Introduction to Radar Signal Processing

Craig Heinselman
University of Alaska Fairbanks

Most of the content from:
Josh Semeter
Boston University

A Few Conventions/Concepts

$$j = \sqrt{-1}$$

Euler's Formula

$$e^{j\theta} = (\cos\theta) + j(\sin\theta)$$

Convolution

$$y(t) = \int_{-\infty}^{\infty} h(u)x(t-u)du = h * x$$

Complex exponentials are Eigen functions of linear, time-invariant systems (of the convolution operator). This is why systems engineers like them so much!

$$\mathrm{Hf} = \lambda \mathrm{f}$$

Autocorrelation

$$r_{xx}(\tau) = E[x^*(t)x(t+\tau)]$$

Wiener-Khinchin theorem

The Autocorrelation function and the Power spectral density function make a Fourier Transform pair for a wide-sense-stationary random process (even though the Fourier Transform of the process itself does not exist).

Fourier Transforms

$$h(t) = \int_{-\infty}^{\infty} H(f) e^{j2\pi f t} df \qquad H(f) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi f t} dt$$

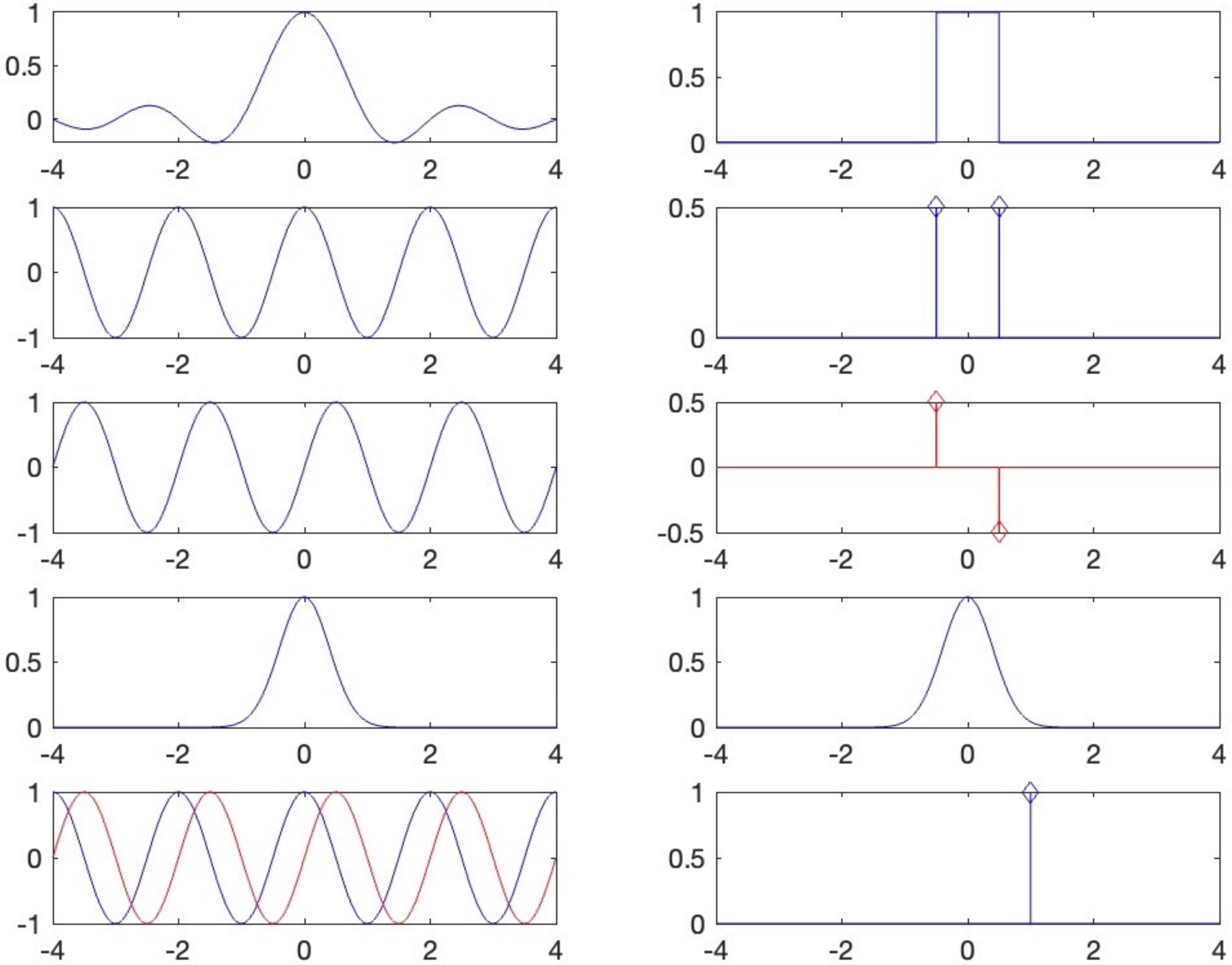
$$sinc(t) = \frac{\sin \pi t}{\pi t} \Leftrightarrow \Pi(f)$$

$$\cos(\pi t) \Leftrightarrow \frac{1}{2}\delta(f + \frac{1}{2}) + \frac{1}{2}\delta(f - \frac{1}{2})$$

$$\sin(\pi t) \Leftrightarrow \frac{j}{2}\delta(f + \frac{1}{2}) - \frac{j}{2}\delta(f - \frac{1}{2})$$

$$e^{-\pi t^2} \Leftrightarrow e^{-\pi f^2}$$

$$e^{j2\pi f_0 t} \Leftrightarrow \delta(f - f_0)$$



Fourier transform properties

Convolution

$$h(t) * x(t) \Leftrightarrow H(f)X(f)$$

$$h(t)x(t) \Leftrightarrow H(f) * X(f)$$

Shifts

$$h(t-t_0) \Leftrightarrow e^{-j2\pi t_0 f} H(f)$$

$$e^{j2\pi f_0 t} h(t) \Leftrightarrow H(f-f_0)$$

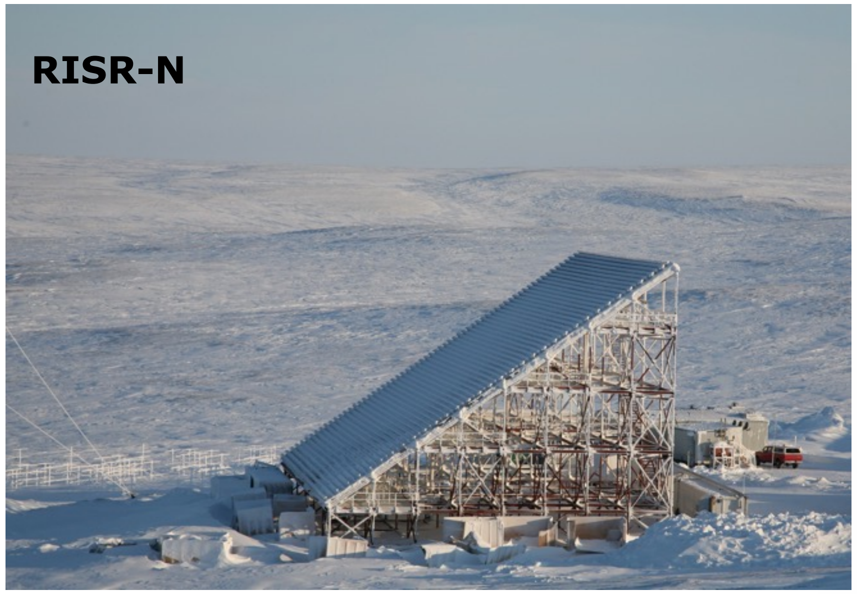
Similarity

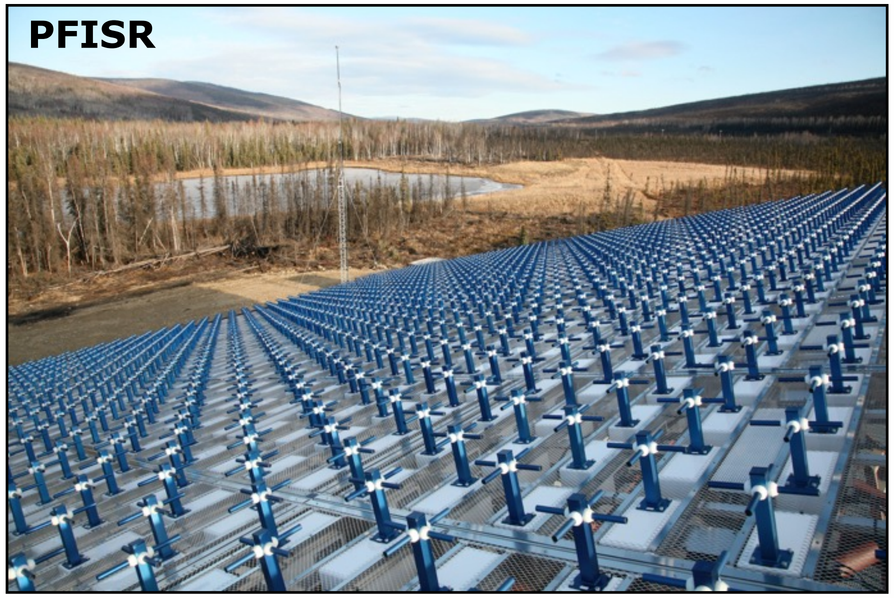
$$h(at) \Leftrightarrow \frac{1}{|a|} H(\frac{f}{a})$$

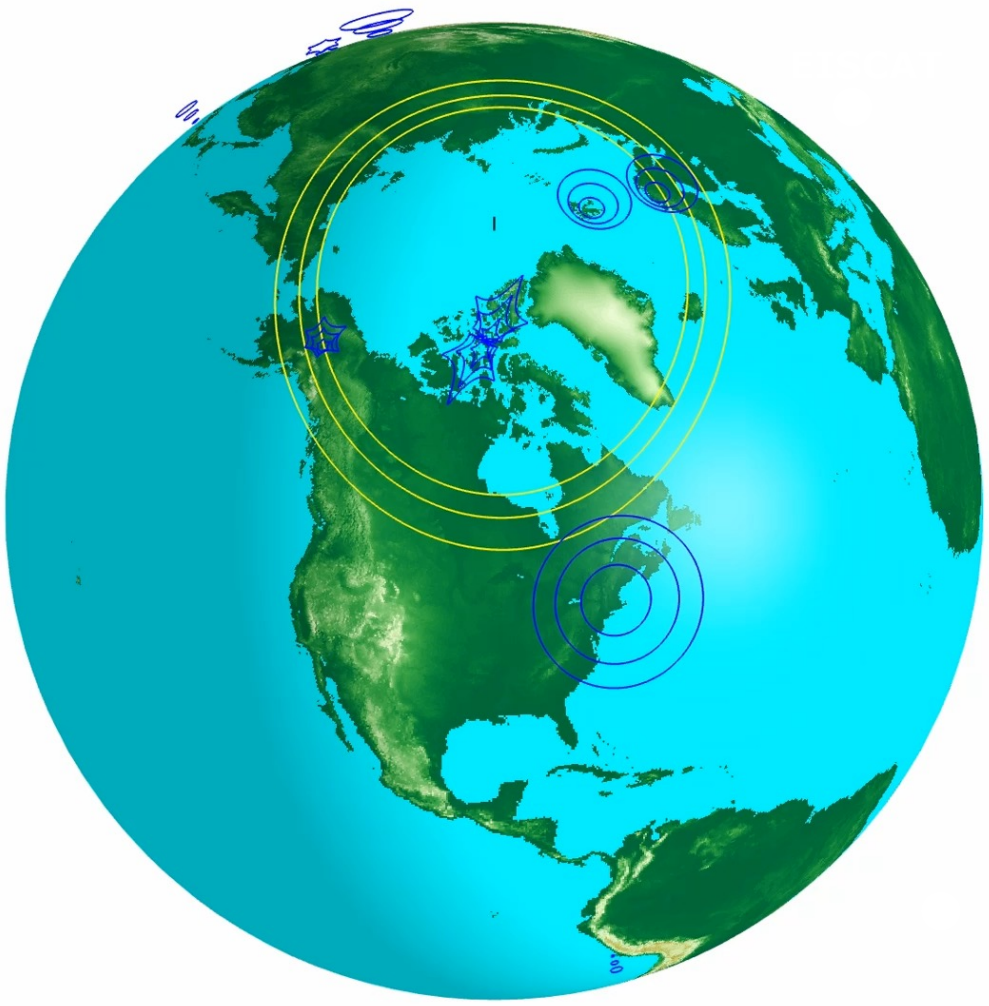
Linearity

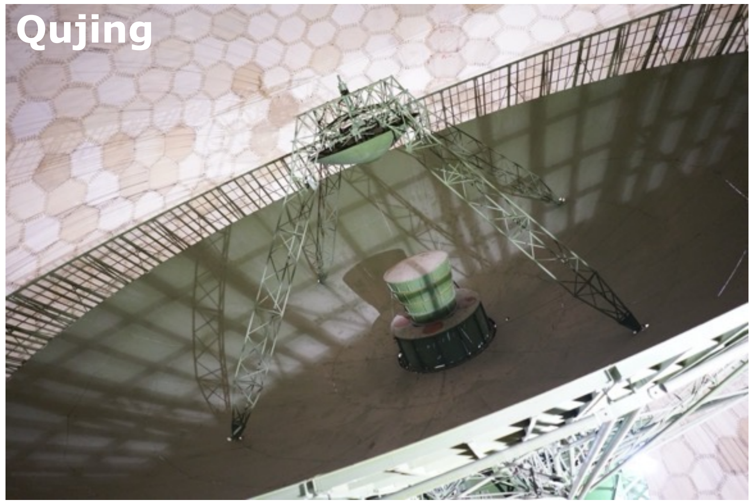
$$ag(t) + bh(t) \Leftrightarrow aG(f) + bH(f)$$

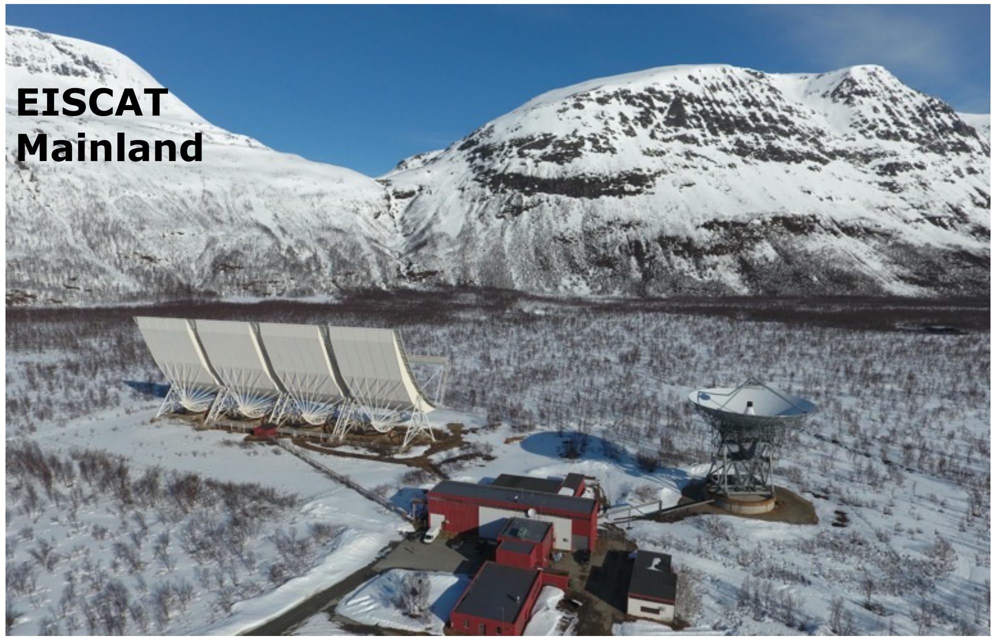
Incoherent Scatter Radars (ISRs)



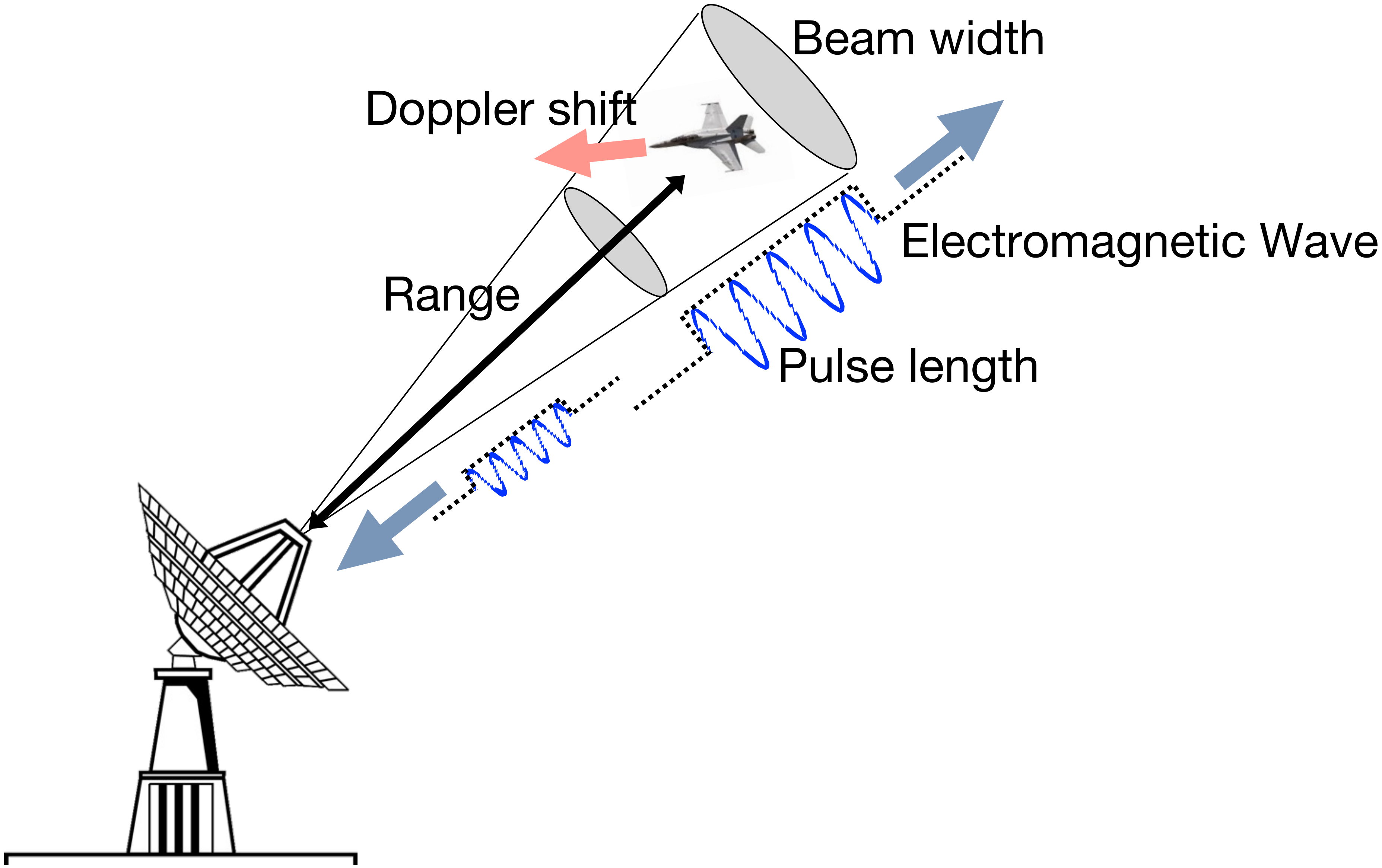




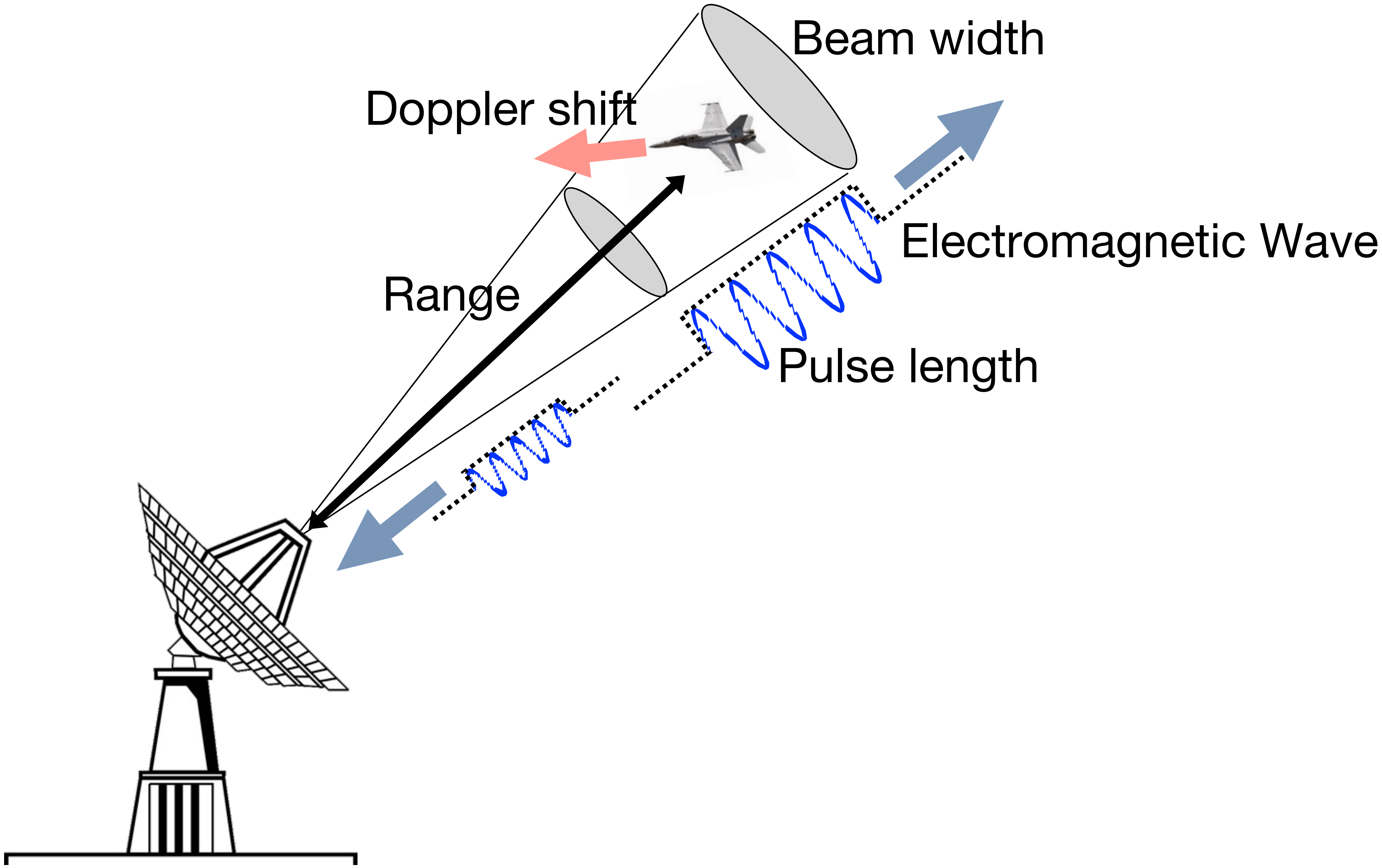




Pulse Doppler Radar



Pulse Doppler Radar



Time delay for the pulse echo to return -> range
Frequency shift of the echo -> velocity component

Traveling Waves

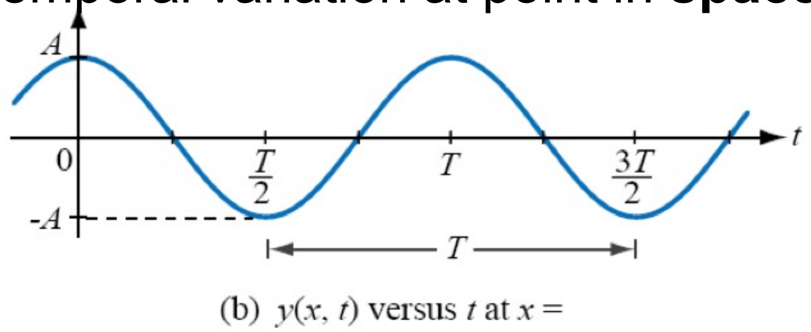
Traveling wave, 1D: $y(x,t) = A\cos(\omega t - kx)$

Angular velocity (radians/s): $\omega = 2\pi f = 2\pi/T$

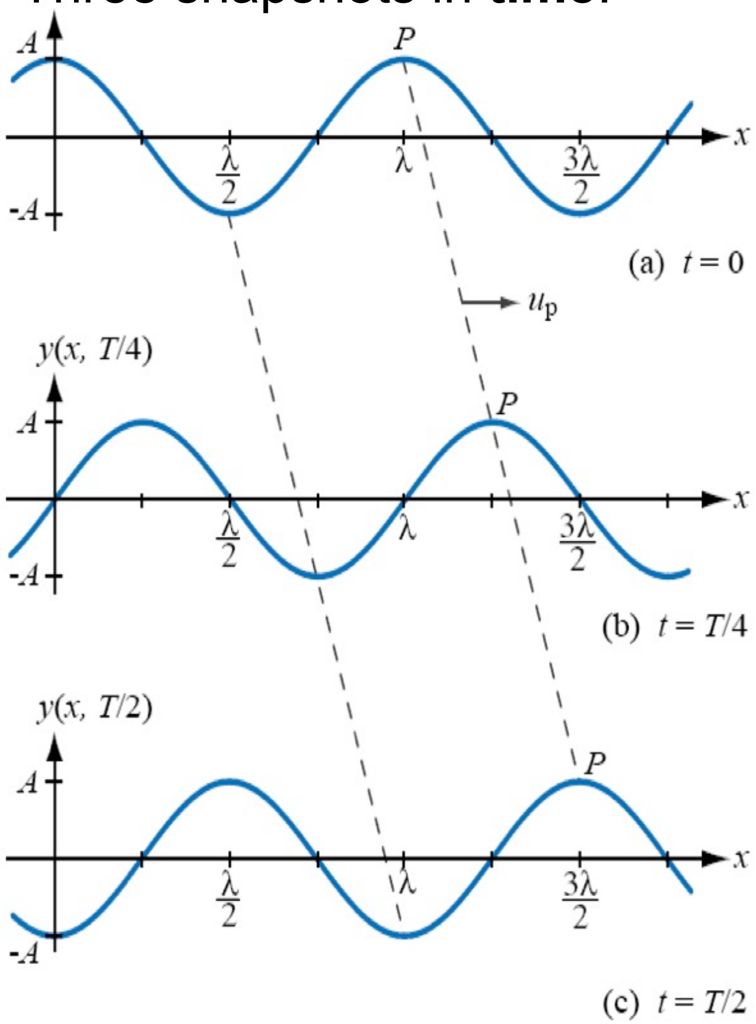
Wave number (spatial frequency): $k = 2\pi/\lambda$

Phase velocity (c in a vacuum): $u_p = \omega/k$

Temporal variation at point in **space**:



Three snapshots in **time**:



The velocity of a point on the wave can be found by setting $\omega t - kx$ = constant. Taking the time derivative we obtain the ***phase velocity,***

$$u_p = \frac{dx}{dt} = \frac{\omega}{k}$$

The functional relationship between ω and k is called a ***dispersion relation.*** It appears ubiquitously in the study of wave phenomena.

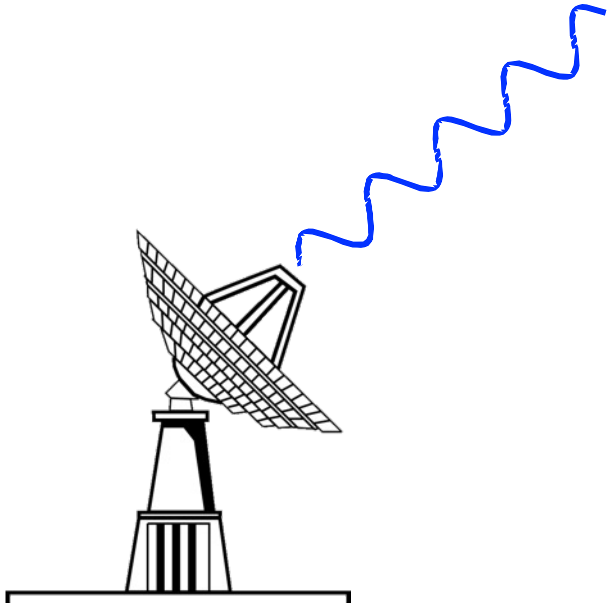
The simplest dispersion relation for an EM wave describes its propagation through free space,

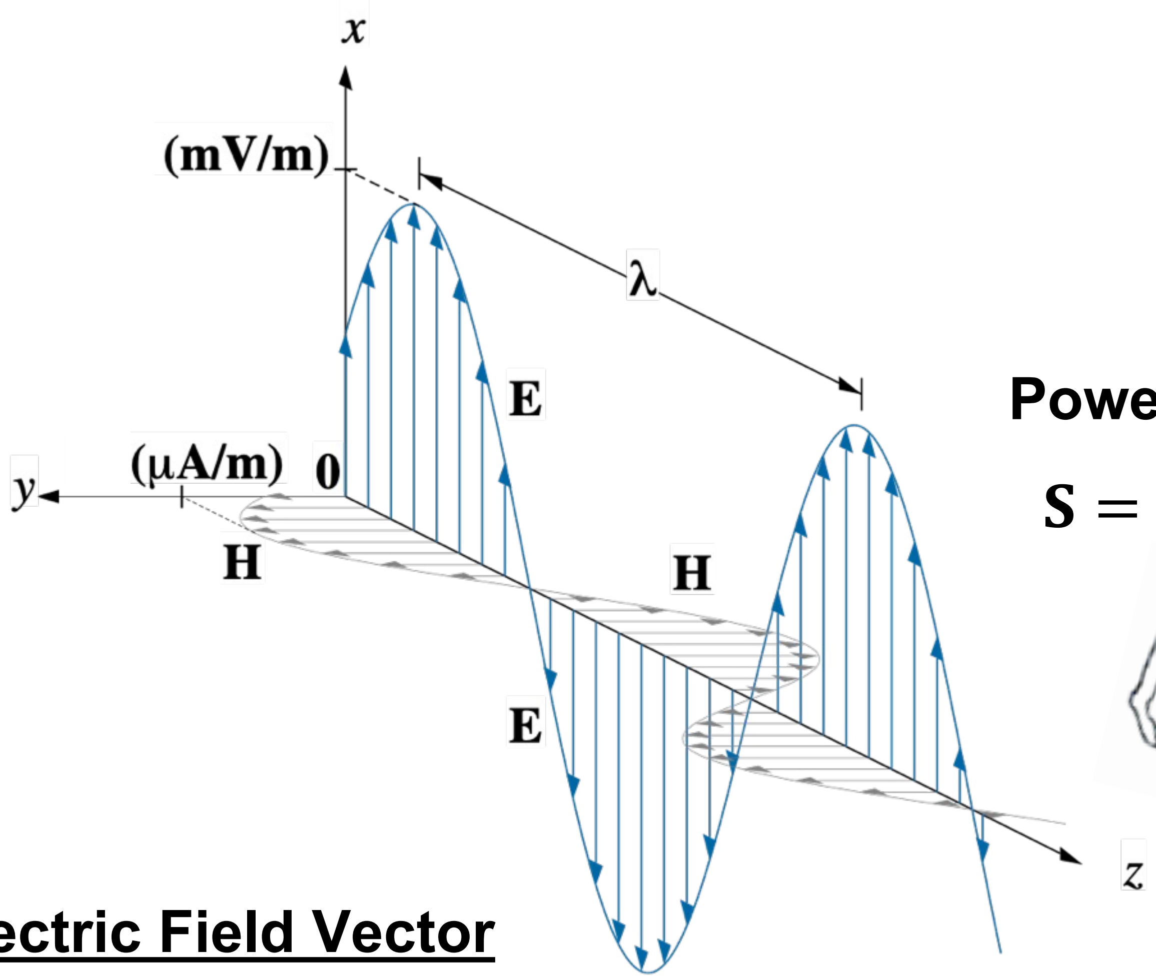
$$\omega = ck$$

where $c \sim 3 \times 10^8 \mathrm{m/s}$. We will encounter more complicated dispersion relations soon!

Transverse Electromagnetic (TEM) Waves

Radars transmit TEM waves and measure the scattered radiation from a target



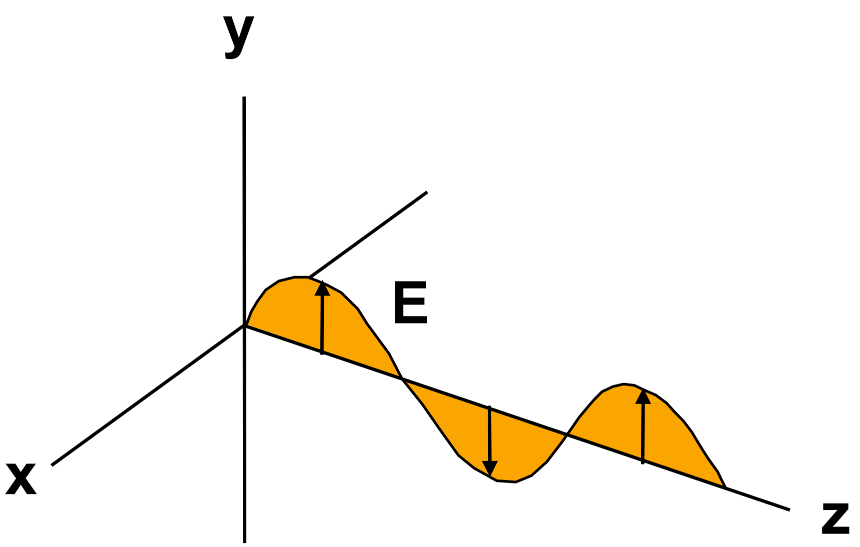


Power flow:

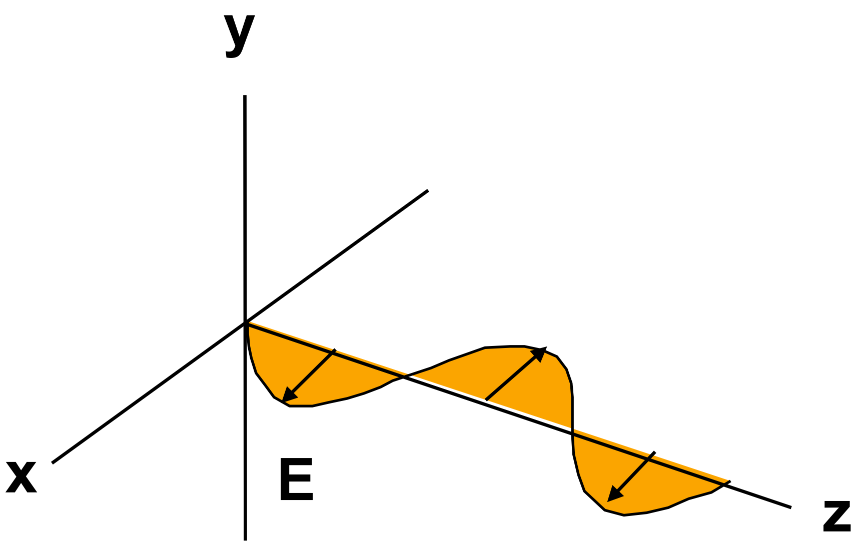
$$\mathbf{S} = \mathbf{E} \times \mathbf{H} \; W/m^2$$

Polarization: Orientation of the Electric Field Vector

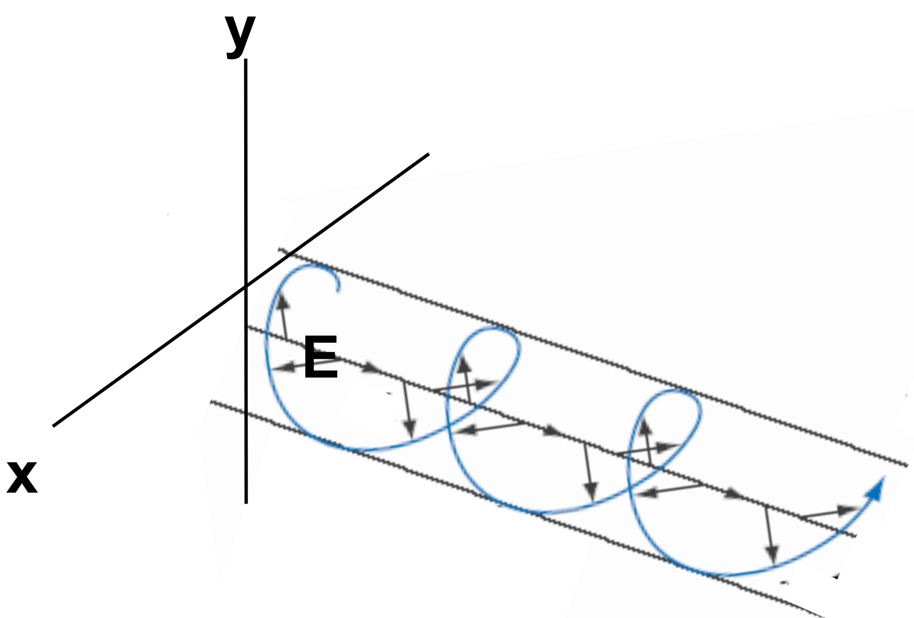
Vertical Polarization



Horizontal Polarization



Circular Polarization

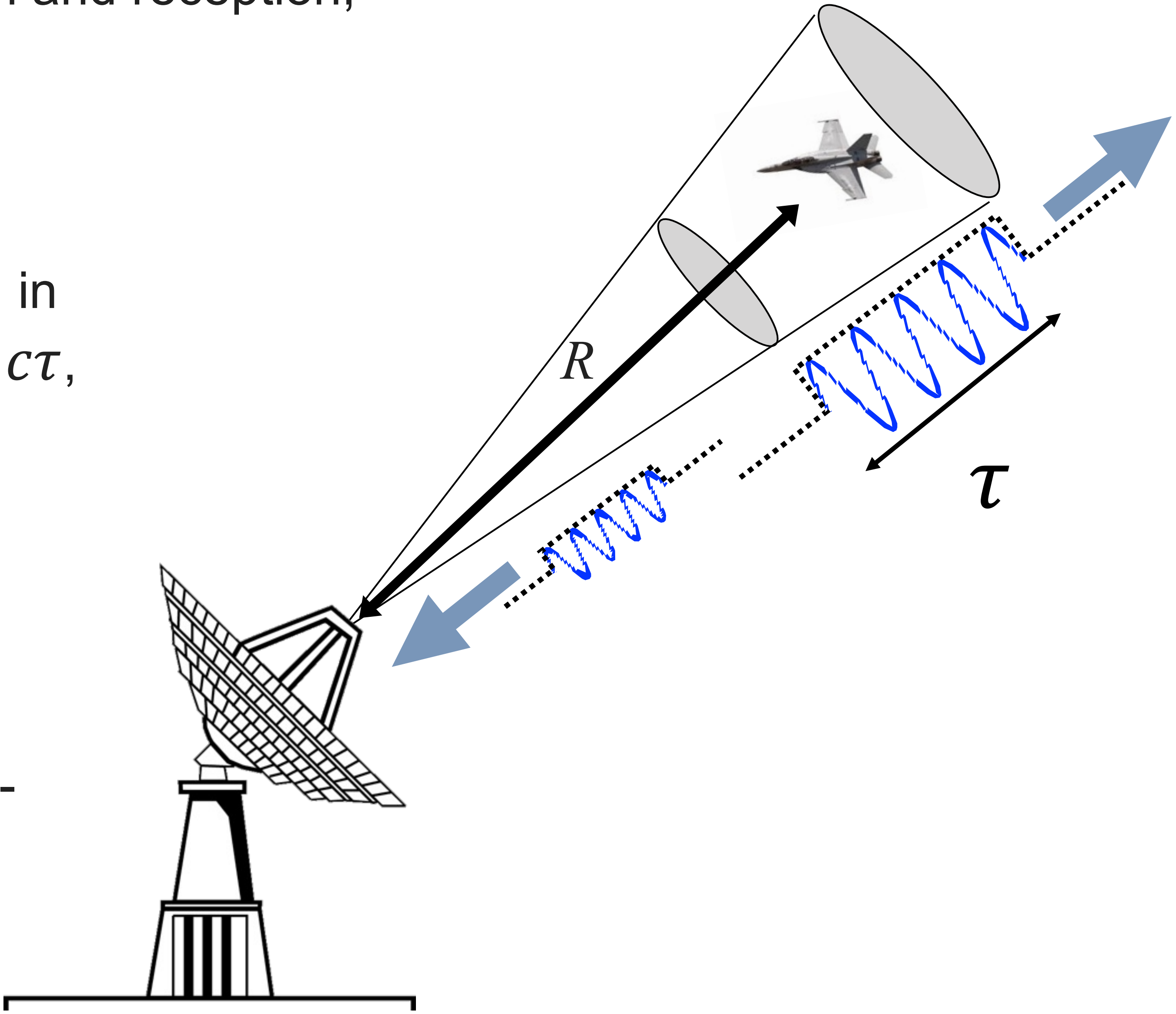


Range

Range R to the target is measured by transmitting a pulse of electromagnetic waves, and measuring the time Δt between transmission and reception,

$$R = \frac{c\Delta t}{2}$$

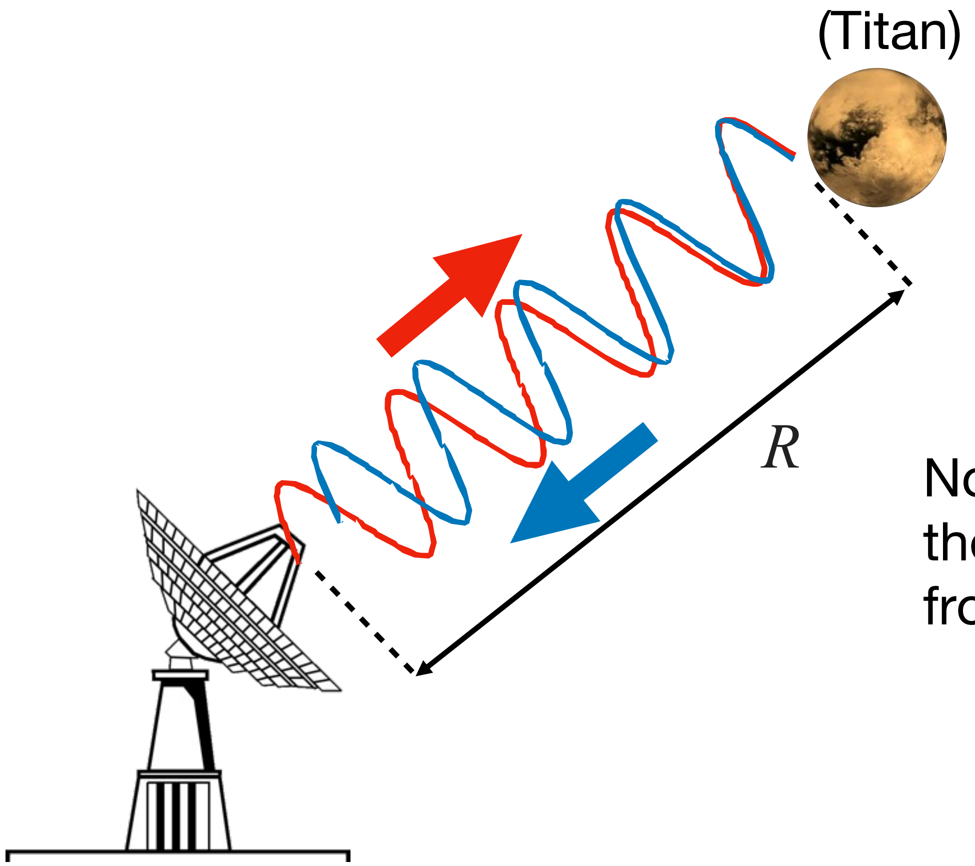
The ***pulse length*** τ is most often expressed in units of time, and corresponds to a distance $c\tau$, where $c \sim 3 \times 10^8\, m/s$.



Range resolution depends on how well we can resolve Δt. For the case of a simple on-off pulse, the optimal approach is to match the sampling period and receiver anti-aliasing filter to the pulse length (the so-called "matched filter" approach).

Range resolution for a simple on-off pulse ("uncoded pulse") is controlled by τ. Shorter τ yields higher range resolution. But a shorter pulse also carry less total energy, and so the reflected signal is more difficult to discriminate from background noise.

Measuring Velocity



Assume a transmitted signal: $\cos(2\pi f_o t)$

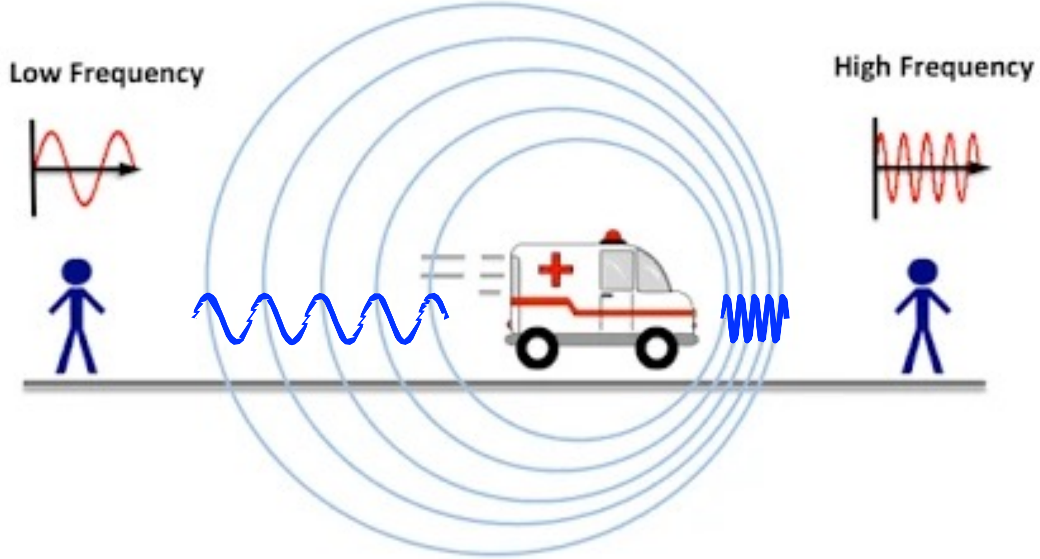
After return from target: $\cos\left[2\pi f_o\left(t-\frac{2R}{c}\right)\right]$

Now let us allow range *R* to vary with time. Let's assume the target moves at a constant velocity, with positive away from the radar and negative toward the radar:

$$R = R_o + v_o t$$

Substituting we obtain:

$$\cos\left[2\pi\left(f_o \underbrace{-f_o\frac{2v_o}{c}}_{f_D}\right)t \underbrace{-\frac{4\pi f_o R}{c}}_{\text{constant}}\right]$$

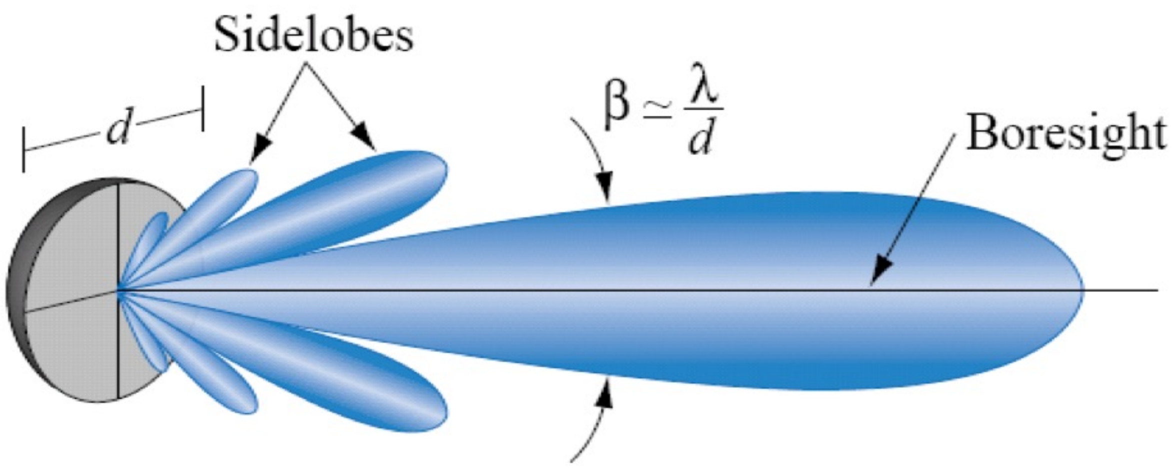


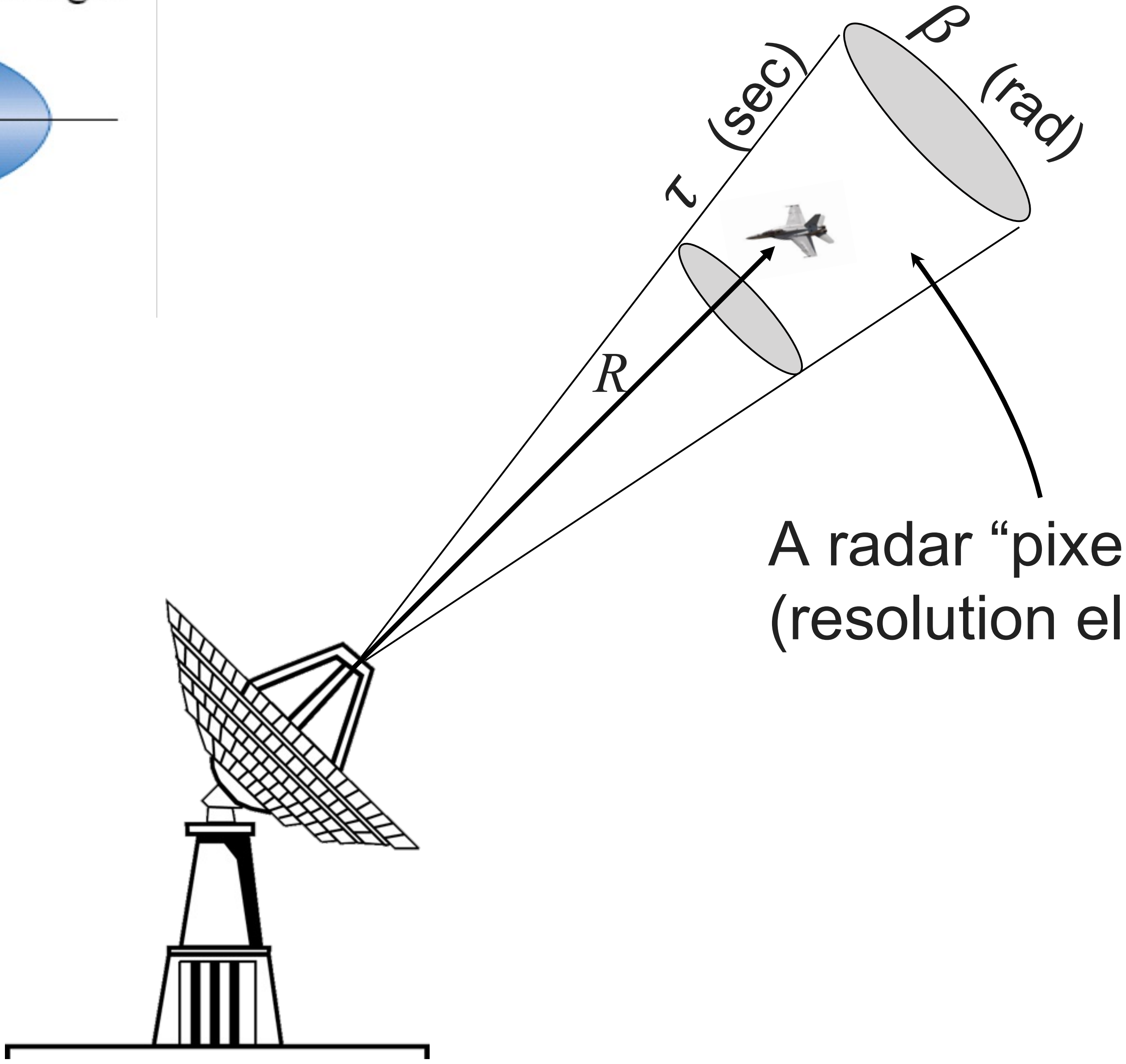
The change in frequency by a moving target is proportional to the *component* of the velocity vector along the radar line of sight:

$$f_D = -\frac{2f_o}{c}v_o = -\frac{2v_o}{\lambda}$$

Frequency resolution is driven, in part, by how long we can make measurements (e.g. pulse length or number of pulses). The longer the better?

Cross-range resolution (beam width)





A radar “pixel”
(resolution element)

The cross-range resolution is usually defined by the angular width of the main lobe of the antenna's power pattern. For a dish antenna this is approximately equal to the ratio of the wavelength to the physical diameter,

$$\beta = \frac{\lambda_o}{d} \qquad \text{(radians)}$$

Millstone Hill ISR has a 46-m dish operating at a frequency of 440 MHz, or $\lambda = 0.68$ m, giving a beam width of $\beta \simeq 0.85°$.

Doppler Radar Summary: “Coherent” hard targets

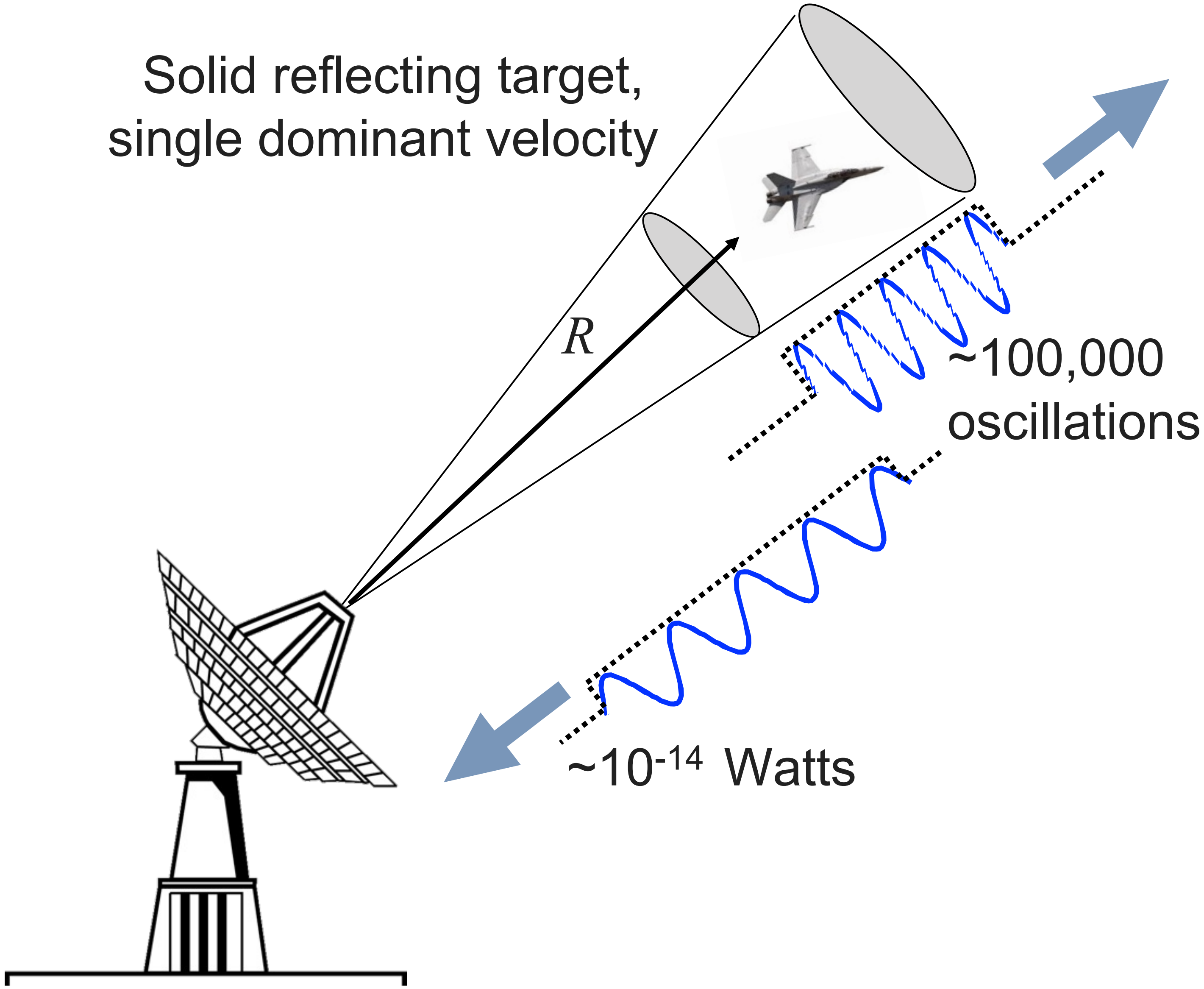
Two key concepts:

Time ⟺ Distance

$$R = \frac{c\Delta t}{2}$$

Frequency ⟺ Velocity

$$f_D = -\frac{2f_o}{c} v_o$$



Solid reflecting target, single dominant velocity

~100,000 oscillations

~10^{-14} Watts

A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

What happens when we have multiple targets in the radar volume, moving at different velocities?

Doppler Radar Summary: Distributed "Incoherent" Targets

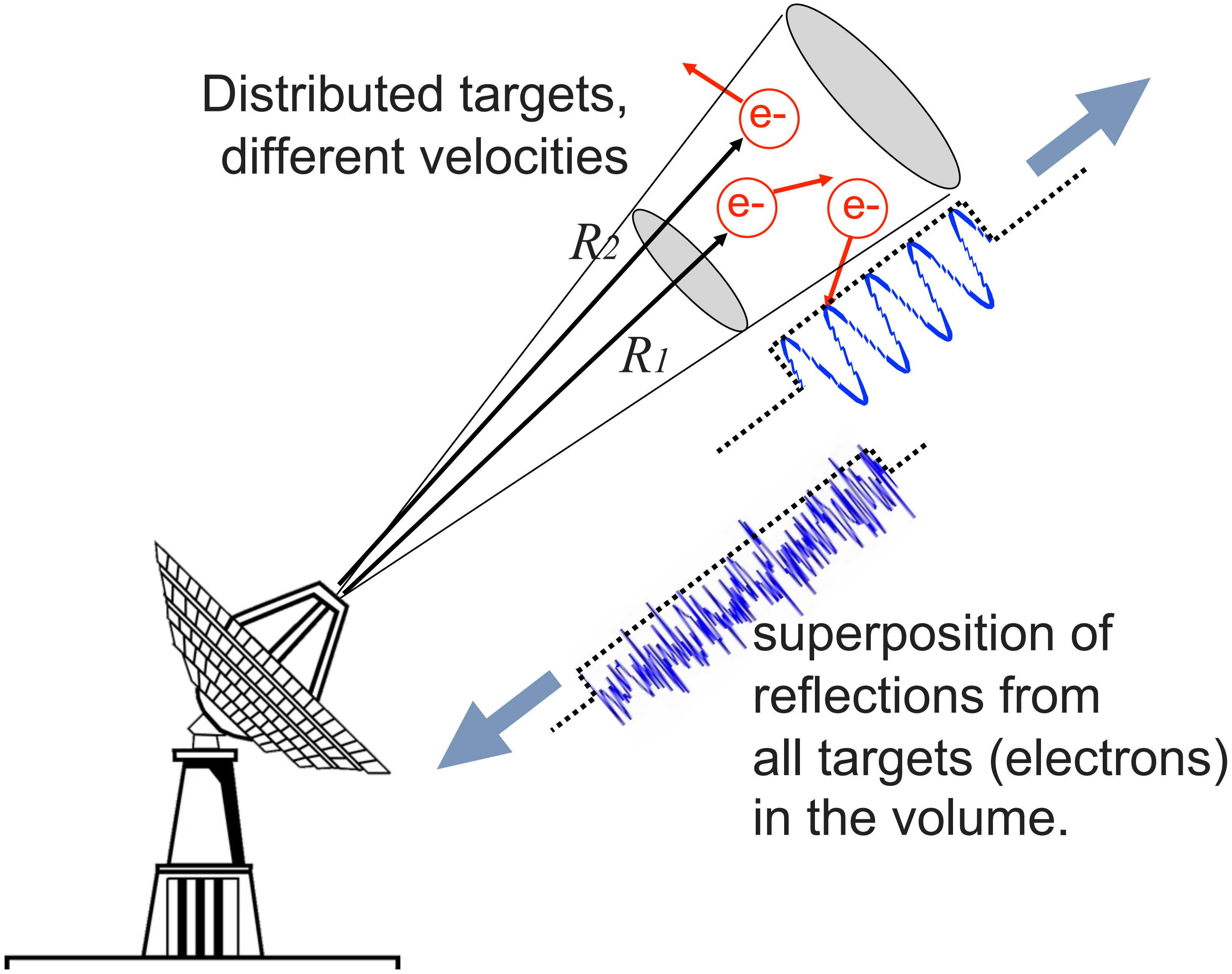
Two key concepts:

Time ⇔ Distance

$$R = \frac{c\Delta t}{2}$$

Frequency ⇔ Velocity

$$f_D = -\frac{2f_o}{c} v_o$$



Distributed targets, different velocities

superposition of reflections from all targets (electrons) in the volume.

A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

What happens when we have multiple targets in the radar volume, moving at different velocities?

Concept of a "Doppler Spectrum"

Superposition of targets moving with different velocities within the radar volume

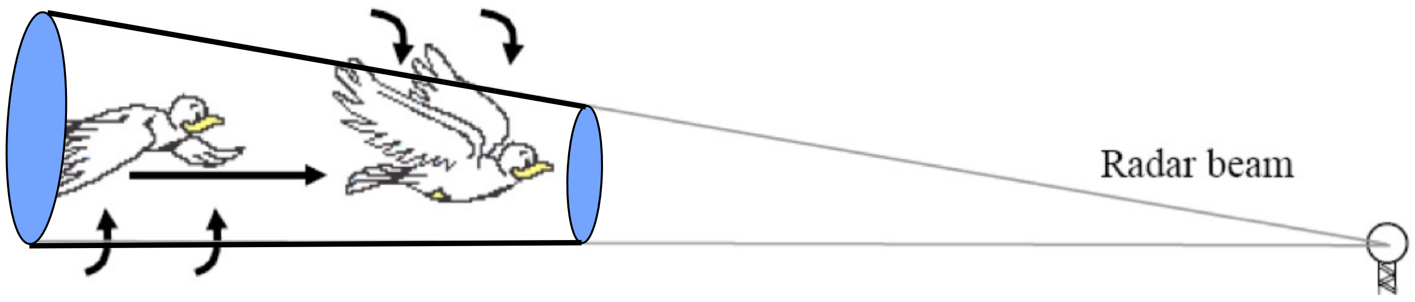
Two key concepts:

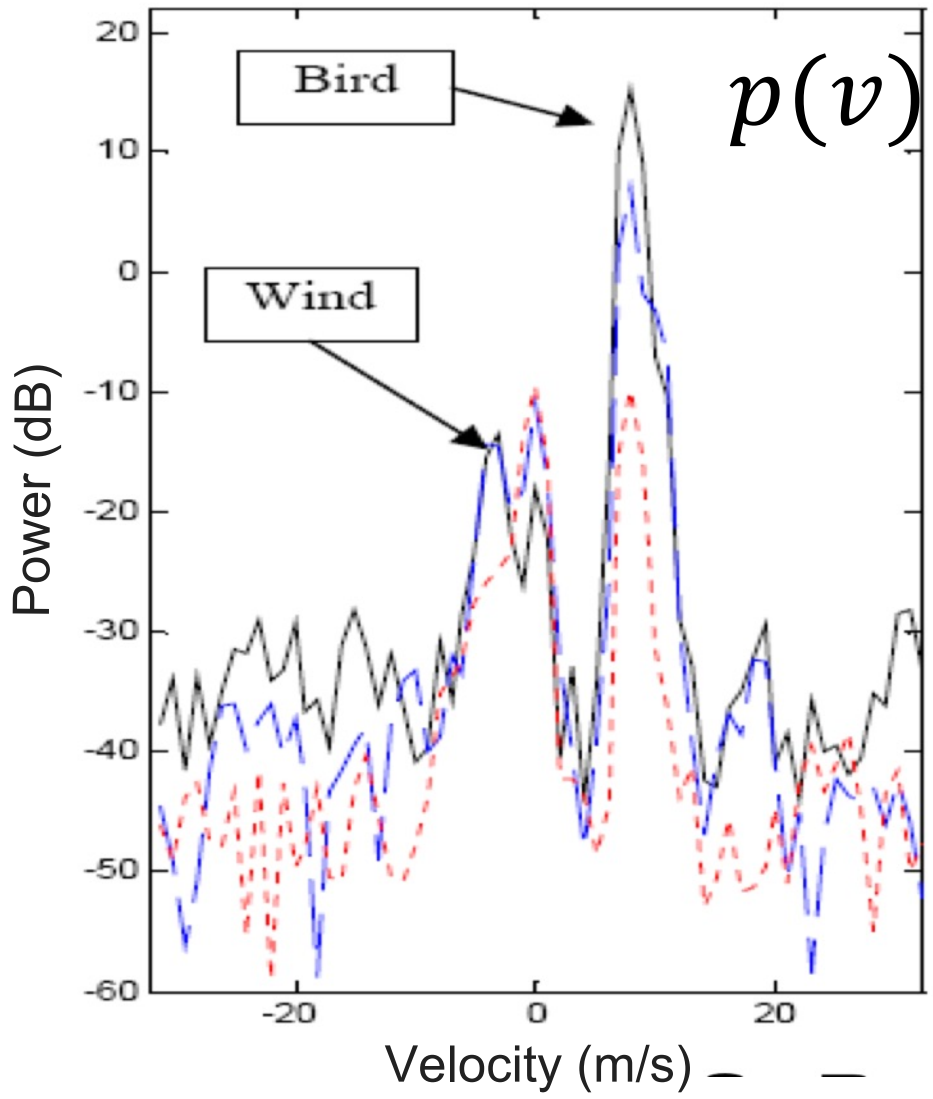
Time ⇔ Distance

$$R = \frac{c\Delta t}{2}$$

Frequency ⇔ Velocity

$$f_D = -\frac{2f_o}{c}v_o$$





Processing: $p(R, f_D) \rightarrow p(R, v)$

If there is a distribution of targets with different velocities (e.g., bird, flapping wings, wind) then there is no single Doppler shift but, rather, a Doppler spectrum.

Distributed “beam filling” Target

A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

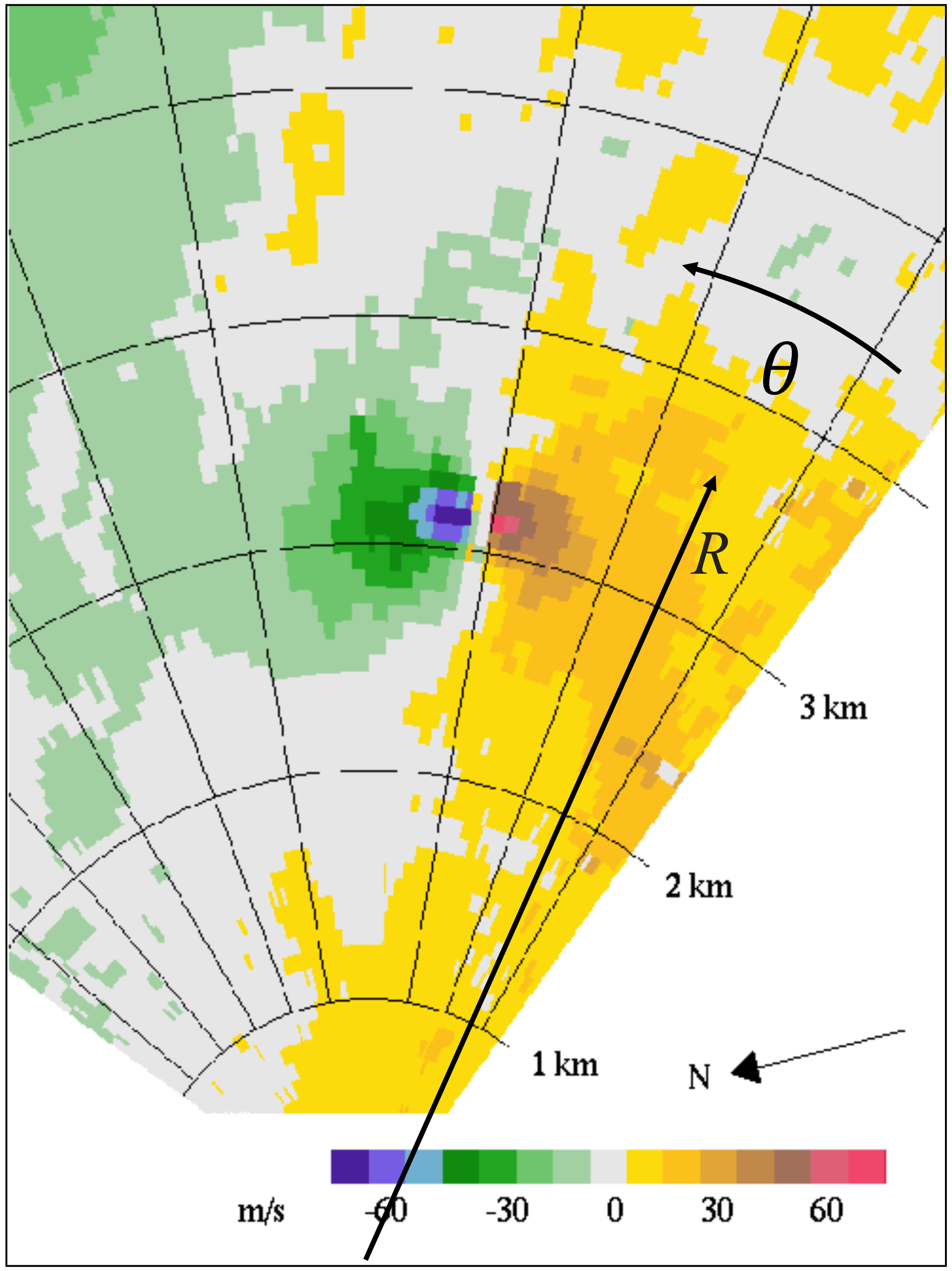
Two key concepts:

Time ⇔ Distance

$$R = \frac{c\Delta t}{2}$$

Frequency ⇔ Velocity

$$f_D = -\frac{2f_o}{c} v_o$$



Processing:

$$p(R, f_D, t) \rightarrow f_D(R, t) \rightarrow v(R, \theta)$$

For a beam-filling target (like water droplets in a tornado), the radar can be used to construct insightful images of velocity relative to the radar.

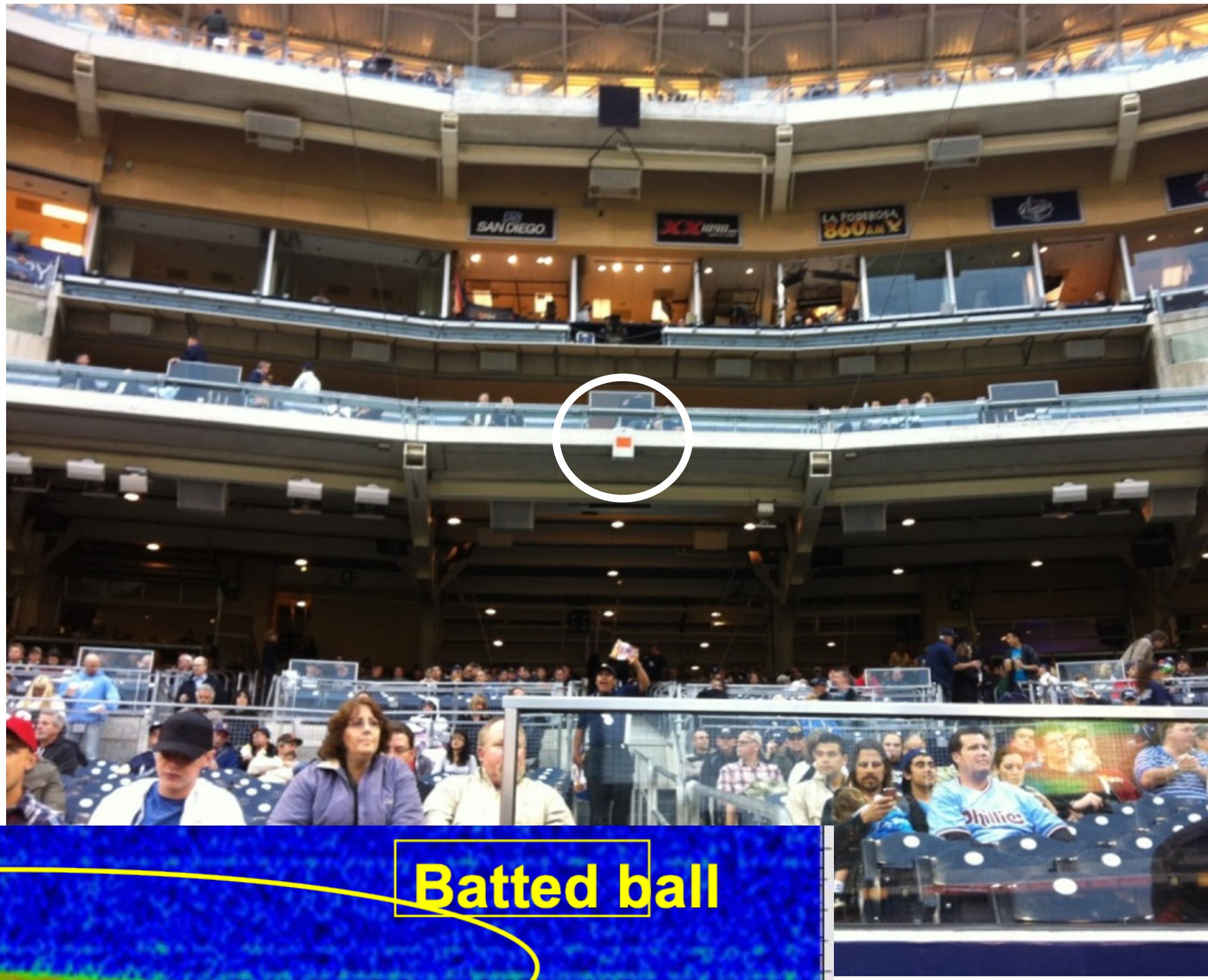
Micro-Doppler Analysis

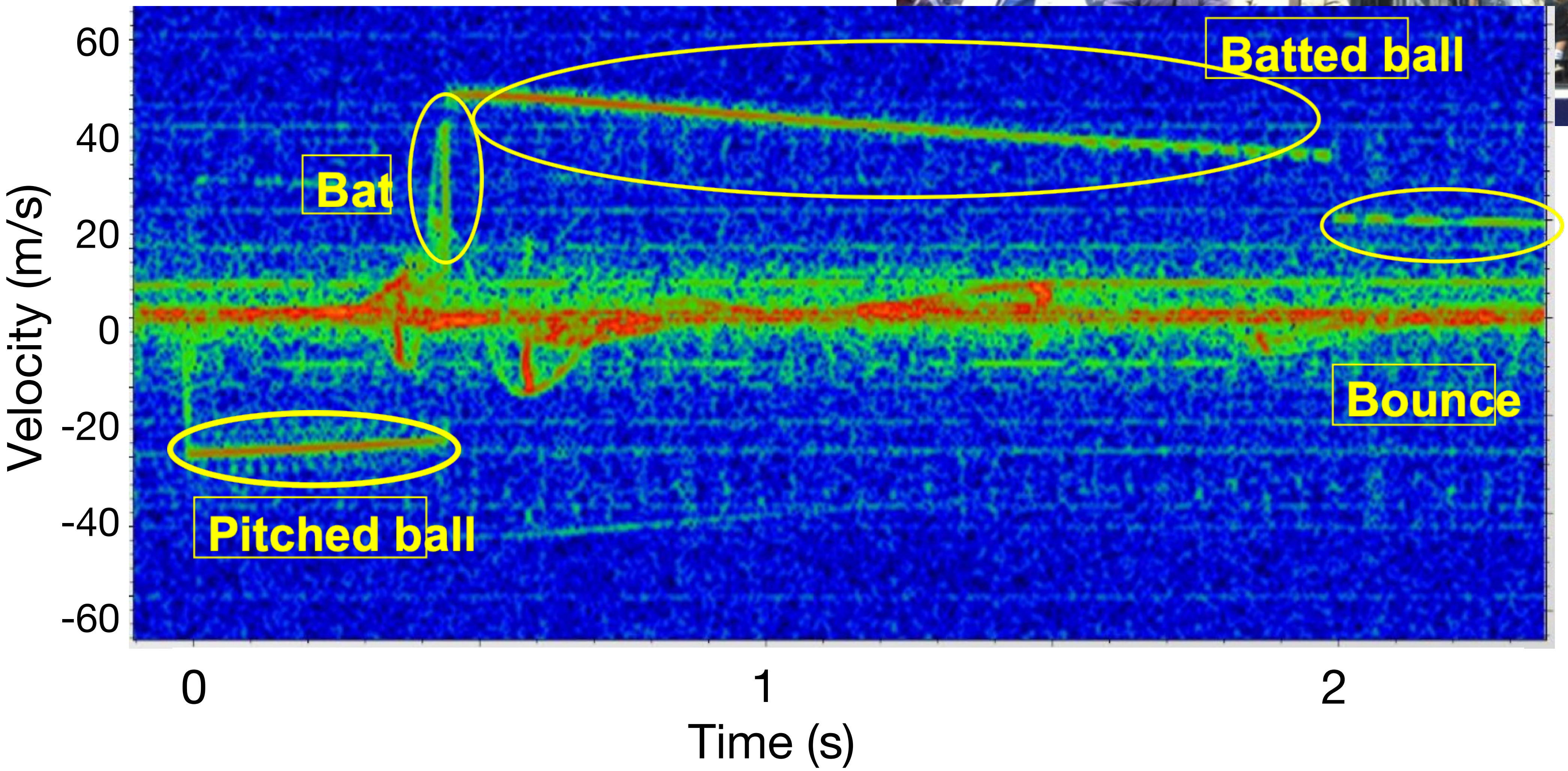
Trackman radar: “continuous wave” (CW) radar: precise Doppler but no range information.

Can identify targets and actions based on Doppler signatures!

Processing:

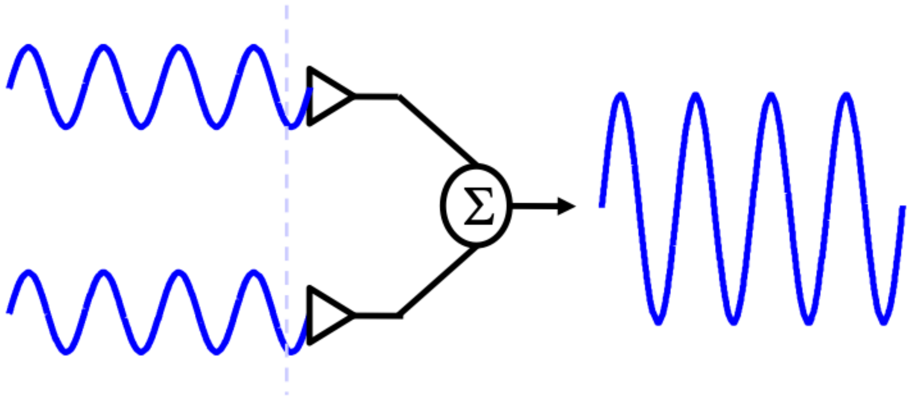
$$p(f_D, t) \rightarrow p(v, t)$$



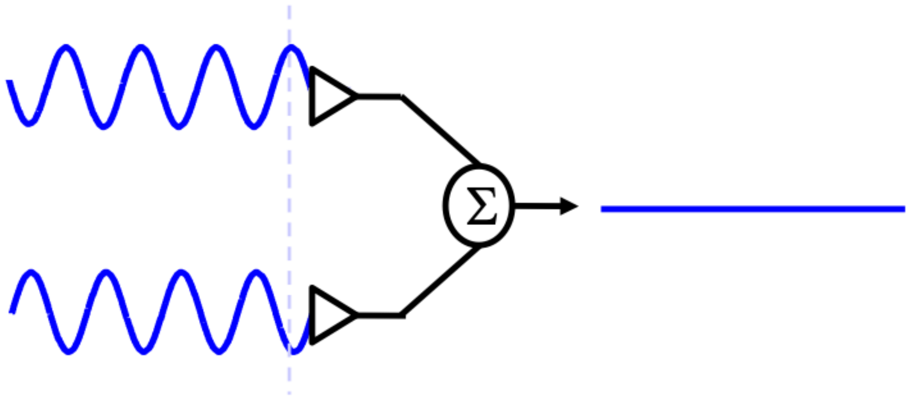


Wave Interference and Bragg Scatter

Consider two waves with the same frequency but different phase.

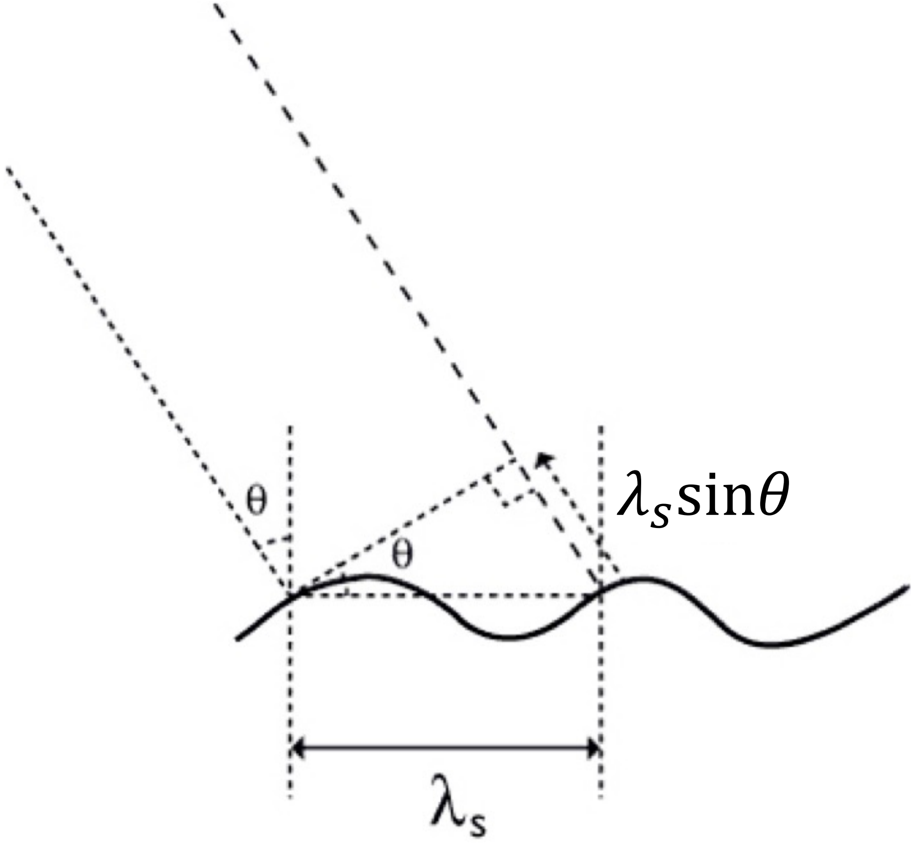


Constructive
(in phase)



Destructive
(180° out of phase)

Consider a wave along the interface between a dielectric and a conducting (reflective) medium, as depicted below. This is representative of an air-ocean boundary.

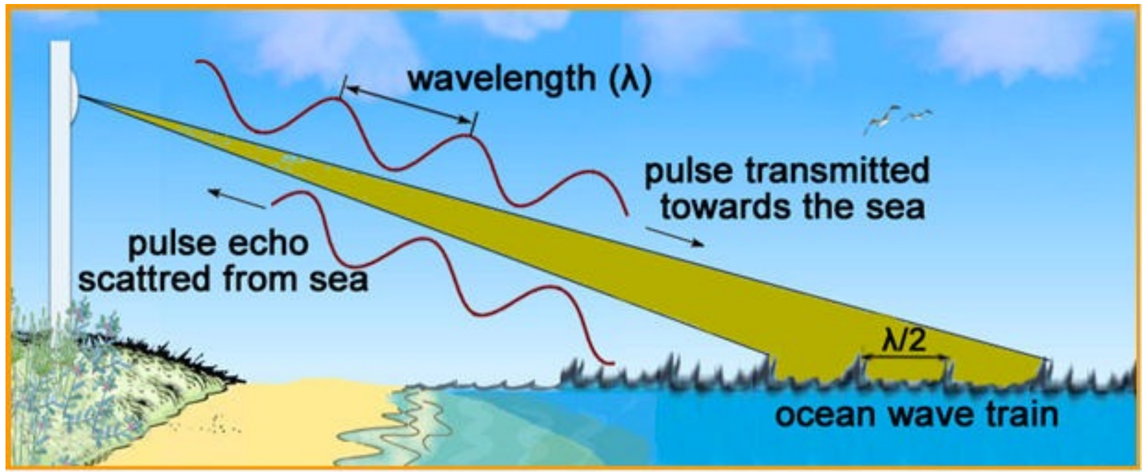


Suppose waves are observed at angle θ using a radar with wavelength λ_o. The condition for maximum constructive interference is

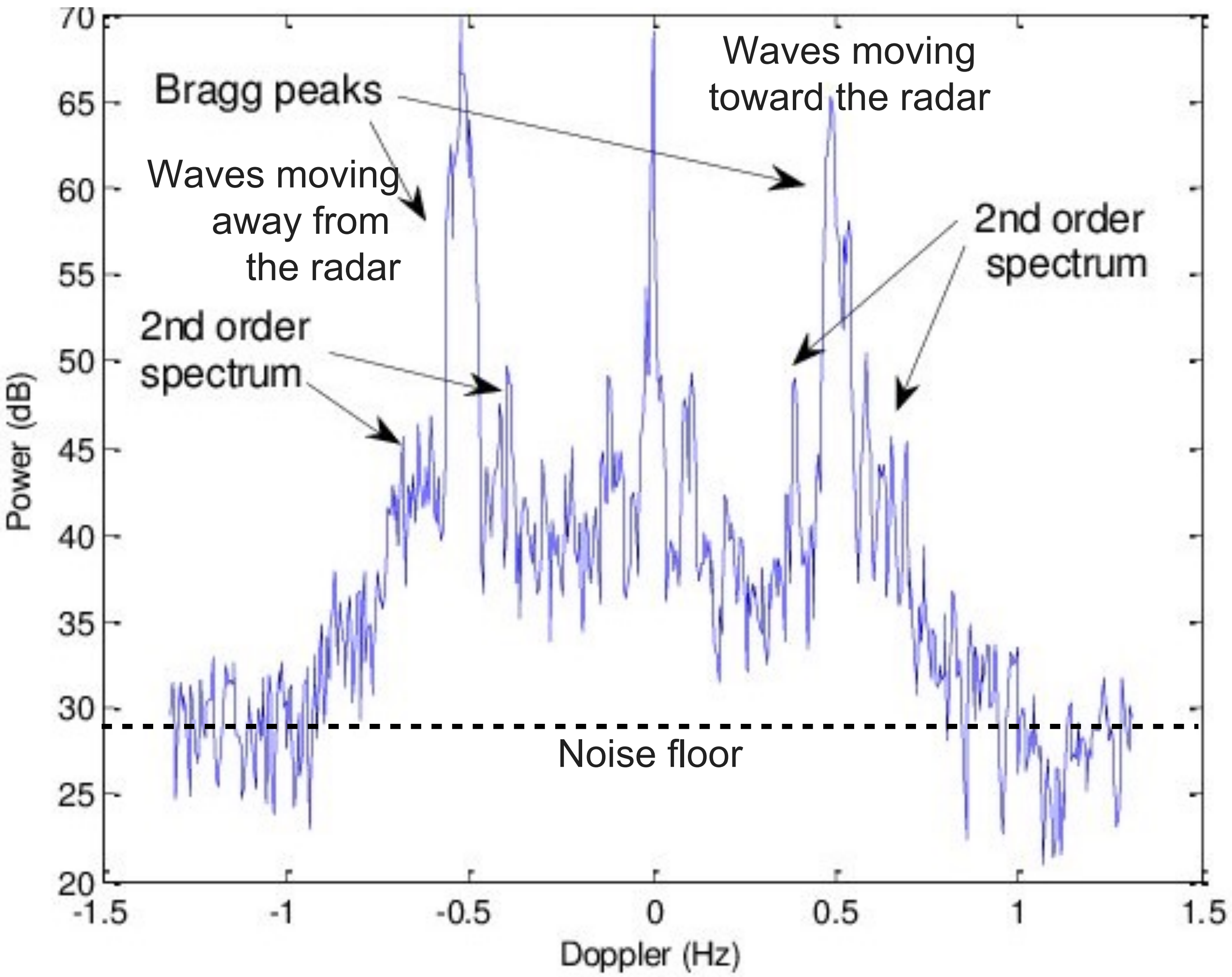
$$n\lambda_0 = 2\lambda_s \sin\theta$$

If $\theta = 90°$ (or if these waves are propagating isotropically), then the Bragg condition is met for $n\lambda_0 = 2\lambda_s$

Doppler spectrum of ocean waves



Backscatter from the ocean at low aspect angle shows peaks in the Doppler spectrum from the subset of waves matching the Bragg condition for the radar (spacing ≃ half the radar wavelength)



Important points:

The target is distributed over the entire radar beam width.
The scattering is from free electrons in the conducting sea water.
The Doppler spectrum has peaks due to Bragg scatter from waves in the medium.
The frequency of the peaks tells us the velocity and direction of the waves.
The height of the peaks tells us something about the amplitude and density of the waves.
The width of the peaks tells us something about the spread in velocity of the waves

Doppler spectrum of the ionosphere

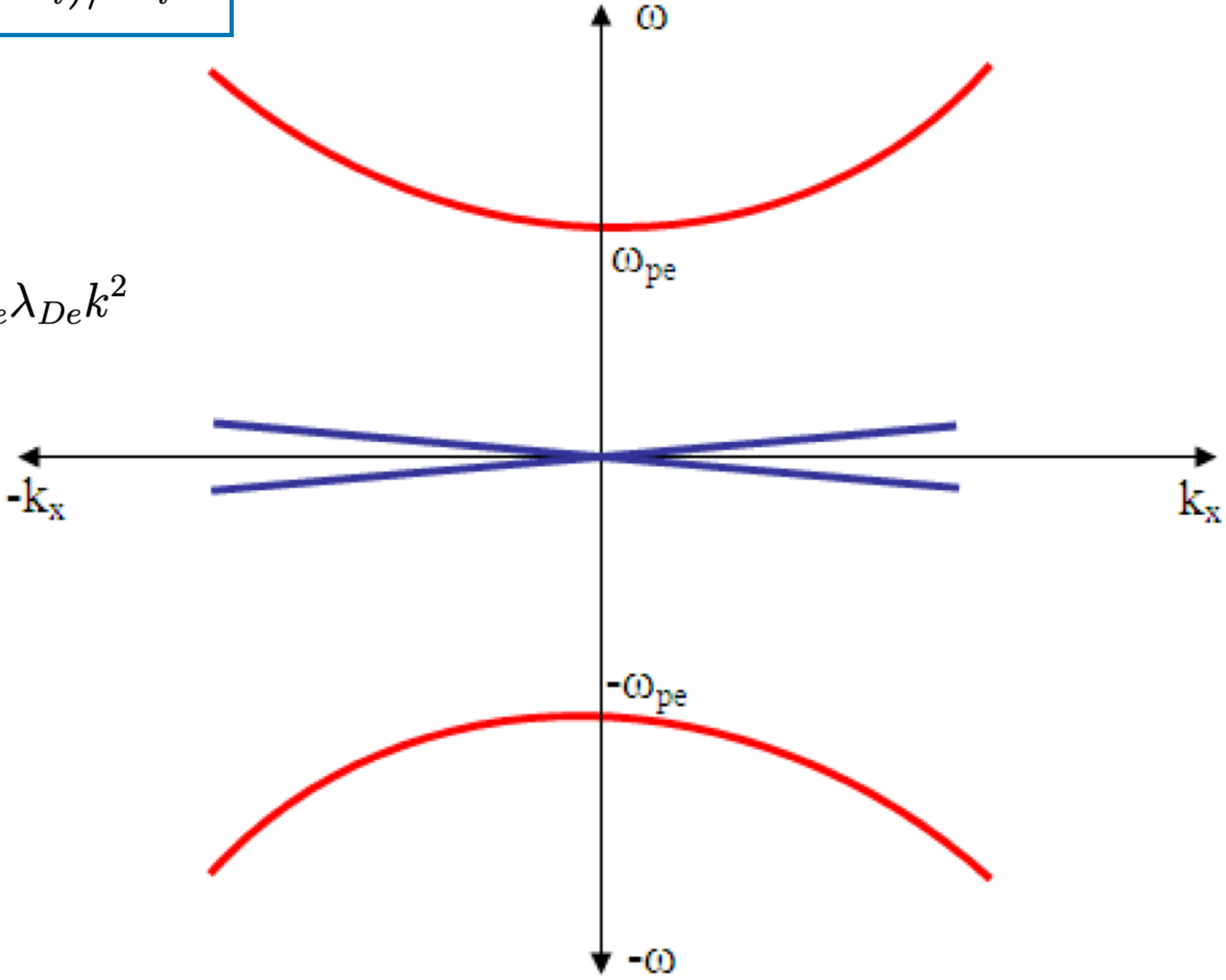
Let's put this all together for the ionosphere.
The two predominant longitudinal modes in a thermal plasma:

Ion-acoustic mode:

$$\boxed{\omega_s = C_s k \qquad C_s = \sqrt{k_B(T_e + 3T_i)/m_i}}$$

Langmuir mode:

$$\omega_L = \sqrt{\omega_{pe}^2 + 3\,k^2\,v_{the}^2} \approx \omega_{pe} + \frac{3}{2} v_{the} \lambda_{De} k^2$$



Computer simulation of the ionosphere

Simple rules yield complex behavior

Particle-in-cell (PIC) simulation:

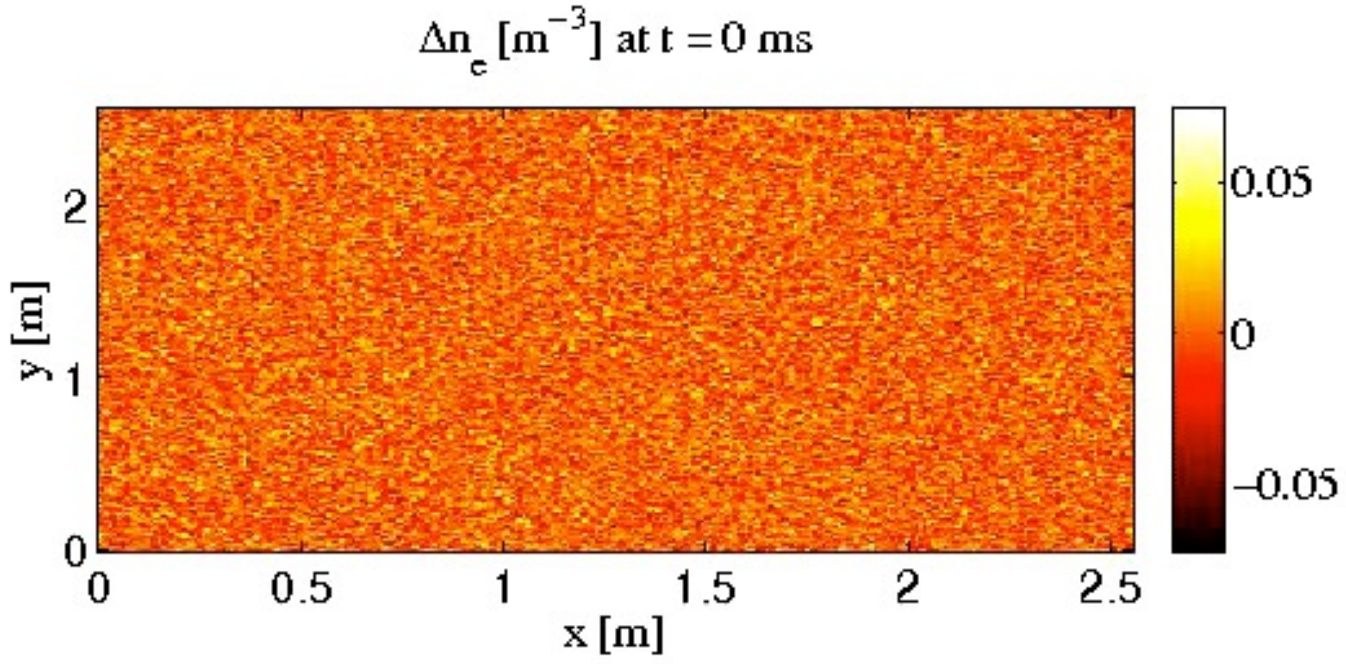
$$\frac{d\,\mathbf{v}_i}{d\,t} = \frac{q_i}{m_i}(\mathbf{E}(\mathbf{x}_i) + \mathbf{v}_i \times \mathbf{B}(\mathbf{x}_i))$$

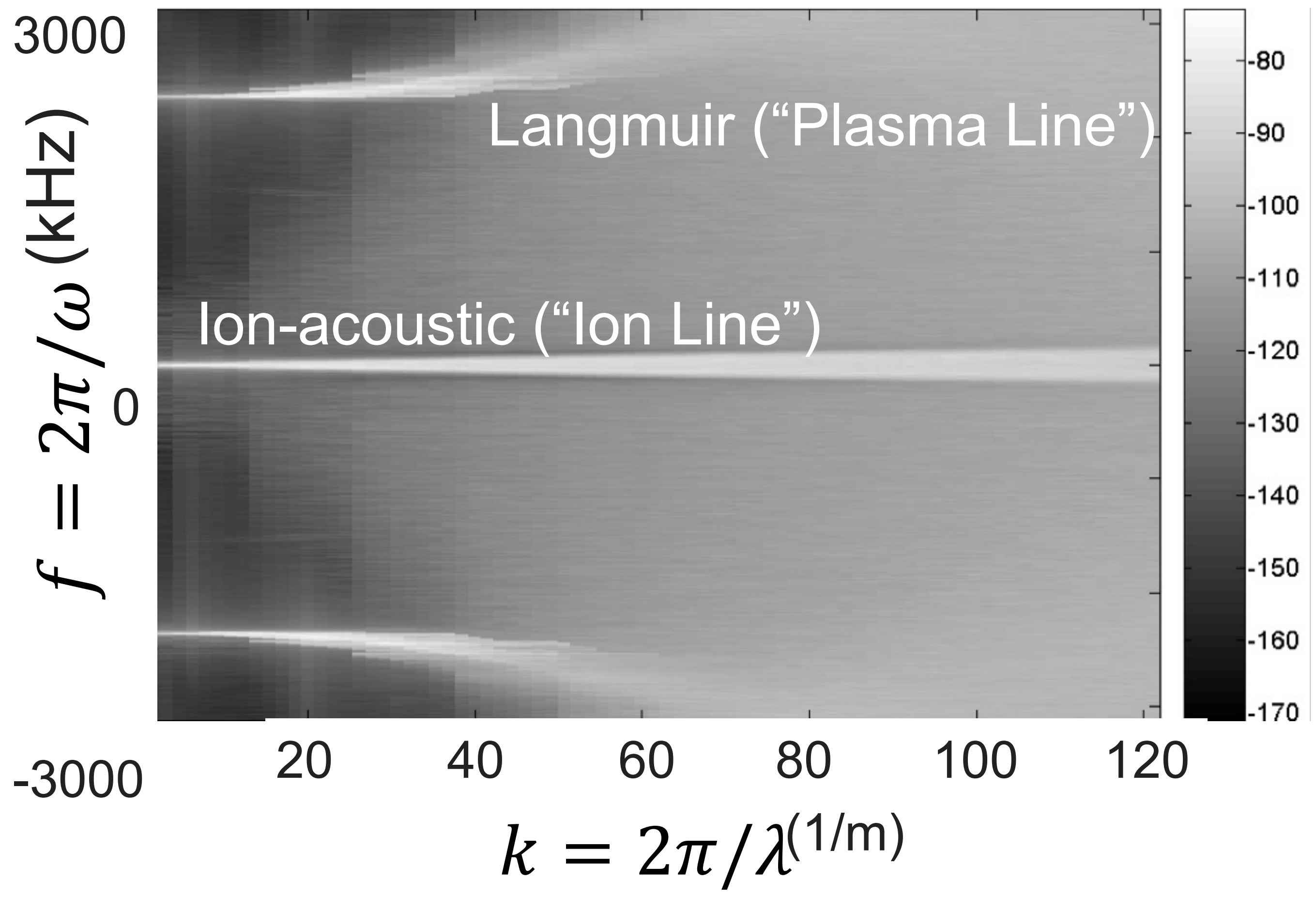
$$\nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

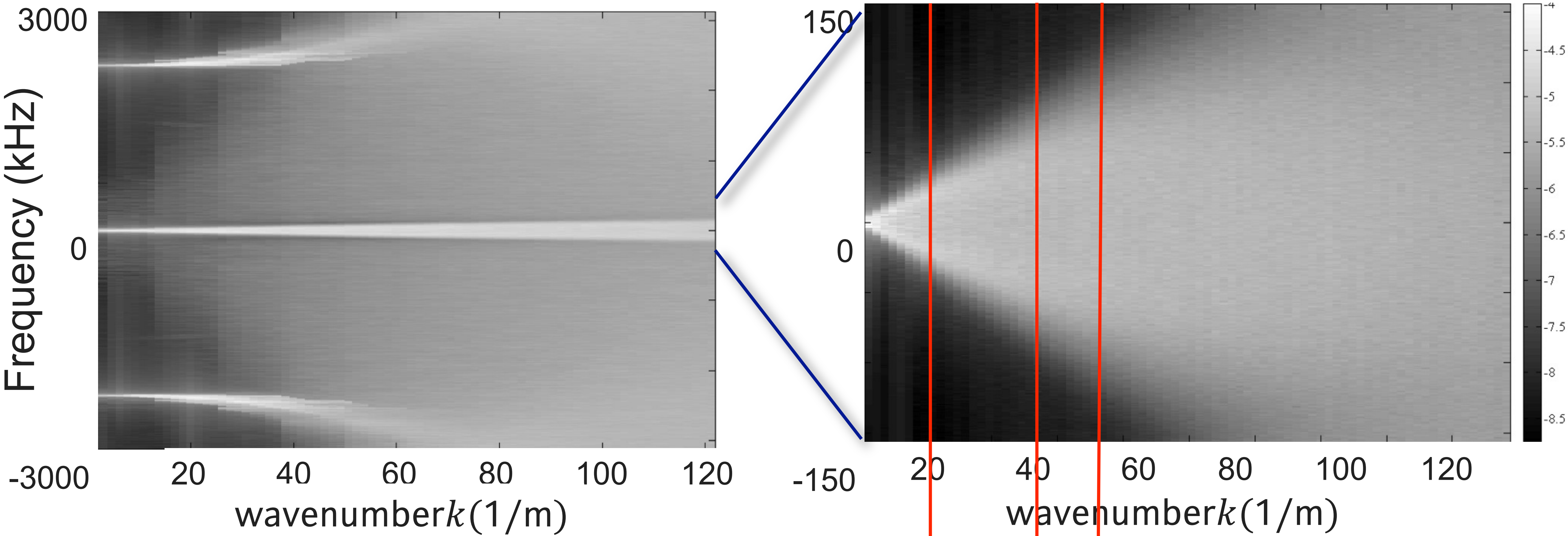
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$





ISR measures a cut through this surface at a particular wave number

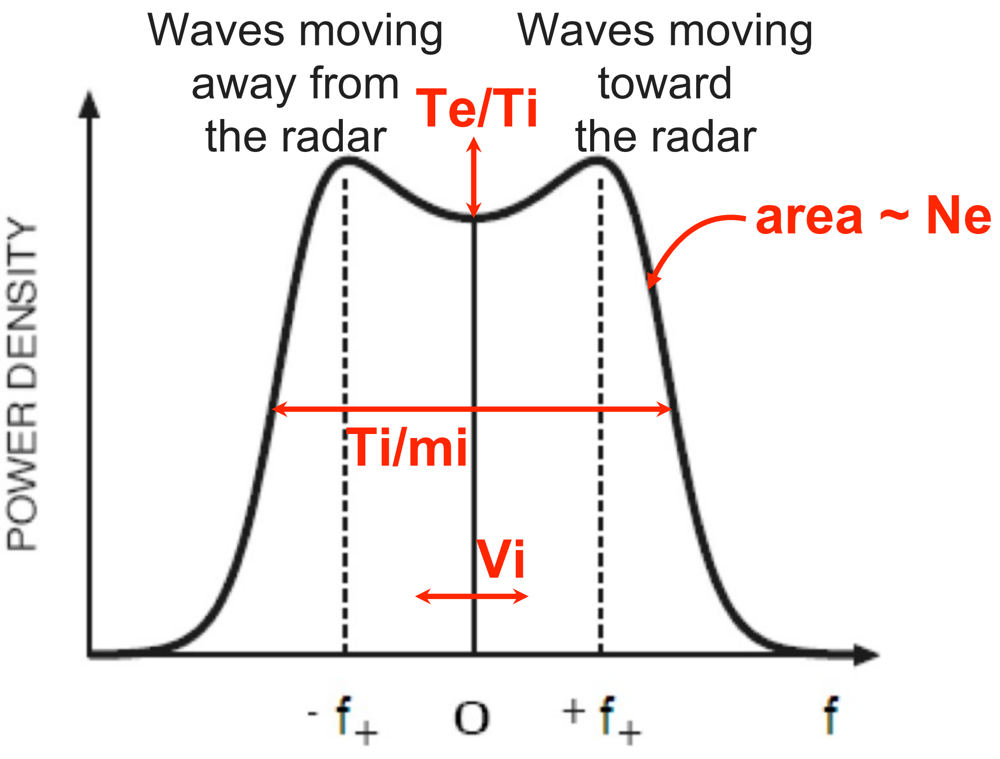


Sondrestrom (1.3 GHz)

EISCAT UHF (930 MHz)

AMISR (450 MHz),
Millstone (440 MHz)

Ion-acoustic "lines" are broadened by Landau damping



Doppler Radar:
“Incoherent” Distributed Target

Two key concepts:

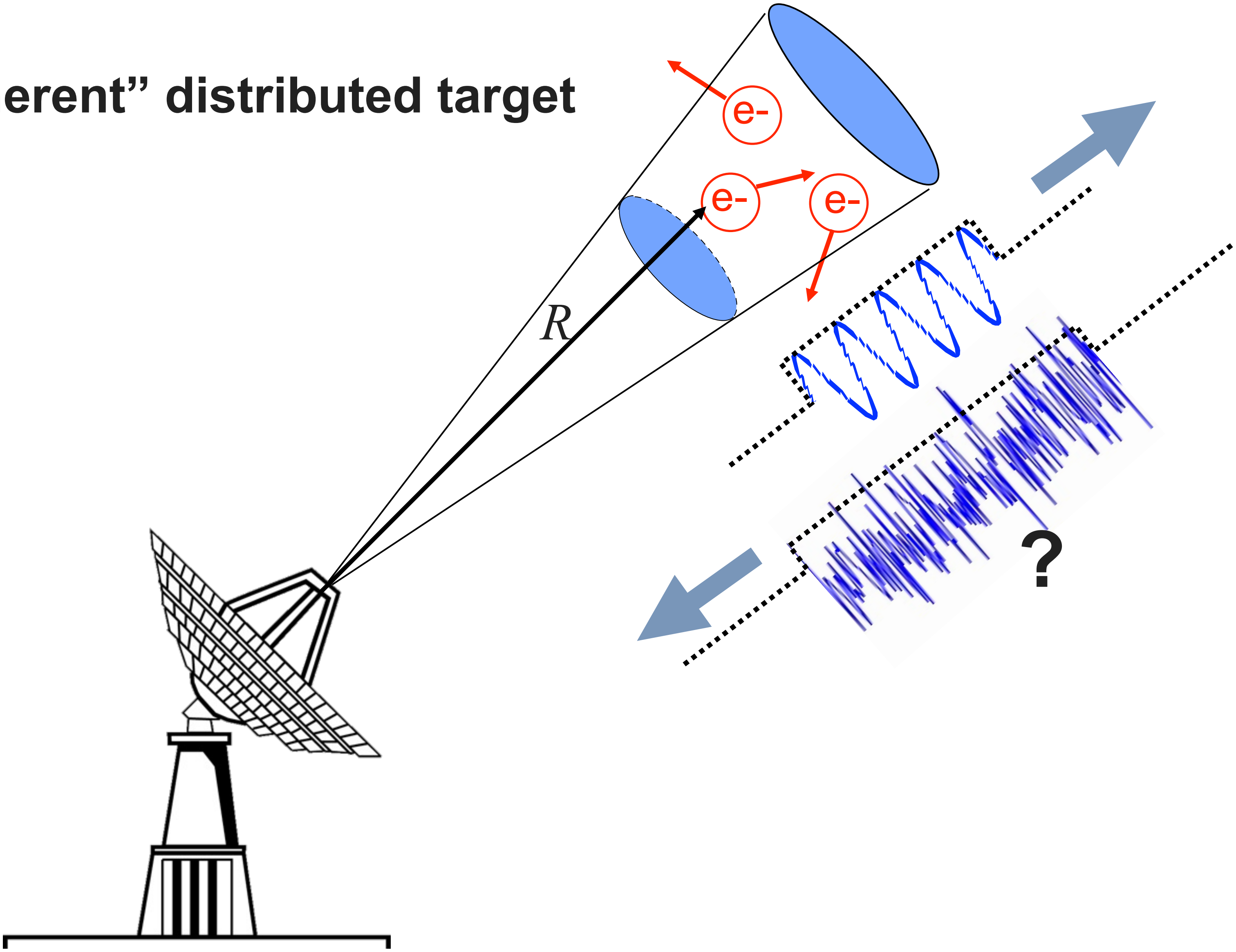
Distant ⟺ Time

$$R = \frac{c\Delta t}{2}$$

Velocity ⟺ Frequency

$$f_D = -\frac{2f_o}{c}v_o$$

“Incoherent” distributed target

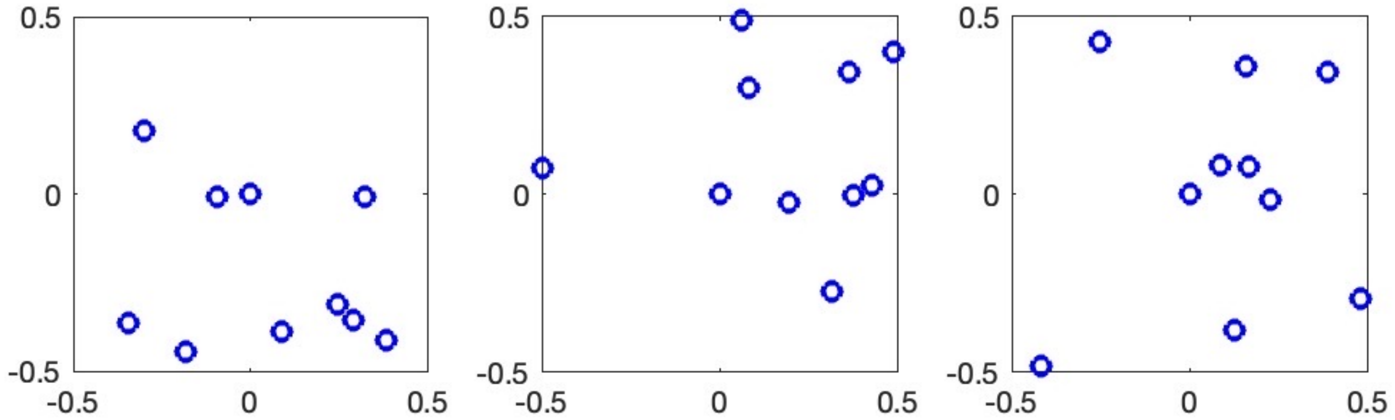


A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

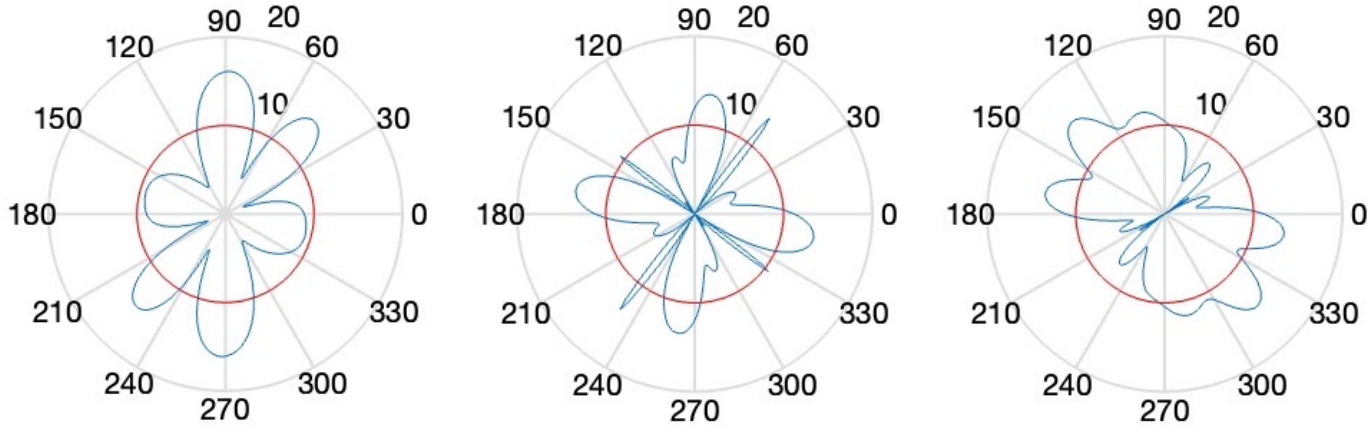
What happens when we have multiple targets in the radar volume, moving at different velocities?

Constructive/destructive volume scatter

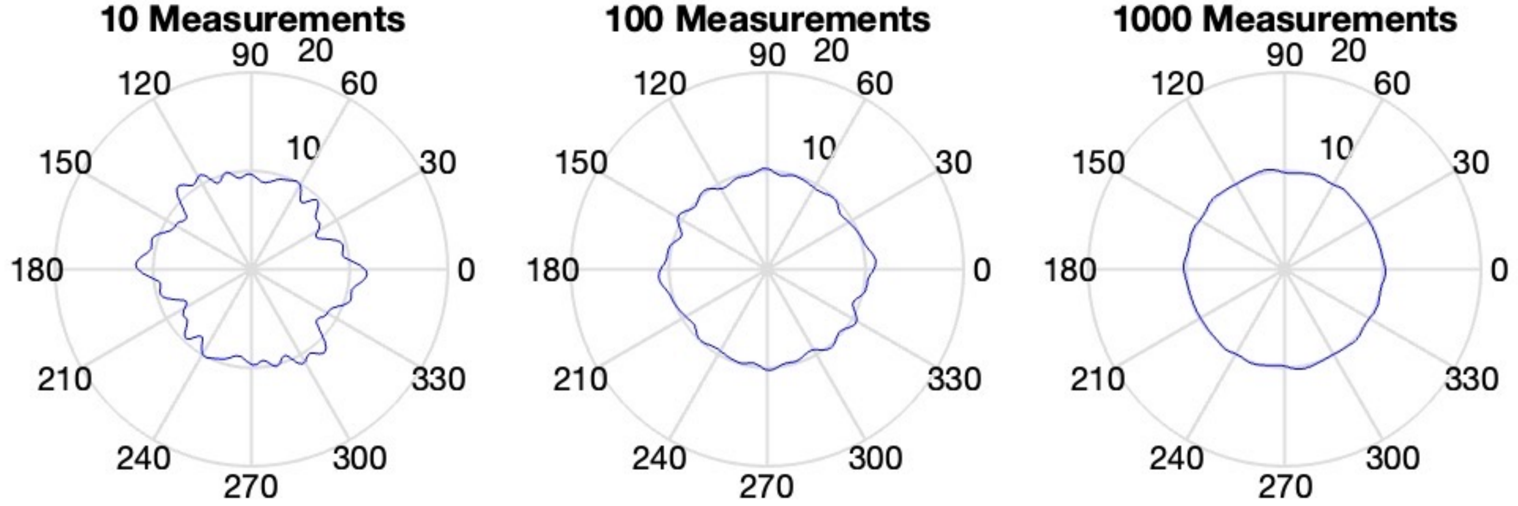
10 electrons / square wavelength



Return as a function of look direction (dB)

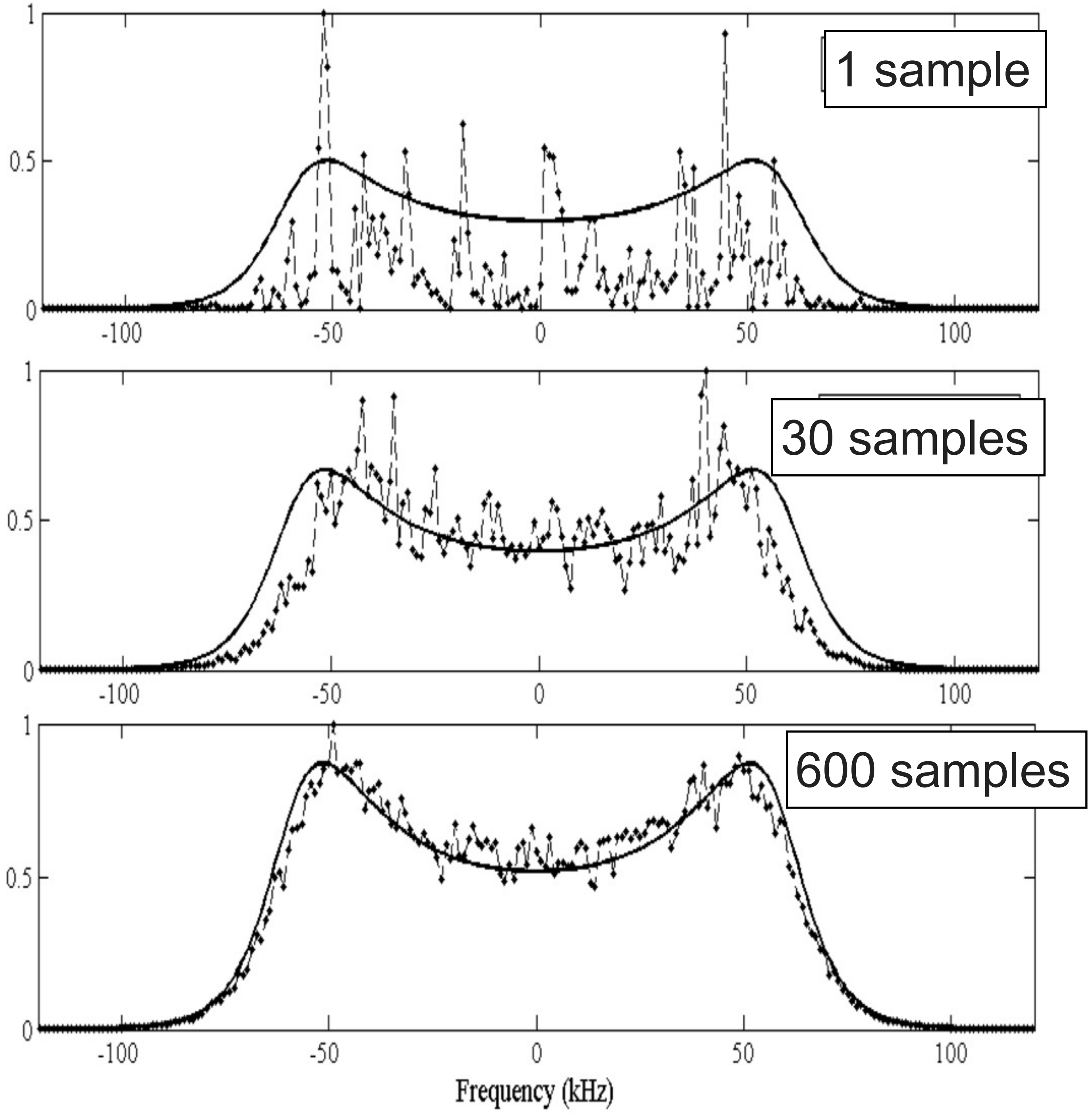


Incoherent averaging



Incoherent Averaging

Normalized ISR spectrum for different integration times at 1290 MHz

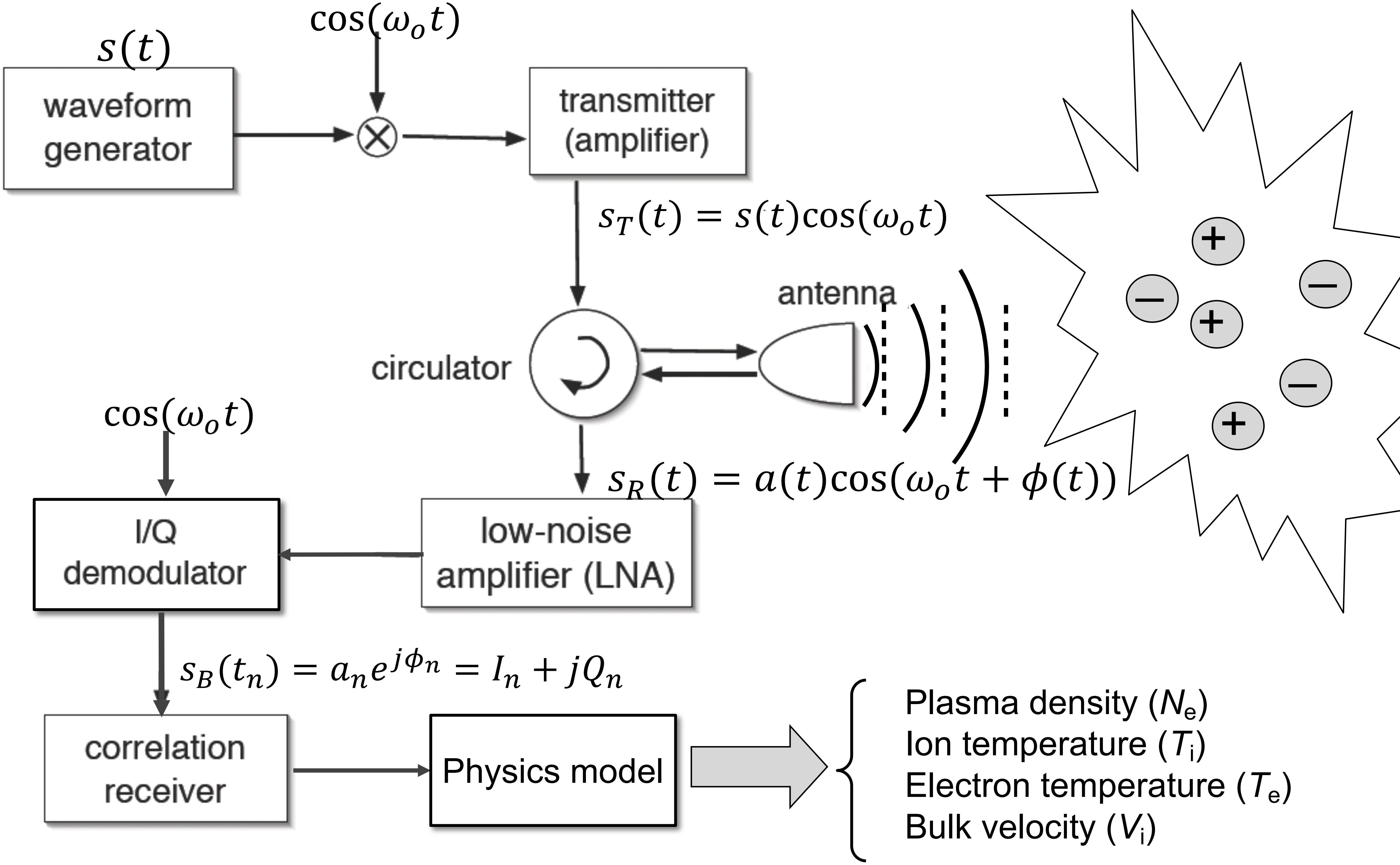


We are seeking to estimate the power spectrum of a Gaussian random process. This requires that we sample and average many independent “realizations” of the process.

$$\rho_e \sim \frac{1}{K}\left(1+\frac{1}{SNR}\right)^2$$

ρ_e =Normalized Mean Square Error
K =number of samples
SNR =per-pulse Signal-to-Noise Ratio

Components of a Pulsed Doppler Radar



$s(t)$

waveform generator

$\cos(\omega_o t)$

transmitter (amplifier)

$s_T(t) = s(t)\cos(\omega_o t)$

circulator

antenna

$s_R(t) = a(t)\cos(\omega_o t + \phi(t))$

$\cos(\omega_o t)$

I/Q demodulator

low-noise amplifier (LNA)

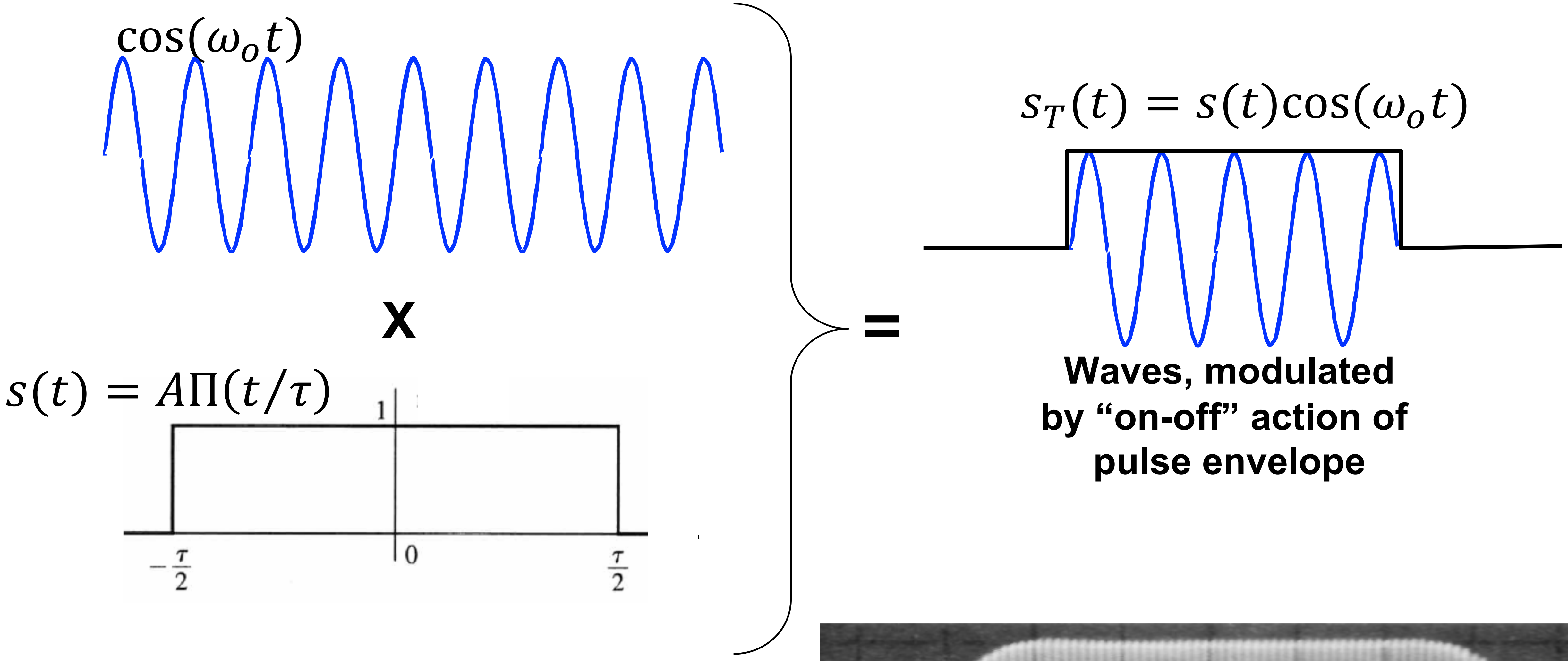
$s_B(t_n) = a_n e^{j\phi_n} = I_n + jQ_n$

correlation receiver

Physics model

Plasma density (N_e)
Ion temperature (T_i)
Electron temperature (T_e)
Bulk velocity (V_i)

A Simple Radar Pulse



$\cos(\omega_o t)$

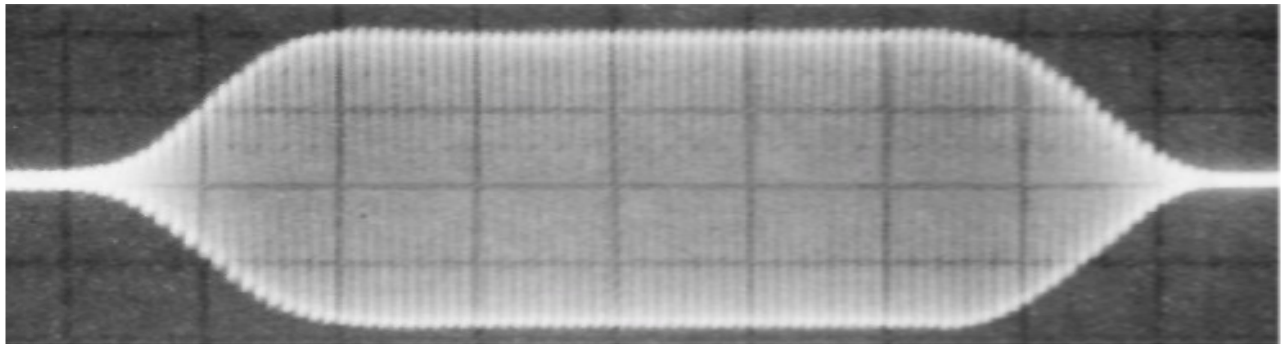
X

$s(t) = A\Pi(t/\tau)$

=

$s_T(t) = s(t)\cos(\omega_o t)$

Waves, modulated by “on-off” action of pulse envelope

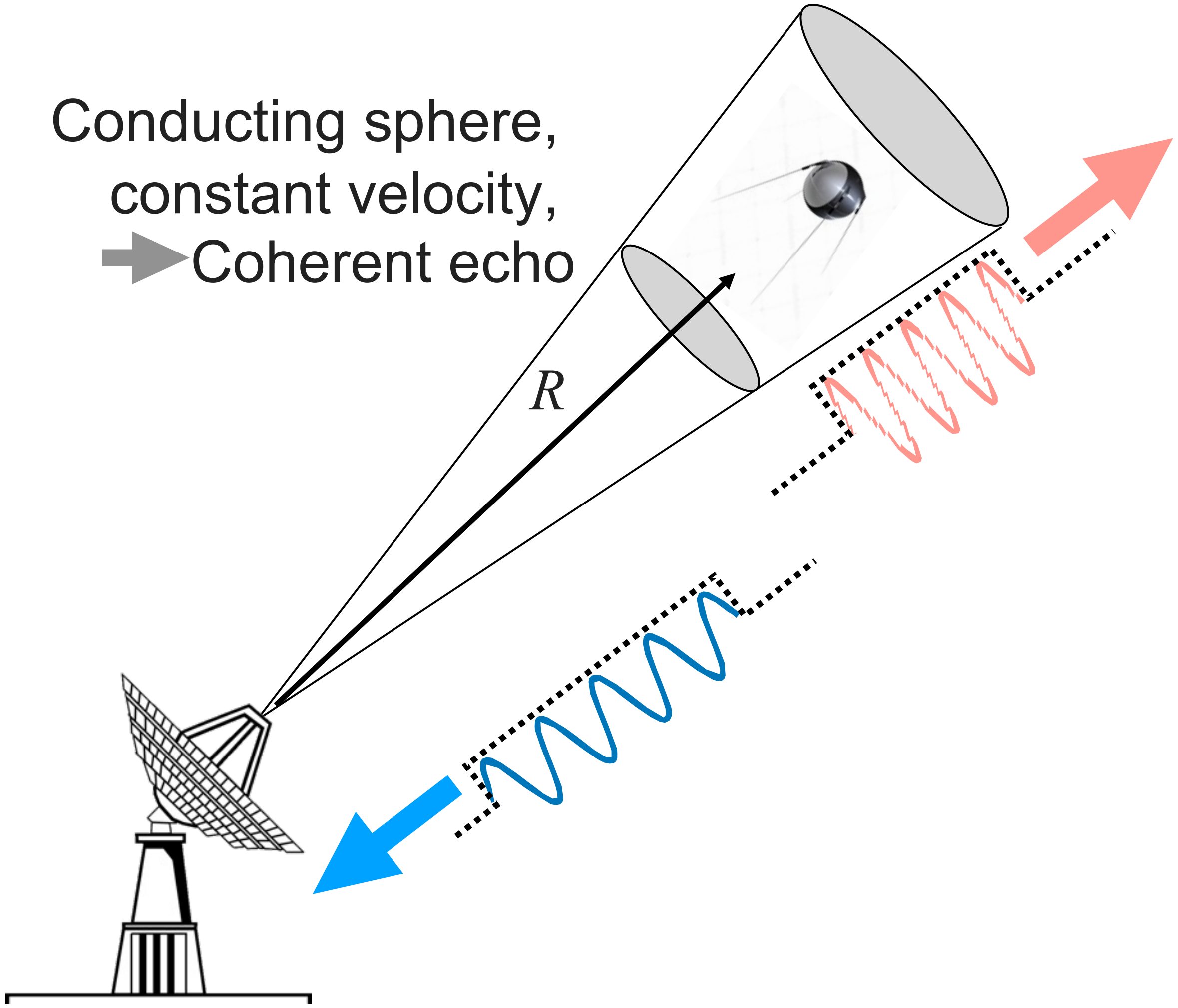


How many cycles are in a typical pulse?

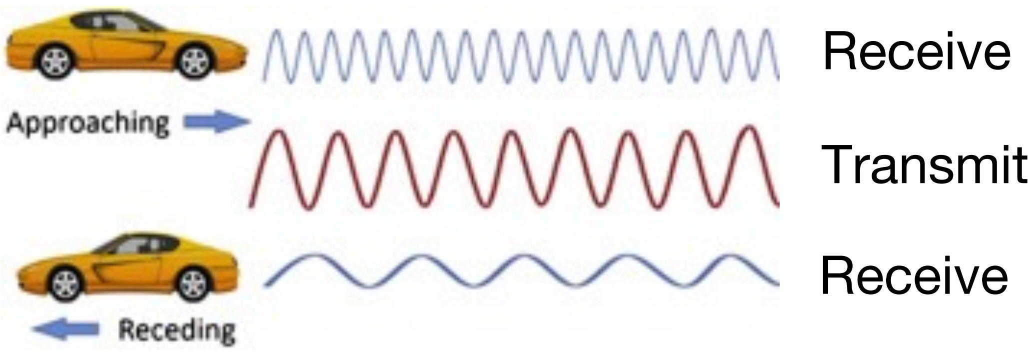
PFISR frequency: 449 MHz
Typical long-pulse length: 480 μs

215,520 cycles!

Measuring Velocity



Conducting sphere,
constant velocity,
➡ Coherent echo



Assume a transmitted signal: $s(t)\cos(2\pi f_o t)$

After return from target: $a(t)\cos\left[2\pi f_o\left(t-\frac{2R(t)}{c}\right)\right]$

Let's assume target moves with constant velocity with respect to the radar during the measurement,

$$R = R_o + v_o t$$

Substituting we obtain:

$$a(t)\cos\left[\underbrace{2\pi f_o}_{\omega_o} t - \underbrace{2\pi f_D t}_{\phi(t)} - \frac{4\pi f_o R_o}{c}\right] \quad f_D = -\frac{2f_o}{c}v_o$$

$$a(t)\cos[\omega_o t + \phi(t)] \quad \omega_D = 2\pi f_D = -\frac{d\phi}{dt}$$

$f_o \sim 500$ MHz, f_D ~ 50 kHz= $0.0001 f_o$

Two issues:

1) How do we discriminate positive from negative f_D?

2) How do we remove f_o , and just sample $a(t)\cos[\phi(t)]$?

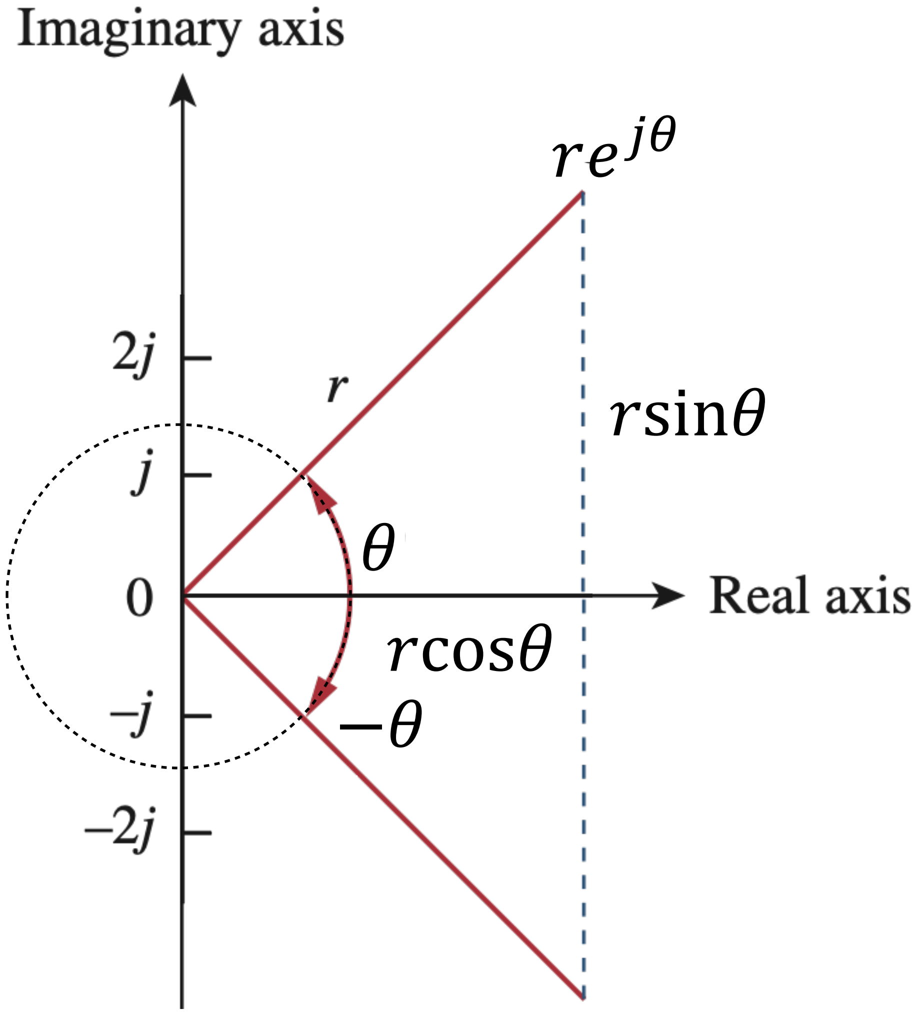
Analytic Signal Model

From Euler's identity

$$re^{j\theta} = (r\cos\theta) + j(r\sin\theta) \qquad j = \sqrt{-1}$$

$$r\cos(\theta) = \Re\{re^{j\theta}\}$$ “real part”

$$r\sin(\theta) = \Im\{re^{j\theta}\}$$ “imaginary part”



Setting $r = a(t)$ and $\theta = \omega_o t + \phi(t)$, we obtain a general complex signal model for radar applications.

$$\boxed{s(t) = a(t)e^{j(\omega_o t+\phi(t))}}$$

AM

Carrier

PM

Or by letting $\omega_d = -d\phi/dt \rightarrow \phi(t) = -\omega_d t$

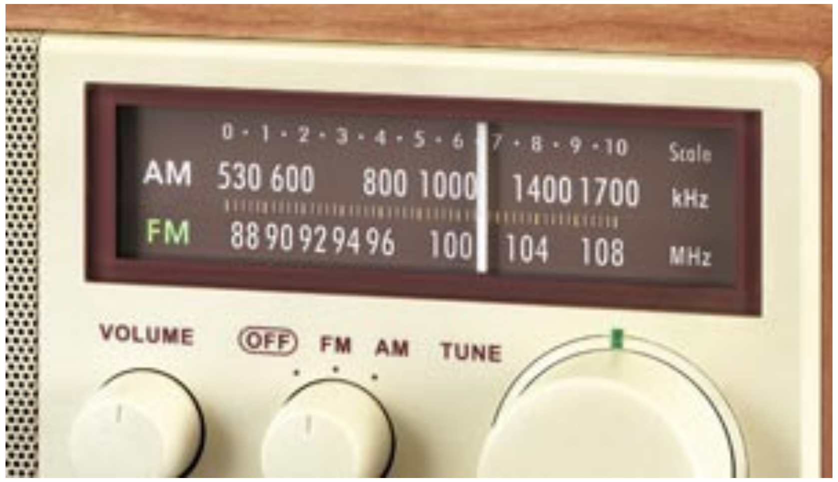
$$s(t) = a(t)e^{j(\omega_o-\omega_d)t}$$

FM

Now through Euler’s identity :

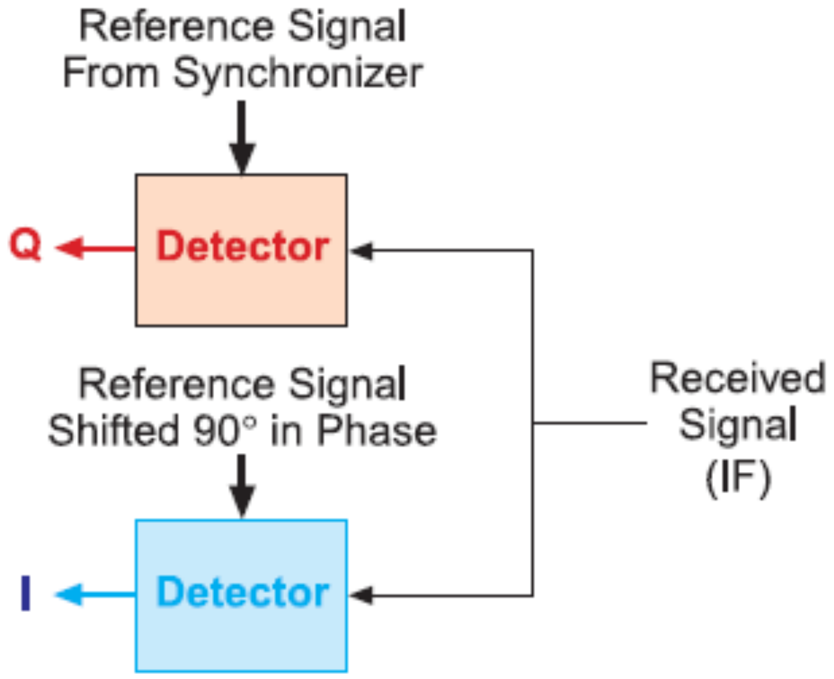
$$\Re\{s(t)\} = a(t)\cos(\omega_o t + \phi(t))$$

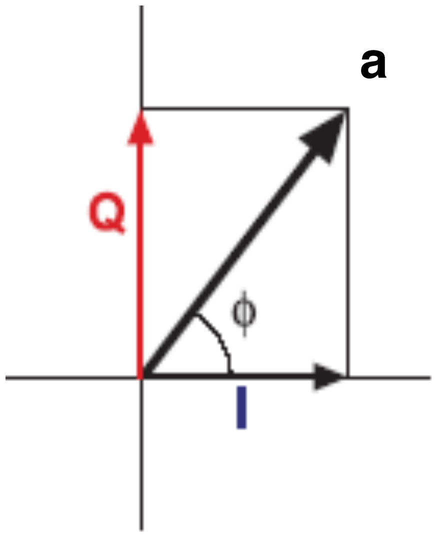
$$\Im\{s(t)\} = a(t)\sin(\omega_o t + \phi(t))$$



I and Q Demodulation

Consider radar transmission of a simple RF pulse. The reflected signal from the target will be the original pulse with some time varying amplitude and phase applied to it:





$$s_R(t) = a(t)\cos(\omega_o t + \phi(t))$$

We compute the analytic signal by “mixing” with cosine and sine.

Mixing with $\cos(\omega_0 t)$ gives the **“in-phase” (I) channel:**

$$\begin{aligned} s_R(t)\cos(\omega_o t) &= a(t)\cos(\omega_o t + \phi(t))[\cos(\omega_o t)] \\ &= a(t)\frac{1}{2}\Big(\underbrace{\cos[2\omega_o t + \phi(t)]}_{\text{filter out}} + \cos[\phi(t)]\Big) \end{aligned}$$

Mixing with $\sin(\omega_0 t)$ gives the “**quadrature**” **(Q) channel:**

$$\begin{aligned} s_R(t)[\sin(\omega_o t)] &= a(t)\cos(\omega_o t + \phi(t))[\sin(\omega_o t)] \\ &= a(t)\frac{1}{2}\Big(\underbrace{\sin[2\omega_o t + \phi(t)]}_{\text{filter out}} + \sin[\phi(t)]\Big) \end{aligned}$$

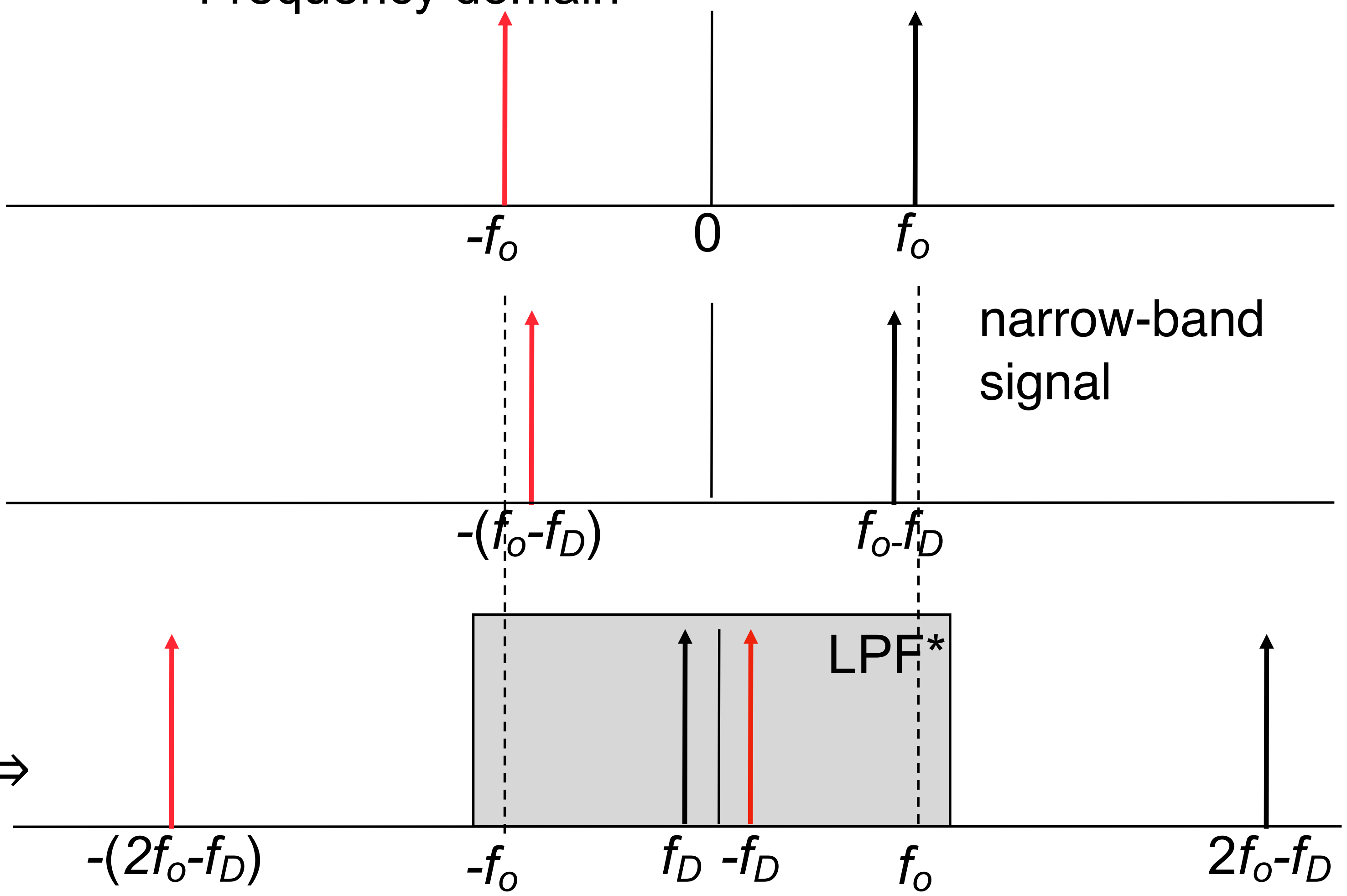
If we include a gain of 2, we retain the original signal energy. Using Euler’s identity we obtain the analytic baseband signal:

$$\boxed{s_B(t) = a(t)e^{j\phi(t)} = a(t)\cos\phi(t) + ja(t)\sin\phi(t) =} I + jQ$$

I/Q demodulation produces a time-series of complex voltage samples (I_n, Q_n) from which we can construct a discrete representation of $s_B(t)$. The Doppler frequency shift is the time rate of change of the phase, $\omega_D = -d\phi/dt$.

I/Q Demodulation: Frequency Domain

Frequency domain



*Low Pass Filter

**Fast Fourier Transform

Transmitted signal:

$$\cos(2\pi f_o t) \Leftrightarrow$$

Reflected signal from moving target

$$\cos(2\pi(f_o - f_D)t) \Leftrightarrow$$

Mixed (multiplied) with oscillator $\cos(2\pi f_o t)$

$$\frac{1}{2}\cos[2\pi(2f_o - f_D)t] + \frac{1}{2}\cos[2\pi f_D t] \Leftrightarrow$$

To resolve both positive and negative Doppler shifts, we need:

$$e^{j2\pi f_D t} = \cos(2\pi f_D t) + j\sin(2\pi f_D t)$$

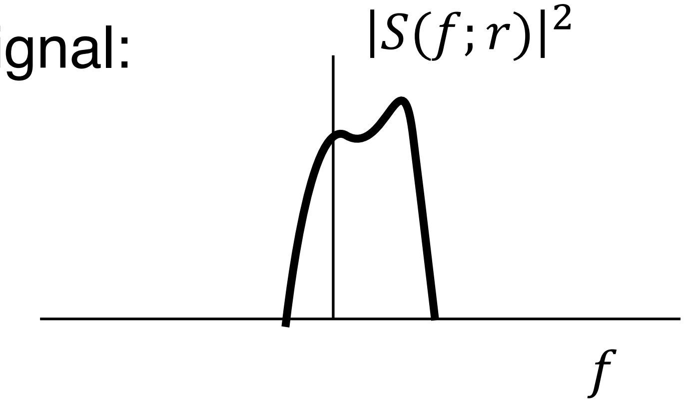
We thus need to mix with a second oscillator at same frequency but $90°$ out of phase. For a cosine reference, the quadrature function is sine. The two components are called “in phase” (*I*) and “quadrature” (*Q*). Together *I* and *Q* represent discrete samples of the baseband analytic signal,

$$s_B(t) = Ae^{j2\pi f_D t} = I(t) + jQ(t) \xLeftrightarrow{\text{FT}} A\delta(f - f_D)$$ (for a single scatterer)

Correlation and the ISR Spectrum

How do we compute the power spectrum from our complex voltages ?

One approach is to compute Fourier transform of the range-resolved signal:



$$s(r,t) = I(r,t) + Q(r,t) \Leftrightarrow S(r,f)$$

from which the power spectrum may be represent as $|S(r,f)|^2$

Based on the stochastic nature of the target, and the way ISR samples the echos, we will take a different approach. We first compute the auto-correlation function (ACF),

$$R_s(r,\tau) = \frac{\langle s(r,t)\overline{s(r,t+\tau)}\rangle}{\langle |s(r,t)|^2\rangle}$$

where the angle brackets denote the ensemble average, or the expected value.
The power spectral density is given by the Fourier transform of the R_s

$$R_s(r,\tau) \Leftrightarrow |S(r,f)|^2 \qquad \text{(Wiener-Khinchin theorem)}$$

The discrete representation of $R_s(r,\tau)$ is constructed through appropriate scaling and multiplication of the complex voltage samples $s(r_k,t_n)$.

Soon we will begin to explore methods for constructing the ACF.

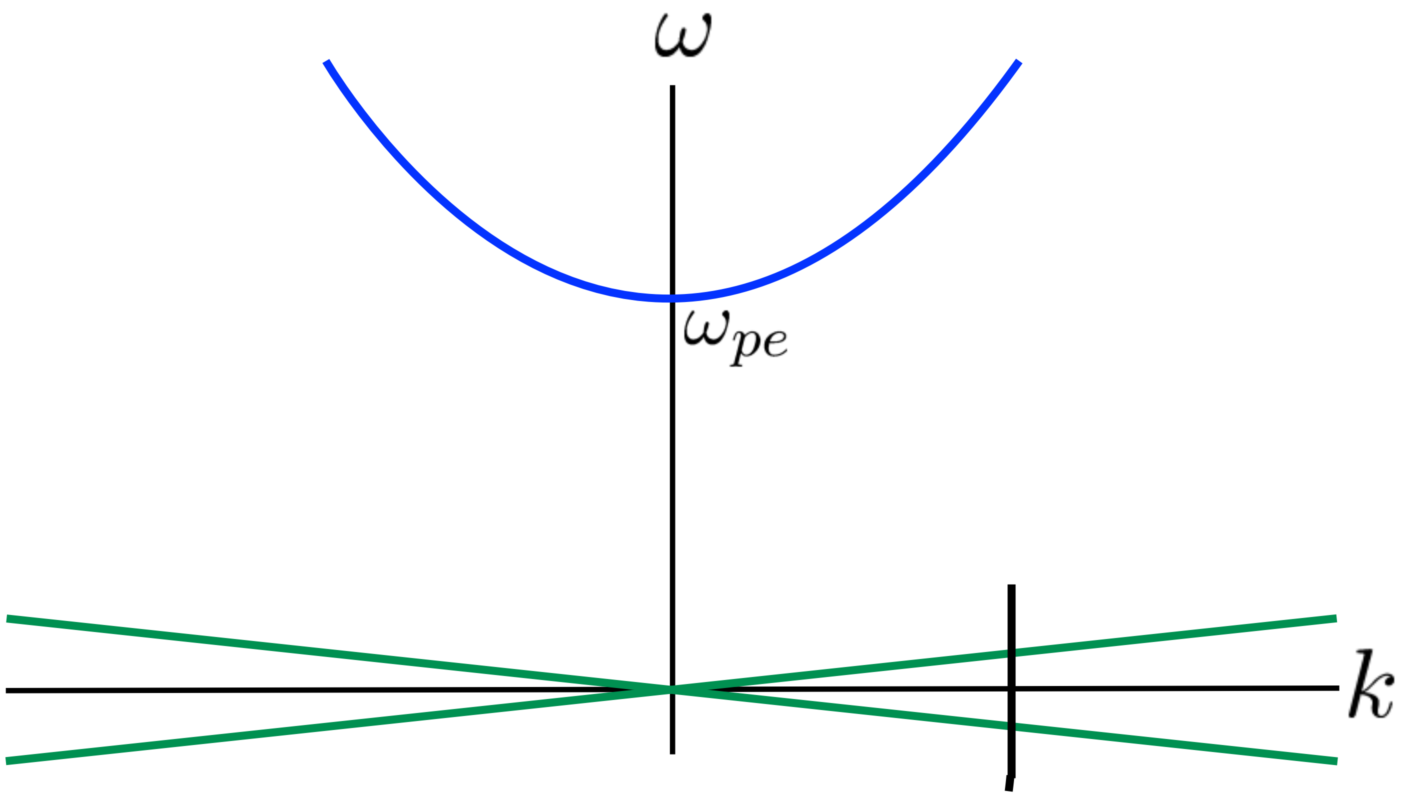
Incoherent Scatter Radar (ISR)

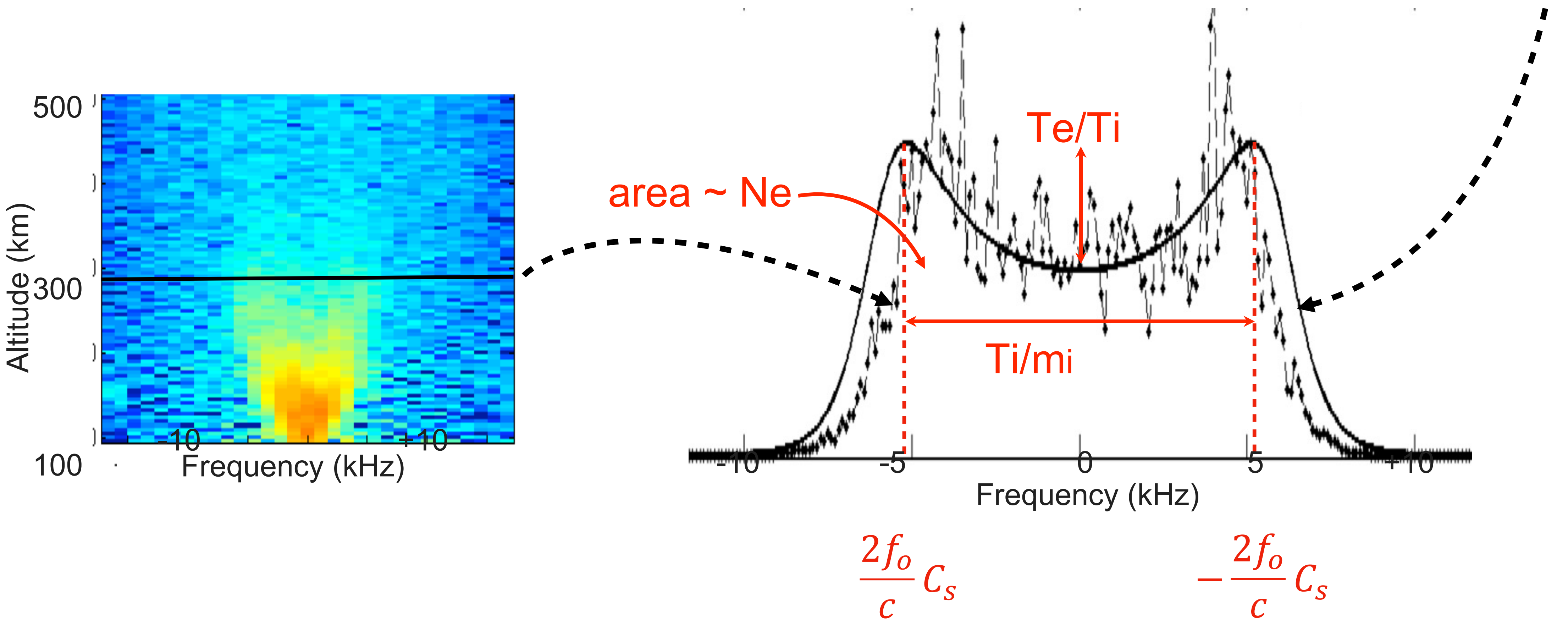
Ion-acoustic

$$\omega_s = C_s k \qquad C_s = \sqrt{k_B(T_e + 3T_i)/m_i}$$

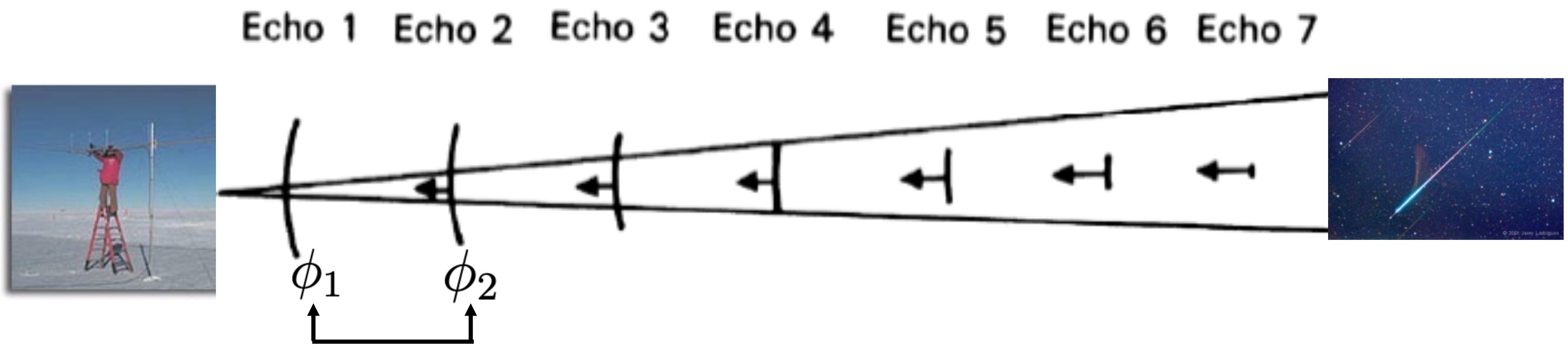
Langmuir

$$\omega_L = \sqrt{\omega_{pe}^2 + 3\,k^2\,v_{the}^2} \approx \omega_{pe} + \frac{3}{2} v_{the} \lambda_{De} k^2$$





Meteor Radar Example

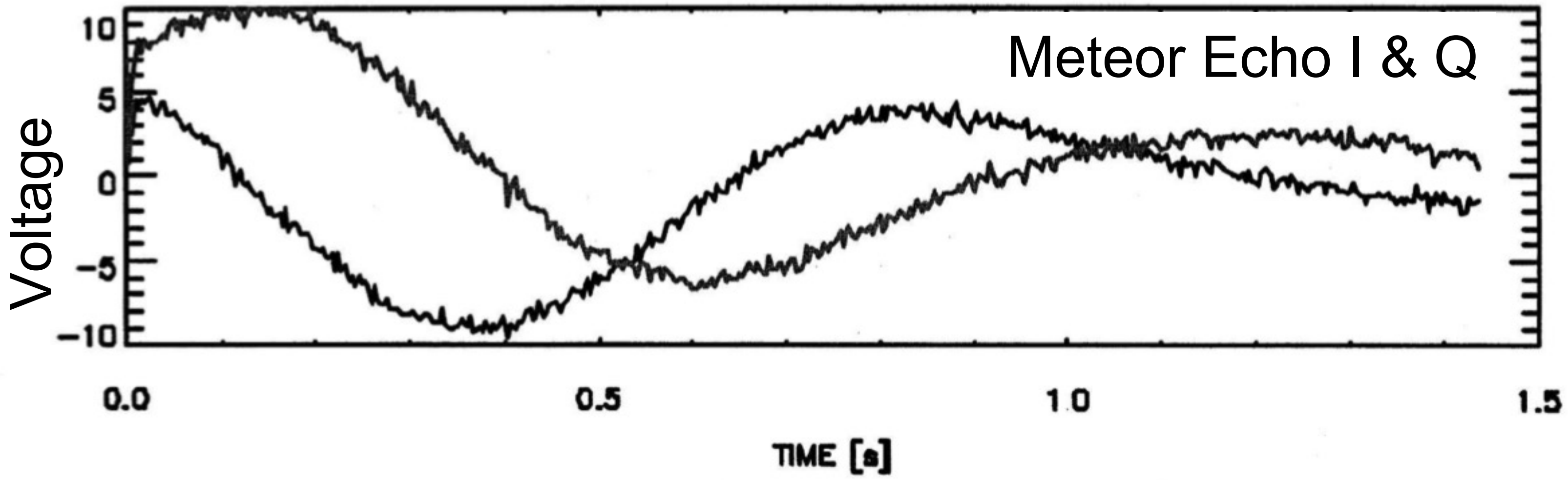


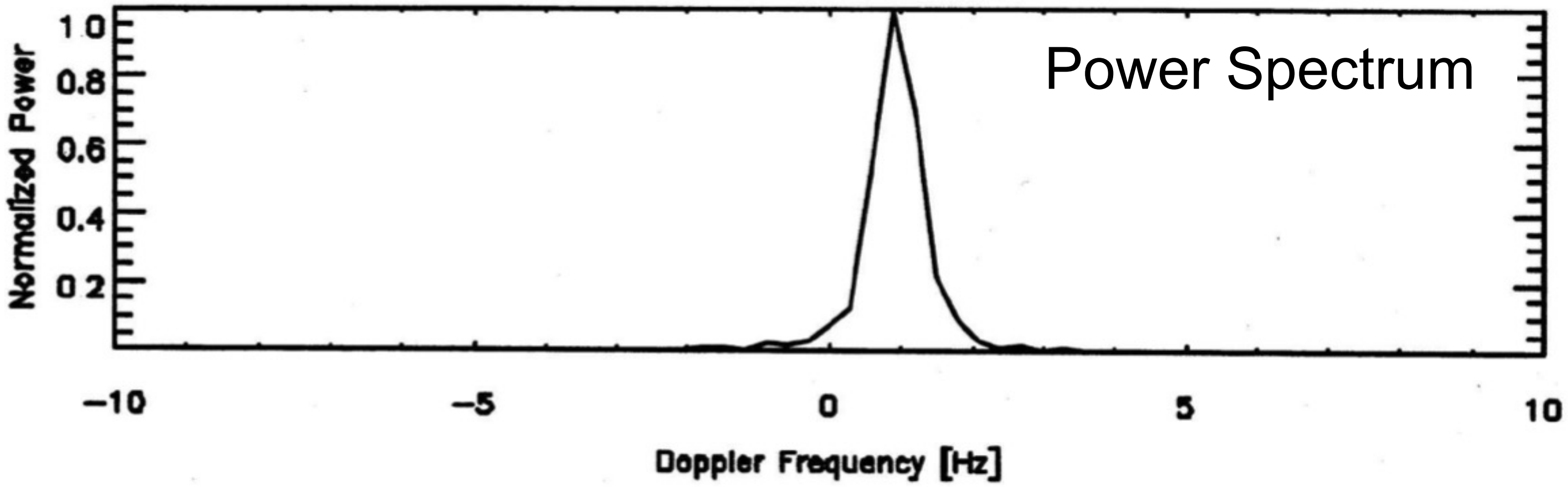
Inter-pulse period (IPP)

Coherent target (meteor ionization trail), with ~constant velocity.

Find velocity (hence, neutral wind velocity along radar line of sight) by sampling I and Q from many pulses, taking the Fourier Transform (FFT), and forming $|S(f)|^2$

Velocity and reflected power are found from the peak in the power spectrum.





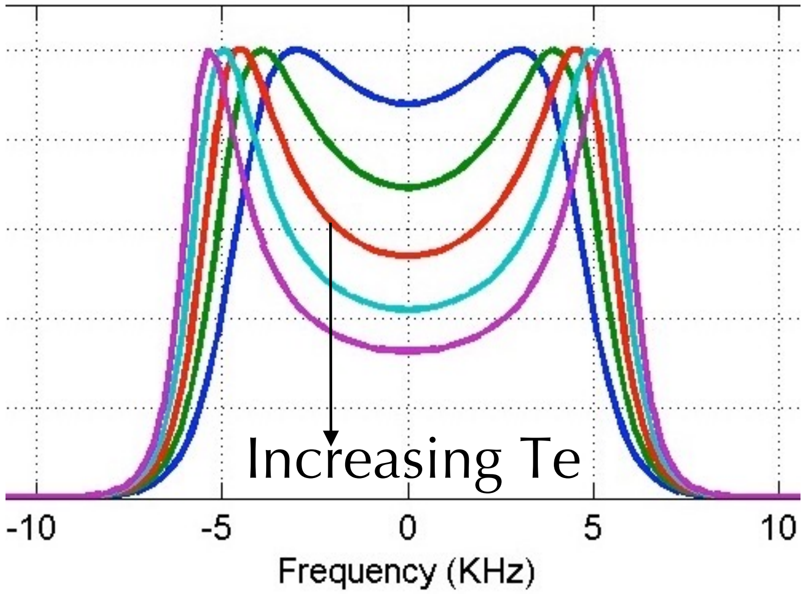
Does this strategy work for ISR?

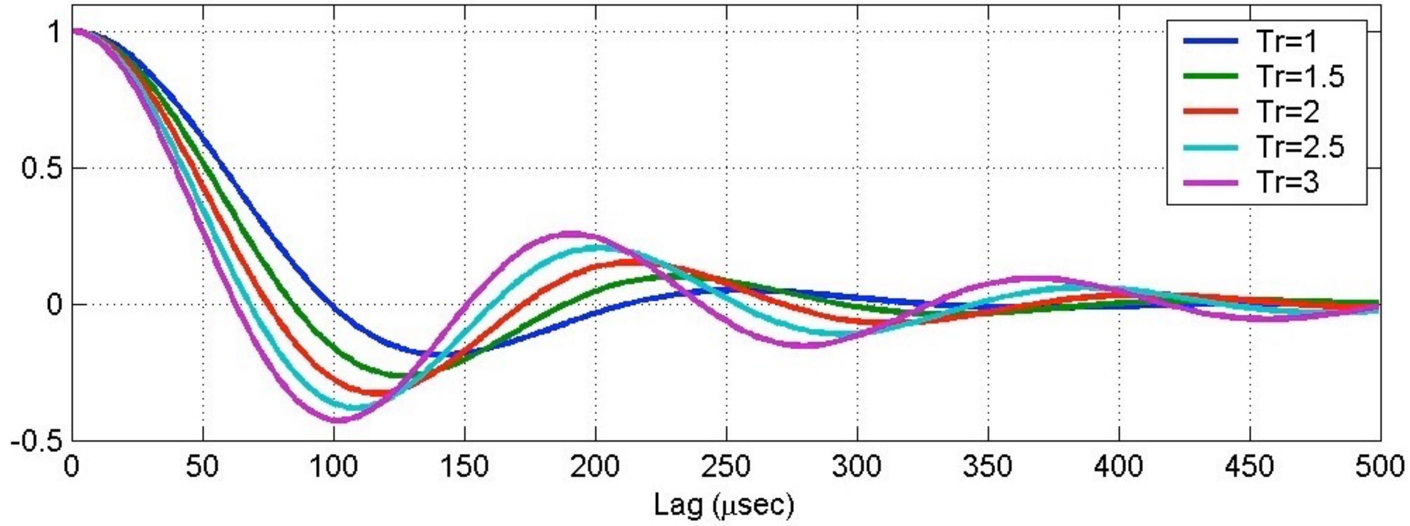
Doppler width at 450 MHz: 10 kHz
de-correlation time (zero crossing): ~1/10kHz = 0.1 ms
Inter-pulse period (IPP) to reach 450 km: $2R/c$ = 3ms

Plasma has de-correlated by the time we send the next pulse.

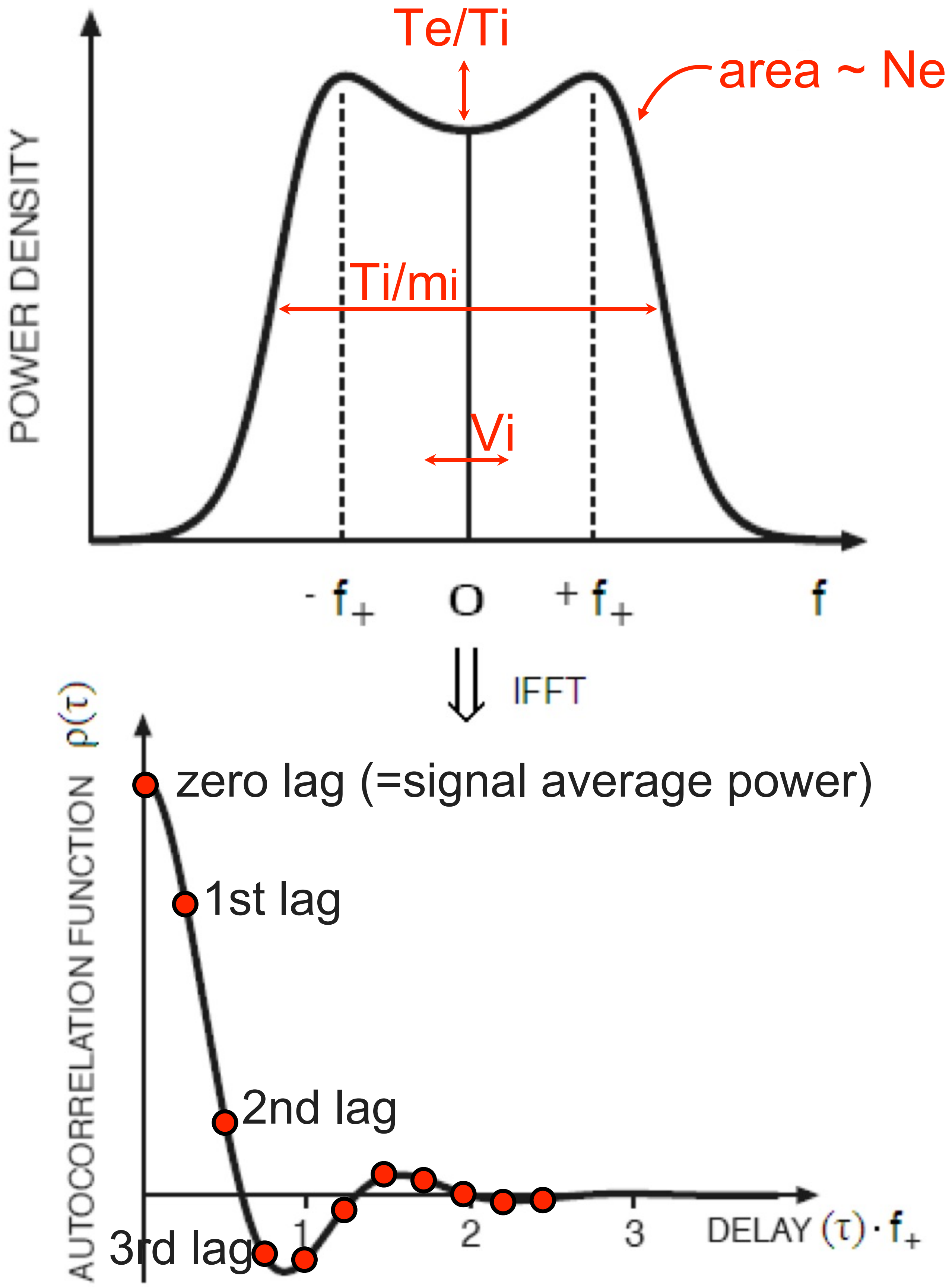
Stated alternately, the Doppler frequency shift of the plasma is much higher than the maximum unambiguous Doppler shift measurable for the pulse-repetition frequency.

ISR spectrum ⟺ Autocorrelation function (ACF)





Autocorrelation function and power spectrum



Ion temperature (Ti) to ion mass (mi) ratio from the width of the spectra

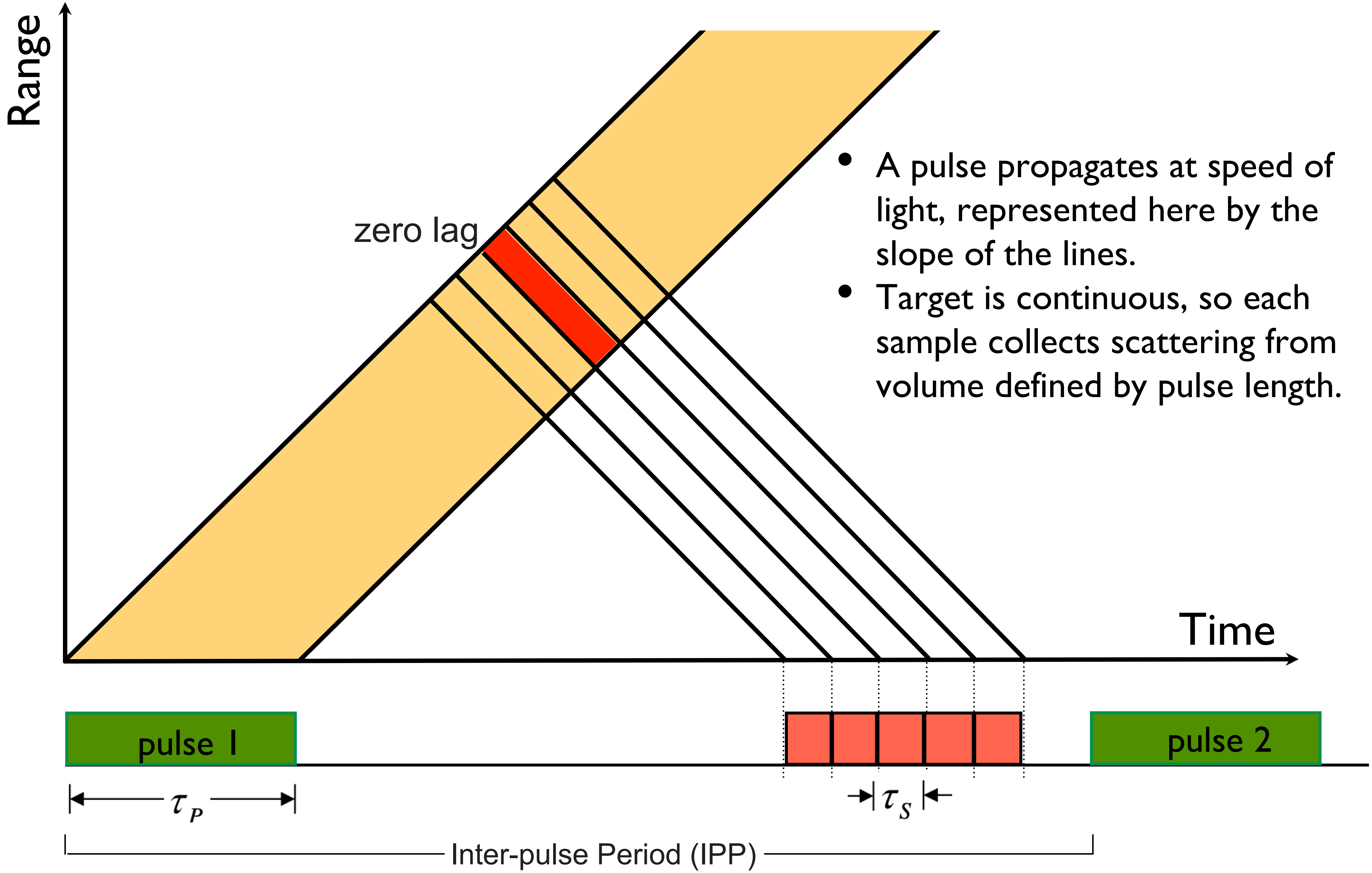
Electron to ion temperature ratio (Te/Ti) from “peak-to-valley” ratio

Electron (= ion) density from total area (corrected for temperatures)

Line-of-sight ion velocity (Vi) from bulk Doppler shift

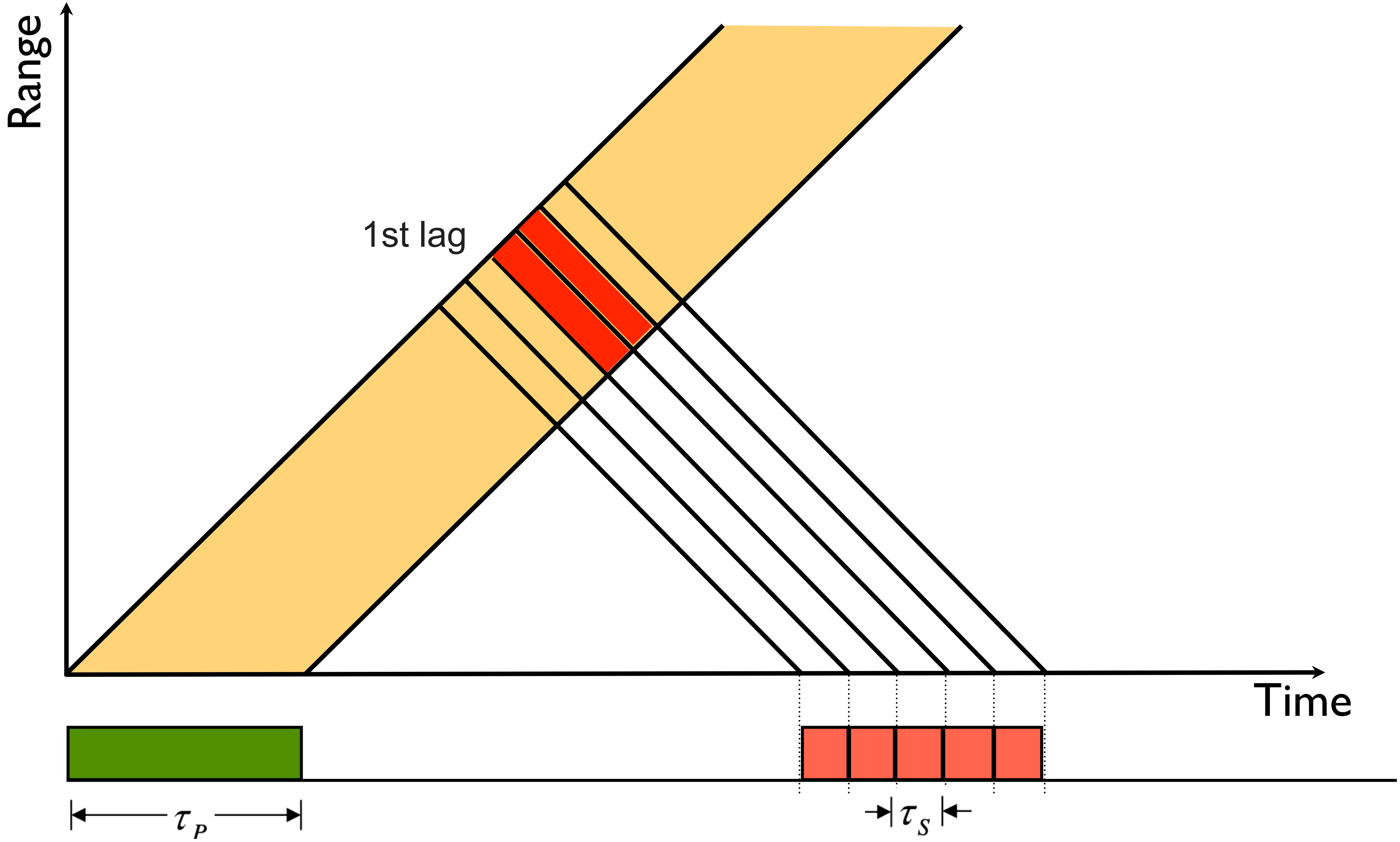
Our goal is to sample lags with sufficient fidelity to provide meaningful estimates of plasma parameters

Computing the ACF (and, hence, spectrum)

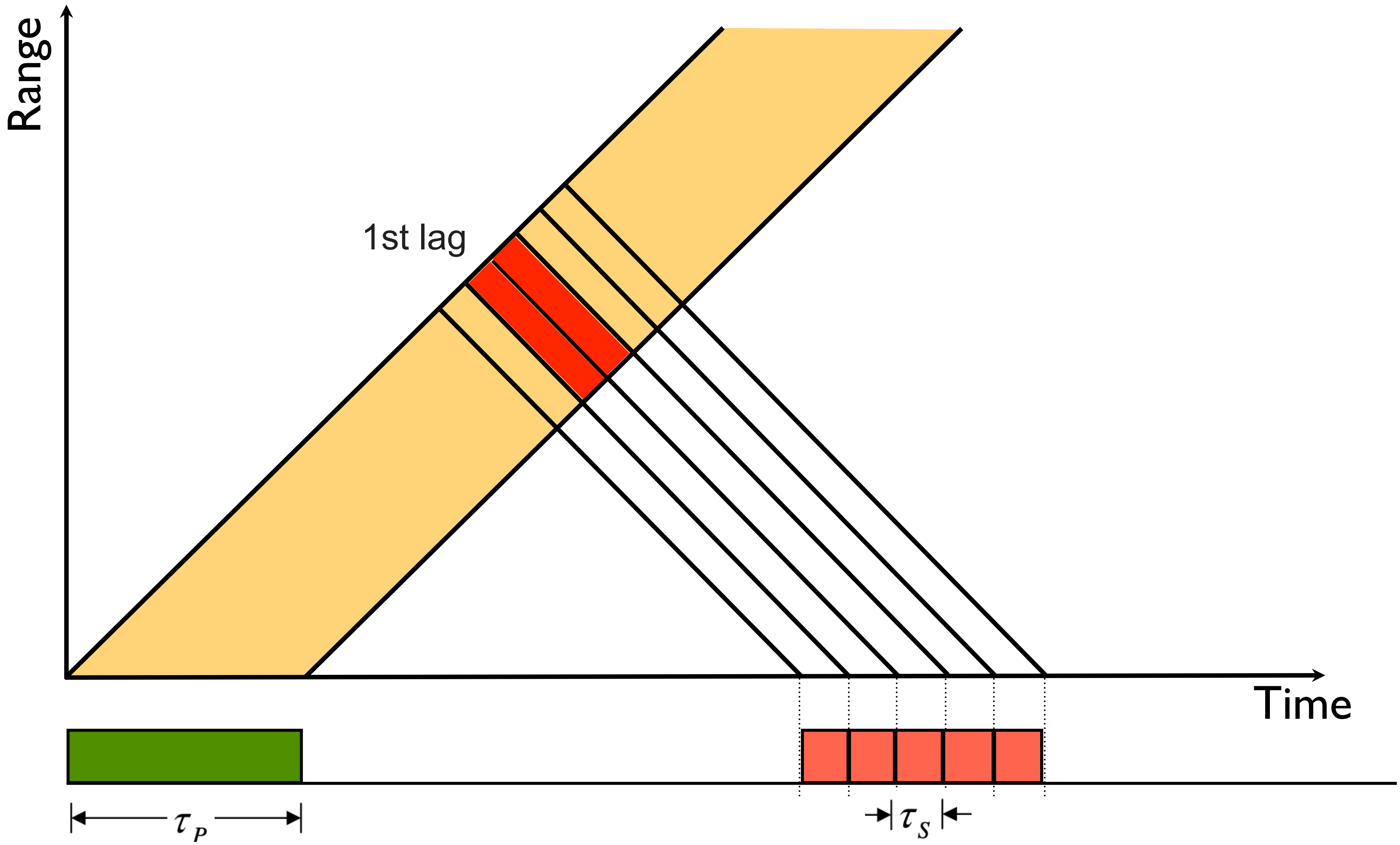


- A pulse propagates at speed of light, represented here by the slope of the lines.
- Target is continuous, so each sample collects scattering from volume defined by pulse length.

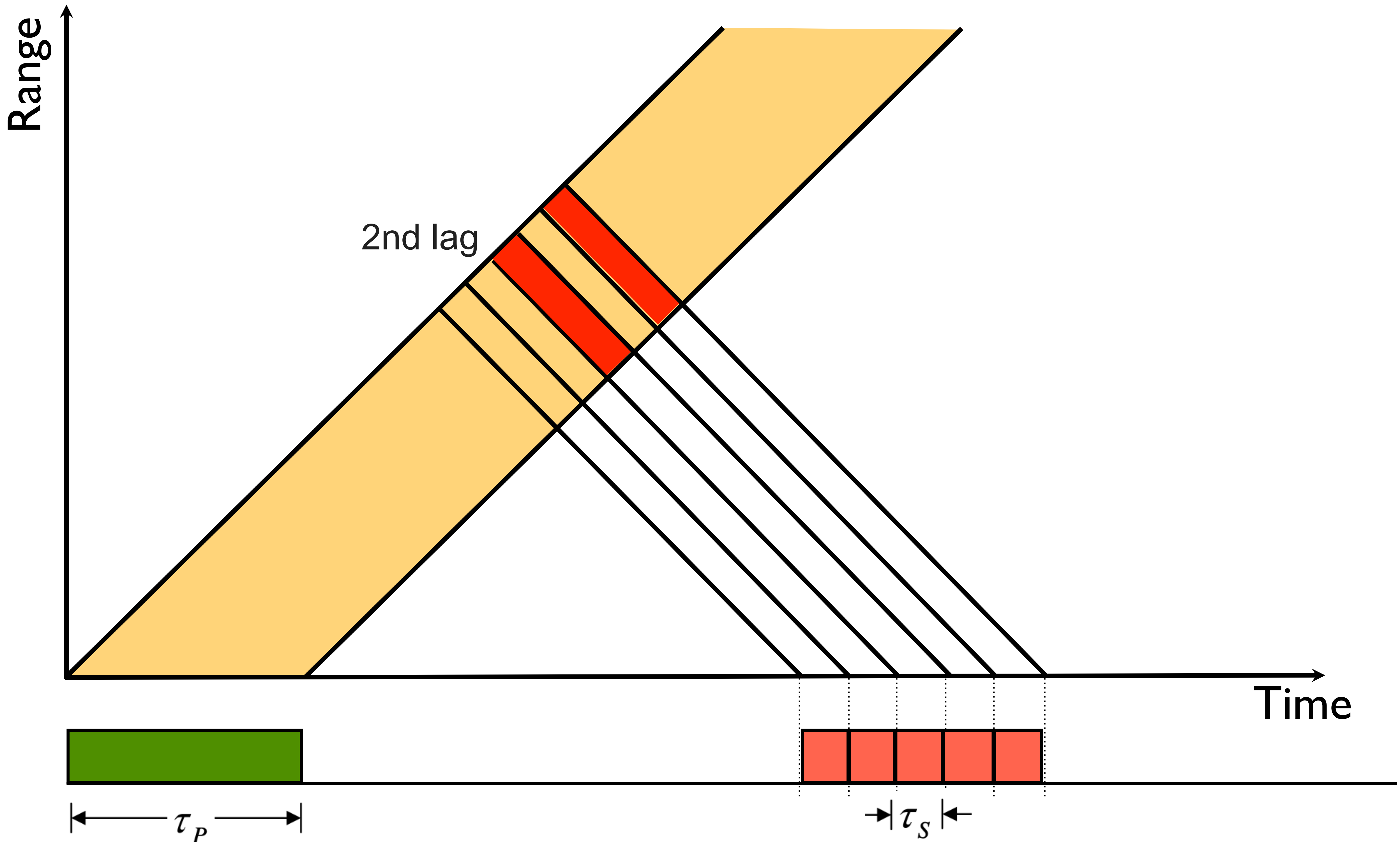
Computing the ACF (and, hence, spectrum)



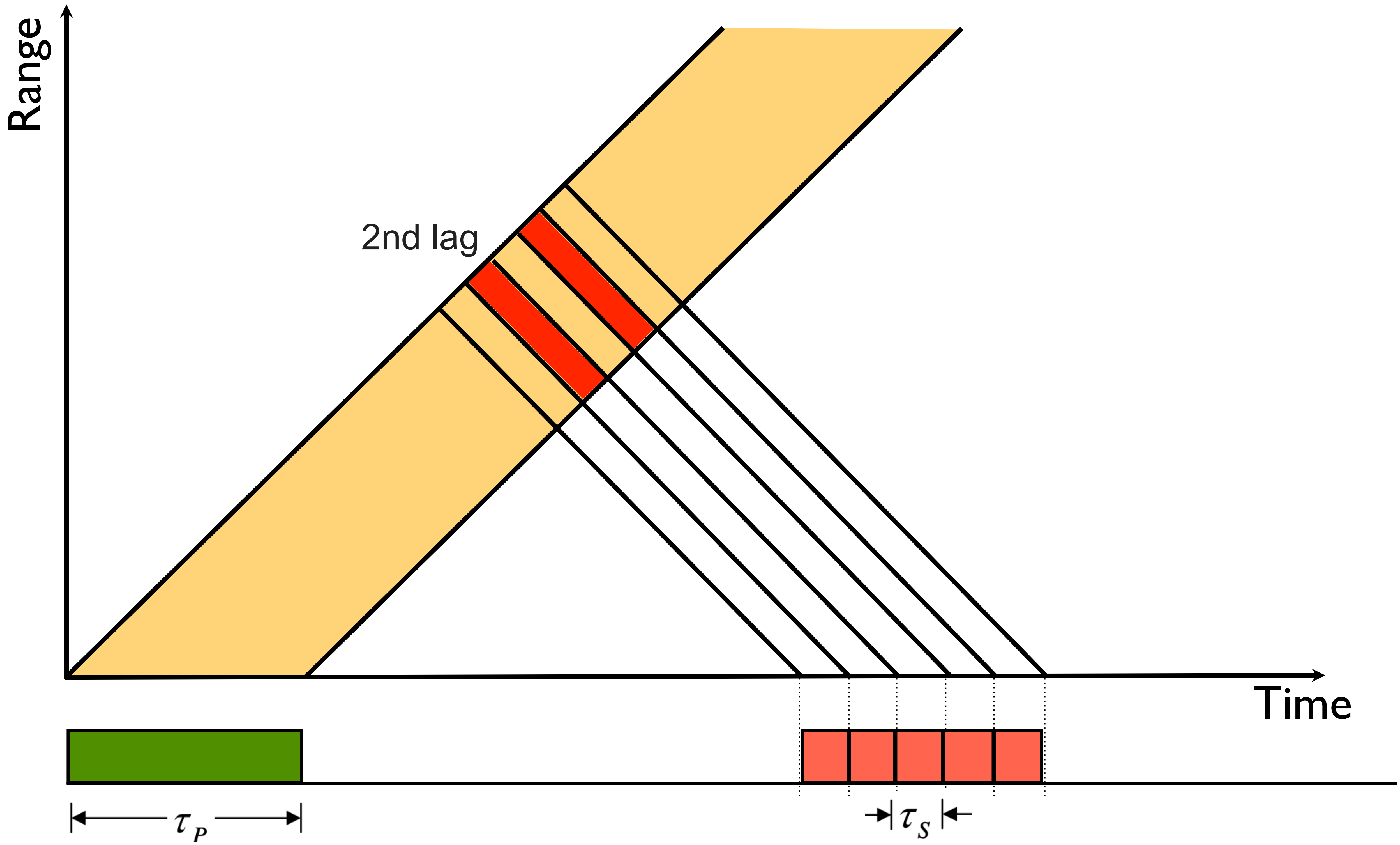
Computing the ACF (and, hence, spectrum)



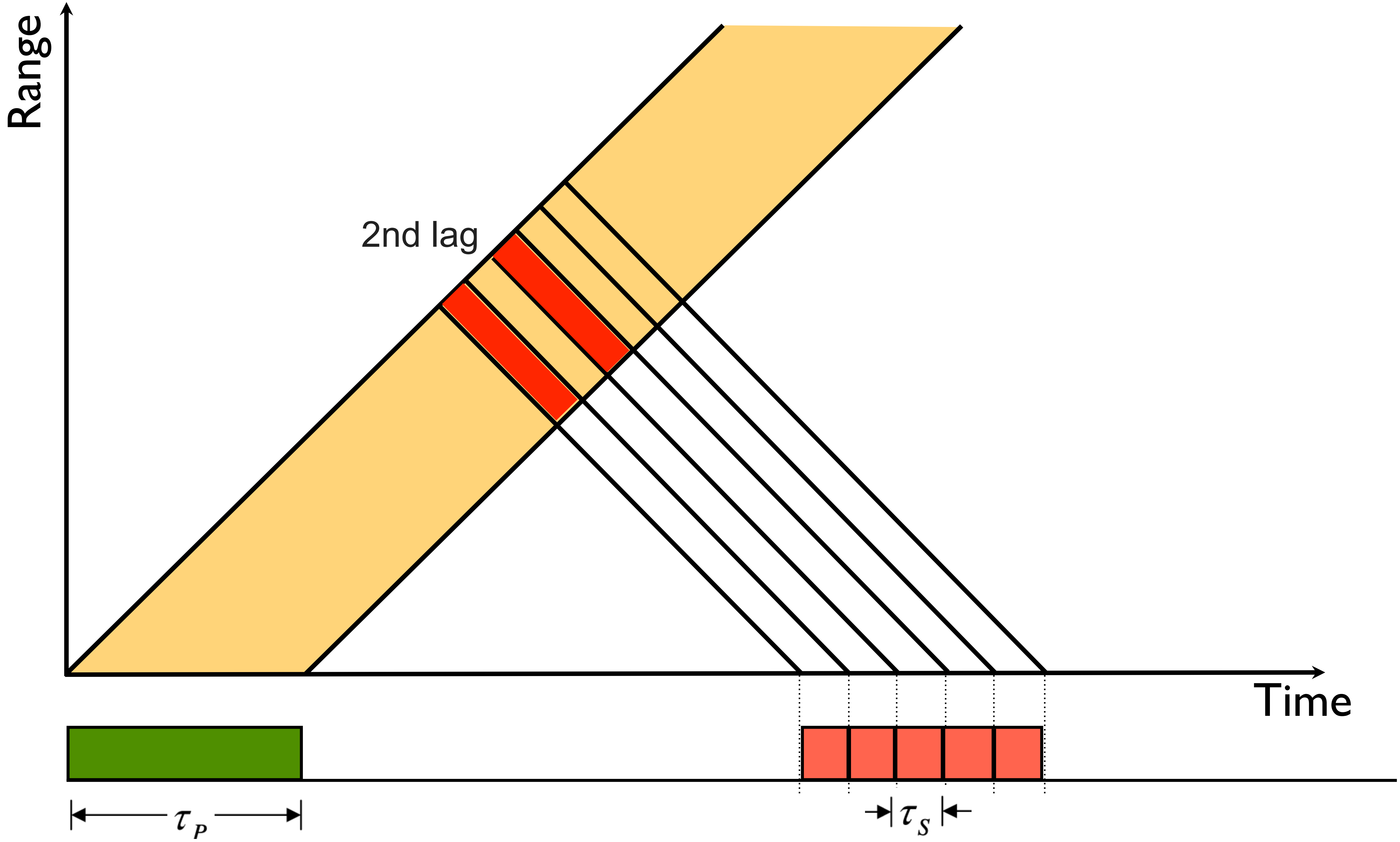
Computing the ACF (and, hence, spectrum)



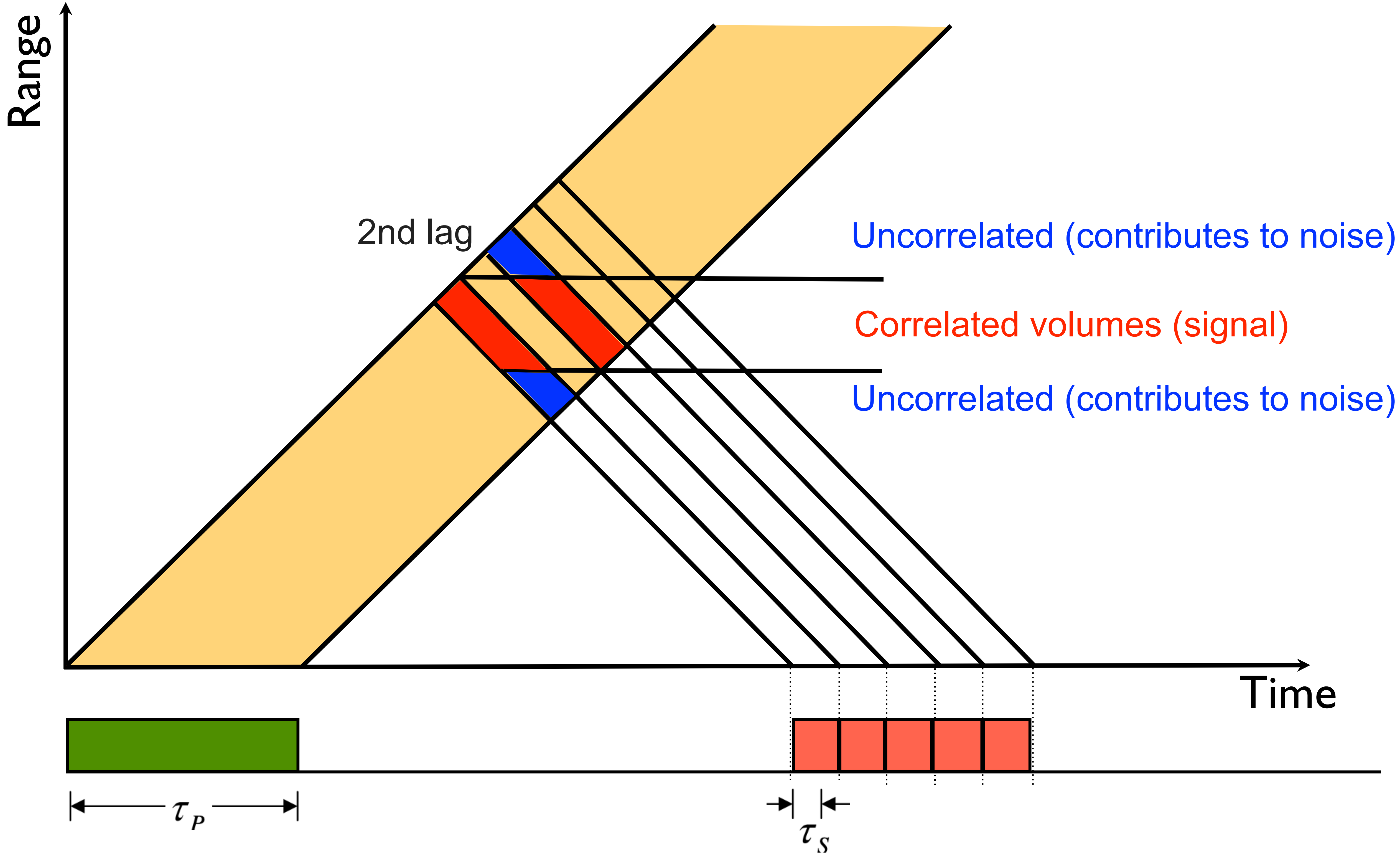
Computing the ACF (and, hence, spectrum)



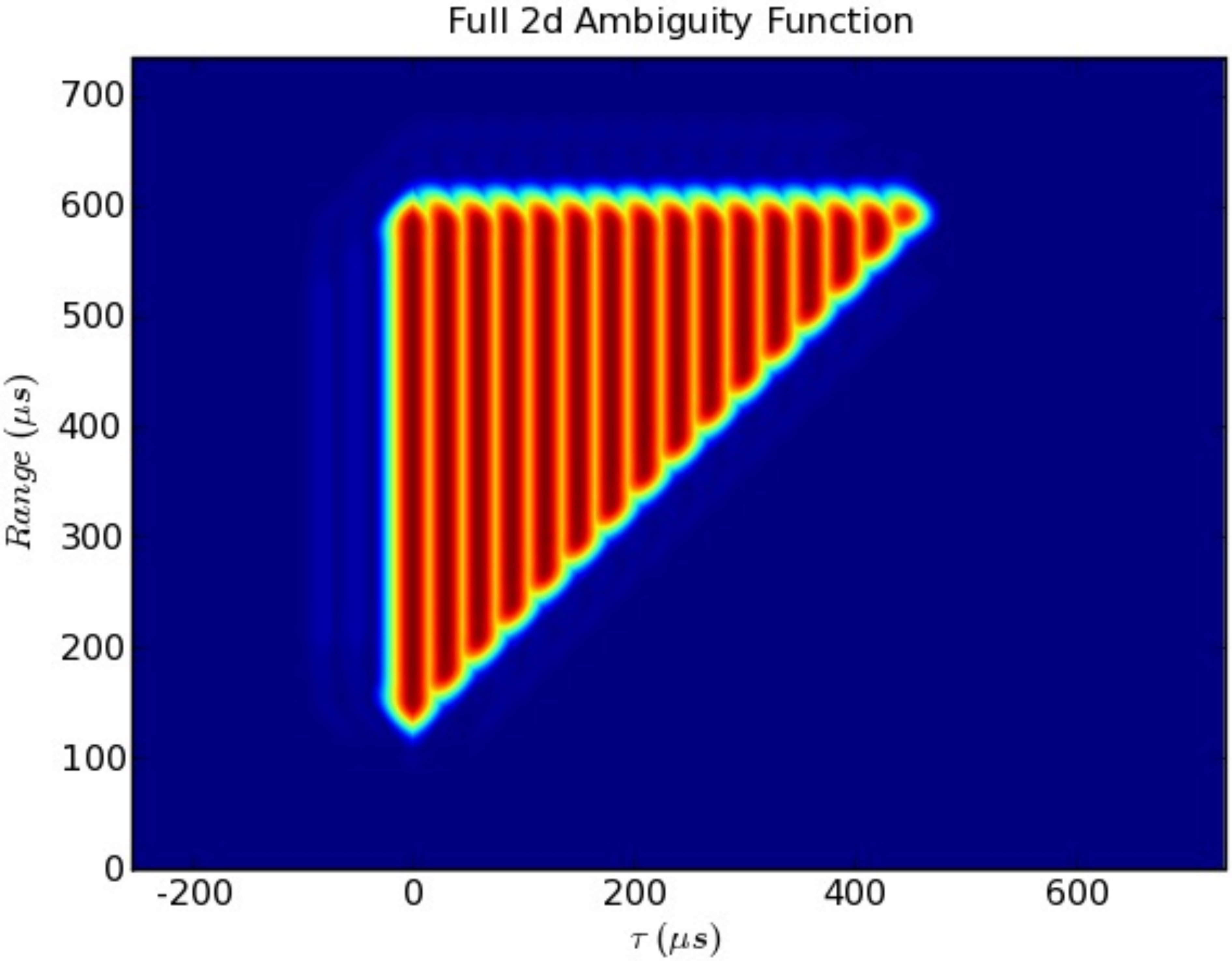
Computing the ACF (and, hence, spectrum)



Computing the ACF (and, hence, spectrum)



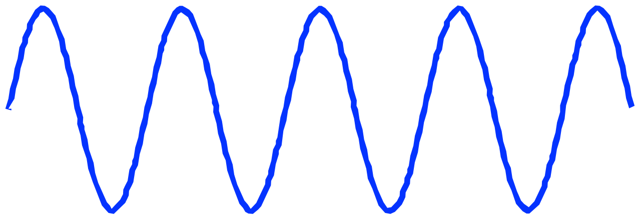
Ambiguity function

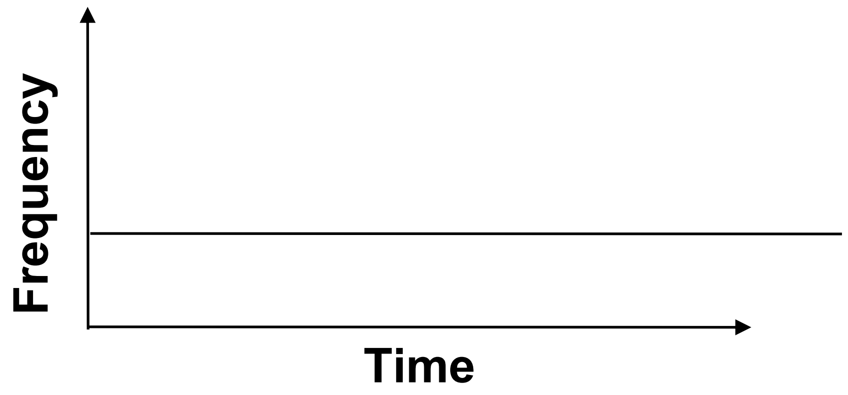


480 microsec pulse
30 microsec sampling
Anti-aliasing filter

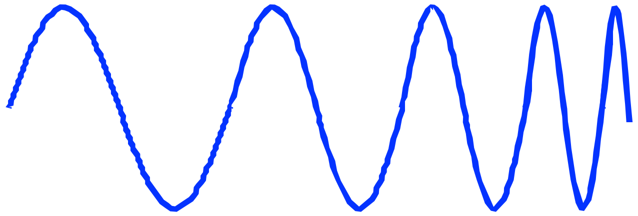
Radar Waveforms

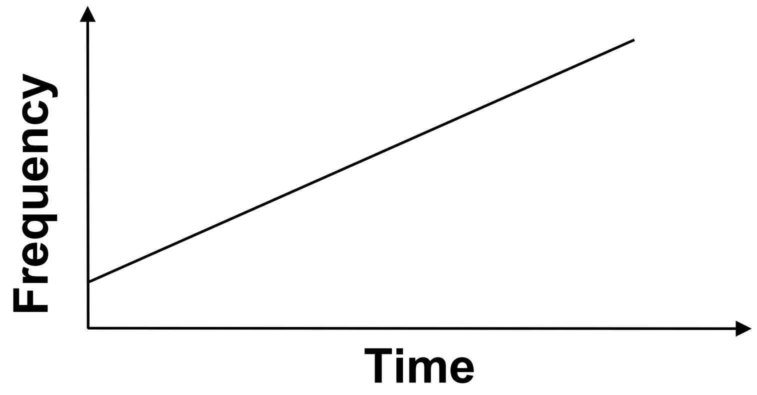
Pulse at single frequency





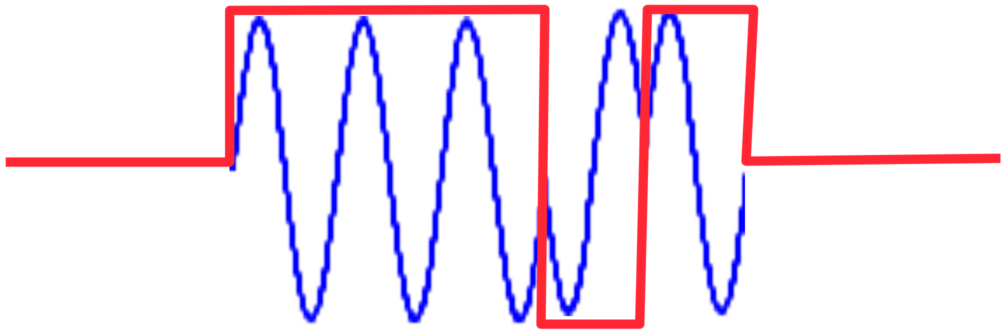
Pulse with changing frequency

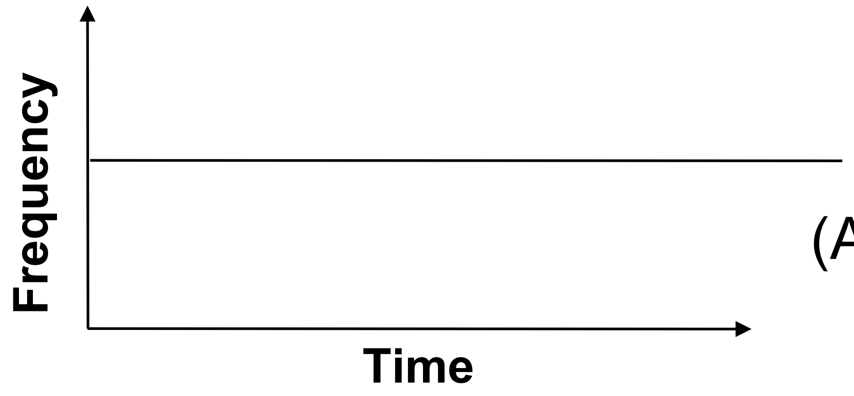




Linear Frequency-Modulated (LFM) Waveform

Pulse at single frequency, but variable phase



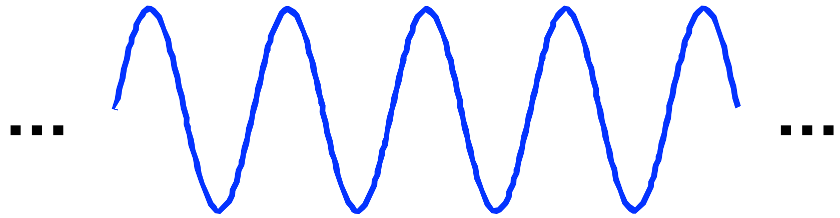


Phase-coded Waveform (Alternating codes Barker Codes)

Radar Waveforms

$$s(t) = A(t)\cos[2\pi f_o t + \phi(t)]$$

Unmodulated RF signal



$$s(t) = A_o e^{j2\pi f_o t}$$

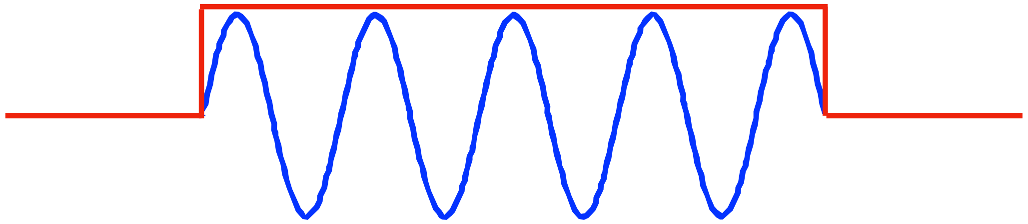
$$e^{j0} = 1$$

$$e^{j\pi} = -1$$

$$e^{j\pi/2} = j$$

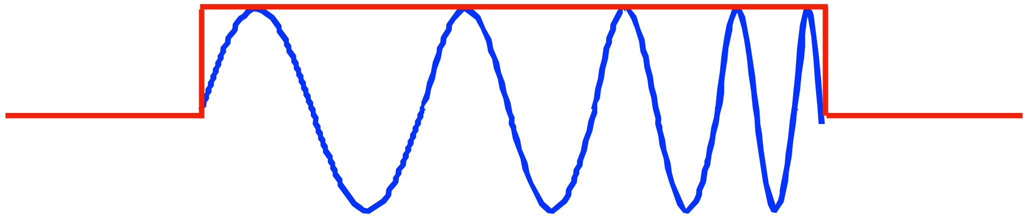
$$e^{-j\pi/2} = -j$$

RF pulse at a single frequency



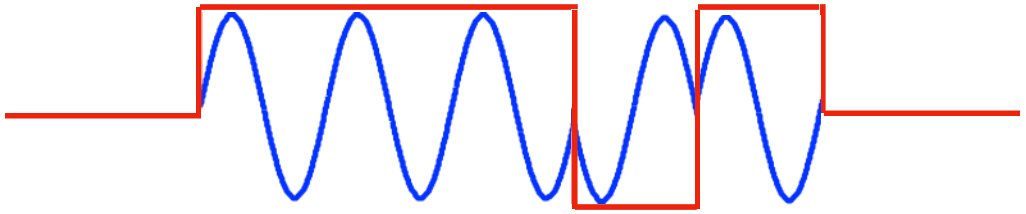
$$s(t) = A(t)e^{j2\pi f_o t}$$

RF Pulse with changing frequency



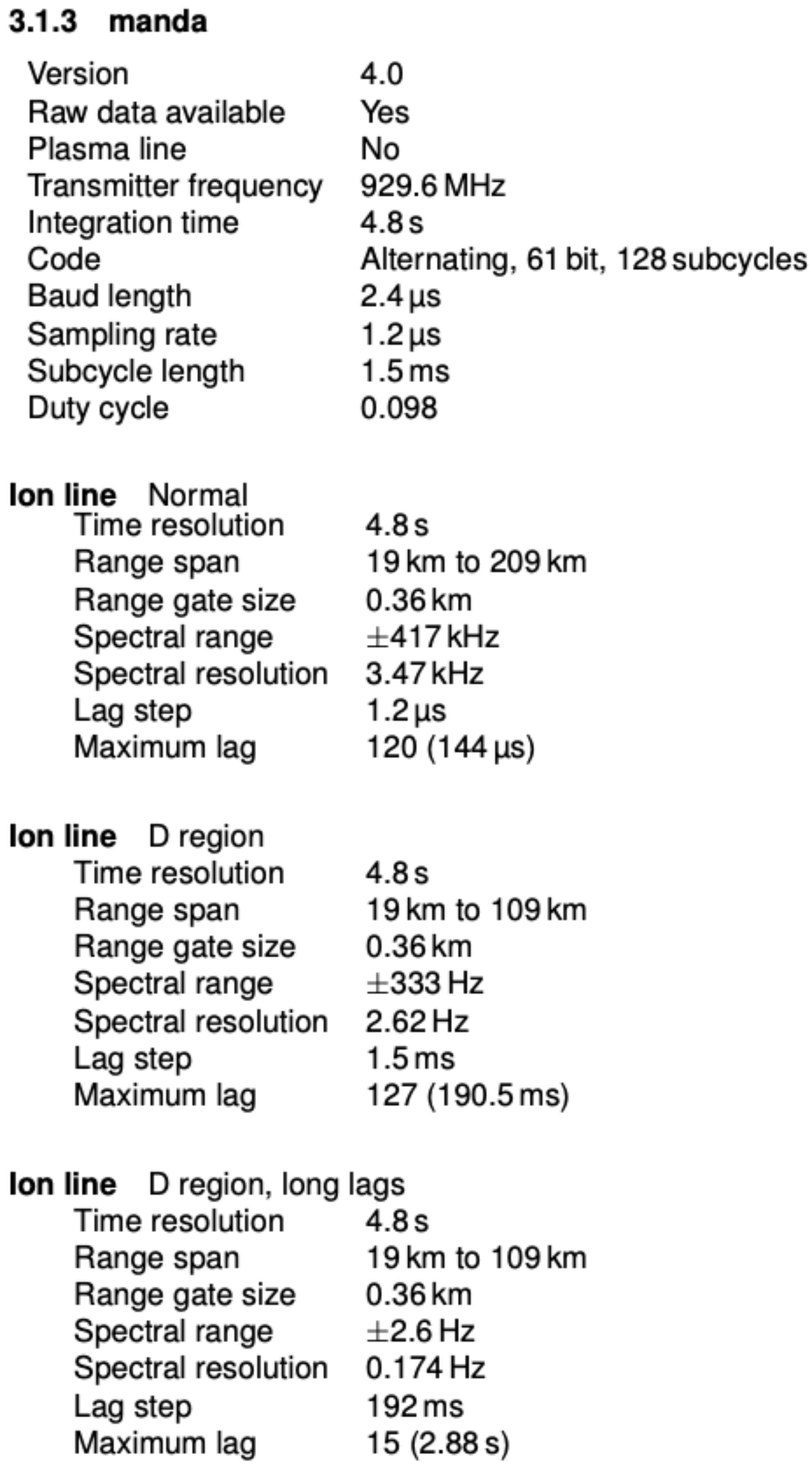
$$s(t) = A(t)e^{j2\pi(f_o+\Delta f(t)t}$$

RF Pulse, single frequency, changing phase



$$s(t) = A(t)e^{j2\pi f_o t}e^{j\phi(t)}$$

Example Radar Waveform Set



3.1.3 manda

Version	4.0
Raw data available	Yes
Plasma line	No
Transmitter frequency	929.6 MHz
Integration time	4.8 s
Code	Alternating, 61 bit, 128 subcycles
Baud length	2.4 µs
Sampling rate	1.2 µs
Subcycle length	1.5 ms
Duty cycle	0.098

Ion line Normal

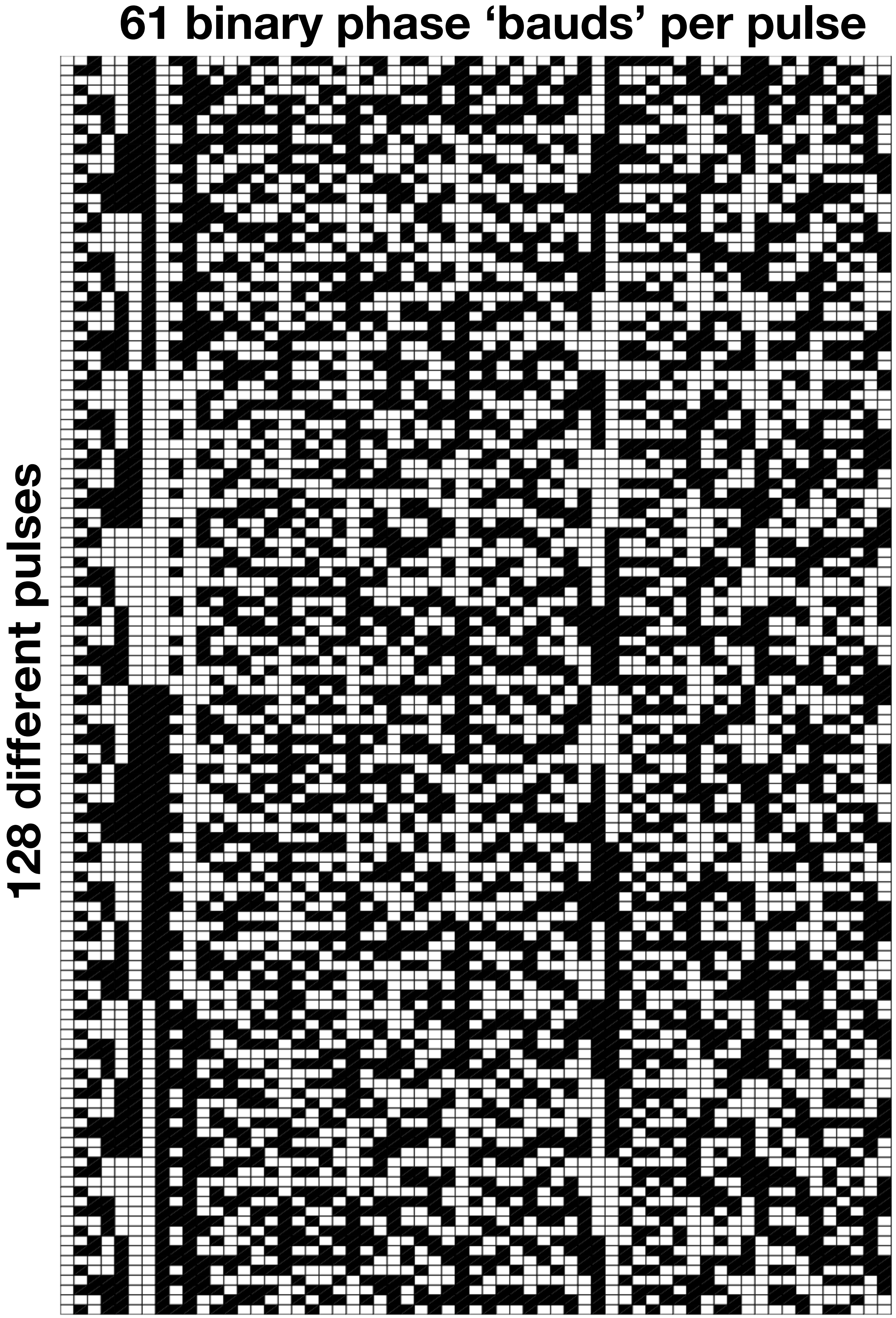
Time resolution	4.8 s
Range span	19 km to 209 km
Range gate size	0.36 km
Spectral range	±417 kHz
Spectral resolution	3.47 kHz
Lag step	1.2 µs
Maximum lag	120 (144 µs)

Ion line D region

Time resolution	4.8 s
Range span	19 km to 109 km
Range gate size	0.36 km
Spectral range	±333 Hz
Spectral resolution	2.62 Hz
Lag step	1.5 ms
Maximum lag	127 (190.5 ms)

Ion line D region, long lags

Time resolution	4.8 s
Range span	19 km to 109 km
Range gate size	0.36 km
Spectral range	±2.6 Hz
Spectral resolution	0.174 Hz
Lag step	192 ms
Maximum lag	15 (2.88 s)



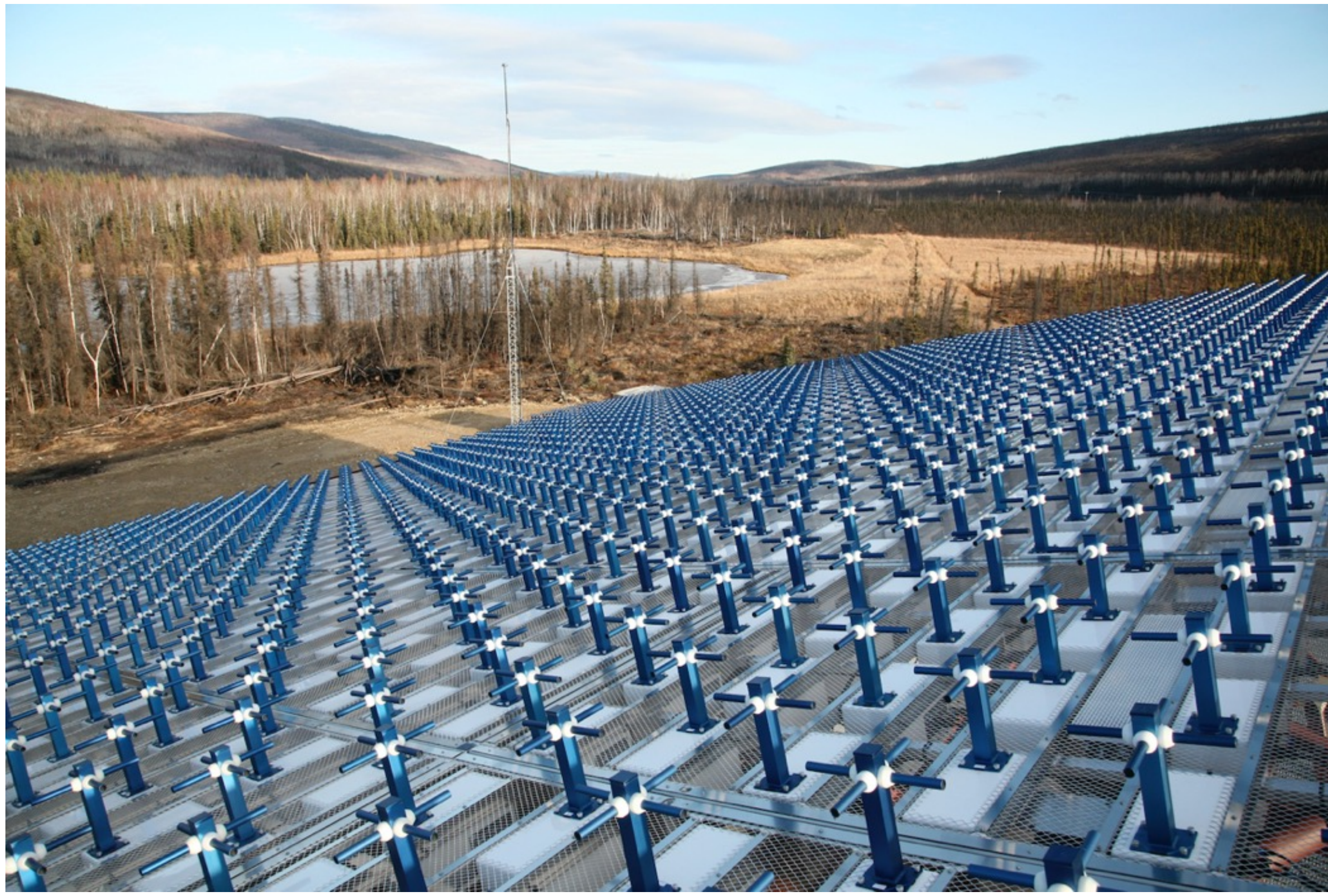
Dish Versus Phased-array

-FOV: Elevation angles > 30 deg

-Integration constrained by antenna motion

-Power concentrated at Klystron

-Significant mechanical complexity



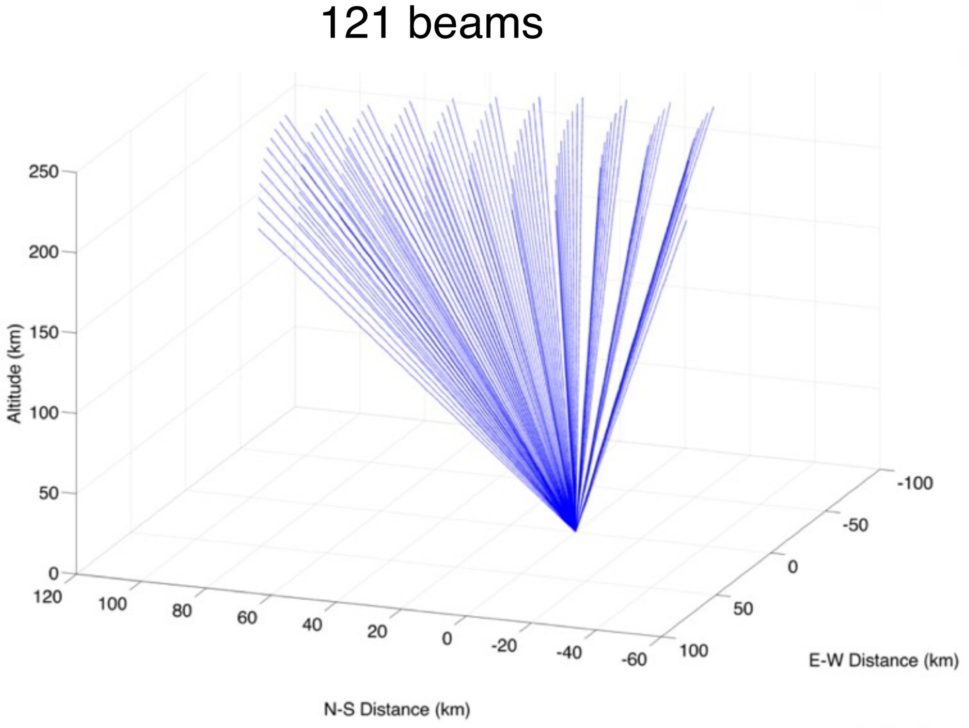
-FOV: +/- 25 degrees from boresight

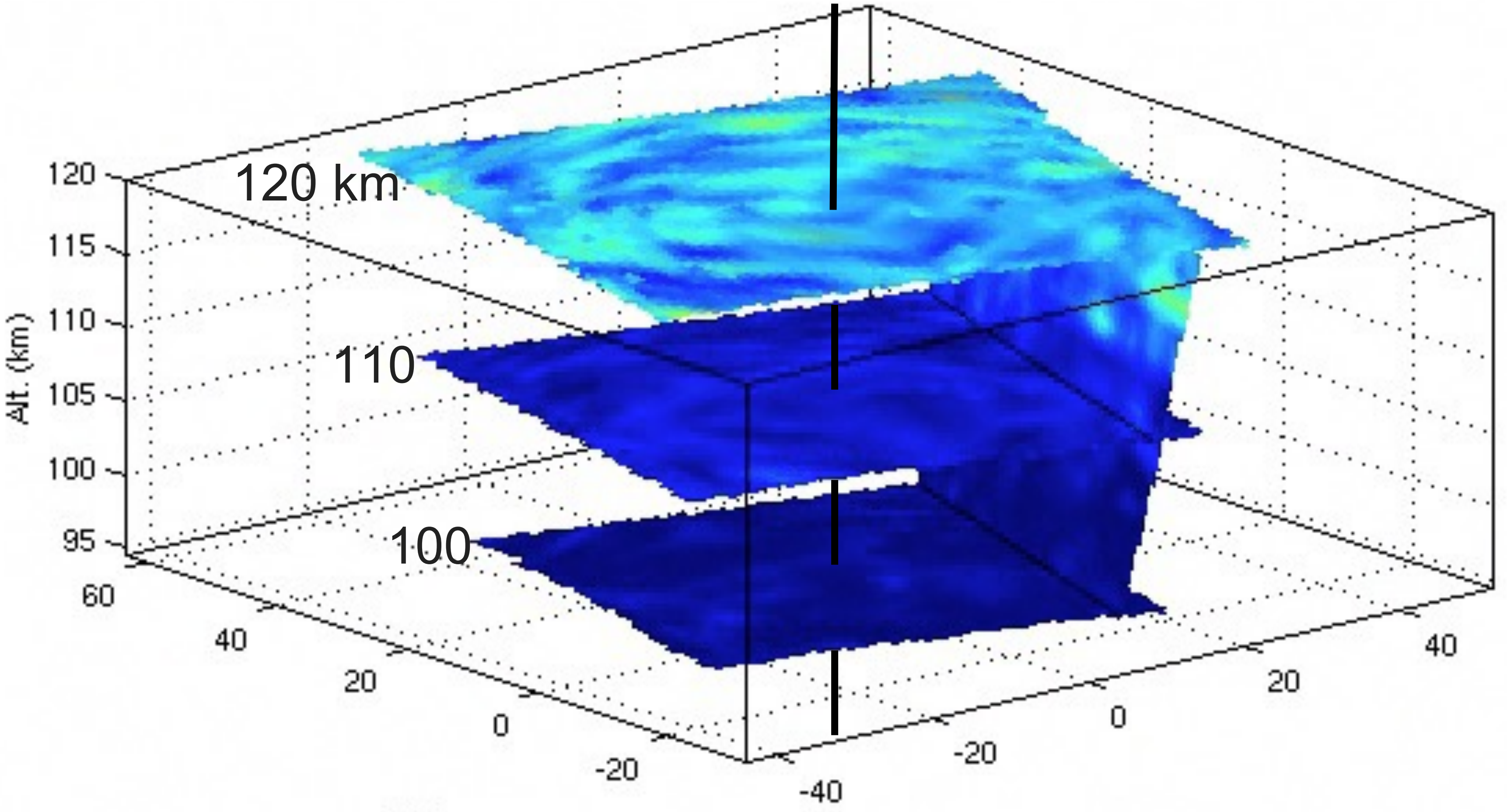
-Integration over all positions simultaneously

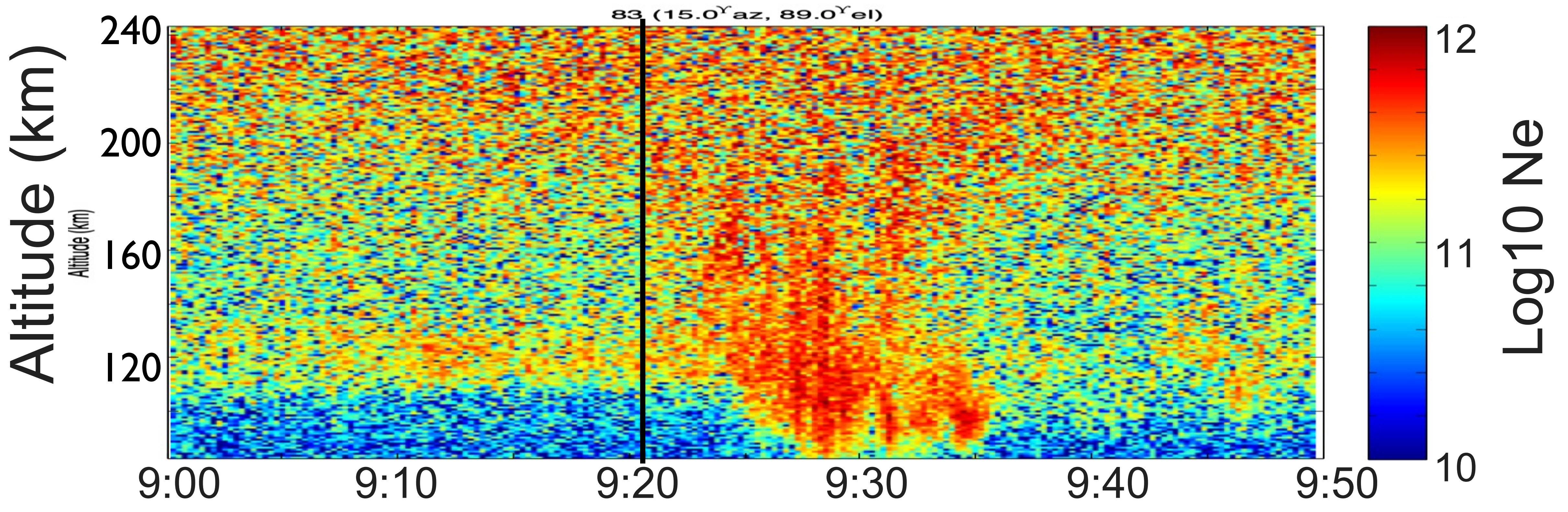
-Power distributed

-No moving parts

Three-dimensional ionospheric imaging







Semeter et al., JASTP 2009