Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

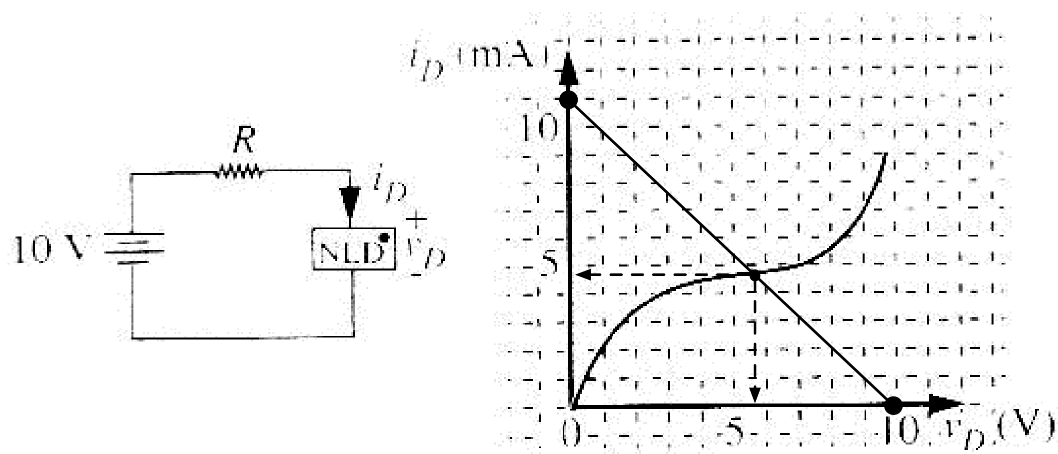
6.002 – Electronic Circuits
Fall 2005

Homework 4 Solutions

Exercise 4-1: Do KVL around the circuit to get $-10 + i_d(910) + v_d = 0$. Solve for i_d.

$$i_d = \frac{10 - v_d}{910}[A] \tag{1}$$
$$i_d = 11 - 1.1v_d[mA] \tag{2}$$

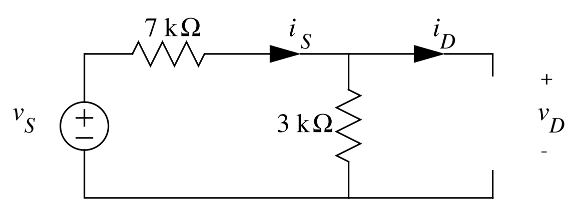
Plot this i_d vs v_d relation on same plot of the i_d vs v_d characteristics of the non linear device. The intersection point is the operating point of the circuit, which is approximately 4.8 [mA] and 5.8 [V].



Exercise 4-2: Worst case power consumption is that the MOSFET is on and power is lost through the R_L and R_{ON} resistor in series.

$$\begin{aligned} R_L + R_{ON} &= 10 + 1[k\Omega] && (3)\\ &= 11[k\Omega] && (4)\\ P &= IV && (5)\\ &= \frac{V^2}{R} && (6)\\ &= \frac{5^2}{11}[mWatts] && (7)\\ &= 2.3[mWatts] && (8) \end{aligned}$$

Problem 4.1: The zener diode is off when $-4.5 < v_d < 0.6[V]$. The circuit looks like



When the diode is off, solving for v_s in terms of v_d is a simple voltage divider relationship .

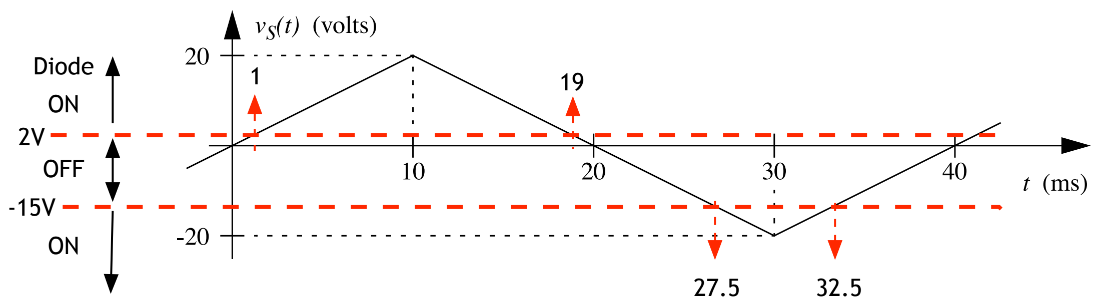
$$\begin{aligned} v_d &= \frac{3}{3+7}v_s && (9)\\ &= \frac{3}{10}v_s && (10) \end{aligned}$$

Now solving for the input conditions when the diode is off becomes a simple substitution.

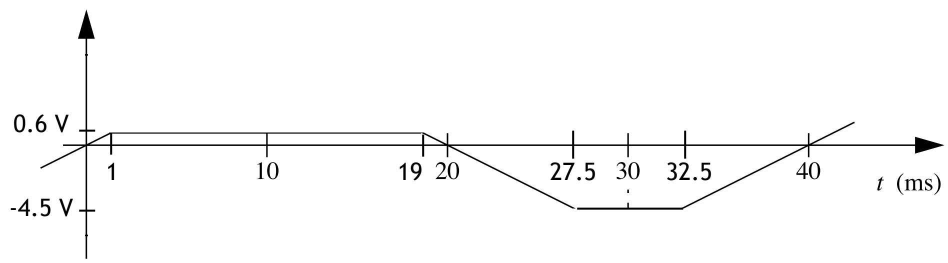
$$\begin{aligned} -4.5 < &\quad v_d &< 0.6[V] && (11)\\ -4.5 < &\quad \tfrac{3}{10}v_s &< 0.6[V] && (12)\\ -15 < &\quad v_s &< 2[V] && (13) \end{aligned}$$

The diode is on when $v_s < -15$ [V] or $v_s > 2$ [V].

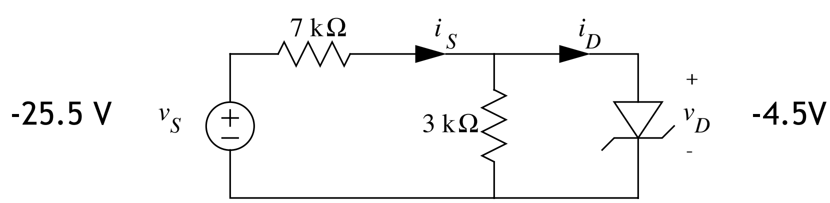
a) Looking at the graph, we see that the slope of the graph is either + or - 2 [V/ms]. Therefore, the diode begins in the off state at 0 [ms] until v_s reaches 2 [V] at 1 [ms]. The diode stays on until $v_s(t) = 2$ [V] at 19 [ms]. The diode stays off until $v_s(t) =$ -15 [V] at 27.5 [ms]. The diode stays on until $v_s(t) =$ -15 [V] at 32.5 [ms] and stays off until 40 [ms].



b) Sketch $v_d(t)$



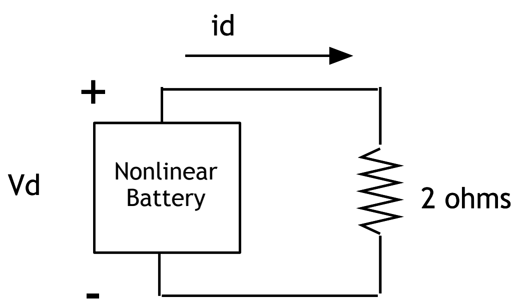
c) When $v_s = -25.5$ volts, the diode is on.

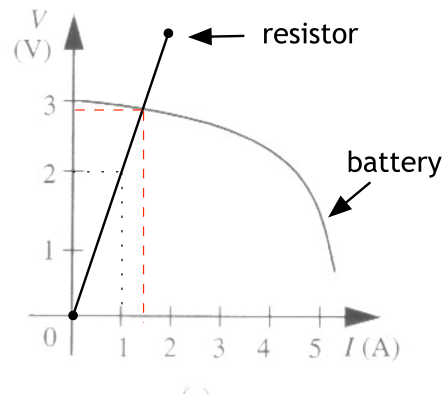


$$
\begin{aligned}
i_s &= \frac{v_s - v_d}{7}[mA] && (14)\\
&= \frac{-25.5 - -4.5}{7}[mA] && (15)\\
&= -3[mA] && (16)
\end{aligned}
$$

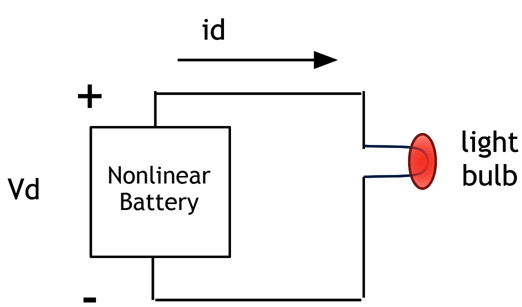
Problem 4.2:

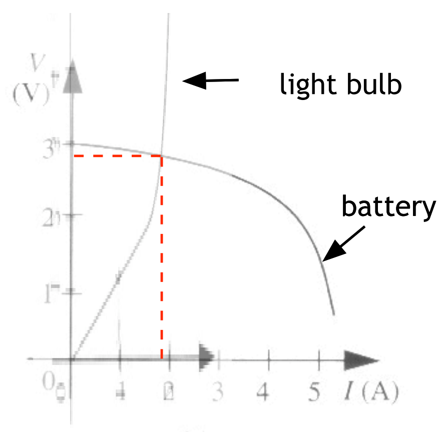
a) Note that $v_d = i_d R$, or $v_d = i_d 2$. Superimpose IV plots of the resistor and non linear battery to find the operating point $v_d \approx 2.8[V]$ and $i_d \approx 1.4[A]$





b) Superimpose IV plots of battery and lightbulb to find operating point $v_d \approx 2.7[V]$ and $i_d \approx 1.8[A]$





c) For the battery $i_d = 0[A], v_d = 3[V]$. On the graph, the battery is pretty much linear up until the point $i_d = 2[A], v_d = 2.7[V]$.

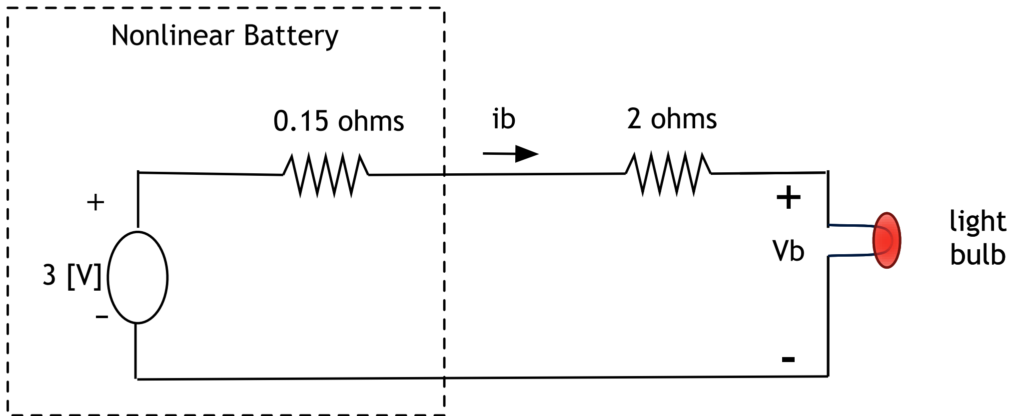
$$\frac{dv}{di} = \frac{2.7 - 3[V]}{2 - 0[A]} \tag{17}$$

$$= -0.15\frac{[V]}{[A]} \tag{18}$$

$$v_d = (\frac{dv}{di})i_d + v_{oc} \tag{19}$$

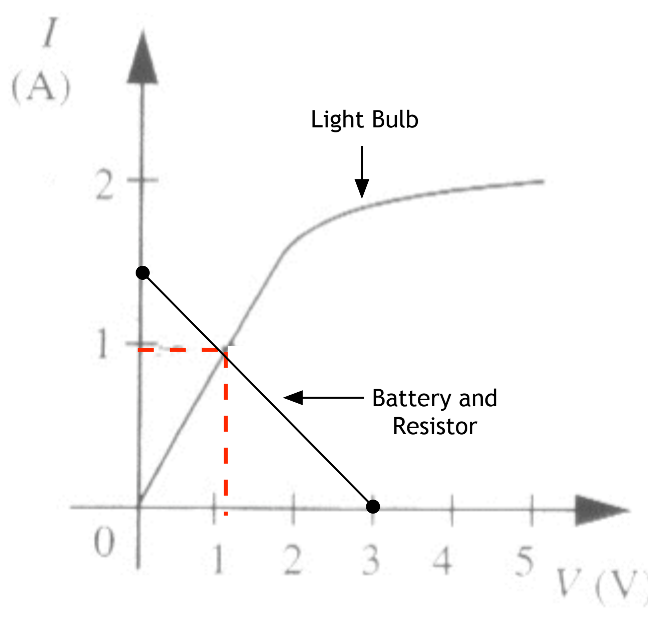
$$v_d = -0.15 i_d + 3[V] \tag{20}$$

d) The circuit now looks like the figure below.



Using KVL the circuit equation becomes

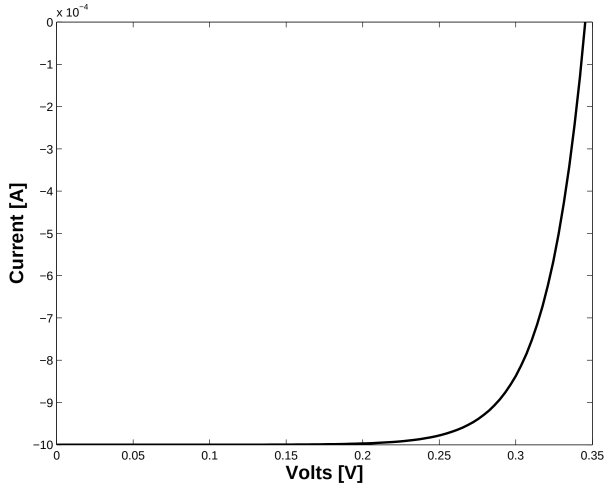
$$-3V + 2i_b + 0.15 i_b + v_b = 0 \tag{21}$$
$$v_b = 3 - 2.15 i_b [V] \tag{22}$$



The operating point is $v_b \approx 1.3[V]$ and $i_b \approx 1[A]$. The battery voltage is $3[V] - 1[A] * 0.15[Ohms] = 2.85[V]$.

Problem 4.3:

a) Plot i-v



b) Expression for power in terms of solar cell

$$
\begin{aligned}
P &= IV && (23)\\
&= (I_1(e^{\frac{v}{v_{th}}} - 1) - I_2)v && (24)\\
&= I_1 e^{\frac{v}{v_{th}}} v - (I_1 + I_2)v && (25)
\end{aligned}
$$

To maximize power, take derivative of power with respect to voltage and set equal to zero.

$$
\begin{aligned}
\frac{dP}{dV} &= 0 && (26)\\
&= I_1 e^{\frac{v}{v_{th}}} + \frac{I_1}{v_{th}} e^{\frac{v}{v_{th}}} v - (I_1 + I_2) && (27)\\
\frac{I_1 + I_2}{I_1} &= e^{\frac{v}{v_{th}}} (1 + \frac{v}{v_{th}}) && (28)
\end{aligned}
$$

The exact solution for v in this equation can be computed using the Lambert W function, that solves for w in equations of the form $we^w = x$.

Let $x = \frac{v}{v_{th}}$ and $c = \frac{I_1+I_2}{I_1}$.

$$
\begin{aligned}
c &= e^x(1 + x) && (29)\\
ec &= e^{(1+x)}(1 + x) && (30)\\
1 + x &= lambertW(ec) && (31)\\
x &= lambertW(ec) - 1 && (32)
\end{aligned}
$$

Plug back in for x and c.

$$\frac{v}{v_{th}} = lambertW(e\frac{I_1 + I_2}{I_1}) - 1 \quad (33)$$

$$v = v_{th}[lambertW(e\frac{I_1 + I_2}{I_1}) - 1] \quad (34)$$

$$v = 0.283[V] \quad (35)$$

Solar cell power:

$$P = IV \quad (36)$$

$$= (I_1(e^{\frac{v}{v_{th}}} - 1) - I_2)v \quad (37)$$

$$= (10^{-9}(e^{\frac{0.283}{0.025}} - 1) - 10^{-3})(0.283)[Watts] \quad (38)$$

$$= -2.6 * 10^{-4}[Watts] \quad (39)$$

Solar cell delivers $2.6 * 10^{-4}[Watts]$ to the resistor.

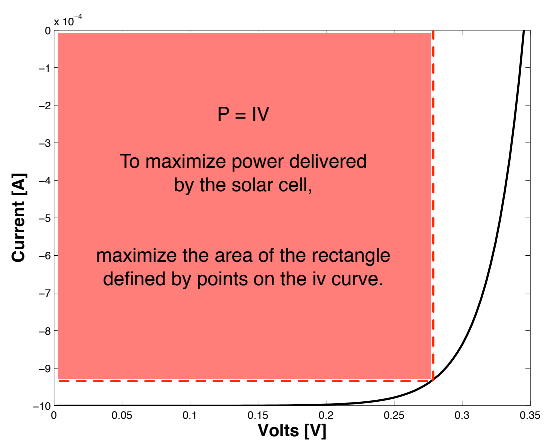
$$P = V^2/R \quad (40)$$

$$R = \frac{V^2}{P} \quad (41)$$

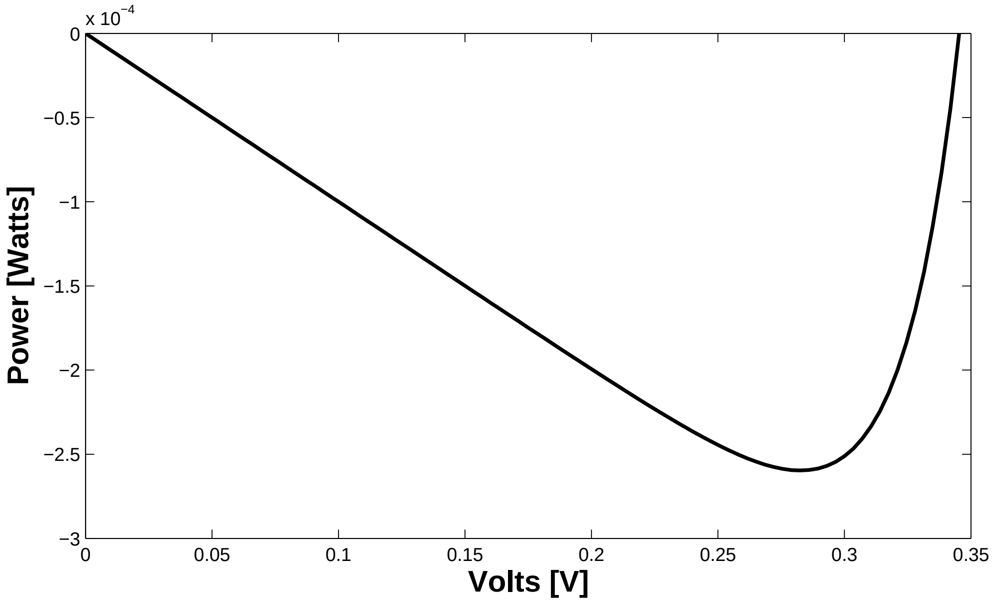
$$R = \frac{0.283^2}{2.6 * 10^{-4}}[Ohms] \quad (42)$$

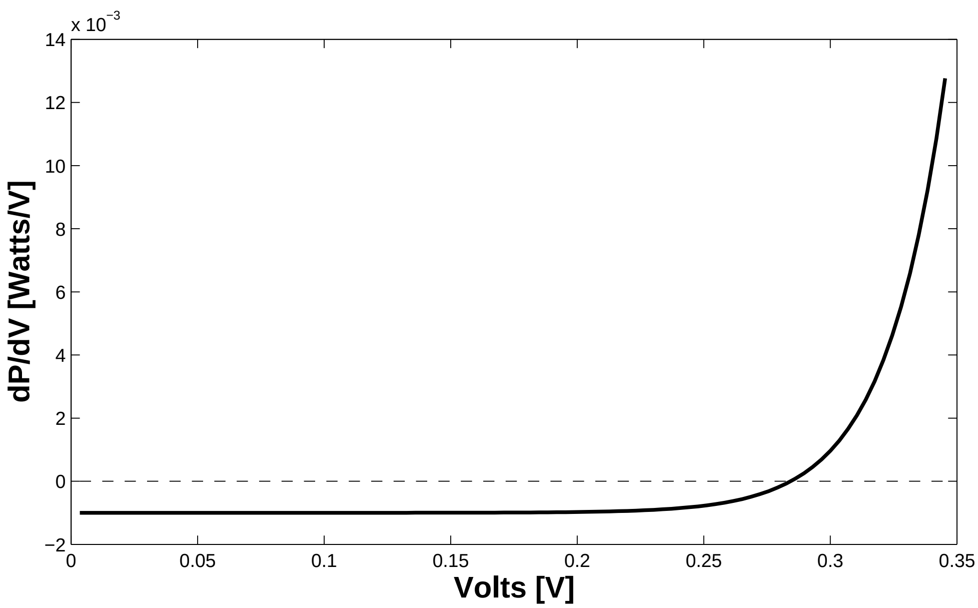
$$R = 308[Ohms] \quad (43)$$

Note that maximizing power is equivalent to finding the largest area of a rectangle made of the iv plot in the figure in part (a).



Another way to solve this problem is to plot the power with respect to voltage and look for a minimum value. This can be done by simply eyeing the graph, or actually plotting the derivative of power with respect to voltage. This will give you the same answer for v if you zoom in and find the voltage which corresponds to $\frac{dP}{dV} = 0$.





c)

$$
\begin{aligned}
P &= I^2 R && (44)\\
&= (I_1(e^{\frac{v}{v_{th}}} - 1) - I_2)^2 R && (45)\\
&= (I_1^2(e^{\frac{v}{v_{th}}} - 1)^2 - 2I_1(e^{\frac{v}{v_{th}}} - 1)I_2 + I_2^2)R && (46)
\end{aligned}
$$

Take the derivative of P with respect to I_2.

$$
\begin{aligned}
\frac{dP}{dI_2} &= (0 - 2I_1(e^{\frac{v}{v_{th}}} - 1) + 2I_2)R && (47)\\
\Delta P &= (\frac{dP}{dI_2}|_{v,I_1,I_2})(\Delta I_2) && (48)\\
&= [(-2I_1(e^{\frac{v}{v_{th}}} - 1) + 2I_2)R](\Delta I_2) && (49)\\
&= [(-2 * 10^{-9}(e^{\frac{0.2826}{0.025}} - 1) + 2 * 10^{-3})308](\Delta I_2)[Watts] && (50)\\
&= 0.616\Delta I_2[Watts] && (51)
\end{aligned}
$$

If I_2 can change by 10%, then

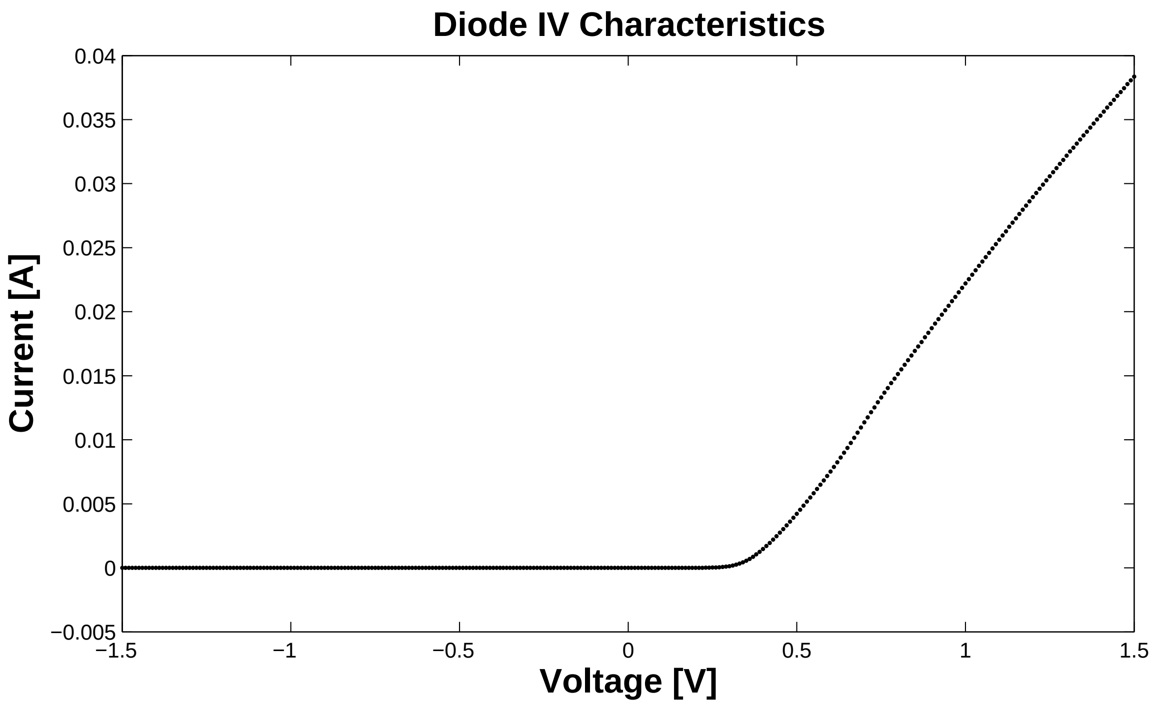
$$
\begin{aligned}
\Delta I_2 &= .1 * 10^{-3}[A] && (52)\\
&= 10^{-4}[A] && (53)\\
\Delta P &= 0.616 * 10^{-4}[Watts] && (54)\\
&= 6.16 * 10^{-5}[Watts] && (55)
\end{aligned}
$$

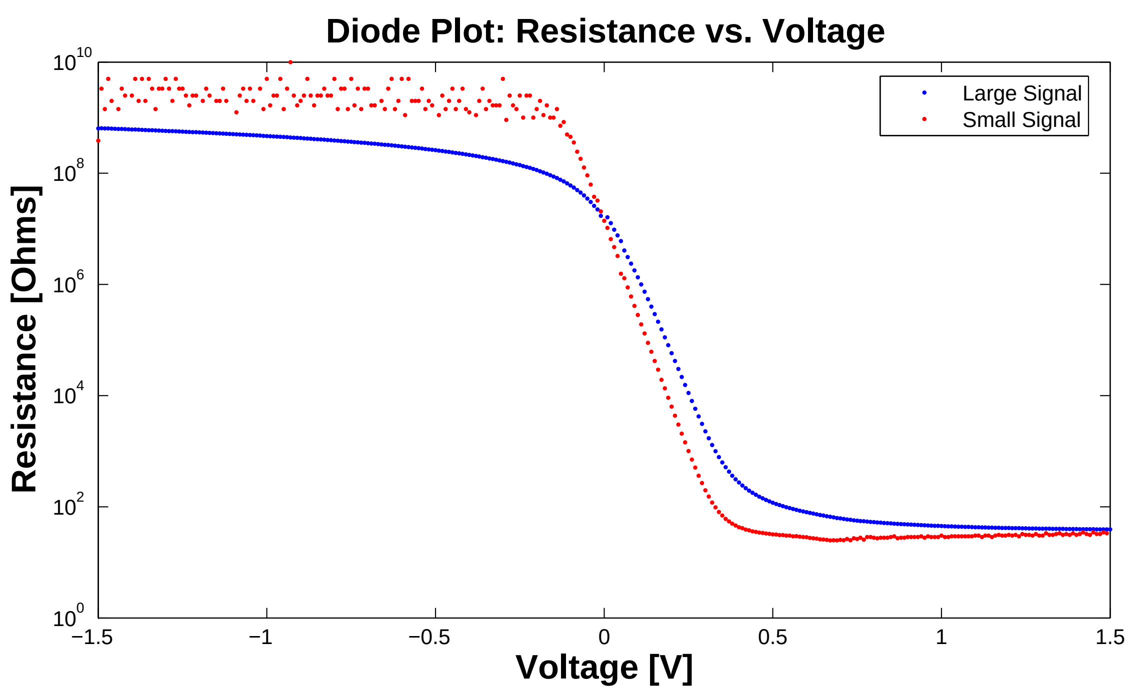
Problem 4.4: a) and b) See attached plots

c) The plots follow the same trend of having high resistance when the diode is off and low resistance when the diode turns on. Notice that when the diode is off (negative voltage), it essentially acts like an open circuit and does not support a current. Therefore, we would expect the resistance in this region to be high and flat. Likewise, when the diode is on (high enough positive voltage), the diode acts like a short and supports any current. We expect the resistance of the diode in this region to be flat and low.

In the large signal model, the resistance is calculated by $\frac{V}{I}$ so the plot of resistance varies in these regions. However, in the small signal model, we see that the plot of resistance is flat (albeit some noise) these regions.

d) The simplest way to do this measurement in 6.002 lab is to use the function generator and multimeter. First, connect the function generator as a voltage source across the diode. Then connect the multimeter in series with the diode to measure the current. Program the function generator to supply -1.5[V], take a measurement, and then increment this voltage by 10 [mV] until 1.5 [V]. This is very time consuming measurement and would require you to write down each data point by hand.


Diode IV Characteristics
Current [A]
Voltage [V]
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
−0.005
−1.5
−1
−0.5
0
0.5
1
1.5


Diode Plot: Resistance vs. Voltage
Resistance [Ohms]
Voltage [V]
Large Signal
Small Signal
10^10
10^8
10^6
10^4
10^2
10^0
−1.5
−1
−0.5
0
0.5
1
1.5