

**Modeling Learning When Alternative Technologies Are Learning & Resource
Constrained: Cases In Semiconductor & Advanced Automotive Manufacturing**

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Abstract

When making technology choice decisions, firms must consider technology costs over time. In many industries, technology costs have been shown to decrease over time due to (a) improvements in production efficiency and the accumulation of worker experience accompanying production, known as “learning-by-doing,” and (b) firm investments in research and development, worker training and other process improvement activities, known as “learning-by-investing.”

Rapid technological progress may mean that new technologies become available while existing technologies still exhibit learning-related cost reductions. In these cases, switching to a new technology means giving up these ongoing benefits while also incurring new technology introduction costs. Additionally, In some industries, high switching costs, regulatory compliance and/or the risks associated with new technologies may require firms to continue allocating production volume and investments to an existing technology whether or not a new technology is introduced. In these cases, firms must decide how to allocate finite production volume and investment resources between technologies. Learning is driven by resource allocation. Therefore, sharing finite resources among multiple learning technologies may reduce the learning-related benefits associated with each. This may lead firms to underestimate technology costs, leading to sub-optimal technology choice and resource allocation decisions.

A methodology is presented which couples technology costs over time via capacity and investment resource allocation to characterize the impacts of (1) learning in an incumbent technology, and (2) resource allocation constraints, on technology choice and resource allocation decisions. Case studies in the semiconductor and automotive industries are examined using this method in combination with process based cost modeling. We find that (1) when the existing technology is still learning, diverting resources to a new technology results in an opportunity cost in both technologies which diminishes the benefits of switching technologies; (2) this effect can persist over a wide range of learning rates and technology costs; (3) capacity allocation constraints can significantly change the conditions under which the firm should choose a new technology, and (4) cumulative production volume and investment based learning differentially impact technology costs, leading to different cost-minimizing resource allocation decisions.

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Table of Contents

1	Introduction	13
2	Background and Contributions	16
2.1	Learning in Manufacturing	17
2.2	Other Factors Impacting Technology Costs Over Time	21
2.2.1	Knowledge Transfer	21
2.2.2	Knowledge Depreciation	22
2.2.3	Economies of Scale	23
2.2.4	Uncertainty	23
2.3	Learning in Technology Choice & Gap Analysis	25
2.4	Gap Analysis Summary	30
2.5	Research Contributions	33
2.5.1	Contributions to the Literature	33
3	Methodology Overview	33
4	Cumulative Production Volume Driven Learning	35
4.1	Decision Contexts	36
4.2	Unit Costs Over Time	36
4.3	Production Costs Over Time	43
4.4	Total Production Costs	48
4.5	Total Production Cost Function Characteristics	52
4.6	Impact of Decision Context on Technology Decisions	57
4.7	Impact of Factors Driving Technology Costs on Technology Decisions	61
4.7.1	Initial Technology Costs and Learning Behavior	62
4.7.2	Timeframe Over Which Costs Are Accrued	62
4.7.3	Total Production Volume Over Time	63
4.8	Cumulative Production Volume Learning Example	64
4.8.1	Learning Functions	64
4.8.2	Production Function	66
4.8.3	Baseline Cost Functions	66
4.8.4	Baseline Scenario Considered	67
4.8.5	Baseline Results by Decision Context	68

4.8.6	Discussion of Base Case Results	75
4.9	Sensitivity Analyses	78
4.9.1	Factors considered	78
4.9.2	Factor value ranges considered	80
4.9.3	Cost functions	81
4.9.4	Results and discussion	82
5	Learning-by-Doing Case Study: Semiconductor Fabrication	93
5.1	Background	93
5.2	Overview of Approach	94
5.3	Process-Based Cost Model Description	95
5.3.1	PBCM Without Learning	96
5.3.2	Learning and Dynamic PBCM	99
5.4	Production and Cost Parameter Values	101
5.4.1	Production Function and Parameter Values	102
5.4.2	Chip and Wafer Geometry Parameter Values	102
5.4.3	Capital and Variable Cost Parameter Values	102
5.4.4	Learning Parameter Values	103
5.4.5	Labor Parameter Values	104
5.5	Scenarios Considered	104
5.5.1	Scenario Parameter Values	106
5.6	Results	107
5.7	Identification of Key Cost Drivers	111
5.8	Sensitivity Analyses	112
5.8.1	Parameter Values Considered	113
6	Cumulative Investment and Production Volume Based Learning	119
6.1	Decision Contexts	120
6.2	Cumulative Investment Driven Unit Costs Over Time	121
6.3	Cumulative Investment and Cumulative Production Volume Driven Unit Costs Over Time	122
6.4	Total Production Costs	122
6.5	Total Production Cost Function Characteristics	123

6.6	Cumulative Production Volume and Cumulative Investment Learning Example	127
6.6.1	Learning Functions	127
6.6.2	Production and Investment Functions	128
6.6.3	Cost Functions	129
6.6.4	Baseline Scenario Considered	131
6.6.5	Baseline Results by Decision Context	133
7	CPV and Investment Driven Learning Case Study: Novel Automotive Sub-Component Assembly	146
7.1	Background	146
7.2	Process Based Cost Model	146
7.2.1	Learning and Dynamic PBCM	148
7.3	Production and Cost Parameter Values	150
7.3.1	Production and Investment Functions	150
7.3.2	Cost Parameter Values	151
7.3.3	Learning Parameter Values	151
7.3.4	Labor Parameter Values	152
7.4	Scenarios Considered	152
7.4.1	Scenario Parameter Values	153
7.5	Results by Decision Context	157
7.6	Sensitivity Analyses	160
8	Conclusions and Future Work	170
8.1	Conclusions	170
8.2	Future Work	171
8.2.1	Incorporating Uncertainty	171
8.2.2	Incorporating Knowledge Depreciation	172

List of Figures

Figure 1: Learning curve for labor hours as a function of cumulative production volume (Argote and Epple 1990)	18
Figure 2: Distribution of progress ratios observed across multiple industries (Dutton and Thomas 1984)	24
Figure 3: Overview of factors and decisions considered	35
Figure 4: Potential A and B unit cost curves as a function of total cumulative production volume for a single product when (i) A is no longer learning when $CPV \geq CPV^{T_0}$, (ii) A continues to learn when $CPV \geq CPV^{T_0}$	39
Figure 5: Potential learning curves, capacity allocation and realized unit cost curves for each technology for decision context (a) in Table 3	40
Figure 6: Potential learning curves, capacity allocation and realized unit cost curves for each technology for decision context (c) in Table 3	42
Figure 7: Capacity allocation, realized total cost and realized versus potential total cost curves as a function of time when technology A no longer exhibits learning for $t \geq T_0$	45
Figure 8: Capacity allocation, realized total cost and realized versus potential total cost curves as a function of time when technology A continues to exhibit learning for $t \geq T_0$	47
Figure 9: Total production costs as a function of capacity allocated to B for allocation scenarios in Figure 7 (row (1)), and Figure 8 (row (1))	50
Figure 10: Examples of total production costs as a function of F when (1) A no longer exhibits learning for $t \geq T_0$, and (2) A exhibits learning for $t \geq T_0$	55
Figure 11: Impacts of allocation constraints on technology decisions when (1) some production must continue to be allocated to the A, and (2) some production must be allocated to B	59
Figure 12: Total production cost behavior as a function of F and technology decisions for each decision context in Table 3 for the example technologies in Figure 5 through Figure 11	61
Figure 13: Base case results by decision context	76

Figure 14: Technology decision as a function of the learning rate for technology A and the ratio of initial costs of B to initial costs of A for the baseline case	83
Figure 15: Technology decision as a function of the learning rate for technology A and the ratio of initial costs of B to initial costs of A for the case when $g = 5\%$	84
Figure 16: Technology decision as a function of the learning rate for technology A and the ratio of initial costs of B to initial costs of A for the case when $d = 5\%$	86
Figure 17: Technology decision as a function of the learning rate for technology A and the ratio of initial costs of B to initial costs of A for the case when $T_i = 2$ years	87
Figure 18: Technology decision as a function of the learning rate for technology A and the ratio of initial costs of B to initial costs of A for the case when $T_i = 5$ and $T_f - T_i = 15$ years	88
Figure 19: Baseline case threshold capacity constraint, \hat{F}_i , below which the firm should stay with A as a function of the A learning rate and initial cost ratio	91
Figure 20: Baseline case threshold capacity constraint, \hat{F}_i , above which the firm should switch to B as a function of the A learning rate and initial cost ratio	92
Figure 21: Static PBCM framework (Field and Kirchain 2007)	96
Figure 22: Dynamic process-based cost modeling incorporating learning in both processing requirements (wafer starts per month and yield), and factor costs (raw wafer costs)	100
Figure 23: Total production costs as a function of the production capacity allocated to 450mm for the base case (decision context (a))	107
Figure 24: Determination of the capacity constraint at which introducing 450mm technology is economically feasible.	108
Figure 25: Total production costs as a function of the production capacity allocated to 450mm when continued learning is expected in 300mm (decision context (c))	109
Figure 26: Determination of the range over which the firm should only use 300mm technology	110
Figure 27: 450mm vs. 300mm decision results as a function of introduction year and the capacity fraction allocated to 450mm when 300mm does not exhibit learning in any operational parameters	114

Figure 28: 450mm vs. 300mm decision results as a function of introduction year and the capacity fraction allocated to 450mm when all three 300mm operational parameters exhibit 98% progress ratios	115
Figure 29: Context for technology choice decisions when considering both CPV and cumulative investment based learning	120
Figure 30: Stylized example of the total production cost curve as a function of both capacity and investment allocations	126
Figure 31: (i) Total production costs as a function of capacity allocation to B for decision context (b.3); (ii) – (iv) TPC as a function of investment allocation to B when (ii) $F = 25\%$, and (iii) $F = 50\%$ and (iv) $F = \hat{F}_i = 76\%$	140
Figure 32: (i) Total production costs as a function of capacity allocation to B for decision context (d.3); (ii) – (iv) TPC as a function of investment allocation to B when (ii) $F = \hat{F}_{ii} = 27\%$, (iii) $F = 25\%$, and (iii) $F = 50\%$ and (iv) $F = 75\%$	143
Figure 33: Dynamic process-based cost modeling incorporating learning in cycle time, tool life and reject rate	149
Figure 34: Total production costs as a function of the investment allocated to B for the base case	158
Figure 35: Total production costs as a function of the investment allocated to B when the existing technology still exhibits learning-by-investing (decision context d.2)	160
Figure 36: Total production costs as a function of the investment allocated to technology B for the initial investment years, T_{INV} in Table 36 when $F = 80\%$	162
Figure 37: Regions over which sharing investments results in reduced total production costs when the firm begins investing in B in 2011 and 2014 and $F = 80\%$	163
Figure 38: Optimal investment allocation to technology B as a function of the initial year the firm allocates investment to B when $F = 80\%$	164
Figure 39: Optimal investment allocation to technology B as a function of the initial year the firm allocates investment to B when $F = 25\%$, 50% , 80% , and 90%	165
Figure 40: Optimal investment allocation to technology B and resulting total production costs for the “even split” investment strategy	170

List of Tables

Table 1: Summary of previous literature on learning and technology choice	32
Table 2: Contexts for technology choice decision making considered	34
Table 3: Context for technology choice decisions when considering only CPV learning	36
Table 4: Baseline case parameters and values considered	68
Table 5: Semiconductor case sensitivity analysis parameters and value ranges considered	81
Table 6: Baseline sensitivity analysis parameters	82
Table 7: Results of growth in production volume sensitivity compared to the baseline case	85
Table 8: Results of discount rate sensitivity compared to the baseline case	86
Table 9: Results of sensitivity to B introduction year compared to the baseline case	88
Table 10: Wafer size transitions and industry costs	93
Table 11: Chip and wafer geometry parameter values considered	102
Table 12: Capital costs for baseline 300mm and 450mm facilities	103
Table 13: Variable costs per wafer	103
Table 14: Initial and limiting values for learning parameters	104
Table 15: Labor parameter values	104
Table 16: Scenario parameter values	106
Table 17: Summary of technology decisions for the semiconductor case	111
Table 18: 300mm learning parameter values tested to determine which factor(s) are most important in impacting the technology decision	112
Table 19: 450mm vs. 300mm sensitivity analyses parameter values considered	113
Table 20: Parameters and technology decisions for decision scenarios (3) – (5) in Figure 28	118
Table 21: Baseline case parameters and values considered	132
Table 22: Capacity and investment allocations resulting in minimum total production costs for decision contexts (a) and (a.3)	135
Table 23: Capacity and investment allocations resulting in minimum total production costs for decision contexts (c) and (c.3)	136

Table 24: Total production costs and investment allocation decisions for fixed capacity allocations	139
Table 25: Sample capacity and investment allocations resulting in minimum total production costs for decision contexts (d) and (d.3)	142
Table 26: Sample investment allocations and corresponding capacity allocations resulting in minimum total production costs under decision context (c.4)	145
Table 27: Relative weighting on cost elements for each welding technology	151
Table 28: Initial and limiting CPV driven learning parameter values	152
Table 29: Labor parameter values	152
Table 30: Production and capacity allocation parameter values that remain constant for the scenarios considered	154
Table 31: Progress ratios and learning exponents held constant for the scenarios considered	155
Table 32: Investment budget and portfolio parameter values used for all the scenarios considered	156
Table 33: Investment portfolio for each technology	156
Table 34: Parameter values for the decision context considered	157
Table 35: Summary of technology decisions in the automotive case decision contexts (d.1) and (d.2)	160
Table 36: Sensitivity parameters and values considered	161
Table 37: Optimal investment strategies for different initial years the firm invests in B	161
Table 38: Fraction of investment in technology A allocated to each factor exhibiting learning	167
Table 39: Total production costs as a function of the first year the firm invests in technology B for the investment profiles in Table 38 assuming the firm begins investing in B in 2011	167
Table 40: Total investment allocations to A and B and the resulting total production costs for each of the investment strategies in Table 38 when the firm begins investing in B in 2011	169

1 Introduction

Firms change technologies to acquire competitive advantage, keep up with innovation at other firms and comply with new policies among other reasons. Effective technological change over time can enable competitive advantage, decreasing manufacturing costs while increasing effective capacity and product quality. When making technology choice decisions, firms compare the production costs associated with a new technology against those of the current technology: if the long-term benefits of switching to a new technology outweigh the associated short-run introduction costs relative the incumbent technology, then the firm should change technologies. Therefore, it is important that decision makers accurately characterize these costs and benefits over time.

Empirical observations across myriad industries have demonstrated that cumulative production volume and firm investments in activities that increase the stock of knowledge can drive down production costs over time. The concept of “learning” has been developed as a mechanism to characterize these effects. Learning refers to the production cost reductions associated with the accumulation of knowledge and/or manufacturing experience. The former, often referred to as “learning-by-doing, or “autonomous learning,” reflects the observation that, as more “widgets” produced, workers become more familiar with required tasks and minor process improvements can be made, reducing waste while increasing efficiency. A second form of learning, referred to as “learning-by-investing” or induced learning,” has been identified as an important mechanism to characterize how firm investments in activities improving technical knowledge reduces technology costs over time. Examples include research and development, ongoing worker training, and experimentation. This type of learning can involve investments both before and after the launch of a new technology, and may impact technology introduction costs in addition to the ongoing cost reductions seen in cumulative production related learning. Recent research has concluded that incorporating both cumulative production volume and cumulative investment driven learning enables firms to better characterize the factors impacting technology costs and identify additional opportunities to influence these factors.

In some industries, rapid technological progress may make new technologies available while existing technologies still exhibit learning-related cost reductions. Additionally, high switching costs or the risks associated with new technologies drive firms to employ multiple technologies simultaneously over extended time periods. In these cases, finite resources will be shared between multiple technologies. Sharing finite resources among multiple learning technologies results in less “movement down the learning curve” associated with that resource for each technology. For example, diverting production volume away from a technology exhibiting CPV based learning will result in higher associated unit costs due to a reduction in realized learning-by-doing. As we will demonstrate, under certain conditions this can result in higher than expected unit costs for *both* the incumbent *and* new technology over time. This *foregone learning* represents an opportunity cost to the firm. As a result, firms may *overvalue* new technologies and *undervalue* existing technologies, leading firms in some industries to make incorrect and costly decisions about which technologies to choose and how to allocate resources over time.

The semiconductor fabrication industry provides an illustrative example. Demand for integrated circuits increases every year as does the expectation of more performance per chip, commonly known as “Moore’s Law” (McClean, Matas et al. 2008). Semiconductor manufacturers meet these goals in two ways: decreasing the feature size per chip increases transistor density (increasing performance); and, increasing the area of the silicon wafers enables better-than-linear increases in chip density per wafer (increasing returns to scale). Each strategy requires novel processing technologies and significant capital investments by chipmakers, and each new technology takes time to implement and ramp. As a result, individual firms often employ multiple processing technologies simultaneously. Rapid innovation in the semiconductor space means that new technologies become available while “continuous cost improvements are sure things for existing technologies and platforms” (Chien, Wang et al. 2007). This means that new *and* existing processing technologies exhibit ongoing learning related cost reductions as a function of cumulative production volume. Given that each firm produces a finite

production volume in any time period, chipmakers are faced with the following technology choice decision: when, if at all, does it make sense to introduce new technologies and how should finite production volume be allocated among new and existing technologies?

This work extends existing methods to help answer these kinds of questions by explicitly incorporating the impacts of shared resources and among multiple technologies exhibiting learning related cost reductions over time. We then couple the resulting formalism with process-based cost modeling to examine two case studies in detail. The first study explores the semiconductor industry's transition from the current 300mm silicon wafer to a larger 450mm wafer. Via extensive data collection in conjunction with industry, we are able to exercise the methodology in a "real world" setting in which the pace of technological innovation changes rapidly. Our results suggest that the cost implications of shared production capacity can significantly impact the choice of when to introduce the new wafer size processing technology and how to allocate production resources over time. The second case study enables us to characterize the impacts of both learning by doing and learning by investing in the automotive industry. In this case, we worked with a major North American car manufacturer to collect 24-hour production data and investment decisions and outcomes over a two-year period for novel welding technologies for advanced automotive propulsion systems. These cases enable us not only to determine how capacity and investments should be allocated between new and existing technologies, but also to identify the primary production factors driving learning and explore the conditions under which our method changes technology choice decisions.

This thesis is structured as follows. Chapter 2 provides a survey of the existing literature and identifies how this work contributes. Chapter 3 provides an overview of our methodological approach. In chapters 4 through 7 we develop a methodology and case studies to explore these questions impacts. Chapter 4 examines the case of learning-by-doing, in which technology costs decrease over time only as a function of cumulative production volume. Chapter 6 builds on chapter 4, incorporating learning-by-investing, enabling both capacity allocation and firm investment resources to impact technology

costs. In each chapter, we derive a general expression for total production costs over time which couples individual technology costs via resource allocation. Then, we characterize the behavior of production cost as a function of the decision context, time interval over which costs are accrued and initial technology costs. We then select functional forms to model technology learning and develop cost functions integrating these forms. Finally, we select production functions and parameter values to characterize both how the decision context impacts technology decisions, and the conditions under which the factors driving technology costs impact these decisions within each context.

Chapters 5 and 7 provide detailed case studies. Chapter 5 presents a case from the semiconductor fabrication industry, in which enormous switching costs require that new technologies are ramped up over time, while rapid technological innovation results in new technologies becoming available while existing technologies still exhibit learning-by-doing related cost reductions. Chapter 7 presents a case from the automotive industry, in which the existing and new technologies exhibit both learning-by-doing and learning-by-investing and will be produced simultaneously. In both cases, we identify the key production factors driving learning and develop process based cost models to characterize how these factors impact technology costs. In each case, we characterize the conditions under which learning in the existing technology and resource allocation change the technology decision.

Finally, Chapter 8 discusses conclusions and opportunities for future work to extend this research.

2 Background and Contributions

Characterizing how the addition of learning in multiple simultaneously produced technologies impacts technology choice requires understanding the mechanisms by which learning impacts production costs and how previous work has characterized learning as a factor influencing technology decision-making. This chapter discusses these issues and highlights the areas where this research adds value.

2.1 Learning in Manufacturing

The concept of learning was first introduced by Wright, and later confirmed by Alchain, as a way to explain the reductions in the number of direct labor hours observed as cumulative production volume (CPV) increased in aircraft manufacturing (Wright 1936; Alchain 1963). Specifically Wright observed that, in aircraft frame production, each doubling of cumulative production volume resulted in a uniform decrease in the number of direct labor hours required to produce each frame. This cumulative production volume based learning would later become known as cumulative “learning-by-doing” (Arrow 1962) or “autonomous learning” (Levy 1965; Dutton and Thomas 1984). Many forms have historically been used to model learning in an effort to explain experimental production data in different industries. Several authors provide excellent summaries of learning curve functional forms (see for example (Badiru 1992; Yelle 2007)). However, the power law form first introduced by Wright remains the most commonly used expression to model learning:

$$y(x) = ax^n \quad (1)$$

where x is the cumulative number of units produced; y is the number of direct labor hours required to produce the “ x th” unit; a is the direct labor hours required to produce the first unit; and n is the learning exponent, which characterizes the rate at which direct labor hours are reduced with increasing CPV. Figure 1 represents one possible “learning curve” for assembly labor hours to construct aircraft as a function of cumulative number of aircraft produced.

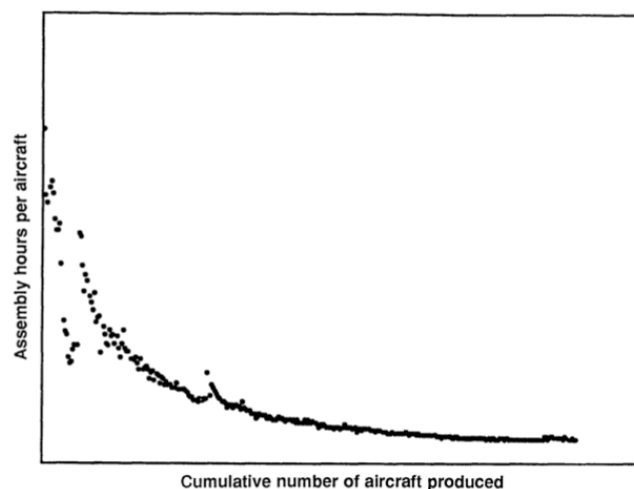


Figure 1: Learning curve for labor hours as a function of cumulative production volume (Argote and Epple 1990)

The slope of the learning curve is determined by the learning exponent, b , characterized in terms of the progress ratio, p : the percentage reduction in unit cost achieved with every doubling of CPV (Lloyd 1979; Montgomery and Day 1983; Lieberman 1987):

$$p = \frac{a \cdot [2x]^n}{a \cdot [x]^n} = 2^n \quad (2)$$

The learning exponent, b , is then defined as:

$$n = \frac{\ln(p)}{\ln(2)} \quad (3)$$

For example, in the Wright model, an 80% progress ratio would mean that the number of labor hours required to produce an aircraft frame decreases by 20% with each doubling of CPV. In this case, the learning exponent is given by:

$$n = \frac{\ln(0.8)}{\ln(2)} = -0.32 \quad (4)$$

Initially, small efficiency gains largest cost reductions are realized early in production. A higher production levels however, these gains become more difficult and expensive to achieve, leading to the asymptotic behavior shown in Figure 1. This matches our intuition that the most significant cost-reducing discoveries will be made early in the production process, and that, over time, as the production process becomes more refined, it becomes more and more difficult to make further improvements.

Since Wright's first formulation, learning curves have been used extensively to explain the empirical relationship between increasing cumulative production volume and decreasing production costs across a wide range of industries, including: aircraft manufacturing (Asher 1956; Alchain 1963; Argote and Epple 1990; Frischtak 1994), automobiles (Baloff 1971), apparel and textiles (Baloff 1971; Jarmin 1994), ships (Argote, Beckman et al. 1990), power generation (Sultan 1974; Zimmerman 1982; Kouvaritakis, Soria et al. 2000; Colpier and Cornland 2002) metals products (Dudley 1972; Ayres and Martinas 1992), chemical processing (Lieberman 1984; Sinclair,

Klepper et al. 2000) and semiconductors (Dick 1991; Gruber 1992; Growchowski and Hoyt 1996; Hatch and Mowery 1998; Chung 2001). CPV based learning curves have been also used to model alternative energies in a number of industries including fuel cells for automotive drive trains (Tsuchiya and Kobayashi 2004), ethanol production (Goldemberg, Coelho et al. 2004) and wind and solar power (Neij 1997; Harmon 2000; IAEA 2000; Trancik 2006; Trancik and Zweibel 2006). The impacts of learning-by-doing based knowledge acquisition has been explored as a driver for more effective and efficient process technology change by Carrillo and Gaimon (Carrillo and Gaimon 2000; Carrillo and Gaimon 2002), and learning curves have been combined with process-based cost modeling to explore how learning impacts process technology costs for hydroforming, wire drawing and assembly by Nadeau et al. (Nadeau, Kar et al. 2010).

Cumulative firm investments (CI), in non production-specific activities have also been identified as an important factor influencing technology costs and product or process quality over time. Research suggests that as firms invest in a technology, the resulting increase in the stock of knowledge associated with that technology can lower technology introduction and production costs and/or increase product/process technology performance/quality. This “learning-by-investing” or *induced* learning (Dutton and Thomas 1984; Fine 1985; Fine 1986; Tapiero 1987; Fine and Porteus 1988; Li and Rajagopalan 1998; Zangwill and Kantor 1998; Carrillo and Gaimon 2000; Zangwill and Kantor 2000; Carrillo and Gaimon 2002) has been observed for multiple investment activities, including: worker training (Baloff 1970; Adler and Clark 1991), pre-production planning activities (Levy 1965; Ramamurthy 1995; Pisano 1996), experimentation (Bohn 1995; Terwiesch and Bohn 2001) and R&D investments (Baloff 1966; Cohen 1989; Argote and Epple 1990; Adler and Clark 1991; Pisano 1996; Thomke 1997; Li and Rajagopalan 1998; Kouvaritakis, Soria et al. 2000; Barreto and Kypreos 2004; Miketa and Schratzenholzer 2004; Jamasb 2007; Yelle 2007; Li and Rajagopalan 2008). As with CPV driven learning, the power law functional form is most often selected to model CI driven learning. For example, to characterize the impact of CI on unit cost (1) becomes:

$$C(CI) = C_0 \cdot [CI]^{n_2} \quad (5)$$

where $C(CI)$ is the unit cost at a level of cumulative investment, CI , C_0 is the initial unit cost, and n_2 is a unique learning exponent corresponding to the expected investment-related progress ratio via (2) and (3).

Recent research has found that including both CPV and cumulative investment enables firms not only to better characterize the factors impacting technology costs, but also to identify additional opportunities to influence these factors. For example, Li and Rajagopalan incorporate both forms of learning to model the impacts of learning on production costs and quality, citing that “the history of automobile manufacturing provides convincing evidence that most production processes benefit not only from autonomous learning but also induced learning” (Li and Rajagopalan 1998). Recent research has introduced the two-factor learning curve (2FLC) (Kouvaritakis, Soria et al. 2000), which couples CPV and CI, enabling independent characterization of how learning in each factor impacts technology costs. The 2FLC follows “a standard (Wright) learning-curve scheme driven by two main factors, namely cumulative capital deliveries and cumulative R&D flows” (Kouvaritakis, Soria et al. 2000). The 2FLC functional form combines (1) and (5):

$$C(CPV, CI) = C_0 \cdot [CPV]^{n_1} \cdot [CI]^{n_2} \quad (6)$$

Jamasb (Jamasb 2007) and Miketa (Miketa and Schratzenholzer 2004) employ the 2FLC to explore how resources should be spread among different energy technological sectors worldwide in the presence of R&D and production volume based learning, while Barreto and Kypreos (Barreto and Kypreos 2004) use a variation of the 2FLC to explore the effects of innovation and technological knowledge diffusion via spillover in the presence of learning.

In practice, learning takes place in a number of areas simultaneously during manufacturing, often at different rates. Recent work in our group has differentiated different elements of the production process where learning may occur (Kar 2007; Nadeau, Kar et al. 2010). Their results suggest that the dynamics of these independent processes can dramatically impact the observed technology learning rate, and they are

able to identify specific process levers, including cycle time, downtime and reject rates, where additional investment will significantly impact the realized cost reductions due to learning. For example, an increase in the number of rejected products in manufacturing causes firms to produce more units to reach a target production volume, while a decrease in cycle time reduces the total production time to reach this target. Some of these factors may provide disproportionate learning-related benefits relative to others. For example, in electric vehicle battery manufacturing, individual battery cells are often welded together to form cell groups before these groups are assembled into complete battery packs. Cell material costs make up a large portion of total production costs. Therefore, reducing the rate at which welds are defective is more important to reducing production costs than decreasing the weld rate. This suggests that the firm will be best served by focusing process improvements and/or investments on decreasing weld defects. An analogous example in the semiconductor industry is increasing the total quantity of chips produced by decreasing the cycle time to process silicon wafers versus increasing the yield of useable chips on each wafer. Grubler has explored the role learning in the yield rate plays in reducing costs in the semiconductor industry (Gruber 1992; Gruber 1994), while Bohn and Terwiesch examine the yield versus cycle time tradeoff resulting from firm investments on in-line experimentation during production ramp up (Bohn 1995; Bohn and Terwiesch 1999; Terwiesch and Bohn 2001).

2.2 Other Factors Impacting Technology Costs Over Time

This work focuses on the two learning mechanisms most often cited as having the largest impacts on production costs over time: learning-by-doing and learning-by-investing. However, research has identified other learning mechanisms and factors impacting the perceived benefits of learning. Future work by the authors will focus on characterizing the impacts these factors may have on the results presented in this research.

2.2.1 Knowledge Transfer

Research suggests that firms may benefit from the transfer of technical knowledge in two ways: between technologies within the firm and between firms (Montgomery and Day 1983; Cohen 1989; Epple, Argote et al. 1991; Darr, Argote et al. 1995; Epple, Argote et al. 1996; Barreto and Kypreos 2004). Intra-firm knowledge transfer occurs when worker

skills and other production knowledge gained by employing one technology can be reused on another. As a result, “an organization with previous experience in a related product appearing to have a faster rate of learning than an organization without prior experience” (Argote, Beckman et al. 1990; Epple, Argote et al. 1991). Carillo and Gaimon also find evidence for the transfer of knowledge between processing technologies, finding that *learning synergies* “may contribute to process change effectiveness” (Carrilo and Gaimon 2000; Carillo and Gaimon 2002). Knowledge may also be transferred between firms via the movement of workers, technological reverse engineering, conferences etc. Research suggests that as a result of this type of transfer may provide competitive advantage in some industries to firms that are late to employ a given technology via “higher (initial) productivity levels than their counterparts with early start dates” (Argote and Epple 1990).

2.2.2 Knowledge Depreciation

While the transfer of technical knowledge can benefit firms, other research has found that this knowledge can depreciate over time, especially in industries with intermittent production (Keachie and Fontana 1966; Baloff 1970; Sule 1983; Dar-El 2000). Several mechanisms have been identified which contribute to knowledge depreciation, including individual forgetting, misplaced manuals/records and worker turnover (Smunt 1987; Argote, Beckman et al. 1990; Elm'Aghraby 1990; Darr, Argote et al. 1995; Jaber and Bonney 1996; Jaber and Silstrom 2004). Of these, individual workers forgetting production related tasks have been shown to have the largest negative impact on learning (Nembhard and Osothsilp 2001). Multiple models have been developed to explore the specific mechanisms by which forgetting occurs, including the type of production activity (Bailey 1989; Arzi and Shtub 1997; Globerson 1998; Dar-El 2000), length of the break between production cycles (Carlson and Rowe 1976), and the level of learning achieved prior to a production break (Wickelgren 1981; Globerson 1998). Several authors have proposed implementing a “knowledge stock function” to characterize depreciation in knowledge gained via investments and incorporate a time lag between when investments are made and learning occurs (Griliches 1984; Griliches 1995; Wantanabe 1995; Wantanabe 1999). These models result in a “forgetting-by-not-doing” feature, which

results in increases in a technology's production costs without continual R&D investments. Barreto and Kypreos incorporate the knowledge stock concept into the 2FLC in (6) to examine how R&D expenditures over time impact the competitiveness of renewable energy technologies in the global energy market. They suggest that including knowledge depreciation creates "an incentive to invest in R&D to counteract the forgetting effect," and conclude that as a result, "faster knowledge depreciation may favor allocating more funds to currently competitive technologies in order to avoid or mitigate their "forgetting" process, rather than allocating them to currently expensive technologies that are promising only in the long run (Barreto and Kypreos 2004).

2.2.3 Economies of Scale

Economies of scale (EoS), result in the same faster-than-linear drops in production costs with increasing production volume resulting from learning-by-doing. Montgomery and Day surveyed literature on the role of learning and EoS in manufacturing, concluding "a far more powerful predictor of cost declines than was scale of production" (Montgomery and Day 1983). This result was observed in many industries, including textiles (Hollander 1965), heavy equipment manufacturing (Preston and Keachie 1964), and chemical processing (Stobaugh and Townsend 1975; Lieberman 1984) and shipbuilding (Argote, Beckman et al. 1990).

2.2.4 Uncertainty

The empirical and case literature has highlighted the important role uncertainty plays in predicting the impacts of learning on technology costs. Two primary sources of uncertainty impact learning: uncertainty in production volume and uncertainty in learning rates. Uncertainty about production volume is an inherent characteristic of attempts to forecast demand for products. Because learning-by-doing depends on cumulative production volume, this type of uncertainty will impact the learning related cost reductions realized by the firm by controlling "movement down the learning curve." Although learning has been observed across a wide range of industries, in any industry these rates vary dramatically. Figure 2 illustrates the variation in progress ratios across industries observed in over 100 field studies (data from (Dutton and Thomas 1984), graph from (Farmer and Trancik 2007)).

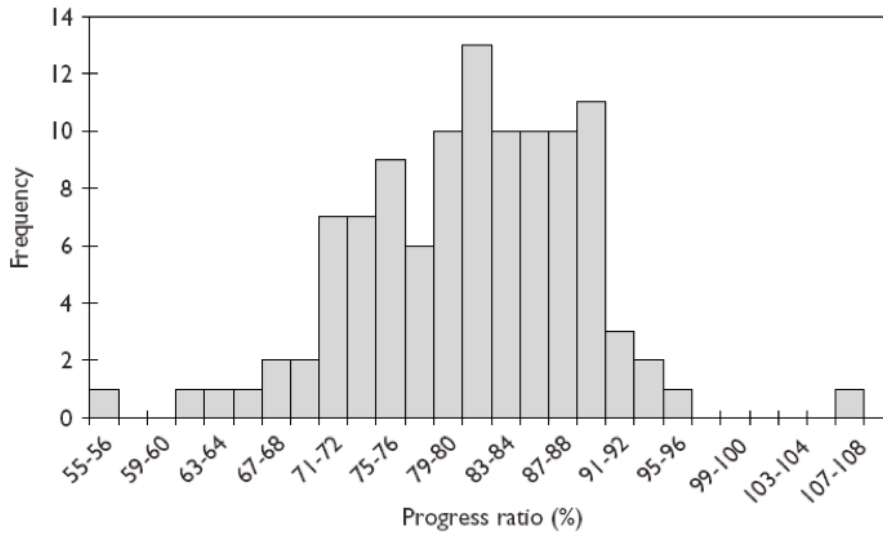


Figure 2: Distribution of progress ratios observed across multiple industries (Dutton and Thomas 1984)

Variation in learning rates has also been observed across firms in the same industry and within the same firm for different products (Conway and Schultz 1959; Alchain 1963; Nadler and Smith 1963; Porter 1980; Gruber 1992). As a result, attempting to predict technology learning rates based on historical figures has proven unreliable. As a result, firms may select rates resulting in significant cost errors, which could lead to suboptimal technology choices.

Methods exist to characterize uncertainty in learning. However, no single approach has been widely adopted. Fine presents stochastic dynamic programming and markov decision models to explore the impacts of investments in quality improvements over time (Fine 1988; Fine and Porteus 1988). Tang (Tang 1990) develops a discrete time model of a multi-stage production system incorporating both demand and output rate uncertainty to help provide insights for production planning and inventory control. Grubler and Gritsevskiy (Grubler and Gritsevskiy 2000) present a stochastic optimization model, which incorporates uncertain returns on R&D learning. Carillo and Gaimon (Carrillo and Gaimon 2000; Carillo and Gaimon 2002), suggest strategies to limit the impact of uncertainty on process technology changes. Mazzola and McCardle (Mazzola and McCardle 1996; Mazzola and McCardle 1997) present a Bayesian decision model

enabling characterization of the impacts of random processing variation on learning rates. Harpaz (Harpaz, Lee et al. 1982) and Thompson and Horowitz (Thompson and Horowitz 1993) characterize the impacts of uncertain demand in the presence of learning, and Lippman and McCardle (Lippman and McCardle 1991) examine technology choice where costs of each technology are initially unknown and discovered by investing in experimentation. More recently, Farmer and Trancik (Farmer and Trancik 2007) suggest that exploring technology selection as a problem in dynamic portfolio allocation, in which the return on investments in competing technologies is uncertain.

We recognize the important role uncertainty plays in the modeling technology costs and therefore the choice of technologies. In this work however, we focus on addressing two structural components of the question of learning as a driver for technology choice which have not been fully addressed: the impact of learning in multiple technologies and resource constraints on technology choice over time. We believe that employing a simple deterministic model makes it easier to focus the discussion on the implications of these structural elements on technology choice.

2.3 Learning in Technology Choice & Gap Analysis

Current literature on learning technology choice focuses on informing decision-making at two levels: industrial sector and individual firm. At the industry level, this research focuses on informing governmental policy makers and often includes social welfare metrics in addition to cost as a basis for technology decision-making. Recent examples have primarily come from the energy sector, exploring the cost and environmental tradeoffs of renewable versus existing energy technologies (Kouvaritakis, Soria et al. 2000; Barreto and Kypreos 2004; Miketa and Schrattenholzer 2004; Trancik 2006; Albrecht 2007; Farmer and Trancik 2007; Jamasb 2007). These models explore how policy changes at the macro level can lead to technology diffusion at the national or global level. However, some authors have concluded that this level of resolution is insufficient to characterize the impact of learning on technology decisions at the firm level. For example, Alchain found that using a single learning rate for the aircraft manufacturing industry led to large errors in predicting the learning behavior of

individual technologies (Alchain 1963), while Dutton found that learning curves can vary dramatically between firms within an industry (Dutton 1984; Dutton and Thomas 1984). At the level of individual firms, research exploring impacts of learning on technology choice has primarily focused on the decision of whether or not to replace an existing technology that has exhausted its associated learning related cost reductions (Parente 1994). In these cases technological change is seen as a necessity, as it is assumed that “further growth can occur only by switching to a better technology” (Jovanovic and Nyarko 1996). The strategy underlying these approaches is an extension of results in a seminal paper by Spence, in which he explored optimal pricing strategies for firms in the presence of learning. He concluded that the firm can achieve higher long-term profits by increasing production to move down the learning curve more rapidly than competing firms and pricing as if the significant learning-related cost reductions have already been achieved (Spence 1981). This conclusion suggests that expected long-run costs are the most important metric to consider when comparing technologies: if a new technology is expected to become less expensive in the future, then the firm should switch as soon as possible and allocate as much production as possible to move down the learning curve as quickly as possible. This technology strategy assumes a long enough time horizon exists between technology introductions that long-run cost savings associated with the new technology will more than offset any introduction costs. This approach makes sense in industries marked by long periods between technology changes. In these cases, technologies have time to mature before they are replaced. However, “shorter product lifecycles and faster product obsolescence are increasingly evident for high-technology products as well as products not typically regarded as high technology” (Franza and Gaimin 1998). In these industries, “rapid changes in technology fuel the need to create knowledge and drive constant changes in manufacturing” (Carrillo and Gaimon 2002). Rapid technological change means that new technologies become available before existing technologies have reached maturity. Additionally, large investments in existing technologies means that new technologies are often phased in over time. As a result, finite investment and capacity budgets need to be allocated to multiple technologies exhibiting learning simultaneously. Learning is driven by the allocation of these resources. Therefore, sharing these them between multiple technologies exhibiting

learning necessarily translates to less “movement down the learning curve” associated with each technology. This may result in higher production costs for *both* technologies. Current methods that assume the new technology is replacing a mature technology treat technology costs independently. For example, Grubler (Grubler and Gritsevskii 2002) treats the case of multiple plants producing multiple technologies sharing a common R&D investment pool; however, allocating resources to one technology does not impact the learning related cost reductions observed in the other. This approach makes sense when there are no learning-related opportunity costs associated with diverting resources away from the existing technology. However, when the existing technology continues to exhibit learning, diverting resources reduces does introduce opportunity costs. These costs may alter technology choice decisions: the cost penalties for diverting resources from an existing technology may outweigh the benefits from allocating these resources to a new technology, even if the new technology is expected to “learn faster” than the incumbent.

Methods examining the impacts of finite resources on technology choice suggest that a single technology will dominate, “locking out” other technologies. This is the result of a “virtuous cycle” whereby resources allocated to a technology enable it to learn, reducing costs and enabling the firm to reduce prices. This stimulates demand, which increases production volume, speeding up the realized cost reductions enabled by learning (Wantanabe 1995; Wantanabe 1999). The majority of this work focuses on technology choice at the industry level. Barreto and Kypreos (Barreto and Kypreos 2004), explore the impacts of learning by investing in R&D and capacity allocation on technology choice in the energy sector, finding that “a situation where only one of the mechanisms acts is not observed. Either both of them act “hand-in-hand” or none of them is set in motion.” As a result, “If a given technology has enough “learning potential” ...the model will try to install it at the maximum rate possible to exhaust such potential. If not, it will very likely leave it ‘locked-out’.” Farmer and Trancik (Farmer and Trancik 2007) explore the question of investment as a driver for technology choice in situations where “public and private investors supply capital for R&D and the manufacture of new technologies, and managers, engineers, and workers create new technologies” They suggest using

portfolio theory to examine the question of how to allocate investments in different technologies in the energy sector, with the goal of “maximiz(ing) the probability of achieving a socially desirable outcome such as cheap, carbon free energy.” They also find evidence for technology lock-in, as there is a strong pressure to limit the number of technologies in a portfolio via consolidation due to the technology cost and performance improvements resulting from investments.

At the firm level, technology lock-in can only occur if the firm is able to allocate all resources to a single technology. However, when the firm must introduce a new technology over time, multiple technologies will be produced simultaneously. When these technologies exhibit learning, the opportunity costs associated with diverting resources from one technology to another complicate the resource allocation decision. To date, little research has examined the impacts of the opportunity costs resulting from constraints requiring investment and capacity resources to be shared on technology choice and resource allocation.

A large body of research has concluded that both investment driven and cumulative production volume driven learning play critical roles in technological change across a wide range of industries (Dutton and Thomas 1984; Bohn 1995; Thomke 1997; Li and Rajagopalan 1998; Bohn and Terwiesch 1999; Carrilo and Gaimon 2000; Goulder and Mathai 2000; Kouvaritakis, Soria et al. 2000; Terwiesch and Bohn 2001; Carrillo and Gaimon 2002; Miketa and Schrattenholzer 2004; Klassen, Miketa et al. 2005; Trancik 2006; Trancik and Zweibel 2006; Farmer and Trancik 2007; Li and Rajagopalan 2008). Kouvaritakis et. al. introduced the two factor learning curve (2FLC) presented in (6) as a means to directly explore the tradeoffs between investments and capacity allocation on the diffusion of renewable energy technologies in the energy sector (Kouvaritakis, Soria et al. 2000). This work was later expanded by other authors to include the impacts of knowledge depreciation over time and finite R&D budgets on decisions (Barreto and Kypreos 2004; Miketa and Schrattenholzer 2004; Jamasb 2007). While work has examined the role the coupling of these two sources of learning play in technology choice, to date little work has been done examining these effects at the firm level.

Learning has been shown to occur in a wide array of operational characteristics across myriad industries, including production yield (Gruber 1994; Bohn 1995; Bohn and Terwiesch 1999; Chung 2001; Terwiesch and Bohn 2001), production line speed (Dar-El and Rubinovitz 1991; Terwiesch and Bohn 2001; Alamri, Balkhi et al. 2007), and the quantity of rework required after manufacturing (Jaber and Guiffrida 2004; Jaber and Guiffrida 2008). Despite these insights, the literature on learning as a strategic tool has largely “foregone discussions of a mechanism by which different aspects of learning and operational performance improvements could be prioritized within a facility” (Nadeau, Kar et al. 2010). Instead, learning is most often modeled using a single, macroscopic learning rate, often based on historical rates observed in similar industries.

Several authors have considered combining learning models with more detailed production models to explore the dynamics underlying technological learning. Nadler and Smith proposed a method to decompose the manufacturing process into multiple sub-processes and applies individual learning curves, where the aggregate technology learning function is the time weighted sum of these individual learning functions (Nadler and Smith 1963). Farmer and Trancik discuss incorporating learning in the individual inputs and processes of a technology via the decomposition of inputs (Farmer and Trancik 2007). Other authors have integrated learning parameters into production functions in empirical studies (Preston and Keachie 1964; Rapping 1965). Terwiesch and Bohn explore how yield can provide insights into the tradeoff between using production capacity for experimentation, which contributes to learning by investing, versus the learning by doing that could be gained by using that capacity for regular production (Terwiesch and Bohn 2001). More recently, Baretto and Kypreos incorporate learning curves into an energy-systems optimization cost model to explore the impacts of R&D and capacity allocations on technological diffusion over time (Barreto and Kypreos 2004). However, these studies do not explore the effects of differentiated learning across different operational characteristics, or how these effects combine to result in aggregate technology costs. As a result, these studies are unable to fully characterize the underlying dynamics driving technology costs.

Process based cost modeling (PBCM), provides a useful framework to derive production costs from the technical and operational parameters of an underlying technology (Kirchain and Field 2001). The PBCM approach has been employed to inform technology decisions across a wide range of industries, including automotive manufacturing (Han and Clark 1995; Johnson and Kirchain 2009), e-waste (Gregory and Kirchain 2006), and microphotonics (Singer and Wzorek 2006), and materials technologies (Field and Kirchain 2007). Recent work has extended the PBCM approach to incorporate learning across multiple operational characteristics (Nadeau, Kar et al. 2010). The resulting dynamic PBCM maps “the effect of learning in multiple process parameters on the cost of a given technology.” Using this method, firms are able to study the impact of learning on the evolution of technology costs over time. To date, this work has focused on modeling individual technology costs. No study has examined the effects of learning in different operational characteristics, or how those effects combine and translate into aggregate financial behavior in the context of technology choice.

2.4 Gap Analysis Summary

Table 1 presents a summary of the literature on learning and technology choice presented in this section and the features relevant to this analysis:

- Incorporating both cumulative production volume and investment driven learning
- Modeling the choice between technologies
- Modeling learning in multiple technologies
- Modeling technology choice at the level of an individual firm
- Modeling the impacts of shared investment and capacity resources
- Modeling constraints requiring simultaneous production of and investment in multiple technologies
- Characterizing technology costs at the operational level
- Characterizing learning at the operational level
- Incorporating learning and cost at the operational level to derive technology costs

Although previous work has considered elements of this list, no single study has examined how firms can transform investment and production volume driven learning at the operational level into technology choice and resource allocation decisions. Specifically, little work has been done to examine the impacts of production and investment constraints on technology choice when multiple technologies exhibit learning, and no work to date has characterized the impacts of learning in multiple operational factors on technology choice.

Literature	Features								
	Incorporate capacity and investment based learning	Model the choice between technologies	Model learning in multiple technologies	Model technology choice at the firm level	Model the impacts of sharing finite resources	Model the impacts of constraints requiring shared resources	Characterize production costs at operational level	Characterize learning at operational level	Incorporate learning in operational parameters to derive technology costs
Nadler and Smith 1963							X	X	
Preston and Keachie 1964							X	X	
Rapping 1965							X	X	
Dutton and Thomas 1984							X	X	
Dar-el and Rubinovitz 1991							X	X	
Gruber 1994	X	X		X					
Bohn 1995							X	X	
Wantanabe 1995	X	X							
Han and Clark 1995		X		X			X		
Wantanabe 1996	X	X							
Thomke 1997	X								
Li and Rajagopalan 1998	X								
Kourvaritakis, Soria et.al. 2000	X	X	X		X				
Carillo and Gaimon 2000	X						X	X	
Carillo and Gaimon 2002	X						X	X	
Terwiesch and Bohn 2001	X				X	X	X	X	
Kirchain and Field 2001							X		
Chung 2001							X	X	
Grubler and Gritsevskii 2002	X	X		X					
Barreto and Kypreos 2004	X	X	X		X	X			
Miketa and Schratzenholzer 2004	X	X	X		X	X			
Klassen and Miketa 2005	X	X	X		X				
Jaber and Guiffida 2004							X	X	
Gregory and Kirchain 2006		X		X			X		
Singer and Wzorek 2006		X		X			X		
Trancik and Zweibel 2006	X	X							
Trancik 2006	X	X	X						
Jamasb 2007	X	X	X						
Alamri, Balkhi et. al. 2007							X	X	
Farmer and Trancik 2007	X	X	X				X	X	
Field and Kirchain 2007		X		X			X		
Jaber and Guiffida 2008							X	X	
Li and Rajagopalan 2008	X								
Johnson and Kirchain 2009		X		X				X	
Nadeau et. al. 2010							X	X	X

Table 1: Summary of previous literature on learning and technology choice

2.5 Research Contributions

This research hopes to compliment and extend past work on the role of learning in technology choice by providing both a formal methodology and practical modeling tools firms can use when making these decisions.

Specifically, we develop a formalism and simulation model enabling characterization of how the technology decision *context* in which:

- technologies may be produced simultaneously
- technologies may be learning-by-doing
- technologies may be learning-by-investing

can change technology choice decisions about:

- when, if at all to introduce a new technology
- how to allocate finite production capacity over time
- how to allocate finite investment over time

2.5.1 Contributions to the Literature

To explore these questions, we extend existing learning models to explicitly incorporate learning in multiple operational parameters when capacity and investment resources must be shared between multiple technologies. We then characterize the conditions under which these factors change technology choice and resource allocation decisions. A dynamic process based cost modeling approach is developed to study the evolution of technology costs over time when considering learning from multiple sources (investment and production capacity) and in multiple production factors (cycle time, yield etc.). The goal of the model is to identify opportunities where firms can make operational changes that will impact changes in production costs, and characterize these impacts over time

3 Methodology Overview

We seek to develop a method to characterize the conditions under which learning in the existing technology and shared production and/or investment budgets impact technology choice decision making. Specifically, we are interested in addressing the question:

Under what conditions does consideration of multiple learning technologies produced simultaneously change technology choice decisions over time?

- If and when to introduce new technologies?
- How to allocate production resources over time?
- How to allocate investment resources over time?

We assume that an existing technology, A, and a new technology, B can be used to produce a single product. We define a *decision context* as a “state of the world” facing the firm when making the decision of whether or not to introduce B, and if so, how to allocate resources among technologies. Each context is defined by two factors. The first is whether or not A still exhibits learning related cost reductions after the introduction of B. The second factor is whether or not the firm must continue to allocate some fraction of production capacity and/or investment resources to A even if it introduces B. Table 2 defines the decision contexts considered.

Resource Allocation

Technology Learning	(a) <i>Only B learning</i> <i>Unconstrained resource allocation</i>	(b) <i>Only B learning</i> <i>Constrained resource allocation</i>
	(c) <i>A and B learning</i> <i>Unconstrained resource allocation</i>	(d) <i>A and B learning</i> <i>Constrained resource allocation</i>

Table 2: Contexts for technology choice decision making considered

Scenarios (a) and (b) provide the contexts for decisions in the existing literature, in that the existing technology is assumed to have exhausted the potential learning related cost reductions enabled by allocating increased resources. In (a) the firm is free to share the finite resource or devote it either technology, while in (b) constraints exist on how the firm can allocate resources. Scenarios (c) and (d) consider cases where the existing technology still exhibits learning related cost reductions. In (c) as in (a), the firm is free to allocate resources, while in (d) as in (b), some resources allocation is constrained. This factor reflects contexts in which the firm has large capital and/or research resources tied

up in existing technologies, or where the costs associated with implementing a new technology make it economically infeasible to shift

Within each decision context, we seek to characterize (1) the set of factors facing the firm when making technology choice and resource allocation decisions, and (2) the conditions under which these factors change these decisions. Figure 3 provides an overview of the factors and decisions considered within a decision context.

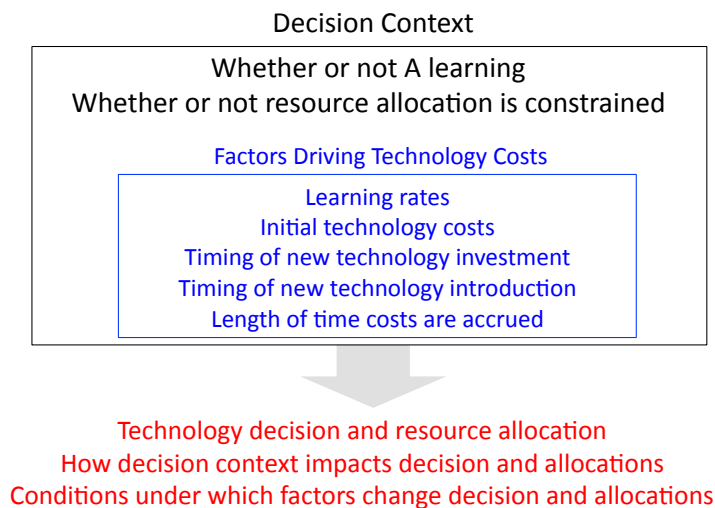


Figure 3: Overview of factors and decisions considered

4 Cumulative Production Volume Driven Learning

In this chapter, we are interested in exploring three questions:

1. Does learning-by-doing in the existing technology change technology choice and allocation decisions?
2. If so, what are the conditions under which the decision changes?
3. How do constraints on capacity allocation impact decision-making?

We first define the decision context facing the firm. Next, we describe our assumptions and develop expressions for individual technology costs as a function of technology learning, initial costs, potential new technology introduction year, timeframe over which costs are accrued and production volume allocation. We then derive an expression for

total production costs over time and attempt to find an expression minimizing these costs. Finally, we explore the impact of decision context on total production costs and comment on the conditions under which changes in the factors driving technology costs impact capacity allocation decisions.

4.1 Decision Contexts

The resource to be allocated is production volume. The resulting decision contexts are defined in Table 3.

Technology Learning	(a) <i>Only B learning</i> <i>Unconstrained capacity allocation</i>	(b) <i>Only B learning</i> <i>Constrained capacity allocation</i>
	(c) <i>A and B learning</i> <i>Unconstrained capacity allocation</i>	(d) <i>A and B learning</i> <i>Constrained capacity allocation</i>

Table 3: Context for technology choice decisions when considering only CPV learning

In scenarios (a) and (b), the existing technology has “finished” learning: no further cost reductions are expected with increasing cumulative production volume. In (c) and (d), allocating production volume to either technology is expected to reduce the associated technology costs via learning-by-doing. Scenarios (a) and (c) assume the firm can allocate production to either or both of the technologies, while (b) and (d) assume that both technologies must be produced simultaneously.

4.2 Unit Costs Over Time

We assume that an existing technology, A, and a new technology, B can be used to produce a single product. We define T_0 as the first period in which the firm will consider introducing B, and T_i as the period in which production volume is first allocated to B. If B is introduced in the first period under consideration, then $T_i = T_0$. The firm is deciding whether or not to introduce B, and if so, how to allocate production volume among the technologies to minimize total production costs from T_0 to an analysis horizon T_f . We

define the unit costs associated with A at T_0 as C_A^0 , and the unit costs associated with B at introduction in T_i as $C_B^{T_i}$. We assume that these initial unit costs are known, and that technology B exhibits higher unit costs at T_i than A does at T_0 : $C_A^0 < C_B^{T_i}$. However, after T_i , B is assumed to (a) exhibit greater cost reductions than A for each additional unit of production (a larger progress ratio), and (b) become the less expensive option with increasing cumulative production volume over time.

We define $F(t)$ as the percentage of total production volume allocated to technology B in every period, where $0 \leq F \leq 1 \forall t$ and $(1-F(t))$ as the percentage of production volume allocated to technology A. Prior to T_i , $F(t)=0$, as all production is handled using the existing technology. The production volume allocated to each technology in period t is given by:

$$\begin{aligned}
 T_0 \leq t < T_i : \\
 P_A(t) &= P(t) \\
 P_B(t) &= 0 \\
 \\
 T_i \leq t < T_f : \\
 P_A(t) &= (1 - F(t)) \cdot P(t) \\
 P_B(t) &= F(t) \cdot P(t)
 \end{aligned} \tag{7}$$

Where the total production volume in an individual period is a constant:

$$P(t) = P_A(t) + P_B(t) \tag{8}$$

The cumulative production volume for each technology from is therefore:

$$\begin{aligned}
T_0 \leq t < T_i : \\
CPV_A^t &= \int_{T_0}^t P(t) dt \\
CPV_B^t &= 0
\end{aligned} \tag{9}$$

$$\begin{aligned}
T_i \leq t \leq T_f : \\
CPV_A^t &= \int_{T_0}^{T_i} P(t) dt + \int_{T_i}^t (1 - F(t)) \cdot P(t) dt \\
CPV_B^t &= \int_{T_i}^t F(t) \cdot P(t) dt
\end{aligned}$$

We assume that the learning function for each technology is a monotonically decreasing function of increasing cumulative production volume¹. Additionally, we assume that the new technology exhibits a larger progress ratio than the incumbent (technology A “learns faster” than B). The learning function for each technology and assumptions are given by:

$$\begin{aligned}
\phi_A(t) &= f(CPV_A^t) \quad \text{where} \quad 0 \leq \phi_A(t), \phi_B(t) \leq 1 \\
\phi_B(t) &= f(CPV_B^t) \quad \frac{d(\phi_B(t))}{d(CPV_B^t)} < \frac{d(\phi_A(t))}{d(CPV_A^t)} \leq 0
\end{aligned} \tag{10}$$

We define the unit cost for each technology at time t as:

$$\begin{aligned}
C_A(t) &= C_A^0 \cdot \phi_A(t) \\
C_B(t) &= C_B^{T_0} \cdot \phi_B(t)
\end{aligned} \tag{11}$$

Figure 4 illustrates these assumptions for stylized technology A and B unit cost curves as a function of the cumulative production volume for a single product. In (i), technology A has exhausted the learning cost reductions by CPV^{T_0} . This case corresponds to the learning behavior of decision contexts (a) and (b) in Table 3. In (ii), technology A costs continue to decrease with increasing CPV past CPV^{T_0} . This scenario illustrates the learning behavior expected in decision contexts (c) and (d).

¹ A function which always decreases or remains constant but never increases

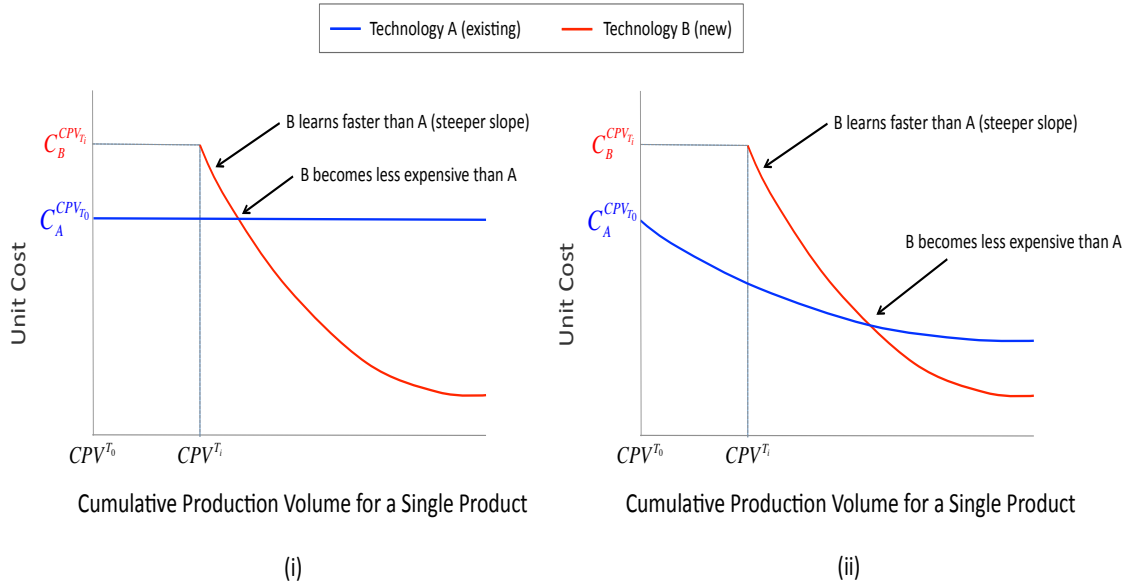


Figure 4: Potential A and B unit cost curves as a function of total cumulative production volume for a single product when (i) A is no longer learning when $CPV \geq CPV^{T_0}$, (ii) A continues to learn when $CPV \geq CPV^{T_0}$

In general, the shape of the learning curves in Figure 4 depends on the progress ratios associated with each technology. These learning curves represent the *potential* to reduce production costs. However, *realizing* these cost reductions depends on how production volume is allocated to each technology. Figure 5 and Figure 6 illustrate this idea for decision contexts (a) and (c) in Table 3, where, for explanatory clarity we focus on cases in which the fraction of capacity allocated to B is constant over time: $F(t) = F(T_i) = F \forall t \geq T_i$. In both decision contexts, the firm is free to allocate capacity to either or both technologies. Each row in the figures corresponds a different allocation strategy. In (1), the firm stays with A ($F=0$). In (2), allocation is shared between technologies, with technology B used to produce some fraction, F , of total production ($0 < F < 1$). In (3), the firm switches all production over to B at T_i ($F=1$). The solid lines in the “realized learning” column represent the unit costs the firm experiences based on the learning characteristics of each technology and the capacity allocation. The dashed lines correspond to the learning potential for each technology, and the difference between the solid and dashed lines is a measure of *foregone learning*: potential learning that the

firm will not realize as a result of a capacity allocation decision. For example, in row (1), the firm stays with existing technology. As a result, none of the potential learning in B will be realized.

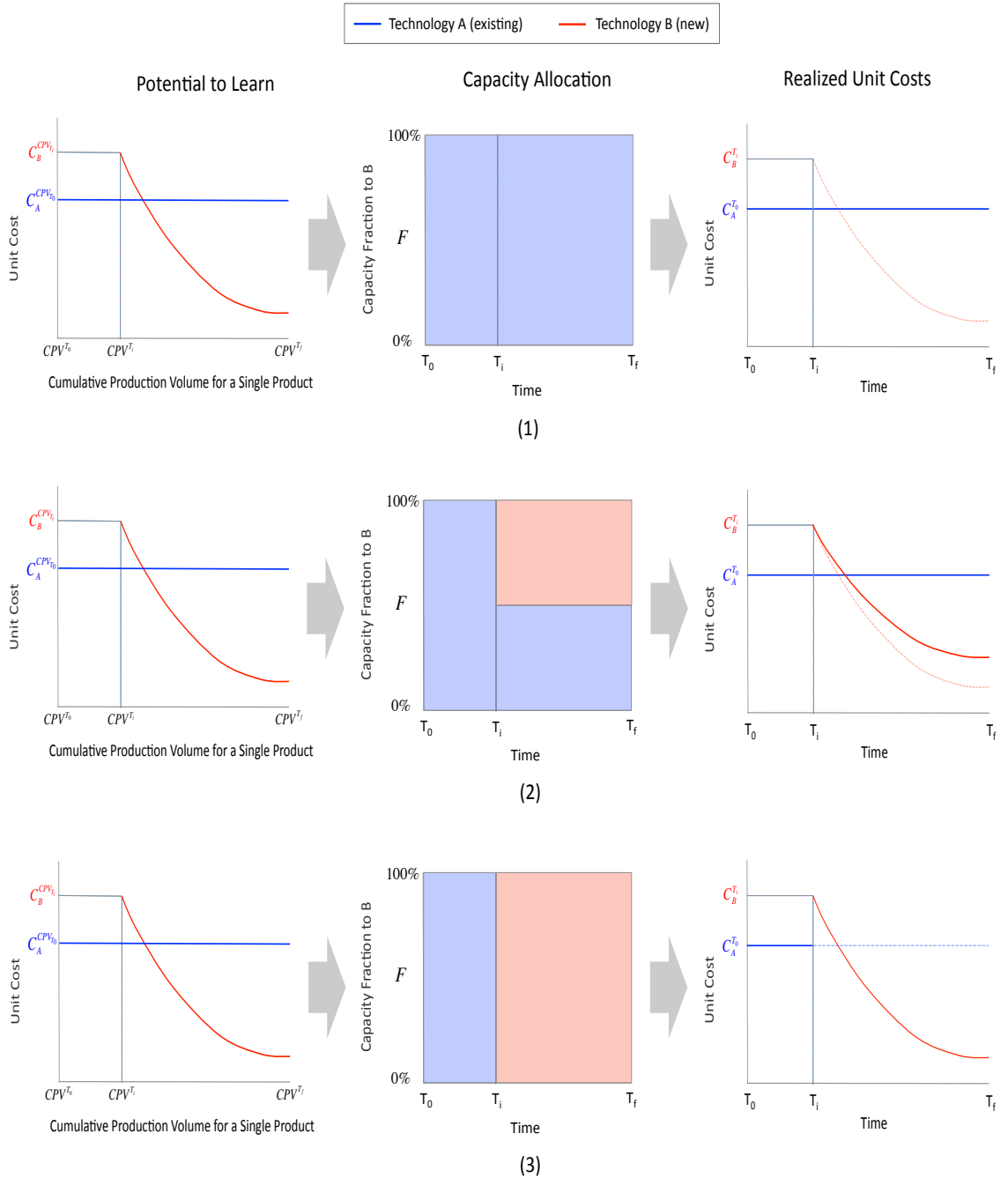


Figure 5: Potential learning curves, capacity allocation and realized unit cost curves for each technology for decision context (a) in Table 3

Row (1) in Figure 5 represents a “worst of both worlds” cost outcome. Technology A is no longer learning. Therefore, allocating production volume to A does not result in any realized cost reductions over time. Additionally, the firm forsakes the potential learning related cost reductions associated with B by allocating all production volume to A. In row (2), allocating some capacity to B results in some realized learning. In this case, the firm still foregoes some cost reductions in B due to the fraction of capacity still allocated to A. However the firm benefits from continued production using A because A is less expensive initially. In row (3), allocating all capacity to B enables the firm to realize the full potential of learning in B as quickly as possible. However, shifting all production to B results in higher realized unit costs until B becomes the less expensive technology.

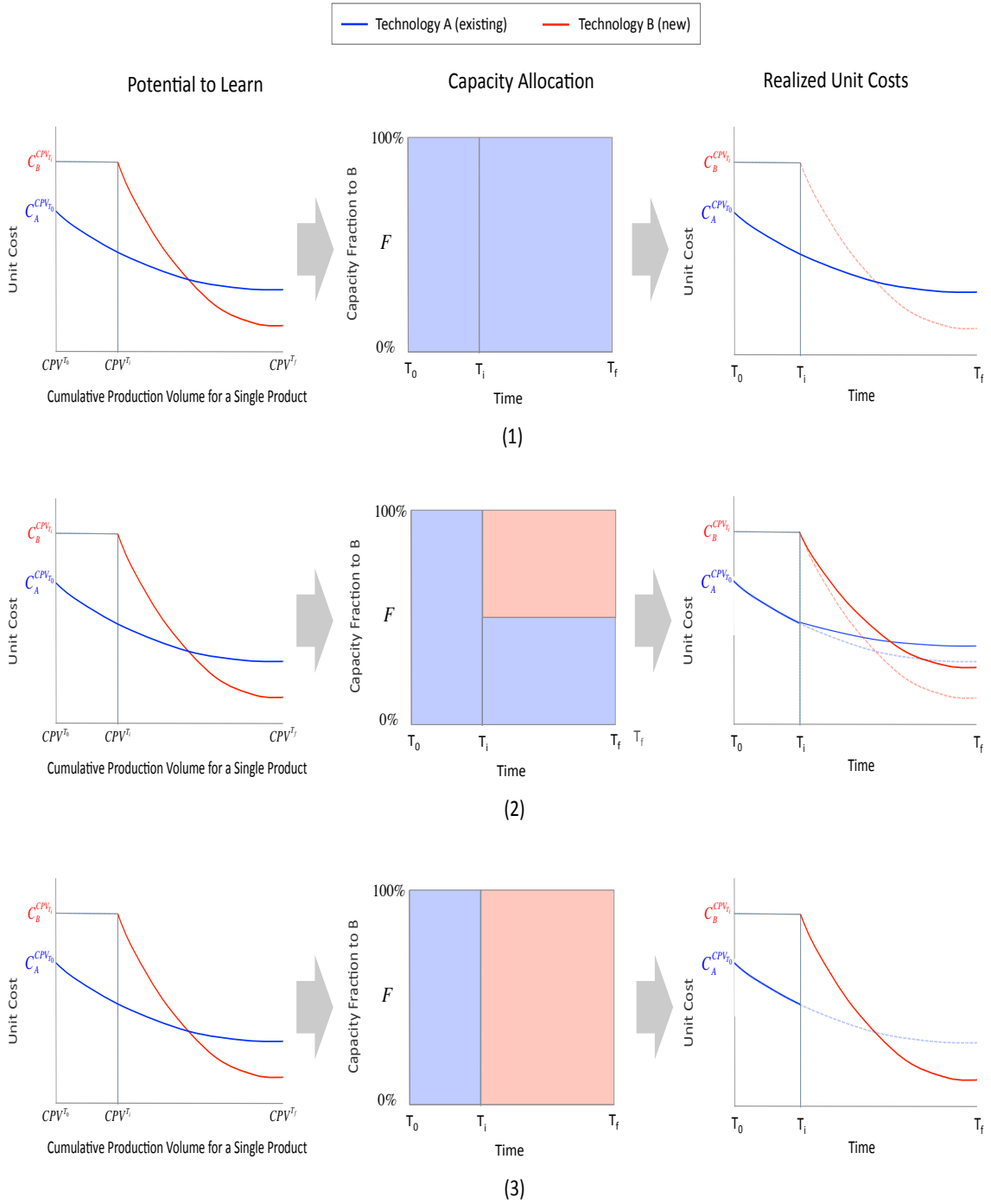


Figure 6: Potential learning curves, capacity allocation and realized unit cost curves for each technology for decision context (c) in Table 3

In Figure 5, the firm benefits from continued production using A due to continued learning-by-doing. As a result, in (1), the firm realizes cost reductions when staying with

A. In (2) sharing capacity means both technologies will experience realized learning. However, sharing also *reduces the realized learning in each* via less movement down the expected learning curve. This effect will be larger for B, because B "learns faster" than A: diverting production comes at a higher penalty. This foregone learning results in *higher realized unit costs for both technologies*. This suggests that the firm should choose to either stay with A, or switch to B to minimize costs. The cost behavior in Figure 5 (3) remains unchanged from that observed in Figure 4, as the firm elects to switch production to B. However, unlike in Figure 4 (3), in this case the firm is foregoing the benefits associated with continued learning in A by switching to B.

We can use the behavior observed in Figure 4 and Figure 5 (decision contexts (a) and (c)), to comment on the behavior of unit costs in the two remaining decision contexts, (b) and (d). Unlike (a) and (c), the technology decision in (b) and (d) is whether to stay with A or to share allocation between A and *given that some fraction, (1-F)*, of total production must remain allocated to A. This situation is analogous to row (2) of Figure 4 and Figure 5, in that production is shared. In (b), as in Figure 4, technology A no longer exhibits learning related cost reductions after T_0 . In this case, the larger (1-F) is, the larger the foregone learning in B. Because A is no longer learning, diverting production volume to B does not result in any foregone A learning. In (d), as in Figure 5, both technologies exhibit learning-by-doing at cumulative production volumes greater than T_0 . In this case, sharing production volume results in foregone learning in both technologies, leading to higher unit costs. As more capacity is diverted away from a particular technology more learning is foregone in that technology. However, that capacity contributes to learning in the other technology.

4.3 Production Costs Over Time

Now that we have examined unit costs, we can address total production costs over time. Individual technology production costs in each period are defined as the product of the unit cost in that period and the total units produced using that technology:

$$\begin{aligned} TC_A(t) &= C_A(t) \cdot P_A(t) \\ TC_B(t) &= C_B(t) \cdot P_B(t) \end{aligned} \tag{12}$$

Total technology costs in period t is the sum of these costs:

$$TC(t) = TC_A(t) + TC_B(t) \quad (13)$$

Figure 7 and Figure 8 illustrate total production costs over time. In Figure 7, A no longer exhibits learning after B is introduced, while in Figure 8, A continues to exhibit learning behavior for $CPV > CPV^{T_0}$. Each row represents a different capacity allocation scenario: F_1 , F_2 and F_3 , where $0\% < F_1 < F_2 < F_3 < 100\%$. Comparing rows within a figure highlights how total production costs change over time as a function of production volume allocation. Comparing each row of Figure 7 with Figure 8 highlights the impact of learning in the existing technology on total production costs over time. The second column in each figure presents the realized total costs the firm experiences in each period resulting from this capacity allocation $TC(t)$, and the individual technology costs that add up to this total, $TC_A(t)$ and $TC_B(t)$. The final column in each figure compares the realized total production costs, $TC(t)$ (solid black line), against the total costs if the firm instead chose to either stay with the existing technology, in which case $F=0\%$ and $TC(t) = TC_A(t)$ (blue dashed line), or switch all production to the new technology, in which case $F=100\%$ and $TC(t) = TC_B(t)$ (red dashed line). In these graphs, the difference between curves represents the variation in costs due to the allocation decision.

Technology B is initially more expensive. Therefore, any capacity allocated to B will cause an increase in $TC(t)$ at T_1 relative to staying with A. This can be seen in the third column, as the black line representing $TC(t)$ initially increases above the dashed blue line representing the total costs associated with staying with A for all three allocation scenarios. Over time however, B becomes less expensive relative to A due to learning. As a result, $TC(t)$ is reduced over time relative to staying with A. The *magnitude* and *duration* of these changes in $TC(t)$ depend on three factors: the production allocation scenario, the technology learning behavior and the initial technology unit costs.

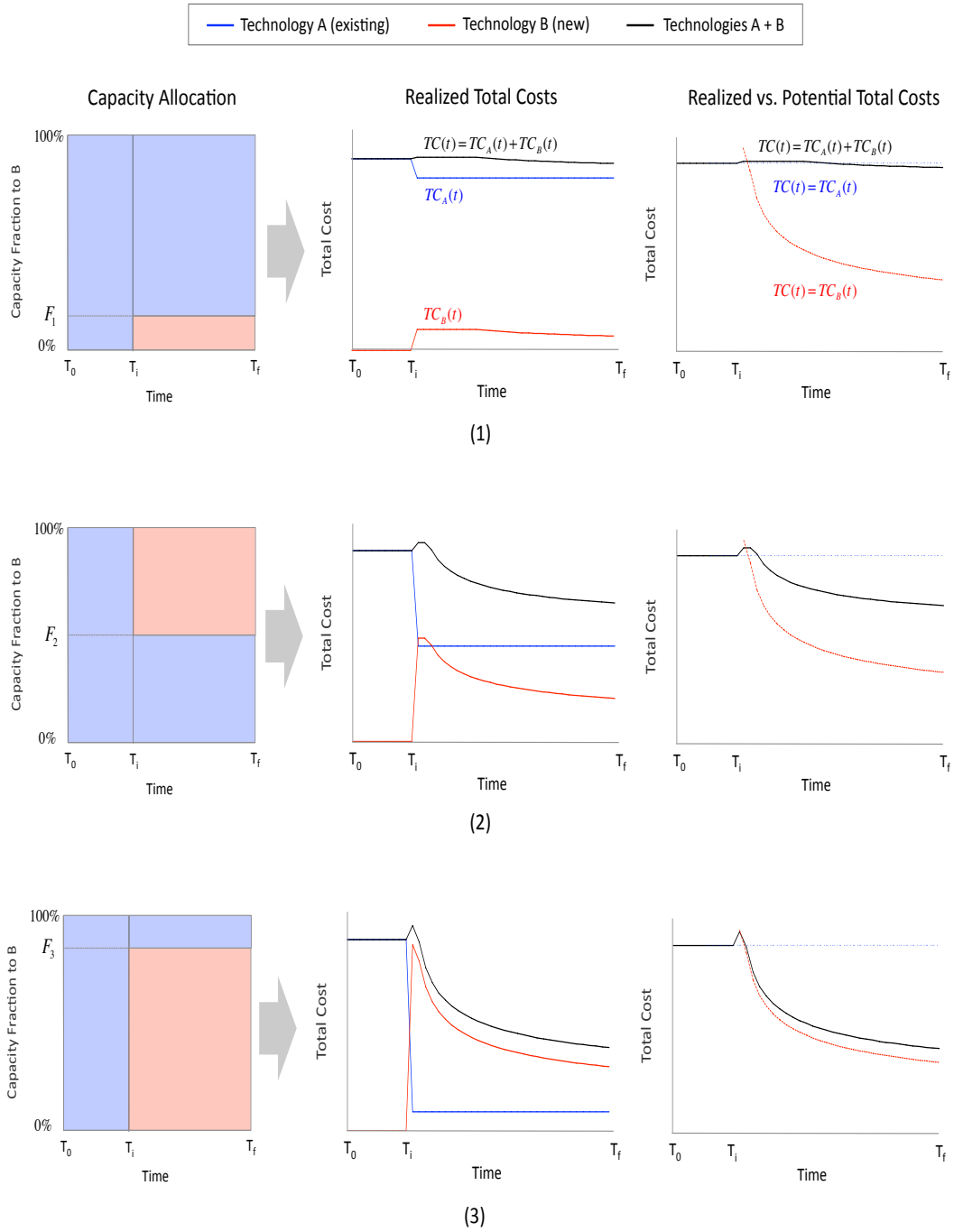


Figure 7: Capacity allocation, realized total cost and realized versus potential total cost curves as a function of time when technology A no longer exhibits learning for $t \geq T_0$

Comparing rows (1) to (3) in Figure 7, as the allocation to B increases from F_1 to F_2 to F_3 beginning at T_i , the firm is producing an increasing fraction of total production using a more expensive technology. As a result, *increasing the allocation to B increases the magnitude* of the initial spike in $TC(t)$ relative to staying with A. These increases could be offset by cost reductions in A; however, in this case A no longer exhibits learning after T_0 . As the allocation to B increases, the costs associated with B decrease more rapidly. As a result, B becomes cost competitive with A sooner. Therefore, *increasing the allocation to B decreases the duration* over which these additional costs are incurred, and *increases the duration over which B decreases $TC(t)$* relative to staying with A. This can be seen in the third column of Figure 7. As F increases, the duration until $TC(t)$ crosses $TC(t) = TC_A(t)$ decreases.

In Figure 8, the mechanisms driving changes in $TC(t)$ are the same as in Figure 7. However, continued learning in A results in different outcomes. In both cases, the initial magnitude of the increase in $TC(t)$ at T_i is the same. However, unlike in Figure 7, continued learning in A helps to offset these initial costs over time. As in Figure 7, as F increases, the duration until $TC(t)$ crosses $TC(t) = TC_A(t)$ decreases. However, continued learning in A counteracts this effect, making $TC(t) = TC_A(t)$ the preferred option over a longer period. Comparing the third column in row (1) in Figure 7 and Figure 8 highlights this effect. In Figure 7, learning in B is enough to drive down costs such that $TC(t)$ is less than $TC(t) = TC_A(t)$ after some period of time. However, in Figure 8, continued learning in A causes $TC(t) = TC_A(t)$ to be less than $TC(t)$ over the timeframe of interest. This is because it takes longer for B to become the less expensive option *and* longer for the firm to recoup the additional costs associated with introducing B. In Figure 7, the benefits of allocating F_1 to B may provide enough cost reductions to offset the costs associated with introducing B. In Figure 8 however, allocating F_1 to B results in increased total production costs, suggesting the firm should stay with A.

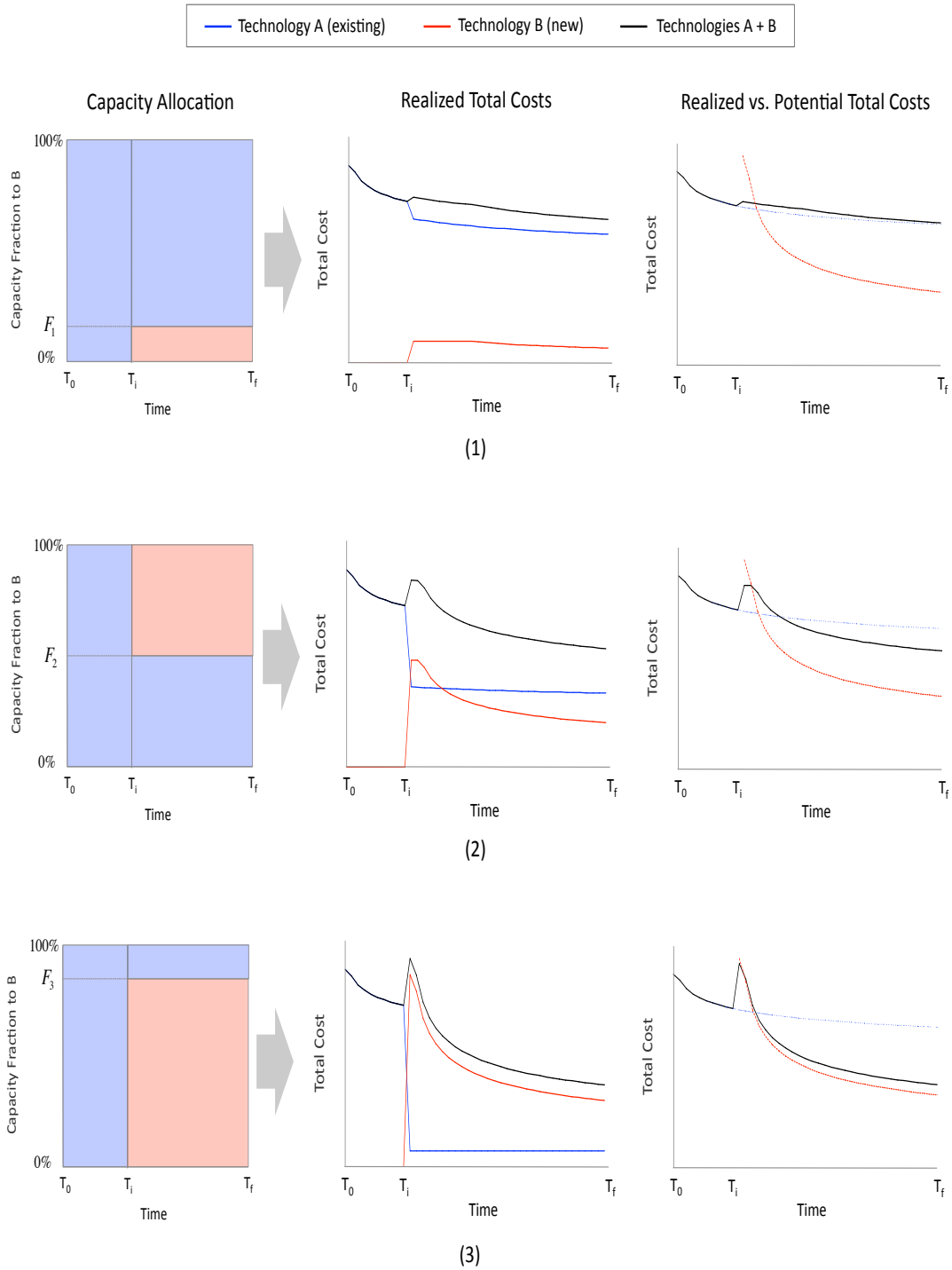


Figure 8: Capacity allocation, realized total cost and realized versus potential total cost curves as a function of time when technology A continues to exhibit learning for $t \geq T_0$

The graphs thus far have been used to examine total costs over time when the firm is free to allocate production to either or both technologies. Figure 7 corresponds to decision context (a) in Table 3, while Figure 8 illustrates total cost behavior under context (c). However, we can also use these figures to comment on the remaining two contexts, then the firm must decide is whether to stay with A or to share allocation between A and *given that some fraction, (1-F)*, of total production must remain allocated to A. In this interpretation, Figure 7 corresponds to decision context (b), while Figure 8 represents context (d). In these cases, although the dashed red line in the third column of Figure 7 and Figure 8 still represents the total costs due to switching all production to B, this is no longer a strategy available to the firm. From this point of view, the allocation $(1-F_1)$ would correspond the minimum fraction of total capacity that must remain allocated to A. In these cases, the firm would compare the total costs of staying with A with total cost resulting from allocations to B up to F_3 (assuming that $F_1 = (1-F_3)$).

These examples also suggest how the timeframe over which costs are accrued impacts decision-making. Technology B is initially the more expensive option. Therefore, it will take some amount of time for B to become less expensive than A, and then an additional amount of time for the firm to recoup the additional introduction costs associated with B. In order for the firm to opt to introduce B, the analysis timeframe (from T_0 to T_f), must be long enough for the firm to recoup these costs. Therefore, shorter timeframes always disadvantage B.

4.4 Total Production Costs

The firm will ultimately make decisions based on the single metric of total production costs from T_i to T_f . Total production costs for each technology are found by summing the contributions from (12) in each period:

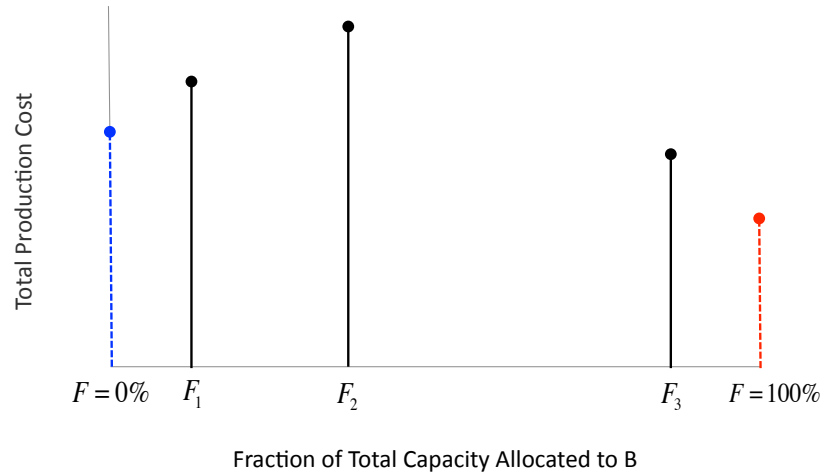
$$\begin{aligned}
 TC_A &= \int_{T_0}^{T_f} C_A(t) \cdot P_A(t) dt \\
 TC_B &= \int_{T_i}^{T_f} C_B(t) \cdot P_B(t) dt
 \end{aligned}
 \tag{14}$$

By analogy with (13), the total production cost the firm experiences from T_0 to T_f is defined as the sum of total production costs corresponding to each technology:

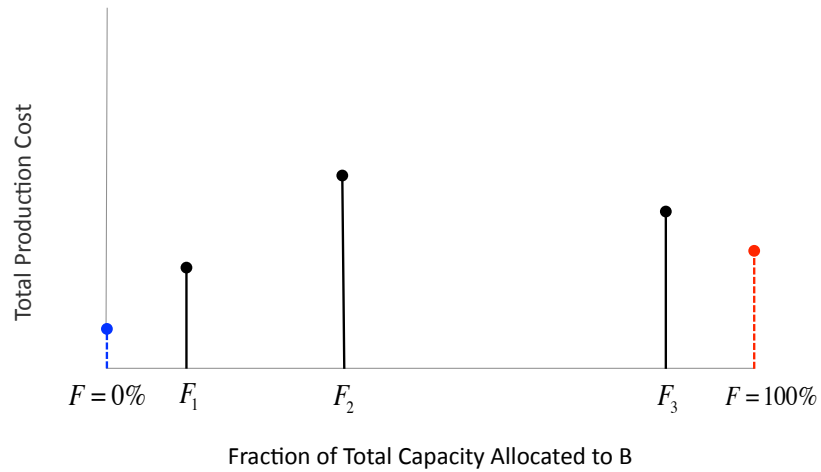
$$TPC = TC_A + TC_B \quad (15)$$

In decision contexts (a) and (c) in which the firm can choose to allocate production to either or both technologies, the firm will compare TPC against the total cost for each technology, TC_A and TC_B . In contexts (b) and (d), in which some fraction of production must continue to be produced with A, the comparison is between TPC and TC_A .

Figure 9 rows (1) and (2) illustrate the impact of capacity allocation on total production costs for the allocation scenarios in Figure 7 and Figure 8 (decision context (a) and (c) in Table 3). Each bar in Figure 9 represents a solution to (15), and is the sum of production costs in each period from T_0 to T_f . The dashed blue line at $F=0\%$ represents the TPC resulting from the decision to staying with technology A. The lines at F_1 , F_2 , and F_3 represent TPC when capacity is shared as shown in Figure 7 and Figure 8. The dashed red line at $F=100\%$ corresponds to the TPC associated with switching all production to B.



(1)



(2)

Figure 9: Total production costs as a function of capacity allocated to B for allocation scenarios in Figure 7 (row (1)), and Figure 8 (row (1))

Row (1) in Figure 9 indicates that switching all production to B results in the minimum TPC. This is what we would expect, as in this case there are no additional learning related benefits to be extracted from continuing to produce using A. As F decreases, TPC increases until F_1 . The increase in TPC has two causes. First, a larger fraction of TPC is produced using A, the more expensive technology. Second, and more subtly, diverting capacity away from B reduces the benefits of learning in B. As a result, each unit of B is more expensive to produce. As the allocation to B decreases from F_1 to $F=0\%$, learning

proceeds so slowly in B that the associated unit costs are unable to overcome the initial unit cost advantage of producing with A. As a result, TPC decreases in this allocation regime.

In Row (2) in Figure 9, TPC is reduced at every allocation relative to row (1), and minimum TPC now corresponds to staying with A ($F=0\%$). In the region T_0 to T_i , learning in A reduces the contribution of A to TPC irrespective of if B is launched. This explains why TPC is reduced in the case when the firm switches all production to B at T_i ($F=100\%$). Learning in A also reduces the contribution of TC_A to TPC at all shared allocation levels. As production is allocated to A, there is a tradeoff between the benefit of learning in A, and the penalty of foregone learning in B. Additionally, when A still exhibits learning, each unit allocated to B leads to foregone learning in A. Technology B is expected to “learn faster” relative to A. Therefore, each unit allocated to B is expected to result in a greater reduction in unit cost. This would suggest that the firm should allocate all resources to B to maximize this benefit. This is true in row (1), when A no longer exhibits learning related benefits. However, B is also the more expensive technology initially. When A continues to learn, the period over which B is more expensive technology increases. Therefore, as production is shifted to A, although the *reduction in unit cost* is greater for B, the *actual unit cost* may still be higher than A. In row (2), the maximum TPC observed at allocation F_2 is the result of the maximum combined cost penalty due to foregone learning in both technologies. Moving to F_3 reduces the costs associated with B while increasing the foregone learning in A, while moving to allocation F_1 has the opposite effect. The fact that TPC is greater at F_3 than F_1 , indicates that the benefits associated with continued learning in A outweigh the penalty of foregone learning in B.

These results indicate that shared capacity leads to increase costs via the mechanism of foregone learning. This suggests (a) minimum TPC will occur at either maximum or minimum F, and (b) maximum TPC will correspond to a shared capacity allocation in which the combined foregone learning in both technologies is a maximum. In order to

explore these hypotheses, we need to more accurately characterize how allocation decisions impact total production costs.

4.5 Total Production Cost Function Characteristics

We are interested in characterizing how TPC changes as a function of F . We begin by determining the extrema of TPC with respect to F from T_i to T_f Expanding (15):

$$\begin{aligned} TPC &= \int_{T_0}^{T_f} (C_A^{T_0} \cdot \phi_A(t) \cdot P_A(t) + C_B^{T_i} \cdot \phi_B(t) \cdot P_B(t)) dt \\ &= C_A^{T_0} \cdot \int_{T_0}^{T_f} (\phi_A(t) \cdot P_A(t) + \eta \cdot \phi_B(t) \cdot P_B(t)) dt \end{aligned} \quad (16)$$

where $\eta = \frac{C_B^{T_i}}{C_A^{T_0}}$

The first order condition to minimize (16) with respect to F is²:

$$\begin{aligned} \frac{\partial TPC}{\partial F} &= \frac{\partial}{\partial F} \left(C_A^{T_0} \cdot \int_{T_0}^{T_f} (\phi_A(t) \cdot P_A(t) + \eta \cdot \phi_B(t) \cdot P_B(t)) dt \right) = 0 \\ &= \int_{T_0}^{T_f} \left[\left(\frac{\partial \phi_A(t)}{\partial F} \cdot P_A(t) + \phi_A(t) \cdot \frac{\partial P_A(t)}{\partial F} \right) + \eta \cdot \left(\frac{\partial \phi_B(t)}{\partial F} \cdot P_B(t) + \phi_B(t) \cdot \frac{\partial P_B(t)}{\partial F} \right) \right] dt = 0 \end{aligned} \quad (17)$$

Setting the integrand to zero enables solving for extrema:

$$\left(\frac{\partial \phi_A(t)}{\partial F} \cdot P_A(t) + \phi_A(t) \cdot \frac{\partial P_A(t)}{\partial F} \right) + \eta \cdot \left(\frac{\partial \phi_B(t)}{\partial F} \cdot P_B(t) + \phi_B(t) \cdot \frac{\partial P_B(t)}{\partial F} \right) = 0 \quad (18)$$

We assume that the total production at time t , $P(t)$, is independent of F . Therefore, from (8):

$$\begin{aligned} \frac{\partial P(t)}{\partial F} &= \frac{\partial P_A(t)}{\partial F} + \frac{\partial P_B(t)}{\partial F} = 0 \\ \therefore \frac{\partial P_A(t)}{\partial F} &= -\frac{\partial P_B(t)}{\partial F} \end{aligned} \quad (19)$$

We can compute these partial derivatives using the production functions defined in (7):

² In the region T_0 to T_i , $F=0$, so we are only concerned with the region from T_i to T_f .

$$\begin{aligned}\frac{\partial P_A(t)}{\partial F} &= \frac{\partial}{\partial F}[(1-F) \cdot P(t)] = -P(t) \\ \frac{\partial P_B(t)}{\partial F} &= P(t)\end{aligned}\quad (20)$$

Incorporating these definitions into (18):

$$\left(\frac{\partial \phi_A(t)}{\partial F} \cdot (1-F) \cdot P(t) - \phi_A(t) \cdot P(t) \right) + \eta \cdot \left(\frac{\partial \phi_B(t)}{\partial F} \cdot F \cdot P(t) + \phi_B(t) \cdot P(t) \right) = 0 \quad (21)$$

Additionally, since we are only interested in cases where $P(t) \neq 0$:

$$\frac{\partial \phi_A(t)}{\partial F} \cdot (1-F) - \phi_A(t) + \eta \cdot \left(\frac{\partial \phi_B(t)}{\partial F} \cdot F + \phi_B(t) \right) = 0 \quad (22)$$

Collecting like terms:

$$\frac{\partial \phi_A(t)}{\partial F} - \phi_A(t) + \left(\eta \cdot \frac{\partial \phi_B(t)}{\partial F} - \frac{\partial \phi_A(t)}{\partial F} \right) \cdot F + \eta \cdot \phi_B(t) = 0 \quad (23)$$

We define F^* as the extremal value of F :

$$F^* = \frac{\frac{\partial \phi_A(t)}{\partial F} - \phi_A(t) + \eta \cdot \phi_B(t)}{\frac{\partial \phi_A(t)}{\partial F} - \eta \cdot \frac{\partial \phi_B(t)}{\partial F}} \quad (24)$$

We can rewrite (24) using the technology cost function definitions given in (11)

$$F^* = \frac{\frac{\partial C_A(t)}{\partial F}}{\frac{\partial C_A(t)}{\partial F} - \frac{\partial C_B(t)}{\partial F}} - \frac{C_A(t) - C_B(t)}{\frac{\partial C_A(t)}{\partial F} - \frac{\partial C_B(t)}{\partial F}} \quad (25)$$

F^* defines the constant fraction of production volume the firm should allocate to B in each time period T_i to T_f which results in an extremal value of total production costs over this period.

We can make several observations about (25). Because both $C_A(t)$ and $C_B(t)$ monotonically decrease with t , the magnitude of the influence of a given change in F must also be decreasing with t :

$$\lim_{t \rightarrow \infty} \frac{\partial C_A(t)}{\partial F}, \frac{\partial C_B(t)}{\partial F} = 0 \quad (26)$$

The denominators in (26) are decreasing with t because their absolute magnitudes are decreasing with t and:

$$\frac{\partial C_A(t)}{\partial F} \geq 0 \quad \text{and} \quad \frac{\partial C_B(t)}{\partial F} \leq 0 \quad (27)$$

Therefore, an incremental increase in F should result in a larger cost reduction for technology A than B, as the incumbent technology is already further “down” its associated learning curve at T_i (due to prior production using this technology):

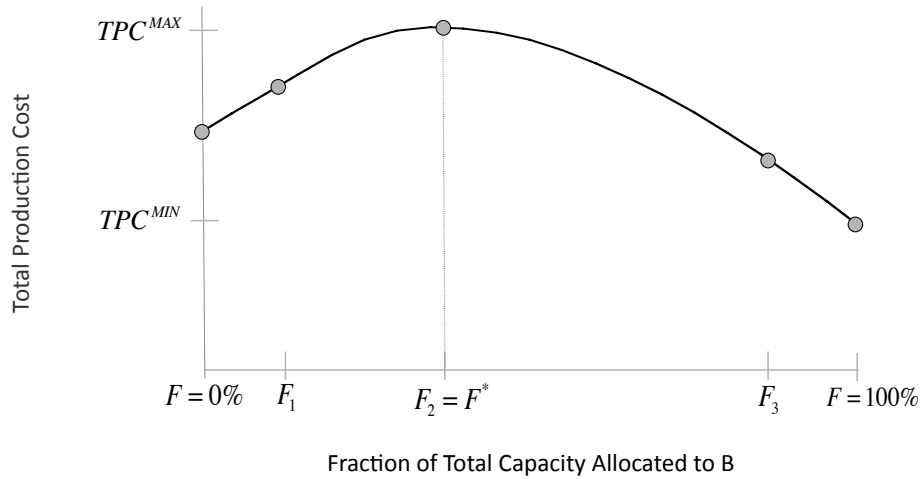
$$\left| \frac{\partial C_A(t)}{\partial F} \right| < \left| \frac{\partial C_B(t)}{\partial F} \right| \quad (28)$$

These observations enable several insights:

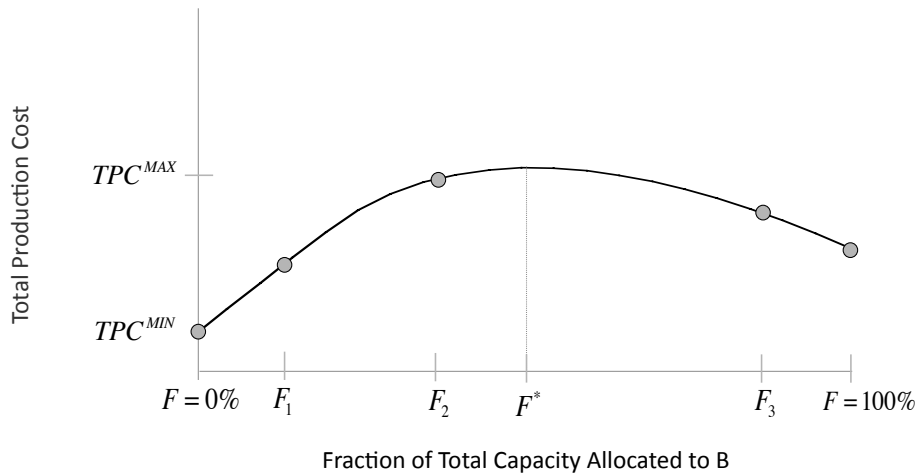
- The first term on the right hand side of (25) must be either decreasing or approaching a constant value. Both derivatives are approaching zero (albeit from different directions), and the numerator is smaller than the denominator; therefore, the numerator is getting smaller faster than the denominator.
- In the cases of interest (conditions specified in (10)), the numerator in the second term of equation (25) is increasing.
- As a result, the first term in (25) is decreasing, and the second term is increasing (the value of the numerator is increasing and the denominator is approaching zero). Therefore, F is decreasing with increasing t .
- Because F is decreasing with increasing t , it represents a solution that seeks to *increase* the use of the technology whose cost is *decreasing more slowly*, while decreasing the rate of cost reduction associated with technology B, which we know should ultimately become the less costly option.

These insights lead us to conclude that F^* defines the unique, (constant) production volume allocation to B (and therefore A) in every period from T_0 to T_f that *maximizes* total production costs, TPC^{MAX} . This result enables us to conclude that TPC is a concave down function with respect to F , with a maximum at F^* . Therefore, minimum total production costs, TPC^{MIN} , will occur at either the *minimum or maximum* possible value of F . This result indicates that the least costly technology decision is always either to

switch as much capacity as possible to B or to stay with A. These results confirm the our expectations base on the behavior observed in Figure 9, and suggest that when CPV is the only factor driving learning, firms only need to compare TPC in (15) at the points $F = F^{MIN}$ and $F = F^{MAX}$. Figure 10 illustrates TPC as a function of F for the example allocations in Figure 9.



(1)



(2)

Figure 10: Examples of total production costs as a function of F when (1) A no longer exhibits learning for $t \geq T_0$, and (2) A exhibits learning for $t \geq T_0$

This parabolic behavior results from a tradeoff between the foregone learning in A and B resulting from shared capacity allocation. Because A is assumed to continue exhibiting learning-by-doing after B is introduced, allocating production to B introduces an opportunity cost in the form of foregone learning in A. However, capacity allocated to A is not available to contribute to learning in B. As a result, sharing production volume reduces the benefits of learning in both technologies.

We can better understand how these tradeoffs impact total production costs by breaking the graph in into two regions. The first considers the allocations to B in the region $F^* < F < 100\%$. In this region, when A is assumed to continue exhibiting learning-by-doing, shifting production to A results in additional realized learning in A. However, because the progress ratio corresponding to B is greater than A, each unit of production diverted to A decreases the realized learning in B more than it increases realized learning in A. As a result, the additional cost of each unit of B produced outweighs the corresponding cost savings in each unit produced with A. This causes total production costs to increase in this region. As more production is allocated to A, the share of total production costs due to A is rising but the cost of each unit produced with A is falling, while the opposite is true for B. However, because diverting capacity to A results in a larger relative cost penalty in B, and B still represents a sizable portion of total production, total costs continue to rise.

The second region considered covers allocations to B such that $0\% \leq F < F^*$. In this region, the benefits associated with learning in A and increasing production allocation to A outweighs the additional costs associated with decreased learning in B and production using B. As a result, total production costs begin to decrease. The rate of this decrease increases (cost curve slope becomes steeper) as more and more production is allocated to A. This is because a larger and larger fraction of production is being allocated to a technology exhibiting decreasing unit costs.

4.6 Impact of Decision Context on Technology Decisions

Figure 10 (1) and (2) are examples of decision contexts (a) and (c) in Table 3. These examples illustrate the different behavior of total production costs as a function of learning in the existing technology. We can use these as a baseline to comment on how the remaining decision contexts, (b) and (d), impact technology decisions.

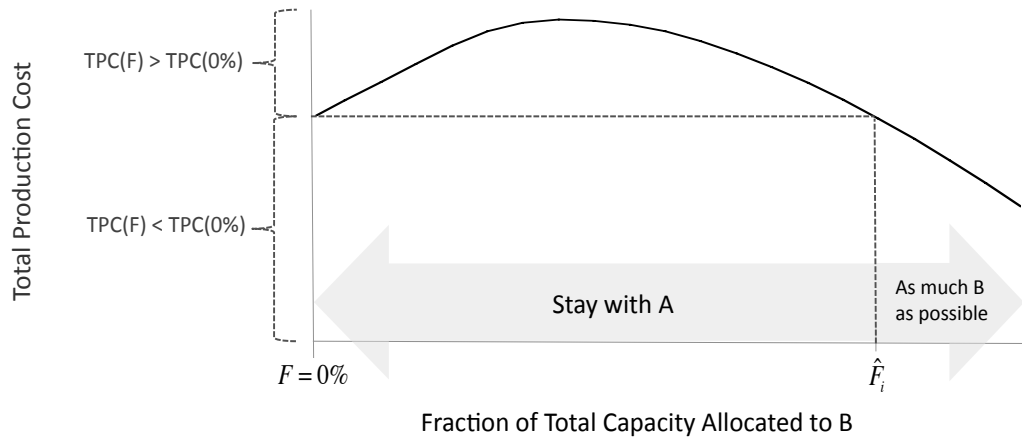
Decision contexts (b) and (d) represent scenarios in which the production allocation is constrained. We define two types of allocation constraints. The first, (i), occurs when some fraction of production will continue to be allocated to A even if B is introduced. In this case, only a fraction of total capacity can be allocated to B. For example, when building new fabrication facilities, semiconductor manufacturers can choose to invest in existing processing technology or in new technology which is expected to drive down production unit costs. However, these facilities take time to build and the firm has sunk significant investment in existing equipment and facilities. As a result, some production will continue to be allocated to A. The second type of capacity constraint, (ii), corresponds to the case when some fraction of capacity must be allocated to B. An example of this case would be when new technologies that must be implemented to comply with new policies or regulations.

In the absence of allocation constraints, the concavity of TPC means that TPC^{MIN} occurs either when the firm stays with A or switches all production to B. However, when facing constraint (i), switching all production to B is not feasible, while in (ii) staying with only A is not possible. Therefore, when facing constraint (i), the firm must decide between staying with A or sharing production between technologies, while under (ii) the decision is between switching to B or sharing capacity.

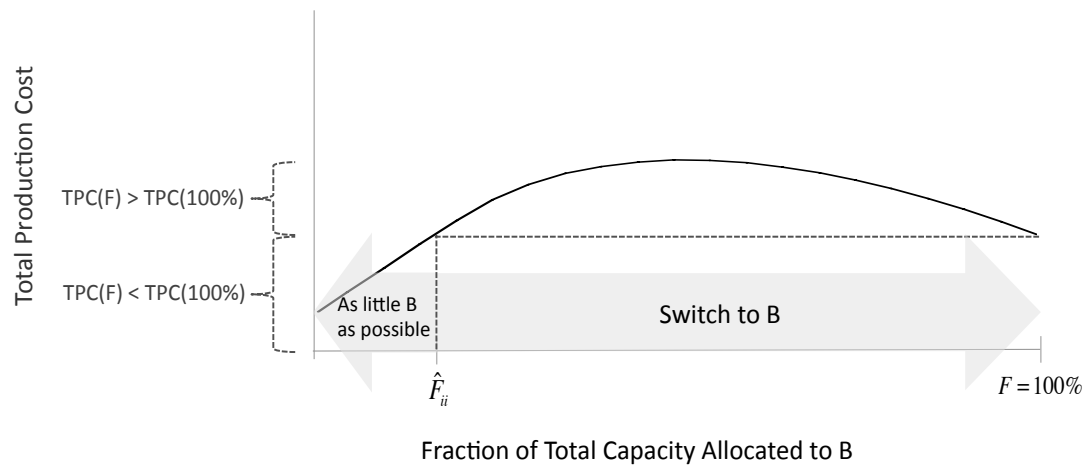
When facing constraint (i), the firm is able to allocate all production to A. Therefore, if staying with A is the least costly option then the constraint has no impact. However, in cases where switching to B would have resulted in TPC^{MIN} , the firm must now characterize the range of conditions under which it is still economically feasible to introduce B, given that some capacity must still be allocated to A. Conversely, when

faced with constraint (ii), if B results in TPC^{MIN} then decision is trivial. However, when staying with A would result in TPC^{MIN} in the absence of the constraint, then the firm needs to characterize how introducing B impacts TPC. Let \hat{F}_i represent the minimum capacity that must be allocated to B to make introducing B cost effective. If the firm cannot allocate at least \hat{F}_i production to B, then it should stay with A. If the firm can allocate at least to \hat{F}_i to B, then it should allocate as much as possible. Let \hat{F}_{ii} be the maximum capacity allocated to B before the firm should switch all production to B. In this case, if the firm must allocate more than \hat{F}_{ii} to B, then it is more cost effective to switch all production to B. If the firm can introduce B at allocations below \hat{F}_{ii} , then it should share production but allocate as little to B as possible.

Figure 11 illustrates how the constraints can impact the technology decision. Row (1) corresponds to constraint (i) while row (2) corresponds to type (ii).



(1)



(2)

Figure 11: Impacts of allocation constraints on technology decisions when (1) some production must continue to be allocated to the A, and (2) some production must be allocated to B

Once initial technology costs and learning behaviors are specified, we can determine the value of the constraint (capacity allocation) that causes a change in the technology decision:

Constraint (i):

$$\text{Set } TPC(F = \hat{F}_i) = TPC(F = 0)$$

$$\text{Solve for } \hat{F}_i$$

(29)

Constraint (ii):

$$\text{Set } TPC(F = \hat{F}_{ii}) = TPC(F = 1)$$

$$\text{Solve for } \hat{F}_{ii}$$

Figure 12 combines the insights gathered thus far to illustrate the total production cost curves in each decision context for the example technologies presented in Figure 5 through Figure 11. The technology choice decisions for each context are also presented. It is important to note that Figure 11 illustrates only example behavior. The specific behavior in each decision context will depend not only on the parameters of the specific context, but also on the factors driving individual technology costs over time.

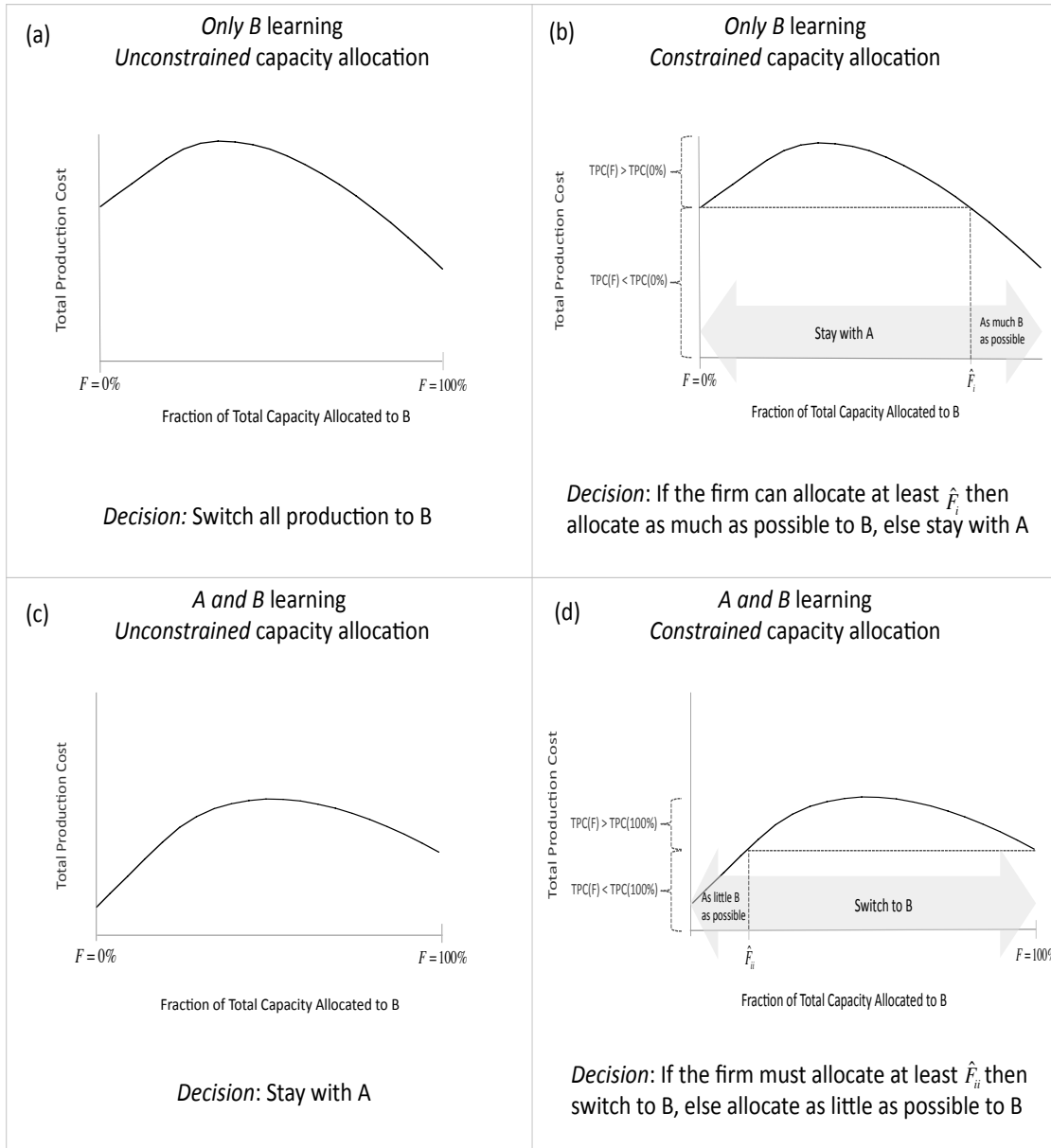


Figure 12: Total production cost behavior as a function of F and technology decisions for each decision context in Table 3 for the example technologies in Figure 5 through Figure 11

4.7 Impact of Factors Driving Technology Costs on Technology Decisions

Irrespective of decision context, total production costs over time are a function of initial technology costs, learning behavior, production volume over time and the timeframe of

the analysis. Initial technology costs and learning behavior determine unit costs as a function of production volume over time. Once these factors are specified, production volume over time and the analysis timeframe will determine the total production costs.

4.7.1 Initial Technology Costs and Learning Behavior

We assume that B is initially more expensive than A, but that the production costs associated with B drop more rapidly with increasing CPV. As a result, technology B becomes less expensive than A on a unit cost basis at some time as a function of CPV over time. This cost crossover is determined by the tradeoff between initial technology costs and the associated learning behavior. If B is significantly more expensive than A initially, then this crossover will only occur if a high progress ratio is expected for B. Conversely, if B is only marginally more expensive initially, then a much smaller progress ratio will enable B to become less expensive. In either case, larger progress ratios associated with B and/or smaller unit cost differentials between A and B will decrease the period over which B is more expensive.

4.7.2 Timeframe Over Which Costs Are Accrued

Given technology learning behavior and initial costs, total production costs for each technology will be determined by the timeframe over which production costs accrue, $(T_f - T_0)$. This timeframe has two components: the period from when the firm first considers introducing B until the time it is introduced $(T_i - T_0)$, and the period from new technology introduction until the final year costs are summed, $(T_f - T_i)$.

If costs are summed over an infinite horizon, $(T_f - T_0) = \infty$, then the firm should always choose to switch to B. This is because B will eventually become the less expensive technology and these savings will continue to accrue ad infinitum. However, we are interested in cases in which rapid technological progress means that technology choice decision points occur frequently. Therefore, we assume $T_f \ll \infty$.

The unit cost benefits to switching to B begin to accrue once B becomes the less expensive technology. However, these benefits will not influence the technology decision

until the introduction costs associated with introducing B are recouped. The shorter the timeframe, the less time the firm has to recoup these costs. Therefore, shorter timeframes favor staying with the existing technology. This effect is enhanced when the existing technology still exhibits learning after T_1 , as this moves the unit cost crossover to higher cumulative production volumes (and therefore longer timeframes), and increases the time until the firm recoups B introduction costs.

Discounting production costs effectively decreases the timeframe over which costs are accrued, as near-term costs are weighted more heavily when calculating TPC. As a result, the cost penalty associated with introducing B plays a larger role in TPC than the longer term cost savings due to learning in B. Therefore, discounting also favors staying with the existing technology.

4.7.3 Total Production Volume Over Time

Recall that production volume drives realized learning. Production volume growth has the same effect as allocating more fixed production to a technology: it increases the rate at which the firm realizes learning-related cost reductions for that technology for a given progress ratio. Given technology learning behavior and an analysis timeframe, growth in production volume will lead to faster cost reductions for both technologies. However, because the new technology exhibits faster learning relative to A, production growth can increase the attractiveness of a new technology by decreasing the time until the new technology is less expensive than the incumbent.

These factors are independent, in that changing one does not cause a change in any another. However, they combine to influence technology cost. For example, as the timeframe over which costs are accrued gets shorter, the new technology is disadvantaged because less time exists for the firm to recoup the additional introduction costs. However, if the costs associated with B are only marginally more than A, and/or B exhibits greater than expected cost reductions via learning (a higher progress ratio), then the firm can recoup these costs over a much shorter timeframe once B is introduced.

4.8 Cumulative Production Volume Learning Example

In this section, we model technology choice in each decision context for two example technologies. We begin by selecting functional forms for learning and production functions. We then use these functions to construct general expressions for technology costs over time. Then, we explore the impact of decision context on technology choice and capacity allocation decisions, and characterize the conditions under which the factors driving technology costs impact these decisions via sensitivity analyses. Finally, we discuss the implications of the results on technology decision making and compare these results to the literature.

4.8.1 Learning Functions

Although several forms for learning curves have been presented since Wright's original formulation (Wright 1936), the Wright model is still the most prevalent form in both the learning literature and in practice (Badiru 1992; Yelle 2007). Therefore, we adopt this form to model learning. We begin by rewriting (1):

$$y(t) = Y_0 \cdot [CPV^t]^n \quad (30)$$

Where CPV^t represents the cumulative production volume up to time t , $y(t)$ is a parameter which is decreasing with increasing CPV and Y_0 is the initial value of this parameter.

Unlike aircraft manufacturing, industries like semiconductor fabrication produce billions of units per year. Simply incorporating these large quantities into (1) would quickly drive costs to zero for any reasonable value of the learning exponent³. Therefore, we introduce a normalized CPV to enable usage of the Wright model for large-scale manufacturing⁴:

³ For example, if a technology exhibits almost no learning, e.g. $p = 0.99$ which leads to $n = -0.015$, a CPV of 1B units results in a 27% decrease in $y(t)$ relative to $y(T_i)$.

⁴ Nadeau et al reach a similar conclusion, see Nadeau, M. C., A. Kar, et al. (2010). "A dynamic process-based cost modeling approach to understand learning effects in manufacturing." *International Journal of Production Economics*(128): 223-234. Trancik also adopts this form Trancik, J. and K. Zweibel (2006). *Technology choice and cost*

$$y(t) = Y_0 \cdot \left[\frac{CPV^t}{P^{T_0}} \right]^n \quad (31)$$

Where the normalization constant, P^{T_0} , is the production volume in the period $t=0$. From (11) we see that:

$$\phi(t) = \left[\frac{CPV^t}{P^{T_i}} \right]^n \quad (32)$$

We use the cumulative production functions from (9) (where we assume $T_f > T_i$ and $F(t)=F$):

$$\begin{aligned} CPV_A^t &= \int_{T_0}^{T_i} P(t) dt + \int_{T_i}^t (1-F) \cdot P(t) dt \\ CPV_B^t &= \int_{T_i}^t F \cdot P(t) dt \end{aligned} \quad (33)$$

The learning functions for the existing and new technologies are therefore given by:

$$\begin{aligned} \phi_A(t) &= \left[\frac{\int_{T_0}^{T_i} P(t) dt + \int_{T_i}^t (1-F) \cdot P(t) dt}{P^{T_0}} \right]^a \\ \phi_B(t) &= \left[\frac{\int_{T_i}^t F \cdot P(t) dt}{P^{T_0}} \right]^b \end{aligned} \quad (34)$$

Where a and b are the learning exponents for technologies A and B, and $b < a < 0$. Once the progress ratio for each technology is defined, the learning exponent is derived using (3).

4.8.2 Production Function

The baseline case assumes constant production volume:

$$P(t) = P^{T_0} \quad (35)$$

4.8.3 Baseline Cost Functions

Incorporating this production function, the learning functions in (34) are given by:

$$T_0 < t < T_i :$$

$$\phi_A(t) = \int_{T_0}^t \left[\frac{P^{T_0}}{P^{T_0}} \right]^a dt = [t - T_0]^a$$

$$T_i \leq t \leq T_f :$$

$$\phi_A(t) = \left[\frac{\int_{T_0}^{T_i} P^{T_0} dt + \int_{T_i}^t (1-F) \cdot P^{T_0} dt}{P^{T_0}} \right]^a = [(T_i - T_0) + (1-F) \cdot (t - T_i)]^a \quad (36)$$

$$\phi_B(t) = \left[\frac{\int_{T_i}^t F \cdot P^{T_0} dt}{P^{T_0}} \right]^b = [F \cdot (t - T_i)]^b$$

From (11), the unit costs for each technology in period t are given by:

$$T_0 < t < T_i :$$

$$C_A(t) = C_A^{T_0} \cdot [t - T_0]^a$$

$$T_i \leq t \leq T_f :$$

$$C_A(t) = C_A^{T_0} \cdot [(T_i - T_0) + (1-F) \cdot (t - T_i)]^a$$

$$C_B(t) = C_B^{T_i} \cdot [F \cdot (t - T_i)]^b$$

(37)

From (14) the total costs for each technology from T_0 to T_f are defined as:

$$\begin{aligned}
TC_A &= \int_{T_0}^{T_i} C_A(t) \cdot P(t) + \int_{T_i}^{T_f} C_A(t) \cdot (1-F) \cdot P(t) dt \\
&= \int_{T_0}^{T_i} C_A^{T_0} \cdot P^{T_0} \cdot [t-T_0]^a dt + \int_{T_i}^{T_f} C_A^{T_0} \cdot [(T_i-T_0) + (1-F) \cdot (t-T_i)]^a \cdot (1-F) \cdot P^{T_0} dt \\
&= C_A^{T_0} \cdot P^{T_0} \cdot \left(\frac{[T_i-T_0]^{a+1}}{a+1} + \frac{\left([(T_f-T_0) - F \cdot (T_f-T_i)]^{a+1} - [T_i-T_0]^{a+1} \right)}{a+1} \right) \\
&= C_A^{T_0} \cdot P^{T_0} \cdot \left(\frac{\left([(T_f-T_0) - F \cdot (T_f-T_i)]^{a+1} \right)}{a+1} \right)
\end{aligned} \tag{38}$$

and

$$\begin{aligned}
TC_B &= \int_{T_i}^{T_f} C_B(t) \cdot F \cdot P(t) dt \\
&= \int_{T_i}^{T_f} C_B^{T_i} \cdot [F \cdot (t-T_i)]^b \cdot F \cdot P^{T_0} dt \\
&= \frac{C_B^{T_i} \cdot P^{T_0} \cdot F^{b+1} \cdot [T_f-T_i]^{b+1}}{b+1}
\end{aligned} \tag{39}$$

The total production costs facing the firm from T_0 to T_f is the sum of these technology costs:

$$TPC = C_A^{T_0} \cdot P^{T_0} \cdot \left(\frac{\left([(T_f-T_0) - F \cdot (T_f-T_i)]^{a+1} \right)}{a+1} + \frac{\eta \cdot F^{b+1} \cdot [T_f-T_i]^{b+1}}{b+1} \right) \tag{40}$$

Where, as in (16), $\eta = C_B^{T_i} / C_A^{T_0}$.

4.8.4 Baseline Scenario Considered

Table 4 presents the parameters and values considered as the baseline scenario. We define $T_0 = 0$ to be the first period in which costs are accrued.

Symbol	Definition	Value
$T_i - T_0$	Periods from T_0 until B introduced	5
$T_f - T_0$	Periods from T_0 over which costs are summed	15
P^{T_0}	Production volume in T_0	10,000
$C_A^{T_0}$	A costs in first period B considered	\$10
η	Ratio of B to A initial costs	1.3
n_A	Progress ratio for A (contexts (b) and (d))	90%
n_B	Progress ratio for B	80% ⁵
a	Learning exponent for A (contexts (b) and (d))	-0.15
b	Learning exponent for B	-0.32

Table 4: Baseline case parameters and values considered

Once B is introduced, we also assume that the capacity allocated to B remains constant over time: $F(t) = F$. Additionally, we assume that all production costs are captured in the unit cost for each technology (there are no additional fixed costs for example). Total production costs are determined using (40).

4.8.5 Baseline Results by Decision Context

For each decision context, we first summarize the technology questions the firm faces in each decision context and the metrics used to answer these questions. Then, we calculate the relevant required quantities of interest and discuss the resulting technology decision.

Decision Context (a)

Summary

Given:

⁵ This value represents the median progress ratio observed for new technologies as observed in Dutton, J. M. and A. Thomas (1984). "Treating progress functions as a managerial opportunity." *The Academy of Management Review* 9(2): 235-247.

- A no longer exhibits learning related cost reductions after $t=0$
- The firm is free to allocate capacity to either technology

Technology Question:

- Should the firm stay with A or switch to B?

Metric:

- Total production costs from T_0 to T_f , (TPC)

Method:

- Compare TPC at $F=0$ and $F=1$

Decision:

- If $TPC(0) < TPC(1)$ then stay with A, else switch to B

When $F=0$, B is never introduced. Therefore $TPC_B=0$ and $TPC=TC_A$. Additionally, because A no longer exhibits learning, $n_A = 100\%$ and therefore $a = 0$. In this case, the total production costs are just the product of the initial unit cost of A and the constant production volume summed from T_0 to T_f :

$$\begin{aligned}
 TC_A &= C_A^{T_0} \cdot P^{T_0} \cdot \left(\frac{[(T_f - T_0) - F \cdot (T_f - T_i)]^{a+1}}{a+1} \right) \\
 &= C_A^{T_0} \cdot P^{T_0} \cdot (T_f - T_0) \\
 &= \$10 \cdot 10,000 \cdot (15 - 0) = \$1.5M
 \end{aligned} \tag{41}$$

When $F=1$, the firm decides to switch all production to B in T_i . However, prior to T_i , the firm will still use A to produce goods. Therefore, we need to add the costs of A from T_0 to T_i to get the total production costs associated with switching to A in T_i . Using (40)

$$\begin{aligned}
 TPC &= C_A^{T_0} \cdot P^{T_0} \cdot \left(\frac{[(T_f - T_0) - F \cdot (T_f - T_i)]^{a+1}}{a+1} + \frac{\eta \cdot F^{b+1} \cdot [T_f - T_i]^{b+1}}{b+1} \right) \\
 &= \$10 \cdot 10,000 \cdot \left(\frac{[(15 - 0) - 1 \cdot (15 - 5)]^1}{1} + \frac{1.3 \cdot 1 \cdot [15 - 5]^{0.68}}{0.68} \right) = \$1.4M
 \end{aligned} \tag{42}$$

In this case, switching to B saves the firm 7% relative to staying with A over the 15 years during which costs will be summed. This result means that, over the timeframe of the

analysis, learning in B enables the firm to both recoup the additional introduction costs associated with B and save an additional \$0.1M by switching to B.

Decision Context (b)

Given:

- A no longer exhibits learning related cost reductions after T_0
- The firm faces constrained allocation

Technology Question:

- If allocation constraint (i): should the firm stay with A or share production between technologies?
- If allocation constraint (ii): should the firm switch to B or share production between technologies?

Metrics:

- Total production costs from T_0 to T_f , (TPC)
- Minimum allocation to B, \hat{F}_i under constraint (i) or \hat{F}_{ii} under (ii)

Method:

- Determine \hat{F}_i or \hat{F}_{ii}
- If constraint type (i): compare $\text{TPC}(F=\hat{F}_i)$ against $\text{TPC}(F=0)$
- If constraint type (ii): compare $\text{TPC}(F=\hat{F}_{ii})$ against $\text{TPC}(F=1)$

Decision:

- Constraint type (i)
 - If $\text{TPC}(0) > \text{TPC}(1)$ and the firm *can* allocate more than \hat{F}_i to B, then share production and allocate as much as possible to B
 - If $\text{TPC}(0) > \text{TPC}(1)$ and the firm *cannot* allocate more than \hat{F}_i to B, then stay with A
- Constraint type (ii)
 - If $\text{TPC}(0) < \text{TPC}(1)$ and the firm *must* allocate more than \hat{F}_{ii} to B, then *switch to B*

- If $TPC(0) < TPC(1)$ and the firm *does not need to* allocate more than \hat{F}_i to B, then share production and *allocate as little as possible* to B

The learning behavior is the same in contexts (a) and (b). Therefore, the expected production costs do not change. However, in (b), the firm is operating under a capacity allocation constraint. If the constraint is of type (ii), then the decision is trivial, as switching to B is the least costly option. However, if the firm faces constraint type (i), then some fraction of capacity will continue to be allocated to A even if B is introduced. In this case, introducing B only makes sense if the amount the firm can allocate to B results in smaller total production costs than staying with A. Therefore, the firm needs to determine the minimum allocation to B, \hat{F}_i , that results in total production cost parity with the costs of staying with A:

$$TPC(\hat{F}_i) = TPC(F = 0) \quad (43)$$

If the firm can allocate at least \hat{F}_i , to B, then it should share production volume between A and B, allocating as much as possible to B, else it should choose to stay with A. Using (38) and (39):

$$TPC(\hat{F}_i) = C_A^{T_0} \cdot P^{T_0} \cdot \left(\frac{[(T_f - T_0) - \hat{F}_i \cdot (T_f - T_i)]^{a+1}}{a+1} + \frac{\eta \cdot (\hat{F}_i)^{b+1} \cdot [T_f - T_i]^{b+1}}{b+1} \right) \quad (44)$$

$$TPC(0) = C_A^{T_0} \cdot P^{T_0} \cdot (T_f - T_0)$$

Using (43):

$$\frac{[(T_f - T_0) - \hat{F}_i \cdot (T_f - T_i)]^{a+1}}{a+1} + \frac{\eta \cdot (\hat{F}_i)^{b+1} \cdot [T_f - T_i]^{b+1}}{b+1} = (T_f - T_0) \quad (45)$$

Where we assume $C_A^{T_0}, P^{T_0} > 0$. For the case when A is no longer learning ($a=0$), this becomes:

$$(T_f - T_0) - \hat{F}_i \cdot (T_f - T_i) + \frac{\eta \cdot (\hat{F}_i)^{b+1} \cdot [T_f - T_i]^{b+1}}{b+1} = (T_f - T_0) \quad (46)$$

Solving for provides the generalized constraint value:

$$\hat{F}_i = \left(\frac{b+1}{\eta \cdot (T_f - T_i)^b} \right)^{\frac{1}{b}} \quad (47)$$

Incorporating the baseline values:

$$\hat{F}_i = \left(\frac{0.68}{1.3 \cdot (10)^{-0.32}} \right)^{-0.32} = 0.76 \quad (48)$$

This result means that, when the firm must continue to allocate production volume to A, it to allocate at least F=76% of production to B in each period to make introducing B cost effective. Conversely, this result suggests that, if the firm needs to continue allocation more than (1-F)=24% of production to A, then it should not introduce B.

Decision Context (c)

Given:

- A continues to exhibit learning related cost reductions after $t=0$
- The firm is free to allocate capacity to either technology

Technology Question:

- Should the firm stay with A or switch to B?

Metric:

- Total production costs from $t=0$ to T_f , (TPC)

Method:

- Compare TPC at F=0 and F=1

Decision:

- If $TPC(0) < TPC(1)$ then stay with A, else switch to B

As in in decision context (a), the choice is between staying with A or switching to B in T_i . However, in this case A continues to exhibit learning after T_0 . As in (a), to decide between A and B, the firm needs to calculate total production costs at the points F=0 and F=1. When F=1, all production is done using B. Therefore, learning in A will not change the total production costs found in (42) As in decision context (a), when F=0, B is never introduced. Therefore $TPC_B=0$ and $TPC=TC_A$. However, unlike (a), in this case, $n_A = 90\%$. We use (41) to find TC_A :

$$\begin{aligned}
TC_A &= C_A^{T_0} \cdot P^{T_0} \cdot \left(\frac{[(T_f - T_0) - F \cdot (T_f - T_i)]^{a+1}}{a+1} \right) \\
&= \frac{C_A^{T_0} \cdot P^{T_0} \cdot [(T_f - T_0)]^{a+1}}{a+1} \\
&= \frac{\$10 \cdot 10,000 \cdot [(15 - 0)]^{0.85}}{0.85} = \$1.17M
\end{aligned} \tag{49}$$

Continued learning in A now makes staying A the least costly decision, saving the firm \$1.4M - \$1.17 = \$0.23M.

Decision Context (d)

Given:

- A continues to exhibit learning related cost reductions after T_0
- The firm faces constrained allocation

Technology Question:

- If allocation constraint (i): should the firm stay with A or share production between technologies? If sharing capacity, what fraction should be allocated to each technology?
- If allocation constraint (ii): should the firm switch to B or share production between technologies? If sharing capacity, what fraction should be allocated to each technology?

Metric:

- Total production costs from T_0 to T_f , (TPC)
- Minimum allocation to B, \hat{F}_i under constraint (i) or \hat{F}_{ii} under (ii)

Method:

- Determine \hat{F}_i or \hat{F}_{ii}
- If constraint type (i): compare $TPC(F=\hat{F}_i)$ against $TPC(F=0)$
- If constraint type (ii): compare $TPC(F=\hat{F}_{ii})$ against $TPC(F=1)$

Decision:

- If $TPC(0) > TPC(1)$ and the firm *can* allocate more than \hat{F}_i to B, then share production and allocate as much as possible to B
- If $TPC(0) > TPC(1)$ and the firm *cannot* allocate more than \hat{F}_i to B, then stay with A
- If $TPC(0) < TPC(1)$ and the firm *must* allocate more than \hat{F}_{ii} to B, then *switch to B*
- If $TPC(0) < TPC(1)$ and the firm *does not need to* allocate more than \hat{F}_{ii} to B, then share production and *allocate as little as possible* to B

In this case, $TPC(0) < TPC(1)$. Therefore, the only constraint that could impact the technology decision is type (ii), in which the firm must allocate some fraction of production to B, even though staying with A is the least costly option. Although $F=0$ results in minimum costs, the nature of the total production cost function suggests there will be a “tipping” point allocation to B above which switching to B becomes the less costly option. This allocation, \hat{F}_{ii} , is found by:

$$TPC(\hat{F}_{ii}) = TPC(F = 1) \quad (50)$$

If the firm must produce more than \hat{F}_{ii} , then it should switch all production to B, else it should share production between A and B but produce the minimum amount of B possible. In this case, technology A still exhibits learning after T_0 . Therefore, (50) is given by:

$$\begin{aligned} & \frac{\left[(T_f - T_0) - \hat{F}_{ii} \cdot (T_f - T_i) \right]^{a+1}}{a+1} + \frac{\eta \cdot (\hat{F}_{ii})^{b+1} \cdot [T_f - T_i]^{b+1}}{b+1} \\ & = \frac{\left[(T_f - T_0) - (1) \cdot (T_f - T_i) \right]^{a+1}}{a+1} + \frac{\eta \cdot (1)^{b+1} \cdot [T_f - T_i]^{b+1}}{b+1} \end{aligned} \quad (51)$$

Where, as in decision context (a), we assume $C_A^{T_0}, P^{T_0} > 0$. This expression does not yield a closed solution for \hat{F}_{ii} . However, we can calculate \hat{F}_{ii} for the baseline scenario by substituting in the values from Table 4. The result is that $\hat{F}_{ii} = 27\%$. This means that if the firm must allocate more than 6% of production volume to B, then it should switch all

production to B. If however, the firm is able to shift less than 27% to B, then it should share capacity between the technologies while trying to produce the minimum possible B.

4.8.6 Discussion of Base Case Results

Figure 13 (a) – (d) presents the total production cost curves as a function of F for the corresponding decision context, and highlights the quantities relevant to decision-making. The decision resulting from each context is also presented.

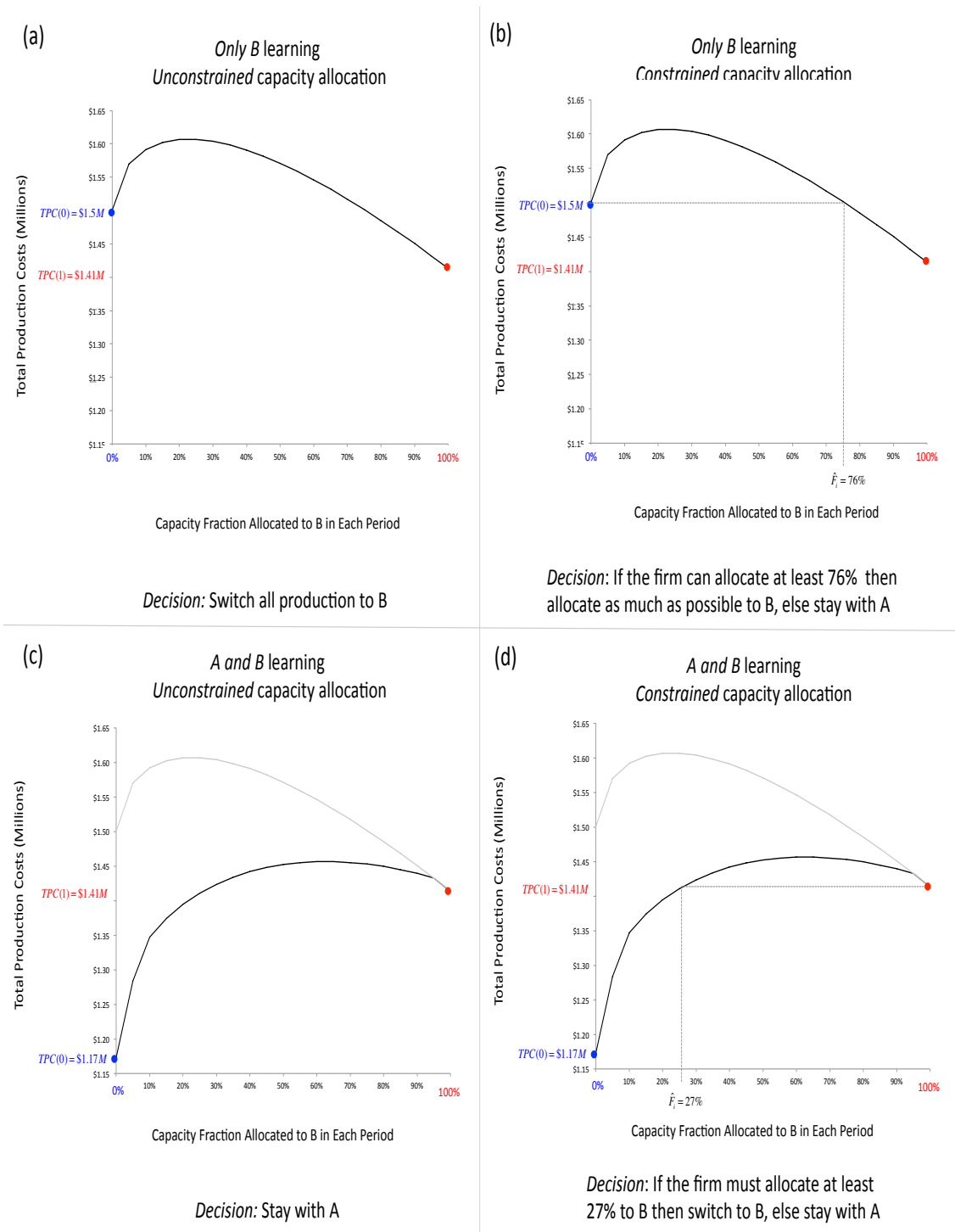


Figure 13: Base case results by decision context

The base case results suggest that the decision context can change the technology decision for realistic values of the underlying factors. Specifically, they suggest that: (i) continued learning in an existing technology can change the technology decision and, (ii)

although allocating all production volume to particular technology may result in minimum production costs, deviations from this allocation due to production constraints may result in significantly higher production costs, (even when the existing technology no longer exhibits learning-by-doing). For example, switching to B results in minimum cost in decision context (a) in Figure 13. Current thinking on learning would suggest that, because switching is the best strategy, if the firm is unable to switch 100% of production to B, then the next best solution is to allocate as much as possible. However, as Figure 13 (b) indicates, this strategy is best only when the firm can allocate at least 76% of production volume to B. The next best solution below this point is not to allocate 75% to B, but rather to *not allocate any to B*.

These non-monotonic allocation preferences are due to the impacts of foregone learning. This effect occurs whenever production volume is shared between technologies, irrespective of whether or not the existing technology still exhibits learning. When only the new technology exhibits learning, as in decision context (b) in Figure 13, every unit still allocated to the existing technology is unavailable to drive learning in the new technology. Although fewer units of the existing technology are being produced, each unit exhibits a higher unit cost. As the firm allocates more production to A, it becomes the less expensive option for longer, decreasing total production costs. However, at allocations less than 76% to B, this benefit is more than offset by the additional costs associated with foregone learning in B. In decision context (d) in Figure 13, continued learning in A changes the decision, so that now staying with A results in minimum production costs. However, while sharing production between technologies was the least costly strategy over a 24% range of allocations to B in context (b) (from $F = 76\%$ to 100%), this strategy only holds over a 6% range when A continues to exhibit learning in (d). This is because B exhibits a higher progress ratio than A. As a result, the cost reductions corresponding to each unit produced with A are not sufficient to offset the cost increases due to foregone learning in B. At 27% allocation to B, the impact of these additional costs more than offsets the cost reductions afforded by A. Therefore, the firm is better off switching all production to B to reduce costs as quickly as possible via learning.

These results suggest small changes in the underlying factors impacting individual technology costs can have large impacts on technology choice and capacity allocation decisions. We would like to characterize the both the magnitude of these impacts, and the values of each factor over which these impacts occur.

4.9 Sensitivity Analyses

We would like to characterize the region over which the decision changes as a function of the factors that drive technology costs: progress ratios, initial costs, when the new technology is introduced, the timeframe over which costs are accrued and production volume growth over time. We can use our assumptions, the results from the base case, and what we know about the problem to define and limit the value range for each factor.

4.9.1 Factors considered

1. Initial technology cost ratio and existing technology progress ratio: (η , n_A)

We are interested in characterizing the conditions under which continued learning in the existing technology changes technology choice decisions. Therefore, we hold the progress ratio for new technology constant at the base case value, 20%. The base case results suggest that the change in the progress ratio in A from 0% to 10% changes the technology decision from stay with A to switch to B. However, as decision context (d) in Figure 13 illustrates, although staying with A results in minimum production costs when $n_A = 10\%$, allocating production volume to A only provides cost savings from allocations $F = 0\%$ to $F = 2\%$. This suggests that a slight reduction in the progress ratio associated with A will result in switching to B becoming the minimum cost solution.

While higher progress ratios mean faster learning, higher initial costs disadvantage the new technology. Therefore, we expect that small initial cost multipliers for B, coupled with a small progress ratio for A will result in the decision to always switch to B (or to allocate as much as possible to B when switching is not an option due to allocation constraints). Conversely, we expect that large cost multipliers for B introduction costs and large progress ratios for A will always lead the firm to stay with A. Therefore, we are interested in combinations of progress ratios both above and below $n_A = 10\%$, but not

values too close to $n_A = 0\%$, and cost multipliers that are not so large as to always favor A irrespective of the progress ratio.

2. Period until new technology is introduced: ($T_i - T_0$)

The longer this period extends, the further into the future the firm defers the introduction costs associated with B. However, this also means that learning in B is also deferred. As a result, for a fixed and finite analysis horizon, T_f , increasing this period reduces the time over which the cost benefits of producing with B accrue. However, when A continues to exhibit learning, deferring the introduction of B leads to additional cost savings in A. The base case is at one extreme of this range, $T_i - T_0 = 0$. This means that the firm is implementing B “today.” This results in the maximum number of periods during which B is the less expensive technology in terms of unit cost. As $T_i - T_0$ increases, the benefits associated with B decrease and the benefits associated with staying with A increase. For these reasons, if the decision is initially to switch to B, increasing the period until introduction will eventually make it impossible for the firm to recoup the costs of introducing B. We are only interested in characterizing the region where the decision changes. Therefore, we model a limited set of introduction periods.

3. Timeframe over which costs are incurred: ($T_f - T_0$)

The longer the timeframe, the longer benefits due to cost savings in B can accrue. Therefore, longer timeframes favor switching to B. Once the timeframe is long enough that the best decision is to switch to B, it will never switch back to A. Because we are interested here in characterizing the region where the decision changes, this enables us to limit the timeframes explored.

Discounting future costs effectively reduces $T_f - T_0$ by reducing the contributions of longer-term costs total production costs. The benefits associated with introducing B take time to realize. Therefore, discounting decreases the attractiveness of switching to B. For a fixed set of the other factors considered, once discounting reduces costs to the point where the best decision is to stay with A, the decision will never change. We assume

exponential discounting of the form. In this case, the total cost functions in (14) are given by:

$$\begin{aligned}
 TC_A &= \int_{T_0}^{T_f} C_A(t) \cdot P_A(t) \cdot e^{-it} dt \\
 TC_B &= \int_{T_i}^{T_f} C_B(t) \cdot P_B(t) \cdot e^{-it} dt
 \end{aligned}
 \tag{52}$$

Where i is the inflation-adjusted or nominal discount rate, given by the Fisher equation:

$$i = (1+r) \cdot (1+E(I)) \tag{53}$$

where r is the discount rate, and $E(I)$ is the expected inflation rate.

4. Production volume growth over time: g

Production volume drives learning-by-doing. Therefore, growth in production volume will drive down technology unit costs. If A continues to exhibit learning, growth reduces the cost of B faster than A. This means that the benefits due to B are experienced sooner than in the base case, where we assume no growth. Therefore, growth favors switching to the new technology. When including growth, the production function in (35) becomes:

$$P(t) = P^{T_0} \cdot (1+g)^{t-T_0} \tag{54}$$

4.9.2 Factor value ranges considered

Table 5 presents the value ranges considered for each factor. The remaining parameter values remain the same as in the base case (Table 4). We use the learning rate for clarity of explanation in place of the progress ratio for the sensitivity analysis. The learning rate is defined as:

$$l = 1 - n_A \tag{55}$$

where n_A is the progress ratio corresponding to A.

Parameter	Value Range
l	0% - 20%
η	1.0 – 2.0
$(T_i - T_0)$	2 yrs, 5 yrs
$(T_f - T_i)$	10 yrs, 15 yrs
i	0%, 5%
g	0%, 5%

Table 5: Semiconductor case sensitivity analysis parameters and value ranges considered

4.9.3 Cost functions

In the simulation, we assume that all production costs associated with a technology can be represented in the corresponding unit costs. Incorporating discounting and growing production, the learning functions for each technology in (36) become:

$T_0 \leq t < T_i$:

$$\phi_A(t) = \left[\frac{\int_{T_0}^t P^{T_0} \cdot (1+g)^{t-T_0} dt}{P^{T_0}} \right]^a = \left[\frac{(1+g)^{t-T_0} - 1}{\ln(1+g)} \right]^a$$

$T_i \leq t \leq T_f$:

$$\phi_A(t) = \left[\frac{\int_{T_0}^{T_i} P^{T_0} \cdot (1+g)^{t-T_0} dt + \int_{T_i}^t (1-F) \cdot P^{T_0} \cdot (1+g)^{t-T_i} dt}{P^{T_0}} \right]^a = \left[\frac{(1+g)^{T_i-T_0} + (1-F) \cdot (1+g)^{t-T_i} - 2}{\ln(1+g)} \right]^a \quad (56)$$

$$\phi_B(t) = \left[\frac{\int_{T_i}^t F \cdot P^{T_0} \cdot (1+g)^{t-T_i} dt}{P^{T_0}} \right]^b = \left[\frac{F \cdot ((1+g)^{t-T_i} - 1)}{\ln(1+g)} \right]^b$$

The total cost functions from (38) and (39) are given by (where $g > 0$):

$$\begin{aligned}
TC_A &= C_A^{T_0} \cdot P^{T_0} \cdot \left(\int_{T_0}^{T_i} \left[\frac{(1+g)^{t-T_0} - 1}{\ln(1+g)} \right]^a \cdot (1+g)^{t-T_0} \cdot e^{i(t-T_0)} dt + \int_{T_i}^{T_f} \left[\frac{(1+g)^{T_i-T_0} + (1-F) \cdot (1+g)^{t-T_i} - 2}{\ln(1+g)} \right]^a \cdot (1-F) \cdot (1+g)^{t-T_i} \cdot e^{i(t-T_0)} dt \right) \\
TC_B &= C_B^{T_i} \cdot P^{T_0} \cdot \left(\int_{T_i}^{T_f} \left[\frac{F \cdot ((1+g)^{t-T_i} - 1)}{\ln(1+g)} \right]^b \cdot F \cdot e^{i(t-T_0)} dt \right)
\end{aligned} \tag{57}$$

These functions are not solvable in closed form, so we use a discrete simulation to perform the sensitivity analyses, where the production volume step size is selected to capture the relevant changes in technology costs.

4.9.4 Results and discussion

We define a baseline production growth, discount rate, and analysis timeline for comparison. Additionally, the baseline case assumes that the firm is free to allocate capacity to either or both technologies. The baseline parameter values are given in Table 6.

Parameter	Baseline Case
l	0% - 20%
η	1.0 – 2.0
$(T_i - T_0)$	5 yrs
$(T_f - T_i)$	10 yrs
i	0%
g	0%

Table 6: Baseline sensitivity analysis parameters

Figure 14 presents the technology choice regimes corresponding to the baseline values as a function of the learning rate of the existing technology and the ratio of initial technology costs. The dependent axis is the ratio of the initial cost of B (in T_i), to A (in T_0). The independent axis is the learning rate for A. Larger values mean that A is “learning faster.” The maximum learning rate for A is set to match that for B, 20%, and the maximum cost ratio is set so that the initial cost of B is 200% the initial cost of A. Smaller values of either parameter favor technology B, while larger values favor A.

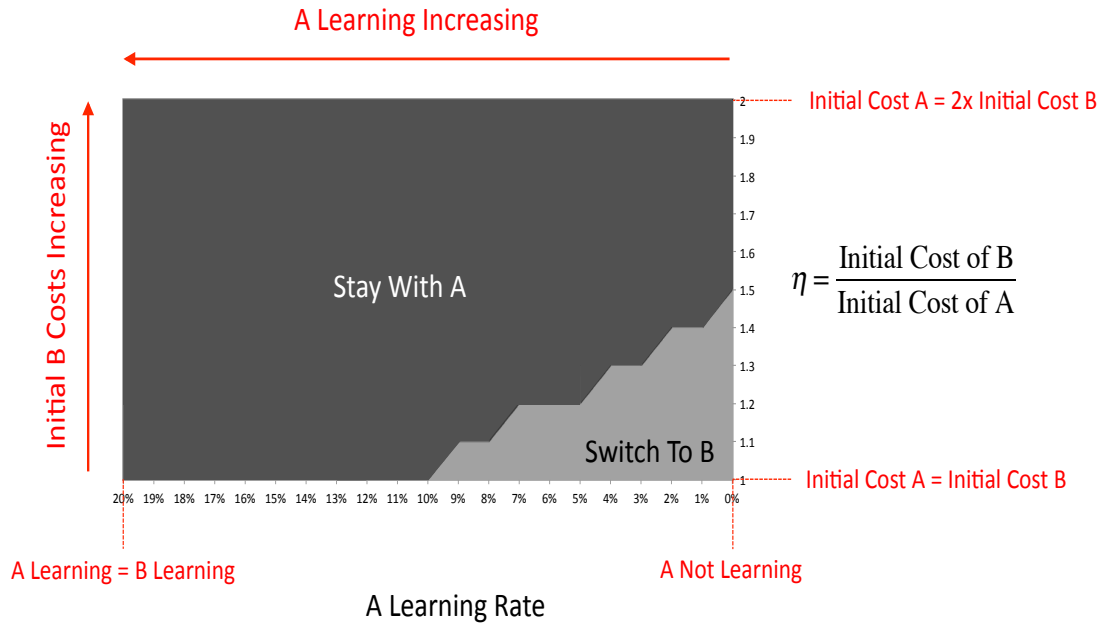


Figure 14: Technology decision as a function of the learning rate for technology A and the ratio of initial costs of B to initial costs of A for the baseline case

The results suggest that, when A is no longer learning ($l = 0\%$), the firm should switch to B for initial costs of B up to 1.5x the initial costs of A at T_0 . As the learning rate for A increases however, the additional expense the firm is willing to accept to implement B decreases. The baseline results also suggest that, if the learning rate for A is larger than 10%, the firm should stay with A even if the initial cost of B equals the initial cost of A. Figure 14 suggests there exists a rate of substitution between how much more expensive B is at introduction and learning in A.

The upper and lower bounds determine the range of acceptable B initial costs and A learning rates over which switching to B is the least costly option. The rate of substitution, S , provides an estimate of the impacts on the decision of the tradeoff between initial costs and continued learning in A. We can interpret the substitution rate directly from Figure 14 in the baseline case as $S = -1$. This means that each 1% increase in the learning rate of A results in a 10% decrease in additional cost of B the firm can accept and still switch to B. This can be seen in Figure 14: increasing the A learning rate from 0% to 1% reduces

the maximum additional introduction costs the firm is willing to pay to switch to B, η^{\max} , from 1.5x (150% the initial cost of A), to 1.4x (140% the initial cost of A).

Sensitivity Case 1: Production volume growth over time

In this case, we vary the growth rate, g , while holding all other parameters constant to characterize the impact of production growth over time on the decision. In this case, the growth rate is set to 5%. Figure 15 presents the results. The baseline results are included for comparison

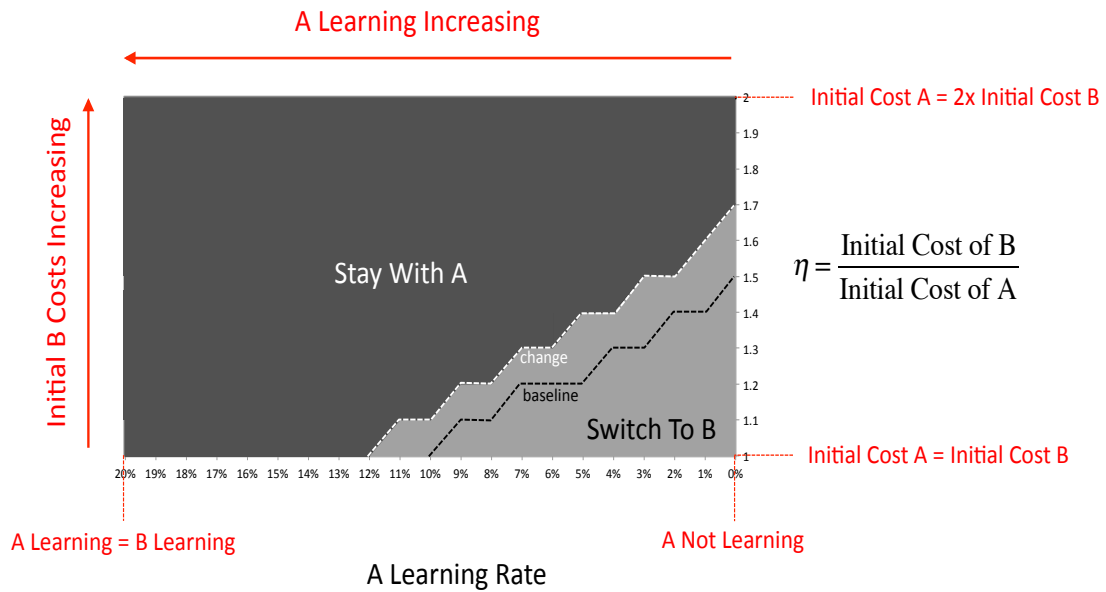


Figure 15: Technology decision as a function of the learning rate for technology A and the ratio of initial costs of B to initial costs of A for the case when $g = 5\%$

As production increases, each technology moves down the associated learning curve faster. This drives down total production costs irrespective of which technology is selected. However, because B is associated with a larger learning rate, increasing production makes B cost competitive with A sooner. As a result, switching to B is the least costly decision over a larger range, both in terms of the A learning rate and the initial technology cost ratio. The results suggest that, if A no longer exhibits learning, the firm should switch for initial B costs up to 1.7x A initial costs, an increase in 13% over the baseline. Even for small learning rates in A however, the initial B cost the firm is

willing to accept to switch decreases significantly. For example, if the A learning rate increases to 2% (1/10 the learning rate for B), the maximum initial cost of B the firm is willing to accept to switch drops to 1.5x the initial cost of A. If the technology A learning rate is greater than 12% then the firm should stay with A, irrespective of the cost of B. Table 7 highlights the differences between the baseline and sensitivity case 1, where we define η^{\max} as the additional premium the firm is willing to pay to introduce B if A is no longer learning, and l_A^{\max} as the progress ratio of A for which the firm should not introduce B irrespective of the initial cost.

Case	Parameter Changed	η^{\max}	l_A^{\max}	S
Baseline	$g = 0\%$	1.5	10%	-1
Sensitivity 1	$g = 5\%$	1.7	12%	-1.2
% Change	+5%	+13%	+20%	-20%

Table 7: Results of growth in production volume sensitivity compared to the baseline case

In this case, $S > -1$. This means that increases in the progress ratio of A have a larger impact on the premium the firm is willing to accept to introduce B. This means that production volume growth makes the technology decision more sensitive to the learning rate of A.

Sensitivity Case 2: discounting cash flows

In this case, we explore the impacts of discounting future cash flows on technology choice. Discounting weights short-term costs more heavily than long-term costs in total production cost calculations. Because the new technology costs more initially and takes time to become cost competitive we expect that discounting will limit the range of conditions under which the firm will adopt B. The effect of discounting is to “flatten” the total production cost curve at all levels of capacity allocation due to smaller contributions of later costs. This effect does not change the fundamental behavior of the production cost curve however, in that the extrema is still a maxima, and the least costly option is

still either to stay with A or switch to B. Figure 16 presents the results for a discount rate of 5%.

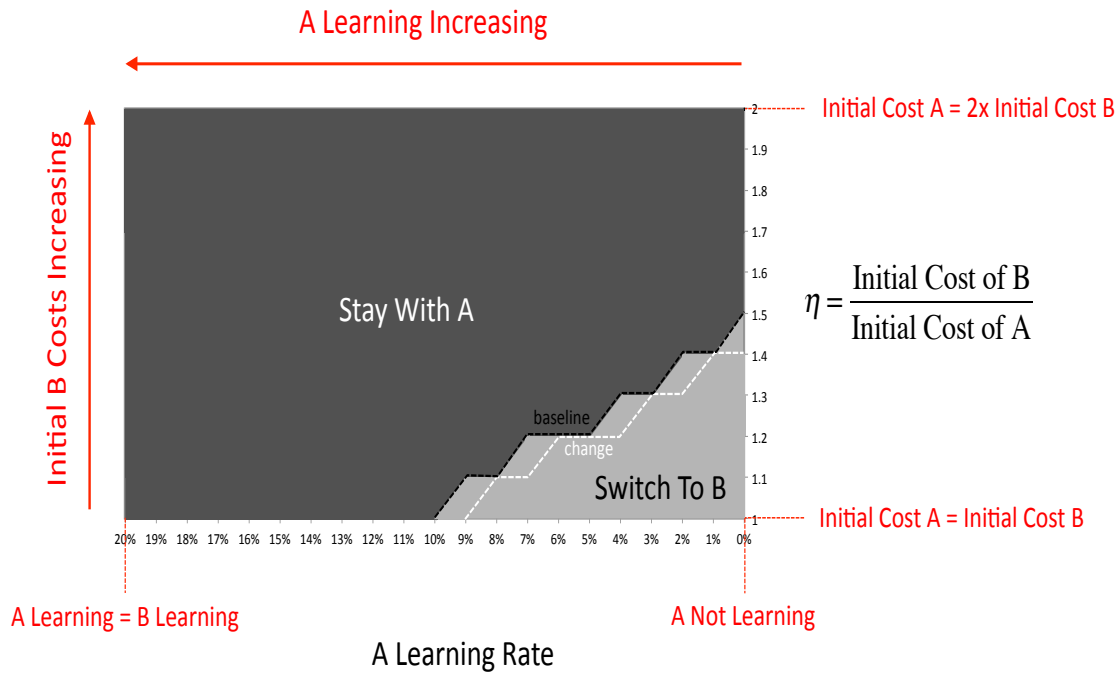


Figure 16: Technology decision as a function of the learning rate for technology A and the ratio of initial costs of B to initial costs of A for the case when $d = 5\%$

The results confirm our intuition that discounting reduces the region over which switching to B is preferred. Specifically, as the result of a 5% discount rate, if A is no longer learning, the firm should switch to B for initial costs up to 140% the initial costs of A. Additionally, if A exhibits a learning rate greater than 9% (less than $\frac{1}{2}$ the B rate) then the firm should stay with A irrespective of the cost of B.

Case	Parameter Changed	η^{\max}	l_A^{\max}	S
Baseline	$d = 0\%$	1.5	10%	-1
Sensitivity 1	$d = 5\%$	1.4	9%	-1
% Change	+5%	-7%	-10%	0%

Table 8: Results of discount rate sensitivity compared to the baseline case

In this case, $S = -1$. This means that, although discounting reduces the total region over which the firm should switch to B, it does not change the relative tradeoff between increases in the learning rate for A and the initial cost ratio.

Sensitivity Case 3: technology B introduction year

In this case, we examine the impacts of the year B is introduced on the technology decision, by assuming $T_i = 2$ years, compared with $T_i = 5$ years in the baseline case. Figure 17 illustrates the results.

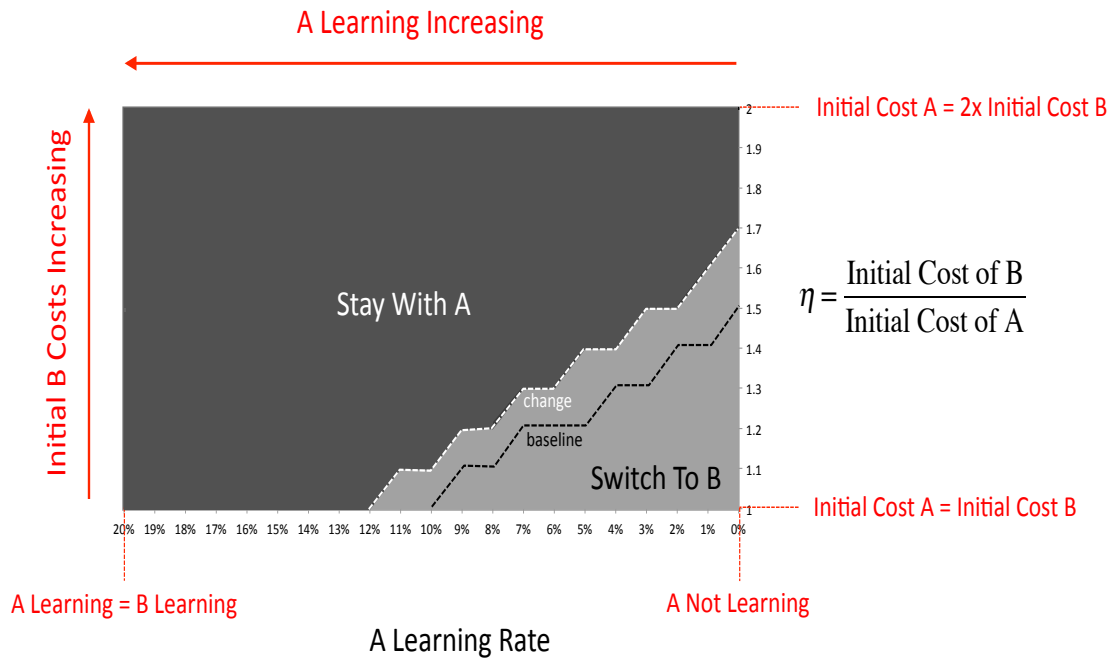


Figure 17: Technology decision as a function of the learning rate for technology A and the ratio of initial costs of B to initial costs of A for the case when $T_i = 2$ years

Introducing B earlier means that the costs and benefits are experienced sooner. As a result, switching to B is the least costly decision over a larger range, both in terms of the A learning rate and the initial technology cost ratio. In this case the results suggest that introducing B 3 years earlier increases the premium the firm will pay to introduce B if A is no longer learning from 1.5x to 1.6x. This is because the firm will have 3 more years to recoup these costs. The results also show that if the A progress ratio is greater than 14%, the firm should stay with A irrespective of the initial cost of B. Once B becomes the less expensive technology, every additional year results in a cost savings compared with

producing using A. Additionally, because B “learns” faster than A, this savings grows over time. As a result, the region over which continued learning in A changes the decision gets smaller (larger A learning rates required to change the decision).

Table 9 presents the results and changes between this case and the baseline.

Case	Parameter Changed	η^{\max}	l_A^{\max}	S
Baseline	$T_i = 5$ yrs	1.5	10%	-1
Sensitivity 1	$T_i = 2$ yrs	1.6	14%	-0.86
% Change	-60%	+7%	+40%	+14%

Table 9: Results of sensitivity to B introduction year compared to the baseline case

In this case, $S > -1$. This means that, increases in the progress ratio of A have a smaller impact on the premium the firm is willing to accept to introduce B. This means that production volume growth makes the technology decision less sensitive to the learning rate of A. This means that A must “learn faster” than in previous cases in order for the firm to decide to stay with A.

Sensitivity Case 4: analysis horizon

In this case, we extend the timeline of the analysis, from $T_f = 15$ to $T_f = 20$ years while holding the B introduction year at the baseline figure, $T_i = 5$.

Figure 18: Technology decision as a function of the learning rate for technology A and the ratio of initial costs of B to initial costs of A for the case when $T_i = 5$ and $T_f - T_i = 15$ years

Increasing the timeframe of the analysis has the opposite effect as introducing discounting. Because it takes time for B to become the less expensive technology, the longer the horizon the more periods over which the firm saves production costs by switching to B. This means that the firm has more time to recoup the initial costs associated with introducing B. As a result, it will accept a higher initial cost penalty than

in the baseline case. Additionally, this longer timeframe makes the learning rate of A less important, as every additional year B learns faster than A, leading to additional cost savings.

Sensitivity Case 5: constrained capacity allocation

Thus far, we have explored cases in which the firm is free to allocate all production volume to one or both technologies and A may or may not exhibit learning. These correspond to decision contexts (a) and (c) in Table 3. However, we are also interested in characterizing the how constraints on capacity impact the technology choice in decision contexts (b) and (d). Specifically, we would like to characterize how the threshold at which the firm should switch from A to B changes as a function of the learning rate for A and the ratio of initial costs.

The previous analyses have characterized the binary decision to either stay with A or switch to B. However, when capacity is constrained, these decisions have an extra component. Recall that capacity constraints are one of two types. Type (i) is only relevant when the optimal decision is to switch to B, but the firm is unable to allocate 100% of capacity to B. This constraint changes the technology decision from simply stay with A or switch to B. The technology question in this case is: given that the firm must continue to allocate some level of production to A what level of capacity, \hat{F}_i , does the firm need to allocate to B in order to make B cost effective? If the firm cannot introduce B at at least this threshold level, then it should stay with A. The type (ii) constraint is relevant when staying with A is the best decision, but the firm must produce some level of B. The technology question in this case is: given that the firm must allocate some level of production to B, what is the threshold, \hat{F}_{ii} , at which the firm should switch to B?

We start with constraint type (i). This constraint becomes important in the “Switch To B” region when the firm must either share production capacity or stay with A. For each combination of A learning rate and initial cost ratio we can identify the minimum amount of capacity the firm must allocate to B make it cost effective. Figure 19 illustrates the

baseline scenario results in Figure 14 where we focus in on the “Switch To B” region: $1 \leq \eta \leq 1.5$, $0\% \leq n_A \leq 10\%$. The points overlaid on the graph represent the threshold percentage of total capacity at which the firm needs to introduce B to make it cost effective, \hat{F}_i , where we find the \hat{F}_i values using (43) in the simulation model. For example, at the yellow point in Figure 19, the A learning rate is 0%, indicating that the existing technology is no longer learning, and the initial cost ratio is 1.1. This point indicates that the firm must be able to introduce at least 5% of B to make B cost effective. This makes sense, as A is initially less expensive, so in order to make B the less expensive technology, the firm would need to produce only a small amount, in this case 5%. This scenario, as well as all scenarios on the $n_A = 0$ line, are examples of decision context (b), in which A is no longer learning but the firm needs to share capacity. When the A learning rate increases to 1% however (the blue point in Figure 19), the threshold jumps significantly, to 30%, even though technology B is still only 10% more expensive than A at the time of introduction. This point, and all points not on the $n_A = 0$ line, are examples of decision context (d) in Table 3.

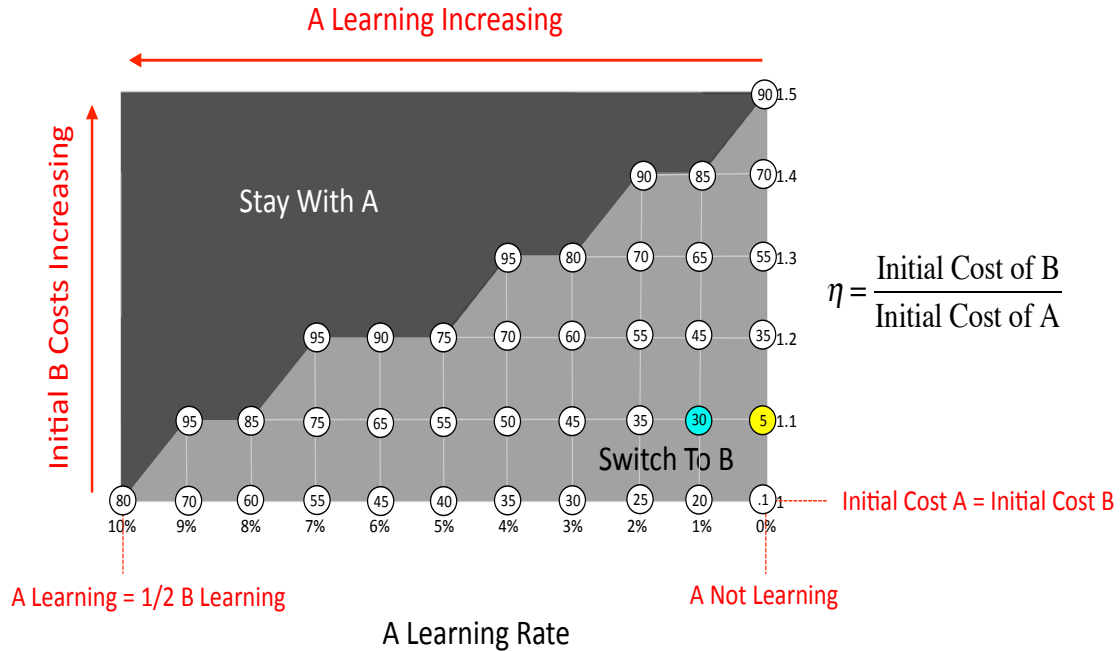


Figure 19: Baseline case threshold capacity constraint, \hat{F}_i , below which the firm should stay with A as a function of the A learning rate and initial cost ratio

The threshold for B viability gets larger with increasing learning rate for A and/or the initial cost ratio. Intuitively this is what we would expect, as increases in these factors disadvantage B. The large threshold values at the decision interface indicate that the firm must introduce almost 100% B in order to make B viable. Because the firm cannot introduce 100% B, at A learning rate and initial cost ratio values above this interface staying with A becomes the less expensive option.

The second constraint becomes important in the “Stay With A” decision region in Figure 19. In these cases, for each combination of A learning rate and initial cost ratio we identify the allocation threshold to B at which switching to B becomes the less costly option. Figure 20 illustrates the baseline case where this time we focus on a subsection of the “Stay With A” region: $1 \leq \eta \leq 1.7$, $0\% \leq n_A \leq 10\%$. The points overlaid on the graph represent the threshold percentage of total capacity above which the firm should switch to B, \hat{F}_{ii} . We calculate the \hat{F}_{ii} values using (50) in the simulation model. For example, the yellow point in Figure 20 corresponds to an A learning rate of 0%, indicating that the

existing technology is no longer learning, and an initial cost ratio is 1.6, indicating that B is 60% more expensive than A in their respective introduction years. Under these conditions, the firm should switch to B if it must allocate more than 5% of total production volume to B. This indicates that, although B is significantly more expensive, if the firm must allocate some production to B, it is better to offset the initial expense by driving down the learning curve as quickly as possible. This scenario, as well as all scenarios on the $n_A = 0$ line, is an example of experiencing a type (ii) capacity constraint in decision context (b). As in the case of constraint (i) however, learning in A significantly increases the threshold at which the decision changes. When the A learning rate increases to 1% (the blue point in Figure 19), the threshold again jumps significantly, this time to 12%. This indicates there are non-trivial benefits to continuing to produce with A up to $\hat{F}_{ii} = 12\%$ that will be lost if the firm switches to B. As in the case of constraint (i), all points not on the $n_A = 0$ line, are examples of decision context (d) in Table 3.

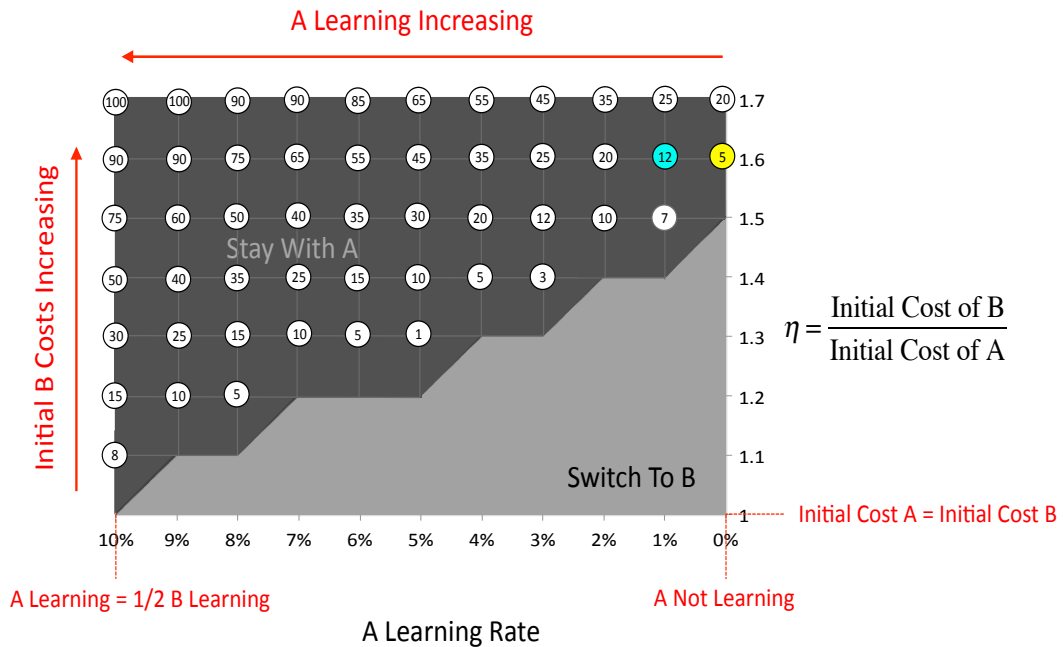


Figure 20: Baseline case threshold capacity constraint, \hat{F}_{ii} , above which the firm should switch to B as a function of the A learning rate and initial cost ratio

The results from both capacity constraint examples indicate that both learning in the existing technology and both types of capacity constraint can have significant impacts on the technology decision for the firm. This suggests that by providing mechanisms to characterize these impacts, the methodology presented adds value to firm technology choice decision-making.

5 Learning-by-Doing Case Study: Semiconductor Fabrication

5.1 Background

Chipmakers in the semiconductor industry are constantly thinking of ways to meet rapidly growing demand. One way to accomplish this is to increase the size of the silicon wafer substrates so that more chips can be processed per wafer. Increasing wafer size enables economies of scale, as a linear increase in wafer diameter results in a squared increase in the available wafer area. However, wafer size transitions are extremely expensive, requiring a redesign of virtually every piece of equipment in the factory and significantly increasing raw wafer costs.

Table 10 presents a list of the wafer size transitions and the corresponding industry costs over the last 20 years, and current best estimates for the change to 450mm wafers.

Wafer Diameter	Transition Period	Transition Costs
200mm	Early 1990s	\$1B
300mm	1998 - 2003	\$5B
450mm	2014 - 2018	>\$20B (estimate)

Table 10: Wafer size transitions and industry costs

As Table 10 illustrates, wafer size transitions⁶ have occurred approximately once a decade, with the current largest wafer having a diameter of 300mm. Chipmakers are currently considering a move to 450mm wafers with a processing area per wafer that is

⁶ As opposed to feature size reductions which occur very 2-3 years and are associated with Moore's Law

approximately 2.25 times (1.50 squared) that provided by the current 300mm wafers. Because not all processing equipment scales linearly with wafer size, the costs associated with 450mm processing technology is expected to scale at a slower rate than 2.25, providing some economies of scale. Table 10 also shows that the cost of wafer transitions has gone up exponentially, as the technology required to grow and process larger and larger wafers gets increasingly expensive. Historically, “these transitions have been unpleasant experiences for the lead company in that it had to bear the burden of development costs, manufacturing delays, and poor equipment performance, all at little or no cost benefit” (Seligson 1998). The switch to 450mm is expected to be, by far, the most expensive transition thus far, so chipmakers are carefully weighing the decision to equip new fabrication facilities with either the existing 300mm or new and more expensive 450mm technologies.

These prohibitively high switching costs, coupled with significant capital expense already sunk in existing 300mm fabrication facilities mean that even if the transition to 450mm wafers occurs, it will exist side by side with 300mm for some time. Additionally, 300mm production costs continue to fall due to continued learning and process improvements.

5.2 Overview of Approach

In the general formalism and example already presented, we assumed a single progress ratio for each technology. However, within a given technology multiple operational characteristics, such as reject rate and cycle time, may exhibit learning. Research has suggested that considering these multiple sources of learning may result in significantly different learning behavior than assumed at the aggregate level (Nadler and Smith 1963; Montgomery and Day 1983; Terwiesch and Bohn 2001) Additionally, characterizing learning at the operational may enable firms to identify where process improvements may have the largest impacts on production costs (Fuchs, Bruce et al. 2006; Nadeau, Kar et al. 2010).

Our methodological goals are not only to provide a mechanism to characterize the impacts of learning on technology choice, but also to provide a means for decision

makers to identify the important drivers of these impacts. A method has been developed within our research group which performs these functions by coupling learning with Process-Based Cost Modeling (PBCM) to map the effect of learning in multiple operational parameters on technology cost. PBCM relates final product characteristics (size, shape etc.) to the technical parameters of the process required to produce it, and derives technology costs from the technical and operational parameters to estimate total production costs (Kirchain and Field 2001).

Our approach incorporates technical knowledge about products and processes from experts with working knowledge and enables data collection in terms of processing variables that are tracked, rather than relying on historical estimates of learning rates. This enables identification of the key cost drivers and characterization of the relationships between processing variables.

In the remainder of this chapter, we follow the method described in Nadeau et. al. (Nadeau, Kar et al. 2010), in that we first define the static PBCM model, which derives technology costs from operational characteristics, and then define the learning functions considered and incorporate them into the static PBCM to enable technology costs to be modeled dynamically over time. We then define the parameters and scenarios of interest in the wafer transition case. Finally, we explore technology decision-making in these scenarios, and comment how the decision context and underlying factors driving technology costs impact these decisions.

5.3 Process-Based Cost Model Description

PCBMs have been employed to inform technology decisions across a wide range of industries, including automotive manufacturing (Han and Clark 1995; Johnson and Kirchain 2009), e-waste (Gregory and Kirchain 2006), and microphotonics (Singer and Wzorek 2006). Incorporating learning curves into this approach enables identification of the effect of learning on individual processing parameters and the impact of this learning on technology decisions (Nadeau, Kar et al. 2010).

Process-based cost modeling relates final product or part characteristics such as size, shape, and material to the technical parameters of the process required to produce that product. These parameters can include cycle time, reject rate and downtime. The process model also characterizes the relationships and constraints between various processing variables. For example, increases in downtime and reject rates can limit the technical feasibility of reductions in cycle time.

5.3.1 PBCM Without Learning

The static PBCM framework, introduced by Field and Kirchain (Field and Kirchain 2007), is presented in Figure 21.

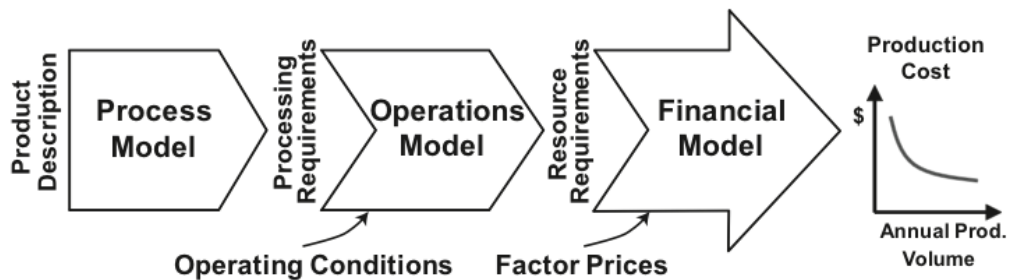


Figure 21: Static PBCM framework (Field and Kirchain 2007)

In the model processing requirements are passed on to an operational sub-model along with production operating conditions, which take into account the production shift schedule, working hours, and production volume. These inputs are translated into the total amount of equipment, materials, labor, and other resources needed to achieve the desired product output. The financial sub-model applies factor prices to the resource requirements determined by the operations model, and allocates costs over time and across products in order to output a unit production cost. This figure can be broken down in terms of fixed and variable costs or into individual contributions from labor, equipment, tooling, and material costs. Although this cost is not time-dependent or cumulative volume-dependent, the underlying relationships implemented by the model enable the analysis of variations in production costs as operating and processing parameters change. Such sensitivity analyses allow identification of primary cost drivers that can be targeted for improvement.

In this case, the product considered (the functional unit), a wafer. In the semiconductor industry, decreases in feature size enable higher densities of transistors per chip every 2-3 years. This results in multiple different products at the chip level, and an increasing number of chips per wafer over time. However, wafer size transitions occur much more slowly, on the order of 8-10 years. Because we are interested in modeling multiple technologies used to produce a single product, we need this product to remain constant over this time period. To accomplish this, we assume a constant 45nm feature size⁷ for all transistors on both 300mm and 450mm wafers. This ensures the same number of transistors per wafer over the timeframe of interest.

We need to define several factors specific to semiconductor fabrication to develop an expression for total costs. The total yield (Y), is a measure of the percentage of usable transistors per wafer. The total yield is the product of two sub-components, the yield per die and the yield per wafer. The former tracks the efficacy of building transistors on the silicon substrate, while the latter characterizes imperfections etc. in the wafer itself. The novelty of 450mm wafer processing and the larger substrates are expected to decrease yields initially compared to 300mm processing. The wafer starts per month, (WSPM), is a measure of the total number of wafers of each size that can be produced by a single fabrication facility. This parameter determines the cycle time per wafer. Although the total WSPM is not expected to change significantly for 450mm wafers, the same WSPM will result in approximately 2.25x as many transistors. The third parameter tracks the costs of raw wafers. As wafer diameters get larger, it becomes more difficult to grow them at the purities and structural integrity required to make high-performance semiconductors. As a result, 450mm wafers are expected to be considerably more expensive than 300mm wafers.

The total wafer volume to be produced, V_{gross} will exceed the target volume, V_{net} due to yield losses:

⁷ Although many features sizes are produced at any point in time, 45nm represents the feature size currently considered for production on the majority of 450mm wafers

$$V_{gross} = \frac{V_{net}}{Y} \quad (58)$$

Where Y is the total yield, $0\% \leq Y \leq 100\%$. We assume that rejected components are not reworked. We assume a baseline facility that can produce a quantity of wafers per month, $WSPM_{base}$. The firm can build bigger or smaller facilities to meet new demand. The wafer starts per period required per new facility is defined in terms of multiples of this baseline. Given V_{gross} , we determine this multiplier N using:

$$N = \frac{V_{gross}}{WS_{base}} \quad (59)$$

We assume the production time for each baseline facility is given by:

$$PT_{base} = PPF_{base} \cdot DPP \cdot NS \cdot (HPS - PD) \quad (60)$$

Where PT_{base} is the production time period, PPF_{base} is the personnel per facility, DPP is the days per period, NS is the number of shifts per day, HPS is the hours per shift and PD is paid downtime.

The financial model applies factor prices to these resources. It outputs a cost per period for each technology. The costs per period are divided into six categories:

$$C(t) = C(t)_{wafer} + C(t)_{tools} + C(t)_{bldg} + C(t)_{var} + C(t)_{labor} \quad (61)$$

Total materials costs are the raw wafers, RWC :

$$C(t)_{wafer} = V_{gross} \cdot RWC \quad (62)$$

We consider building, equipment and tooling to be capital expenses. Incorporating these into a unit cost requires distributing them across time by determining the sum of payments in each period that is functionally equivalent to the initial investment. We distribute these payments over the useful life of the building, equipment and tools. In order to determine the payment per period, we define the capital recovery factor, CRF :

$$CRF = \frac{i \cdot (1+i)^{UL}}{(1+i)^L - 1} \quad (63)$$

where i is the inflation adjusted discount rate and UL is the useful life in years. We assume an initial tooling investment for the baseline facility, CAP_{tool} . Therefore, the cost per period is:

$$C_{tool} = N \cdot CAP_{tool} \cdot CRF \quad (64)$$

We assume an initial building and equipment investment CAP_{bldg} . Therefore, the cost per period is:

$$C_{bldg} = N \cdot CAP_{bldg} \cdot CRF \quad (65)$$

We classify additional variable costs as all non-wafer costs required to process each wafer (chemicals etc.) as VC. Total variable costs are:

$$C(t)_{var} = V_{gross} \cdot VC \quad (66)$$

Finally, total labor costs is given by the product of the total production time for each facility, scaled by the baseline factory size multiplier, N, and the hourly labor rate, LR:

$$C(t)_{labor} = PT_{base} \cdot N \cdot LR \quad (67)$$

The unit cost in each period, UC, for a technology is given by:

$$UC(t) = \frac{TC(t)}{V_{net}} \quad (68)$$

5.3.2 Learning and Dynamic PBCM

The PBCM considers process and price parameters in its cost calculation. Therefore, it enables investigation of the specific impact on cost of variation in these parameters over time. To integrate learning into the PBCM framework in Figure 21, we incorporate individual learning curves for processing and cost components.

Learning has been primarily observed in the three operational parameters explained above: yield, wafer starts per month and raw wafer costs. We can easily incorporate individual learning curves for each factor into the PBCM to investigate how each factor impacts technology costs over time. Figure 22 highlights the factors for which we model learning considered in the PBCM in Figure 21. Wafer starts per month and yield represent processing related factors, while raw wafer costs represent a factor price.

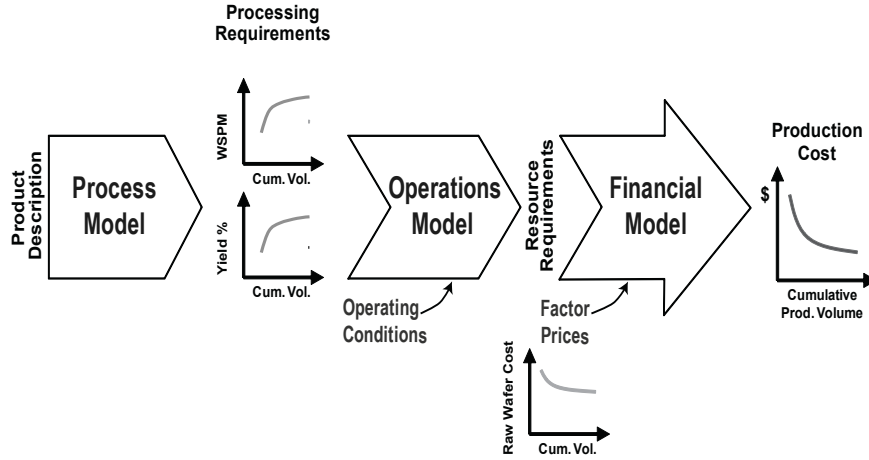


Figure 22: Dynamic process-based cost modeling incorporating learning in both processing requirements (wafer starts per month and yield), and factor costs (raw wafer costs)

For each parameter, learning is assumed to follow the power law formulation in (31). We assume that each parameter has an initial value and that this value declines with increasing cumulative production volume over time. We set a final value for each parameter beyond which the curve becomes flat and learning no longer occurs.

Unlike traditional parameters whose values decrease as CPV increases, learning in the yield and WSPM parameters increase their values. Therefore, we need to transform the learning exponent to reflect this modification. In this case, the progress ratio represents the percentage increase in the parameter value for each doubling of cumulative production volume. For example, if a progress ratio of $p = 80\%$ corresponds to a decrease in the parameter value of 20% for every doubling of CPV, then we need to transform this such that the parameter value increases by 20% for the same doubling. Let p^* be the new progress ratio that performs this function. Then, in this example:

$$\begin{aligned}
 p &= 0.8 \\
 p^* &= 1.2 = 1 + (1 - p) = 2 - p
 \end{aligned}
 \tag{69}$$

using (2), and defining n^* as the learning exponent for the increasing parameter:

$$\begin{aligned}
 p &= 2^{-n} \\
 p^* &= 2^{-n^*} = 2 - p
 \end{aligned}
 \tag{70}$$

The learning exponent is therefore given by:

$$n = \frac{\ln(p)}{\ln(2)} \quad \therefore \quad n^* = \frac{\ln(2-p)}{\ln(2)} \quad (71)$$

We define the learning functions for each parameter from (31) and the fact that each parameter has a limiting value. For total yield this is given by:

$$Y(t) = \min \left(Y_{initial} \cdot \left[\frac{CPV^t}{P^{T_0}} \right]^{y^*}, Y_{max} \right) \quad (72)$$

where $Y_{initial}$ is the total yield in the initial period, y^* is the learning exponent corresponding to (increasing) yield and Y_{max} is the maximum possible yield. For wafer starts per period the function is:

$$WSPM(t) = \min \left(WSPM_{initial} \cdot \left[\frac{CPV^t}{P^{T_0}} \right]^{w^*}, WS_{max} \right) \quad (73)$$

where $WSPM_{initial}$ is the baseline wafer starts per month in the initial period, w^* is the learning exponent corresponding to (increasing) wafer starts per period and WS_{max} is the maximum possible wafer starts per month. For raw wafer costs, the learning function is:

$$RWC(t) = \max \left(RWC_{initial} \cdot \left[\frac{CPV^t}{P^{T_0}} \right]^r, RWC_{min} \right) \quad (74)$$

where $RWC_{initial}$ is the initial raw wafer cost, r is the learning exponent corresponding to (decreasing) wafer costs per period and RWC_{min} is the minimum possible raw wafer costs.

5.4 Production and Cost Parameter Values

We derive the parameter values used in the analyses from several sources for which data were available on the economics of chipmakers and semiconductor industry dynamics (Jones 2005; Choi 2008; Dance, Layben et al. 2008; Havel, Kailiang et al. 2008; Insights 2008; SEMI 2008; Fine, Gregory et al. 2009; Insights 2009), and in consultation with industry professionals. We consider $T_0 = 2011$ to be the first year in which the firm is considering the introduction of 450mm technology, as this is the beginning of the period over which data was collected.

5.4.1 Production Function and Parameter Values

We use a spreadsheet based cash flow model simulates production costs from T_0 to T_f . In each period (year), the total production volume of chips required is given by:

$$P(t) = P^{T_0} \cdot \left((1+g)^{t-T_0} + (1+g)^{T_0-A} \right) \quad (75)$$

where P^{T_0} is the production volume in year T_0 , g is the annual production growth rate and A is the amortization period in years. We assume that once building and equipment costs are fully amortized they are retired. The second term in (75) reflects the new production that must be brought online to replace this retired capacity.

5.4.2 Chip and Wafer Geometry Parameter Values

Once the production to be built in each year is identified, the model calculates the fraction of capacity to be produced using each technology. The total number of wafers required for each technology to produce this fraction is different, as each wafer size supports a different quantity of chips. Table 11 presents the chip and wafer geometry parameter values considered.

Wafer and Chip Parameters	300mm	450mm	Unit
Node size modeled	45	45	nm
Total wafer area	70,686	159,043	mm ²
Fraction wafer usable Area	90%	95%	of wafer
Total usable area	63,617	151,090	mm ²
Chip length at this node size	4	4	mm
Chip width at this node size	4	4	mm
Total chip area	16	16	mm ²
Chips per wafer at this node size	3,976	9,443	chips

Table 11: Chip and wafer geometry parameter values considered

5.4.3 Capital and Variable Cost Parameter Values

We assume 45,000 wafer starts per month for the baseline fabrication facility. Equipment and tooling costs are classified as either “beam” or “non-beam.” The amount of beam equipment required (for example photo-lithography machines), scales linearly with

production. This means that moving from 300mm to 450mm wafers does not confer any economies of scale benefits. Therefore, equipment costs will scale linearly with production. For example, 90,000 WSPM requires twice as much beam equipment to process as 45,000 WSPM. Non-beam equipment scales sub-linearly with production volume. For example, a deposition chamber that processes a 450mm wafer only needs to be 1.5x bigger than once to process a 300mm wafer even though the 450mm wafer produces ~2.25x as many chips once processed. Table 12 presents the total capital costs for the baseline facility with a capacity of 45,000 WSPM and the contribution of beam and non-beam equipment and tools to the total.

	300mm	450mm	Unit
Total Bldg Investment	\$891,000,000	\$1,250,000,000	Per fab
Total equipment and tooling	\$2,475,000,000	\$5,875,000,000	Per fab
Total beam	\$742,500,000	\$2,025,000,000	Per fab
Total non-beam	\$1,732,500,000	\$3,850,000,000	Per fab
Total capital costs per baseline facility	\$3,366,000,000	\$7,125,000,000	Per fab

Table 12: Capital costs for baseline 300mm and 450mm facilities

The variable costs required to process each wafer are broken down into beam and non-beam components. Table 13 presents these values.

	300mm	450mm	Unit
Variable beam	\$318	\$715	Per wafer
Variable Non beam	\$481	\$961	Per wafer
Total variable costs	\$799	\$1,676	Per wafer

Table 13: Variable costs per wafer

5.4.4 Learning Parameter Values

Each learning parameter is assigned an initial value and a limiting value, representing the best possible value the parameter can achieve through learning. The limiting values for

yield and wafer starts per month represent the maximum, while for raw wafer costs the value represents a minimum. Table 14 presents the initial and limiting values considered for these parameters.

	300mm		450mm	
	Initial	Limit	Initial	Limit
Wafer starts per month per fab	45k	60k	45k	60k
Total yield	83.3%	89.1%	60%	89.1%
Raw wafer costs	\$300	\$250	\$3000	\$1,500

Table 14: Initial and limiting values for learning parameters

5.4.5 Labor Parameter Values

Table 15 presents the labor parameter values considered

Capital Cost Parameters	Value
Work days per year	235
Shifts per day	2
Hours per shift	8
Paid downtime per shift	1
Labor rate per hour	\$22

Table 15: Labor parameter values

5.5 Scenarios Considered

We are seeking to characterize the impacts of continued learning in the existing 300mm technology and simultaneous production using 300mm and 450mm technologies on two questions facing chipmakers:

1. When capacity allocation is unconstrained, should chipmakers stay with 300mm or switch to 450mm?

2. When capacity is constrained, how much production capacity do chipmakers need to build using 450mm processing technology to make introduction cost effective?
3. What are the primary operational drivers of learning within each technology and how do these influence technology decisions?

Developing scenarios corresponding to the decision contexts in Table 3 enables us to explore these questions. The baseline scenario corresponds to decision context (a), in which none of the parameters in Table 14 exhibit learning for the 300mm technology and the firm is free to allocate capacity to either or both technologies. Next, we develop scenarios for each of the remaining decision context and compare the results against the baseline to examine how changes in learning in the existing technology and the requirement of simultaneous production impact the baseline results. Finally, we investigate how learning in the individual factors impacts the technology decision to identify the factor(s) where improvements lead to the largest benefits.

5.5.1 Scenario Parameter Values

The parameters considered for the scenarios considered are given in Table 16.

Symbol	Definition	Decision Contexts	
		(a), (b)	(c), (d)
$T_i - T_0$	Time from T_0 until B introduced	3 yrs	3 yrs
$T_f - T_0$	Time from T_0 over which costs are summed	15 yrs	15 yrs
P^{T_0}	Production volume in T_0	2 bil	2 bil
g	Annual production growth	4%	4%
d	Discount rate	0%	0%
A	Amortization period	10 yrs	10 yrs
P_{300}^w	300mm WSPM progress ratio	100%	98%
P_{300}^y	300mm yield progress ratio	100%	98%
P_{300}^r	300mm RWC progress ratio	100%	98%
w_{300}^*	300mm WSPM learning exponent	0%	1.03
y_{300}^*	300mm yield learning exponent	0%	1.03
r_{300}	300mm RWC learning exponent	0%	-0.03
P_{450}^w	450mm WSPM progress ratio	85%	85%
P_{450}^y	450mm yield progress ratio	85%	85%
P_{450}^r	450mm RWC progress ratio	85%	85%
w_{450}^*	450mm WSPM learning exponent	-0.23	-0.23
y_{450}^*	450mm yield learning exponent	-0.23	-0.23
r_{450}	450mm RWC learning exponent	-0.23	-0.23

Table 16: Scenario parameter values

5.6 Results

Figure 23 presents the total production costs as a function of the capacity allocation to the 450mm technology for the base case.

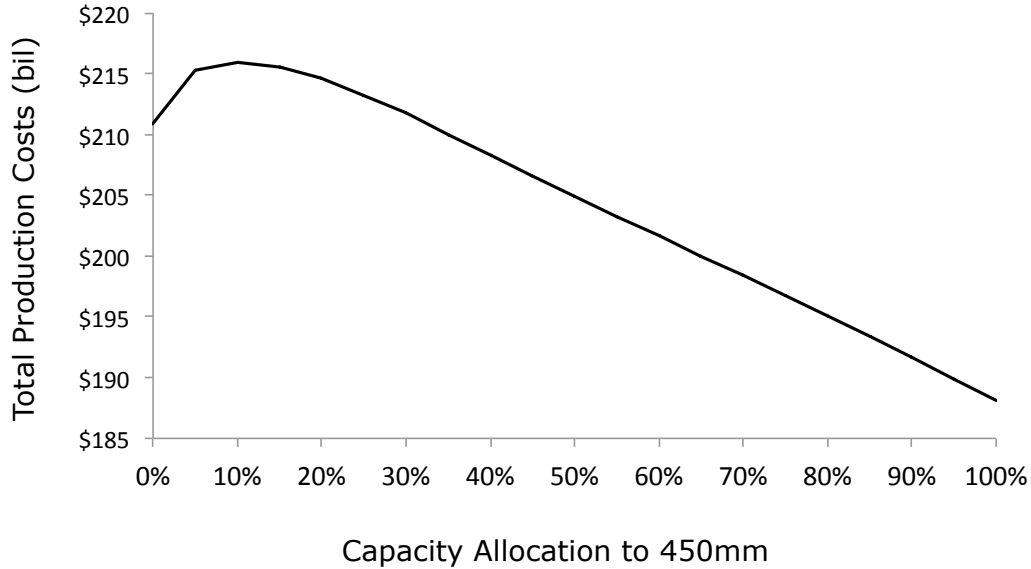


Figure 23: Total production costs as a function of the production capacity allocated to 450mm for the base case (decision context (a))

As expected, the results indicate that minimum cost corresponds to an extremal value of capacity allocation. The base case results indicate that the firm should introduce 450mm technology and devote as much production volume as possible. Compared to continued production using only 300mm technology, switching to 450mm saves the firm approximately 10%, or \$20 billion.

As previously discussed however, the firm will be unable to shift all production to 450mm wafer technology. Given this constraint, the firm must determine the level of production volume at which it becomes cost effective to introduce the new technology. The introduction of the constraint means that the firm is now operating in decision context (b), and the firm trying to determine the value of the constraint, \hat{F}_i . Due to the complex nature of the cost function, we are unable to use (48) to determine the value of \hat{F}_i . However, using the results from the base case, we can approximate its value. Figure 24 presents the results for decision context (b) for the cost function in Figure 23.

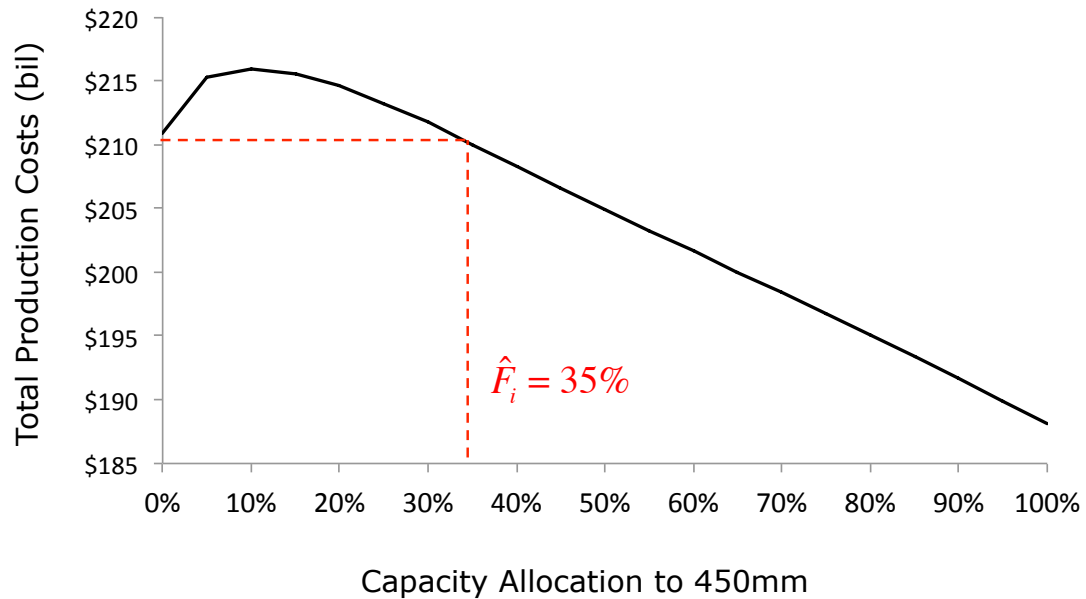


Figure 24: Determination of the capacity constraint at which introducing 450mm technology is economically feasible.

The results suggest that, if the firm is unable to allocate at least 35% of capacity to production using the 450mm technology, then it will not be economically feasible to introduce it at any level. Conversely, if 450mm can be introduced at capacities greater than 35%, then the firm should introduce the new technology and allocate as much capacity as possible to the new technology.

Thus far, we have only considered decision contexts in which the existing technology no longer exhibits learning. However, some in the industry have suggested that 300mm technology may still exhibit learning-by-doing due to ongoing process improvements (Chien, Wang et al. 2007; Fine, Gregory et al. 2009). Therefore, we would like to characterize the technology decision in this case. Figure 25 presents the results assuming the 300mm technology still exhibits learning in all three factors, with the progress ratios provided in Table 16.

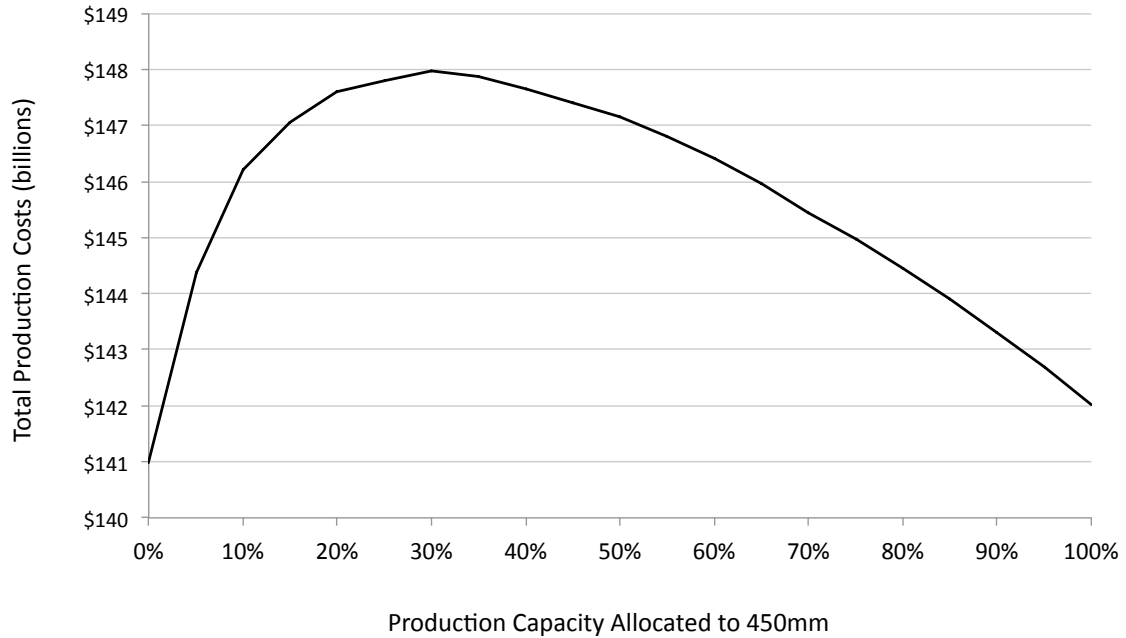


Figure 25: Total production costs as a function of the production capacity allocated to 450mm when continued learning is expected in 300mm (decision context (c))

The introduction of continued learning in the 300mm technology changes the technology decision such that the least costly option is for the firm not to introduce 450mm. However, this is the least costly decision over a small range of potential allocations. We can determine the size of this range by finding the value of the capacity allocated to 450mm at which the costs of staying with 300mm equal the cost of switching to 450mm (this is the same method used to determine the value of constraint type (ii), \hat{F}_{ii} , in the example in Figure 13). Figure 26 illustrates this range.

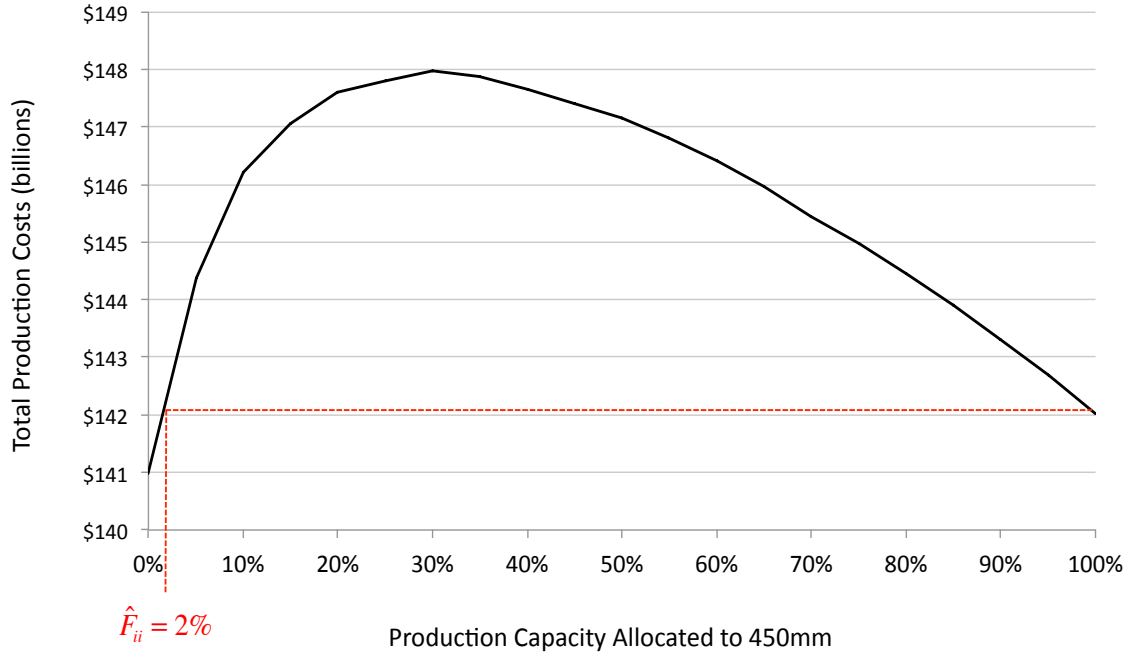


Figure 26: Determination of the range over which the firm should only use 300mm technology

Continued learning in the 300mm has significantly decreased total production costs for all shared capacity allocations. The results suggest that the firm should stay exclusively with the 300mm technology only when it cannot introduce at least 2% of production capacity to 450mm. This is a significantly different result that observed in Figure 24. In that case, the firm should stay with 300mm unless it can allocate at least 68% to 450mm.

Table 17 provides a summary of the technology decisions for the semiconductor case.

Decision context	Technology Decision
(a)	Switch to 450mm
(b)	Only introduce 450mm for $F \geq 35\%$
(c)	Stay with 300mm
(d)	Introduce 450mm at allocations greater than 2%

Table 17: Summary of technology decisions for the semiconductor case

5.7 Identification of Key Cost Drivers

We have identified that continued learning in 300mm can lead to a different decision. However, we would also like to know which learning factor or set of factors that has the biggest impacts on the results. We can do this by changing one factor at a time and observing the impact on the decision. Table 18 presents the resulting combinations of 300mm progress rates tested and the corresponding technology decision. The base case (no 300mm factors learning), and decision context (c) (all three factors learning at the same amount) are included for reference.

Trial ID	300mm Learning Rates (1- progress ratio)			Technology Decision
	WSPM:	Yield:	RWC:	
	P_{300}^w	P_{300}^y	P_{300}^r	
1 (base case)	0%	0%	0%	Switch to 450mm
2	5%	0%	0%	Switch to 450mm
3	0%	5%	0%	Switch to 450mm
4	0%	0%	5%	Switch to 450mm
5	5%	5%	0%	Stay with 300mm
6	0%	5%	5%	Switch to 450mm
7	5%	0%	5%	Stay with 300mm
8 (context c)	5%	5%	5%	Stay with 300mm

Table 18: 300mm learning parameter values tested to determine which factor(s) are most important in impacting the technology decision

The results suggest that, in this case, continued learning in 300mm wafer starts per month is the most important factor driving the technology decision, as it is the only factor in both trials. However, trial 2 enables us to conclude that changes in this factor alone are not enough to change the decision, as in this trial it is the only factor exhibiting learning, yet the decision is still to switch to 450mm. Interestingly however, the results suggest that learning in wafer starts and learning in either yield or raw wafer costs is enough to change the decision.

5.8 Sensitivity Analyses

In this section, we use sensitivity analyses to address two major chipmakers concerns: (1) how the timing of 450mm introduction impacts the decision of whether or not to introduce, and (2) how the constraint on the amount of new capacity that can be allocated to 450mm impacts the technology decision. We also characterize the impacts of changes in the operational factors exhibiting learning on these questions to provide insight into where improvements can have the biggest impact on the technology decision.

5.8.1 Parameter Values Considered

Table 19 presents the introduction year and 300mm learning parameter value ranges considered for the sensitivity analyses. All other parameters are assumed to be the same as those in Table 11 through Table 15.

Parameter	Definition	Sensitivity Range
T_i	450mm Introduction year	2011 - 2017
l_{300}^w	300mm monthly wafer starts progress ratio	100%, 98%
l_{300}^y	300mm yield rate progress ratio	100%, 98%
l_{300}^r	300mm raw wafer cost progress ratio	100%, 98%

Table 19: 450mm vs. 300mm sensitivity analyses parameter values considered

Figure 27 represents the technology decision space as a function of the 450mm introduction year and the capacity fraction allocated to the 450mm technology when we assume that the 300mm technology no longer exhibits learning in any operational parameter. This corresponds to decision contexts (a) and (b) in Table 3. Decision context (a), in which the firm is free to either stay with 300mm or switch to 450mm, corresponds to the edges of the decision space in Figure 27, when the capacity allocated to 450mm is either 0% or 100%. We know however from our earlier analyses that the introduction of a capacity changes the binary technology decision “stay with A” or “switch to B” to more nuanced conditional questions. Introducing 450mm technology requires sharing capacity, an example of a type (i) constraint. The technology question therefore, becomes: given that the firm must continue to allocate some level of production to 300mm what level of capacity, \hat{F}_i , does the firm need to allocate to 450mm in order to make 450mm cost effective? If the firm cannot introduce 450mm at at least this threshold level, then it should stay with 300mm. The yellow dashed line represents the minimum amount of capacity the firm will need to allocate to production using the 450mm technology to make introducing 450mm economically viable (the value of the type (i) constraint). The base case results suggest that the firm needs to allocate at least 20% of capacity to the 450mm technology to make it worth while to introduce at all.

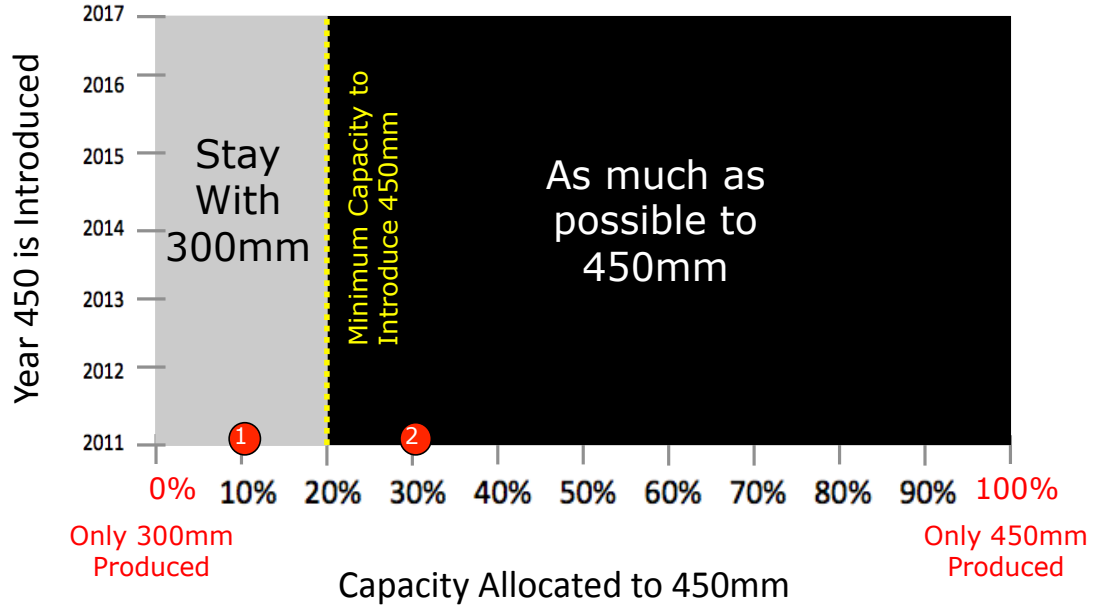


Figure 27: 450mm vs. 300mm decision results as a function of introduction year and the capacity fraction allocated to 450mm when 300mm does not exhibit learning in any operational parameters

Each point in Figure 27 represents a set of decisions for the firm. For example, the red circles labeled (1) and (2) correspond to the firm’s decision to introduce 450mm in 2011 and allocate a maximum of either 10% or 30% of total production volume to 450mm. In scenario (1), the firm should not introduce 450mm, while in (2), it should introduce. Figure 27 also indicates that the minimum threshold at which the firm should introduce 450mm, 20%, is not a function of the introduction year. Recall that in Table 16, the final analysis year is defined as 15 years after the introduction of the new technology. This means that the firm will experience the same costs and benefits associated with the new technology irrespective of when it is introduced. If the final analysis year is independent of when 450mm is introduced, introducing later would truncate the period over which 450mm is the less expensive technology. As a result, the firm would need to introduce more 450mm to make the technology cost competitive, and the yellow dashed line in Figure 27 would move to higher values with later introduction years.

Figure 28 presents the same decision space when we assume a 98% progress ratio for all three 300mm operational parameters. The 20% minimum 450mm introduction threshold and decision scenarios (1) and (2) from Figure 27 are included for reference.

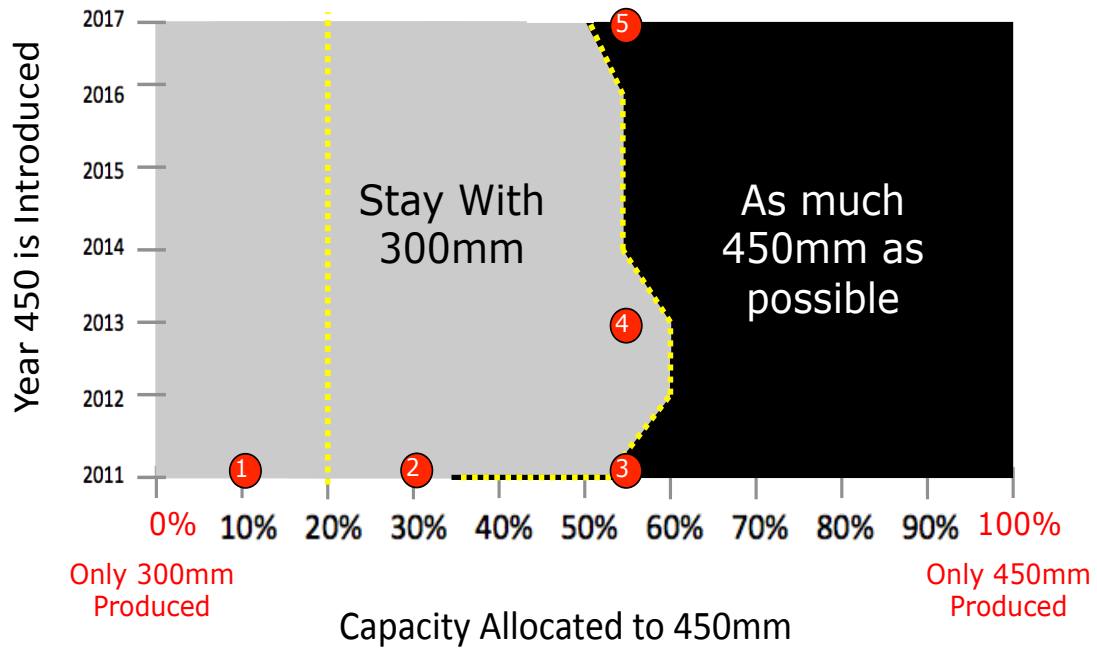


Figure 28: 450mm vs. 300mm decision results as a function of introduction year and the capacity fraction allocated to 450mm when all three 300mm operational parameters exhibit 98% progress ratios

The three additional firm decision scenarios, (3) – (5), are also presented for discussion, where each scenario includes two components: a 450mm introduction year and the maximum amount of capacity the firm can allocate to 450mm beginning in that year. For example, the red circle corresponding to scenario (3) in Figure 28 represents the scenario in which the firm will introduce 450mm in 2011 and can allocate a maximum of 55% of total capacity to 450mm beginning in 2011.

Continued learning in 300mm results in three significant changes to the decision space. First, the region over which the least costly alternative is to stay with the 300mm technology has expanded considerably. This occurs because when the existing technology continues to learn, reducing the capacity allocated to this technology results

in foregone learning. This effect manifests as a cost penalty due to increased unit costs associated with the existing technology. In order to make 450mm economically viable, the firm needs to compensate for this penalty. They can only do this by allocating more capacity to the 450mm technology so that it becomes the less expensive technology sooner. Conversely, continued 300mm technology learning makes 300mm the less expensive technology over a longer time period. Therefore, it takes longer for 450mm to become the less expensive technology. This combination of increased costs and decreased benefits increases the minimum threshold required to introduce the 450mm technology for all 450mm introduction years (the difference between the two yellow dashed lines in Figure 28). For example, whereas in Figure 27, if the firm introduces 450mm in 2011 at 30% of total capacity the least costly decision is to introduce 450mm (decision scenario (2)), in Figure 28 the firm should now stay with 300mm unless it can allocate at least 35% to the 450mm technology (this 35% is the constraint value observed earlier in Figure 24).

The second important change is that the minimum threshold at which the firm should introduce the 450mm technology is now strongly dependent on the 450mm introduction year. For example, although the firm should allocate at least 35% of capacity to production using 450mm technology if it intends to introduce 450mm in 2011, this figure increases to 60% if the firm cannot introduce until 2012. This is due to a combination of 300mm becoming even less expensive relative to 450mm in the 2011-2012 period due to continued learning, and because it takes 450mm longer to become the less expensive technology. This second effect would suggest that later 450mm introduction years would mean that the firm would have to allocate more and more capacity to 450mm to make it economically viable to introduce the 450mm technology. However, we observe that the dependence of the minimum 450mm capacity threshold on 450mm introduction is non-monotonic. This is the third significant change in the decision space when the existing technology still exhibits learning. For example, consider firm decision scenarios (3) – (5) in Figure 28. In all three scenarios, the firm can allocate up to 55% of total capacity to the 450mm technology. Therefore, if the minimum threshold required to introduce 450mm is greater than 55%, the chipmaking firm should continue to use only the 300mm wafers.

In scenario (3), the firm introduces 450mm at the earliest possible time, in 2011. This means that the unit costs associated with the 450mm technology will start to decrease in 2011. As a result, the 450mm technology will become less the expensive option sooner. This means that the benefits associated with introducing 450mm will accrue over a longer period. However, because 300mm still exhibits learning, diverting capacity to the 450mm technology results in a cost penalty in the form of foregone 300mm learning. The minimum threshold capacity that must be allocated to 450mm (35% in scenario (3)), is the capacity that results in the cost savings in 450mm required to offset this penalty. The firm can allocate up to $F_{MAX} = 55\%$ of capacity to 450mm. In decision scenario (3), this is greater than the 35% required to offset the foregone 300mm learning penalty. Therefore, the firm should introduce 450mm. Additionally, because the 450mm technology is learning faster than the 300mm technology, capacity in excess of this threshold results in additional cost savings. Therefore, the firm should allocate the maximum possible capacity, 55%, to 450mm.

In decision scenario (5), the firm will introduce 450mm at the latest possible introduction year, 2017 (although recall that costs are accrued over a 15 year period after 450mm introduction). In this case, although the decision is also to introduce 450mm, the drivers are different. Introducing the 450mm technology late enables the firm to maximize the 300mm learning related cost reductions. This results in the minimum cost penalty due to foregone learning in 300mm when the firm introduces 450mm. However, continued learning in 300mm means that in 2017 (when 450mm is introduced), 300mm unit costs are significantly less than the introduction costs associated with 450mm. As a result, it will take longer for 450mm to become the less expensive technology. The minimum capacity allocation threshold for 450mm introduction in this case, 50%, represents the amount that results in 450mm learning fast enough to offset these additional introduction costs. As in scenario (3), the firm should allocate the maximum possible capacity, 55%, to 450mm.

In scenario (4), the firm will introduce 450mm in an intermediate year, 2013. However, the results suggest that the 55% the firm can allocate to 450mm production will not be enough to make introduction economically viable. This is a counterintuitive result, as the previous results would suggest that the firm should introduce 450mm in this year, and that the minimum threshold for introduction should be between the 35% in scenario (3) and the 50% in scenario (5). However, the observed threshold is 60%. This scenario represents a “worst of both worlds” outcome: introducing 450mm later when 300mm is still learning means that it takes longer for 450mm to become the less expensive option, *and* diverting capacity from 300mm results in a foregone learning cost penalty.

Table 20 presents a summary of these results for the 450mm introduction year, T_i , the maximum capacity the firm can allocate to 450mm beginning in that introduction year, F_{MAX} , the minimum threshold capacity allocation to 450mm to make introduction in this year economically feasible, \hat{F}_i , and the resulting technology decisions for decision scenarios (3) – (5) in Figure 28.

ID	T_i	F_{MAX}	\hat{F}_i	Technology Decision
(3)	2011	55%	35%	As much 450mm as possible
(4)	2013	55%	60%	Stay with 300mm
(5)	2017	55%	50%	As much as possible to 450mm

Table 20: Parameters and technology decisions for decision scenarios (3) – (5) in Figure 28

These results are due to the combination of continued learning in the 300mm technology and the infeasibility of chipmakers to switch all production volume to the 450mm technology. Therefore, failing to incorporate these factors into the technology choice analysis may lead firms to introduce the new technology at an inopportune time and/or at an insufficient capacity to benefit from introduction.

6 Cumulative Investment and Production Volume Based Learning

In this chapter, we extend our earlier framework to include investment driven learning. Investment in this context includes all activities which are not directly related to production volume. Examples include R&D, experimentation and worker training. We are interested in exploring two questions:

1. Does the inclusion of learning-by-investing change the technology decision compared resulting from only considering learning-by doing?
2. Does a constraint on how investments are allocated between technologies change the capacity allocation minimizing total production costs?
3. If so, what are the conditions under which the decision changes?

As in case of CPV driven learning, we assume that a finite pool of resources exists that must be allocated between an existing and new technology. We assume that the firm must produce some amount of B in order to experience cumulative investment (CI) driven learning. If the firm invests in a technology but never produces it, then we assume that CI has no benefit. Learning can take place as a function of both cumulative production volume and cumulative investments over time. As in Chapter 4, we first expand the decision contexts facing the firm to include investment related learning and allocation constraints. Next, we describe our assumptions and develop and technology cost curves as a function of CI technology learning, the period in which investments are first allocated to technology B and the timeframe over which costs are accrued. Then, we derive complete cost functions that couple capacity and investment learning. We then derive an expression for total production costs over time and attempt to find an expression minimizing these costs. Finally, we explore the impact of decision context on total production costs and comment on the conditions under which changes in the factors driving technology costs impact capacity and investment allocation decisions.

6.1 Decision Contexts

The resources to be allocated are now both production volume and investments. This results in two sets of decision contexts under which the firm makes decisions. Figure 29 illustrates both sets of decision contexts.

		Production Allocation	
Technology Learning	(a)	<i>Only B learning</i> <i>Unconstrained capacity allocation</i>	(b)
	(c)	<i>A and B learning</i> <i>Unconstrained capacity allocation</i>	(d)
		Investment Allocation	
Technology Learning	(1)	<i>Only B learning</i> <i>Unconstrained investment allocation</i>	(2)
	(3)	<i>A and B learning</i> <i>Unconstrained investment allocation</i>	(4)

Figure 29: Context for technology choice decisions when considering both CPV and cumulative investment based learning

The set of decision contexts is now 16, as each of the investment contexts, (1)-(4), can occur for each of the production contexts, (a)-(d). For example, under decision context (a.1), the new technology exhibits learning as a function of both cumulative production volume and cumulative investments over time, while the existing technology no longer exhibits learning in either area. Additionally, the firm is free to allocate both production volume and investments to either or both technologies. In contrast, in decision context (d.4), both the existing and new technologies exhibit learning driven by cumulative investments and cumulative production volume. In this case, the firm also experiences either a type (i) or (ii) constraint in both the production volume and investment allocation dimensions. As a result, production volume and investments may be shared between technologies.

Some of these contexts provide the same results as when we considered only learning-by-doing. For example, in context (a.1), technology A no longer exhibits either learning-by-investing or learning-by-doing. In this case, the technology and allocation decisions will be the same as under context (a). Similarly, the results under context (b.2) will be the same as (b), and the results under constraint (c.1) will be the same as (c).

6.2 Cumulative Investment Driven Unit Costs Over Time

We assume that the firm can invest in either A or B. In this case, T_0 is the period in which the firm is first considering B. However, the firm now has to decide not only when, if ever, to introduce B, (T_i) but also when, if ever, to begin investing in B. We define the first period when the firm invests in B as T_{inv} , where we assume $T_0 \leq T_{inv} \leq T_i < T_f$.

We define $G(t)$ as the percentage of investment allocated to technology B in every period, where $0 \leq G(t) \leq 1 \forall t$ and $(1-G(t))$ as the percentage of production volume allocated to technology A. Prior to T_{invest} , $G(t)=0$, as all production is handled using the existing technology. The investment allocated to each technology in period t is given by:

$$\begin{aligned}
 &T_0 \leq t < T_{inv} : \\
 &\quad I_A(t) = I(t) \\
 &\quad I_B(t) = 0 \\
 & \\
 &T_{inv} \leq t < T_f : \\
 &\quad I_A(t) = (1 - G(t)) \cdot I(t) \\
 &\quad I_B(t) = G(t) \cdot I(t)
 \end{aligned} \tag{76}$$

Where the total investment in an individual period is a constant:

$$I(t) = I_A(t) + I_B(t) \tag{77}$$

As with CPV driven learning, we assume that the investment learning function for each technology is a monotonically decreasing function of increasing cumulative investments. Additionally, we assume that the new technology exhibits a larger progress ratio than the incumbent (technology A “learns faster” than B). The learning function for each technology and assumptions are given by:

$$\begin{aligned} \psi_A(t) &= f(CI'_A) \quad \text{where} \quad 0 \leq \psi_A(t), \psi_B(t) \leq 1 \\ \psi_B(t) &= f(CI'_B) \quad \frac{d(\psi_B(t))}{d(CI'_B)} < \frac{d(\psi_A(t))}{d(CI'_A)} \leq 0 \end{aligned} \quad (78)$$

The unit cost functions considering learning-by-investing are given by:

$$\begin{aligned} C_A(t) &= C_A^{T_0} \cdot \psi_A(t) \\ C_B(t) &= C_B^{T_0} \cdot \psi_B(t) \end{aligned} \quad (79)$$

6.3 Cumulative Investment and Cumulative Production Volume Driven Unit Costs Over Time

We incorporate learning-by-doing to arrive at an expression for total unit costs over time. We use the two-factor cost curve presented in, which models technology costs in the presence of both CPV and investment driven learning (Kouvaritakis, Soria et al. 2000; Miketa and Schratzenholzer 2004; Jamasb 2007):

$$\begin{aligned} C_A(t) &= C_A^{T_0} \cdot \phi_A(t) \cdot \psi_A(t) \\ C_B(t) &= C_B^{T_0} \cdot \phi_B(t) \cdot \psi_B(t) \end{aligned} \quad (80)$$

6.4 Total Production Costs

The total production costs for each technology as a function of time are given by:

$$\begin{aligned} TC_A(t) &= C_A^{T_0} \cdot \phi_A(t) \cdot \psi_A(t) \cdot P_A(t) \\ TC_B(t) &= C_B^{T_0} \cdot \phi_B(t) \cdot \psi_B(t) \cdot P_B(t) \end{aligned} \quad (81)$$

The corresponding total production costs up to time t are therefore:

$$\begin{aligned} TC_A &= \int_{T_0}^t C_A^{T_0} \cdot \phi_A(t) \cdot \psi_A(t) \cdot P_A(t) dt \\ TC_B &= \int_{T_i}^t C_B^{T_0} \cdot \phi_B(t) \cdot \psi_B(t) \cdot P_B(t) dt \end{aligned} \quad (82)$$

Where the integral for the new technology now begins in the first period in which the firm invests, T_{inv} . Total production costs is the sum of the total costs of each technology:

$$TPC = TC_A + TC_B + CI \quad (83)$$

Where CI is the total investments from T_0 to T_f .

6.5 Total Production Cost Function Characteristics

As in the case of CPV only learning, for explanatory clarity we focus on cases in which the fraction of capacity and investment allocated to B are constant over time: $F(t) = F(T_i) = F$ and $G(t) = G(T_{inv}) = G \quad \forall t \geq T_{inv}$. Additionally, we assume that F and G are independent decisions made by the firm. We are interested in characterizing how TPC changes as a function of G, *given a capacity allocation F*. Therefore, we seek extrema of the total production cost function with respect to G. We begin by expanding (83):

$$\begin{aligned} TPC &= CI + \int_{T_0}^{T_f} \left(C_A^{T_0} \cdot \phi_A(t) \cdot \psi_A(t) \cdot P_A(t) + C_B^{T_i} \cdot \phi_B(t) \cdot \psi_B(t) \cdot P_B(t) \right) dt \\ &= CI + C_A^{T_0} \cdot \int_{T_0}^{T_f} \left(\phi_A(t) \cdot \psi_A(t) \cdot P_A(t) + \eta \cdot \phi_B(t) \cdot \psi_B(t) \cdot P_B(t) \right) dt \end{aligned} \quad (84)$$

where, as before, $\eta = C_B^{T_i} / C_A^{T_0}$. The first order condition to minimize (84) with respect to G is⁸:

$$\begin{aligned} \frac{d(TPC)}{dG} &= \frac{d}{dG} \left(CI + C_A^{T_0} \cdot \int_{T_{inv}}^{T_f} \left(\phi_A(t) \cdot \psi_A(t) \cdot P_A(t) + \eta \cdot \phi_B(t) \cdot \psi_B(t) \cdot P_B(t) \right) dt \right) = 0 \\ &= C_A^{T_0} \cdot \int_{T_{inv}}^{T_f} \phi_A(t) \cdot P_A(t) \cdot \frac{\partial \psi_A(t)}{\partial G} + \eta \cdot \phi_B(t) \cdot P_B(t) \cdot \frac{\partial \psi_B(t)}{\partial G} dt = 0 \end{aligned} \quad (85)$$

Setting the integrand to zero:

$$\phi_A(t) \cdot P_A(t) \cdot \frac{\partial \psi_A(t)}{\partial G} + \eta \cdot \phi_B(t) \cdot P_B(t) \cdot \frac{\partial \psi_B(t)}{\partial G} = 0 \quad (86)$$

We can use the “second derivative test” to confirm whether the extrema is a min or a max:

⁸ The derivative is with respect to G, and G=0 in the region T₀ to T_{inv}, and CI does not depend on G, as it represents total investments which will be made irrespective of whether or not B is introduced.

$$\begin{aligned} & \frac{\partial}{\partial G} \left(\phi_A(t) \cdot P_A(t) \cdot \frac{\partial \psi_A(t)}{\partial G} + \eta \cdot \phi_B(t) \cdot P_B(t) \cdot \frac{\partial \psi_B(t)}{\partial G} \right) \\ & = \phi_A(t) \cdot P_A(t) \cdot \frac{\partial^2 \psi_A(t)}{\partial G^2} + \eta \cdot \phi_B(t) \cdot P_B(t) \cdot \frac{\partial^2 \psi_B(t)}{\partial G^2} \end{aligned} \quad (87)$$

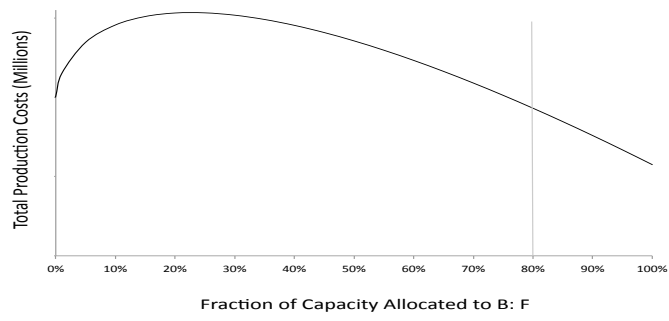
In general we assume that the learning-by-doing and learning-by-investing functions are monotonically decreasing. As a result, all learning functions exhibit positive or zero curvature. Therefore, all the terms in (87) are positive. This means that the extrema is a total production cost *minimizing* solution. This means that, given a capacity allocation F , the value of G that satisfies (85), which we define as $G^*(F)$, is the investment allocation to the new technology that minimizes production costs at that F , TPC_F . This behavior is the opposite of what we observed in the case of capacity allocation. In that case, F^* maximized total production costs.

To understand this difference, recall that in order to realize CPV driven learning the firm must make products. Each of these products has an associated unit cost which the firm must therefore pay. As a result, there is a “cost to learn” associated with learning-by-doing. This cost pushes the firm to allocate all production volume to a single technology so that this technology can “learn” as quickly as possible, as learning makes each unit cost less to produce.

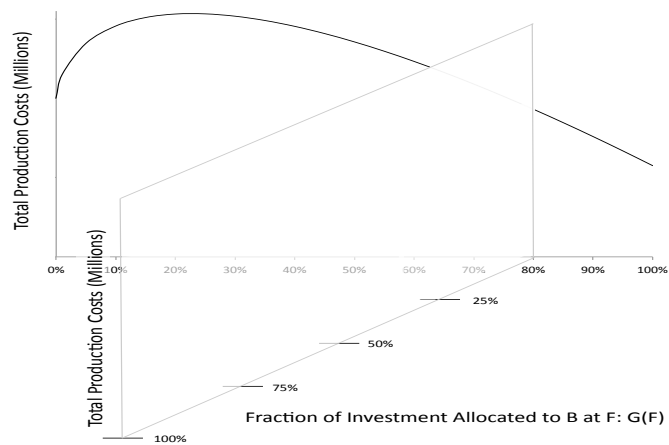
In contrast, there are no marginal costs associated with learning-by-investing. The firm has a total investment budget which will be spent in each period irrespective of the capacity allocation decision. When the existing technology is still learning, investments directly translate into cost reductions without the need to pay additional costs. As a result, there is a tradeoff between continuing to capture the benefits associated with investing in A and using investment dollars to drive down the costs associated with B. Every dollar allocated to B is a dollar not available for A. This results in foregone learning in A, which is effectively an additional cost associated with B.

Figure 30 presents a stylized example to help visualize how capacity and investment allocations impact total production costs. Row (1) illustrates a total production cost curve

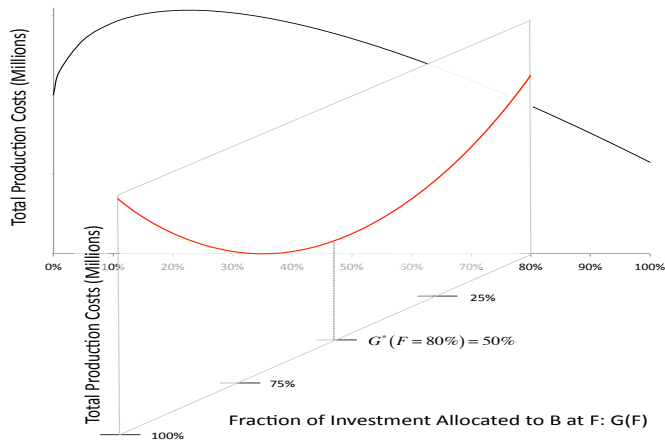
as a function of the capacity allocated to B. The firm has decided to allocate $F = 80\%$ to B. Given this allocation, row (2) illustrates the orthogonal investment allocation dimension, which indicates that the investment and capacity allocation decisions are independent. Row (3) in Figure 30 illustrates a total production cost curve as a function of the investment allocation to B given $F = 80\%$. This TPC curve, in red, represents the actual total production costs the firm will incur. In this example, minimum total production costs correspond to an investment allocation of $G^*(F = 80\%) = 50\%$. This means that in this case, at a capacity allocation strategy of 80% to B and 20% to A, splitting the investment in each period evenly between A and B results in minimum total production costs.



(1)



(2)



(3)

Figure 30: Stylized example of the total production cost curve as a function of both capacity and investment allocations

6.6 Cumulative Production Volume and Cumulative Investment Learning Example

In order to explore these ideas, we extend the example presented in section 4.8 to include learning-by-investing. As before, we model technology choice in each decision context for two example technologies. We begin by selecting functional forms for learning and production functions. We then use these functions to construct general expressions for technology costs over time. Then, we explore the impact of decision context on technology choice and investment allocation decisions, and characterize the conditions under which the factors driving technology costs impact these decisions via sensitivity analyses. Finally, we discuss the implications of the results on technology decision-making and compare these results to the literature.

6.6.1 Learning Functions

We assume both the CPV and CI learning functions follow a power law, and that both CPV and CI are normalized as in(32):

$$\begin{aligned}\phi(t) &= \left[\frac{CPV^t}{P^{T_0}} \right]^{n_1} \\ \psi(t) &= \left[\frac{CI^t}{I^{T_0}} \right]^{n_2}\end{aligned}\tag{88}$$

Where P^{T_0} and I^{T_0} are the production volume and investment in the period T_0 , and n_1 and n_2 are the CPV and CI learning exponents. We assume the simple, no growth production function in (35), $P(t) = P^{T_0}$, resulting in the cumulative production volume learning functions as in (36). By The cumulative investment functions for each technology are given by:

$$\begin{aligned}
T_0 \leq t < T_{inv} \\
CI_A^t &= \int_{T_0}^{T_{inv}} I(t) dt \\
CI_B^t &= 0
\end{aligned}
\tag{89}$$

$$\begin{aligned}
T_{inv} \leq t \leq T_f \\
CI_A^t &= \int_{T_0}^{T_{inv}} I(t) dt + \int_{T_{inv}}^t (1-G) \cdot I(t) dt \\
CI_B^t &= \int_{T_{inv}}^t G \cdot I(t) dt
\end{aligned}$$

The investment learning functions are therefore given by:

$$\begin{aligned}
T_0 \leq t < T_{inv} \\
\psi_A(t) &= \left[\frac{\int_{T_0}^t I(t) dt}{I^{T_0}} \right]^{a_2}
\end{aligned}
\tag{90}$$

$$\begin{aligned}
T_{inv} \leq t \leq T_f \\
\psi_A(t) &= \left[\frac{\int_{T_0}^{T_{inv}} I(t) dt + \int_{T_{inv}}^t (1-G) \cdot I(t) dt}{I^{T_0}} \right]^{a_2} \\
\psi_B(t) &= \left[\frac{\int_{T_{inv}}^t G \cdot I(t) dt}{I^{T_0}} \right]^{b_2}
\end{aligned}$$

where a_2 and b_2 are the learning by investing exponents for technologies A and B.

6.6.2 Production and Investment Functions

The production function assumed is the same as in (35). The investment function is given by:

$$I(t) = I^{T_0} \quad (91)$$

6.6.3 Cost Functions

Incorporating this investment function, the learning functions in (90) are given by:

$$T_0 \leq t < T_{inv}$$

$$\psi_A(t) = \left[\frac{\int_{T_0}^t I^{T_0} dt}{I^{T_0}} \right]^{a_2} = [t - T_0]^{a_2}$$

$$T_{inv} \leq t \leq T_f$$

$$\psi_A(t) = \left[\frac{\int_{T_0}^{T_{inv}} I^{T_0} dt + \int_{T_{inv}}^t (1-G) \cdot I(t) dt}{I^{T_0}} \right]^{a_2} = [(T_{inv} - T_0) + (1-G) \cdot (t - T_{inv})]^{a_2} \quad (92)$$

$$\psi_B(t) = \left[\frac{\int_{T_{inv}}^t G \cdot I^{T_0} dt}{I^{T_0}} \right]^{b_2} = [G \cdot (t - T_{inv})]^{b_2}$$

To arrive at the unit cost functions in (80), we incorporate the CPV based learning functions from (36):

$$T_0 < t < T_i :$$

$$\phi_A(t) = \int_{T_0}^t \frac{P^{T_0}}{P^{T_0}} dt = (t - T_0)$$

$$T_i \leq t \leq T_f :$$

$$\phi_A(t) = \left[\frac{\int_{T_0}^{T_i} P^{T_0} dt + \int_{T_i}^t (1-F) \cdot P^{T_0} dt}{P^{T_0}} \right]^a = \left[(T_i - T_0) + (1-F) \cdot (t - T_i) \right]^a \quad (93)$$

$$\phi_B(t) = \left[\frac{\int_{T_i}^t F \cdot P^{T_0} dt}{P^{T_0}} \right]^b = \left[F \cdot (t - T_i) \right]^b$$

The unit costs in each region are given by:

$$T_0 \leq t < T_{inv}$$

$$C_A(t) = C_A^{T_0} \cdot [t - T_0]^{a_1} \cdot [t - T_0]^{a_2} = [t - T_0]^{a_1 + a_2}$$

$$T_{inv} \leq t < T_i$$

$$C_A(t) = C_A^{T_0} \cdot [t - T_0]^{a_1} \cdot \left[(T_{inv} - T_0) + (1-G) \cdot (t - T_{inv}) \right]^{a_2} \quad (94)$$

$$T_i \leq t \leq T_f$$

$$C_A(t) = C_A^{T_0} \cdot \left[(T_i - T_0) + (1-F) \cdot (t - T_i) \right]^{a_1} \cdot \left[(T_{inv} - T_0) + (1-G) \cdot (t - T_{inv}) \right]^{a_2}$$

$$C_B(t) = C_B^{T_i} \cdot \left[F \cdot (t - T_i) \right]^{b_1} \cdot \left[G \cdot (t - T_{inv}) \right]^{b_2}$$

Using (82) we can calculate the total production costs associated with each technology (where we are interested in the timeframe T_0 to T_f):

$$TC_A = C^{T_0} \cdot P^{T_0} \cdot \left(\int_{T_0}^{T_{inv}} [t - T_0]^{a_1 + a_2} dt + \int_{T_{inv}}^{T_i} [t - T_0]^{a_1} \cdot [(T_{inv} - T_0) + (1 - G) \cdot (t - T_{inv})]^{a_2} dt + \int_{T_i}^{T_f} [(T_i - T_0) + (1 - F) \cdot (t - T_i)]^{a_1} \cdot [(T_{inv} - T_0) + (1 - G) \cdot (t - T_{inv})]^{a_2} dt \right) \quad (95)$$

and

$$TC_B = C_B^{T_i} \cdot P^{T_0} \cdot \left(\int_{T_i}^{T_f} [F \cdot (t - T_i)]^{b_1} \cdot [G \cdot (t - T_{inv})]^{b_2} dt \right) \quad (96)$$

Total production costs from T_0 to T_f are the sum of (95), (96) and the total investment over this time, CI (as shown in (83)).

6.6.4 Baseline Scenario Considered

We are interested in exploring the impacts of the introduction of investment based learning on the technology decision and capacity and investment allocations. We assume the same parameter values as in the base line case for CPV only learning (Table 3). Table 4 presents the parameters and values considered as the baseline scenario. We define $T_0 = 0$ to be the first period in which costs are accrued. The new parameters and values required to model investment based learning are shaded grey in Table 21.

Symbol	Definition	Value
$T_i - T_0$	Periods from T_0 until B introduced	5
$T_f - T_0$	Period from T_0 over which costs are summed	15
$T_i - T_{invest}$	Period before launch of B firm invests in B	2
P^{T_0}	Production volume in T_0	10,000
I^{T_0}	Investment budget in I_0	\$10,000
$C_A^{T_0}$	A costs in first period B considered	\$10
η	Ratio of B to A initial costs	1.3
$(n_A)_1$	CPV learning progress ratio for A	90%
$(n_B)_1$	CPV learning progress ratio for B	80%
a_1	CPV learning exponent for A	-0.15
b_1	CPV learning exponent for B	-0.32
$(n_A)_2$	CI learning progress ratio for A	90%
$(n_B)_2$	CI learning progress ratio for B	80%
a_2	CI learning exponent for A	-0.15
b_2	CI learning exponent for B	-0.32

Table 21: Baseline case parameters and values considered

As in the previous example, once B is introduced, we also assume that the capacity and investments allocated to B remain constant over time: $F(t) = F$ and $G(F(t)) = G(F)$. We also again assume that all production costs are captured in the unit cost for each technology (there are no additional fixed costs for example). When calculating total production costs, we do not include the total cost of investments in the period T_0 to T_f . This is because we assume the firm will be making these investments irrespective of the learning behavior of the underlying technologies, or how the investments are allocated

between technologies. Therefore, including this factor will not change technology choice or resource allocation decisions.

6.6.5 Baseline Results by Decision Context

As in section 4.8.5, for each decision context, we first summarize the technology questions the firm faces in each decision context and the metrics used to answer these questions. Then, we calculate the relevant required quantities of interest and discuss the resulting technology decision. Rather than present the results for all 13 of the remaining contexts, we focus on the contexts in which A exhibits continued CI driven learning and investment allocations are constrained. Each context result is compared against the corresponding context that only considered CPV driven learning. For example, the results from context (a.2), (a.3) and (a.4) would all be compared against context (a) to highlight the changes due to incorporating learning-by-investing.

Context (a.3)

From Figure 29, the decision context is given by:

Production Allocation	
Technology Learning	<div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 5px;">(a)</div> <div style="text-align: center;"> <p><i>Only B learning</i></p> <p><i>Unconstrained capacity allocation</i></p> </div> </div>
	<div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 5px;">(3)</div> <div style="text-align: center;"> <p><i>A and B learning</i></p> <p><i>Unconstrained investment allocation</i></p> </div> </div>
Investment Allocation	

Technology Questions:

- Does incorporating CI based learning change the capacity allocation decision compared to context (a)?
- How should capacity be allocated?
- How should investments be allocated?

Metrics:

- Total production costs from T_0 to T_f , (TPC)
- Capacity allocated to B, (F)

- Investment allocated to B, (G)

Method:

- Determine F and G that minimize TPC

In this case, we will compare the total production costs of decision context (a) against (a.3) to characterize the impacts of CI learning we want to know (i) what is the best capacity allocation decision, given that A still exhibits investment driven learning, (and is it the same or different than when only CPV learning was considered), and (ii) what allocation of investments results in minimum production costs? Because the parameter values are slightly different in this case, we recalculate the technology costs corresponding to context (a) (recall that in this context the cost minimizing solution will correspond to either staying with A or switching to B).

$$\begin{aligned}
 TC_A &= C_A^{T_0} \cdot P^{T_0} \cdot \left(\frac{[(T_f - T_0) - F \cdot (T_f - T_i)]^{a+1}}{a+1} \right) \\
 &= C_A^{T_0} \cdot P^{T_0} \cdot (T_f - T_0) \\
 &= \$10 \cdot 10,000 \cdot (15 - 0) = \$1.5M
 \end{aligned} \tag{97}$$

$$\begin{aligned}
 TC_B &= TC_B|_{T_i}^{T_f} + TC_A|_{T_0}^{T_i} \\
 &= C_A^{T_0} \cdot P^{T_0} \cdot \left(\frac{\eta \cdot F^{b+1} \cdot [T_f - T_i]^{b+1}}{b+1} + \frac{[T_i - T_0]^{a+1}}{a+1} \right) \\
 &= \$10 \cdot 10,000 \cdot \left(\frac{1.3 \cdot 1 \cdot [15 - 5]^{0.68}}{0.68} + \frac{[5 - 0]^1}{1} \right) = \$1.4M
 \end{aligned}$$

As observed earlier, the firm minimizes TPC by switching to B. We seek to characterize the impacts of investment based learning on both TPC and the capacity allocated to B, F.

Therefore, we want to find the values of F and G that minimize TPC:

$$\min(TC_A + TC_B) \text{ w.r.t } \{F, G\} \tag{98}$$

The forms of TC_A and TC_B in (95) and (96) do not easily lend themselves to closed form solutions with respect to F and G . Therefore, we simulate the cost functions. Table 22 presents the results.

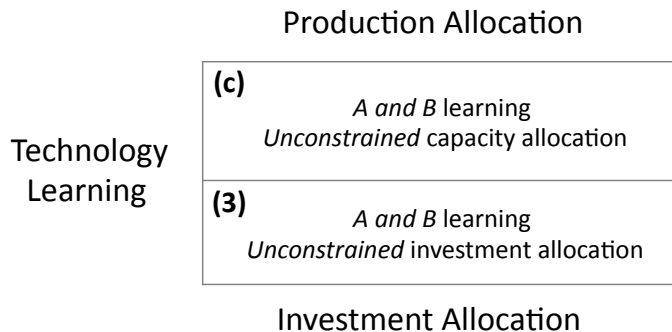
Decision Context	TPC^{MIN}	F	G	Decision
(a)	\$1.4M	100%	N/A	Switch to B
(a.3)	\$691k	100%	100%	Switch to B

Table 22: Capacity and investment allocations resulting in minimum total production costs for decision contexts (a) and (a.3)

By investing in B prior to launch, the firm is able to drive down the initial unit cost of B. This means that B becomes cost competitive with A sooner, and the cost savings associated with B are experienced longer. The significant cost savings associated with this effect outweigh the foregone learning in A due to diverting investment to B. Before considering CI learning, the best decision was to switch all capacity to B. The inclusion of CI learning reinforces this decision.

Context (c.3)

In this context, A exhibits capacity and investment driven learning and the firm is free to allocate both resources to either or both technologies.



Technology Questions:

- Does incorporating CI based learning change the capacity allocation decision compared to context (c)?
- How should capacity be allocated?
- How should investments be allocated?

Metrics:

- Total production costs from T_0 to T_f , (TPC)
- Capacity allocated to B, (F)
- Investment allocated to B, (G)

Method:

- Determine F and G that minimize TPC

In this case we are comparing against context (c), in which minimal TPC corresponds to staying with A. We can see this by comparing the total costs of A against the costs associated with switching to B in (97):

$$\begin{aligned}
 TC_A &= C_A^{T_0} \cdot P^{T_0} \cdot \left(\frac{\left[(T_f - T_0) - F \cdot (T_f - T_i) \right]^{a+1}}{a+1} \right) \\
 &= \frac{C_A^{T_0} \cdot P^{T_0} \cdot \left[(T_f - T_0) \right]^{a+1}}{a+1} \\
 &= \frac{\$10 \cdot 10,000 \cdot \left[(14 - 0) \right]^{0.85}}{0.85} = \$1.1M
 \end{aligned} \tag{99}$$

As in the previous case, we seek to minimize total production costs with respect to F and G. Table 23 presents the results for both contexts.

Decision Context	TPC^{MIN}	F	G	Decision
(c)	\$1.1M	0%	N/A	Stay with A
(c.3)	\$692k	100%	100%	Switch to B

Table 23: Capacity and investment allocations resulting in minimum total production costs for decision contexts (c) and (c.3)

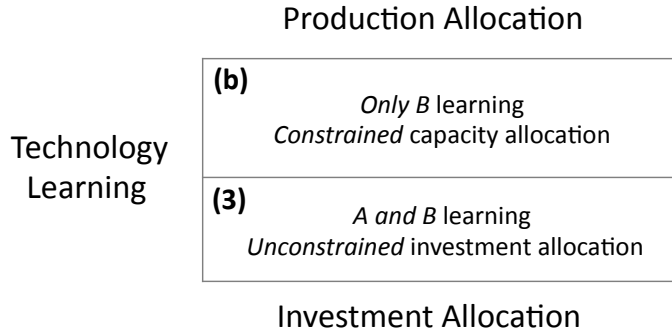
Total costs are reduced further than in the previous case due to the additional CPV learning in A prior to the launch of B. As with the previous case, the introduction of CI learning in (c.3) reduces the costs associated with B more than it increases the costs for A relative to (c). This effect is enough to change the technology decision such that the firm should now switch to B.

The results for these cases suggests that, when the firm is free to allocate all resources to a single technology then the minimum cost solution is always either to stay with A or switch to B. Additionally, the introduction of CI learning in B makes switching to B more attractive, as it lowers the initial costs associated with B and enables B to become the less expensive technology sooner than when only considering CPV driven learning.

We are also interested in cases in which capacity and investment allocations are constrained. These cases correspond to decision contexts (b) and (d) in the CPV learning space, and (2) and (4) in the CI learning space. This includes contexts in which the resource may be constrained in a single learning dimension. For example, when the firm is free to allocate investment resources but must share production capacity or vice versa. These cases may provide insight into the “value of substitution” of one type of learning for another. For example, when we examined decision context (b) in section 4.8.5, the firm would need to introduce at least 76% B to make switching to be economically feasible. However, investment driven learning may decrease this figure by decreasing the total costs associated with B. In this case, the firm may be able to substitute investment learning for capacity driven learning to be able to introduce B at a smaller fraction of total production.

Context (b.3)

In this context, we examine the situation where the firm has a capacity allocation constraint resulting in shared production, but it is free to allocate investments to either or both technologies.



Technology Questions:

- How does incorporation of CI driven learning change total production costs when capacity is constrained?
- How should investments be allocated for a given capacity allocation?

Metrics:

- Total production costs from T_0 to T_f , (TPC)
- Capacity constraint, \hat{F}_i or \hat{F}_ii
- Investment allocated to B, (G)

Method:

- Given the capacity constraint, determine G that minimizes TPC

In this case, we know that the firm will face capacity constraint type (i), as minimum TPC corresponds to allocating all production volume to B (recall that this is the same solution as in context (a)). We will determine the capacity constraint, \hat{F}_i , using (47), and then use this as the fixed value of F for which we will evaluate what investment allocation results in TPC^{MIN} . From (47):

$$\hat{F}_i = \left(\frac{b+1}{\eta \cdot (T_f - T_i)^b} \right)^{\frac{1}{b}} = \left(\frac{0.68}{1.3 \cdot (14 - 4)^{-0.32}} \right)^{\frac{1}{-0.32}} = 76\% \quad (100)$$

In addition to investment behavior at the constraint, we would also like to understand characterize how investment allocation behavior changes at values below this constraint. For example, if the firm can only allocate 50% of capacity to B rather than 76%, can investment driven learning help mitigate the resulting increase in total production costs?

Table 24 presents the total production cost and investment allocations decisions resulting from capacity allocations both at the constraint and for two allocations below the constraint: $F = 25\%$ and $F = 50\%$. Figure 31 compares the results for context (b) and (b.3) to highlight the impact of CI driven learning on production costs.

Decision Context	TPC^{MIN}	F	G	Investment Decision
(b)	\$1.61M	25%	N/A	N/A
(b.3)	\$1.22M	25%	50%	Share investments
(b)	\$1.57M	50%	N/A	N/A
(b.3)	\$1.05k	50%	50%	Share investments
(b)	\$1.4M	76%	N/A	N/A
(b.3)	\$910k	76%	100%	All investment to B

Table 24: Total production costs and investment allocation decisions for fixed capacity allocations

The introduction of CI learning reduces total production costs for all three capacity allocations, with total production costs declining in all cases as the capacity allocation moves towards the constraint. The results suggest that when the firm allocates less than the constraint to B, sharing investments results in minimal total production costs. To explore why this is the case, Figure 31 illustrates the total production cost curves as a function of the investment allocated to B for the capacity allocation scenarios in Figure 31 (i) presents total production costs as a function of capacity allocation for the base case, while (ii) through (iv) illustrate total production costs as a function of investment allocation corresponding to the three capacity allocation “slices:” (ii) at the capacity constraint $F = \hat{F}_i = 76\%$, (iii) $F = 25\%$, and (iv) $F = 50\%$.

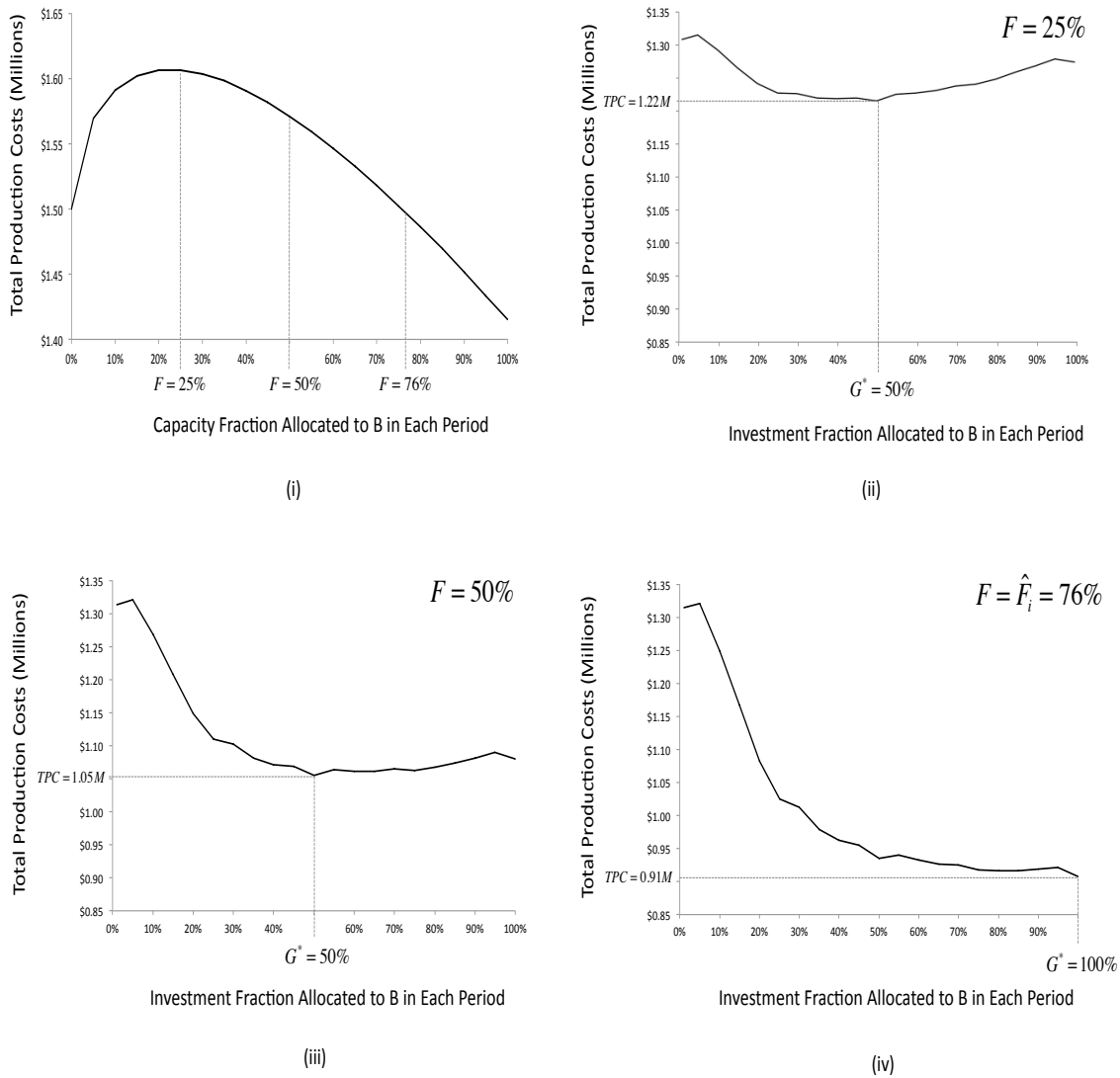


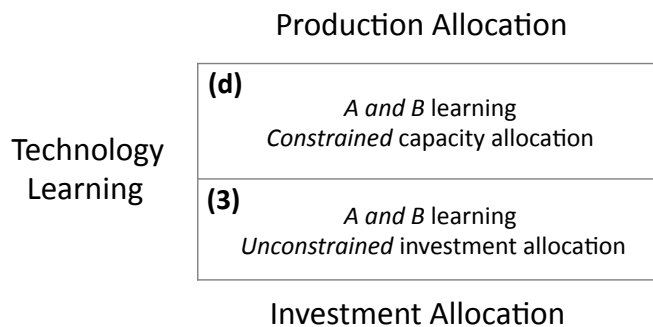
Figure 31: (i) Total production costs as a function of capacity allocation to B for decision context (b.3); (ii) – (iv) TPC as a function of investment allocation to B when (ii) $F = 25%$, and (iii) $F = 50%$ and (iv) $F = \hat{F}_i = 76%$

As Figure 31 illustrates, investment driven learning results in reduced total production costs at all three levels of capacity allocation. As the fraction of capacity allocated to B decreases, (F getting smaller), the cost of each unit of B increases and it takes longer for B to become the less expensive technology. As a result, total production costs rise. However, investment driven learning reduces this penalty by reducing the introduction costs associated with B and decreasing B costs over time. In this decision context, A also

exhibits CI driven learning. As the firm produces less of the more expensive technology, B, and more of the less expensive technology, A, the contribution of production costs due to A to total production cost increases. The minima observed in Figure 31 (ii) and (iii) reflect the cost tradeoff between the penalty of foregone learning in A and the benefits of learning in B. Once the capacity allocation reaches the constraint at $F = 76\%$, the firm should allocate all investments to B. This makes sense, as 76% represents the capacity at which the firm should start producing all B. As a result, allocating investment reduces the costs associated with B even faster, while also reducing introduction costs. At capacities greater than the constraint, this effect will be even more pronounced, Therefore, the firm should allocate $G = 100\%$ of investments to B for all capacity allocations greater than the constraint.

Decision context (d.3)

This context differs from (b.3) in that technology A continues to exhibit both CPV and CI driven learning.



Technology Questions:

- How does the addition of continued CPV driven learning in technology A change the investment allocation decision?
- Does incorporating CI based learning change the capacity allocation decision compared to context (d)?
- Given that capacity allocation is constrained, how should investments be allocated?

Metrics:

- Total production costs from T_0 to T_f , (TPC)

- Capacity constraint, \hat{F}_i or \hat{F}_{ii}
- Investment allocated to B, (G)

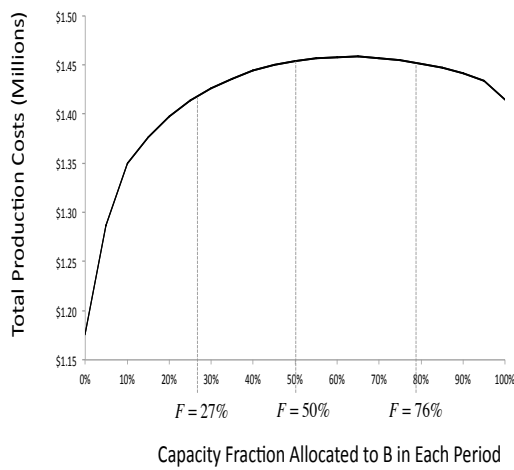
Method:

- Given the capacity constraint, determine G that minimizes TPC

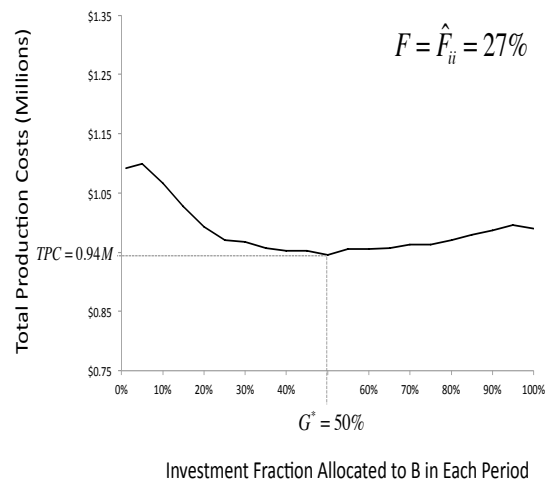
In context (d), the firm faces capacity constraint type (ii), in that some fraction of B must be produced, even though the least costly solution is to stay with A. Recall that the constraint represents the allocation to B at which the total production costs of sharing capacity are the same as switching all capacity to B (see Figure 13 for illustration). In the decision context (d) example, we calculated a constraint value of $\hat{F}_{ii} = 27\%$ by incorporating the baseline parameters values into (51). As in the last example, we would like to explore how the firm should allocate investments both at this constraint, and as capacity allocation changes. In this case, we want to explore how investment allocation impacts total production costs for capacity allocations to B greater than the constraint. To do this, we compare the total production costs considering CI driven learning against the costs when only CPV learning is considered for sample capacity allocations $F = 50\%$ and $F = 75\%$.

Decision Context	TPC ^{MIN}	F	G	Investment Decision
(d)	\$1.17M	27%	N/A	N/A
(d.3)	\$945	27%	50%	Share investments
(d.3)	\$904	50%	75%	Share investments
(d.3)	\$808	75%	100%	All investment to B

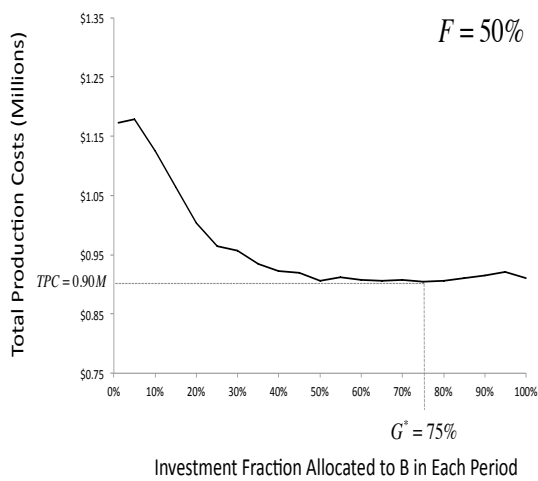
Table 25: Sample capacity and investment allocations resulting in minimum total production costs for decision contexts (d) and (d.3)



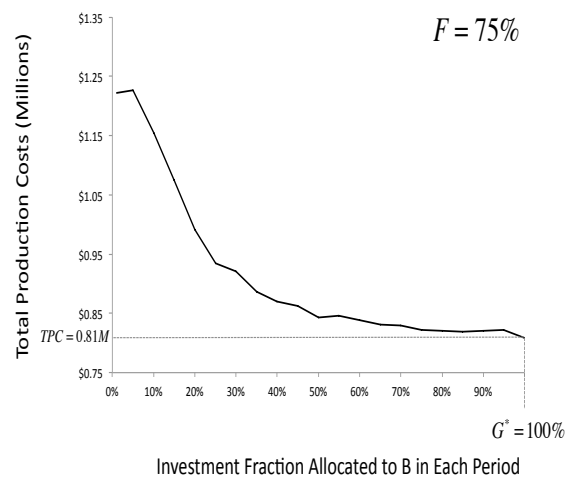
(i)



(ii)



(iii)



(iv)

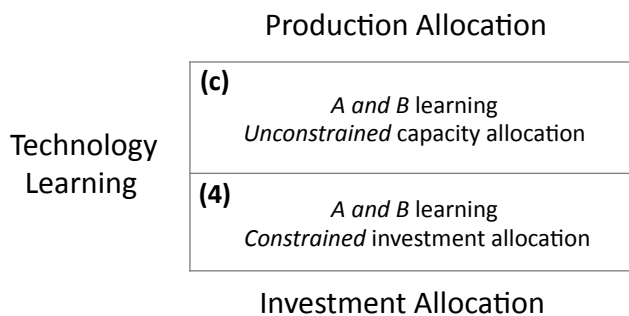
Figure 32: (i) Total production costs as a function of capacity allocation to B for decision context (d.3); (ii) – (iv) TPC as a function of investment allocation to B when (ii) $F = \hat{F}_{ii} = 27\%$, (iii) $F = 25\%$, and (iii) $F = 50\%$ and (iv) $F = 75\%$

Suggests that this allocation results in maximum benefits from the tradeoff over a wide range of capacity allocations. However, the total production costs are different. When allocating less than 27% to B, in this case 15%, the firm is still producing some B, but

each unit is more expensive due to foregone learning. This causes an increase in TPC. Conversely, allocating more

Decision Context (c.4)

We have examined cases where the investment decision was a function of constrained capacity allocation. We would also like to characterize the impacts of constrained investments on capacity allocation. In this example, we assume that both technologies exhibit CPV and CI driven learning and that the firm can allocate production capacity to either or both technologies.



Technology Questions:

- How do investment allocation constraints impact the capacity allocation decision when both technologies exhibit both CPV and CI driven learning?

Metrics:

- Total production costs from T_0 to T_f , (TPC)
- Capacity allocated to B, (F)

Method:

- Given the investment allocation, determine F that minimizes TPC

As in the previous context, we examine several allocation scenarios and then explore the full range of possible outcomes. However, in this case we are fixing the investment allocation levels to B then determining the capacity allocation that minimizes total production costs. Once we have explored these examples, we perform a more detailed sensitivity analysis to characterize the range of possible outcomes for every investment allocation decision. We expect at low levels of investment to observe results similar to

context (c)—staying with technology A is the least costly option. However, as the investment allocation to B gets larger, we expect that the benefits in the form of cost reductions in B will outweigh the penalty of forgone learning in A. We know that once an investment allocation is reached that makes switching all production to B the least costly option, this decision will not change for any larger investment allocations. The investment allocations sampled and the resulting total production costs and capacity allocation are presented in Table 26. Decision context (c) which only considers CPV driven learning is included for reference.

Decision Context	TPC^{MIN}	F	G	Capacity Decision
(c)	\$1.17M	0%	N/A	N/A
(c.4)	\$918k	0%	5%	Stay with A
(c.4)	\$923k	0%	10%	Stay with A
(c.4)	\$871k	100%	30%	Switch to B
(c.4)	\$767k	100%	50%	Switch to B

Table 26: Sample investment allocations and corresponding capacity allocations resulting in minimum total production costs under decision context (c.4)

The results indicate that (1) the decision changes from keeping all production volume allocated to A to switching to B in the region $10\% \leq G \leq 25\%$, and (2) that this coincides with a total production cost maximum. This suggests that there are values of investment allocation which result in an increase in production costs due to foregone learning in both technologies. To better characterize these effects, we can plot the minimum total production cost path as a function of the investment allocated to B over the range $0 \leq G \leq 1$.

7 CPV and Investment Driven Learning Case Study: Novel Automotive Sub-Component Assembly

7.1 Background

In this case, multiple welding technologies will be used simultaneously to produce a single product: an existing technology, A, and a new technology, B. The firm has already decided in which year the new technology will be introduced, and has already developed a production schedule which defines how capacity will be allocated between the technologies.

7.2 Process Based Cost Model

In this case, the product considered (the functional unit), is the weld used to join two propulsion system components during assembly. As in the case of semiconductors, we need to define the factors specific to automotive assembly to develop an expression for total costs. In this case, we are tracking reject rate, cycle time, and tool life. We assume that each weld is completed in cycle time CT. We assume a single welding processing step (although the cycle time is calculated as the time from the end of a weld to the end of the following weld, which includes any necessary time to move the parts to be welded). Production volume is defined in terms of the assembled sub-components needed, but we define costs in terms of welds. We define:

$$W(t)_{net} = V(t)_{net} \cdot N \quad (101)$$

where $V(t)_{net}$ is the annual production volume of batteries required and N is the number of welds per battery. We derive the total number of welds required in a given year, $W(t)_{gross}$, based on the target volume of welds in that year, $W(t)_{net}$, and the process reject rate, RR ($0 \leq RR \leq 1$):

$$W(t)_{gross} = \frac{W(t)_{net}}{1 - RR} \quad (102)$$

Because the component cost is a large component of sub-assembly costs, we assume that a large percentage of rejected welds are reworked. We amend (102) to include this factor, where, rw ($0 \leq rw \leq rr$) presents the fraction that are reworked:

$$W(t)_{gross} = \frac{W(t)_{net}}{1 - RR + rw} \quad (103)$$

The total operating time, T , per year to produce this many welds is then given by:

$$T(t) = CT \cdot W(t)_{gross} \quad (104)$$

The operating time of a single production line, UT , is given by:

$$UT = DY \cdot SD \cdot (HS - UD - PB - UB) - Idle \quad (105)$$

Where DY is the days per year the facility is operating, SD it the shifts per day, HS is the hours per shift, UD is unplanned downtime per shift, PB is paid break time per shift, UB is unpaid break time per shift and $Idle$ is the total time in hours per year that the facility is idle (due to lack of demand for example).

We define the integer number of lines required, L , as:

$$L(t) = \left\lceil \frac{T(t)}{UT} \right\rceil \quad (106)$$

We assume that workers receive wages for paid breaks, unplanned downtime and also when the line is idle. Therefore, the total paid time per year, PTY , is given by:

$$PTY(t) = L(t) \cdot (UT - DY \cdot SD \cdot UB) \quad (107)$$

The integer number of tools required per year to produce W_{gross} welds, NT , is given by:

$$NT(t) = \left\lceil \frac{W(t)_{gross}}{TL} \right\rceil \quad (108)$$

where TL is the tool life.

The financial model applies factor prices to these resources. It outputs a unit cost for each technology. Total annual cost, TC , is divided into six categories:

$$UC(t) = C(t)_{materials} + C(t)_{labor} + C(t)_{bldg} + C(t)_{equipment} + C(t)_{tools} \quad (109)$$

Total materials costs are given by:

$$C(t)_{materials} = W(t)_{gross} \cdot MW \cdot CM \quad (110)$$

where MW is the material used per weld and CM is the cost of material. Labor cost is the product of the total time required to produce W_{gross} and the wage per hour, WPH :

$$C(t)_{labor} = PTY(t) \cdot WPH \quad (111)$$

We consider building, equipment and tools to be capital expenses. Incorporating these into a unit cost requires distributing them across time by determining the sum of payments in each period that is functionally equivalent to the initial investment. We distribute these payments over the useful life of the building and equipment. In order to determine the payment per period, we define the capital recovery factor, CPV:

$$CRF_j = \frac{d \cdot (1+d)^{UL_j}}{(1+d)^{UL_j} - 1} \quad (112)$$

where d is the inflation adjusted discount rate and UL is the useful life in years of the j th element. We assume an initial building and equipment investment $CAP_{bldg.}$ to house the production line. Therefore, the annual cost is:

$$C(t)_{bldg} = L(t) \cdot CAP_{bldg} \cdot CRF_{bldg} \quad (113)$$

Equipment capital investment is the sum of the equipment capital required for each line ($CAP_{equipment}$), multiplied by the number of lines. The annual equipment cost is:

$$C(t)_{equipment} = L(t) \cdot CRF_{equipment} \cdot CAP_{equipment} \quad (114)$$

Similarly, we assume total tooling costs are given by:

$$C(t)_{tooling} = NT(t) \cdot CRF_{tooling} \cdot CAP_{tooling} \quad (115)$$

where $CAP_{tooling}$ is the capital required for each tool.

With all the individual factors calculated, we define the cost per weld in a given year, $WC(t)$ as:

$$WC(t) = \frac{TC(t)}{W(t)_{net}} \quad (116)$$

7.2.1 Learning and Dynamic PBCM

We assume learning in cycle time, reject rate and tool life. Table 35 highlights where these factors occur in the PBCM from Figure 21.

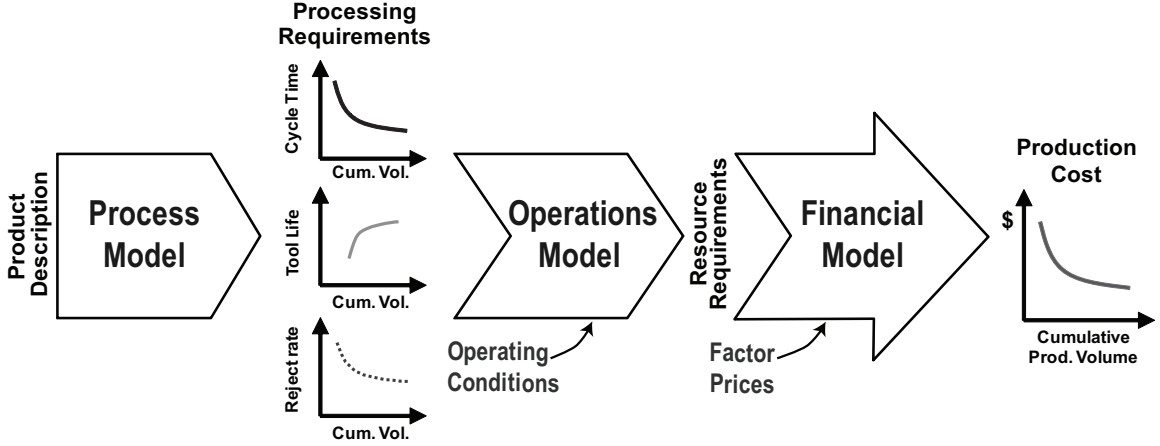


Figure 33: Dynamic process-based cost modeling incorporating learning in cycle time, tool life and reject rate

For each parameter, learning is assumed to follow the power law formulation in (31). We assume that each parameter has an initial value and that this value declines with increasing cumulative production volume over time. We set a final value for each parameter beyond which the curve becomes flat and learning no longer occurs. In this case, learning occurs as the result of both cumulative production volume and investments. Each parameter exhibiting learning will have a learning function for each driver.

As with wafer starts per month and yield in the semiconductor case, learning increases tool life. Therefore, we use the same transformation as in (71) was used to arrive at the tool life learning exponent. We define the learning functions for each parameter from (31) and the fact that each parameter has a limiting value. For tool life, the CPV and CI learning functions are given by:

$$\begin{aligned}
 TL(t) &= \min \left(TL_{initial} \cdot \left[\frac{CPV^t}{P^{T_0}} \right]^{tl_{CPV}^*}, TL_{max} \right) \\
 TL(t) &= \min \left(TL_{initial} \cdot \left[\frac{CI^t}{I^{T_0}} \right]^{tl_{CI}^*}, TL_{max} \right)
 \end{aligned} \tag{117}$$

where $TL_{initial}$ is the total yield in the initial period, tl_{CPV}^* and tl_{CI}^* are the CPV and CI learning exponents corresponding to (increasing) tool life and TL_{max} is the maximum possible tool life. For cycle time the learning function is function is:

$$CT(t) = \max \left(CT_{initial} \cdot \left[\frac{CPV^t}{P^{T_0}} \right]^{ct_{CPV}}, CT_{min} \right) \quad (118)$$

$$CT(t) = \max \left(CT_{initial} \cdot \left[\frac{CI^t}{I^{T_0}} \right]^{ct_{CI}}, CT_{min} \right)$$

where $CT_{initial}$ is the initial cycle time per weld, ct_{CPV} and ct_{CI} are the CPV and CI learning exponents, and CT_{min} is the minimum possible cycle time. For reject rate, the learning function is:

$$RR(t) = \max \left(RR_{initial} \cdot \left[\frac{CPV^t}{P^{T_0}} \right]^{rr_{CPV}}, RR_{min} \right) \quad (119)$$

$$RR(t) = \max \left(RR_{initial} \cdot \left[\frac{CI^t}{I^{T_0}} \right]^{rr_{CI}}, RR_{min} \right)$$

where $RR_{initial}$ is the initial reject rate, rr_{CPV} and rr_{CI} are the CPV and CI learning exponents and RR_{min} is the minimum possible reject rate for the process.

7.3 Production and Cost Parameter Values

We derive the parameter values used in the analyses from data collected from a major North American car manufacturer over a 6-month period during 2010-2011.

7.3.1 Production and Investment Functions

As in the semiconductor case, we use a spreadsheet based cash flow model simulates production costs from T_0 to T_f . In each period (year), the total production volume required is given by:

$$P(t) = P^{T_0} \cdot \left((1+g)^{t-T_0} + (1+g)^{T_0-A} \right) \quad (120)$$

where P^{T_0} is the production volume in year T_0 , g is the annual production growth rate and A is the amortization period in years. We assume that once building and equipment costs are fully amortized they are retired. The second term in (75) reflects the new production that must be brought online to replace this retired capacity.

The investment function is given by:

$$I(t) = I^{T_0} \cdot (1+h)^{t-T_0} \quad (121)$$

where I^{T_0} is the total investment budget in year T_0 and h is the annual rate of growth of the investment budget.

7.3.2 Cost Parameter Values

Due to the confidential nature of the information collected, we normalize all cost values. We assume that the normalization factor is the costs associated with a single sub-assembly produced using welding technology A in the first year of the analysis. Each of the cost components is then expressed as fractions of this value. Total initial technology B cost is defined as a multiple of this value. Table 27 provides the relative cost composition of each welding technology.

	A	B
Materials fraction	1%	0%
Labor fraction	16%	36%
Equipment fraction	37%	63%
Tooling fraction	46%	1%

Table 27: Relative weighting on cost elements for each welding technology

The cost compositions are quite different between the technologies. Therefore, we would expect learning to differentially impact the technology production costs. For example, the portion of production costs due to equipment and building for technology B is twice that for A. Therefore, we would expect learning in cycle time to play a more important role in reducing the production costs of B than A. Alternatively, tooling costs make up a much larger fraction of total costs for A than B, so we would expect learning in tool life to benefit A more than B.

7.3.3 Learning Parameter Values

Each learning parameter is assigned an initial value and a limiting value, representing the best possible value the parameter can achieve through learning. The limiting value for tool life represents a maximum, while for cycle time and reject rate the value represents a

minimum. Table 28 presents the initial and limiting values considered for these parameters.

	A		B	
	Initial	Limit	Initial	Limit
Tool life (welds)	60k	100k	N/A	N/A
Cycle time (seconds)	9	6	4	2
Reject rate (% of total welds)	0.15%	0.08%	1%	0.15%

Table 28: Initial and limiting CPV driven learning parameter values

7.3.4 Labor Parameter Values

Table 15 presents the labor parameter values considered

	Value
Employees per line	2
Work days per year	235
Shifts per day	2
Hours per shift	8
Paid downtime per shift	1
Labor rate per hour	\$22

Table 29: Labor parameter values

7.4 Scenarios Considered

We are seeking to characterize the impacts of continued learning technology A and the constraint that capacity must be allocated to both technologies to address two questions:

1. Given that capacity allocation is constrained, how should the firm's investment resources be allocated between the technologies?
2. What are the primary operational drivers of learning within each technology and how do these influence the investment allocation decision?

To explore these questions, we first develop a base case in which the capacity allocation is constrained, technology A exhibits learning-by-doing not learning-by-investing, and the firm is free to allocate investment to either or both technologies. In our taxonomy, this corresponds to decision context (d.1) in Figure 29. We then develop three other scenarios to compare against the base case. Finally, we investigate how learning in the individual factors impacts the technology decision to identify the factor(s) where improvements lead to the largest benefits.

7.4.1 Scenario Parameter Values

In this case, there are both learning-by-doing and learning-by-investing learning rates associated with each factor for each technology. Additionally, the capacity allocation to each technology has already been decided by the firm, as has the year of technology B introduction, 2014, and the year in which the firm will start investing in B, 2011. Table 30 presents the production and capacity allocation parameters that stay constant for all the decision contexts considered.

Symbol	Definition	Value
$T_i - T_0$	Time from T_0 until B introduced	3 yrs
$T_f - T_0$	Time from T_0 over which costs are summed	10 yrs
T_{INV}	Initial year the firm invests in B	2011
P^{T_0}	Production volume in T_0	200k
$1 - F$	Fraction of production allocated to A	20%
F	Fraction of production allocated to B	80%
g	Annual production growth	1%
d	Discount rate	0%
UL	Useful life of building and equipment	10 yrs

Table 30: Production and capacity allocation parameter values that remain constant for the scenarios considered

We are focusing on exploring the impacts of learning in A and investment allocation on the technology choice. Therefore, we hold the learning parameters for B fixed, and assume that A exhibits the same level of learning-by-doing in all three factors. Table 31 presents the values considered

Symbol	Definition	Value
$P_{A CPV}^{tl}$	CPV Tech A tool life progress ratio	95%
$P_{A CPV}^{ct}$	CPV Tech A cycle time progress ratio	95%
$P_{A CPV}^{rr}$	CPV Tech A reject rate progress ratio	95%
$tl_{A CPV}^*$	CPV Tech A tool life learning exponent	1.07
$ct_{A CPV}$	CPV Tech A cycle time learning exponent	-0.07
$rr_{A CPV}$	CPV Tech A reject rate learning exponent	-0.07
$P_{B CPV}^{tl}$	CPV Tech B tool life progress ratio	N/A
$P_{B CPV}^{ct}$	CPV Tech B cycle time progress ratio	85%
$P_{B CPV}^{rr}$	CPV Tech B reject rate progress ratio	85%
$tl_{B CPV}^*$	CPV Tech B tool life learning exponent	N/A
$ct_{B CPV}$	CPV Tech B cycle time learning exponent	-0.23
$rr_{B CPV}$	CPV Tech B tool life progress ratio	-0.23
$P_{B CI}^{tl}$	CI Tech B tool life progress ratio	N/A
$P_{B CI}^{ct}$	CI Tech B cycle time progress ratio	85%
$P_{B CI}^{rr}$	CI Tech B reject rate progress ratio	85%
$tl_{B CI}^*$	CI Tech B tool life learning exponent	N/A
$ct_{B CI}$	CI Tech B cycle time learning exponent	-0.23
$rr_{B CI}$	CI Tech B tool life progress ratio	-0.23

Table 31: Progress ratios and learning exponents held constant for the scenarios considered

We also need to define the investment budget and parameters that will drive learning-by-investing in the scenarios. Table 32 presents the investment budget and parameter values used for all the scenarios considered.

Symbol	Definition	Value
$T_{inv} - T_0$	Time from T_0 until firm invests in B	2 yrs
I^{T_0}	Initial annual investment	\$1M
h	Annual investment growth	0%

Table 32: Investment budget and portfolio parameter values used for all the scenarios considered

In addition to the total budget to be allocated between technologies, the firm must decide when to begin investing in the new technology, and how to share the allocation amongst activities that to better each of the learning factors within each technology. For example, the firm may choose to invest 50% of the budget to research and development focused on increasing the tool life and the other 50% to process improvements to reduce cycle time. We define this set of allocations as the investment portfolio. In this analysis, we assume the investment portfolio is fixed by the firm. Table 33 presents the investment portfolios considered.

Learning Parameter	A	B
Cycle time	33.3%	0%
Reject rate	33.3%	100%
Tool life	33.3%	0%

Table 33: Investment portfolio for each technology

Table 34 presents the parameter values that vary between scenarios, and the values considered in each scenario.

Symbol	Definition	Decision Contexts	
		(d.1), (d.2)	(d.3), (d.4)
$P_{A CI}^{tl}$	CI Tech A tool life progress ratio	100%	95%
$P_{A CI}^{ct}$	CI Tech A cycle time progress ratio	100%	95%
$P_{A CI}^{rr}$	CI Tech A reject rate progress ratio	100%	95%
$tl_{A CI}^*$	CI Tech A tool life learning exponent	0%	1.07
$ct_{A CI}$	CI Tech A cycle time learning exponent	0%	-0.07
$rr_{A CI}$	CI Tech A reject rate learning exponent	0%	-0.07
$P_{B CI}^{tl}$	CI Tech B tool life progress ratio	N/A	N/A
$P_{B CI}^{ct}$	CI Tech B cycle time progress ratio	85%	85%
$P_{B CI}^{rr}$	CI Tech B reject rate progress ratio	85%	85%
$tl_{B CI}^*$	CI Tech B tool life learning exponent	N/A	N/A
$ct_{B CI}$	CI Tech B cycle time learning exponent	-0.23	-0.23
$rr_{B CI}$	CI Tech B reject rate learning exponent	-0.23	-0.23

Table 34: Parameter values for the decision context considered

7.5 Results by Decision Context

Decision context (d.1) (base case)

Figure 34 presents total production costs as a function of the investment allocation for the base case, which assumes 80% of production volume is allocated to B, and 20% to A.

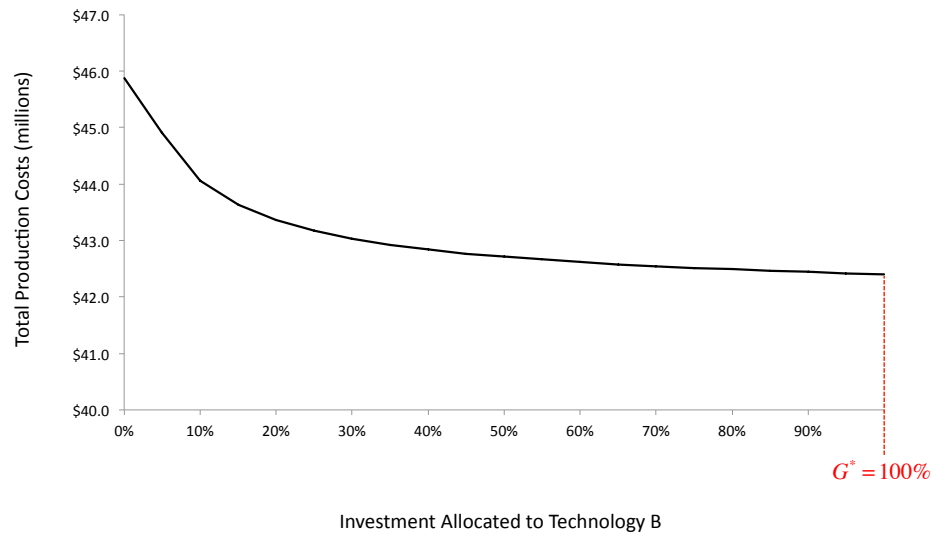


Figure 34: Total production costs as a function of the investment allocated to B for the base case

These results indicate that the best solution is to allocate all investment resources to B. This is because investing in A in the base case does not provide any benefit to the firm, as none of the factors considered exhibit learning-by-investing.

Decision context (d.2)

In this case, technology A still exhibits investment driven learning related cost reductions. This explains the existence of the large region over which sharing investments results in lower total production costs.

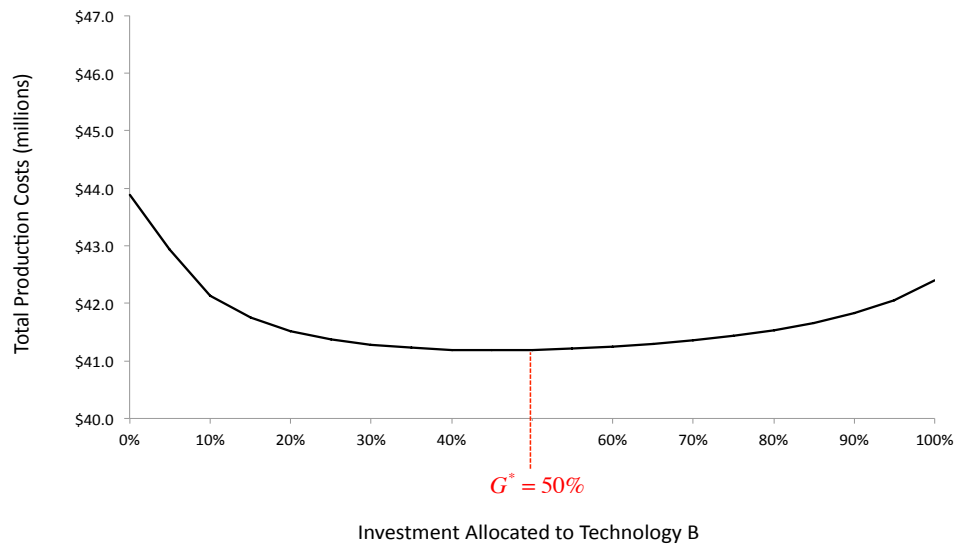


Figure 35 illustrates the resulting total production cost results as a function of the investment allocated to B. The results indicate that, although A only exhibits a progress ratio of 5%, this is enough to significantly impact both total production costs and the investment allocation decision. Although 80% of production capacity is allocated to B, the results suggest that only 50% of the investment resources should be allocated to B. Unlike in learning-by-doing, the firm does not have to pay a unit cost to gain the benefits of learning-by-investing. As a result, the firm is able to realize the maximum amount of cost reductions in both A and B without experiencing a cost penalty. This explains the existence of the large region over which sharing investments results in lower total production costs.

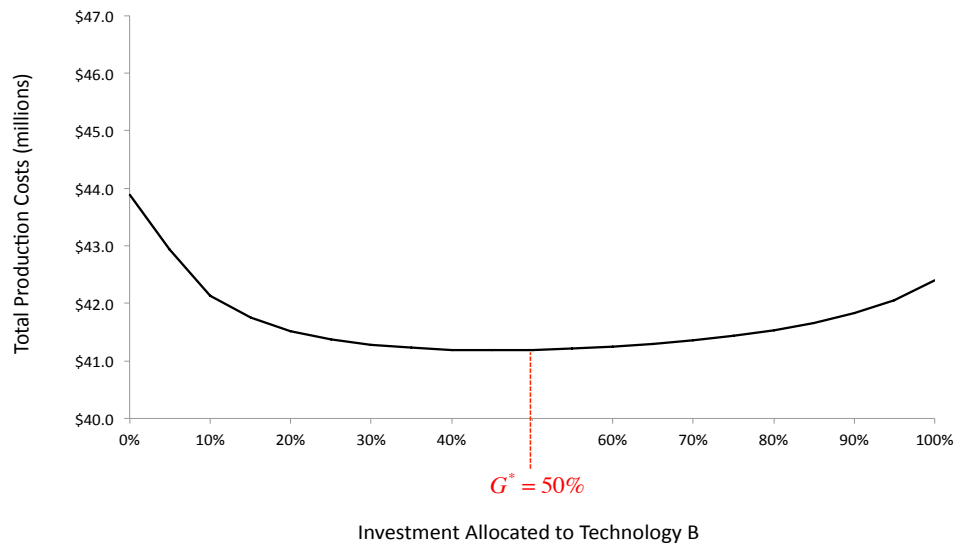


Figure 35: Total production costs as a function of the investment allocated to B when the existing technology still exhibits learning-by-investing (decision context d.2)

Table 17 provides a summary of the technology decisions for the automotive case.

Decision context	Technology Decision
(d.1)	Invest all in technology B
(d.2)	Share investments: 50% to B and 50% to A

Table 35: Summary of technology decisions in the automotive case decision contexts (d.1) and (d.2)

7.6 Sensitivity Analyses

The results in Figure 34 and Figure 35 correspond to a particular production scenario in which the firm allocates 80% of total production to technology B and begins investing in B in 2011. In this section we explore the implications of changes in these constraints on the firm's optimal investment allocation strategy. Table 36 presents the parameters and values considered. We assume that the technology A exhibits 95% progress ratios for all

three learning parameters for both learning-by-doing and learning-by-investing. All other parameters are assumed to be the same as in Table 27 through Table 34.

Parameter	Definition	Values
T_{INV}	First year firm invests in B	2011 - 2014
F	Capacity allocated to B	25%, 50%, 80%, 90%

Table 36: Sensitivity parameters and values considered

Figure 36 presents total production cost curves as a function of the investment allocation to technology B for the T_{INV} range in Table 36, when we continue to assume that 80% of production is allocated to B, ($F = 80\%$). As T_{INV} increases, the optimal investment allocation to the new technology, G^* , also increases. Table 37 tabulates the results for the introduction years in Figure 36.

T_{INV}	G^*
2011	50%
2012	55%
2013	60%
2014	75%

Table 37: Optimal investment strategies for different initial years the firm invests in B

These results suggest that waiting to invest in B means that the firm must invest more in B when the existing technology still exhibits learning by investing. This happens for two reasons. First, when A continues to exhibit learning, it takes longer for B to become the less expensive technology. As a result, the firm needs to facilitate faster movement down the learning-by-investing curve associated with B to try and offset this additional time. Second, the impact of learning-by-investing on the new technology prior to launch is to lower the associated initial unit costs. When the firm waits to begin investing in B, it needs to allocate more investments to attain the same initial costs as would have been

achieved had they begun investing earlier. As a result of these two effects, beginning to invest in B later results in higher total production costs irrespective of how much the firm allocates to B.

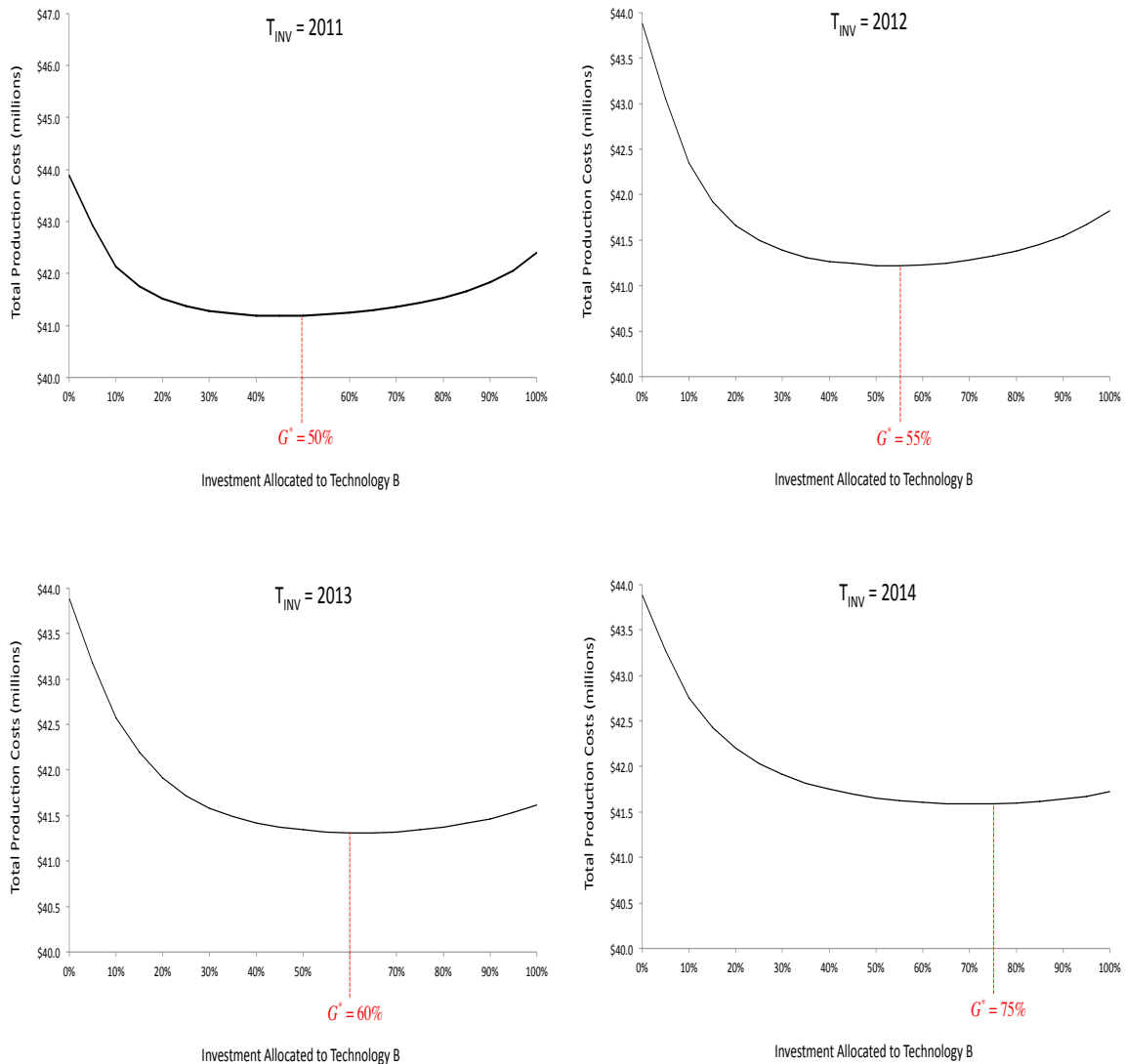


Figure 36: Total production costs as a function of the investment allocated to technology B for the initial investment years, T_{INV} in Table 36 when $F = 80\%$

Beginning to invest in B later also decreases the region over which sharing investments reduces total production costs over an end solution (either do not invest in B at all or invest 100% in B). For example, Figure 37 highlights the region over

which sharing investments between A and B results in lower total production costs relative to allocating all investment to B for two cases in Figure 36. In the first case, $T_{INV} = 2011$, while in the second $T_{INV} = 2014$. When the firm invests earlier, any level of shared investment greater than $G = 8\%$ (meaning that A continues to receive 92% of total investment), results in lower total production costs than shifting all investment to B. When $T_{INV} = 2014$, this increases to $G = 42\%$.

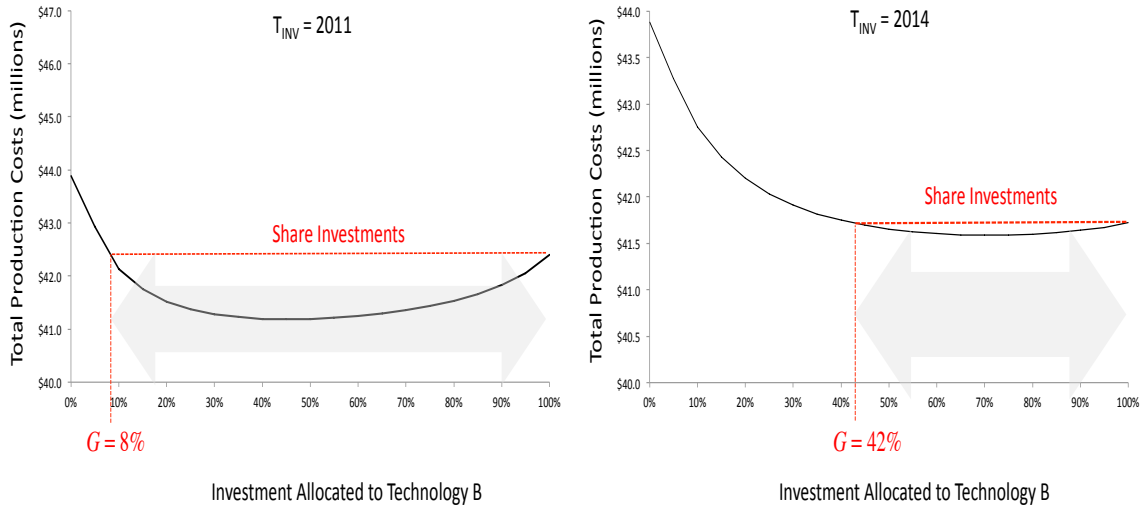


Figure 37: Regions over which sharing investments results in reduced total production costs when the firm begins investing in B in 2011 and 2014 and $F = 80\%$

This increase is due to the fact that the firm must shift more investment resources to B to make up for the penalties associated with beginning to invest later. However, because A is still learning and 20% of capacity will still be produced using A, there is still benefit to sharing investments, even when the firm begins to invest in B late.

Thus far, we have assumed that the firm allocates $F = 80\%$ of total production volume to B. However, we would also like to examine the impacts of changing the production allocation on investment allocations. Figure 38 presents the optimal investment allocation to B, G^* , as a function of the initial year the firm invests in B when $F = 80\%$.

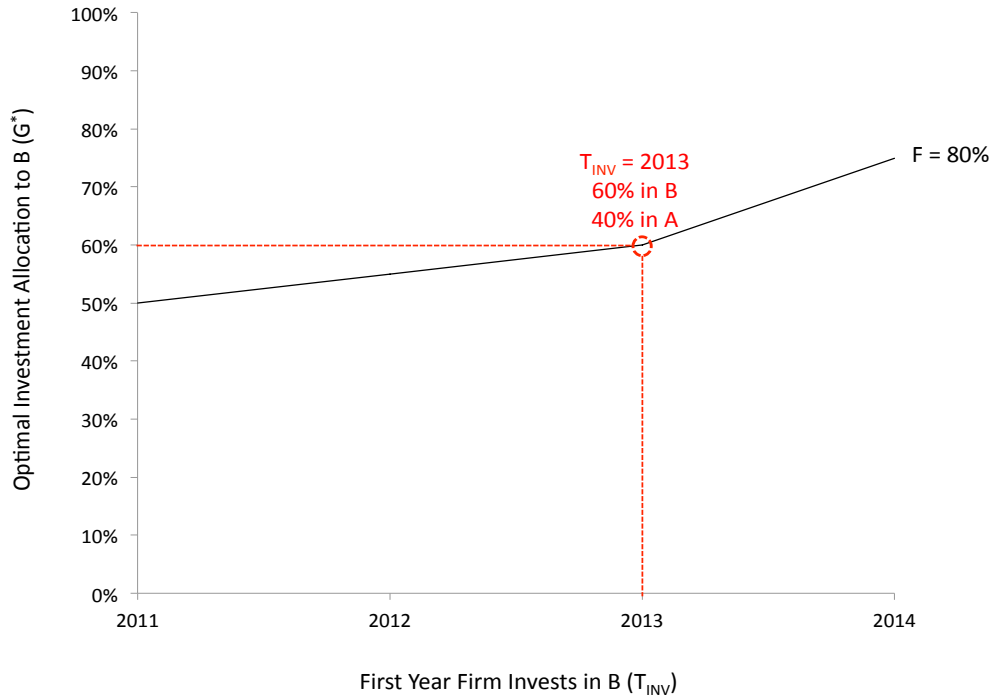


Figure 38: Optimal investment allocation to technology B as a function of the initial year the firm allocates investment to B when $F = 80\%$

The highlighted red circle in Figure 38 represents the case when $T_{INV} = 2013$. In this case, the optimal investment strategy is for the firm to allocate $G^* = 60\%$ of total investments to B beginning in 2013. This optimum corresponds to the minima in Figure 36 when $T_{INV} = 2013$. This data presentation enables us to compare the impacts of varying the capacity allocation strategy on the optimal investment strategy as a function of when the firm initially invests in B. Figure 39 presents the results when the firm allocates 25%, 50%, 80% and 90% of total production to B in 2014.

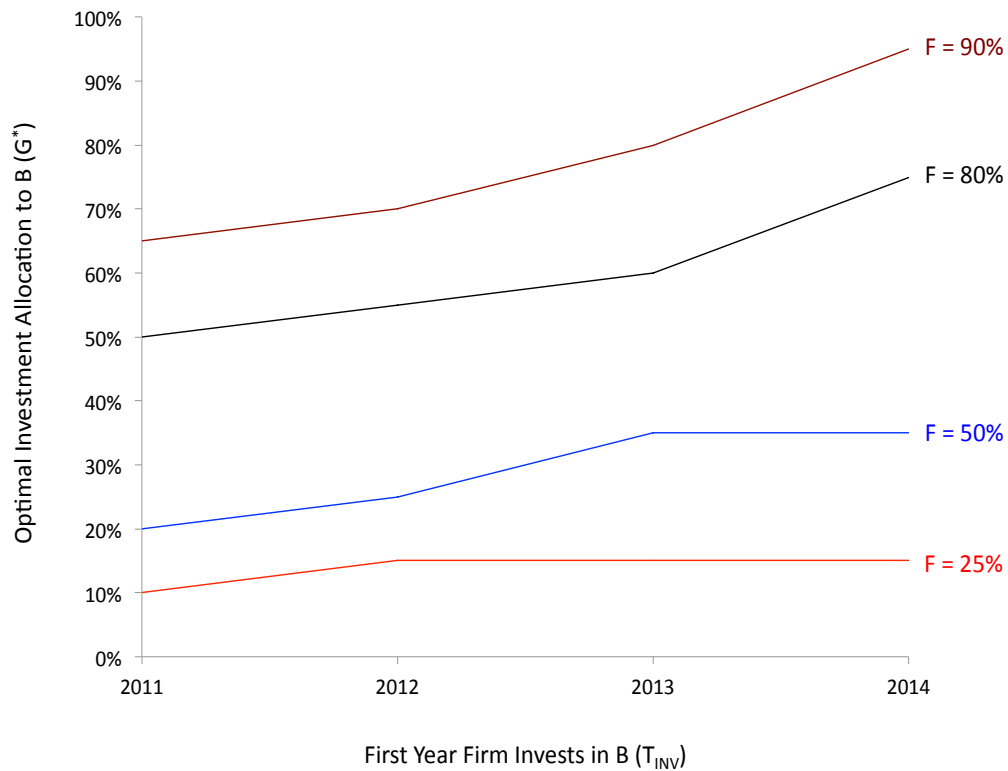


Figure 39: Optimal investment allocation to technology B as a function of the initial year the firm allocates investment to B when $F = 25\%$, 50% , 80% , and 90%

When $F = 90\%$, the firm is allocating almost all production to B. However, the results suggest that the firm will still benefit from sharing investments with A, irrespective of when the investment begins in B. This is due to the aforementioned fact that continued learning in A reduces total production costs. As expected, when the firm begins investing in B later it needs to invest more to try and bring down costs as quickly as possible. However, the increase in this investment is different depending on the capacity allocation decision. For example, when the firm allocates $F = 25\%$ of production to B, the optimal investment to B stays the same after $T_{INV} = 2012$, $G^* = 15\%$, while when the firm allocates $F = 90\%$ of capacity to B, G^* continues to increase with T_{INV} . These results highlight the tradeoff between the benefits from learning in B and foregone learning in A. When $F = 25\%$, the firm is using A to produce 75% of total capacity. As a result, investments in A can significantly reduce total production costs. However, because the

firm is still producing 25% of capacity using B, there is benefit to continuing to reduce the associated unit costs via learning-by-investing. In this case, the optimum investment of $G^* = 15\%$ represents the maximum the firm is willing to sacrifice in foregone learning in A to reduce costs in B. This figure is not strongly dependent on when the firm begins to invest in B because reducing costs in A is more important. As the firm allocates more and more capacity to B however, stimulating learning in B via investment becomes more important, as the costs associated with B make up an increasing fraction of total production costs. The fact that the difference in unit costs between A and B is non-linear in time (due to different learning rates), means that the optimal investment strategy will also be non-linear as a function of when the firm begins to invest in B.

Although we have demonstrated that investment driven learning can change the investment decision, we would also like to inform decision making about how to allocate investments within each technology. For example, the highlighted data point in Figure 38 indicates that, if the firm starts investing in B in 2013, it should allocate $G^* = 60\%$ to B and $(1 - G^*) = 40\%$ to A. However, this result assumed the investment allocation strategy in Table 33, in which the firm is dedicating all the investment resources it puts towards B to reducing the reject rate, while dividing the investment resources allocated to A evenly among cycle time, reject rate and tool life. But, if the firm were to change the relative weightings of investments in these factors, would this change the optimal total investment strategy to A and B? And, if so, what fraction of investment allocated to each factor results in minimum total production costs? The final sensitivity analysis addresses these questions.

We assume that the firm continues to invest all resources allocated to technology B to decrease the reject rate, as in Table 33. We then examine four different investment allocation profiles for technology A. By “turning on and off” investments in each combination of the three production factors, we can characterize the impact of each production factor on total production costs. As before, we assume the firm is allocating $F = 80\%$ of production to B. Table 38 presents the investment allocation to each factor exhibiting learning in technology A. Scenario 1 represents the “even split” allocation

used in our earlier analysis which resulting in the optimal investment allocation curve in Figure 38.

Investment Strategy	Reject Rate	Cycle Time	Tool Life
Even Split	33%	33%	33%
Cycle Time Only	100%	0%	0%
Reject Rate Only	0%	100%	0%
Tool Life Only	0%	0%	100%

Table 38: Fraction of investment in technology A allocated to each factor exhibiting learning

Table 39 presents the resulting total production costs as a function of the investment profiles in Table 38 when we assume the firm begins to invest in technology B in 2011 and that each learning factor exhibits a 95% progress ratio.

Investment Strategy	Total Production Costs
Even Split	\$46,317,766
Cycle Time Only	\$47,388,358
Reject Rate Only	\$47,371,370
Tool Life Only	\$41,825,000

Table 39: Total production costs as a function of the first year the firm invests in technology B for the investment profiles in Table 38 assuming the firm begins investing in B in 2011

The results indicate that investing in improving tool life provides significant cost savings over other investment strategies. Therefore, the automotive manufacturer should focus investment dollars on increasing tool life. This result stems from the relative benefits the firm can achieve by investing in each factor. These were provided in Table 28 and are provided again here:

	Technology A	
	Initial	Limit
Tool life (welds)	60k	100k
Cycle time (seconds)	9	6
Reject rate (% of total welds)	0.15%	0.08%

Investing in improving tool life enables the firm to improve the tool life by 40k welds. As a result, the firm needs to purchase significantly fewer tools, which make up almost 50% of the total production costs (see Table 27). In this case, only 20% of production is done using technology A, as $F = 80\%$. As a result, increases in cycle time are not as valuable, as decreasing the cycle time too much could result in underutilization of equipment. Decreases in the reject rate, although valuable in that they decrease materials and tool usage, do not provide enough of a savings to be significant when such a small amount of total production is made using A.

We have seen that how the investment is allocated within technology A can impact the total production costs. Does this allocation then also impact how the total investment pool should be allocated between A and B? Table 40 presents the optimal investment allocations to A, $(1 - G^*)$, and B, G^* , for each investment strategy in Table 38. The “even split” scenario corresponds to the $G^* = 50\%$ point in Figure 38 when the firm begins investing in B in 2011.

Investment Strategy	Optimal Investment in A ($1 - G^*$)	Optimal Investment in B (G^*)	Total Production Costs
Even Split	50%	50%	\$46,317,766
Cycle Time Only	20%	80%	\$47,388,358
Reject Rate Only	0%	100%	\$47,371,370
Tool Life Only	45%	55%	\$41,825,000

Table 40: Total investment allocations to A and B and the resulting total production costs for each of the investment strategies in Table 38 when the firm begins investing in B in 2011

These results suggest that characterizing learning at the process/operational level can have a significant impact on the firm’s overall investment allocation strategy. For example, the “even split” strategy suggests that, when the firm decides to evenly divide the investments allocated to technology A between cycle time, reject rate and tool life, the optimal allocation to technology A should be 50%, while when the firm decides only to invest in improving tool life, the optimal investment decreases to 45%. In the case where the firm chooses only to invest in decreasing the reject rate, the cost reductions gained by the resulting decrease in reject rate is not enough to offset the penalty of diverting investment away from decreasing the reject rate of technology B. As a result, the firm should not invest any resources in technology A.

We can now provide an investment strategy to the automaker that characterizes how total production costs change as a function of the factors they can directly influence: how much they choose to invest in projects improving each factor exhibiting learning. We know that the firm has already decided to allocate 80% of production to B, and that the firm should invest all resources allocated to A into improving the tool life. Figure 40 presents the updated optimal total investment allocations as a function of the year in which the firm begins to invest in B, and the resulting total production costs. We include the “even split” strategy for comparison.

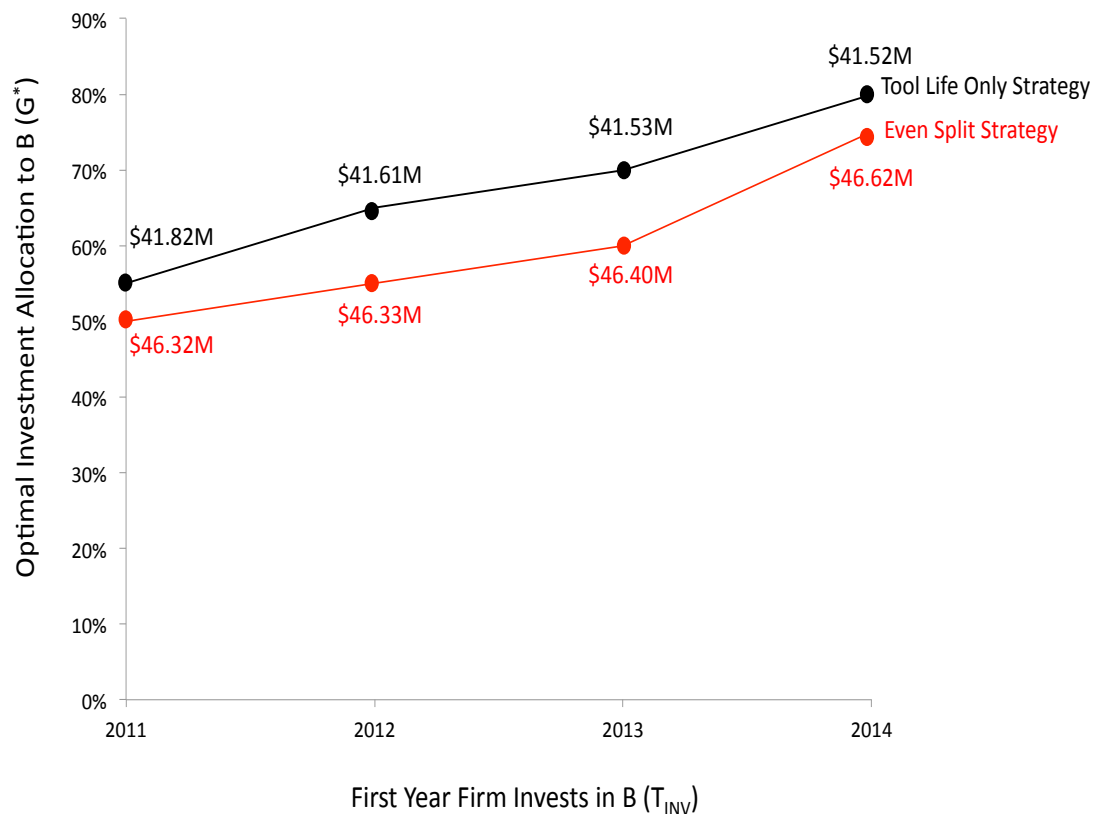


Figure 40: Optimal investment allocation to technology B and resulting total production costs for the “even split” investment strategy

The results show that, investing all resources allocated to A towards increasing tool life results in both allocating *less* investment resources to technology A *and* a reduction in total production costs irrespective of the year in which the firm begins to invest in B.

8 Conclusions and Future Work

8.1 Conclusions

This research has presented work to compliment and extend past work on the role of learning in technology choice, providing both a formal methodology and practical modeling tools firms can use when making these decisions.

Specifically, we developed a formalism and simulation model enabling characterization of how the technology decision *context* in which:

- technologies may be produced simultaneously
- technologies may be learning-by-doing
- technologies may be learning-by-investing

can change technology choice decisions about:

- when, if at all to introduce a new technology
- how to allocate finite production capacity over time
- how to allocate finite investment over time

To explore these questions, we extend existing learning models to explicitly incorporate learning in multiple operational parameters when capacity and investment resources must be shared between multiple technologies. We then characterize the conditions under which these factors change technology choice and resource allocation decisions. A dynamic process based cost modeling approach is developed to study the evolution of technology costs over time when considering learning from multiple sources (investment and production capacity) and in multiple production factors (cycle time, yield etc.). The goal of the model is to identify opportunities where firms can make operational changes that will impact changes in production costs, and characterize these impacts over time

8.2 Future Work

We see two primary areas for future work to extend the methodology: incorporating uncertainty, and the depreciation of knowledge assets over time.

8.2.1 Incorporating Uncertainty

We see two primary areas where uncertainty should be incorporated into the methodology. The first is in forecasts of the production volume. Production volume determines how much learning-by-doing is realized. Overestimating future production may lead the firm to overestimate the learning benefits associated with a new technology, leading to a sub optimal technology decision. Conversely, underestimates of production

may make firms hesitant to adopt new technologies, potentially leading to a loss of market share if competitors adopt these technologies and production exceeds forecasts.

The second area where we see uncertainty playing an important role is in the amount of learning the firm can expect to realize as the result of investment related activities. This can lead firms to both misallocate resources among technologies, and lead to higher production costs.

8.2.2 Incorporating Knowledge Depreciation

Knowledge depreciation has been observed across shifts and plants. This results in an increase in the learning rate associated with a given technology unless the firm continually invests in a technology. This could lead to higher costs, and place additional restrictions on how investments can be allocated among technologies. A more complete representation of the knowledge accumulated through R&D efforts can be obtained using a knowledge stock function, as proposed in the literature (Griliches, 1984, 1995; Watanabe, 1995, 1999). This function adds time lags during which the firm benefits from investments, but then the knowledge gained begins to depreciate.

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