An Introduction to the Terrestrial Ionosphere

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1. Local Photochemistry and Energetics
   - Photochemistry
   - Energetics
2. Transport
   - Thermal Plasma Transport
   - Suprathermal Particle Transport
3. Electrodynamics
   - Dynamo Theory
   - High Latitude Electrodynamics

Hydrostatic Equilibrium

Vertical structure of neutral atmosphere determined by balance between **pressure gradient** and **gravity**.

$$\nabla p = -mng$$
$$\frac{d}{dz} n k_B T = -mng$$

In the simplest case of constant T

$$k_B T \frac{d}{dz} n = -mng$$
$$\frac{1}{n}\frac{dn}{dz} = -\frac{mg}{k_B T} = -\frac{1}{H}$$

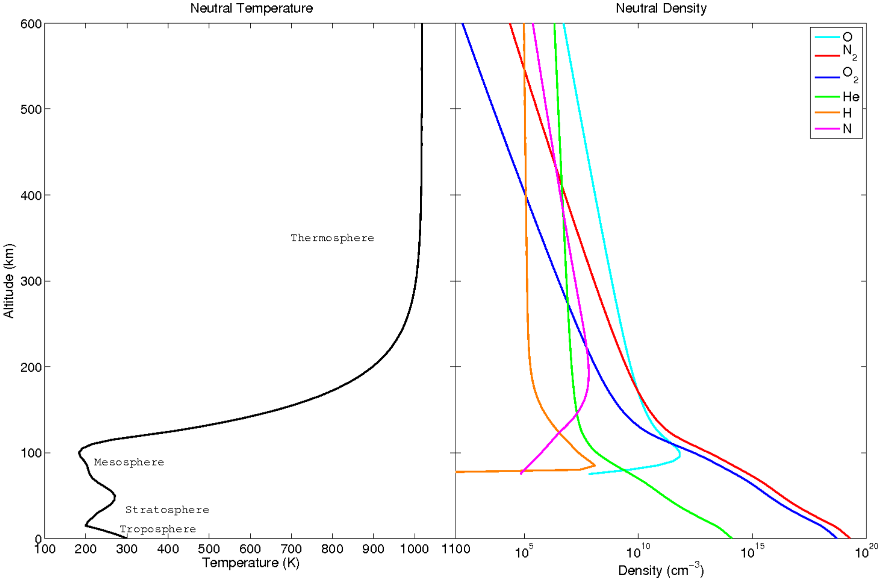
Define the scale height:

$$H = \frac{k_B T}{mg}$$

Solution:

$$n(z) = n\left(z_0\right) \exp\left[-\frac{z - z_0}{H}\right]$$

The Neutral Atmosphere (according to NRLMSISE-00)



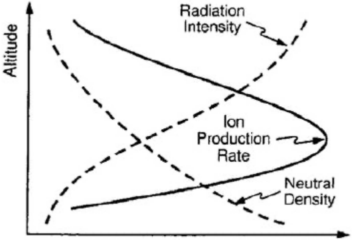
Absorption of EUV Radiation

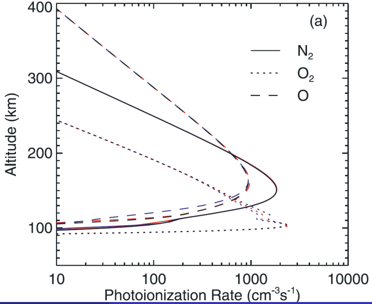
Photoionization Rate:

$$P_i(s) = \int d\lambda \sum_n \sigma_n^{ion}(\lambda) N_n(s) I(s, \lambda)$$

Attenuation of EUV Flux (Lambert's Law):

$$\frac{dI}{ds} = -\sum_n \sigma_n^{abs}(\lambda) N_n(s) I(s, \lambda)$$





Basic Ionospheric Photochemistry

Photoionization:

$$O + h\nu(> 13.6 \text{ eV}) \rightarrow O^+ + e^*$$

Radiative Recombination (SLOW):

$$O^+ + e \rightarrow O + h\nu(13.6 \text{ eV})$$

Atom-Ion Interchange:

$$O^+ + N_2 \rightarrow NO^+ + N$$
$$O^+ + O_2 \rightarrow O_2^+ + O$$

Dissociative Recombination:

$$NO^+ + e \rightarrow N + O$$
$$O_2^+ + e \rightarrow O + O$$

Airglow:

$$O\left(^1D\right) \rightarrow O\left(^3P\right) + h\nu(630.0 \text{ nm})$$
$$O\left(^1S\right) \rightarrow O\left(^1D\right) + h\nu(557.7 \text{ nm})$$

The Ionosphere and Thermosphere

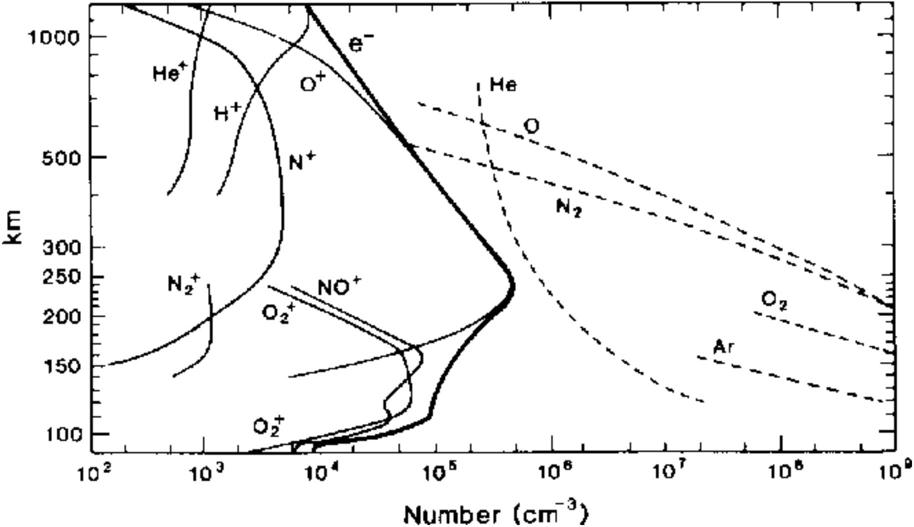
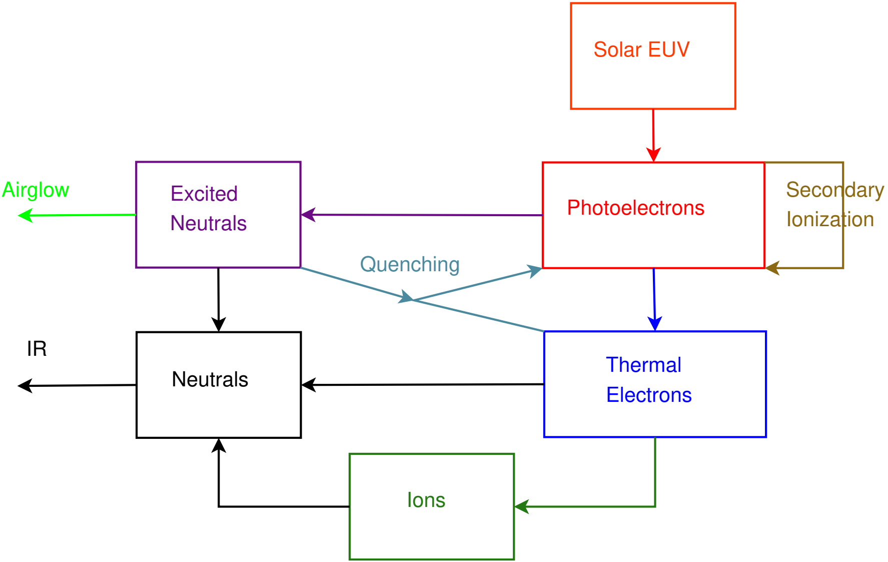
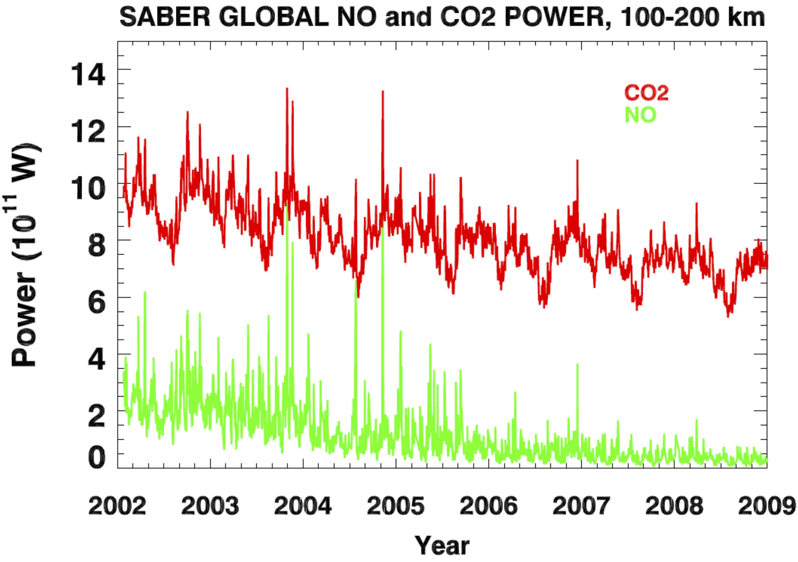


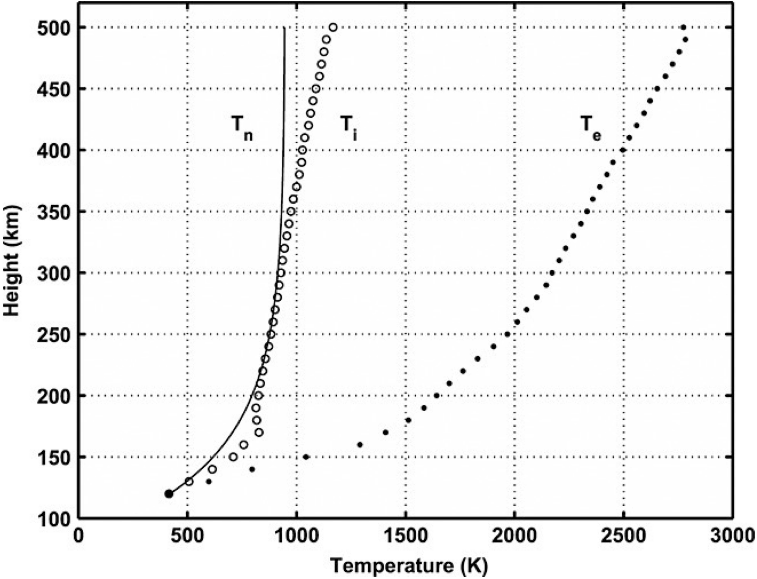
Diagram of Ionospheric Energetics



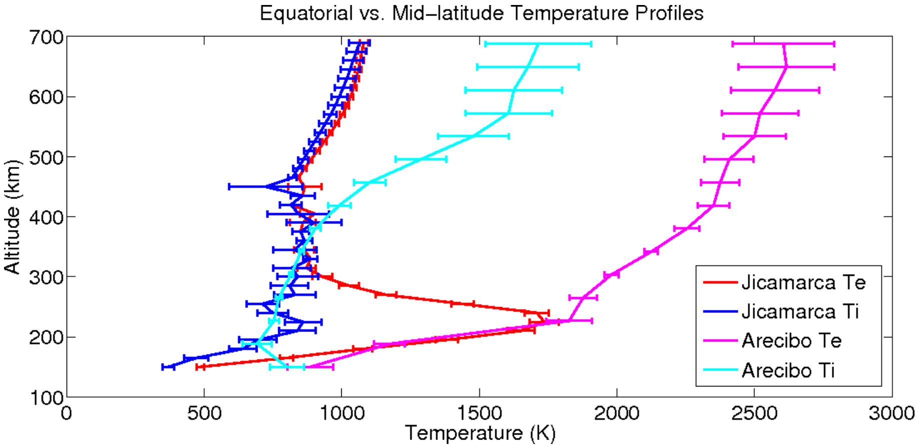
IR Cooling of Thermosphere



Typical Daytime Mid-latitude Temperature Profiles



Temperature Profiles at Different latitudes



Importance of the Magnetic Field

Dynamic Pressure: mnu^2

Thermal Pressure: $nk_BT = mn\left(\sqrt{\frac{k_BT}{m}}\right)^2$

Magnetic Pressure: $\frac{B^2}{2\mu_0} = mn\frac{1}{2}\left(\frac{B}{\sqrt{\mu_0 mn}}\right)^2$

Typical Numbers for the ionosphere:

$$u \approx 100 \text{ m/s Midlatitude Ionosphere}$$

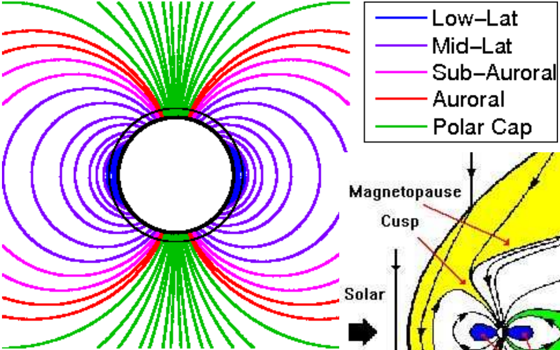
$$u \approx 20 \text{ km/s } \mathrm{H}^+ \text{ Outflow in Polar Wind}$$

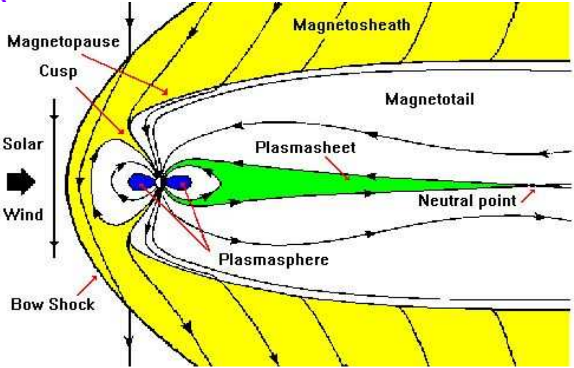
$$T \approx 2000\mathrm{K} \rightarrow \sqrt{\frac{k_BT}{m}} = \begin{cases} 1 \text{ km/s} & \text{for } \mathrm{O}^+ \\ 4 \text{ km/s} & \text{for } \mathrm{H}^+ \end{cases}$$

$$B \approx 3 \times 10^{-5} \text{ T}, \; n \approx 10^{11} \text{ m}^{-3} \rightarrow \frac{B}{\sqrt{\mu_0 mn}} \approx 500 \text{ km/s}$$

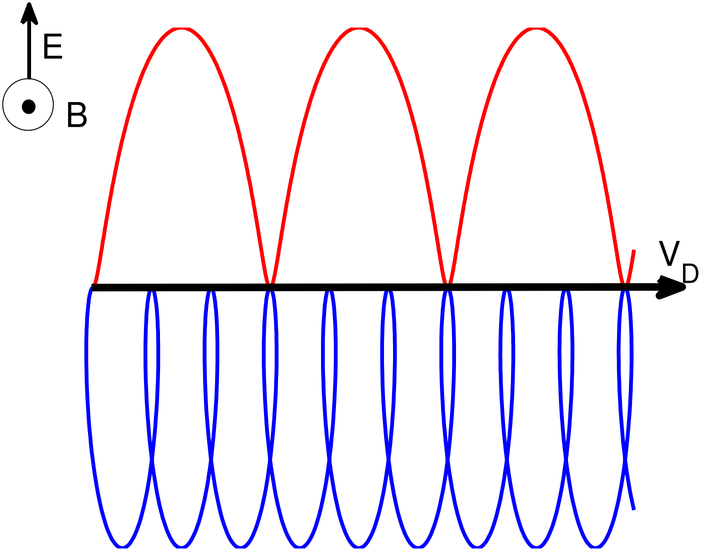
Transport parallel and perpendicular to B are fundamentally different.

Magnetic Structure of the Ionosphere and Magnetosphere





Perpendicular Transport: The $\mathbf{E} \times \mathbf{B}$ Drift



$$v_D = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

Moments of the Boltzmann Equation

Boltzmann Equation:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \left[\frac{e}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) + \mathbf{g}\right] \cdot \frac{\partial f}{\partial \mathbf{v}} = \left.\frac{\delta f}{\delta t}\right|_{\text{collisions}}$$

Continuity Equation: Apply $\int d\mathbf{v}$ to Boltzmann Equation

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial \mathbf{x}} \cdot (n\mathbf{u}) = \frac{\delta n}{\delta t}$$

Momentum Equation: Apply $\int m\mathbf{v}d\mathbf{v}$ to Boltzmann Equation

$$\frac{\partial}{\partial t}(mn\mathbf{u}) + \frac{\partial}{\partial \mathbf{x}} \cdot (mn\mathbf{u}\mathbf{u} + \mathbf{P}) = ne\left(\mathbf{E} + \mathbf{u} \times \mathbf{B}\right) + nm\mathbf{g} + \frac{\delta M}{\delta t}$$

Energy Equation: Apply $\int \frac{1}{2}mv^2 d\mathbf{v}$ to Boltzmann Equation

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial}{\partial \mathbf{x}} \cdot \left[\mathbf{u} \cdot (\epsilon\mathbf{I} + \mathbf{P}) + \mathbf{q}\right] = ne\mathbf{u} \cdot \mathbf{E} + nm\mathbf{u} \cdot \mathbf{g} + \frac{\delta E}{\delta t}$$

Closing the System of Transport Equations

- Easiest way: Assume an isotropic Maxwellian distribution $\rightarrow$ 5-moment approximation

$$\mathbf{P} \rightarrow p\mathbf{I} \qquad \mathbf{q} \rightarrow 0$$

Energy equation reduces to the adiabatic gas law $\frac{D}{Dt}\left(\frac{p}{n^\gamma}\right) = 0$

- Hard way: Assume more complicated distributions (e.g. Maxwellians times truncated series expansions). The 8-, 10-, 13-, 16-, and 20-moment equations are derived this way.
- Middle Ground: Assume higher moments are small and derive steady state limits of high-moment transport equations

$$\mathbf{q} = -\kappa \cdot \nabla T$$

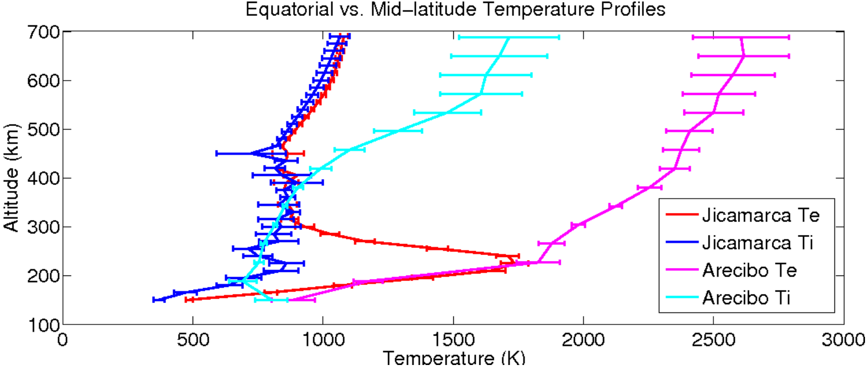
Thermal Conduction

In the F-region $\kappa = \kappa_{\parallel}\hat{\mathbf{b}}\hat{\mathbf{b}} \rightarrow \mathbf{q} = -\kappa_{\parallel}\nabla_{\parallel} T\hat{\mathbf{b}}$

For a fully ionized plasma $\kappa_{\parallel} = 7.7 \times 10^5 \, T_e^{5/2} \ \mathrm{eVcm^{-2}s^{-1}K^{-1}}$

Parallel Temperature Equation:

$$\frac{\partial T}{\partial t} + u_{\parallel}\nabla_{\parallel} T + \frac{2}{3} T\nabla_{\parallel} \cdot \mathbf{u} - \frac{2}{3}\frac{1}{nk_B}\nabla_{\parallel} \cdot \kappa_{\parallel}\nabla_{\parallel} T = \frac{2}{3}\frac{1}{nk_B}(Q - L)$$



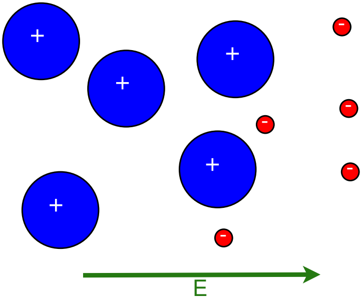
Ambipolar Electric Fields and Ambipolar Diffusion

Steady state parallel electron momentum equation:

$$\cancel{m_e \left[\frac{\partial}{\partial t} (n_e u_e) + \nabla_\parallel \cdot \left(n_e u_e^2 \right) \right]} = -\nabla_\parallel p_e - n_e e E_\parallel \longrightarrow E_\parallel = -\frac{1}{e n_e} \nabla_\parallel p_e$$

Substitute into parallel ion momentum equation:

$$m_i \left[\frac{\partial}{\partial t} (n_i u_i) + \nabla_\parallel \cdot \left(n_i u_i^2 \right) \right] = -\nabla_\parallel p_i - \frac{n_i}{n_e} \nabla_\parallel p_e - m_i n_i g_\parallel$$
$$- m_i n_i \sum_j \nu_{ij} \left(u_i - u_j \right)$$



Energetic Electron Transport

Populations of electrons in the ionosphere:

- Thermal: $k_B T_e \sim 0.2\ \mathrm{eV}$
- Photoelectrons: mostly $< 60\ \mathrm{eV}$, peak energy flux at $\sim 20\ \mathrm{eV}$
- Soft Precipitation (e.g. cusp, polar rain): $100 - 1000\ \mathrm{eV}$
- Auroral Precipitation: $> 1\ \mathrm{keV}$

Simplified kinetic equations derived by

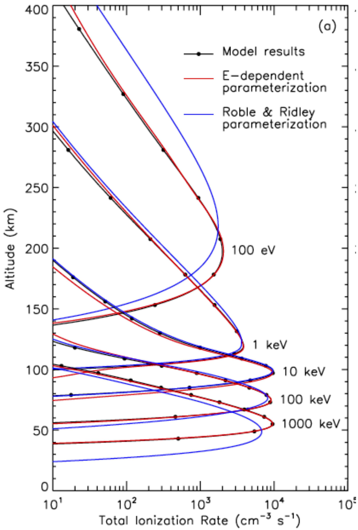
- Assuming suprathermal density $\ll$ thermal density
- Ignoring perpendicular transport
- Assuming gyrotropy (azimuthal symmetry about **B**)
- Assuming steady state ($m_e \to 0$)

Simplest possible form is derived by additionally neglecting $E_{\|}$, $\frac{\partial B}{\partial s}$, and Coulomb collisions and assuming isotropic elastic collisions.

$$\mu \frac{\partial \Phi}{\partial s} = q + \sum_n \left\{ - \left[\sigma_{an}(\mathcal{E}) + \sigma_{en}(\mathcal{E}) \right] N_n \Phi + \frac{\sigma_{en}(\mathcal{E})}{2} N_n \int_{-1}^{1} \Phi(s, \mathcal{E}, \mu')\, d\mu' \right\}$$

This has the same mathematical form as a radiative transfer equator

Auroral Particle Deposition

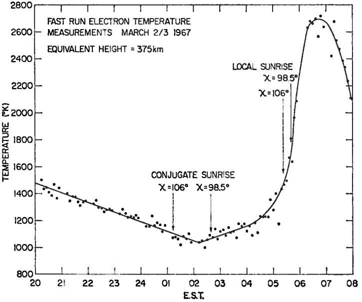


Higher energy particles penetrate deeper into atmosphere.

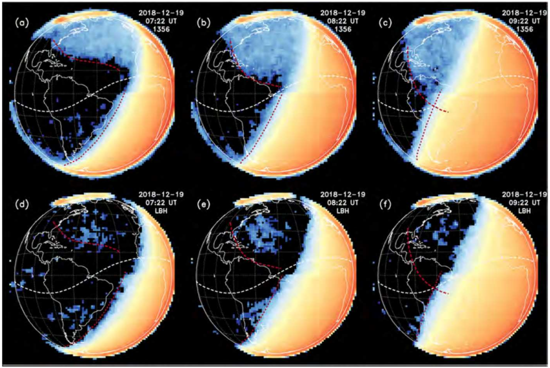
Fang et al. (2008)

Conjugate Photoelectron Transport

Millstone Hill T_e Observations



GOLD Ultraviolet Emissions



Solomon et al. 2020

Fundamentals of Ionospheric Electrodynamics

Electrostatic Limit of Maxwell's Equations:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \cancelto{0}{\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}} \qquad \longrightarrow \nabla \cdot \mathbf{J} = 0$$

$$\nabla \times \mathbf{E} = -\cancelto{0}{\frac{\partial \mathbf{B}}{\partial t}} \qquad \longrightarrow \mathbf{E} = -\nabla \Phi$$

Ohm's Law for the ionosphere:

$$\mathbf{J} = \sigma \cdot \mathbf{E} + \mathbf{J}_0$$

Putting everything together yields a boundary value problem:

$$\nabla \cdot \sigma \cdot \nabla \Phi = \nabla \cdot \mathbf{J}_0$$

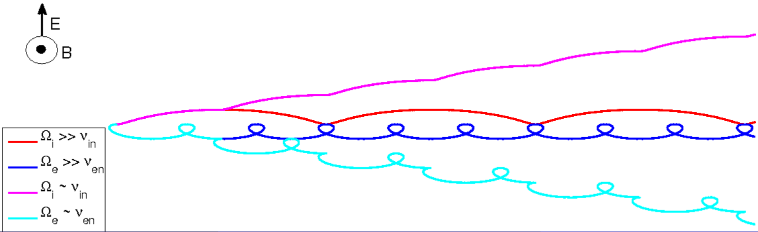
Ohm's Law for the Ionosphere

Steady-state momentum equation for each species (zero neutral wind case):

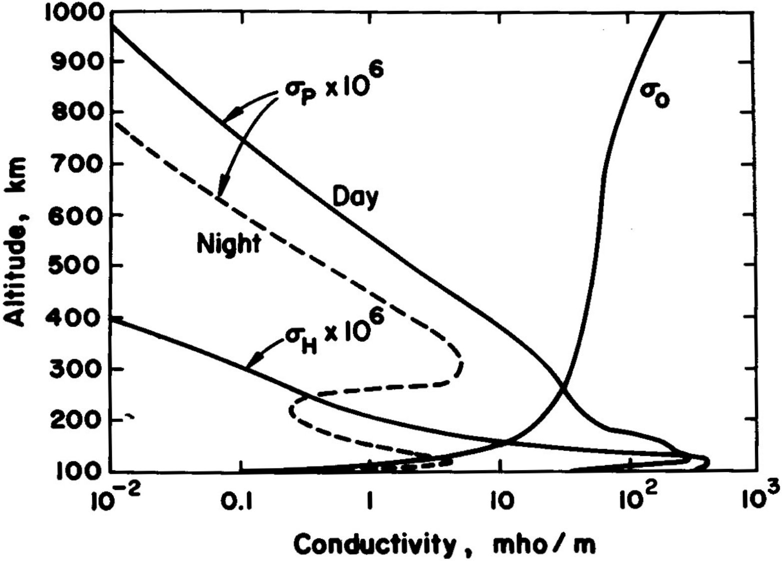
$$0 = n_\alpha q_\alpha (\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B}) - \nu_{\alpha n} m_\alpha n_\alpha \mathbf{u}_\alpha$$

Resulting Ohm's Law:

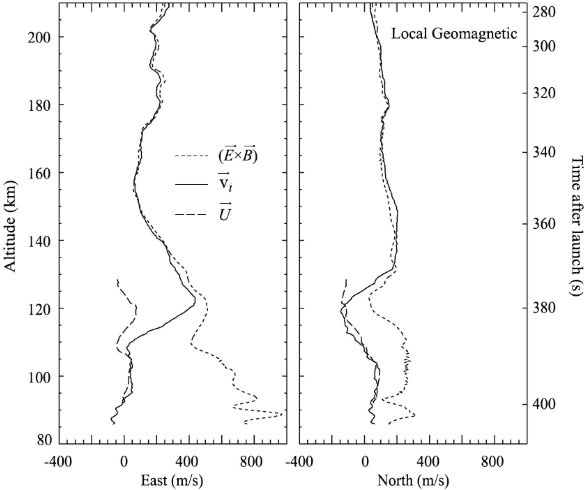
$$\mathbf{J} = \sum_\alpha n_\alpha q_\alpha \mathbf{u}_\alpha \longrightarrow \mathbf{J} = \begin{pmatrix} \sigma_P & -\sigma_H & 0 \\ \sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_0 \end{pmatrix} \cdot \mathbf{E}$$



Conductivity Profiles



Ion Velocity Rotation

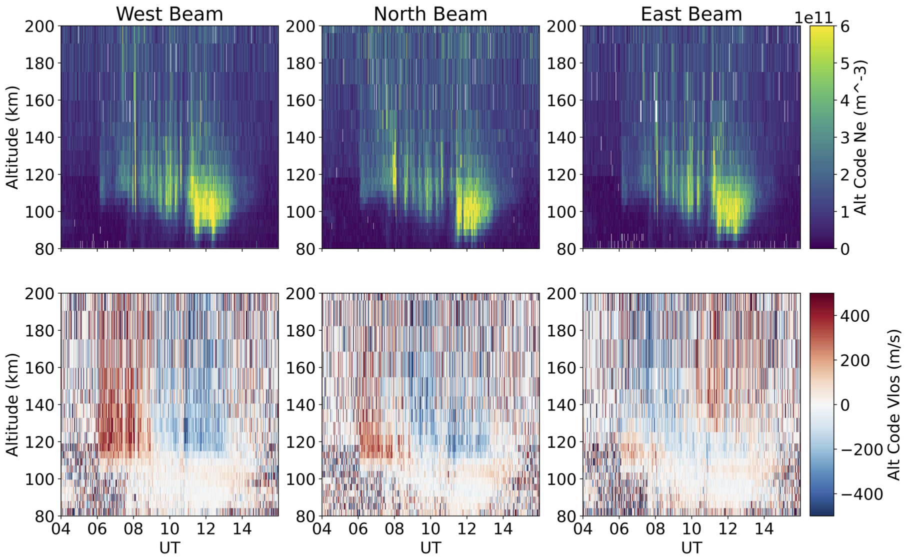


Rocket measurements comparing

- **E** from double-probe
- $\mathbf{v}_i$ from ion imager
- $\mathbf{u}_n$ from TMA chemical release

Sangalli et al. (2009) doi:10.1029/2008JA013757

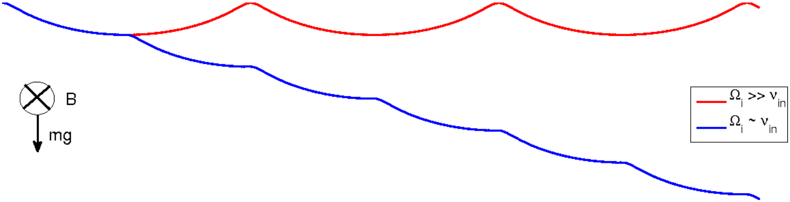
PFISR E-region Velocity Measurements



Other Kinds of Current

Substitute $\mathbf{F}$ for $q_\alpha \mathbf{E}$ in steady state momentum equation.

- Wind drag: $\mathbf{F} = \nu_{\alpha n} m_\alpha \mathbf{u}_n \longrightarrow \mathbf{J} = \sigma \cdot (\mathbf{u}_n \times \mathbf{B})$
- Gravity: $\mathbf{F} = m_\alpha \mathbf{g} \longrightarrow \mathbf{J} = \mathbf{\Gamma} \cdot \mathbf{g}$

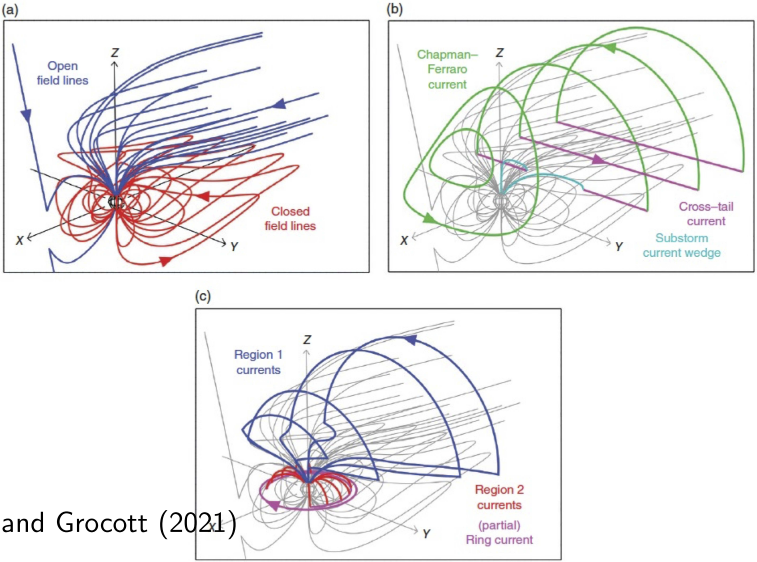


- Pressure Gradients (Diamagnetic Currents):
  $\mathbf{F} = -\frac{1}{n_\alpha} \nabla p_\alpha \longrightarrow \mathbf{J} = \mathbf{D} \cdot \nabla \sum_\alpha p_\alpha$

Complete Dynamo Equation:

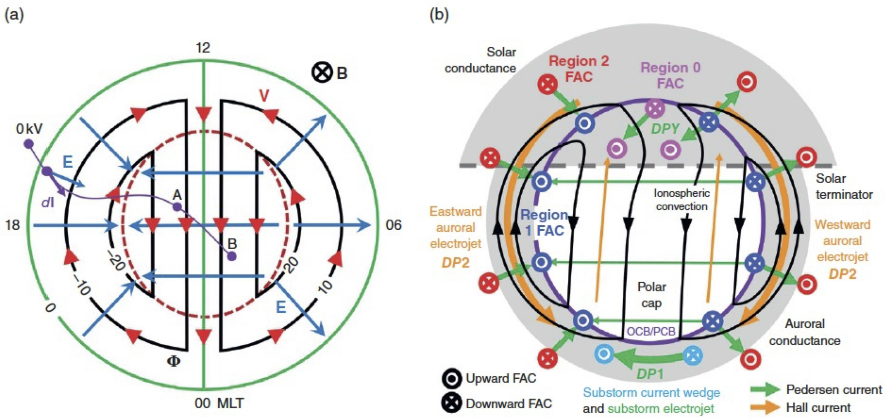
$$\nabla \cdot \sigma \cdot \nabla \Phi = \nabla \cdot \left(\sigma \cdot (\mathbf{u}_n \times \mathbf{B}) + \mathbf{\Gamma} \cdot \mathbf{g} + \mathbf{D} \cdot \nabla \sum_\alpha p_\alpha \right)$$

Current Systems in the Ionosphere and Magnetosphere

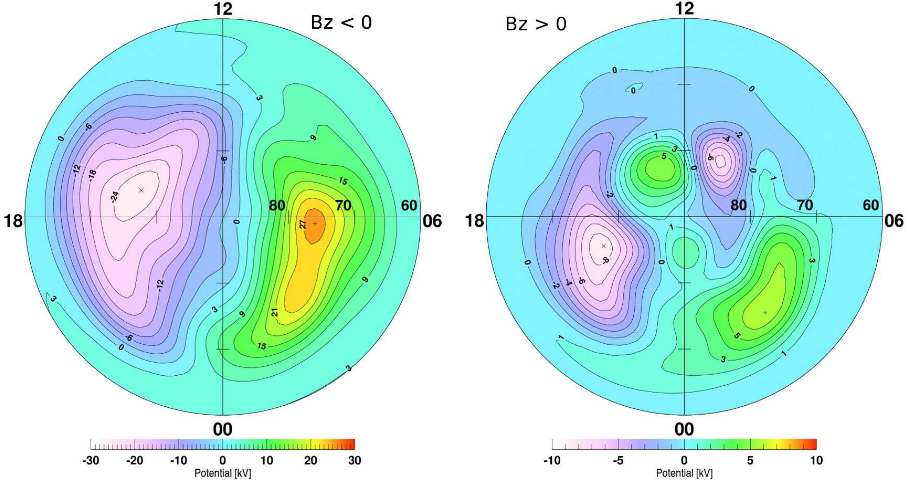


Milan and Grocott (2021)

Current Systems in the Ionosphere and Magnetosphere

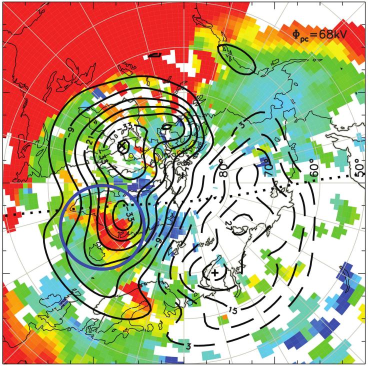


High Latitude Convection Patterns



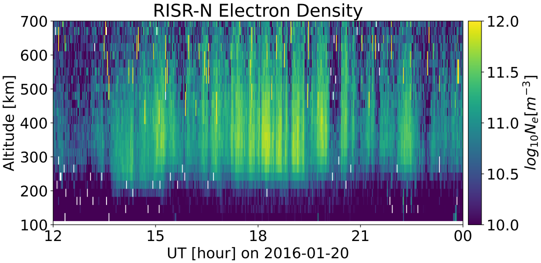
Density Structures in the Polar Cap

GPS TEC Observations

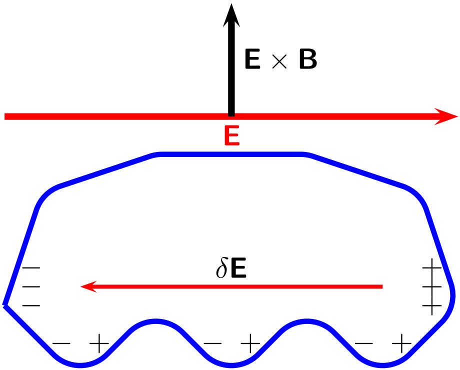


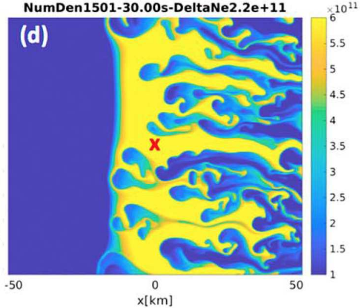
Zhang et al. (2013)

IS Radar Observations



Polarization Electric Fields and Gradient-Drift Instability





Deshpande and Zettergren (2019) 10.1029/2019GL082576

GDI Finger-like Structures