. Although we do not address cost structure and its impact on scarcity assessment in detail in this dissertation, we believe it is an interesting area to explore for future researchers, and since we have built this infrastructure into our model already, we decide to make the documentation. In addition, understanding the copper value chain simply provides one with a better sense of this industry in general, although one does not necessarily need a sophisticated grasp of it to understand our model. After that, we will then investigate a wide spectrum of extant literature on this or related topics. We will categorize these articles into their corresponding groups depending on their modeling attributes. We will discuss the relative strengths and weaknesses of these different types of models. Lastly, we will discuss our model, and why it can serve the purpose of filling in the gaps of existing models and better provide answers to the most critically important questions of our interest.

The Copper Value Chain:

Three types of copper ore, namely porphyry, sediment-hosted, and volcanogenic massive sulfide, have annually represented over 90% of world copper production for the past 80 years. Sulfide ores are ones that copper is linked with sulfur, and oxide ores when copper is linked with either carbon or silicon, and oxygen. Sulfide ores represent about 80% of world's copper production. [14] Before 1900, most of copper are produced from volcanogenic massive sulfide ores due to their relatively easier accessibility and considerably higher ore grade. They are also on average much smaller in size, making it possible for labor to extract. By 1905, Daniel C. Jackling, an engineer working at the Bingham Canyon open-pit mine in Utah introduced mass mining. Mass mining applied large scale machinery in the production process, namely the use of ore crushers, trucks, rail, steam shovels and heavy blasting made profitable the exploitation of low grade sulfide ores through economics of scale. Radetzki notes this "switch was akin to a move from handicraft methods to large-scale industrial processes. [8] The second important development was the flotation process, first introduced in copper in Butte, Montana in 1911 [10]. This process concentrates sulfide ores and significantly improves the recovery rates of metal, in turn lowering the processing costs. By 1935, recovery rates increased to more than 90% from the 75% average recovery rate observed in 1914. [15] Economies of scale coupled with better processing technologies made possible the extraction of copper from much larger deposits such as sulfide porphyry ore bodies. Sulfide porphyry quickly became the type of ore that produced the majority of copper, climbing up from 40% to 70% by 1925, and stabilizing at roughly 60% ever since.[14] A last breakthrough was the introduction of solvent extraction eletrowinning (SX-EW) technology in 1968 for leaching oxide ores. During this process, copper ores are first stacked and irrigated with acid solutions and subsequently cleaned by a solvent extraction process obtaining an organic solution. Next, in the refining process, copper with a grade of 99.9% is recovered from the organic solution by the application of electricity in a process called electrowinning. The process, first used commercially at the Bluebird plant in Arizona, has a number of advantages including lower capital costs, faster startup times, and the ability to process mining waste dumps [16]. This development led to a quick rise in oxide porphyry ore as a major source of global copper production from virtually negligible to more than 20% by 2010.

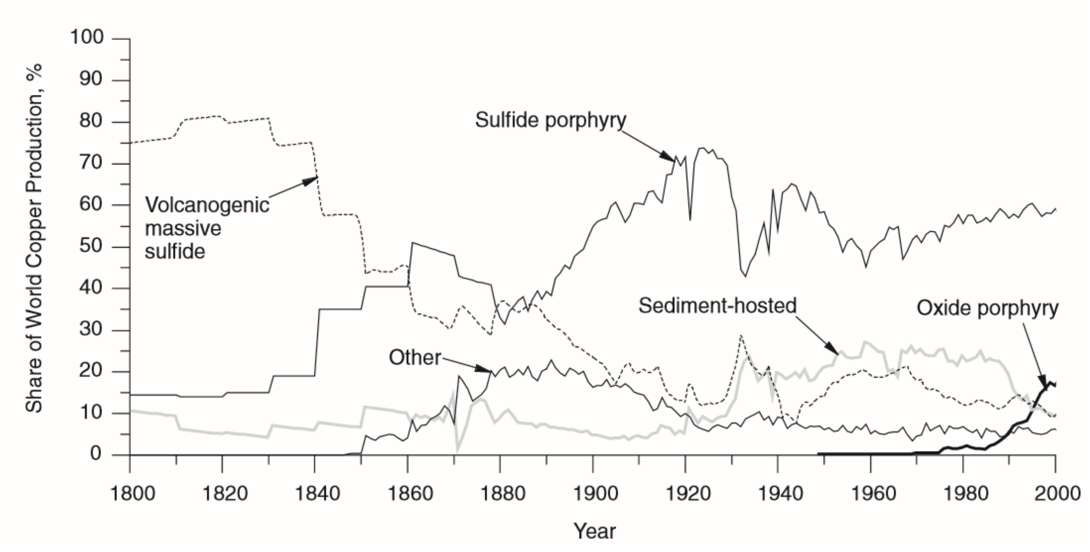


Figure 2: Percent of world production by major ore type from 1800 to 2000. "Other ore type" category includes any ore type not classified by the other four categories (e.g., magmatic, Ni-Cu, lode gold, and native copper). Of especial interest is the long term dominance of production from porphyry copper deposits over the last century.

Copper cathodes from the production stage of electro-winning are subsequently fabricated and manufactured into copper or copper alloy products used in a wide array of industrial sectors. In the electrical sector, copper are used for electric power transmission and distribution, telecommunication (trunk, feeder, or unshielded twisted pair drop cable), broadcasting and TV network (trunk, feeder, or coaxial cable), and streetscapes and traffic lights (grounding wire).[17] In the transportation sector, copper can be found in the electrical systems, brass hardware, and antifouling paint components of automobiles, aircraft, civil transport vessels, rail lines and metro systems, and railroad vehicles. [18] In the building and construction sector, copper is primarily used for residential, commercial, and industrial buildings' plumbing and electrical wiring purposes. Copper is also widely used in the industrial machinery, industrial equipment, personal electronics, household durables, and water distribution sectors. [19] Per capita copper stock is a function of the level of economic development and societal affluence. To provide the readers with a sense of magnitude, North America, OECD countries, and Western Europe have the highest per capita copper in-use stock ranging from 170 Kg/person to 230 Kg/person. Africa and Asia's per capita in-use stock is around 30-40 Kg/person, yet rapidly growing. [20]

After the products where copper resides are retired, copper is either landfilled, or recycled, or dissipated. Studies have shown that about 160 million tonnes of copper was extracted from the lithosphere and two anthropogenic stocks have formed, i.e., in use (70 million tonnes) and in landfills, tailings and slag reservoirs (85 million tonnes) in the 20th century in North America. Over the 100 year period, 40 million tonnes of copper was collected and recycled from post-consumer waste, 56 million tonnes in landfills or was lost through dissipation, and 29 million tonnes of copper in waste was produced in the form of tailings and slag and stored in waste reservoirs. Unless the copper stored in waste repositories can be economically extracted, that resource is lost to society. In addition, it is shown that there has been a significant upward trend in the rate at which copper is placed in landfills from post-consumer waste in the period 1940 (0.27 million tonnes) to 1999 (2.8 million tonnes), intensified by the increasing rate of electronic equipment use and correspondingly shorter residence times, coupled with the absence of an efficient collection and processing infrastructure for retired electronics. [21] Other regions with less environmental awareness might see even lower percentage of old scrap recycling.

The cradle to grave copper cycle can be summarized by the following flow chart.[22]

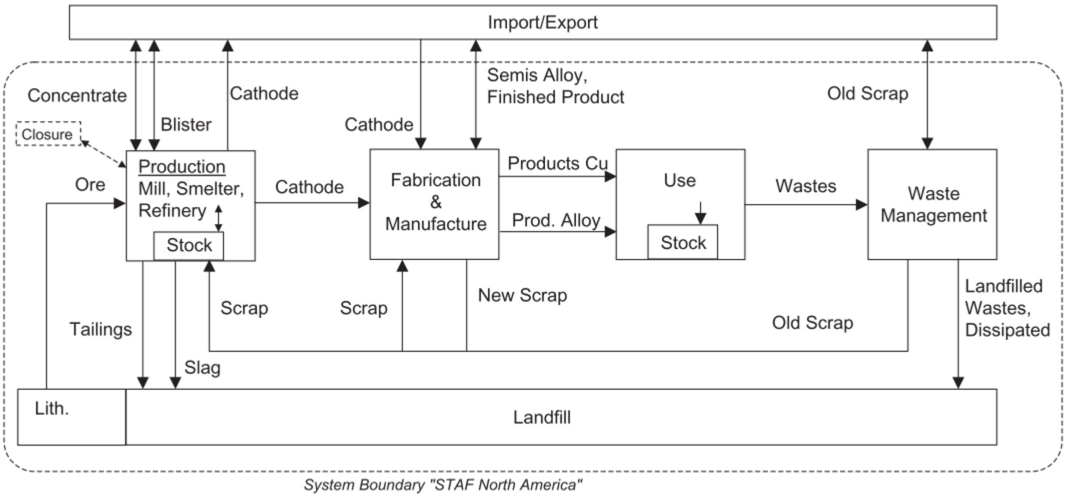


Figure 3: Materials flow diagram summarizing cradle to grave copper cycle.

Existing Literature:

Since our goal is to assess copper scarcity, we need to resort to the copper market where demand and supply interact to back out the amount of copper consumption for each time period going forward. One then compares the total consumption during the time period of interest with the copper reserve, and evaluate whether we run the risk of running out of copper any time soon. The copper market, or more broadly, commodities market analyses are typically approached from two perspectives. One is from the resource economist's, whereas the other is from industrial ecologists. We will introduce existing literatures in this order.

Not many individuals have started a whole new field, and Harold Hotelling is one of the selected few. Harold Hotelling started the entire field of resource economics with a paper named *Economics of Exhaustible Resources*. [23] In that paper, Hotelling also introduced the method called calculus of variation. Like many great works, the significance of his work was little appreciated until the 1970s where commodities prices experienced a huge surge and materials scarcity became a hot topic in the academia.

In his seminal paper, Hotelling laid out a model where a firm would decide the optimal production or extraction path of the resource to maximize the expected and discounted flow of profits subject to a known and finite stock of reserves of the non-renewable resource. The control variable was the amount of extraction, whereas the state variable was the reserve of the resource. One can derive the Euler equation of this model, and it establishes that under the optimal extraction path, the price-cost margin of the natural resource should increase over time at a rate equal to the interest rate. [24] Numerous literatures were published following Hotelling's lead. Empirical studies have mostly rejected Hotelling's hypothesis. [25]–[27] An MIT professor, Robert Pindyck, extended Hotelling's model by allowing firms to make exploration decisions where it can determine the amount of investments to be made for each period during its exploration process. Following this approach, Pindyck predicted that prices follow a U-shaped path. [28] Scholars also considered situations where the state variable such as demand and reserve size is stochastic and uncertain. [29], [30] Thus far, predictions from these models share a common limitation where the data most commonly used in applications consists of aggregate data on output and reserves at the country or firm level – mine level information is limited. These applications assume that the 'in-situ' depletion effects at the mine level can be aggregated to obtain similar depletion effects using aggregate industry data. However, in general, the necessary conditions for this "representative mine" model to work are very restrictive and they do not hold because the characteristics at the deposit level evolve over time as people rationally prefer to mine more economically attractive deposits first. With greatly improved computational power, scholars propose and estimate dynamic structural models of the operations of copper mines using a unique dataset with rich information at the mine level. For instance, Luengo employed a dataset with rich information at the mine level from 334 mines that account for more than 85% of the world production of copper during 1992-2010. They proposed and estimated a dynamic structural model that incorporates information regarding production delay, unit cost heterogeneity, and market concentration. Their estimates show that these extensions of the standard model contribute to explain the observed departures from Hotelling's rule. They also use the estimated model to study the short-run and long-run dynamics of prices and output under different types of changes in demand, costs, and policies. [31] Despite of the advance these authors have made, their model does not incorporate short term and long term price elasticity of demand which is essential in addressing material substitution questions, and granularity on the mine-level does not allow for evolution in technology, demographics, and mine-type.

Literature from the industrial ecologists’ side typically employs static or dynamic materials flow assessment to investigate the pattern of commodity flows and net addition to stock within a particular geographical boundary. [32] Material flow analyses can be used to characterize material and substance flows within a given physical boundary, and global, national or continental analyses can be consequently carried out. Material flow assessment frameworks have been defined and in some cases also incorporated into government policy frameworks. [33] The main objective of material flow assessment is to establish the flow patterns of materials and elemental substances in specific regions. The term material flow assessment is used when referring to specific substances such as copper and zinc. [34]–[36]It is an important and useful tool for identifying sources of hazardous substances that may potentially be released to the environment, for example, lead. It can be used to assess the long-term sustainability of how a set of substances is managed with respect to resource availability and environmental impact. [37] For example, it can identify spatial reservoirs of materials in use should those materials become scarce in the future. It can also be used to analyze consumption patterns of specific materials and the associated energy and environmental impacts that accompany those materials.

The field of materials flow assessment can be broken down into static materials flow assessment (SMFA) and dynamic materials flow assessment (DMFA) in terms of methodology. Static materials flow assessment looks at the stock and flow picture in a snapshot.[38] Dynamic materials flow assessment however, seeks to quantify past material flows, establish the material flow patterns, and apply the lifetimes of such material containing products to these patterns in order to track the temporal changes in the materials flows. [39] Put in another way, SMFA depicts the flow of a certain material during a particular time period, typically one year. DMFA, however, usually involve an assessment across the horizon of time for as short as several years to as long as over a hundred.

This thesis, given our duty to understand the future global consumption and scarcity risk associated with copper on a multi-decade basis, employs dynamic materials flow assessment. Dynamic materials flow assessment typically involves four steps in its analysis: Firstly, it requires a deliberate delineation of system boundary and product classification. Secondly, it requires the determination of the lifetime and its distribution of each commodity product group; thirdly, it projects a trend of scrap generated by each product group in the future. Lastly, result analysis is performed through a quantified material flow. [40] Prof. Thomas Graedel from Yale University is quite a prolific researcher that employed MFA to study various types of minerals. [41]–[47]

After surveying as many papers on this topic as we can, we have been able to abstract six groups of features that may serve to nicely categorize existing literatures:

	Physical Flow Based	Uni-market	Multi-Market
Analytical Based	Purely Materials Flow	Market Centric	Market Centric

	Based Deterministic	Deterministic	Deterministic Interaction Salient
Simulation Based	Purely Materials Flow Based Stochastic	Market Centric Stochastic	Market Centric Stochastic Interaction Salient

Table 1: The characteristics of physical flow based analytical & simulation based models, uni-market analytical & simulation based models, multi-market analytical & simulation based models.

We will first explore the vertical axis, and then discuss the horizontal axis.

The difference between analytical model and simulation model is that analytical model has a greater tendency of trying to obtain closed-form solutions with deterministic characteristics. Some analytical models focus on obtaining reduced form solutions and are therefore more prone to adopting a top-down approach due to tractability concerns. Hotelling's original work is a classic example of an analytical model. Time series analysis, such as autoregressive integrated moving average models (ARMIA) that investigates single price series, vector error-correction models (VECM) that focuses multi price series, and autoregressive distributed lag models (ARDL) that incorporates exogenous variables into price analytics are all reduced form assessments following the analytical vein. [48], [49] Analytical models require mathematical ingenuity, but to boil down complexity into closed-form solution necessarily means making simplifying assumptions. Despite of the elegance of analytical models, they sometimes suffer the Lucas Critique where structural shifts are not incorporated in a robust fashion. A simulation model, on the other hand, seeks to incorporate uncertainties in the model. It tends to focus more on aggregating stochastic agent level characteristics to arrive at macroscopic conclusions. It sometimes sacrifices tractability in exchange of numerical sophistication and reality imitation. One classic example for simulation based model is an agent based model where agents make decisions based on a certain distribution of utilities or preferences. In addition, simulation model results are obtained through running the model numerous times (sometimes as much hundreds of thousands of runs). A simulation model, while lacking elegance and closed form solutions, sometimes provides a more realistic picture with uncertainty embedded in any of its outputs. While it is true that stochasticity can be built into analytical models as well, simulation models distinguish themselves also by possessing a solid micro-foundation where "deep parameters" are deliberated and carefully implemented, and are thus more robust to structural shifts. In addition, a bottom-up approach typically associated with simulation models seems to be better equipped to capture systemic emergence behavior.

The horizontal axis is more straightforward. Depending on the purpose of one's analysis, the researcher might either choose to focus simply on the physical flow of materials (such as dynamic materials flow assessment without socioeconomic indicators such as mineral prices and GDP/capita built into supply and demand determination), or choose to look at one single market with explicit consideration of price as a feedback mechanism that alters behavioral patterns of both consumers and suppliers. Mineral substitution contingent on changing relative prices is also prevalent (i.e., copper vs aluminum in transmission, plastics vs copper in plumbing, and platinum vs palladium in catalytic converters), therefore sometimes one needs to look at two or even three markets to make sure such important substitution mechanism is endogenous. [50], [51]

We would like to introduce three of the most relevant articles that we believe have played instrumental roles in shaping our modeling strategy.

The first article we want to introduce is written by a group of researchers from Australia and published in 2014. We will revisit their work, particularly their results in the sixth chapter. These scholars have compiled a detailed list of all discovered copper reserves in the world, and have assigned them agent-level decision-making capability in terms of opening, closing and ramping up decisions. They have also conducted a century long analysis to investigate scarcity risk that we might face in the future. Last but not least, they conducted scenario analyses to investigate scarcity risk for various levels of reserve size. As we will see later, however, they work has not incorporated price mechanism, and has not allowed for explicit materials substitution as a result of changing commodity prices. Their demand model is a simple multiplication of logistic population growth and a constant per capita intensity of use growth rate.[5]

One serious endeavor in understanding materials substitution has been made in an article published in 2016 on the platinum group metals (PGMs). The scholars conducted an exhaustive literature research and a set of meta-analyses across various Euro standards on emission control, which is the direct driver for using PGMs in automobile catalytic converters. In a nutshell, substitution decisions are made by manufacturers who evaluate platinum, palladium, and rhodium’s respective catalytic performances and their individual market prices to make a balanced evaluation on the particular combination of these metals to optimize their performance while minimize cost. Although interesting, this methodology and its application is constrained due to lack of data about technical limitations for most materials and systems. Quantitative methods have to be developed to extract similar information from more transparent and readily available data sources such as commodity market prices or price elasticities of demand.[50]

Last but not least, one of the most advanced and recent paper has been published in 2015 by a group of researchers from Germany. They have developed a sophisticated copper market model that takes into account of delay in adjustment of supply and price mechanism, both treatments rarely synthetically combined in any prior literature. They first drew on the classical cobweb theorem to show how price volatility has been created because of delayed supply side responding to market prices. This is one of the few studies that had an explicit focus on price mechanism on realistic supplier decision-making processes. These scholars seamlessly combined physical material flow model which takes into account of copper inventory alteration and recycling, and market model which holistically considers GDP/capita, sectorial demand (building & construction, infrastructure, industrial, transport, consumer and electronic), and the entire process of capacity addition (from investment flow to project pooling to capacity building). Their forecast has also been done on a simulation basis. Despite of their work representing a major step towards a more involved and realistic understanding of the copper market dynamics, they have not broken down suppliers into a granular enough basis such that ore grade can have an impact on the overall cost structure of the mine independent of factor costs they have not treated material substitution on a sectorial and regional basis, they have mostly focused on short term (several years) price volatility rather than multi-decade depletion analysis or scarcity evaluation.[31]

Our work aims to integrate the strong points of these studies while eliminate their weaknesses through intelligent and complementary modeling components insertion.

Gerst, Hatayama, and Igarashi etc have based their model on using analytical equations in determining physical demand, and use those demand as a proxy for materials consumption. They do not explicitly consider materials substitution or socio-economic indicators in their model. Arseneau and etc model commodities market such as the oil market by resorting to physical flow based simulations. Their models do not have analytical solutions, and the outputs are in general the outcome of hundreds of thousands of runs. Xiarchos's two articles on using reduced form time series analysis, representative of analytical based uni-market model, rely on parsimonious equations that analyze correlation and autocorrelation among different variables and the lagged terms of the dependent variable itself. Ignaciuk's work on biomass and food production, and Zink's model on primary & secondary aluminum, representative of simulation uni-market models, adopt econometric structural equations and multiple runs to determine the statistical outcome of output parameters. As for analytical based multi-market analysis, we have introduced the Zhang paper already. Considine and Ramberg’s work also focus on fuel or material substitution, and used socio-economic indicator such as price in structuring the analytical model so one can vary the parameters in the model to examine their impact on outputs. Last but not least, Gloser-Chahoud spearheaded simulation based multi-market model in constructing agent based multi-market simulation models in analyzing the copper market, which we have introduced. Other scholars such as Auping and Pruyt incorporated system dynamics into multi-market simulation based model in accomplishing similar ends, namely injecting stochasticity and uncertainty into an intrinsically unpredictable system.

We categorize some of the literatures we have surveyed in the table below.

	Physical Flow Based	Uni-Market	Multi-Market
Analytical Based	Gerst et al., 2008; Hatayama et al, 2012; Igarashi et al, 2007, Saurat et al, 2009; Yamaguchi et al., 2006; Binder et al., 2006 [36], [37], [43], [52]–[54]	Ekvall & Andrae, 2006; I. M. Xiarchos et al., 2005; I. M. Xiarchos et al., 2006; Xin & Zhang 2014; Elshkaki & Van der Voet 2006; Aguirregabiria & Luengo, 2016 [31], [49], [55]–[58]	Zhang et al., 2016; Considine, 1989; Ramberg & Parsons, 2012; [50], [59], [60]
Simulation Based	Arseneau & Leduc, 2013; Blomberg & Soderholm, 2009; Bouman et al., 2000; Frees, 2008; Lin, 2004; Fernandez &	Ignaciuk, Vohringer, Ruijs, & van Ierland, 2006; E. Alonso et al., 2010; Zink et al., 2015; Jones et al., 2002; Kampmann & Sterman,	Pruyt & Kwakkel, 2012; Kwakkel, Auping and Pruyt 2014; Gloser – Chahoud, et al. 2017 [51], [76], [77]

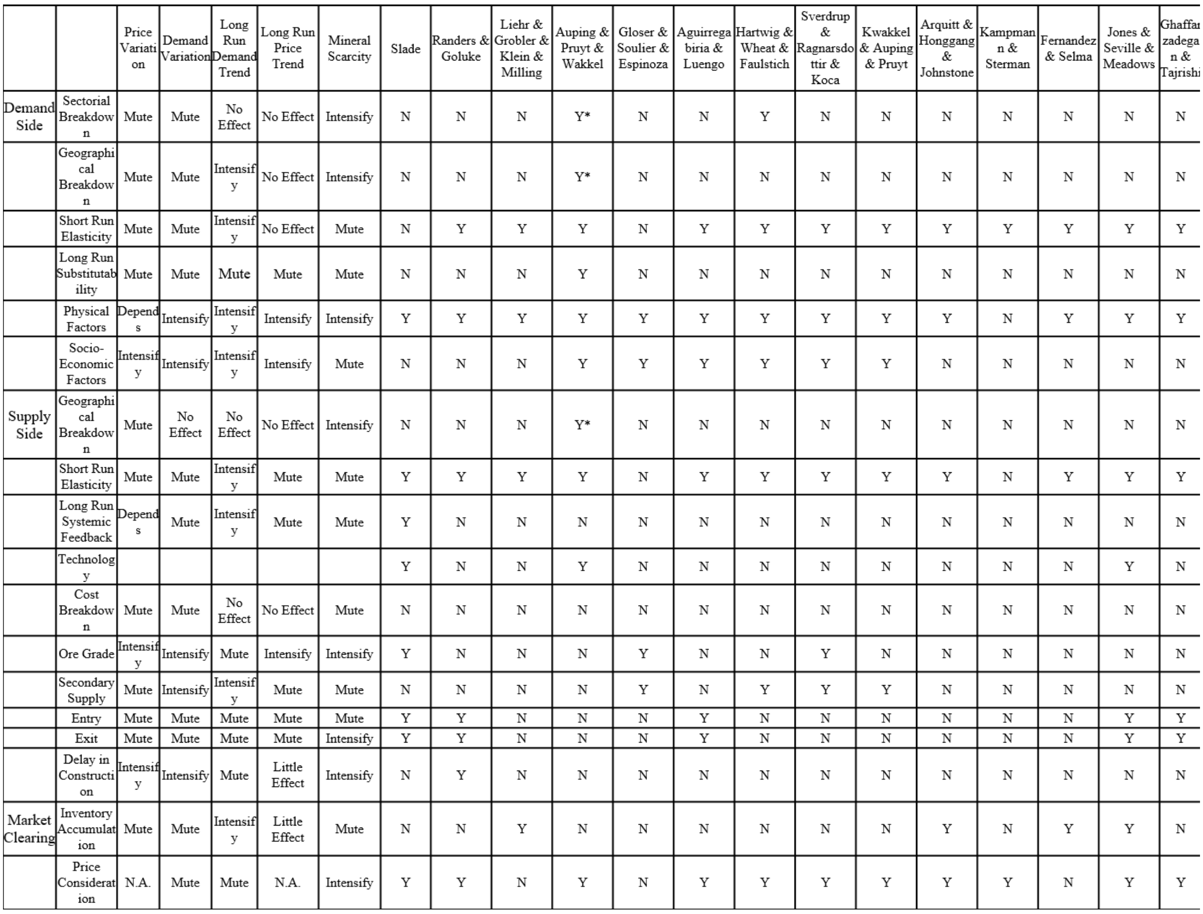
	Selma, 2004; Gloser, Soulier & Espinoza, 2013; Liehr, Grobler, Klein & Milling, 2001; [61]–[68]	2014; Arquitt et al., 2005; Sverdrup et al., 2014 [69]–[75]	

Table 2: Categorization of existing literatures based on whether they are physical based, uni-market or multi-market on the horizontal axis, and whether they are analytical based or simulation based on the vertical axis.

Our model locates in the lower right corner. We prefer simulation based models for the reason that despite of the fact that they frequently give up tractability and elegant closed form solutions, they allow the modeler to incorporate detailed granular level information. We are interested in constructing a model where individual agents from both the supply side and the demand side can evolve and adapt to their environment. Their self-organizing and adaptive behavior, in aggregate, may produce emergent properties that cannot be captured by looking at reduced form solutions. In addition, we understand that the copper market, like any complex systems, is sensitive to initial conditions. Results produced based on 10,000 runs enabled by our simulation model blunt the sensitivity of the model to its initial conditions. We prefer market models to physical models, for we believe price serves as a critically indispensable feedback signal that inform suppliers and consumers decisions in future periods. We prefer market model that consider two or more commodities to uni-market models for we know that substitution is an important yet undervalued issue in analyzing the copper market. Models that ignore substitution tend to overestimate demand in the long term, because as price increases, substitution takes away demand from one metal to another. To conclude, we believe a multi-market simulation based model can adequately address the issue of agent-level spontaneous order forming, can capture feedback mechanism and emergent properties, is more robust against systemic sensitivity regarding initial conditions, and provides us with a tool to take into account of materials substitution. In other words, we believe a multi-market simulation-based model, at least in this particular instance and for this particular application, is the best choice for complex system modeling as it enables one to better understand the most crucial and relevant issues of the copper market.

Filling Out the Gap:

To identify and fill out the academic gap, we first deliberate the most critical aspects that should be addressed by the model, and then look back to the literature to see which aspects have been adequately addressed and which aspects are least visited or ignored. We then assess whether such omission causes tangible biases in the modeling results. Specifically, the modeling results of our greatest interests are short term price variation and demand variation, long term price and demand trend, and resulting influences on the assessment of mineral scarcity. If the answer is yes, we attempt to mitigate such omission by incorporating these aspects in our own model in addition to all other aspects that we deem as important for modeling purposes. We provide a (non-exhaustive) list of existing literature regarding which aspects have or have not been addressed and how we hypothesize the omission will influence modeling outcomes as follows:



		Price Variation	Demand Variation	Long Run Demand Trend	Long Run Price Trend	Mineral Scarcity	Slade	Randers & Goluke	Liehr & Grobler & Klein & Milling	Auping & Pruyt & Wakkel	Gloser & Soulier & Espinoza	Aguirregabiria & Luengo	Hartwig & Wheat & Faulstich	Sverdrup & Ragnarsdottir & Koca	Kwakkel & Auping & Pruyt	Arquitt & Honggang & Johnstone	Kampmann & Sterman	Fernandez & Selma	Jones & Seville & Meadows	Ghaffarzadegan & Tajrishi
Demand Side	Sectorial Breakdown	Mute	Mute	No Effect	No Effect	Intensify	N	N	N	Y*	N	N	Y	N	N	N	N	N	N	N
	Geographical Breakdown	Mute	Mute	Intensify	No Effect	Intensify	N	N	N	Y*	N	N	N	N	N	N	N	N	N	N
	Short Run Elasticity	Mute	Mute	Intensify	No Effect	Mute	N	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	Y	Y	Y
	Long Run Substitutability	Mute	Mute	Mute	Mute	Mute	N	N	N	Y	N	N	N	N	N	N	N	N	N	N
	Physical Factors	Depends	Intensify	Intensify	Intensify	Intensify	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	Y	Y	Y
	Socio-Economic Factors	Intensify	Intensify	Intensify	Intensify	Mute	N	N	N	Y	Y	Y	Y	Y	Y	N	N	N	N	N
Supply Side	Geographical Breakdown	Mute	No Effect	No Effect	No Effect	Intensify	N	N	N	Y*	N	N	N	N	N	N	N	N	N	N
	Short Run Elasticity	Mute	Mute	Intensify	Mute	Mute	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	N	Y	Y	Y
	Long Run Systemic Feedback	Depends	Mute	Intensify	Mute	Mute	Y	N	N	N	N	N	N	N	N	N	N	N	N	N
	Technology						Y	N	N	Y	N	N	N	N	N	N	N	N	Y	N
	Cost Breakdown	Mute	Mute	No Effect	No Effect	Mute	N	N	N	N	N	N	N	N	N	N	N	N	N	N
	Ore Grade	Intensify	Intensify	Mute	Intensify	Intensify	Y	N	N	N	Y	N	N	Y	N	N	N	N	N	N
	Secondary Supply	Mute	Intensify	Intensify	Mute	Mute	N	N	N	N	Y	N	Y	Y	Y	N	N	N	N	N
	Entry	Mute	Mute	Mute	Mute	Mute	Y	Y	N	N	N	Y	N	N	N	N	N	N	Y	Y
	Exit	Mute	Mute	Mute	Mute	Intensify	Y	Y	N	N	N	Y	N	N	N	N	N	N	Y	Y
	Delay in Construction	Intensify	Intensify	Mute	Little Effect	Intensify	N	Y	N	N	N	N	N	N	N	N	N	N	N	N
Market Clearing	Inventory Accumulation	Mute	Mute	Intensify	Little Effect	Mute	N	N	Y	N	N	N	N	N	N	Y	N	Y	Y	N
	Price Consideration	N.A.	Mute	Mute	N.A.	Intensify	Y	Y	N	Y	N	Y	Y	Y	Y	Y	Y	N	Y	Y

Table 3: The aspects of the structural commodity market that the surveyed literature has addressed or left unaddressed. We identify the aspects least addressed by existing papers that play a role in leading to potentially biased estimations.

We summarize the four most frequently omitted yet important aspects of the models in the following table (Note the impacts are our hypotheses derived through deduction and to be tested):

	Price Variation	**Quantity Variation**	**Price Trend**	**Quantity Trend**	**Scarcity**
Long Run Substitution	NA	NA	Over Estimation	Over Estimation	Over Estimation
Rate of Depletion	NA	NA	Under Estimation	Over Estimation	Over Estimation

Long Run Systemic Feedback on Suppliers	Under Estimation	Under Estimation	Under Estimation	Under Estimation	Under Estimation
Cost Breakdown	Influenced	Influenced	Influenced	Influenced	Influenced

Table 4: This is a summary of how are model output and scarcity assessments likely to be influenced by not properly addressing these key features in the model.

Our research aims to understand and evaluate copper scarcity risks by constructing a structural commodity market model that has all the critical components that are essential in capturing the functional mechanism of this complex system. In the next chapter, we will introduce the model in detail. In the remaining part of this chapter, we will discuss the four key features in the table above as they are the distinguishing features of our model.

In the short run, a consumer can refrain from buying a product because its price surges in the short term. In anticipation of price series mean reversion, the consumer patiently waits and makes a purchase only after the price returns to its normal level. However, when structural reasons cause prolonged price excursion (e.g., Russian ban on precious metals export, OPEC’s ban on oil export, shale revolution leading to a prolonged natural gas glut), the consumer, typically a manufacturer that uses that particular mineral source as factor input to produce semi-finished or finished goods, will attempt to either become more thrifty or turn away and use another type of substitutable material that is more economical. In the short run, his manufacturing tools and equipment are tuned to process the original material; his production line, workers, and tools all need adjustments to be able to successfully process the new material. However, once the adjustment is done, it becomes an impediment to switch back to the original material even if its price returns to normal. This somewhat irreversible switch of materials in the process of manufacturing as a result of prolonged relative price shift is called long term substitution. Therefore, based on this definition, any model that considers only short term elasticity of demand is likely to leave out the irreversible component of demand migration, leading to overestimation of both the original material's price and consumption. The overestimation of consumption, in turn, leads to an exaggeration of mineral scarcity caused by the falsely assumed lack of long term consumer response to price excursions.

Ore grade declines over time – just look at the stratospheric ore grade of the Copper Queen Mine of the early 20th century. As ore grade declines, unit production cost of the original material increases as a result. The increase in cost leads to both short term and long term materials substitution, resulting in lower consumption increments. To leave ore grade out of consideration will tend to lead to overoptimistic projection of unit production cost which is passed on to consumers reflected in unit price. Essentially, it leads to underestimation of price growth rate. This underestimation of price growth leads to an overestimation of consumption growth rate, resulting in an overestimation of mineral scarcity, another way of saying how soon we are running out of this mineral. Of course, ore grade decline leads to higher price, which has an impact on both short term and long term demand behavior.

Suppliers respond to incentives, but their responses take time. We know that balancing feedback loop that is delayed will create oscillations. Suppliers take time to work down their inventory, to do their prospect work, to conduct corporate budgeting and feasibility assessment, to construct their plants to ramp up their production, and to close down their plants when the market price of the commodity is depressed. If one does not take these delays into account, the modeling results will tend to be biased.

Cost breakdown allows us to examine geographical related or ore type related changes over time. For example, the cost structure of processing oxide porphyry ore is considerably different from that of sulfide porphyry ore. With cost structure as a controllable lever, one can better assess the impact of changing ore grade, ore type, or geopolitical/socio-economic factors on the modeling outputs.

We will rigorously describe the details of our model in the next chapter. We simply aim to provide a general explanation of how our model is capable of filling in the gap. To address long run substitution issues, we first break down aggregate demand into its volume driver and its intensity of use, and then implement an autoregressive distributed lag model and recast it into its error correction form to tease out the impact of both short run and long run substitution. To address the rate of depletion issue, we gathered data on all operating mines, known and undiscovered deposits to allow us to adjust the size of the pool thereby manipulating as well as tracking the ore grade time series. We also collected information on all the component costs of production for all operating mines over the world. This information allows us to statistically impute cost structures on all future mines, thus providing us with a lever to assess cost breakdown's impact on modeling outcomes. For each mine, we assign a time frame for prospecting & planning, ramp-up, and ramp-down to allow for long term systemic feedback to sink in on our suppliers. Each individual mine makes rational decisions based on prevailing prices, their respective ore grade and cost structure, thus enabling the capture of aggregate emergent phenomenon when observing macroscopic outputs. Before we take a more in-depth look into our model from a logically holistic and mathematically rigorous fashion in the following chapter, we would like to clarify a pivotally important concept – complex systems, along with some particularly relevant technical phrases we have used in this chapter to make sure that our readers understand perfectly the modeling framework we will propose.

Chapter Four: Defining Complex Systems

When your author was in college, his first encounter with “complex systems” was reading a *Science* paper published in 1999 called Complexity in Chemistry. In this paper, George Whitesides and Rustem Ismagilov gave an informal definition to a complex system as “a system that is complicated by some subjective judgment, and is not amenable to exact description, analytical or otherwise”. [78]

Your author was immediately intrigued by the concept of “complex systems”, for he simply does not buy this *Science* paper’s definition of it.

The success of Newton’s laws enabled physicists to solve many problems and is the bedrock of the entire modern science. However, the moment Sir Issac Newton laid out his three timeless principles of mechanics, the genie of chaos is out of the box. According to Newton’s laws, if one perfectly elastic sphere strikes two touching perfectly elastic spheres simultaneously, it is impossible to predict where the three spheres will go. Although one might argue in practice the first sphere will collide with one of the two stationary spheres first, this explanation is not sufficient to mathematically predict the outcome under simultaneity.

A system consists of three spheres is hardly complicated by any standard, yet it is complex enough that any viable analytical solution won’t emerge until a man called Henri Poincare was born in 1854.

In a universe containing just two gravitating bodies, there is no difficulty in describing their orbits. As Newton showed, they move around one another in a completely regular and periodic way. Once we add another body, the motion becomes too complicated to be analytically calculated. Henri Poincare tried to simply the problem by defining a “restricted three body problem” where the third body is so much smaller than the other two that, for the purposes of the calculations, its gravitational effect on the lager bodies can be ignored (referred to as a “dust particle”). It took him 200 pages to solve this problem in 1889. In addition, Poincare has made a mistake in his mathematical deduction, which he has to revise at a later stage. And don’t forget, this is the solution to the “restricted three body problem”. In addition to the “dust particle” assumption, Poincare further assumed that he was to look only at a small part of the appropriate phase space which represents a surface through which the trajectory being investigated had to pass. The three body problem is even more complicated. Hence, why is a three-body system not a complex system?

After your author enrolled in the Engineering System PhD program at MIT, he spent hours to peruse almost every single paper written on the general topic of complex systems, ranging from biological to ecological to political to economic to physical systems. He realizes that analytical difficulty is not the sole determinant of a complex system in an academic sense. The mathematical convolution of the three-body problem does not make this system a complex one. Nevertheless, he believes the conflation of colloquial “complexity” and academic “complexity” might be misleading. Therefore, before we explore the copper system, your author hope to collate the common themes regarding the definition of a complex system based on the literature research that he has duly performed on this particular topic. We will first list the extant definitions of a complex system, and distill the most significant attributes thereafter.

In the same *Science* paper mentioned above, the authors gave another set of definition of a complex system as “one whose evolution is very sensitive to initial conditions or to small perturbations, one in which the number of independent interacting components is large, or one in which there are multiple pathways by which the system can evolve.”[78]

In another *Science* paper published in the same year named “Complexity, Pattern, and Evolutionary Trade-offs in animal aggregation”, Julia Parrish and Leah Edelstein-Keshet maintains that “complexity theory indicates that large populations of units can self-organize into aggregations that generate pattern, store information, and engage in collective decision-making. [79]

In yet another influential *Science* paper named “Mathematics and Complex Systems” published in 2007, Richard Foote lays out his definition of a complex system: In recent years the scientific community has coined the rubric “complex system” to describe phenomena, structure, aggregates, organisms, or problems that share some common theme: (i) They are inherently complicated or intricate; (2) they are rarely completely deterministic; (iii) mathematical models of the system are usually complex and involve non-linear, ill-posed, or chaotic behavior; (iv) the systems are predisposed to unexpected outcomes, the so called emergent behavior. [80]

Last but not least, in yet another *Science* paper termed “Complexity and the Economy”, W. Brian Arthur argues that “common to all studies on complexity are systems with multiple elements adapting or reacting to the pattern these elements create”. [81]

These are four characteristic complex systems definitions. Your author has read numerous other articles on this topic and among all of them, there are four most salient features of any complex system. These are: 1). Sensitivity to initial condition; 2). Feedback; 3). Spontaneous order; 4). Emergence. In the meantime, these five attributes are inextricably correlated among themselves. As these four important attributes of complex systems permeate throughout this dissertation, it is perhaps fitting that we spend some time to explain them in finer detail.

1. Sensitivity to Initial Condition

   This attribute is frequently correlated with another attribute of a great many complex systems – nonlinearity. If I make a small mistake in measuring or estimating in a linear system, this will carry through and lead to a small error by the end. But a nonlinear system will magnify the error, and lead to humongous discrepancy by the end. The same Poincare realized this point when he was analyzing the “restricted three-body problem”. In 1908, he wrote in his *science et method*, that “a very small cause that escapes our notice determines a considerable effect that we cannot fail to see, and then we say that the effect is due to chance. If we knew exactly the laws of nature and situation of the universe at the initial moment, we could predict exactly the situation of that same universe at a succeeding moment. [But] It happens that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon." Remarkably, Poincare presciently related this phenomenon with weather forecasting. [82]

40 years later, an MIT meteorologist, Edward Lorenz, discovered this phenomenon when he tried to use a set of 12 nonlinear differential equations to model the atmosphere. He realized the model was so sensitive to the initial conditions that a one quarter of one tenth of 1 per cent difference will make two runs diverge completely from one another after a short time. He first announced his results in a fairly low-key way at a scientific meeting held in Tokyo in 1960, and formally published a paper termed "Deterministic Non-periodic Flow" in 1963 in the Journal of the Atmospheric Sciences, and with it, the foundation of chaos theory. Lorenz argues that "two states differing by imperceptible amounts may eventually evolve into two considerably difference states… If, then, there is any error whatever in observing the present state – and in any real system such errors seem inevitable – an acceptable prediction of an instantaneous state in the distant future may well be impossible… In view of the inevitable inaccuracy and incompleteness of weather observations, precise very-long-range forecasting would seem to be nonexistent." [83] These

initial discoveries and theorizations led Lorenz to postulate “the butterfly effect” after the title of a paper that Lorenz presented to a meeting in Washington DC in 1972 – “Does the Flap of a Butterfly’s Wings in Brazil Set off a Tornado in Texas?”

Due to potentially lurking nonlinearities and systemic sensitivity due to varying initial conditions, we cannot rely on any single run of our model. As we will see in the modeling part, we need to simulate our models for a statistically significant number of runs before we can state any conclusions with reasonable confidence assisted by the “Law of Large Numbers”.

2. Feedback

When your author first started the program, he has a great focus on stocks of any system that he analyzes. His adviser, Randolph Kirchain, told him that flows are more important than stocks when analyzing a system. He has increasingly discovered the truthfulness of that statement as time goes by. Donella Meadows, the author of Limits to Growth and perhaps one of the most notable disciples of Jay Forrester who is the founder of System Dynamics and a long-time MIT professor, perhaps gives the best explanations regarding the significance of flows: “people monitor stocks constantly and make decisions and take actions designed to raise or lower stocks or to keep them within acceptable ranges. Those decisions add up to the ebbs and flows, successes and problems, of all sorts of systems.

Systems thinkers see the world as a collection of stocks along with the mechanisms for regulating the levels in the stocks by manipulating flows. That meant system thinkers see the world as a collection of “feedback processes”. [84] When a stock grows by leaps and bounds or declines swiftly or is held within a certain range no matter what else is going on around it, it is likely that there is a control mechanism at work. In other words, if you see a behavior that persists over time, there is likely a mechanism creating that consistent behavior. That mechanism operates through a feedback loop. It is the consistent behavior pattern over a long period of time that is first hint of the existence of a feedback loop. The importance of feedback loops cannot be overstated, and they are one of the most important mechanisms that operate in any real life systems. In Prof. Jay Forrester’s own words, “systems of information-feedback control are fundamental to all life and human endeavor, from the slow pace of biological evolution to the launching of the latest space satellite… Everything we do as individuals, as an industry, or as a society are done in the context of an information feedback system.”[85]

There are broadly two types of feedback loops – reinforcing feedback loop, and stabilizing feedback loop. A reinforcing feedback loop happens when, for instance, economies of scale is at play. A firm with massive scale of production is able to spread their fixed cost on capital expenditure to more tonnages of production, raising operating margin, accumulating more capital. It can utilize the additional capital to further expand its production capacity, further lowering per unit fixed cost, raising operating margin, and becoming even more competitive in the market place. An easily observable balancing feedback loop regulates the equilibrium level of the overall market. Surging demand coupled with low demand elasticity quickly translates into higher commodity prices, increasingly profitability of the copper firms. With more retained earnings, the copper firms can expand their production capacity and capture more market share by further capital investment. Seeing high profitability, individuals or corporations from other sectors are incentivized to enter the copper market, further increasing copper supply. When the increase of supply supersedes the increase of demand, price is depressed; profitability drops, firms in the industry ramp down production, and firms that want to enter the industry are deterred. A new cycle is initiated as demand increments exceed supply increments and price is elevated again. Because of budgeting and planning, construction, and production ramp-up/ramp-down all take time, some taking as long as five to seven years, delay is introduced into the balancing feedback loop. As we know, delay in balancing feedback loop causes oscillation. Therefore, we see high price volatilities as an important information-feedback mechanism critical to any modeling efforts, particularly for capital intensive or high fixed cost industries such as commodities, airline, sawmill, and shipment.

[32], [68], [73], [86]

3. Spontaneous Order

It is a fundamental idea in complex systems research that of order in a system's behavior that arises from the aggregation of a large number of uncoordinated interactions between elements. However, it is far from easy to say what order is. Notions that are related include symmetry, organization, periodicity, determinism, and pattern. [87] One prominent Santa Fe Institute scholar Melanie Mitchell argues that spontaneous order through self-organizing is one of the two most important attributes of any complex adaptive systems, the other attribute being emergence, which we will discuss shortly. She further argues that the core question of complexity as a science is the investigation and analysis of why emergence and self-organization transpire. [88]

Individual agents react to the environment around them, in the same process modifying and redefining this environment. Micro-level agent adaptability contributes to the formation of spontaneous order. The aggregation of such behavior *emerges* towards an equilibrium characterized by spontaneous order, as in the case of our copper model. Each agent, where agent can be atoms, individuals, or even nation-states, behave based on a set of predefined rules, and their uncoordinated interactions converge to a local optimum whose outward representation is a spontaneous order. For instance, in the copper model that we will visit, each firm looks at the prevailing and trailing copper price, makes a projection of future prices, evaluate its cost structure, and calculate the profitability of the project based on net present value, payback period, and internal rate of return. Their collective behavior constitute the supply side, which interacts with aggregate demand arrived at in a similar fashion, and exhibits spontaneous order as price and quantity are codetermined in each period. Firm-level decision is based on the environment (namely overall production capacity and market price), and at the same time shapes the price level of the next period through firm-level decision of the current period.

4. Emergence

Phil Anderson's short 1972 *Science* article, "More is Different: Broken Symmetry and the Hierarchical Structure of Science", is perhaps one of the classics in the complexity literature. This article probably reflects the frustration of doing solid-state physics in the 1950's and the 1960's, during the heyday of experimental and theoretical breakthroughs in the study of elementary particles. The elementary particle physicists were wont to describe their own work as "fundamental physics", and at least some of them liked to use the pun "squalid state physics" to describe what their underprivileged solid-state colleagues were doing. [89] Phil Anderson argues that the constructionist hypothesis "breaks down when confronted with the twin difficulties of scale and complexity". At each new “level of complexity”, new kinds of properties arise, and research into these properties is “as fundamental as any other”. That is, the “fundamental laws” of elementary particle physics apply to all matter, but particular configurations of matter require additional laws that are not deductive consequences of the “fundamental laws”. Phil argues that a “level of complexity” is represented by an order sequence – in his terms, a hierarchy – of what he calls “sciences”: “elementary particle physics, solid state or many-body physics, chemistry, molecular biology, cell biology,…, physiology, psychology, social sciences”. Phil’s statement, in my view, is largely ontological rather than epistemological. What he wants to emphasize that the “elementary entities” in each level of this hierarchy “obey the laws of” the science preceding it in the list. “But this hierarchy does not imply that Science [n+1] is ‘just applied [n].’ At each stage, entirely new laws, concepts and generalizations are necessary… Psychology is not applied biology, nor is biology applied chemistry.”[90]

Your author agrees with Phil that because of “emergence”, each level of complexity has to be addressed, at least initially, on their respective level. The repudiation of fundamental physicists’ claim of their work’s unparalleled prestige is to jump across levels of complexity, and see if any meaningful interpretation can be drawn. Once one does this, Phil’s claim becomes almost transparently obvious. Another seminal paper on complexity hierarchy is written by one of your author’s few heroes, Herbert Simon. In “the Architecture of Complexity”, Simon argues stability of emergence relies on lower level interactions. Nevertheless, to completely understand higher level attributes, for instance, international relations, a detailed understanding of molecular movement whose vibration frequency is orders of magnitude higher is unnecessary. Simon, however, does acknowledge the importance of perhaps looking down one or two levels in the hierarchy to better understand higher constructs, as is evidenced by his famed “watchmaker” example, where the watchmaker who designs the watch based on subassemblies is much less prone to lose a big chunk of his work during interruptions, and hence takes on average a fraction of the time it would take if such lower-level subassembly structures are absent. Simon reaffirmed his original view in another lecture he delivered after more than 30 years of the initial publication. [91], [92]

Simon’s “look below one level of complexity hierarchy” is echoed by another hero of your author – Robert Lucas. Both Simon and Lucas won Nobel Prize in Economics, with the former also winning the Turing Award. Lucas believes that “look below one level of complexity hierarchy” serves to capture important emergent economic phenomenon. His famous Lucas Critique is a testable empirical hypothesis. When policy maker’s behavior and private agent’s prediction of the future based on their anticipation of policy maker’s behavior coincide, the forecast of private agent will conformably change as the policy of the policy maker changes. Therefore Lucas argues that “if policy changes occur as fully discussed and understood changes in rules, there is some hope that the resulting structural changes can be forecasted on the basis of estimation of past data.”[93] Note how his statement has potentially influence central bank’s behavior over the past four decades. To state it in a more widely accepted economics principle: macro forecast lacks robustness unless it has a solid micro foundation. The micro foundation should incorporate what Lucas named as “deep parameters” which governs individual behavior, such as preferences, technologies, and resource constraints. [61]

In other words, rationality on an agent level can possibly lead to aggregate level irrationality. For instance, individually profit maximizing agents, under a broader level financial panic, rushes to their local banks to withdraw their capital will result in a bank-run. The insolvency of the bank leads to agent level capital loss. Therefore, individually rational decision, namely to get back his/her capital as soon as possible, leads to a collective paralysis where everyone loses out. We are salient of the Lucas Critique, and therefore, instead of directly conducting reduced form analysis such as time series assessment on historical prices or quantities, opt for a micro-founded structural model where “deep parameters” are carefully implemented in hope of capturing any important emergent systemic properties.

This concludes our very brief touch on the topic of complex systems in the context of copper market modeling. As Melanie Mitchell comments, complex systems studies as a science is still in its infancy, and despite of countless incredibly intelligent individuals' endeavors into this fascinating field making numerous notable contributions that have vastly advanced our intellectual frontier, we are still, in her words, "waiting for our Carnot".

Complex systems studies, in abstract, are powerful methodological tools that can be broadly applied to many disparate disciplines. It is a way of thinking, a way of problem solving.

Complex systems theory lies in the heart of our entire modeling construct. The copper market is in essence a complex system that embraces all four aforementioned critical features. Price mechanism is arguably one of the most important feedback chain in guiding supply and demand behavior to ultimately reach systems equilibrium. Individual supplier make calculated anticipatory decisions without colluding with each other, but spontaneous order is formed as collective decisions lead to a consensus supply curve based on supplier's individual cost structure and ore grade. The interaction between supply and demand, the possibility of short term & long term substitution, altogether lead to an emergence of price and consumption trajectory uniquely representative of the underlying dynamics. Each price and consumption trajectory is susceptible to perturbation in initial condition and the randomness in deposit's selection into incentive mines, therefore Monte Carlo simulation is required to extract essential statistics to describe the pertinent properties of each scenario with its specified assumptions. In the next chapter, we provide a dissected, detailed view regarding the specifics of our model.

Chapter Five: Understanding the Copper Market through Modeling

Any market is composed of a demand side and a supply side, and most of its interesting outcomes are products of the interaction between these two; that is interaction between consumers and suppliers.

In this chapter, we will introduce how we have constructed our bottom-up partial equilibrium model of copper. We will first describe the details of model construction for the demand side, then discuss the nuts and bolts of the supply side, and ultimately delineate how the demand side and supply side interact to reach a dynamic equilibrium.

Demand Side:

To paint a parsimonious demand side picture is by no means trivial. There are, generally speaking, five types of methods in modeling demand, differed by their respective levels of complexity. The first and simplest method is to model demand implicitly. For example, David Ramberg and John Parsons investigated the weak tie between natural gas and oil prices by modeling demand into price, or to put it another way, by having demand related parameters such as GDP/capita in price evolution.[60] The benefit of the method is its utmost simplicity, but it does not allow the modeler to go anywhere beyond a price level consideration of demand. One can go one step further by modeling demand explicitly, without distinguishing between particular regions or industrial sectors. Most of MFA studies fall into this category. For instance, Spatari et.al modeled the contemporary European copper cycle behavior using such a framework.[22] The model is more data intensive and requires more input, but it allows the researcher to track materials flow in a more nuanced and detailed fashion. One can go another step further by breaking down aggregate demand into its regional or/and sectorial components. For instance, Guo Xueyi and Song Yu conducted a substance flow analysis of copper in China by breaking down aggregate demand into building wire, copper tube, vehicle wire, magnet wire, telecom wire, old scrap, and etc.[34] This increases the data intensive requirement of the model, but is one step closer to a granular analysis of the copper demand market through detailed knowledge on its various venues of consumption and application. One natural extension of this method is to go one step further by incorporating socio-economic indicators such as price GDP/capita and population into the demand side equations. This allows to socio-economic feedback to be incorporated. The article we have introduced written by Gloser is of this genre.[51] Last but not least, one can do the analysis from an agent-based demand view as we have seen in the paper written by Luengo et.al.[31] The last method, if one wants to execute it well, requires the most information on a micro-level. Preferably one needs to understand each agent's utility function and preference structure, and one has to be able to model their demand characteristics on a regional and sectorial basis. The article written by Luengo et.al simplifies the demand characteristics and assumes largely homogenous features of agents.

For our analysis, we prefer to reside in the fourth level of complexity, namely regional & sectorial demand breakdown with a holistic consideration of socio-economic indicators. We believe this is the optimal balance given our lack of information on a consumer-specific level and our needs of having a reasonably sophisticated construct that takes into account of growth and price elasticity of demand both in the short term and in the long run. This level of complexity satisfies all the research goals that we have previously stated, and at the same time is feasible given the data and information that we have at hand. We describe our model as follows.

Firstly, aggregate demand is composed of regional demand from many countries. The most granular level of demand breakdown would be to individual consumers. Nevertheless, choosing the level of granularity is subject to two constraints: the availability of data, and the necessity of a particular granular level. We have chosen to stay at region specific sectorial level largely due to unavailability of more granular information. This particular level of breakdown already enables us to separate different regions with different demographics and growth prospects, and different sectors residing in those regions which have different price elasticities of demand characteristics. Therefore we believe this level of breakdown, given information constraints, is sufficient. To retain modeling simplicity in a sense of computational tractability while at the same time ensure granularity, we categorize regions into two broad camps: developing countries including China, Africa, and ROW; developed countries including US, Japan, and Europe. Secondly, regional demand can be further broken down into different industrial sectors. We identify five sectors, namely Electrical (electrical power generation, electric power transmission and distribution, telecommunication, broadcasting & TV network, streetscapes and traffic lights, etc), Transportation (automobiles, aircraft, civil transport vessels, railroad vehicles, etc), Building and Construction (residential, commercial, and industrial buildings), Industrial Machinery (commercial equipment and industrial machinery), and Others (including consumer durables and other copper related applications). Most studies in this space stay at sectorial level. Different from these studies, we believe a further breakdown of this sectorial level demand into its volume driver and intensity of use is illustrative, informative, and necessary if one is to explore materials substitution potential. Specifically, in order to infer an elasticity for each sector and each region, we need to separate the effect from demographic growth and affluence on total material demand. Such effect is incorporated in volume driver growth, whereas elasticity is embedded in intensity of use per unit. We therefore decide to separately analyze these two aspects so we can investigate both the effect of demographics & affluence on volume driver growth and also the price elasticity of demand effect on intensity of use. To illustrate the concept, take automobiles in the transportation sector as an example. Each automobile requires a certain amount of copper in it for electrical wiring, and the total copper demand for all automobiles in the transportation sector is the number of automobiles (volume driver) times the average amount of copper (intensity of use) in each automobile. One important factor that influences volume driver is affluence. As people have more disposable income, they purchase vehicles instead of taking subway to attain more efficiency and flexibility. Intensity of use, however, is a different story. Manufacturers, within their technical constraints, have the incentive to use raw materials with the cheapest cost for the same functionality. Even if volume driver increases as GDP/capita increases, intensity of use for a particular type of material in a unit might *decrease* due to a rapid increase of price. We can describe the systemic hierarchy in the following figure:

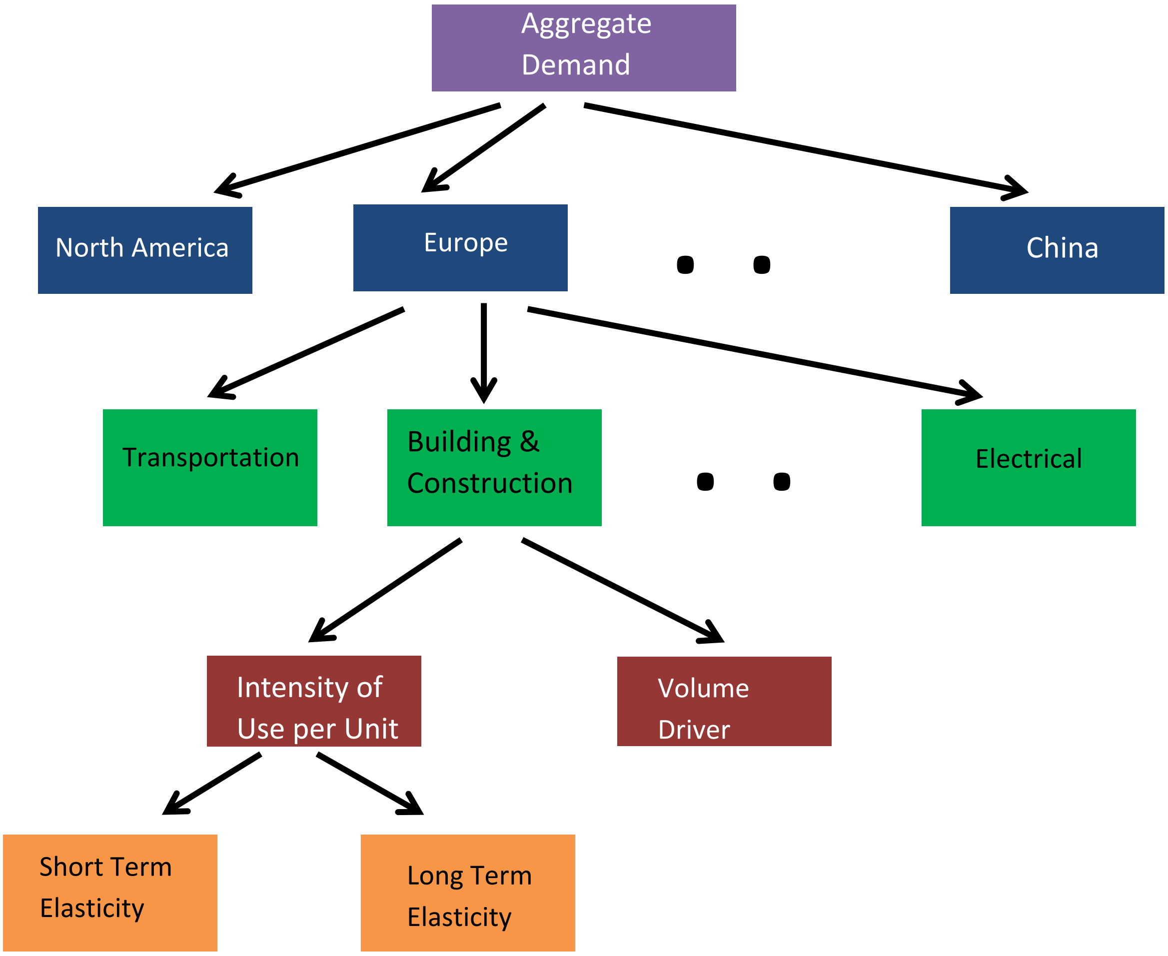


Figure 4: Demand breakdown by region, sector, volume drive vs. intensity of use, and short term vs. long term elasticity.

Intensity of use, which is essentially the amount of copper used in each unit of product, deserves a bit of extra deliberation. As we have described in the previous chapter, there are two types of elasticities. Short run elasticity corresponds to consumers making switching product decisions for the current period; long run elasticity corresponds to manufacturers that changing their processing techniques that to some degree irreversibly accommodate for another type of material. A well-established way to tease out long term elasticity from short term is to use an autoregressive distributed lag model and recast it into its vector error correction form. [94]–[97]

To speak in the language of mathematics, the aforementioned, somewhat convoluted structure can be elegantly expressed by the four following equations.

$$Demand_t = \sum_i \sum_j Intensity_{i,j,t} \cdot Volume_{i,j,t}$$

$$Volume_{i,j,t} = \frac{K}{\exp(a \cdot \exp(-b \cdot t))}$$

$$\Delta \log(Intensity_{i,j,t}) = \beta_{Cu,i,t} \cdot \Delta \log P_{Cu,t} + \beta_{Al,i,t} \cdot \Delta \log P_{Al,t} + \alpha_D \cdot (\log(Intensity_{i,j,t-1}) - \\ (\omega + \gamma_{Cu,i,t} \cdot \log P_{Cu,t-1} + \gamma_{Al,i,t} \cdot \log P_{Al,t-1})) + \beta_{GDP,t} \cdot \log GDP_t + \varepsilon_t$$

$$\Delta \log(\vec{P}_t) = \beta_1 \cdot \Delta \log(\vec{P}_{t-1}) + \beta_2 \cdot \Delta \log(\vec{P}_{t-2}) + \gamma_1 \cdot \Delta \log({}^{GDP}\!/_{cap}) + \gamma_2 \cdot \Delta \log(Pop)$$

Where i represents the sector, j represents the region, and t stands for the time period of interest. The first equation sums the product of intensity of use and volume for each region, each sector, to arrive at aggregate demand for time t. The second equation means the volume of a particular region and sector follows an S curve where K stands for the saturation level of the product. The S curves are mostly borrowed from existing studies on emerging and developed countries for particular industrial sectors, and this form has appeal as most scholars believe a saturation level of units per capita will be reached at some point in the future. The third equation is the autoregressive distributed lag model recast in its error correction form. The beta's in the third equation stand for self and cross-material elasticity of demand in the short term; the gamma's in the third equation stand for self and cross-material elasticity of demand in the long term. Note we do not have an explicit partial equilibrium model for the substitute, which, in this case, is aluminum. We derive the reduced form forecast for aluminum through the fourth structural vector autoregressive model. A vector autoregressive model is by its nature a reduced form model in the sense that the model looks at macro variables without having a micro-foundation. The benefit of such a type of model is its simplicity and its minimal reliance on micro-level data pertaining to the global aluminum industry which we do not have. The drawback, however, is that agent-level behavior cannot be captured in a detailed fashion so that emergence can be studied. Given the lack of data on aluminum and that our main focus is on analyzing copper demand, supply, and equilibrium, we believe such a simplification of the aluminum market model is a best compromise between information unavailability and modeling necessity. Aluminum price series is a function of exogenous factors used by other scholars and believed to be of the most importance that drive demand, namely average global GDP/capita and global population. In addition, current price is influenced by historical price due to autocorrelation. These include both past aluminum price data and copper price data. We follow standard VAR analysis procedure in determining the number of lag terms to be incorporated in our model using Bayesian Information Criteria.

The data regarding the saturation rate, the time series of volume drivers and intensity of use for each region is provided in the appendix. The saturation rates for each sector of the developing countries are modeled after literature based on China;[98]–[100] the saturation rates for each sector of the developed countries are modeled after literature based on US. [101]–[103]We have chosen these studies carefully, so that the volume drivers are physically meaningful in the sense that one unit of increase in the volume driver is mostly likely to lead to demand increments in the material of interest. Our volume driver for the transportation sector is vehicle per thousand individuals, for the building & construction sector is floor space per capita, and for the electrical sector is the amount of electricity generated, which correlates with the amount of overhead or underground cable laid out. The correlations between the amount of volume drivers and affluence are provided by the aforementioned studies. The GDP/capita forecasts are obtained from IMF World Economic Outlook[104], whereas population forecasts are obtained from Population Pyramid.[105] The sector level copper demand for each region is obtained from industrial journals and presentations.[106] The volume drivers for building & construction, electrical, and transportation for China are from National Bureau of Statistics, China Electricity Council (CEC), and Wards Auto, respectively.[107], [108] The volume drivers for building & construction, electrical, and transportation for US are from Moura's studies, Energy Information Agency (EIA), and Wards Auto, respectively.[107], [109], [110]

We report the short term and long term elasticity as follows:

China Buildings & Construction		**China Electrical**		**China Transportation**	
LR Copper	0	LR Copper	-0.066**	LR Copper	-0.049***
LR Aluminum	0	LR Aluminum	0.033**	LR Aluminum	0.022***

SR Copper	0	SR Copper	-0.123**	SR Copper	0
SR Aluminum	0	SR Aluminum	0.083*	SR Aluminum	0
US Buildings & Construction		**US Electrical**		**US Transportation**	
LR Copper	-0.071*	LR Copper	-0.091**	LR Copper	-0.069**
LR Aluminum	0.069**	LR Aluminum	0.072**	LR Aluminum	0.016**
SR Copper	-0.106**	SR Copper	-0.112**	SR Copper	-0.104**
SR Aluminum	0	SR Aluminum	0.071*	SR Aluminum	0.079*

Table 5: The short term and long term elasticity estimates obtained from the ARDL model for buildings & construction, electrical, and transportation sectors for China (representing emerging economies) and the US (representing developed countries).

We observe that elasticity of self and cross material substitution ranges from 0 to 12% across regions and industries. This implies that despite of the prevailing existence of self-price elasticity and cross material substitutability, the effect is salient, but not overly pronounced. This very moderate level of elasticity, coupled with delay in supply, partially explains the strong cyclicality most commodity markets. Only the building and construction sector of China did not exhibit statistically defensible long-run self and cross material substitution which we believe is due to strong demand growth and limited substitution between copper and aluminum for electric wiring and telephony wiring. For simplicity, we further assume that the self-price elasticity and cross material substitution for machinery follows the electrical sector for each region, and others (consumer durables and etc) follow the average of building & construction, electrical, and transportation sectors for each region. We use these elasticities in the intensity of use per unit equations, which are in turn used to determine regional specific and sector specific demand. We sum these numbers across regions and sectors to arrive at short term aggregate demand which is used to determine equilibrium price and quantity. Next we turn to the other critical half for equilibrium determination – supply.

Supply Side:

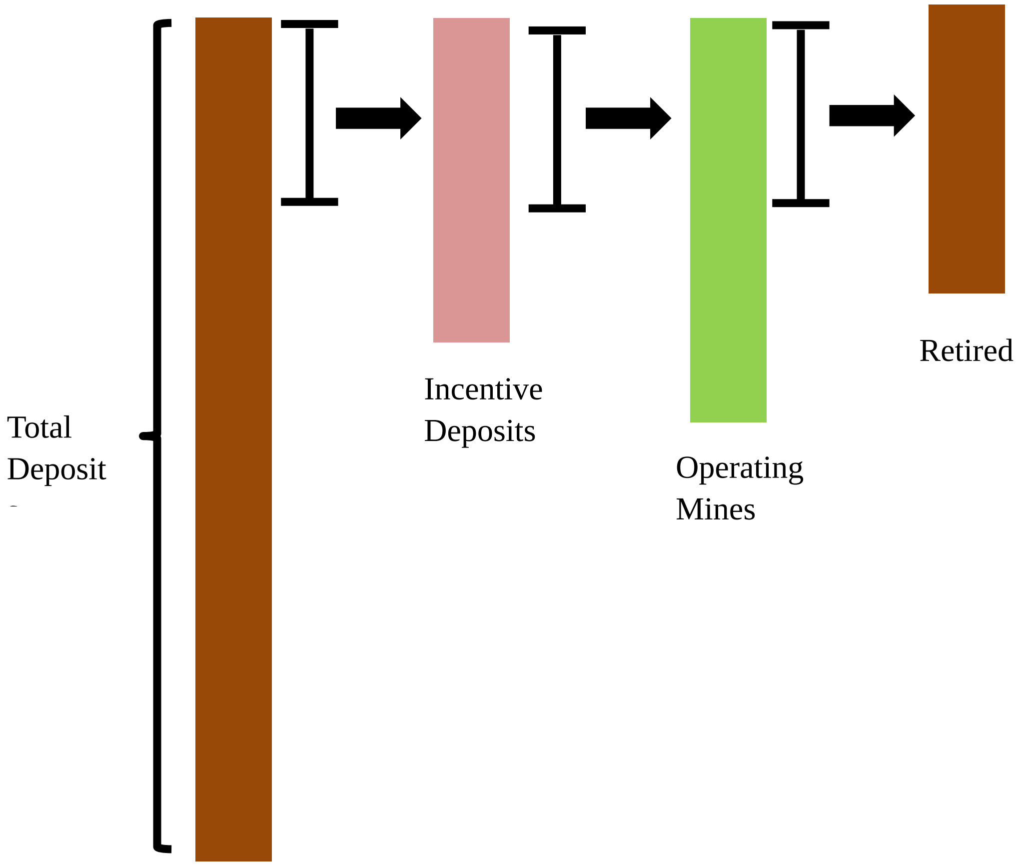


Figure 5: Flow chart representing how copper is produced from prospecting to production to deposit depletion.

On the supply side, we model the entire process of copper production based on extensive literature research and data collection. Procedurally, producers assess the prevailing market condition, conduct prospecting work to identify discovered or undiscovered deposits, and evaluate profitability of these individual deposits based on its particular cost structure and the market price. If those deposits have favorable factor cost structure and copper ore grade characteristics, the producers designate these deposits as incentive deposits, which mean they are potentially feasible for production. The prospecting period, which is also called project selection, can last 8 to 13 years. During this time period, producers perform governmental negotiations, obtain permission and mining rights. They conduct stepwise exploration work and perform project feasibility evaluation. After the deposit moves from total deposits (including discovered and undiscovered deposits) into incentive deposits, producers perform financial feasibility analysis, market analysis, cost analysis, technical and economical project evaluation, and project profitability. This process can take 1-3 years. Once producers make a final decision about the realization of the project and decide they want to produce copper from this incentive deposit, they initiate construction work and infrastructure development. They start construction work, mounting, installation, and develop & excavate the mine. The construction period can take 3-5 years, whereas the ramping up production period can take 2-4 years. Below is a flow chart of a graphic delineation of the lead times for planning and construction of new mining capacities, reproduced from an article by Simon Gloser-Chahoud. [51]

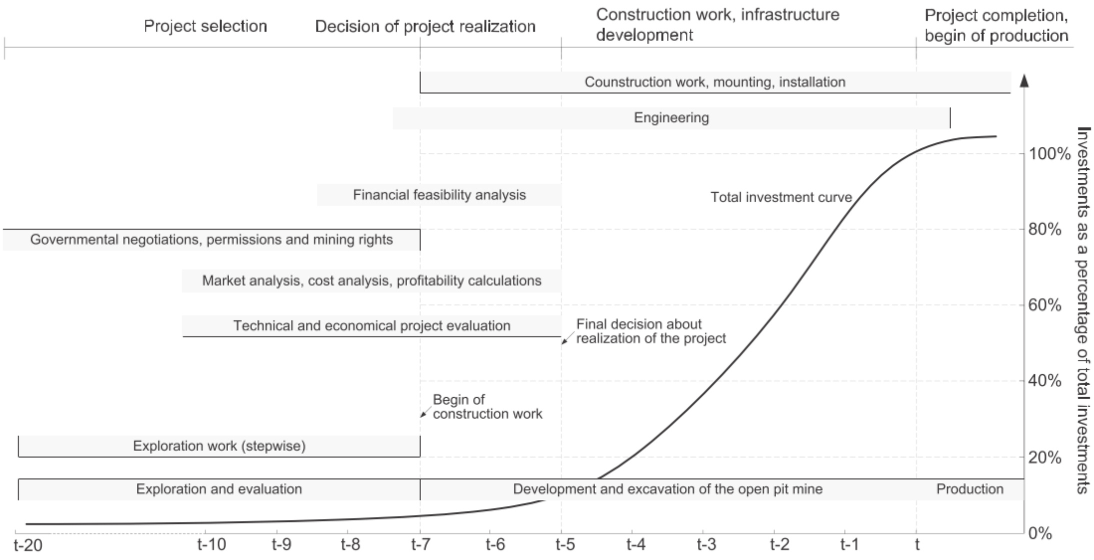
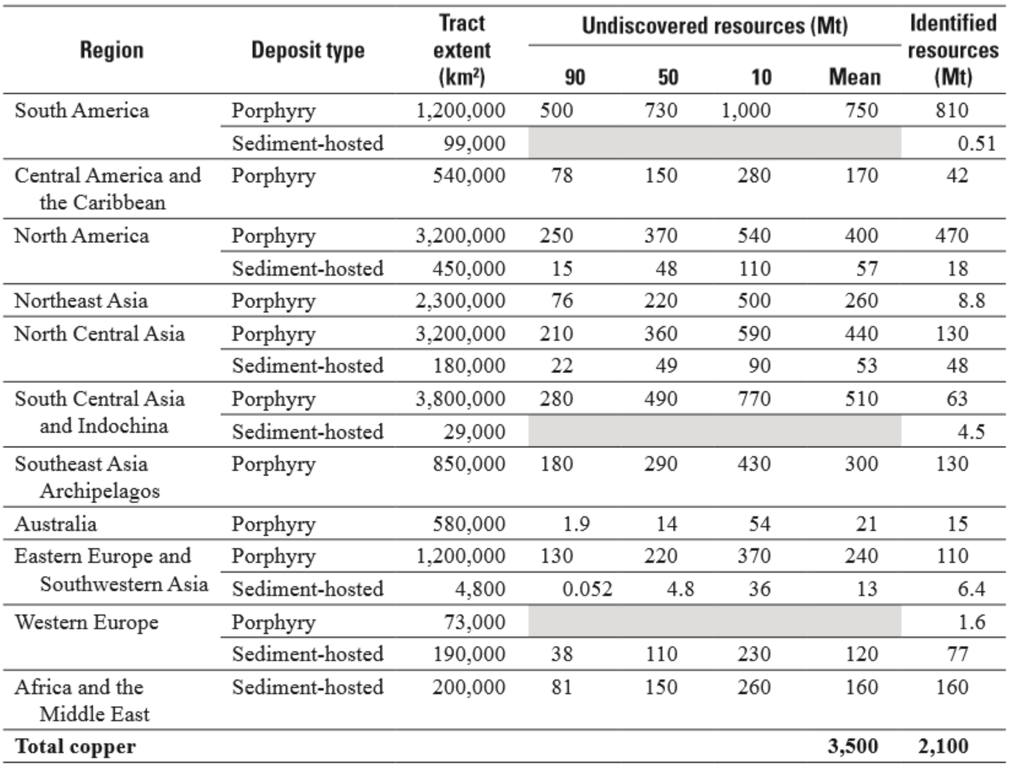


Figure 6: A typical copper project cycle beginning from exploration and evaluation to ultimate production.

In terms of data collection, total deposits can be broken down into discovered deposits and undiscovered deposits. USGS has detailed documentation of ore grade and size of discovered deposits, categorizing deposits into porphyry, VMS, and sediment-hosted.[111]–[113] As for undiscovered deposits, USGS only has tract level data. A tract is defined as “a delineated permissive area for undiscovered copper deposits that may be present in the upper kilometers (minimally) of the Earth’s crust”.[114] Based on tract level information, USGS provides confidence levels at 10%, 50%, 90% in terms of how many copper deposits and how much copper are present.[115] We aggregate both types of copper deposits to form total deposits. Below we showcase the amount of total copper corresponding to three different levels of confidence. Undiscovered copper corresponding to 90% confidence is 1,860 million tonnes, but if one relaxes the confidence level to 10%, there are 5,260 million tonnes of estimated copper reserve.[116] We will investigate how these different levels of confidence impact modeling outcome in the next chapter.



Region	Deposit type	Tract extent (km²)	Undiscovered resources (Mt)				Identified resources (Mt)
			90	50	10	Mean	
South America	Porphyry	1,200,000	500	730	1,000	750	810
	Sediment-hosted	99,000					0.51
Central America and the Caribbean	Porphyry	540,000	78	150	280	170	42
North America	Porphyry	3,200,000	250	370	540	400	470
	Sediment-hosted	450,000	15	48	110	57	18
Northeast Asia	Porphyry	2,300,000	76	220	500	260	8.8
North Central Asia	Porphyry	3,200,000	210	360	590	440	130
	Sediment-hosted	180,000	22	49	90	53	48
South Central Asia and Indochina	Porphyry	3,800,000	280	490	770	510	63
	Sediment-hosted	29,000					4.5
Southeast Asia Archipelagos	Porphyry	850,000	180	290	430	300	130
Australia	Porphyry	580,000	1.9	14	54	21	15
Eastern Europe and Southwestern Asia	Porphyry	1,200,000	130	220	370	240	110
	Sediment-hosted	4,800	0.052	4.8	36	13	6.4
Western Europe	Porphyry	73,000					1.6
	Sediment-hosted	190,000	38	110	230	120	77
Africa and the Middle East	Sediment-hosted	200,000	81	150	260	160	160
Total copper						**3,500**	**2,100**

Table 6: Assessment results for identified and undiscovered copper worldwide, by region. “90” indicates a 90-percent chance of at least the amount shown, with other percentiles similarly defined. Columns may not add to total because of rounding. Gray shading indicates no quantitative assessment.

The cost structure for the total deposits is modeled after the distribution we derived based on operating mines. We fitted the operating mine data using various types of distribution, and we found the best fit was a log-normal distribution. The information regarding operating mines were obtained from industrial presentations. The factor costs include labor cost, electricity cost, royalty, fuel cost, explosive cost, and transportation cost. These costs correspond to moving a unit of rock. We adjust the factor cost per unit of rock through the stripping ratio and ore grade to arrive at the unit cost of production per tonne of copper. Stripping ratio in mining refers to the ratio of the volume of overburden (or waste material) required to be handled in order to extract some tonnage of ore. For instance, a 4;1 stripping ratio with 1% of ore grade means that if we move five tonnes of rocks, four tonnes of them are waste rocks, and the remaining one tonne of rock is ore, and 1% of that ore, which is 10 Kg, is copper.

Each incentive deposit has a particular cost structure. In each period, the manager of the deposit looks at the prevailing market price and assume it persists into the future. It calculates the IRR (internal rate of return) of the project based on endowment, production capacity, and market price, and begins production if and only if the IRR exceeds a certain threshold. After the incentive deposits ramp up their production and become operating mines, they join the existing short run supply curve to produce copper to meet consumption demand. Typically there can be more than one operating mines in an incentive deposit. The size of operating mine is randomly generated based on the size distribution of existing operating mines. If it turns out that the generated size of the operating mine is smaller than half of the deposit size, then the size of the operating mine is subtracted from the incentive deposit, but the incentive deposit still stays in the incentive pool. If the size of the operating mine is greater than half of the deposit size, then the entire deposit becomes an operating mine.

At the beginning of their lifetime, the operating mines are each assigned a randomly generated endowment. Their production capacity and factor costs are statistically generated following lognormality. These lognormal distributions are generated based on existing factor cost information from industrial presentation regarding the characteristics of current operating mines. Note these distributions are also used in generating factor cost distributions for discovered and undiscovered deposits.

The operating mines have the flexibility of over-producing or under-producing 20%

of its nominal production capacity based on the prevailing market price. It produces more than its nominal capacity if price is substantially higher than its cost of production by, for instance, employing extra labor and increase equipment utilization ratio, and it produces less than its nominal capacity if price is lower than its cost of production. This flexibility of short term production, a demonstration of short term price elasticity of supply, is governed by a logistic curve that approaches its upper limit when price is twice its cost of production, and approaches its lower limit when price goes to zero. For each period, we calculate the profit/loss of the mine based on the product of actual production and profit/loss per unit of copper. The profit/loss is accrued on the endowment much like income gets added to retained earnings for a normal corporation. The mine stops production on the fulfillment of one of two types of conditions, an economic one, and a technical one: 1). its endowment falls below a certain percentage of its initial endowment; 2). its ore grade falls below a specific cut-off grade. The economic constrain makes sense as we know companies that keep generating operating losses ultimately lose their ability to raise additional capital to sustain production and have to file for bankruptcy. The technical constraint makes sense as we know when ore grade declines, more rocks have to be removed to recover the same amount of materials, therefore marginal cost of production climbs up to a level that makes production at prevailing market price uneconomical.

Equilibrium and Modeling Output:

The model developed here attempts to reflect the behavior of a system in dynamic equilibrium as short term aggregate supply and short term aggregate demand interact to uniquely determine a price and quantity. As such the market price-clearing mechanism stands at the center of our modeling framework. Price affects suppliers' decision on whether to join operating mines or not, and it also affects demand through self and cross-material elasticity. Quantity is the summation of production across all operating mines. Initially, price is comparatively low due to the fact that most of the incentive deposits that join operating mines have relatively rich ore grade; as deposits of higher ore grades are depleted, overall ore grade declines, and the cost structure of operating mines inflates as a result. The increment of cost is due to more rocks that have to be removed to recover the same amount of copper. This can be captured by the following equation:

$$\mathrm{c}_j = \left(\sum_{i,j} fc_{ij} \right) \bullet \frac{sr_j}{og_j}$$

Where c stands for cost of production for mine j, sr stands for stripping ratio, fc stands for factor cost, and og stands for ore grade. Cost increases when either one or several of factor cot increases, or stripping ratio increases, or ore grade declines.

Market clearing, where short run supply and short run demand simultaneously and uniquely determine a price and quantity of consumption is best described in a procedural fashion:

1) Long term cost structure is transformed into short run supply curve by converting each individual operating mine's nominal capacity into a logistic curve with 20%

of flexibility of production positively correlated with price. More specifically, a much higher prevailing market price compared to the mine's marginal cost of production leads to a maximum of 20% of over production as a percentage of its marginal cost, and vice versa. The aggregation of short run supply curve for each individual operating mine becomes the short run supply curve of the current period.

2) On the bottom left part of the supply curve, we build in inventory release. The size of the inventory ranges from 10% to 25% of total production for a particular period, the particular number is determined by the relationship between prevailing market price and the average of the trailing three years' price. The higher the prevailing market price compared with the average of the trailing three years' price, the more inventories gets released into the short run supply curve of the current period due to higher profitability and bigger incentive, and vice versa.
3) We sum up demand for each region and each sector of the current period to arrive at an aggregate demand for that period, as a function of price for both copper itself and the substitute price.
4) Lastly, we cross the short run demand and supply curve to uniquely determine a price and a quantity of consumption, and the price feeds into suppliers' and consumers' decision making process for the next period as we have described above.

Given such an algorithm, surging demand leads to short term excursion of price, leading to more incentive deposits willing to join operating mines as the IRR of these incentive deposits are higher given same cost structure and higher market price. This can be mathematically represented in the following formula:

$$NPV = \sum_{t=1}^{T} \frac{(p_t - c_t) \bullet q_t}{(1+r)^t} - C_0$$

Where C_0 stands for the upfront investment that has to be made, p_t stands for market price, c_t stands for the cost structure of the mine, and q_t corresponds for the production capacity of the mine. Internal rate of return is defined as the rate r that makes NPV equal to zero. Here p_t has been assumed to be the same and stays constant at the market price of the current period. Naturally, as p_t increases, the internal rate of return will increase as well. Once r increases beyond 10%, which is defined as our threshold internal rate of return as we believe, after extensive conversation with industrial experts that this is a reasonably conservative threshold, the incentive deposit joins operating mines and ramp up production.

As more incentive deposits join operating mines, supply growth outruns demand growth and price mean reverts. As price is lower, substitution effect kicks in as consumers using aluminum or PVC opts for copper, and self-price elasticity also dictates copper demand to increase. Note this effect is reversed when copper price increases substantially. These effects are captured in the previous equation on unit material intensity of use. To restate the general concept, as copper price falls, people tend to use more copper per unit because there it minimizes cost and they are able to do so due to short term price elasticity of demand. When price stays low for several periods, equilibrium level of copper shifts as well, further increasing per unit copper intensity. When increased intensity of use per unit is multiplied by the volume driver, it translates to increased demand. Naturally and intuitively, increased demand further leads to higher price, attracting more incentive deposits to join operating mines as internal rate of return is raised as price increases, and the cycle starts anew. The cyclicality of price and consumption is largely a result of delay in supply due to the considerable amount of time it takes for prospecting, construction, and production ramp-up, and the time it takes for long term elasticity of demand to work towards the equilibrium level. The overall modeling framework can be summarized in the following diagram:

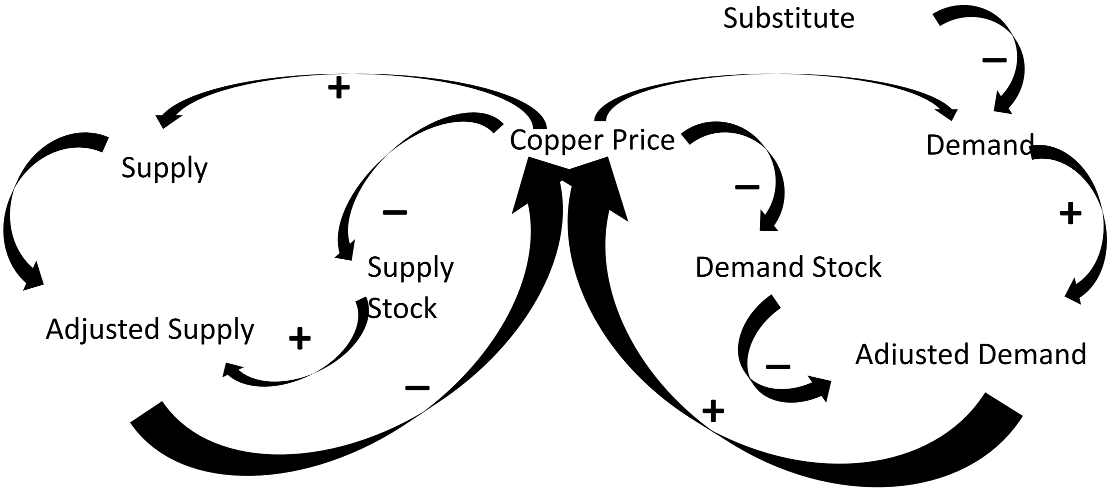


Figure 7: (Simplified) system dynamics diagram of the copper market with copper price and substitute mechanisms highlighted.

There are four modeling outputs that we believe are of paramount importance for us to understand long term and short term equilibrium dynamics.

$$\left(\frac{P_T}{P_1}\right)^{\frac{1}{T}}$$

$$\left(\frac{Q_T}{Q_1}\right)^{\frac{1}{T}}$$

$$\sqrt{\frac{\sum_{t=1}^{T}(\log(\frac{P_{t+1}}{P_t})-\overline{\log(\frac{P_{t+1}}{P_t}}))^2}{T-1}}$$

$$\sqrt{\frac{\sum_{t=1}^{T}(\log(\frac{Q_{t+1}}{Q_t})-\overline{\log(\frac{Q_{t+1}}{Q_t}}))^2}{T-1}}$$

The first two equations are long term price and consumption growth rate, respectively. For our analysis purposes, we run the model for 65 years; therefore T is equal to 65. Of course modeler has the flexibility of deciding whether to extend the modeling period. These two metrics tell us how much price and consumption grow in the long term under a particular setting. Growth alone, however, does not tell us how the path of the growth is characterized. Therefore, we introduce the latter two equations to inform us of the characteristics of the path. We take the logarithm of the ratio between two consecutive years, which is essentially the percentage change of that particular parameter. We calculate the mean percentage change of that particular parameter, and use the standard deviation calculation formula to arrive at the variation of that parameter. The latter two equations are the variation of price and consumption, respectively. These two equations tell us how tortuous or how volatile do price and consumption series evolve. In the next chapter, we will do numerical experiments around the base case to understand the relative impacts of the factors you have considered. We can then run the model thousands of times and calculate the mean of these four parameters of our runs under different scenarios and investigate whether these scenarios are different in a statistically significant fashion. We will also explore the implications of our modeling approach and assumption in a detailed comparison against another result presented in the literature. We will take advantage of visual aids to get a more intuitive sense of the runs to complement our understanding of the scenario parameter outputs. The particular scenarios that we are interested in investigating are subjects for the next chapter.

Chapter Six: Results

Material scarcity, or more simply put, a fear of depleting minerals to such an extent that their prices become infeasibly lofty, has existed for hundreds of years. As early as 1798, Thomas R. Malthus wrote in his *An Essay on the Principle of Population*, that "the power of population is indefinitely greater than the power in the earth to produce subsistence for man".[117]

Scholars of today have more information and computational power to arrive at a more sophisticated and potentially more accurate view on mineral scarcity. The concept of "peak oil" has been explored and debated extensively within the literature. However, there has been comparatively less research examining the concept of "peak minerals", particularly in-depth analyses for individual metals. A group of social scientists from Australia has developed a widely applicable Geologic Resources Supply-Demand Model (GeRS-DeMo). They examined scenarios using cumulative grade-tonnage data to produce estimates of potential rates of copper ore grade decline.

Their results are less than cheering. Their scenarios indicate that there are sufficient identified copper resources to grow mined copper production for at least the next twenty years. The future rate of ore grade decline may be less than has historically been the case, as mined grades are approaching the average resource grade and there is still significant copper endowment in high grade ore bodies. Despite increasing demand for copper as the developing world experiences economic growth, the economic and environmental impacts associated with increased production rates and declining ore grades (particularly those relating to energy consumption, water consumption, and greenhouse gas emissions) will present barriers to the continued expansion of the industry. For these reasons, these authors conclude, peak mined copper production may well be realized during this century.

They collected intelligence on deposit sizes, and compiled them to arrive at ultimately recoverable resources (URR), which is defined by them as the total amount of copper that can be recovered from the earth. They conducted three scenario analyses with 1.5 times URR, 1 times URR, and 0.5 times URR to examine the sensitivity of depletion as a function of endowment size. Even with most optimistic endowment estimates (1.5 times URR), they conclude that we are likely to use most primary copper resources within the first two or three decades of the 22nd century.

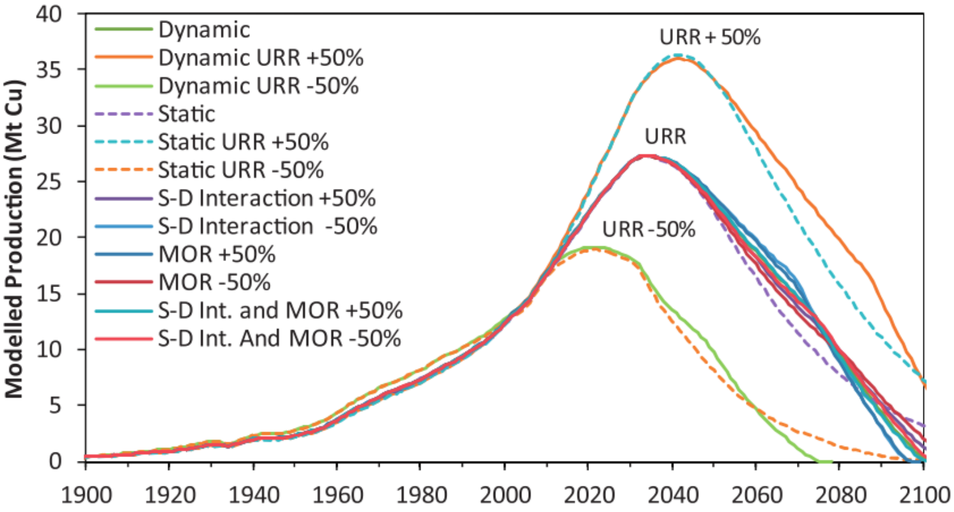


Figure 8: Sensitivity of the deposit model to the ultimately recoverable resource, the supply-demand interaction term, and the mine online rate. (Mudd 2014)

These researchers' work, similar to most of studies in the materials flow assessment space, has not built in an explicit price mechanism. Although demand responds to the gap between supply and demand of that period, it seems the price elasticity of demand is of limited significance in this model.

As we have alluded to before, we believe two important aspects of mineral consumption are potentially critical for any type of mineral depletion forecast: price mechanism and substitution possibility. Feedback is an essential part of any complex adaptive system, and we believe a robust prediction is hard to be obtained when a critical component is left out.

We want to point out another dissonance between these researchers' definition of URR vs. USGS's definition of URR. The authors' analyses seem to be based on identified total resources as we have seen in the USGS table in the previous chapter. Undiscovered resources, however, has not been incorporated into their endowment. A word or two should be said on the definition of "undiscovered", a term that has specific usage in USGS mineral resource assessments. To most people, an undiscovered resource would refer to a quantity of material that is completely unknown. In assessments, the terms "undiscovered mineral resources" refer to a variety of situations in which location, grade, quality, and quantity of mineralized material are not constrained by specific geologic evidence. The presence of mineralized rock might be recognized at a site (location is known) but the grade, quality, and quantity of mineralized material is not sufficiently characterized to estimate mineral resources using industry-standard practices. The estimation of undiscovered copper sizes is expert-opinion based. Specifically, the size of the undiscovered deposits is based on Singer three-part strata-bound models, whereas the number of undiscovered deposits was estimated by an expert panel. After discussion of area geology and deposit models, assessment team members made separate, subjective estimates of the numbers of undiscovered deposits. Estimators were asked for the least number of deposits of a given type that they believed could be present at three specified levels of certainty (the 90%, 50%, and 10% we have seen). For example, on the basis of all available data, a team member might estimate that there is a 90% chance of 1 or more, a 50% chance of 5 or more, and a 10% chance of 10 or more undiscovered deposits occurring in a given permissive tract. Each person made initial estimates without sharing their results until everyone was finished; then, the results were compiled and discussed. Although the undiscovered copper reserves might be temporarily economically inaccessible, they might become dynamically accessible on a run-rate basis. Therefore, we believe by completely ignoring undiscovered reserves, one is overestimating copper scarcity.

As Warren Buffett's right-hand-man, Charlie Munger has sagacious remarked: invert, always invert! We think if we are worried about copper depletion, we should ask ourselves, what extreme situations are likely to deplete copper reserves and exacerbate scarcity. If our model, with its more holistic consideration of substitution potential and price mechanism, more complete incorporation of reserve level information, gives us robust results suggesting copper availability will not be an issue in the foreseeable future, then we can rest assured that mineral scarcity in the copper space might be less of an acute concern. We also present scenarios where scarcity is likely to be mitigated rather than exacerbated. Altogether, these scenarios also allow us to examine the sensitivity of outputs as a response of various assumptions on inputs.

In this chapter, we examine a variety of intriguing scenarios. We separate these scenarios into two groups: individual effects and cumulative effects. For the individual effects, we investigate the impacts of changing one of the critical inputs on output. For the cumulative effects, we hope to extract information regarding how cumulative consumption as a percentage of total reserve changes due to changing reserve size and long term price elasticity of demand.

Before running the scenarios, we calibrate the model from 1970 to 2000 through parameter sweeping, or more specifically, grid-searching for the optimal combination of the size of the incentive pool, size of inventory, flexibility in short term production, the number of total deposits that get screened into the incentive pool for each time period, and other key model input parameters, to minimize the weighted average of mean squared error with respect to both price time series and quantity of production from 1974 to 2000. We use the model to conduct validation through 2001 to 2015, and we further minimize price forecast mean squared error during that time period and the mean squared differential of the supply curve of 2015 which feeds back to improve the goodness of fit and robustness of our model. We use dummy variables in demand to represent the recession during 2008 and 2009.

Individual Effects Analysis:

After we finish calibrating the model, we can categorize individual effects into two parts: Supply and Demand. We discuss supply scenarios first, and then turn to demand scenarios. We pick these scenarios as we believe that, based on our literature research and conversations with experts, these scenarios are likely to develop, and are possible to have tangible impacts on copper consumption, copper scarcity, and price evolution trajectories.

Supply Scenarios:

1. What happens under the three levels of endowments corresponding to varying degrees of confidence provided by USGS
2. What happens under different internal rate of return (IRR) thresholds for copper producers

Demand Scenarios:

3. What happens if the saturation level of per capita housing space in emerging economies are greater than our base case
4. What happens if EV penetration significantly increases
5. What happens under different long term cross and self-price elasticities of demand

Supply Side Scenario Analysis:

Scenario #1: We investigate copper depletion under the three different levels of endowments corresponding to varying degrees of confidence provided by USGS.

	Price Growth Rate (%)	**Quantity Growth Rate (%)**	**Price Variation (%)**	**Quantity Variation (%)**
10% Chance	0.48% NA	2.20% NA	6.79 NA	5.44 NA
Mean Case	1.07% ***	2.15% **	7.01 ***	5.15 **
90% Chance	1.84% ***	2.01% ***	7.56 **	4.72 **

Table 7: Scarcity measures for different assumptions of the quantity of total reserve.

The statistical significance of different scenarios, each with 1,000 runs, is determined by using Wilcoxon rank test, which is robust against varying underlying distributions. We notice larger price growth rate if there is less endowment, a natural deduction since lower ore grade reserves tend to be used quicker for production. Quantity growth rate diminishes as higher price results in substitution. We also report price variation and quantity variation as they pertain to short term dynamics. Smaller reserve size leads to higher price variation as less homogenous reserves come online. Because of substitution, systematically smaller aggregate demand experiences smaller quantity variation. We can further visualize the results.

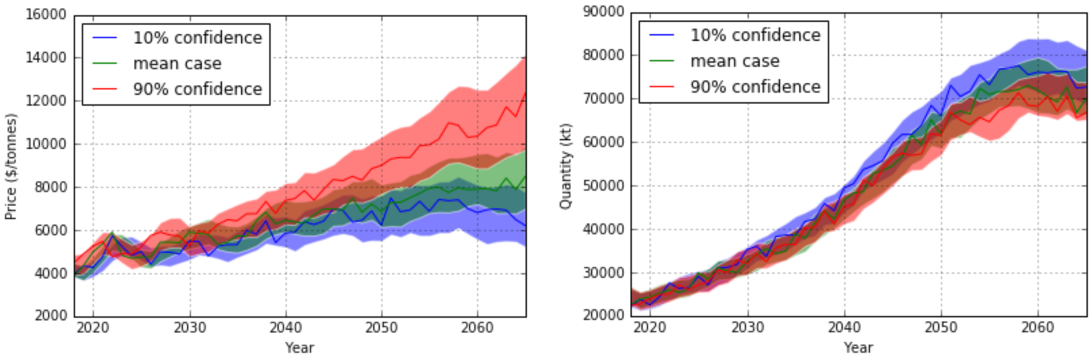


Figure 9: Price and consumption projections given different assumptions on the size of total reserve.

Speaking of scarcity risk, we can investigate two metrics. Firstly, we can ask ourselves how much copper is used up in the next five decades under different reserve sizes. Secondly, we ask ourselves what is the ore grade by the end of the period. Note the cut-off ore grade in the industry is about 0.15%, and this cut-off grade has been declining in the past due to technological progresses.

We report in the mean case, we will use up 41.6% of all copper reserve in the world. In the case where there is the least amount of copper reserve (90% confidence case), we only use up about 53.9% of all copper reserve in the world. In the case where is the most amount of copper (10% confidence case), we report that we use up only 30.2% of all copper reserve in the world. The average ore grade for the 10% case has an ore grade of 0.40%, mean endowment case is 0.35%, whereas the average ore grade for the 90% case still sports 0.30% of ore grade, in all cases considerably above the cut-off grade.

Scenario #2: We investigate the impacts of various internal rates of return on the consumption pattern of copper.

Internal rate of return is the measure used in the model to determine a firm's willingness to initiate new projects given prospective prices and mine-specific cost structure. We investigate three sets of real IRR: 10% (base case), 12%, and 15%. We understand that most firms adopt a 15% nominal IRR. Over the past century, inflation has been roughly 3.5%, so 15% nominal IRR translates to an 11.5% real IRR. When our firms make opening decisions, their price forecast is held constant is assumed to be equal to current year price. We anticipate real price of commodities to increase as ore grade declines, therefore our IRR calculation is conservative. We believe the conservatism in opening decision counteracts the 1.5% of extra IRR premium in the real world, therefore using 10% real IRR as our base case.

	Price Growth Rate （%）	**Quantity Growth Rate （%）**	**Price Variation (%)**	**Quantity Variation (%)**
15% IRR	1.125 NA	2.098 NA	7.92 NA	5.15 NA
12% IRR	1.102 ***	2.121 **	7.45 **	4.93 *
10% IRR	1.070 ***	2.150 **	7.04 **	4.72 *

Table 8: Scarcity measures for different assumptions of copper mine opening criteria (Internal rate of return).

We notice that more strict opening criteria, unsurprisingly, leads to more rapid price growth rate. More rapid price growth rate and higher prices lead to lower quantity growth rate. More strict opening criteria means mines are less likely to open during highly volatile periods to smooth out production, therefore experiencing higher price and quantity variation. We visualize our results in the following graphs:

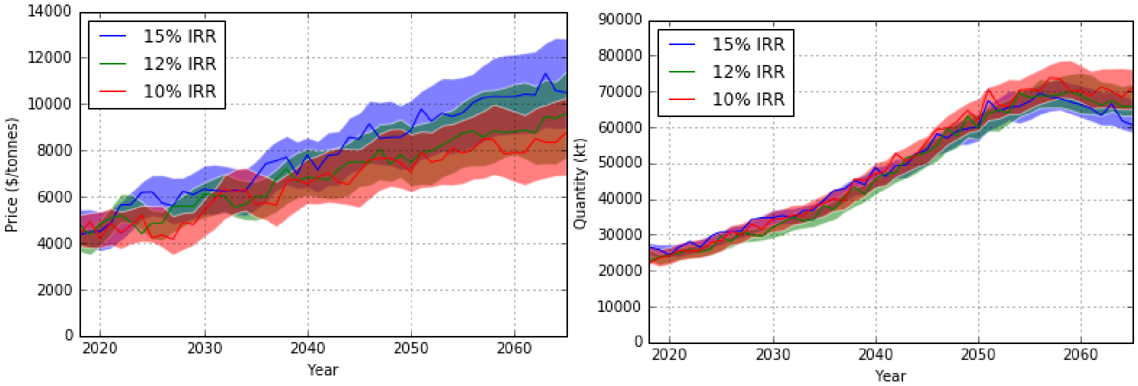


Figure 10: Price and consumption projections given different assumptions on copper mine opening criteria (internal rate of return).

We report in the base case, we will use up 41.6% of all copper reserve in the world. We notice, visually, that IRR variation is unlikely to dramatically change the percentage of reserve that is consumed. For instance, raising IRR from 10% to 15% only reduces the percentage of aggregate consumption of total reserve to 40.5%. The average ore grade for the mean endowment case is 0.350%, whereas a 15% IRR will reduce ore grade depletion, and by the end of period the mean endowment still has an ore grade of 0.357%. We do not report results for an 8% or even 6% IRR as we believe these values are unrealistically low values for decision makers, but we find even with a 6% IRR, only about 42.4% of all copper reserve is used up on a global basis.

Demand Side Scenario Analysis:

Scenario #3: We investigate copper depletion under the three different saturation levels of per capita housing space in the emerging economy.

In our base case, the saturation rate of per capita housing space for folks in the developed economy is 90 square meter, but the saturation rate of per capita housing space for people in the emerging economy is only 70 square meter. We investigate two scenarios where the saturation rate of per capita housing space are at 60 square meter and 80 square meter, respectively. We believe 80 square meter is an aggressive scenario in terms of copper consumption given the population density in some of the largest emerging economies. We conduct the 60 square meter scenario for comparison purpose.

	Price Growth Rate （%）	**Quantity Growth Rate （%）**	**Price Variation (%)**	**Quantity Variation (%)**
Emerging Economy 80 m^2/cap	1.782 NA	2.360 NA	7.81 NA	5.60 NA
Reference Case	1.070 ***	2.150 **	7.04 ***	5.15 **
Emerging Economy 60 m^2/cap	1.010 ***	1.983 **	6.53 ***	4.72 **

Table 9: Scarcity measures for different assumptions of the saturation level of housing space for the emerging economy.

Naturally, if per capita consumption of copper demand increases, quantity growth rate will increase. Because of higher demand growth rate, price will also increase faster, whose impact on substitution and elasticity works through copper intensity and in term influences aggregate demand in the next period. Substitution as a result of higher price tends to tame quantity growth, but its impact is limited. Quicker price and quantity growth rates also lead to higher price and quantity variation. We visualize the results as follows:

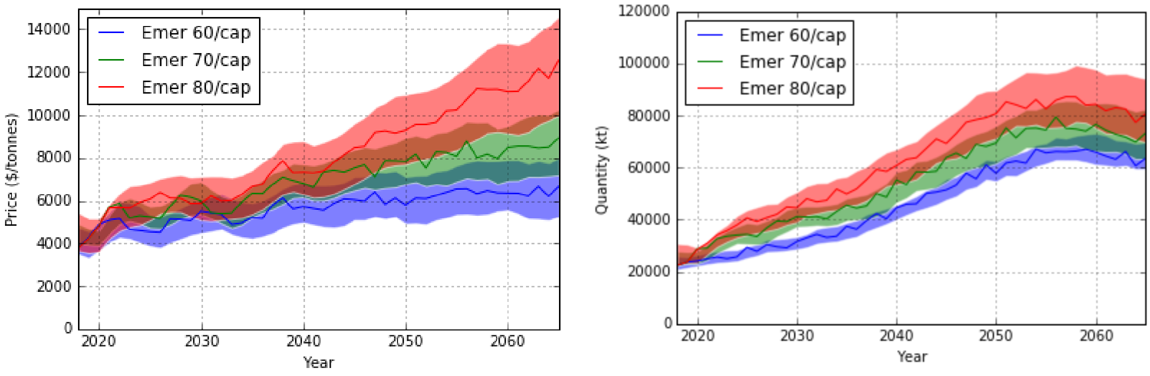


Figure 11: Price and consumption projections given different assumptions on the saturation level of per capita residence space in the emerging economy.

We report in the 70/cap case, we will use up 41.6% of all copper reserve in the world. In the case where there is 80/cap case, we only use up about 45.1% of all copper reserve in the world. The average ore grade for the mean endowment case is 0.35%, whereas the average ore grade for the 80/cap case still sports 0.315% of ore grade, substantially higher than the cut-off grade. One may notice that in both this scenario and the scenario above, percentage difference in price in 2065 for each scenario is substantially greater from percentage difference in quantity in 2065 for each scenario, reflecting the fact that price elasticity of demand exists but its magnitude is reasonably small.

Scenario #4: We investigate the impact of EV penetration on the model outputs.

ICA reports hybrid electrical vehicle uses 40 kg of copper, and a plug-in hybrid electrical vehicle uses 60 kg of copper. The current average copper in a gasoline vehicle is about 20-25 kg. In our model, we assume each EV uses twice as much copper as a gasoline vehicle. Bloomberg New Energy Intelligence predicts that EV penetration will be 56% by 2050. By extrapolation of their predicted data, EV penetration will be 70% by 2065, the end of our prediction period. We further make the simplifying assumption that EV penetration rate, or alternatively the demand for EV, is insensitive to price. We make this assuming as the additional copper required (25 kg) is about $150, less than 1% of the cost of an EV.

	Price Growth Rate （%）	Quantity Growth Rate （%）	Price Variation (%)	Quantity Variation (%)
EV Scenario	1.71 NA	2.271 NA	7.75 NA	5.56 NA
Base Case	1.07 ***	2.150 **	7.04 **	5.15 **

Table 10: Scarcity measures for different assumptions of electric vehicle penetration rate.

With higher EV penetration rate, we notice statistically significant higher quantity growth rate for the EV scenario vs. the base case. The faster quantity growth rate leads to higher price growth rate. We do find that the higher price of copper leads to lower copper content per traditional passenger vehicle, but the substitution part is more than compensated for by rapidly rising EV penetration rate. We visualize the results as follows:

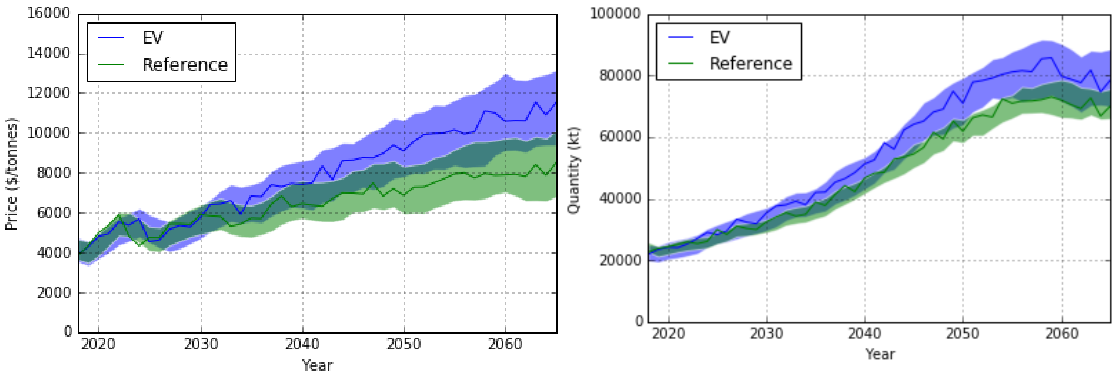


Figure 12: Price and consumption projections given different assumptions on electric vehicle penetration rates.

We report in the base case, we will use up 41.6% of all copper reserve in the world. In the case where there is high EV penetration case, we only use up about 43.8% of all copper reserve in the world. The average ore grade for the mean endowment case is 0.35%, whereas the average ore grade for the high EV penetration case still sports

0.321% of ore grade, substantially higher than the cut-off grade. One may notice that in both this scenario and the scenario above, percentage difference in price in 2065 for each scenario is substantially greater from percentage difference in quantity in 2065 for each scenario, reflecting the fact that price elasticity of demand exists but its magnitude is reasonably small.

Scenario #5: We investigate future copper consumption pattern given different levels of price elasticities of demand.

Recall elasticity of demand is first broken down into long term and short term elasticity, and each can be further broken down into self-elasticity and cross-elasticity; we further break down these two types of elasticities into various sectors which we have described earlier. We investigate three different levels of long term elasticities of demand: high, base, and low. The base case is our reference case using price elasticities of demand we have derived from our ARDL model. The high case assumes both cross and self-elasticities of demand to be twice as much as our base case; the low case assumes both cross and self-elasticities of demand to be only a half of our base case. We also investigated the impacts of short term elasticities of demand on copper consumption. We find higher short term elasticities of demand lead to higher price and quantity variation of copper consumption, but has no statistically significant impact on price and quantity growth rate, therefore we choose not to report the results here.

Long-term Elasticities of Demand	**Price Growth Rate （%）**	**Quantity Growth Rate （%）**	**Price Variation (%)**	**Quantity Variation (%)**
2x Case	1.059 NA	2.083 NA	6.81 NA	5.30 NA
Reference Case	1.070 **	2.150 **	7.04 **	5.15 *
0.5x Case	1.091 **	2.221 **	7.27 **	4.99 *

Table 11: Scarcity measures for different assumptions of the magnitude of long-term elasticities of demand.

We notice higher long term elasticities of demand, which implies higher capabilities of the consumers to use substitutes instead of having no choice but sticking with copper, leads to slower quantity growth rate as logic would dictate. Slower quantity growth rate also leads to slower price growth rate. Higher elasticities also leads to lower price and quantity variation. We can visualize these statistical results as follows:

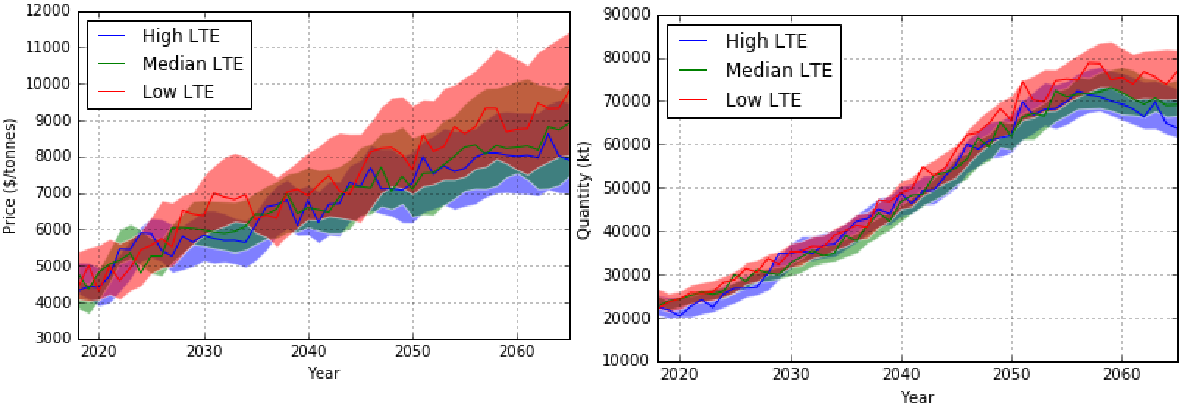


Figure 13: Price and consumption projections given different assumptions on the magnitude of long term elasticity.

We report in the base case, we will use up 41.6% of all copper reserve in the world. In the case where there is high long term price elasticities of demand, we only use up about 40.6% of all copper reserve in the world, whereas when there is low long term price elasticities of demand, we use up 42.7% of all copper reserve in the world. In other words, scarcity risk is only likely marginally increase if the availability of substitution and price elasticities are low. The average ore grade for the mean endowment case is 0.35%, whereas the average ore grade for the low long term price elasticities of demand case still sports 0.34% of ore grade, substantially higher than the cut-off grade.

Cumulative Effects Analysis

In this section we investigate cumulative effects. More specifically, we look at various reserve sizes in combination with different long term price elasticity of demand, and how these conditions affect cumulative consumption as a percentage of total reserve. We summarize the four scenarios we will investigate:

1. Apart from fine tuning our parameters to reasonably replicate the abovementioned paper's results (namely, constrained elasticity of demand and possibility of substitution with their copper endowment level), we hope to conduct three separate experiments to contrast varying results based on different assumptions including the follow:
    a) What if we add in elasticity of demand and possibility of substitution, and use their copper endowment level
    b) What if we add in elasticity of demand and possibility of substitution, and use USGS's mean copper endowment level

c) What if we constrain elasticity of demand and possibility of substitution, and use USGS's mean copper endowment level

We investigate the three scenarios in addition to the Mudd paper's output.

We visualize the results:

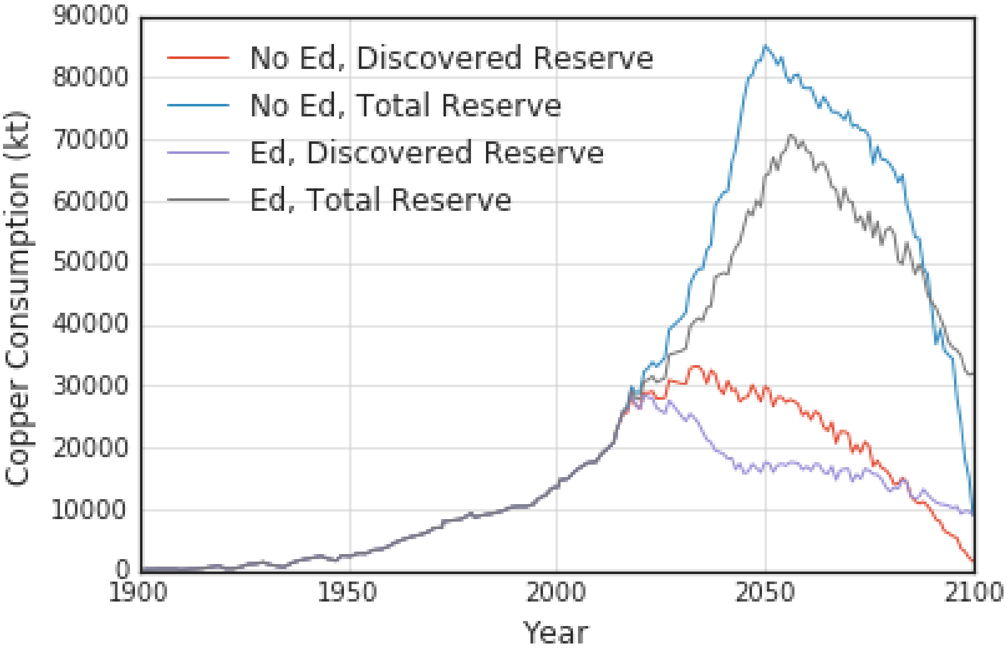


Figure 14: Copper consumption forecast based on whether we use discovered reserve (2,100 million tonnes) or total reserve (5,600 million tonnes which includes undiscovered reserve), and whether we assume there is short term and long term elasticity of demand.

Unsurprisingly, larger reserve sizes unambiguously lead to more copper consumption. As a reminder, our discovered reserve is 2,100 million tonnes, and our total reserve size is 5,600 million tonnes. We found that over the century, having a larger reserve results in an average additional consumption of 2,900 million tonnes of copper. This is more than the entirety of the discovered reserve.

Price elasticity of demand, especially long term price elasticity of demand, also plays a critical role in influencing copper consumption. Total copper consumption for the close-to-zero elasticity and discovered copper reserve base is 94.9% of the entire copper reserve by the end of this century (2100), equaling 1,992.9 million tonnes; total copper consumption for the price elastic and discovered copper reserve base is, in contrast, only 75.1% of total discovered copper reserve, or 1,577.1 million tonnes. In other words, the consideration of elasticity and material substitution potential leads to a reduction of consumption of 20.7%. Total copper consumption for the close-to-zero elasticity and total copper reserve base case is 89.7% of the entire copper reserve for the next century with 5,247.2 million tonnes of consumption. Total copper consumption for the price elastic and total copper reserve base case is 71.2% of the total copper reserve base, or equivalently 4,155.2 million tonnes. This is a reduction of consumption of 20.8%. A common trait across price elastic vs. price inelastic cases is that for the inelastic case, consumption always overshoots the price elastic case because of a lack of regulating price mechanism, and consumption also falls more rapidly as materials become unavailable. In other words, the consideration of long term price elasticity of demand leads to more moderate consumption growth, and is less prone to suffer rapidly falling consumption as a result of more dramatically exacerbated material scarcity issues early on. Therefore, it does not seem to be an overstatement that the incorporation of price mechanism and material substitution potential have a material impact on our understanding of the behavior of this complex commodity market system.

One thing worth pointing out is that our estimation of the percentage of total reserve consumed is likely to be an overestimation for all the aforementioned scenarios for three important, but less addressed reasons in the current version of our model. Firstly we have treated secondary copper, or recycled copper, in a stylized fashion that is likely to be an underestimation of its supply; secondly we have used only USGS's total reserve data, and have not considered other sources of copper that might become economically extractable over time, such as lower ore grade copper deposits not included in USGS and the potential of seabed mining; thirdly we have not taken into account of the emergence of other material substitutes. We will discuss each of these components in more detail. It is important to keep in mind that these three aspects are likely to further strengthen our case that we are unlikely to run out of copper in the foreseeable future.

Copper Recycling:

As has been mentioned before, our model assumes linear extrapolation of existing growth trend of secondary copper in the past three decades will persist into the future. This simplifying assumption might has underestimated the percentage of secondary copper recycled from circulating products by their end of life. As consumption increasingly depletes existing reserve, ore grade declines and drives up copper price, creating more incentive for recycling. In a way, recycled copper can be considered as one of the strongest substitute of primary copper, and higher primary copper price will indubitably create significantly more incentive for constructing more recycling plants and supplying more secondary copper. Energy is a considerable cost component in copper manufacturing, ranging from 20% to 40% of total cost of production. The processing of recycled copper requires much less energy than the processing of new copper from virgin ore, providing a savings of 85-90% of energy requirements.[118]

On a global basis, around 15% of copper was recovered from recycled material with the rest generated from newly mined ore. In developed countries such as US, the number is substantially higher, standing at 34% in 2014.[119] While wire supply is produced predominantly from newly refined copper, nearly two-thirds of the amount used by other segments of industry, including copper and brass mills, ingot makers, foundries and others comes from recycled material, and because of high copper recycling, US is largely sufficient in copper supply. Slightly over one-half of recycled copper scrap is new scrap recovery including chips and machine turnings, with the rest being old post-consumer scrap such as electrical cable, old radiators, and plumbing tube. Given lower cost structure, lower penetration rate in most of the developing world, increasing significance in developed world as environmental concerns become more salient, one should be reasonably confident to argue that with lower ore grade and higher primary production costs, an ever increasing percentage of copper supply will be constituted by secondary copper. Secondary copper as a potent substitute candidate playing an increasingly important role in global copper supply will undoubtedly decrease the criticality risk associated with depleting reserve.

Other Sources of Reserve:

One of the senior industrial experts we have talked with pointed out that in the 1960s, the cut-off ore grade for USGS to consider as reserve is in the vicinity of 1.5%-2.0%. These ore grade numbers are too good to be true nowadays. The average ore grade in 2016 is 0.60%, and USGS employs a cut-off grade of about 0.15%. However, it is likely that the cut-off grade is likely to slip even lower as technology progresses, which will dramatically expand the amount of reserve that is economically extractable. In addition, technological progresses might make previously inaccessible reserves, such as seabed reserve, accessible. This is not a fairy tale. Canada’s Nautilus Minerals is on track to start operations at its Solwara 1 gold, copper, and silver project off the coast of Papua Guinea in early 2019. Its chief executive Mike Johnston said that they expect to have all its undersea mining tools ready to go by mid-2018, so it can kick-off operations at the Bismarck Sea-based project shortly after.[120] This is not to say the road towards accessing previously inaccessible reserve is not bumpy. Since first proposal, Nautilus’ Solwara 1 project has met with some opposition mostly from environmentalists who fear the aquatic ecosystem could be severely affected. The company also faced some hurdles long-dragged dispute with the Papua New Guinea government. Nevertheless, most technological advancements are not without disputes, and humanity ultimately figures out ways to move forward. Therefore, once we expand the definition of reserve base and include previously inaccessible reserve, our reserve base may significantly expand, further mitigating scarcity risks.

Other Emerging Material Substitutes:

Apart from aluminum, which we have adopted as a proxy for material substitution, other material substitutes exist, and some previously nonexistent competitors are rising quickly among the ranks. While it is impossible to enumerate every emerging competitor, we name two that are possibly most salient: plastics for pipe and carbon nanotubes for conductors. While copper has been a material widely used in the U.S. to move drinking water, copper pipes can deliver a highly metallic taste to water, which can become particularly potent and even toxic when those pipes begin to deteriorate. Plastic piping was introduced to the US in the mid-1980s and has become increasingly popular. It is easy to install. Its route through the building can be more versatile and potentially contribute to making the building more holistically designed. It is also less expensive which obviously helps with budgeting and money allocation. It can also be used for 50 years or more.[121] As a potent rival against copper, plastic piping has the capability of potentially grabbing a bigger share of the pie in the building & construction sector previously dominated by copper. Earlier last year, researchers at MIT have found that in certain diameter carbon nanotubes, a solid phase of H_2O can exist at temperatures greater than 378K. The significance of the study is that certain carbon nanotubes already exhibit the property of ballistic conduction (negligible resistivity due to scattering). This means electrons can travel freely along the length of the nanotube until it collides at the end of the tube. The solid H_2O at high temperatures means that protons may be transported as well. While copper is the dominant conductor of choice in the market today and will likely stay that way for quite a while, future generations might see devices and power transmission lines made of carbon nanotubes filled with solid H_2O.[122] In sum, the emergence of other new material substitutes might have an impact on reducing the copper demand in certain areas. This further mitigates copper scarcity risk.

In sum, because of recycling and its sensitivity to price, ever enlarging reserve size due to technological progress, and emergence of other new substitutes, the likelihood of our seeing copper running out within the foreseeable future is further diminished.

Chapter Seven: Future Work

The author understands that despite of his utmost effort, his work is far from perfect in providing the best framework to systematically analyze a commodity market. Nevertheless, he believes his work is an important step towards a holistic description of the commodity market with a deliberate focus on bottom-up granularity that delineates agent-level strategic decision making processes, price mechanism, short term and long term elasticity of demand, and last but not least, the possibility of materials substitution, both in the short run and in the long run. The author believes these are critical pieces to better understand the price and supply/demand mechanism of this complex system, and the typical negligence of these factors in the existing academic community have led to potentially biased estimation of materials scarcity in multiple ways that has been described.

There are, however, much left to be done, and your author will note the several most critical aspects of the model that he believes should be further worked on by later researchers.

Firstly, when we estimate short-term and long-term price elasticity of demand, we used data from 2000 to 2015 for the ARDL parameter estimation. We could not leave out data for cross-validation because of the paucity of available data. If region based industry specific data was available from 1970 to 2000, we could have used these data for training, and the data from 2000 to 2015 for cross validation. We noticed as our research progressed, that one of the major problems facing social sciences research is not the issue of “big data”, but the conundrum of “small data”. We simply did not have granular enough historical information going back multiple decades, since few if any were collecting such intelligence back then. One way of dealing with the “small data” problem is to use Bayesian prior, where prior estimates of the parameters in the ARDL model were obtained through survey interviews conducted with industrial experts. One can then use the prior coupled with data from 2000 to 2010 for training, and use data from 2011-2015 for cross validation, thereby (partially) solving the data paucity issue.

Secondly, we have used a somewhat rough proxy, aluminum, as copper’s substitution. In reality, some applications of copper there are other types of substitution, for instance, polymers, as a plumbing material instead of copper. Therefore, a more nuanced study will probably look at each industrial sector’s main copper substitutes, and evaluate material substitution on a commodity-by-commodity basis. In addition, the evolution of the proxy price follows a vector autoregressive model which is a reduced-form formulation. A potentially more robust method is to develop a bottom-up model as well for that substitute. This implies constructing a separate bottom-up model for that particular type of material as well – in our case, aluminum. Imaginably, this means a lot more work, but dual or maybe triple or even quadruple intermarket dynamic analyses will potentially reveal more sophisticated and intriguing properties of these structural markets as well.

Last but not least, although copper is by and large an international market, each regional market and each particular industry might have quite different dynamics. We have assumed global market clearing, which is certainly a simplifying assumption. Future work on local market clearing and global market dynamics founded on the basis of individual local market aggregation will certainly shed light on the more intricate real world market dynamics of the commodity.

With these improvements that can be made in mind, we are genuinely grateful for the reader’s time in taking this profoundly interesting and absolutely fascinating intellectual journey with us. We welcome any questions or suggestions regarding our work.

References:

[1] T. R. Malthus, “An Essay on the Principle of Population, as It Affects the Future Improvement of Society with Remarks on the Speculations of Mr. Godwin, M. Condorcet, and Other Writers, 1st ed,” *J. Johnson London*, 1798.

[2] M. Milgate, *Goods and Commodities*. 2008.

[3] K. Hubbert, “Techniques of prediction as applied to the production of oil and gas,” *NBS Special Publication*, vol. 631. pp. 16–141, 1980.

[4] A. Regent, “Barrick Gold CEO Finds What’s Precious in Both Business and Community,” 2010. [Online]. Available: http://www.ey.com/us/en/about-us/our-alumni/connect--may-2010---aaron-regent--barrick-gold-ceo.

[5] S. Northey, S. Mohr, G. M. Mudd, Z. Weng, and D. Giurco, “Modelling future copper ore grade decline based on a detailed assessment of copper resources and mining,” *Resour. Conserv. Recycl.*, vol. 83, pp. 190–201, 2014.

[6] A. Cox, *History of Bisbee 1837 to 1877*. University of Arizona, 1938.

[7] R.F.Mikesell, “The World Copper Industry: Structure and Economic Analysis,” *Johns Hopkins Univ. Press. Resour. Futur.*, 2013.

[8] M. Radetzki, “Seven Thousand Years in the Service of Humanity: the History of Copper, the Red Metal,” *Resour. Policy*, vol. 34, no. 4, pp. 176–184, 2009.

[9] H. Evans, *They Made America*. Little, Brown and Company, New York, 2004, 2004.

[10] M. E. Slade, “The rise and fall of an industry: Entry in U.S. copper mining,

1835-1986,” *Resour. Energy Econ.*, vol. 42, pp. 141–169, 2015.

[11] B. Carter, *Boom,Bust,Boom: A Story about Copper, the Metal that Runs the World*. Scribner, New York, 2012.

[12] M. Doggett and R. Leveille, “Assessing the Returns to Copper Exploration, 1989-2008,” *Explor. Min. Geol.*, vol. 19, no. 1–2, pp. 23–33, 2010.

[13] R. B. . B. Gordon M.; Graedel, T. E., “Metal Stocks and Sustainability,” *Proc. Natl. Acad. Sci U.S. A.*, vol. 103 (5), pp. 1209–1214, 2006.

[14] M. Gerst, “Revisiting the Cumulative Grade-Tonnage Relationship for Major Copper Ore Types,” *Econ. Geol.*, no. 1981, pp. 615–628, 2008.

[15] A. McMahon, “Copper: A Materials Survey,” 1965.

[16] U. Congress, “Copper Technology and Competitiveness,” *Off. Technol. Assess.*, 1988.

[17] N. Jiang, “Personal Communication with N.P. Jiang, Manager, Nanjing Recycling Co.,” 2009.

[18] M. Rauch and R. Gordon, “Copper Stock and Copper Old Scrap in the State of Connecticut,” *FES Work. Pap.*, vol. 10, 2007.

[19] L. Zhang, Z. Yuan, and J. Bi, “Estimation of Copper In-use Stocks in Nanjing , China,” vol. 16, no. 2, pp. 191–202, 2011.

[20] M. D. Gerst, “Linking Material Flow Analysis and Resource Policy via Future Scenarios of In-Use Stock : An Example for Copper,” vol. 43, no. 16, pp. 6320–6325, 2009.

[21] S. Spatari, M. Bertram, R. B. Gordon, K. Henderson, and T. E. Graedel,

"Twentieth century copper stocks and flows in North America : A dynamic analysis," vol. 54, pp. 37–51, 2005.

[22] S. Spatari, M. Bertram, K. Fuse, T. E. Graedel, and H. Rechberger, "The contemporary European copper cycle : 1 year stocks and flows," vol. 42, pp. 27–42, 2002.

[23] S. Devarajan and A. C. Fisher, "Hotelling's Economics of Exhaustible Resources 50 Years Later," *J. Econ. Lit.*, vol. 19, no. 1, pp. 65–73, 1981.

[24] H. Hotelling, "Economics of Exhaustible Resources," *J. Polit. Econ.*, vol. 39, pp. 137–175, 1931.

[25] S. Farrow, "Testing the Efficiency of Extraction from a Stock Resource," *J. Polit. Econ.*, pp. 452–487, 1985.

[26] D. Young, "Cost Specification and Firm Behavior in a Hotelling Model of Resource Extraction," *Can. J. Econ.*, pp. 41–59, 1992.

[27] M. E. Slade and H. Thille, "Whither Hotelling: Tests of the Theory of Exhaustible Resources," *Annu. Rev. Resour. Econ.*, vol. 1, no. 1, pp. 239–260, 2009.

[28] R. Pindyck, "The Optimal Exploration and Productionn of Nonrenewable Resources," *J. Polit. Econ.*, pp. 841–861, 1978.

[29] R. Gilbert, "Optimal Depletion of an Uncertain Stock," *Rev. Econ. Stud.*, pp. 47–57, 1979.

[30] R. Pindyck, "Uncertainty and Exhaustible Resource Markets," *J. Polit. Econ.*, pp. 1203–1225, 1980.

[31] V. Aguirregabiria and A. Luengo, “A Microeconometric Dynamic Structural Model of Copper Mining Decisions,” 2015.

[32] S. Spatari, M. Bertram, R. B. Gordon, K. Henderson, and T. E. Graedel, “Twentieth century copper stocks and flows in North America: A dynamic analysis,” *Ecol. Econ.*, vol. 54, no. 1, pp. 37–51, 2005.

[33] T. Graedel, R. Barr, C. Chandler, T. Chase, and C. Zhu, “Methodology of Metal Criticality Determination,” *Environ. Sci. Technol.*, vol. 46, no. 2, pp. 1063–1070, 2012.

[34] G. Xueyi and S. Yu, “Substance flow analysis of copper in China,” vol. 52, pp. 874–882, 2008.

[35] W. Chen, L. Shi, and Y. Qian, “Substance Flow Analysis of Aluminum in Mainland China for 2001, 2004 and 2007: Exploring its Initial Sources, Eventual Sinks, and the Pathways Linking Them,” *Resour. Conserv. Recycl.*, vol. 54, pp. 557–570, 2010.

[36] R. Yamaguchi and K. Ueta, “Substance Flow Analysis and Efficiency Conditions: A Case of Lead,” in *Interfaces for Advanced Economic Analysis, Kyoto University*, 2006.

[37] C. R. Binder, T. E. Graedel, and B. Reck, “Explanatory variables for per capita stocks and flows of copper and zinc: A comparative statistical analysis,” *J. Ind. Ecol.*, vol. 10, no. 1–2, pp. 111–132, 2006.

[38] E. H. Muller Lorenz; Widmer, Rolf; Schluep, Mathias; Faulstich, Martin, “Modeling Metal Stocks and Flows: A Review of Dynamic Material Flow

Analysis Methods,” *Environ. Sci. Technol.*, vol. 48, 2014.

[39] K. Roelich, D. A. Dawson, P. Purnell, C. Knoeri, R. Revell, J. Busch, and J. K. Steinberger, “Assessing the dynamic material criticality of infrastructure transitions: A case of low carbon electricity,” *Appl. Energy*, vol. 123, pp. 378–386, 2014.

[40] L. Yan, A. Wang, Q. Chen, and J. Li, “Dynamic Material Flow Analysis of Zinc Resources in China,” *Resour. Conserv. Recycl.*, vol. 75, pp. 23–31, 2013.

[41] T. E. Graedel and M. Bertram, “Exploratory Data Analysis of the Multilevel Anthropogenic Copper Cycle,” pp. 1253–1261, 2004.

[42] T. Graedel, *Minerals, Critical Minerals, and the US Economy*. US: National Research Council of the National Academies, 2008.

[43] M. D. Gerst and T. E. Graedel, “In-use Stocks of Metals: Status and Implications,” *Environ. Sci. Technol.*, vol. 42, pp. 7038–7045, 2008.

[44] W. Chen and T. E. Graedel, “Anthropogenic Cycles of the Elements : A Critical Review,” 2012.

[45] T. E. Gordon M.; Graedel R B.; Bertram, “Metal Stocks and Sustainability,” *Proc. Natl. Acad. Sci U.S. A.*, vol. 103 (5), pp. 1209–1214, 2006.

[46] X. . G. Du T. E., “Global Rare Earth In-use Stocks in NdFeB Permanent Magnets,” *J. Ind. Ecol.*, vol. 15 (6), pp. 836–843, 2011.

[47] D. B. Mueller, J. Cao, E. Kongar, M. Altonji, P.-H. Weiner, and T. E. Graedel, “Service Lifetimes of Mineral End Uses,” U.S. Geological Survey, Minerals Resources External Research Program, 2007.

[48] D. T. Campbell and H. L. Ross, “The Connecticut Crackdown on Speeding: Time-Series Data in Quasi-Experimental Analysis,” *Law Soc. Rev.*, vol. 3, no. 1, pp. 33–54, 1968.

[49] I. M. Xiarchos, P. D. Candidate, N. Resource, and E. Program, “Steel : Price Links between Primary and Scrap Markets Corresponding Author : I . Introduction,” pp. 1–20, 2005.

[50] J. Zhang, M. Everson, T. Wallington, F. F. III, R. Roth, and R. Kirchain, “Assessing Economic Modulation of Future Critical Materials Use: The Case of Automotive-Related Platinum Group Metals,” *Environ. Sci. Technol.*, vol. 50, no. 14, pp. 7687–7695, 2016.

[51] S. Glöser-chahoud, J. Hartwig, and I. David, “The cobweb theorem and delays in adjusting supply in metals ’ markets,” vol. 32, no. 3, pp. 279–308, 2017.

[52] H. Hatayama, I. Daigo, Y. Matsuno, and Y. Adachi, “Evolution of Aluminum Recycling Initiated by the Introduction of Next-generation Vehicles and Scrap Sorting Technology,” *Resour. Conserv. Recycl*, vol. 69, pp. 35–49, 2012.

[53] Y. Igarashi, I. Daigo, and Y. Matsuno, “Dynamic Material Flow Analysis for Stainless Steels in Japan-reductions Potential of CO2 Emissions by Promoting Closed Loop Recycling of Stainless Steels,” *ISIJ Int.*, vol. 47, pp. 758–763, 2007.

[54] M. Saurat and S. Bringezu, “Metal Flows of Europe, Part II Exploring the Technological and Institutional Potential for Reducing Environmental Impacts,” *J. Ind. Eclo*, pp. 406–421, 2009.

[55] T. Ekvall and A. S.G.Andrae, “Attributional and Consequential Environmental Assessment of the Shift to Lead-Free Solders,” *Int J Life Cycle Assess*, vol. 11, no. 5, pp. 344–353, 2006.

[56] I. M. Xiarchos, J. J. Fletcher, D. Ph, A. R. Collins, W. C. Labys, and W. Virginia, “Three Essays in Environmental Markets : Dynamic Behavior , Market Interactions , Policy Implications,” *Scrap*, 2006.

[57] X. Li and X. Zhang, “Analysis of Aluminum Supply and Demand Development Trend in China and the United States,” *Adv. Mater. Res.*, vol. Vols. 962-, no. Trans Tech Publications, Switzerland, pp. 1936–1942, 2014.

[58] E. Elshkaki, A.;van der Voet, “The Consequences of the Use of Platinum in New Technologies on its Availability and on Other Metals Cycles,” in *Conservation and Recycling of Resources: New Research*, 2006.

[59] T. J. Considine, “Separability , functional form and regulatory policy in models of interfuel substitution,” *Energy Econ.*, vol. 11, pp. 82–94, 1989.

[60] D. J. Ramberg and J. E. Parsons, “The Weak Tie Between Natural Gas and Oil Prices,” vol. 33, no. 2, pp. 13–36, 2012.

[61] D. M. Arseneau and S. Leduc, “Commodity Price Movements in a General Equilibrium Model of Storage,” *IMF Econ. Rev.*, vol. 61, no. 1, pp. 199–224, 2013.

[62] J. Blomberg and P. Söderholm, “The economics of secondary aluminium supply: An econometric analysis based on European data,” *Resour. Conserv. Recycl.*, vol. 53, no. 8, pp. 455–463, 2009.

[63] M. Bouman, R. Heijungs, E. van der Voet, J. C.J.M., van den Bergh, and G. Huppes, “Material Flows and Economic Models: An Analytical Comparison of SFA, LCA, and partial equilibrium models,” *Ecol Econ*, vol. 32, pp. 195–216, 2000.

[64] N. Frees, “Crediting Aluminium Recycling in LCA by Demand or by Disposal,” *Int J Life Cycle Assess*, vol. 13, no. 3, pp. 212–218, 2008.

[65] C.-Y. C. Lin, “Estimating Annual and Monthly Supply and Demand for World Oil: A Dry Hole?,” *J. Econ. Lit.*, 2004.

[66] J. Martínez Fernández and M. A. E. Selma, “The dynamics of water scarcity on irrigated landscapes: Mazarrón and Aguilas in south-eastern Spain,” *Syst. Dyn. Rev.*, vol. 20, no. 2, pp. 117–137, 2004.

[67] S. Gloser, M. Soulier, and L. A. Tercero Espinoza, “Dynamic Analysis of Global Copper Flows. ,” *Environ. Sci. Technol.*, vol. 47, pp. 6564–6572, 2013.

[68] M. Liehr, A. Größler, M. Klein, and P. M. Milling, “Cycles in the sky: Understanding and managing business cycles in the airline market,” *Syst. Dyn. Rev.*, vol. 17, no. 4, pp. 311–332, 2001.

[69] A. Ignaciuk, F. Vöhringer, A. Ruijs, and E. C. van Ierland, “Competition between biomass and food production in the presence of energy policies: A partial equilibrium analysis,” *Energy Policy*, vol. 34, pp. 1127–1138, 2006.

[70] E. Alonso, R. Kirchain, and F. Field, “Platinum Availability for Future Automotive Technologies,” *Environ. Sci. Technol.*, vol. 46, pp. 12986–12993, 2012.

[71] E. Alonso, *Material scarcity from the perspective of manufacturing firms: case studies of cobalt and platinum*. Cambridge, MA: MIT PhD Dissertation, 2010.

[72] T. Zink, R. Geyer, and R. Startz, “A Market-Based Framework for Quantifying Displaced Production for Recycling or Reuse,” *J. Ind. Ecol.*, vol. 0, no. 0, pp. 1–11, 2015.

[73] A. Jones, D. Seville, and D. Meadows, “Resource sustainability in commodity systems: The sawmill industry in the Northern Forest,” *Syst. Dyn. Rev.*, vol. 18, no. 2, pp. 171–204, 2002.

[74] S. Arquitt, H. Xu, and R. Johnstone, “A system dynamics analysis of boom and bust in the shrimp aquaculture industry,” *Syst. Dyn. Rev.*, vol. 21, no. 4, pp. 305–324, 2005.

[75] H. U. Sverdrup, K. V. Ragnarsdottir, and D. Koca, “On modelling the global copper mining rates, market supply, copper price and the end of copper reserves,” *Resour. Conserv. Recycl.*, vol. 87, pp. 158–174, 2014.

[76] W. L. Auping, E. Pruyt, and J. H. Kwakkel, “Analysing the Uncertain Future of Copper with Three Exploratory System Dynamics Models,” *Int. Syst. Dyn. Conf. Delft, Netherlands, March, 2012*, no. Klis 2011, pp. 1–25, 2012.

[77] J. Kwakkel, W. Auping, and E. Pruyt, “Comparing Behavioral Dynamics Across Models: the Case of Copper,” *Proc. 32nd Int. Conf. Syst. Dyn. Soc.*, vol. In many pu, 2014.

[78] G. M. Whitesides and R. F. Ismagilov, “Complexity in Chemistry,” *Science (80-.).*, vol. 284, pp. 89–92, 1999.

[79] J. K. Parrish and L. Edelstein-Keshet, “Complexity, Pattern, and Evolutionary Trade-offs in Animal Aggregation,” *Science (80-.).*, no. 284, pp. 99–101, 1999.

[80] R. Foote, “Mathematics and Complex Systems,” *Science (80-.).*, vol. 318, pp. 410–412, 2007.

[81] W. B. Arthur, “Complexity and the Economy,” *Science (80-.).*, vol. 284, pp. 107–109, 1999.

[82] H. Poincare, “Science et Methode,” 1920.

[83] E. N. Lorenz, “Deterministic Nonperiodic Flow,” *Journal of the Atmospheric Sciences*, vol. 20, no. 2. pp. 130–141, 1963.

[84] D. H. Meadows, *Thinking in Systems: A Primer*. Chelsea Green Publishing, 2008.

[85] J. W. Forrester, “System Dynamics and the Lessons of 35 Years,” *A Syst. Approach to Policymaking*, vol. 3, no. 2, pp. 1–35, 1991.

[86] H. B. Weil, “Why Markets Make Mistakes,” *MIT Sloan Sch. Work. Pap. 4745-09*, 2010.

[87] J. Ladyman, J. Lambert, and K. Wiesner, “What is a Complex System,” *Euro Jnl Phil Sci*, no. 3, pp. 33–67.

[88] M. Mitchell, *Complexity: A Guided Tour*. Oxford University Press; 1 edition (September 1, 2011), 2011.

[89] D. Lane, *Hierarchy in Natural and Social Sciences: Hierarchy, Complexity, Society*. P.O. Box 17, 3300 AA Dordecht, The Netherlands: Springer, 2006.

[90] P. W. Anderson, “More is different.,” *Science*, vol. 177, no. 4047, pp. 393–396, 1972.

[91] H. Simon, “The Architecture of Complexity,” *Sci. Artif. Cambridge, MA MIT Press.*, pp. 192–229, 1969.

[92] H. Simon, *Can There be a Science for Complex Systems*, vol. Chapter 1. 2001.

[93] R. Lucas, “Economic Policy Evaluation: A Critique,” *Carnegie-Rochester Conf. Ser. Public Policy*, pp. 19–46, 1976.

[94] H. Pesaran, Y. Shin, and R. Smith, “Bounds Testing Approaches to the Analysis of Level Relationships,” *J. Appl. Econom.*, vol. 16, no. 3, pp. 289–326, 2001.

[95] S. Pauliuk, T. Wang, and D. B. Müller, “Steel all over the world: Estimating in-use stocks of iron for 200 countries,” *Resour. Conserv. Recycl.*, vol. 71, pp. 22–30, 2013.

[96] J. D. Hamilton, *Time Series Analysis*, 1st ed. Princeton University Press, 1994.

[97] J. D. Hamilton, “Understanding Crude Oil Prices,” *NBER Work. Pap. No. 14492*, 2008.

[98] T. Wu, H. Zhao, and X. Ou, “Vehicle Ownership Analysis Based on GDP per Capita in China: 1963-2050,” *Sustainability*, vol. 6, pp. 4877–4899, 2014.

[99] X. Zhu, S. The, E. Perspectives, and N. Fall, “Understanding China 鈥s Growth : Past , Present , and Future China 鈥s Understanding Present , and Future Growth : Past , Xiaodong,” *Am. Econ. Assoc.*, vol. 26, no. 4, pp. 103–124, 2014.

[100] L. Hong, N. Zhou, D. Fridley, W. Feng, N. Khanna, and L. Berkeley, “Modeling China ’ s Building Floor-Area Growth and the Implications for Building Materials and Energy Demand,” pp. 146–157, 2014.

[101] R. C. Diamond, “An overview of the u.s. building stock 1,” no. January, pp. 1–17, 2001.

[102] E. Reliability, “United States Electricity Industry Primer,” no. July, 2015.

[103] C. for A. Research, “The U . S . Automotive Market and Industry in 2025,” no. June, 2011.

[104] “IMF World Economic Outlook,” 2017. [Online]. Available: https://www.imf.org/external/pubs/ft/weo/2017/01/weodata/index.aspx.

[105] “Population Pyramid,” 2018. [Online]. Available: https://www.populationpyramid.net/world/2017/.

[106] J. Jacques, “Creating leading copper and coal businesses,” *U.S. Geol. Surv. Sci. Investig. Rep.*, no. June, 2010.

[107] W. A. Group, *Ward’s World Motor Vehicle Data 2014*. Wards Communications (September 2014), 2014.

[108] C. Government, “Transportation Sector Update 2015,” *Natl. Bur. Stat. China*, 2015.

[109] M. C. P. Moura, S. J. Smith, and D. B. Belzer, “120 Years of U . S . Residential Housing Stock and Floor Space,” pp. 1–18, 2015.

[110] US Energy Information Administration, “Annual Energy Outlook (Early Release),” no. DOE/EIA-0383er(2013). US Department of Energy,

Washington, D.C., p. 16, 2013.

[111] D. L. Mosier, V. I. Berger, and D. a. Singer, “Volcanogenic massive sulfide deposits of the world; database and grade and tonnage models,” *U.S. Geol. Surv. Open-File Rep. 2009-1034*, pp. 1–46, 2009.

[112] D. P. Cox, D. A. Lindsey, D. A. Singer, B. C. Moring, and M. F. Diggles, “Sediment-Hosted Copper Deposits of the World: Deposit Models and Database: U.S. Geological Survey Open-File Report 03-107,” p. 53, 2007.

[113] D. A. Singer, V. I. Berger, and B. C. Moring, “Porphyry Copper Deposits of the World: Database And Grade and Tonnage,” *Usgs*, p. 45, 2008.

[114] and J. W. Michael L. Zientek, James D. Bliss, David W. Broughton, Michael Christie, Paul D. Denning, Timothy S. Hayes, Murray W. Hitzman, John D. Horton, Susan Frost-Killian, Douglas J. Jack, Sharad Master, Heather L. Parks, Cliff D. Taylor, Anna B. Wilson, Niki E., “Sediment-Hosted Stratabound Copper Assessment of the Neoproterozoic Roan Group, Central African Copperbelt, Katanga Basin, Democratic Republic of the Congo and Zambia,” *USGS*.

[115] USGS, “Global Assessment of Undiscovered Copper Resources.” [Online]. Available: https://mrdata.usgs.gov/sir20105090z/.

[116] G. Mineral, R. Assessment, and T. U. S. G. Survey, “Estimate of Undiscovered Copper Resources of the World , 2013,” no. January, 2014.

[117] T. Malthus and G. Gilbert, *An Essay on the Principle of Population (Oxford World’s Classics)*, First Edit. Oxford World’s Classics, 2008.

[118] R. LeBlanc, “Facts About Copper Recycling-Information About Copper and Copper Recycling,” *Sustain. Businesses,* 2016.

[119] T. H. E. Importance, O. F. Recycling, C. Is, and R. S. Co, “Copper Recycling,” 2013.

[120] C. Jamasmie, “World’s First Seabed Gold, Copper, Silver Mine to Begin Production in 2019,” *Mining.comm,* 2017.

[121] K. Bachner, “Piping for New Construction-Plastic vs. Copper,” *Heal. Build. Sci.,* 2015.

[122] M. Hughes, “Could Carbon Nanotube ‘Ice Wires’ Be the Conductor of the Future?,” *All About Circuits*.