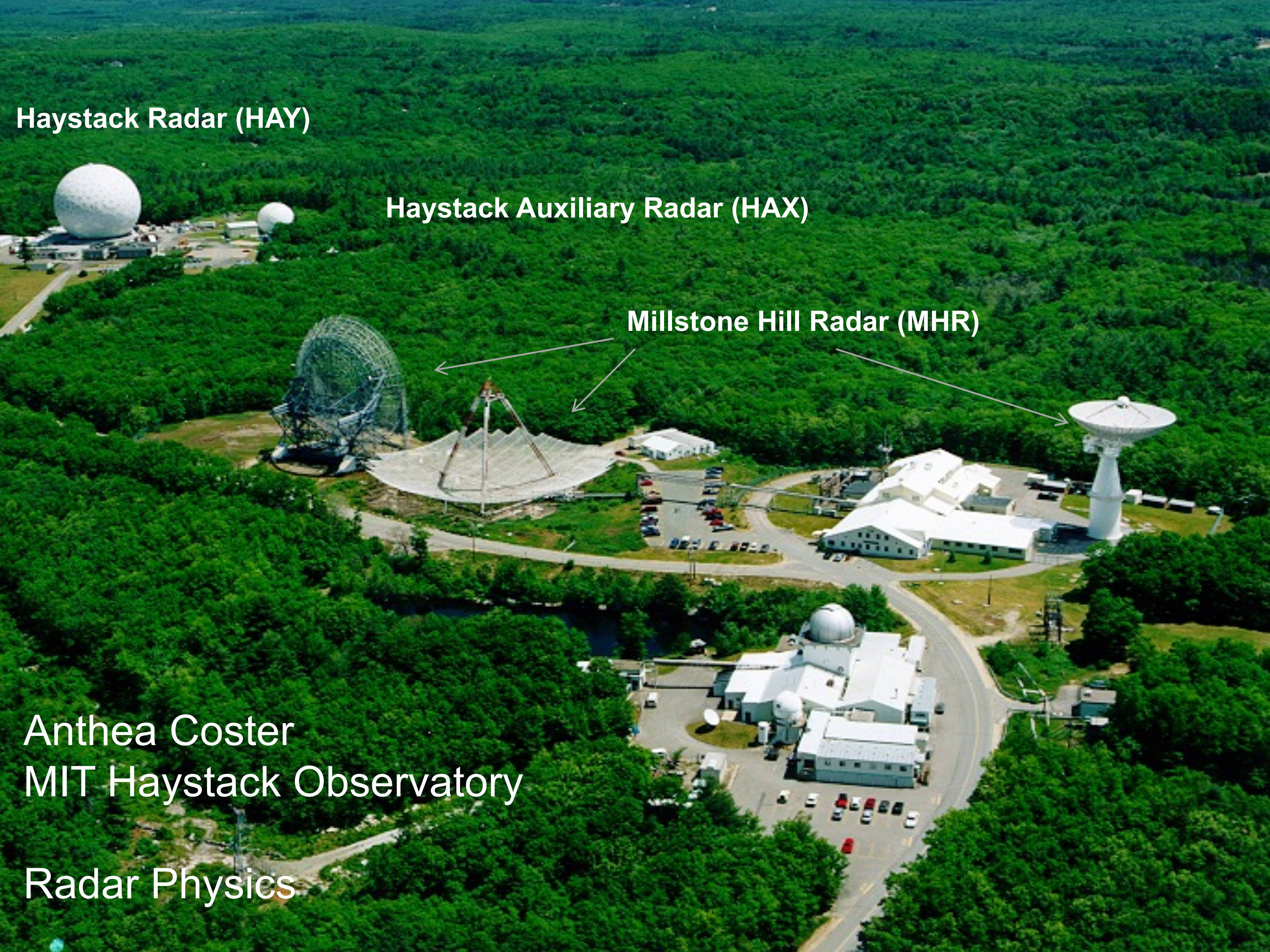

Haystack Radar (HAY)
Haystack Auxiliary Radar (HAX)
Millstone Hill Radar (MHR)
Anthea Coster
MIT Haystack Observatory
Radar Physics

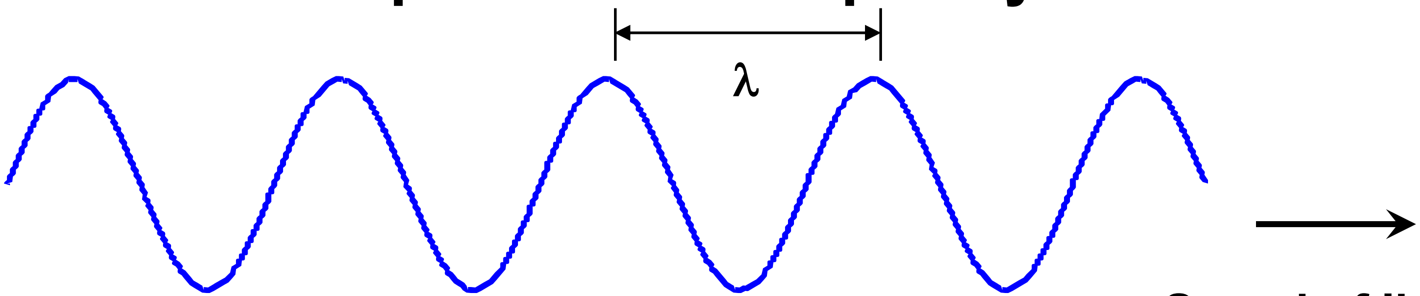
What to know

:

Definition of angular frequency, wave number

Properties of Waves

Relationship Between Frequency and Wavelength



Speed of light, c

$c = 3 \times 10^8$ m/sec
= 300,000,000 m/sec

$$\text{Frequency (1/s)} = \frac{\text{Speed of light (m/s)}}{\text{Wavelength } \lambda \text{ (m)}}$$

Examples:

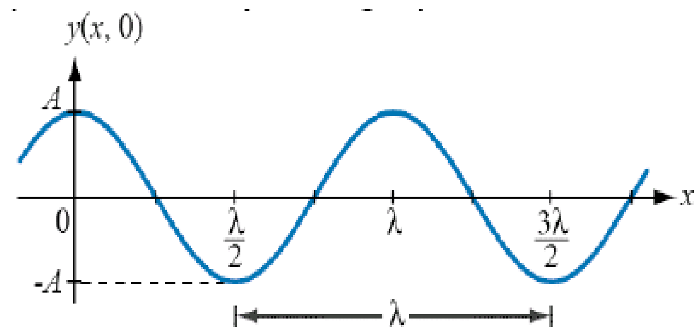
Frequency	Wavelength
100 MHz	3 m
1 GHz	30 cm
3 GHz	10 cm
10 GHz	3 cm

Radio Waves

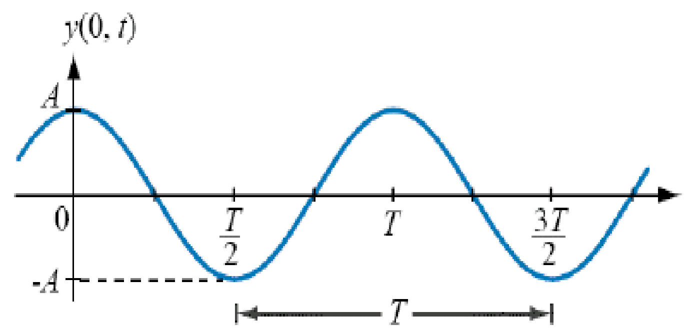
$y(x, t) = A\cos(\omega t - kx + \phi_o)$

Angular frequency
$\omega = 2\pi f = 2\pi/T$

Wavenumber
$k = 2\pi/\lambda$



(a) y(x, t) versus x at t = 0



(b) y(x, t) versus t at x = 0

Wave phase velocity
$c = f\lambda = \omega/k = 3\text{x}10^8\ m/s$

Frequency (1/s) = Speed of light (m/s) / Wavelength l (m)

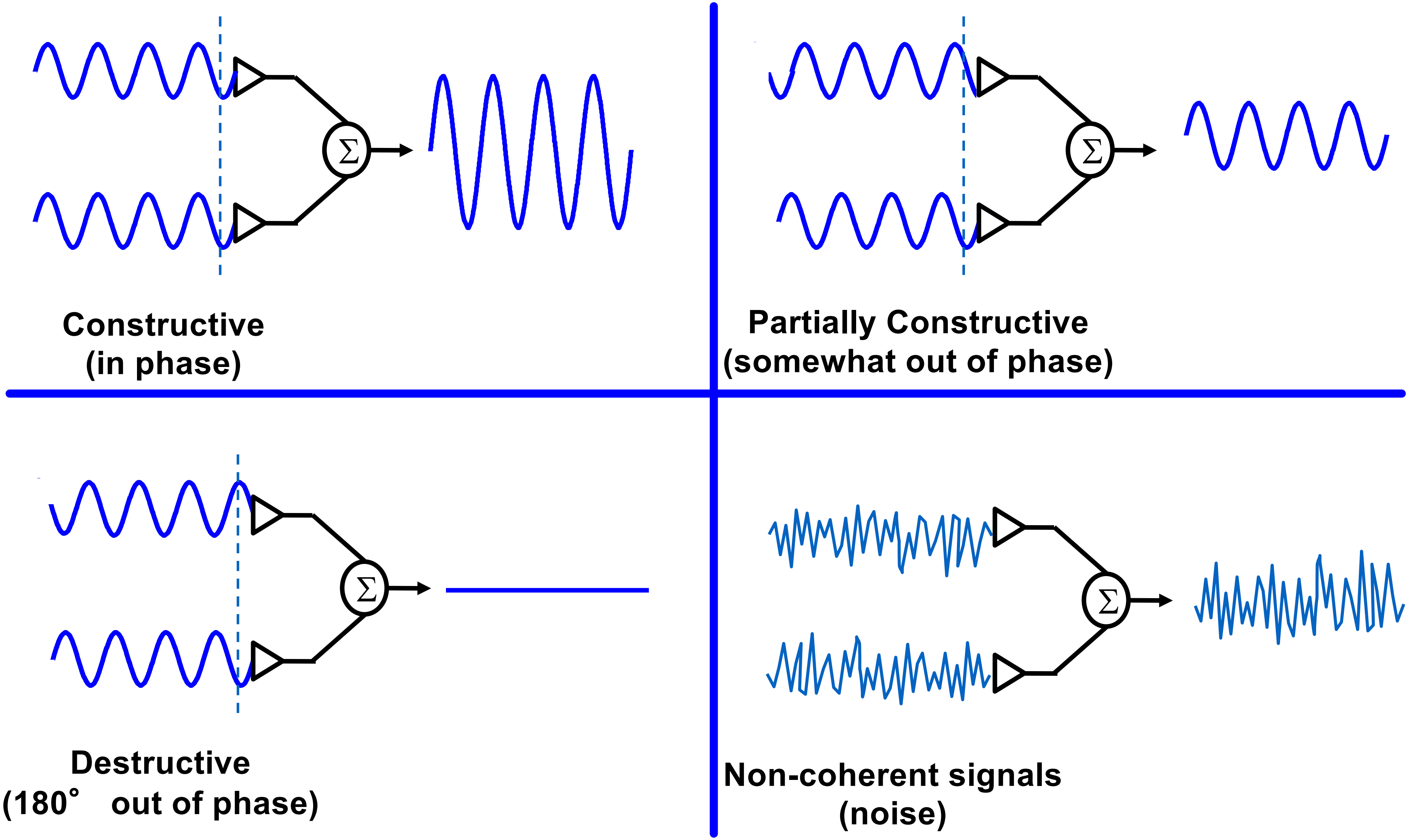
What to know

:

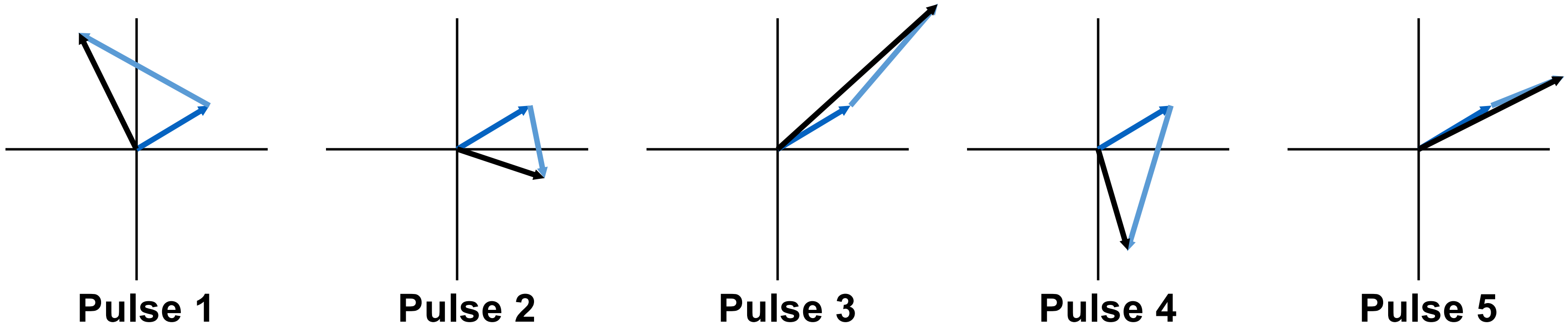
Meaning of constructive and destructive addition

Properties of Waves

Constructive vs. Destructive Addition

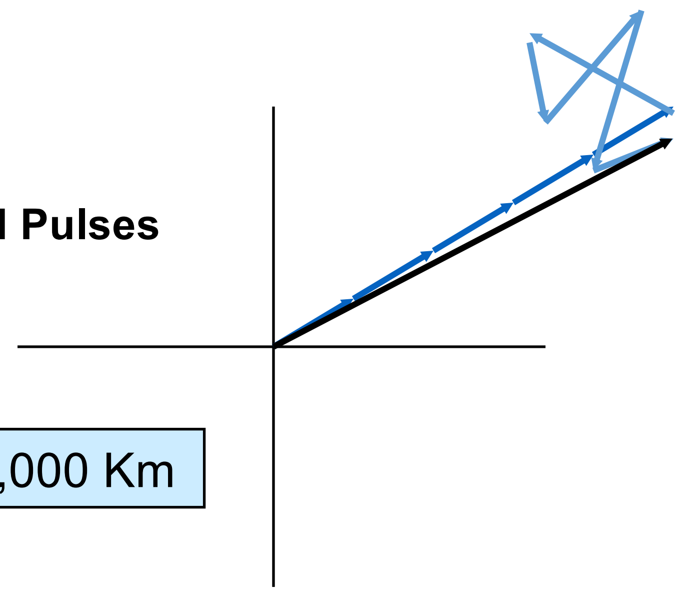


Coherent Integration



- Coherent target returns
- Noise samples at low SNR
- Resultant signal

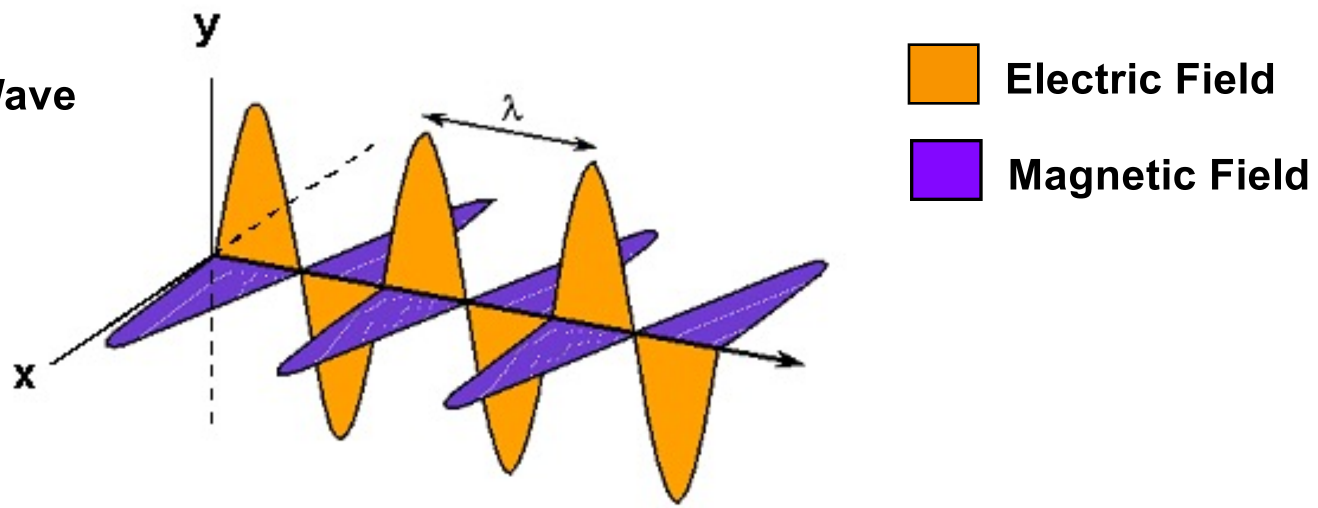
Coherently Integrated Pulses



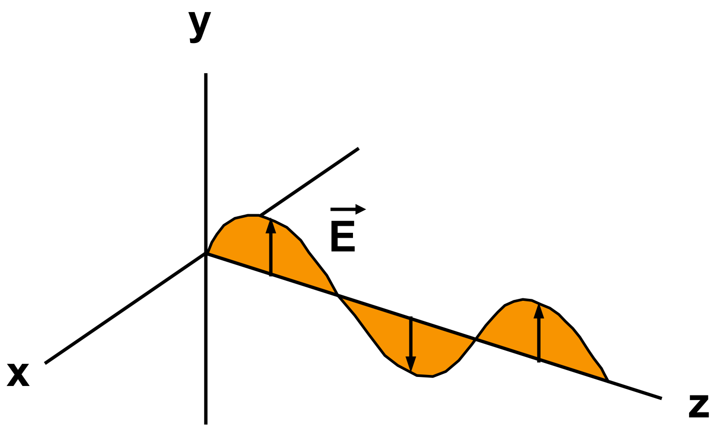
Deep space targets at 30,000 – 40,000 Km

Polarization

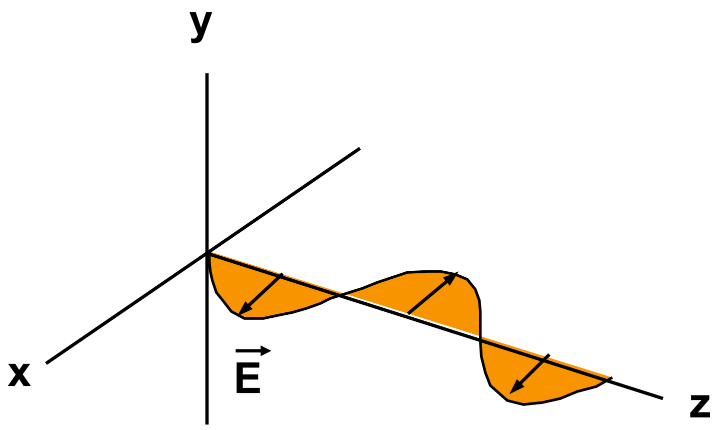
Electromagnetic Wave



Vertical Polarization



Horizontal Polarization

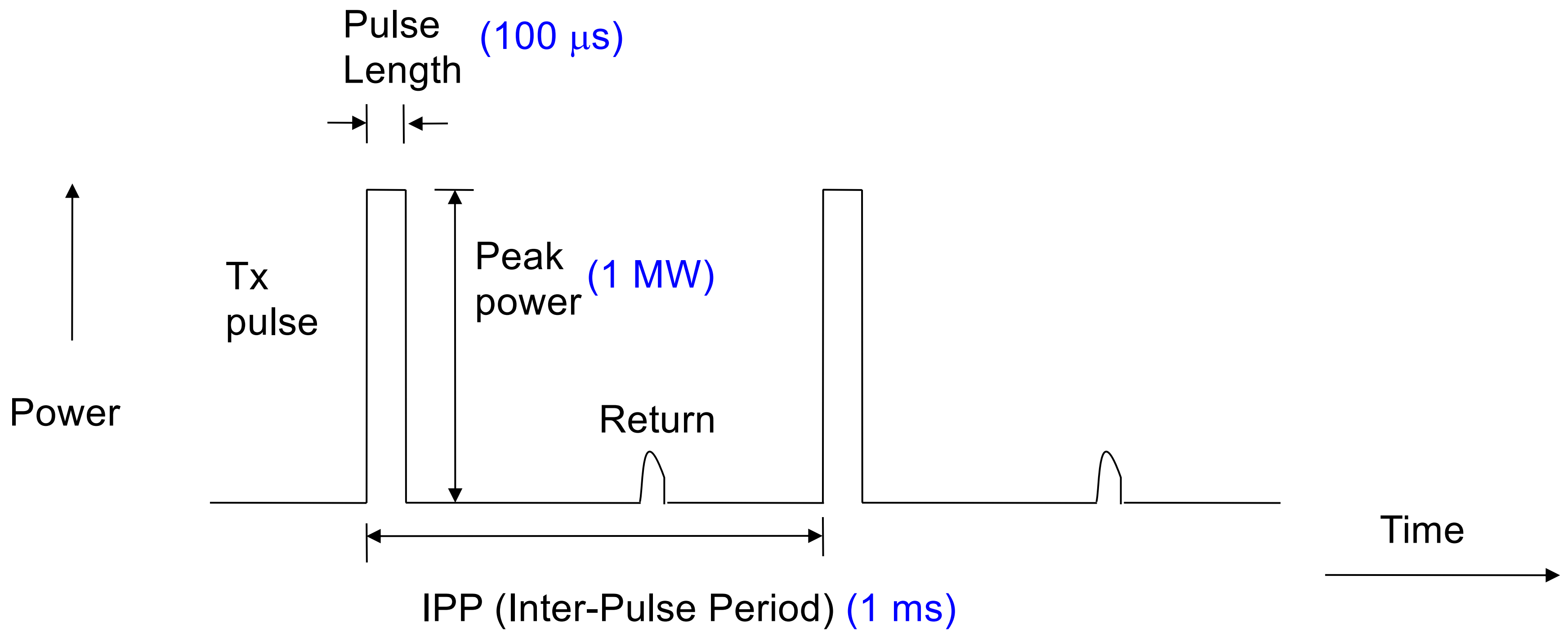


What to know

What does Duty cycle refer to?

What does IPP stand for?

Pulsed Radar



Duty cycle = Pulse Length/IPP (10%)
Average power = Peak power x Duty cycle (100 kW)
PRF (Pulse Repetition Frequency) = 1/IPP (1kHz)

Duty cycle for a CW (continuous wave) radar 100%

What to know

What is the meaning of radar?

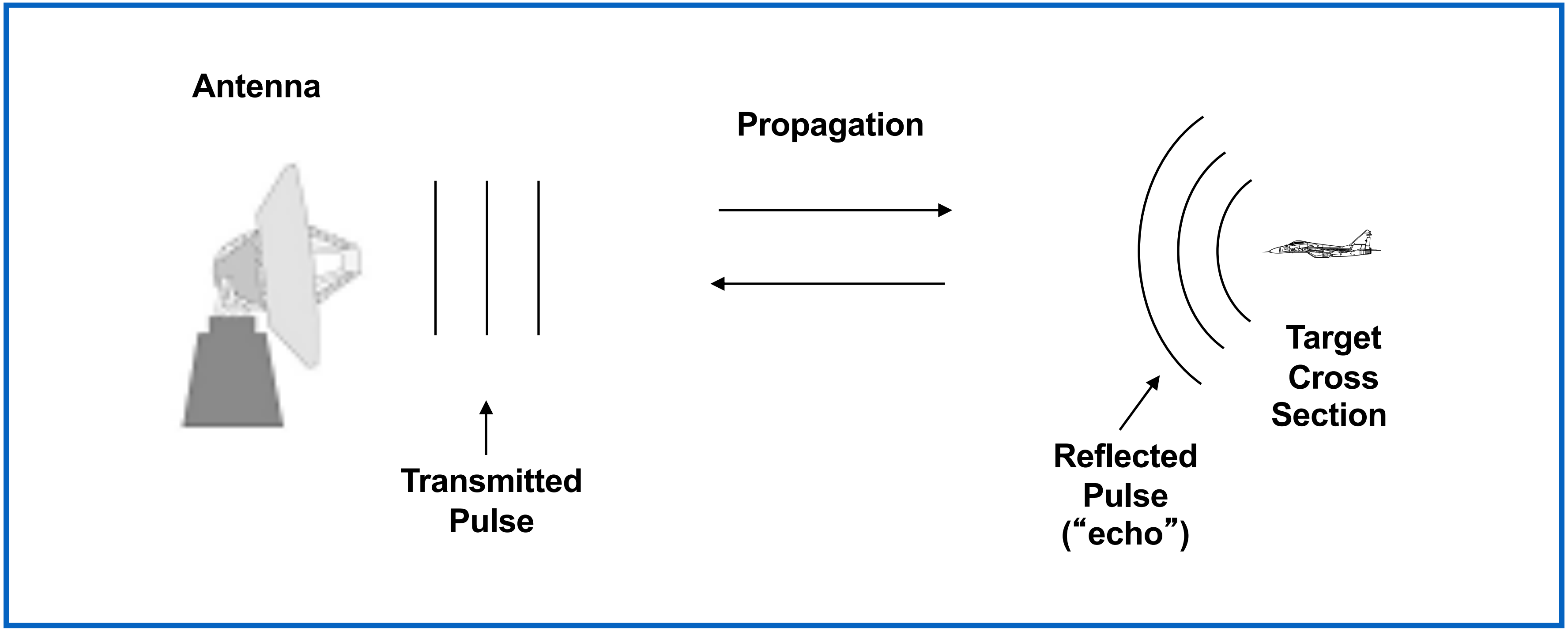
What is the meaning of radar range?

What are the main parts of the radar equation?

W
h
a
t

RADAR

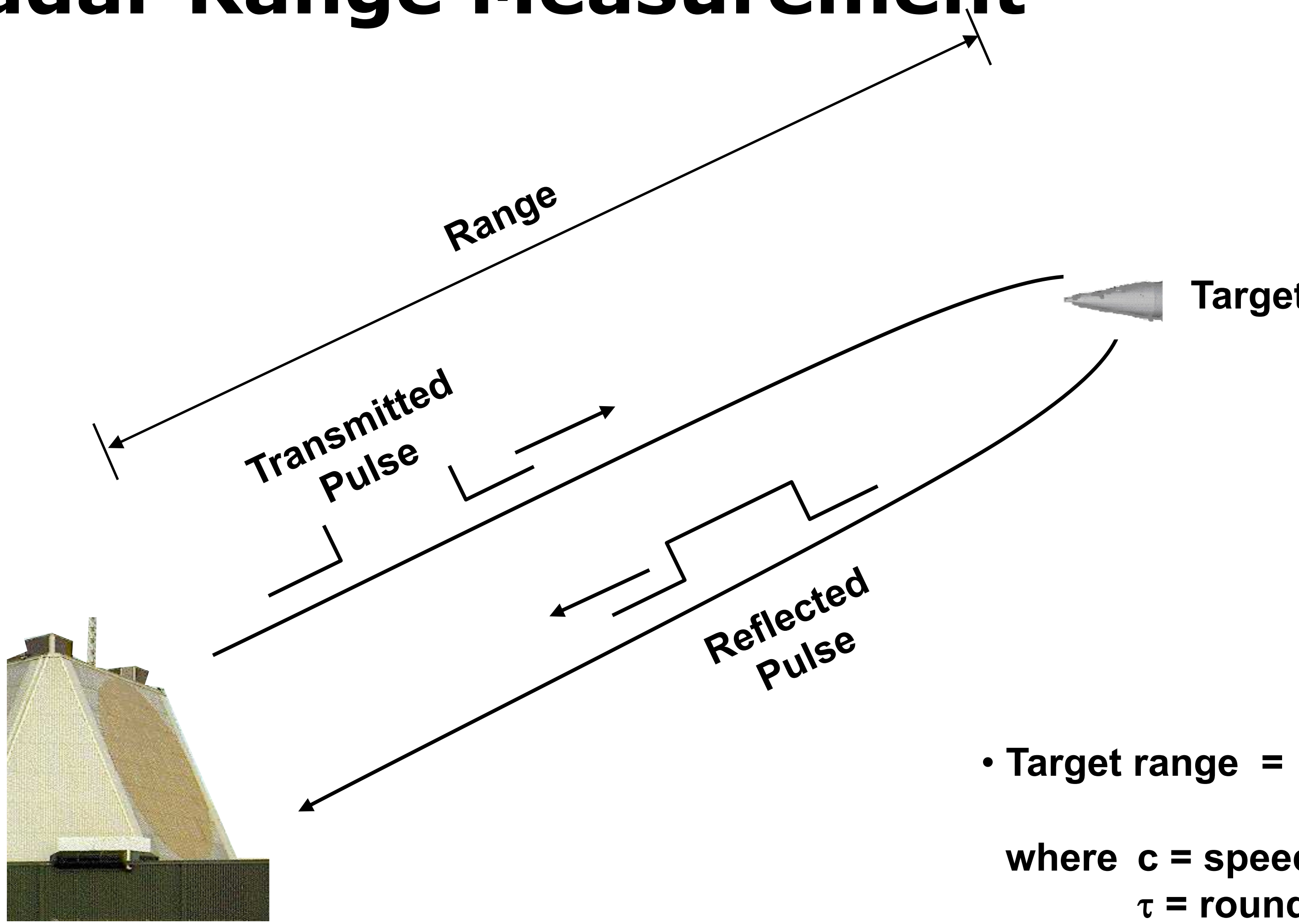
RAdio Detection And Ranging



Radar observables:

- Target range
- Target angles (azimuth & elevation)
- Target size (radar cross section)
- Target speed (Doppler)
- Target features (imaging)

Radar Range Measurement



- Target range $= \frac{c\tau}{2}$

  where c = speed of light
  τ = round trip time

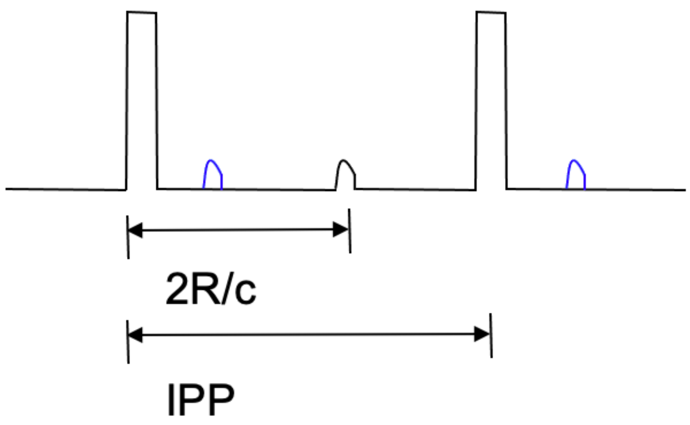
Range Resolution

Or is it ??

Range resolution is set by pulse length

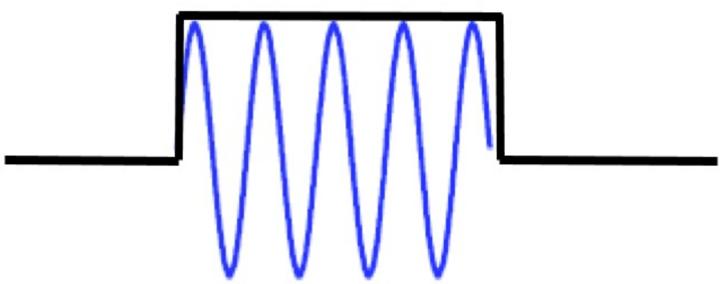
Pulse length = τ_p , Range resolution = $c\tau_p/2$ for a single target.

Maximum unambiguous range



MUR = c*IPP/2

What the radar transmits: Pulses and waves

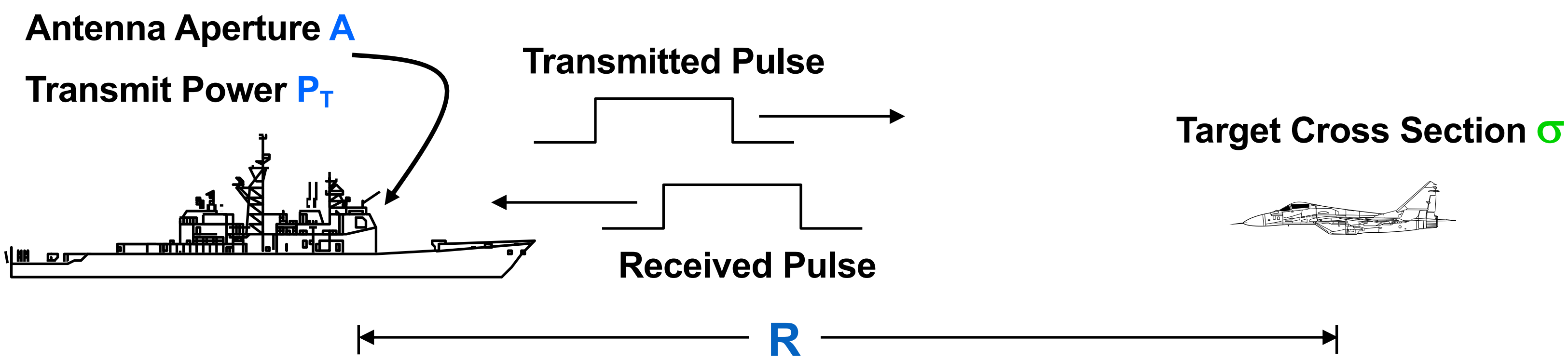


Cycles in a pulse.

PFISR frequency = 449 MHz
Long pulse length = 480 μs
# of cycles = 215520 !

Radar waveforms modulate the waves with on-off sequence

Radar Range Equation

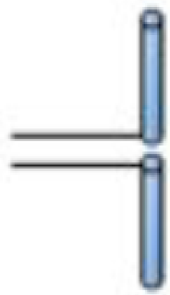


Transmit Power	Transmit Gain	Spread Factor	Losses	Target RCS	Spread Factor	Receive Aperture	Dwell Time

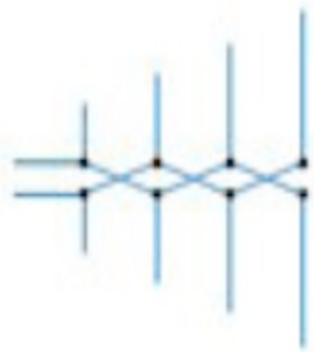
$$\text{Received Signal Energy} = \left[P_T \right] \left[\frac{4\pi A}{\lambda^2} \right] \left[\frac{1}{4\pi R^2} \right] \left[\frac{1}{L} \right] \left[\sigma \right] \left[\frac{1}{4\pi R^2} \right] \left[A \right] \left[\tau \right]$$

Antennas

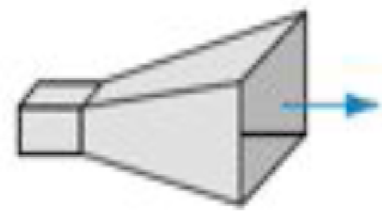
Most basic form of antennas – a wire element with a time varying current flowing in it



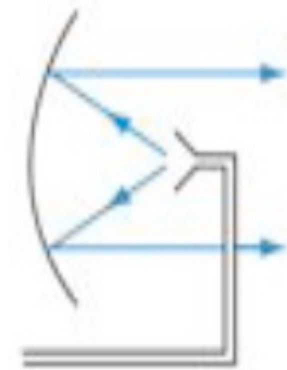
Dipole antenna



Log periodic

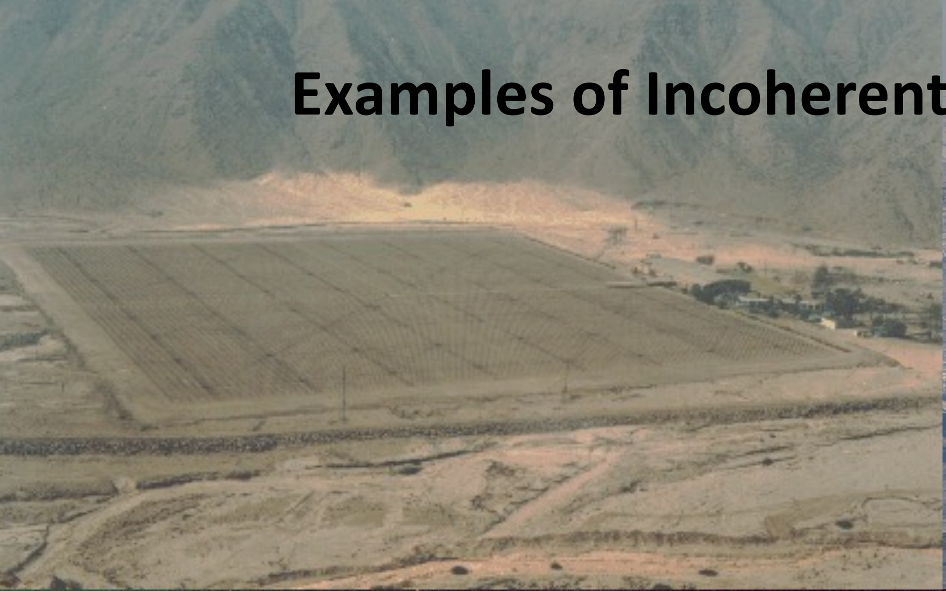


Horn antenna



Parabolic dish
Reflector antenna

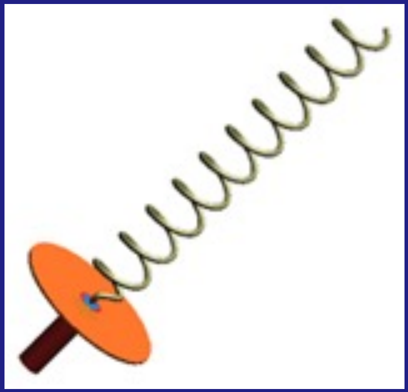
Examples of Incoherent Scatter Radar Antennas



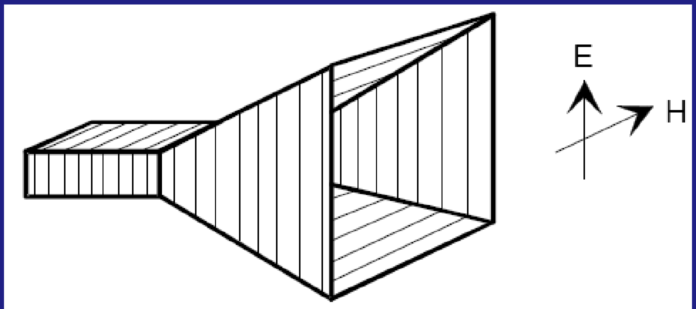
Antennas

Four primary functions of an antenna for radar applications:

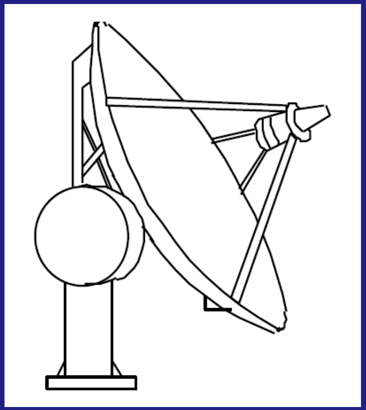
- Impedance transformation (free-space intrinsic impedance to transmission-line characteristic impedance)
- Propagation-mode adapter (free-space fields to guided waves)
- Spatial filter (radiation pattern – direction-dependent sensitivity)
- Polarization filter (polarization-dependent sensitivity)



Helical antenna



Horn antenna



Parabolic reflector antenna

Impedance transformer

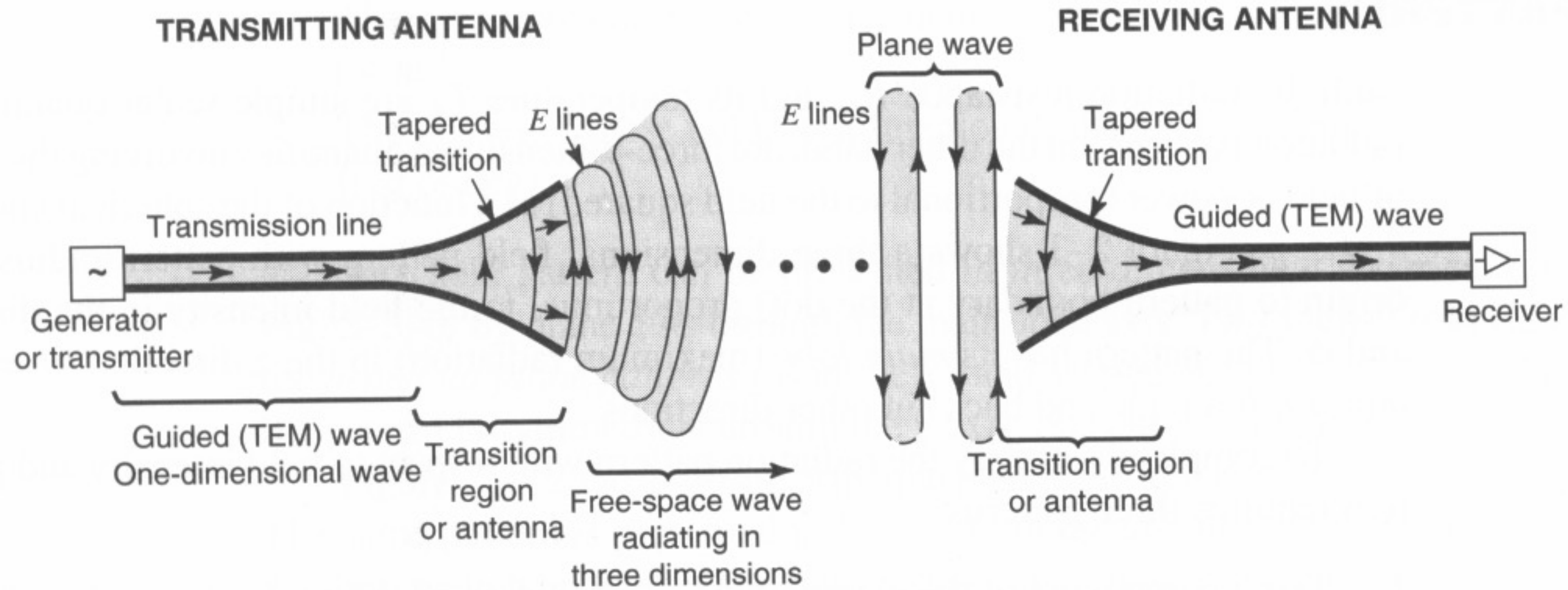
- Intrinsic impedance of free-space, $\eta_o \equiv E/H$ is

$$\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 120\,\pi \cong 376.7\,\Omega$$

- Characteristic impedance of transmission line, $Z_o = V/I$
- A typical value for Z_o is 50 Ω.
- Clearly there is an impedance mismatch that must be addressed by the antenna.

Propagation-mode adapter

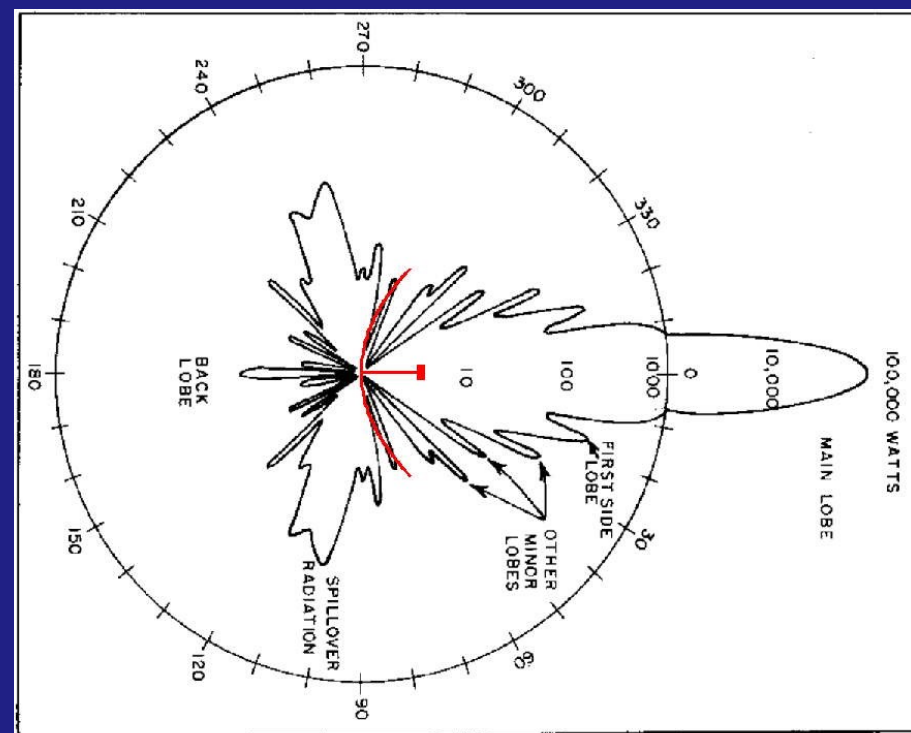
During both transmission and receive operations, the antenna must provide the transition between these two propagation modes.



Spatial filter

Antennas have the property of being more sensitive in one direction than in another, which provides the ability to spatially filter signals from its environment.

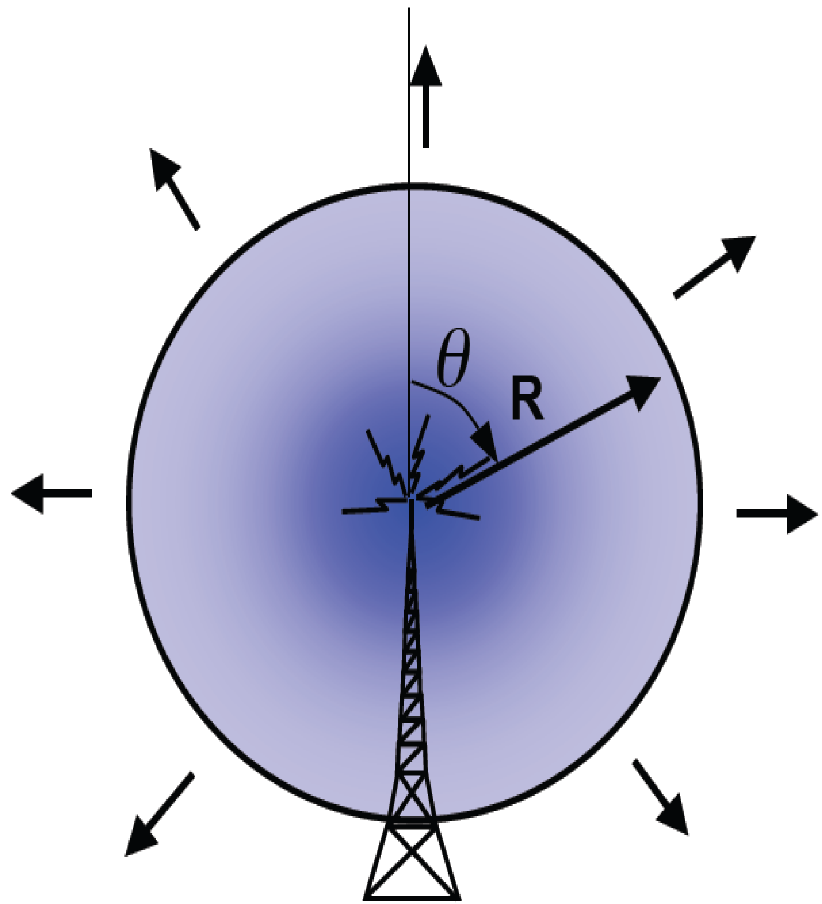
Directive antenna.



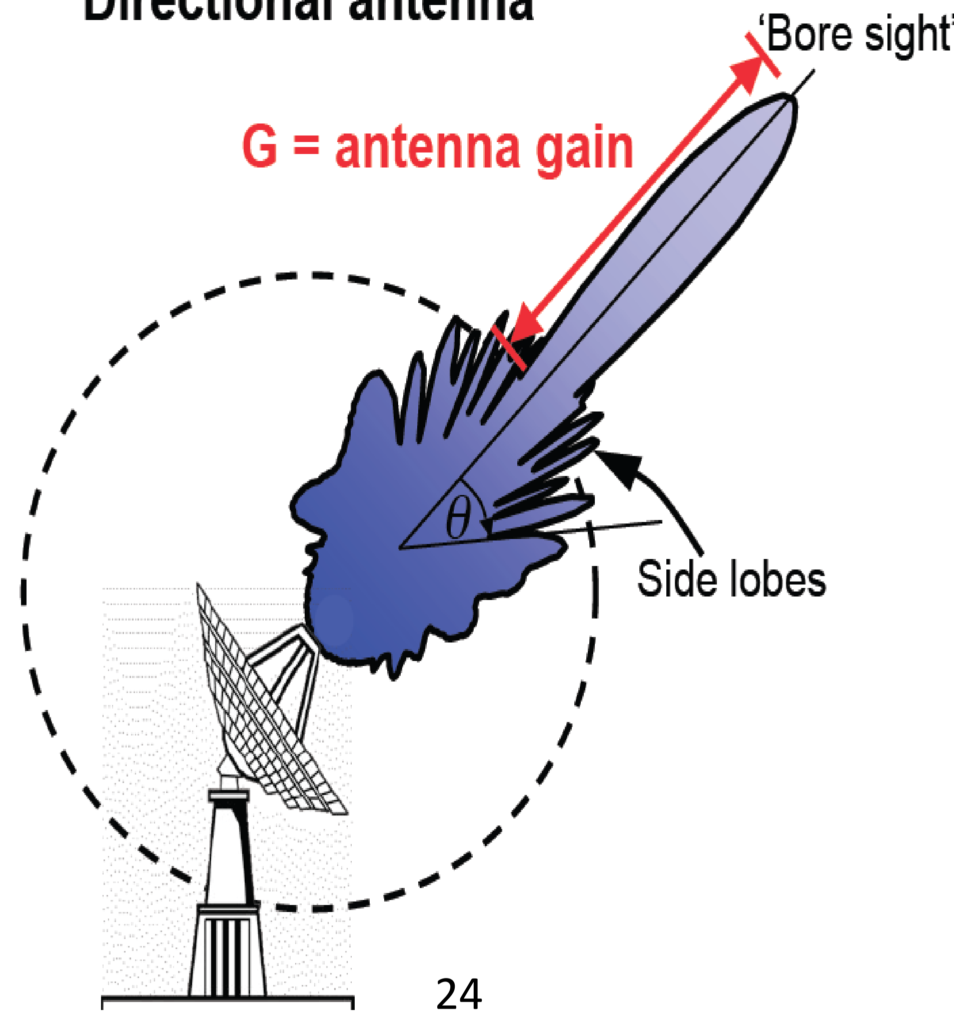
Radiation pattern of directive antenna.

Radiation pattern – antenna gain

Isotropic antenna



Directional antenna



Polarization filter

Antennas have the property of being more sensitive to one polarization than another. This provides the ability to filter signals based on its polarization.

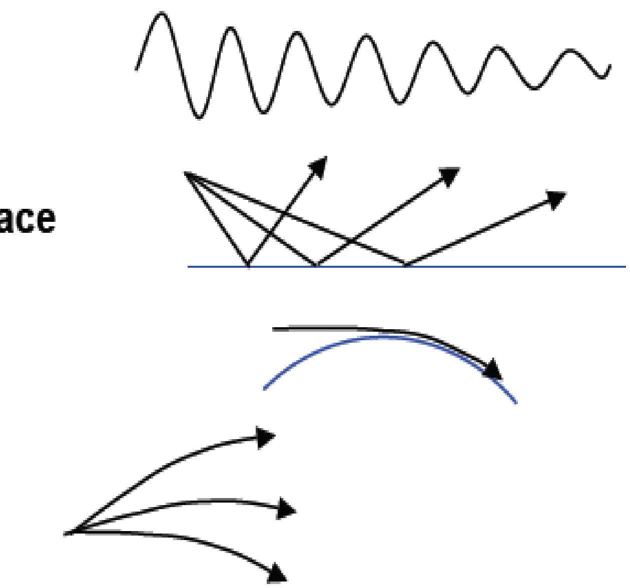
Example: Satellite tracking receive on both right-circular and left-circular

Propagation medium – Losses

Radio waves are affected by the medium they propagate in. Effects dependent on the refractive index of the medium and wave frequency

Radio waves are also reflected off of the surface

- **Atmospheric attenuation**
- **Reflection off of earth's surface**
- **Over-the-horizon diffraction**
- **Atmospheric refraction**



Attenuation usually measured in dB

$$\text{SNR dB} = 10\log_{10}\frac{\textit{signal power}}{\textit{noise power}}$$

dB value	times by
+30 dB	1000
+20 dB	100
+3 dB	2
-10 dB	0.1
-20 dB	0.01

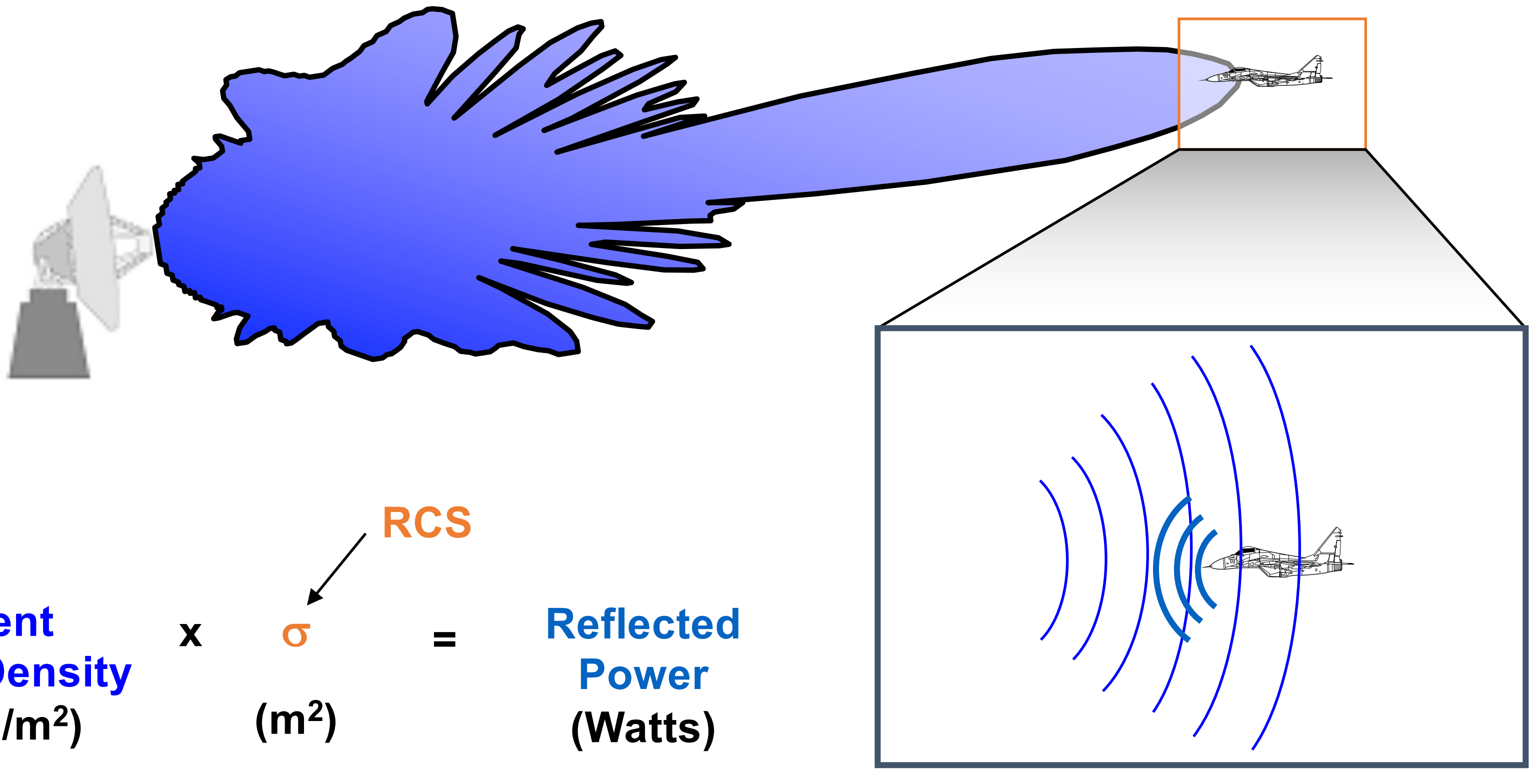
Radar equation

$$\text{Received Signal Energy} = \overset{\text{Transmit Power}}{\left[P_T\right]} \overset{\text{Transmit Gain}}{\left[\frac{4\pi A}{\lambda^2}\right]} \overset{\text{Spread Factor}}{\left[\frac{1}{4\pi R^2}\right]} \overset{\text{Losses}}{\left[\frac{1}{L}\right]} \overset{\text{Target RCS}}{\left[\sigma\right]} \overset{\text{Spread Factor}}{\left[\frac{1}{4\pi R^2}\right]} \overset{\text{Receive Aperture}}{\left[A\right]} \overset{\text{Dwell Time}}{\left[\tau\right]}$$

Radar cross section tells us about the target properties

It is the effective target cross section as seen by the radar

Radar Cross Section (RCS)



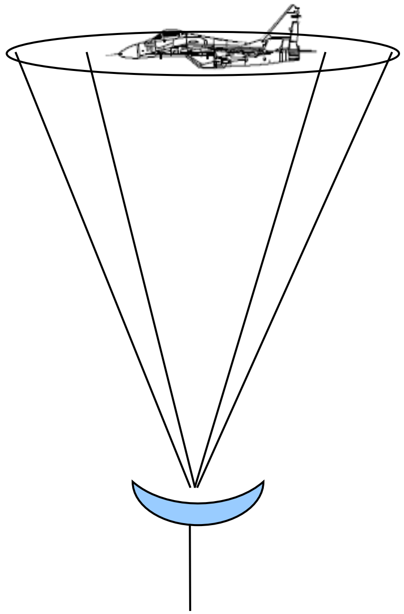
RCS

Incident Power Density (Watts/m^2) **x** σ (m^2) **=** **Reflected Power** (Watts)

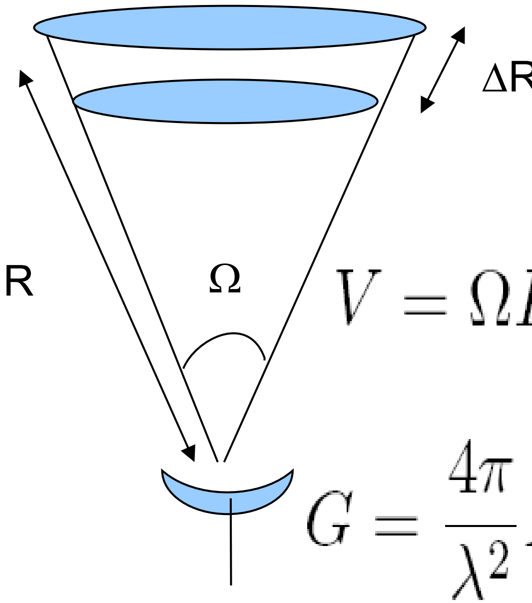
Radar Cross Section (RCS, or s) is the *effective* cross-sectional area of the target as seen by the radar

measured in m^2, or dBm^2

Hard targets vs. Soft targets



vs.



$$V = \Omega R^2 \, \Delta R$$

$$G = \frac{4\pi}{\lambda^2} A = \frac{4\pi}{\Omega}$$

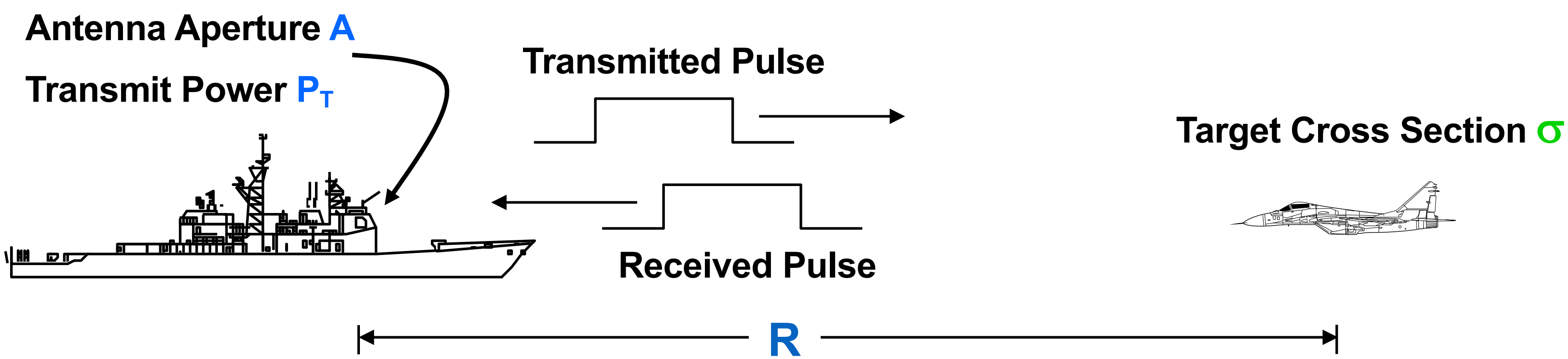
$$P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4}$$

$$P_r = \frac{P_t A \sigma_v \Delta R}{4\pi R^2}$$

Volume scattering - Ionosphere

- Volume scattering cross section σ_v has area/volume units
- Signal is proportional to range resolution
- What about the ionosphere ?
  - Cross section of a single electron = 10^{-28} m^2
  - Cross section of a bunch of electrons in a 10 km^3 volume in the ionosphere assuming electron density = 10^{12} /m^3, is $10^{10} \times 10^{12} \times 10^{-28} = 10^{-6}$ m^2 !!)
  - **CAN be measured by an incoherent scatter radar.**

Radar Range Equation



	Transmit Power	Transmit Gain	Spread Factor	Losses	Target RCS	Spread Factor	Receive Aperture	Dwell Time
Received Signal Energy =	$[P_T]$	$\left[\frac{4\pi A}{\lambda^2}\right]$	$\left[\frac{1}{4\pi R^2}\right]$	$\left[\frac{1}{L}\right]$	$[\sigma]$	$\left[\frac{1}{4\pi R^2}\right]$	$[A]$	$[\tau]$

What to know

Define phase velocity and group velocity

Define refraction and dispersion

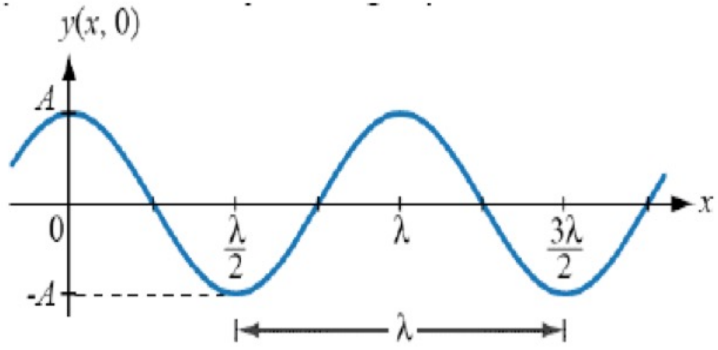
Explain concept of dispersion relation

Radio Waves

$$y(x, t) = A\cos(\omega t - kx + \phi_o)$$

Angular frequency
$\omega = 2\pi f = 2\pi/T$

Wavenumber
$k = 2\pi/\lambda$



(a) $y(x, t)$ versus x at $t = 0$

Phase velocity defined as $v_{\mathrm{p}} = \frac{\omega}{k}$

The phase velocity **the velocity with which phase fronts propagate in a medium**.

The group velocity of a wave is the **velocity** with which the overall envelope shape of the wave's amplitudes—known as the modulation or envelope of the wave—propagates through space.

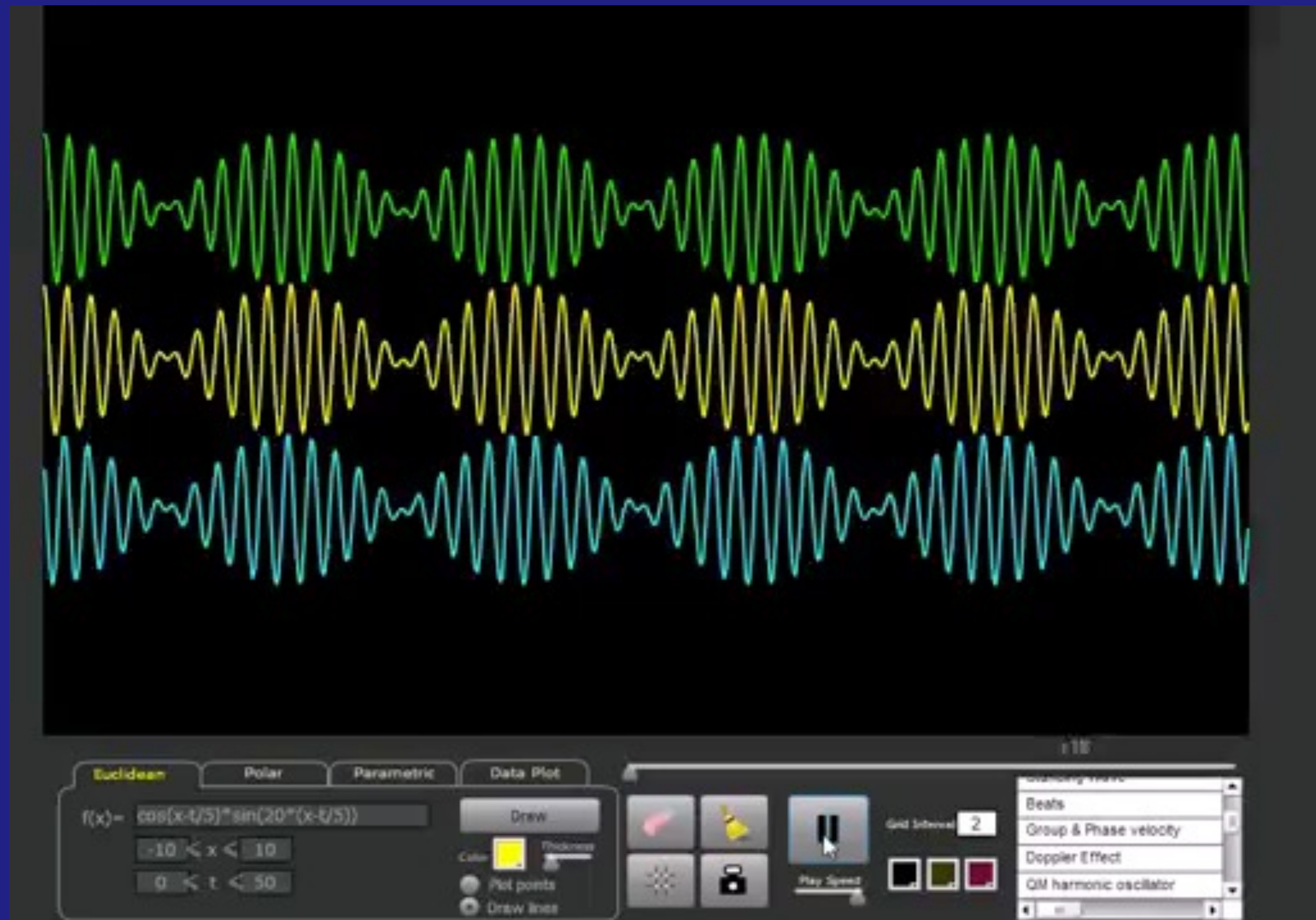
$$v_g \equiv \frac{\partial \omega}{\partial k}$$

Phase Velocity, Group Velocity, Index of Refraction

$$v_\mathrm{p} = \frac{\omega}{k}$$

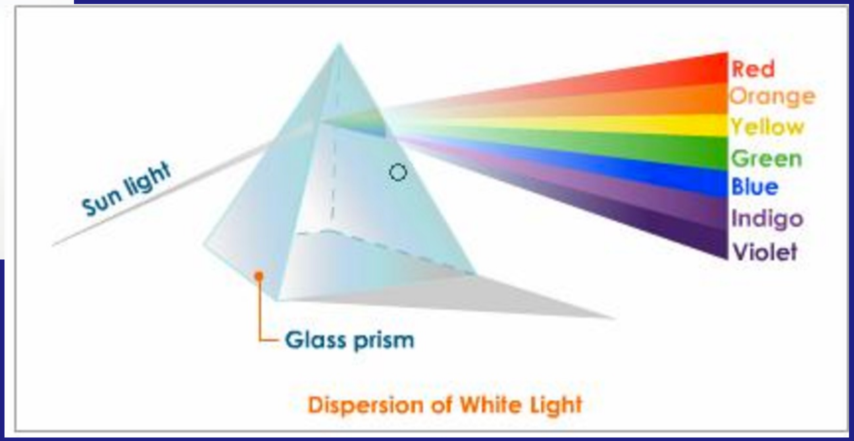
$$v_g \equiv \frac{\partial \omega}{\partial k}$$

$$n = \frac{c}{v_\mathrm{p}}.$$



Refraction and Dispersion

$$n = \frac{c}{v_{\mathrm{p}}}.$$



Index of Refraction $n = \frac{c}{v_p}$. in the Ionosphere

$$n^2 = 1 - \frac{X}{1 - iZ - \frac{\frac{1}{2}Y^2\sin^2\theta}{1-X-iZ} \pm \frac{1}{1-X-iZ}\left(\frac{1}{4}Y^4\sin^4\theta + Y^2\cos^2\theta(1-X-iZ)^2\right)^{1/2}}$$

where

n is the index of refraction

$$X = \frac{\omega_{pe}^2}{\omega^2} \qquad Y = \frac{\omega_c}{\omega} \qquad Z = \frac{\nu}{\omega} \qquad \omega_{pe} = \left(\frac{Ne^2}{\varepsilon_0 m_e}\right)^{1/2} \qquad \omega_c = \frac{e|B|}{m_e}$$

ω = the angular frequency of the radar wave,

$Y_L = Y\cos\theta$, $Y_T = Y\sin\theta$,

θ = angle between the wave vector $\bar{k}$ and $\bar{B}$,

$\bar{k}$ = wave vector of propagating radiation,

$\bar{B}$ = geomagnetic field, N = electron density

e = electronic charge, m_e = electron mass, ν = electron collision frequency

and ε_0 = permittivity constant.

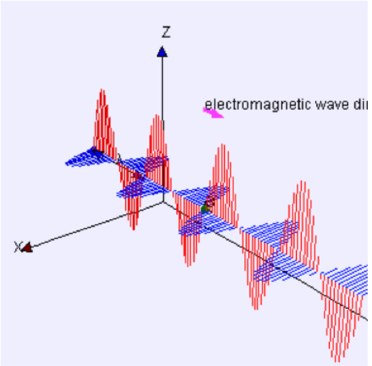
Dispersion relation: the concept

Key concept for wave behavior within a propagation medium.

Describes the relationship between SPATIAL frequency (wavelength) and TEMPORAL frequency.

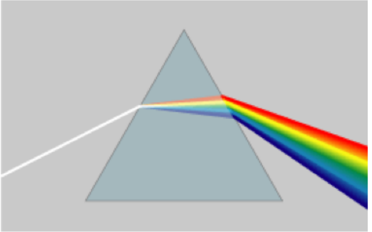
Some wave modes relate wavelength to frequency **linearly,** but waves in most media have **nonlinear** relation between wavelength and frequency.

Linear dispersion example:
EM radiation propagation through free space
(wavelength / velocity = c)



http://weelookang.blogspot.com/2011/10/ejs-open-source-propagation-of.html

Nonlinear dispersion example:
splitting of light through a prism
(effective speed of light depends on wavelength due to glass' non-unity index of refraction)



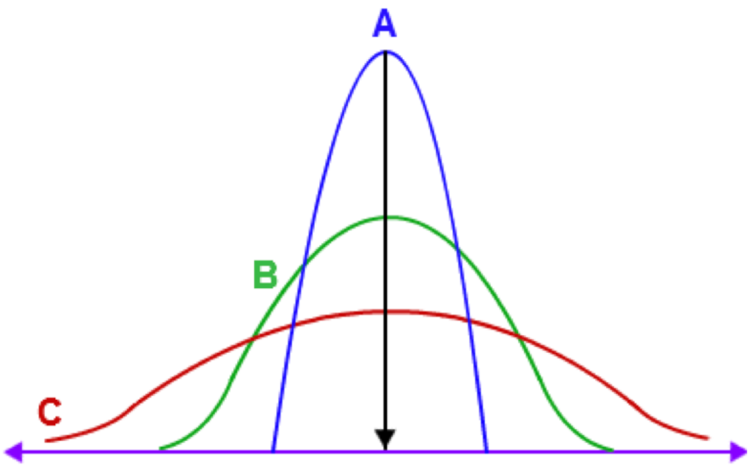
Dispersion relation: the concept

Simple linear case: uniform phase velocity

$$\omega(k) = c\ k$$

Most propagation speeds depend nonlinearly on the wavelength and/or frequency.

NB: for a **nonlinear** dispersion relation, the pulse will typically spread in either spatial frequency or temporal frequency as a function of time.



Example of pulse spreading spatially from time A to B to C.

http://www.mathcaptain.com/statistics/dispersion-statistics.html

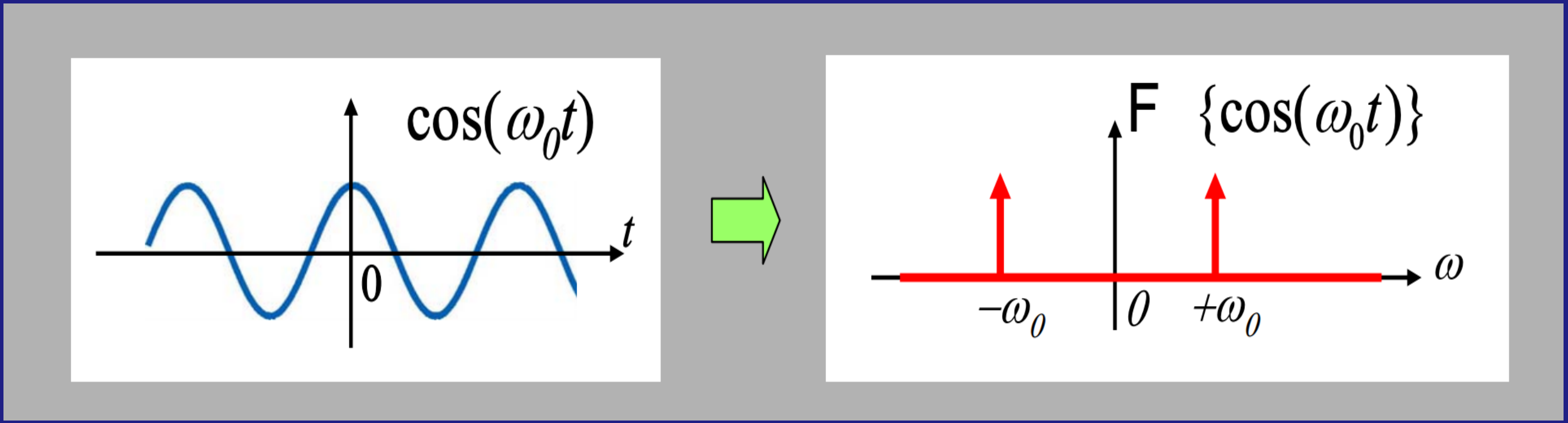
What to know

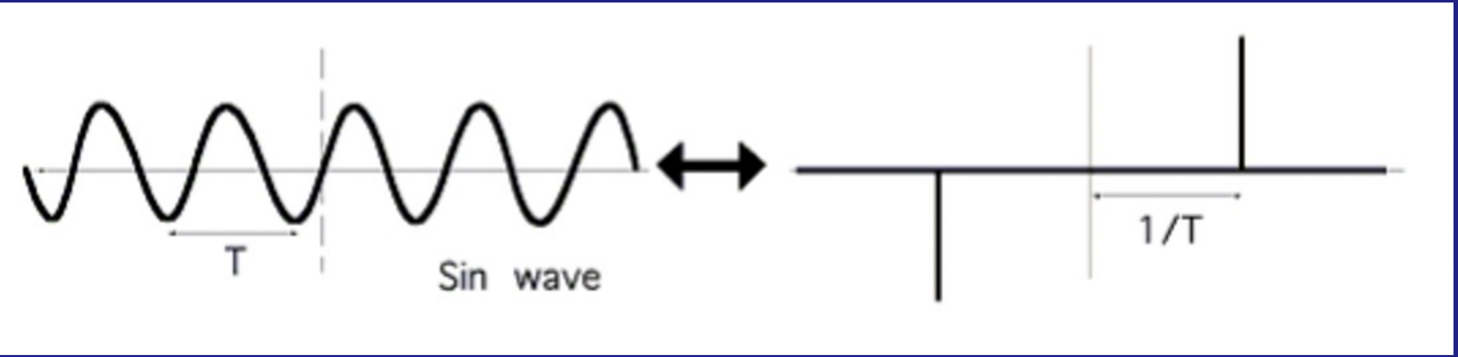
What is the Fourier transform of cosine wave?

What is the Fourier transform of a sine wave?

Write out $e^{i(kx-wt)}$ in the form of sine and cosines

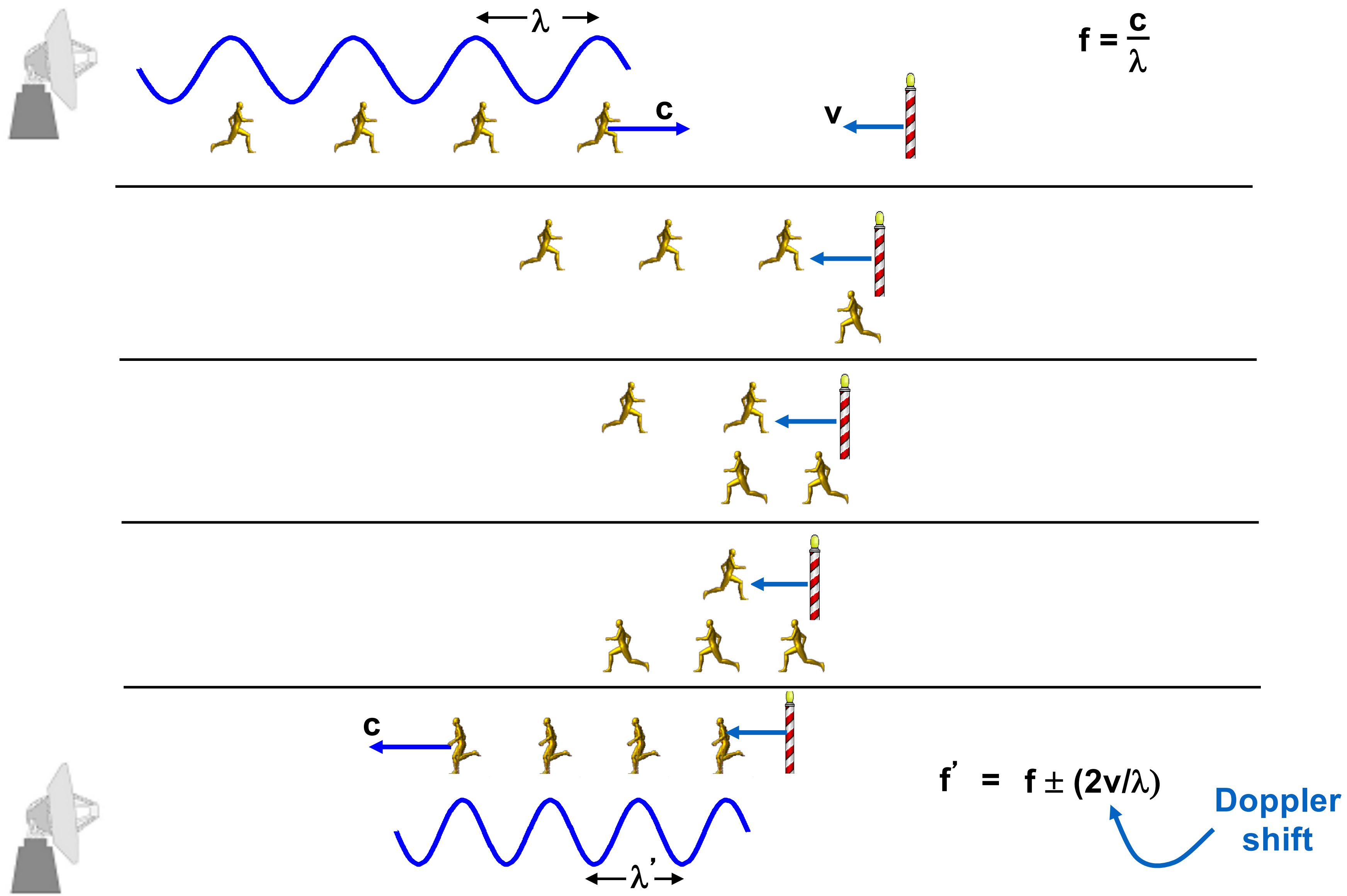
How does one measure the direction of Doppler phase shift?





$$e^{ix} = \cos(x) + i\sin(x)$$

Doppler Shift Concept



Doppler shift frequency

Tx signal: $\cos(2\pi f_o t)$

Return from a moving target: $\cos[2\pi f_o(t + 2R/c)]$

If target is moving with a constant velocity: $R = R_o + v_o t$

then,

Return: $\cos[2\pi(f_o + f_o 2v_o/c)t + 2\pi f_o R_o/c]$

Doppler frequency:
$-2f_o v_o/c = -2v_o/\lambda_o$