6.012 Homework #5 Solutions

① n-MOSFET $V_{SB}=0$ $L_g = 1\mu m$ $X_{ox} = 10\ nm$

a) @ $V_{GS} = 1V$, device is on, but @ $V_{GS} = 0.5V$, almost no current ⇒ device is just turning on.

$\therefore \boxed{V_T \approx 0.5V}$

b) in saturation, $I_D = \frac{W}{2L}\mu_n C_{ox}(V_{GS}-V_T)^2$

so $\frac{2L}{W}\frac{I_D}{C_{ox}}\frac{1}{(V_{GS}-V_T)^2} = \mu_n$

Choose $V_{GS} = 2.5$ data

$$2L\frac{X_{ox}}{\epsilon_{ox}}\cdot\frac{I_D}{W}\frac{1}{(V_{GS}-V_T)^2} = 2(1\times10^{-4}cm)\frac{10\times10^{-7}cm}{(3.9)(8.85\times10^{-14}F/cm)}\frac{310\times10^{-6}A}{10^{-4}cm}\frac{1}{(2.5-0.5V)^2}$$

$\boxed{\mu_n \approx 450\ cm^2/V\cdot s}$

c) $V_{GS} = 3V$, $V_{DS} = 3V$ ⇒ $\frac{I_D}{W} \simeq 470\ \mu A/\mu m$ $W = 10\mu m$

$$g_m = \frac{2I_{Dsat}}{V_{GS}-V_T} = \frac{2W}{(V_{GS}-V_T)}\left(\frac{I_{Dsat}}{W}\right) = \frac{2\cdot10\times10^{-4}cm}{(3-0.5)V}\frac{470\times10^{-6}A}{10^{-4}cm}$$

$\boxed{g_m = 3.76mS}$

d) $V_{gs} = 3V$, $V_{DS} = 3V$, ⇒ saturation

$$C_{gs} = \frac{2}{3}WLC_{ox}$$
$$= \frac{2}{3}(10\times10^{-4}cm)(1\times10^{-4}cm)\left(\frac{3.9\times8.85\times10^{-14}F/cm}{10\times10^{-7}cm}\right)$$

$\boxed{C_{gs} = 2.3\times10^{-14}F = 23fF}$

② n-MOSFET $L = 1\,\mu m$, $W = 5\,\mu m$, $X_{ox} = 20\,nm$ $N_a = 10^{17}\,cm^{-3}$

a) $V_{T_{on}} = V_{FB} - 2\phi_p + \gamma\sqrt{-2\phi_p}$

$$\phi_p = -60mV \log \frac{N_a}{n_i} = -60mV \log \frac{10^{17}}{10^{10}} = -0.42\ V$$

$$V_{FB} = -\phi_B = -(\phi_{n^+} - \phi_p) = -(0.55 - (-0.42)) = -0.97\ V$$

$$\gamma = \frac{1}{C_{ox}}\sqrt{2\epsilon_s q N_a} = \frac{(20\times10^{-7}\,cm)}{(3.9)(8.85\times10^{-14}\,F/cm)}\sqrt{2(11.7)(8.85\times10^{-14}\,F/cm)(1.6\times10^{-19}C)(10^{17}\,cm^{-3})}$$

$$= 1.05\ V^{1/2}$$

so $V_T = -0.97 - 2(-0.42) + 1.05(\sqrt{-2\cdot(-0.42)})$

$\underline{V_{T0} = 0.83\ V}$

in this case ($V_{GS} = 3\ V$, $V_{DS} = 4\ V$), $V_{GS} > V_T$, and

$V_{DS} > V_{GS} - V_T \Rightarrow$ <u>saturation</u>

b) n @ source-end of channel

↳ $V_C(0) = 0$

$$n = \frac{Q_N}{-q} = \frac{-C_{ox}(V_{GS} - V_T)}{-q} = \frac{(3.9)(8.85\times10^{-14}\,F/cm)}{(20\times10^{-7}\,cm)(1.6\times10^{-19}\,C)}(3 - 0.83\ V)$$

$$\boxed{n = 2.34\times10^{12}\ cm^{-2}}$$

c) in inversion → $X_d = X_{dmax}$

$$X_{dmax} = \sqrt{\frac{2\epsilon_s(-2\phi_p)}{qN_a}} = \sqrt{\frac{2(11.7)(8.85\times10^{-14}\,F/cm)(-2(-0.42V))}{(1.6\times10^{-19})(10^{17}\,cm^{-3})}}$$

$$\boxed{X_{dmax} = 1.04\times10^{-5}\ cm} = \underline{1.04\ nm}$$

d) $V_T = V_{T0} + \gamma\left(\sqrt{-V_{BS} - 2\phi_p} - \sqrt{-2\phi_p}\right)$

cutoff → $V_{GS} \leq V_T = 3.0\ V$

$$V_{GS} = V_{T0} + \gamma\sqrt{-V_{BS} - 2\phi_p} - \gamma\sqrt{-2\phi_p}$$

now just solve for V_{BS}

(2) d) ~continued~

$$V_{GS} - V_{TO} + \gamma\sqrt{-2\phi_p} = \gamma\sqrt{-V_{BS} - 2\phi_p}$$

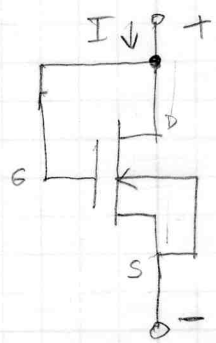
$$\left(\frac{V_{GS} - V_{TO} + \gamma\sqrt{-2\phi_p}}{\gamma}\right)^2 = -V_{BS} - 2\phi_p$$

$$V_{BS} = -2\phi_p - \left(\frac{V_{GS} - V_{TO} + \gamma\sqrt{-2\phi_p}}{\gamma}\right)^2$$

$$= -2(-0.42) - \left(\frac{3 - 0.83 + 1.05\sqrt{0.84}}{1.05}\right)^2$$

$$\boxed{V_{BS} = -9.76\ V}$$ very large reverse bias

(3) MOSFET diode → gate & drain shorted → $V_{GS} = V_{DS} = V$



for $V < V_T$ → no inversion layer ⇒ cutoff ($I = 0$)

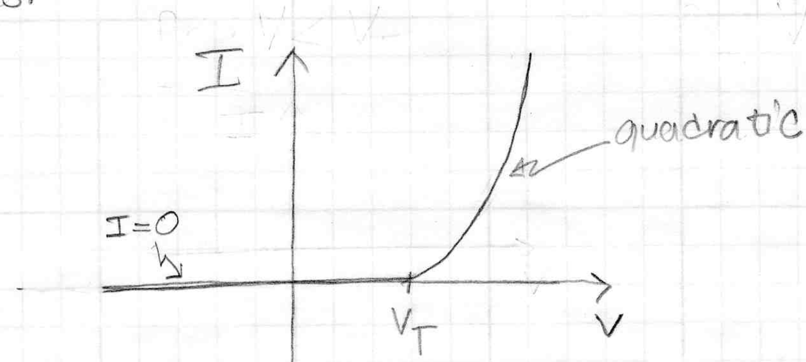
for $V > V_T$ → inversion regime

& since $V_{DS} = V_{GS}$, $V_{DS} > V_{GS} - V_T = V_{Dsat}$

↳ thus always in saturation

$$\therefore I = \frac{W}{2L}\mu_n C_{ox}(V - V_T)^2$$

Thus:



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(4) for graphs for 1), 2), 3) see attached pages

4) use transfer characteristics

pick $V_{DS} = 4V$. Define V_{TO} as V_{GS} such that $I_D = 5\mu A$

from the data, we get that $\boxed{V_{To} = 0.95V}$

model for I_D in saturation:

$$I_D = \frac{W}{2L} \mu_n C_{ox} (V_{GS} - V_T)^2$$

$W = 20\mu m$
$L = 3\mu m$

now just scale $\mu_n C_{ox}$ to get best fit to data.

First can pick a point (V_{GS}, I_D) & solve for $\mu_n C_{ox}$.

Choose $V_{GS} = 1.5V$. $(I_D = 65.3\mu A)$

solve for $\mu_n C_{ox} = \frac{2LI_D}{W(V_{GS}-V_T)^2} = 2\left(\frac{3\mu m}{20\mu m}\right)\frac{65.3\times10^{-6}}{(1.5-0.95)^2}$

$= 6.5\times10^{-5}\ C/V\cdot s$

Now can try using this value in model of I_D in saturation. Can then try varying this value a bit to try to find best fit to measured data. See attached graph for plot of $I_D - V_{GS}$ characteristics with different fits (using different values of $\mu_n C_{ox}$).

best fit → $\mu_n C_{ox} = 6.0\times10^{-5}\ C/V\cdot s$

5) from backgate characteristics, find V_T as function of V_{BS} (use V_{GS} value needed for $I_D = 5\mu A$).

V_{BS} [V]	V_T [V]
0	$0.95 = V_{To}$
-0.5	1.25
-1.0	1.47
-1.5	1.65
-2.0	1.85
-2.5	2.00
-3.0	72.0 (?)

Now plot V_T as a function of V_{BS}. Try to fit model to extract γ & ϕ_p

$$V_T = V_{To} + \gamma\left(\sqrt{-2\phi_p - V_{BS}} - \sqrt{-2\phi_p}\right)$$

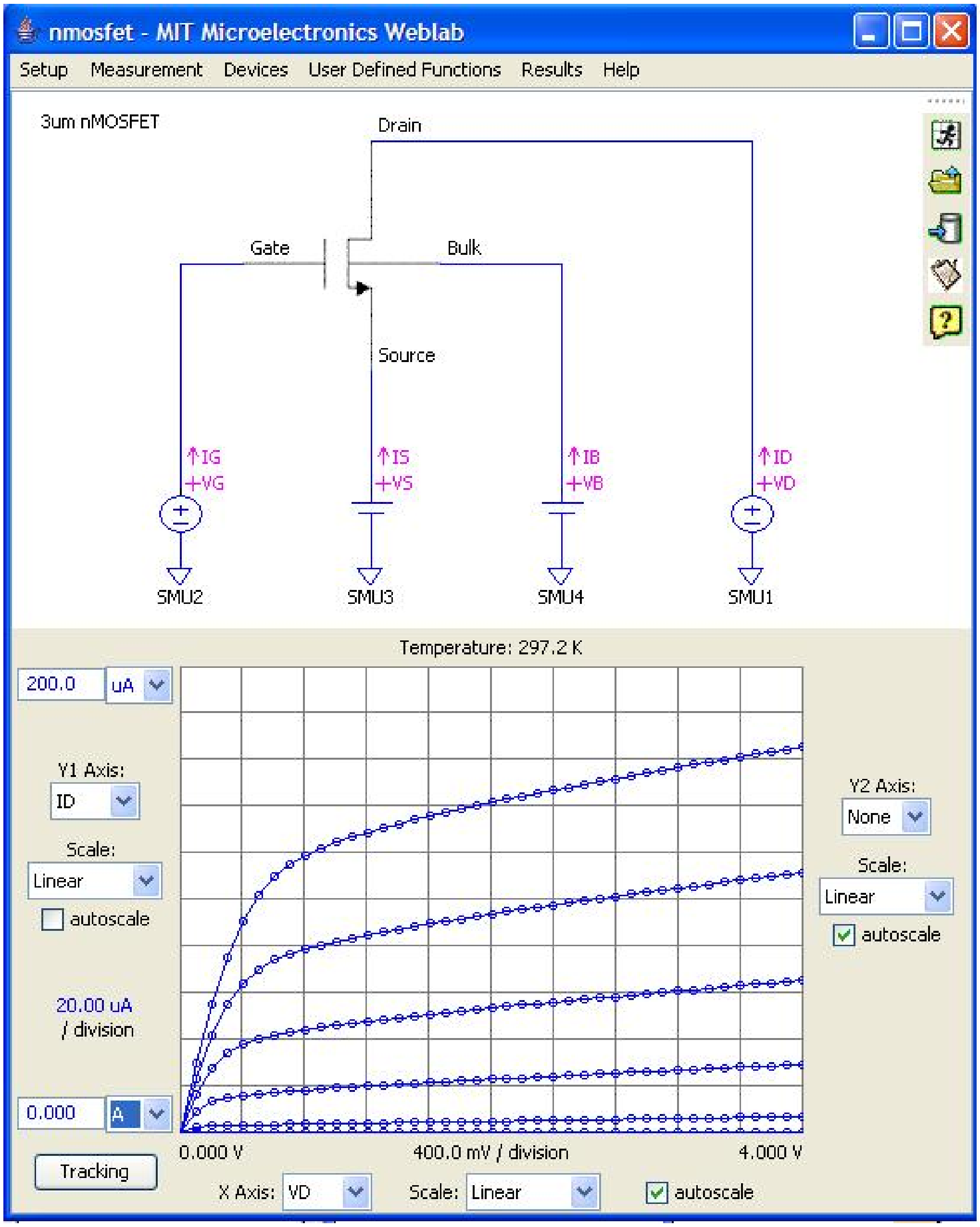
try values of ϕ_p between -0.3 - -0.5V range and solve for γ

$$\gamma = \frac{V_T - V_{To}}{\sqrt{-2\phi_p - V_{BS}} - \sqrt{-2\phi_p}}$$

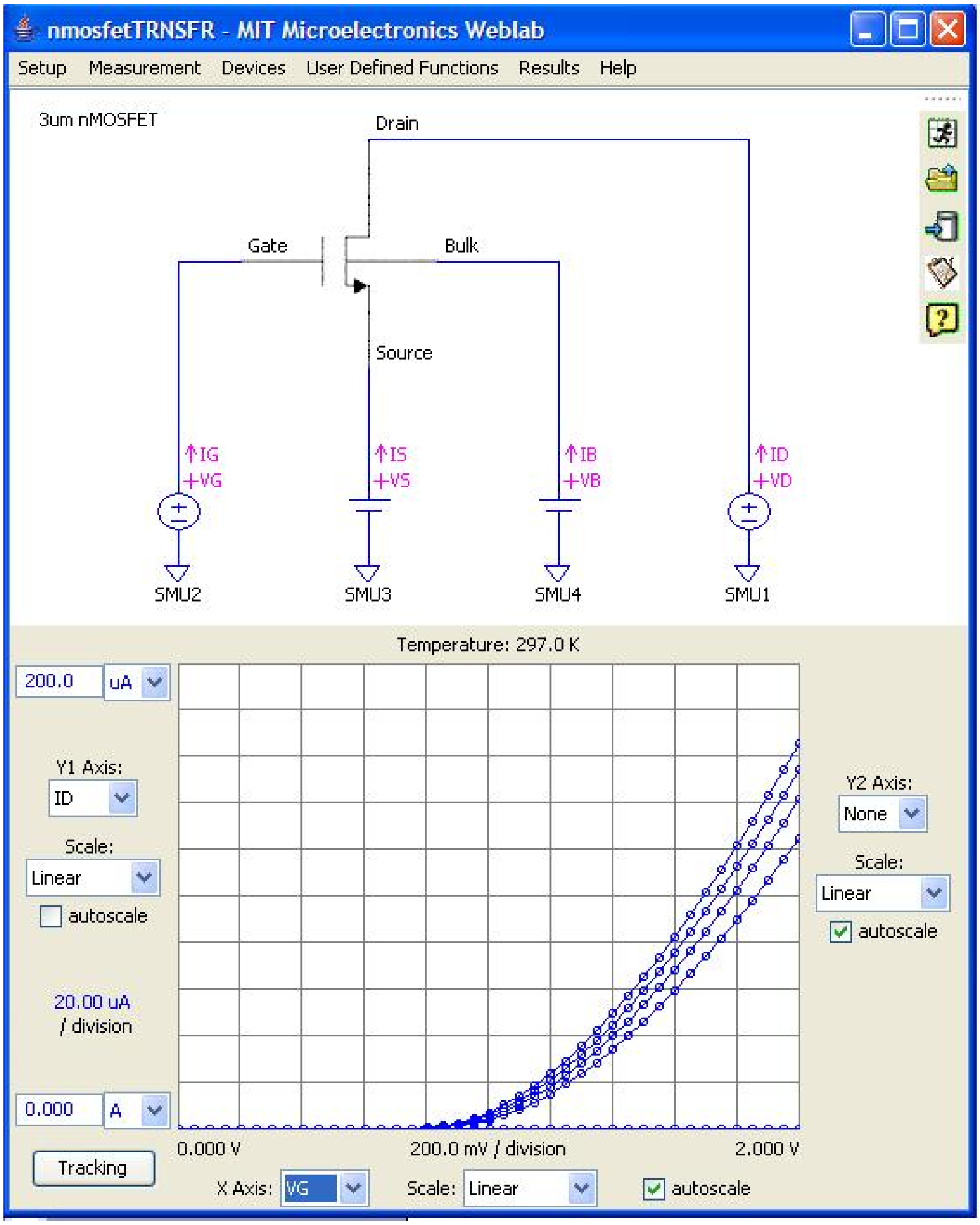
pick γ that is most consistent in whole data set (smallest variation) see table.

best fit: $\boxed{\phi_p = 0.3V \quad \gamma = 1.07\ V^{1/2}}$

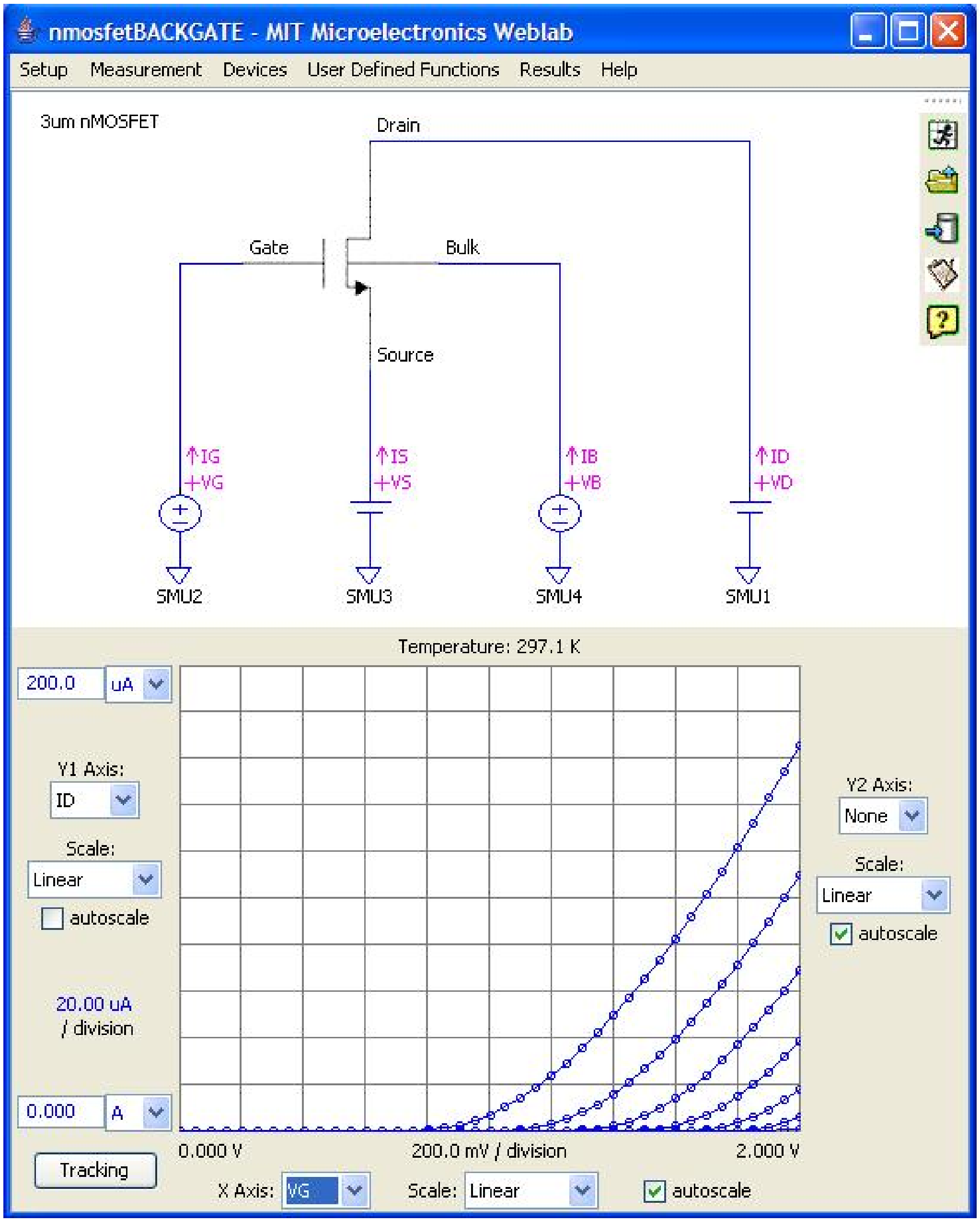
(1) Output Characteristics



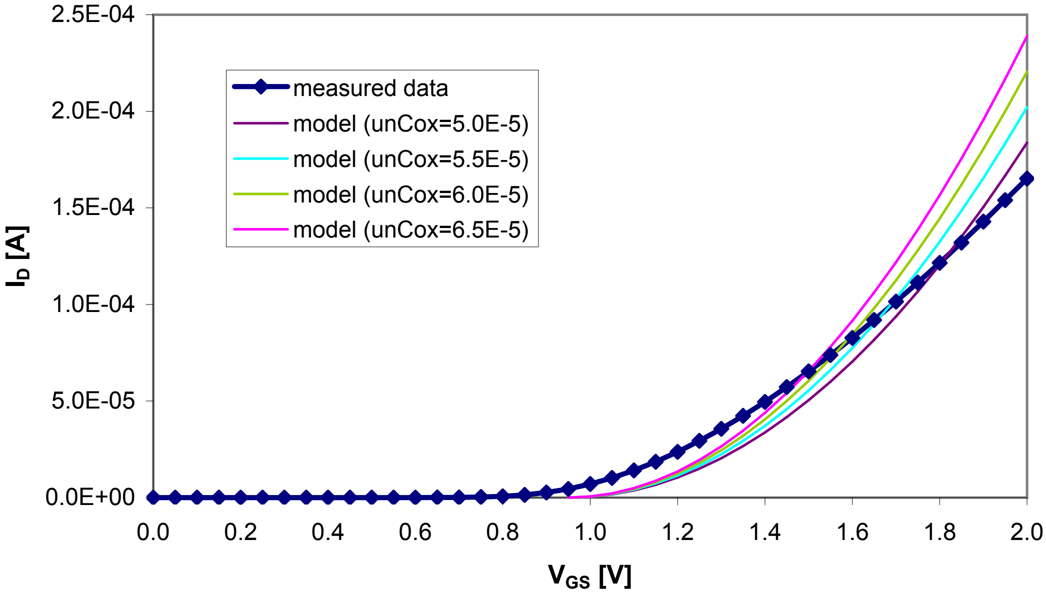
(2) Transfer Characteristics



(3) Backgate Characteristics

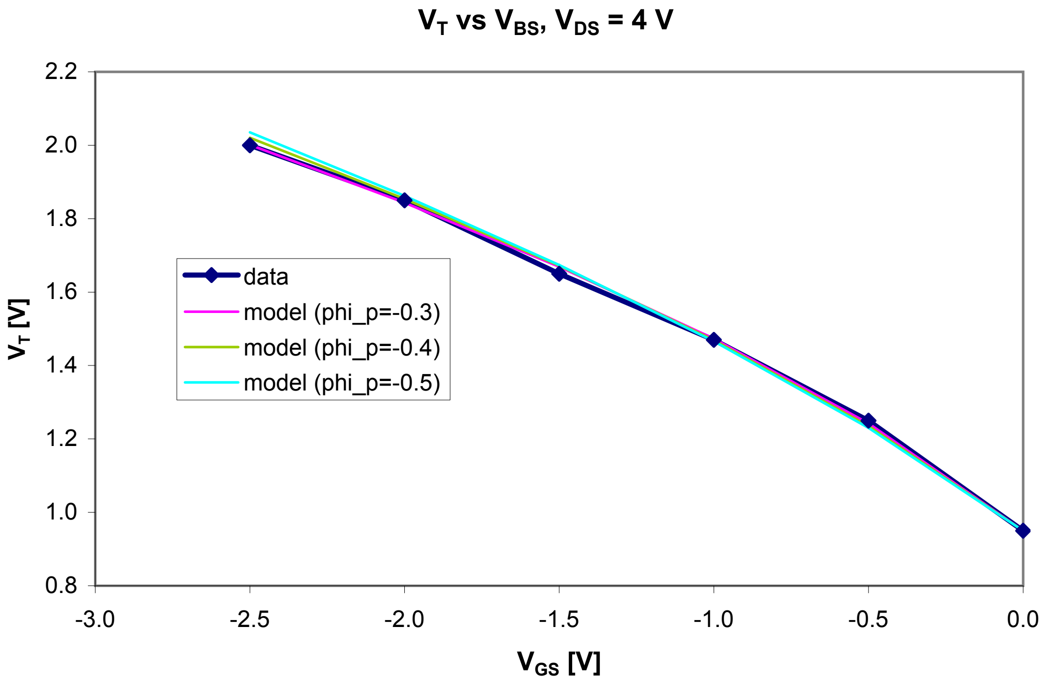


MOSFET Transfer Characteristics, V_{DS} = 4 V (V_{BS} = 0)



Extraction of γ and φ_p from backgate characteristics

		φ_p [V]	φ_p [V]	φ_p [V]	φ_p [V]	φ_p [V]
V_{BS} [V]	V_T [V]	-0.30	-0.35	-0.40	-0.45	-0.50
0.0	0.95					
-0.5	1.25	1.094043	1.159263	1.220762	1.27914	1.334847
-1.0	1.47	1.060544	1.11306	1.162755	1.210086	1.255391
-1.5	1.65	1.037743	1.08262	1.125134	1.165676	1.204531
-2.0	1.85	1.074172	1.115922	1.155486	1.19323	1.229423
-2.5	2	1.064817	1.102716	1.138627	1.172889	1.205748
average	**γ:**	1.066264	1.114716	1.160553	1.204204	1.245988
std dev	**σ:**	0.020509	0.028119	0.036711	0.045375	0.053844



V_T vs V_BS, V_DS = 4 V
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
V_T [V]
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0.0
V_GS [V]
data
model (phi_p=-0.3)
model (phi_p=-0.4)
model (phi_p=-0.5)