An Introduction to the Terrestrial Ionosphere

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Hydrostatic Equilibrium

Vertical structure of neutral atmosphere determined by balance between **pressure gradient** and **gravity**.

$$\nabla p = -mng$$
$$\frac{d}{dz} n k_B T = -mng$$

In the simplest case of constant T

$$k_B T \frac{d}{dz} n = -mng$$
$$\frac{1}{n}\frac{d}{dz} = -\frac{mg}{k_B T} = -\frac{1}{H}$$

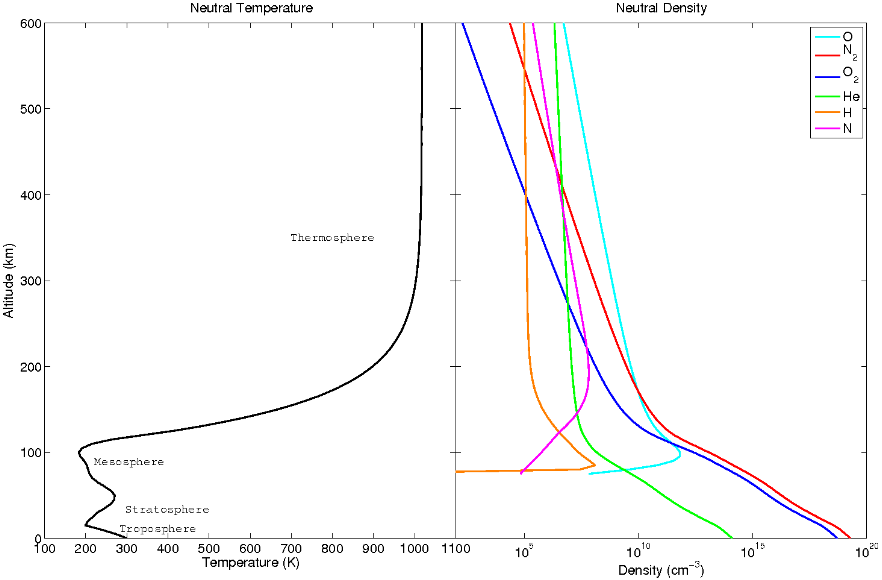
Define the scale height:

$$H = \frac{k_B T}{mg}$$

Solution:

$$n(z) = n\left(z_0\right) \exp\left[-\frac{z - z_0}{H}\right]$$

The Neutral Atmosphere (according to NRLMSISE-00)



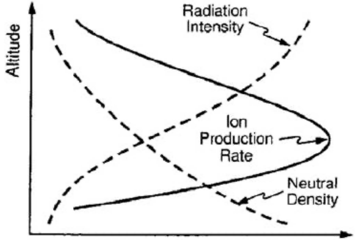
Absorption of EUV Radiation

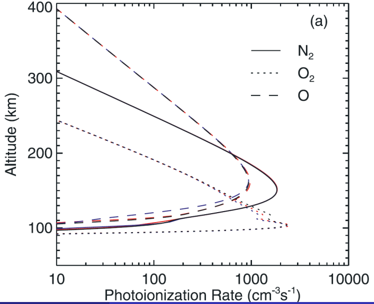
Photoionization Rate:

$$P_i(s) = \int d\lambda \sum_n \sigma_n^{ion}(\lambda) N_n(s) I(s,\lambda)$$

Attenuation of EUV Flux (Lambert's Law):

$$\frac{dI}{ds} = -\sum_n \sigma_n^{abs}(\lambda) N_n(s) I(s,\lambda)$$





Basic Ionospheric Photochemistry

Photoionization:

$$\mathrm{O} + h\nu(> 13.6\ \mathrm{eV}) \rightarrow \mathrm{O}^+ + e^*$$

Radiative Recombination (SLOW):

$$\mathrm{O}^+ + e \rightarrow \mathrm{O} + h\nu(13.6\ \mathrm{eV})$$

Atom-Ion Interchange:

$$\mathrm{O}^+ + \mathrm{N}_2 \rightarrow \mathrm{NO}^+ + \mathrm{N}$$
$$\mathrm{O}^+ + \mathrm{O}_2 \rightarrow \mathrm{O}_2^+ + \mathrm{O}$$

Dissociative Recombination:

$$\mathrm{NO}^+ + e \rightarrow \mathrm{N} + \mathrm{O}$$
$$\mathrm{O}_2^+ + e \rightarrow \mathrm{O} + \mathrm{O}$$

Airglow:

$$\mathrm{O}\left({}^1\mathrm{D}\right) \rightarrow \mathrm{O}\left({}^3\mathrm{P}\right) + h\nu(630.0\ \mathrm{nm})$$
$$\mathrm{O}\left({}^1\mathrm{S}\right) \rightarrow \mathrm{O}\left({}^1\mathrm{D}\right) + h\nu(557.7\ \mathrm{nm})$$

The Ionosphere and Thermosphere

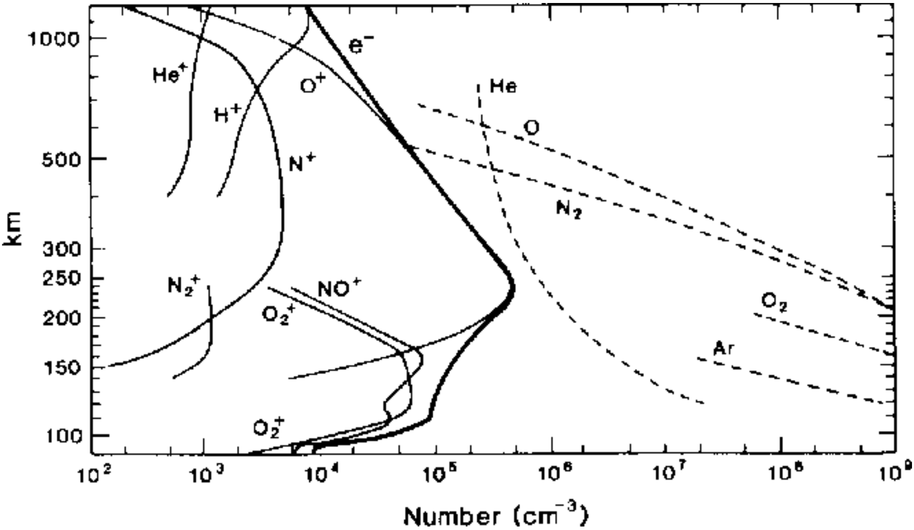
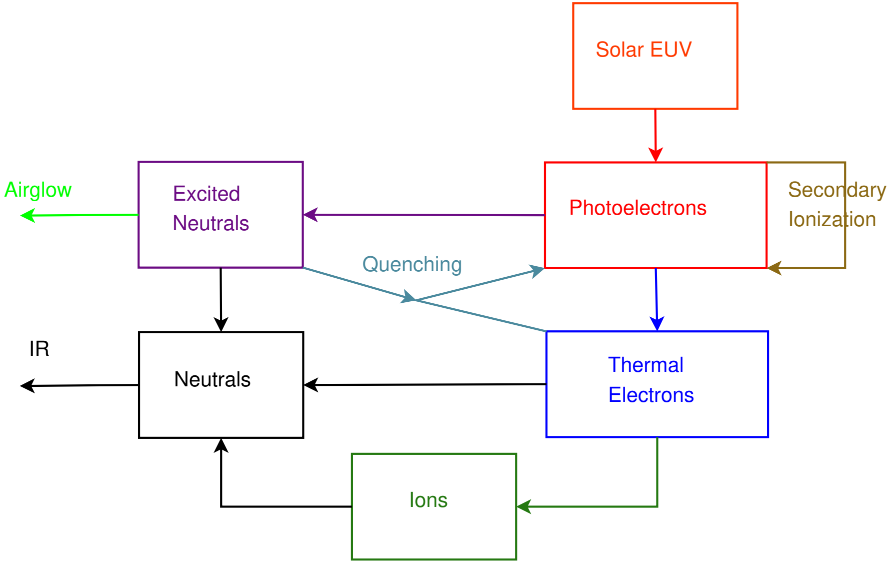
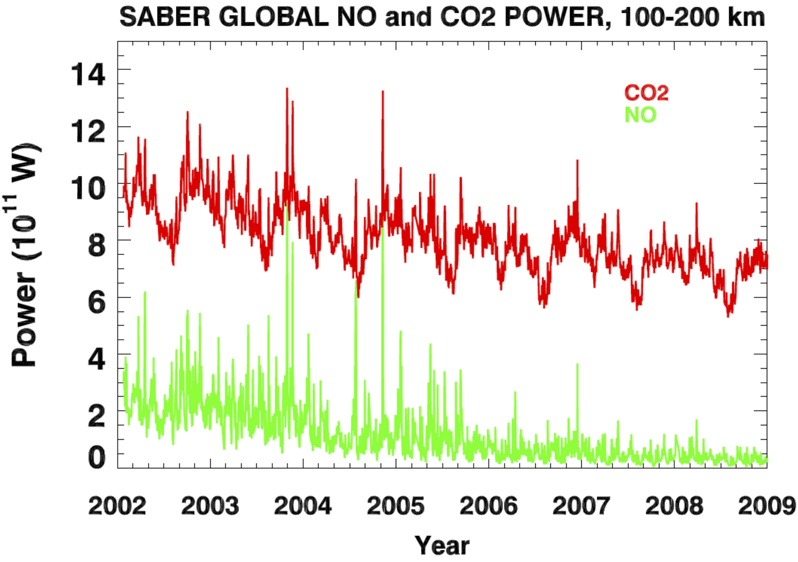


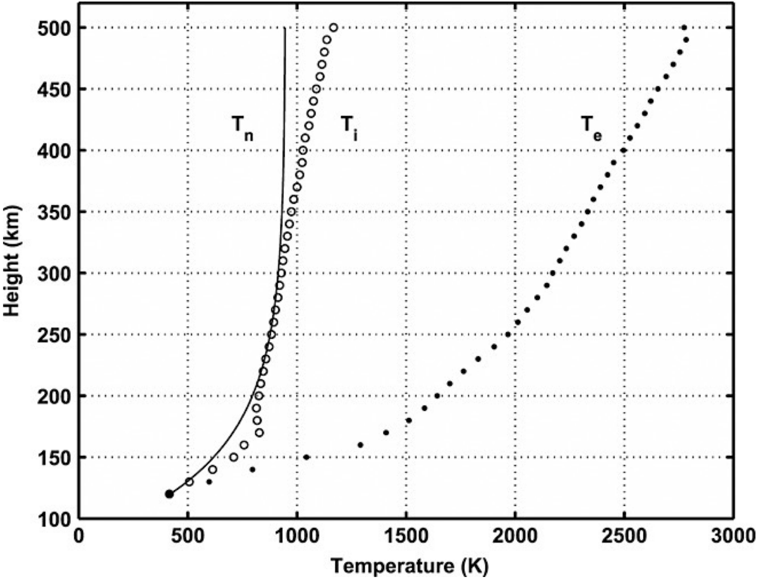
Diagram of Ionospheric Energetics



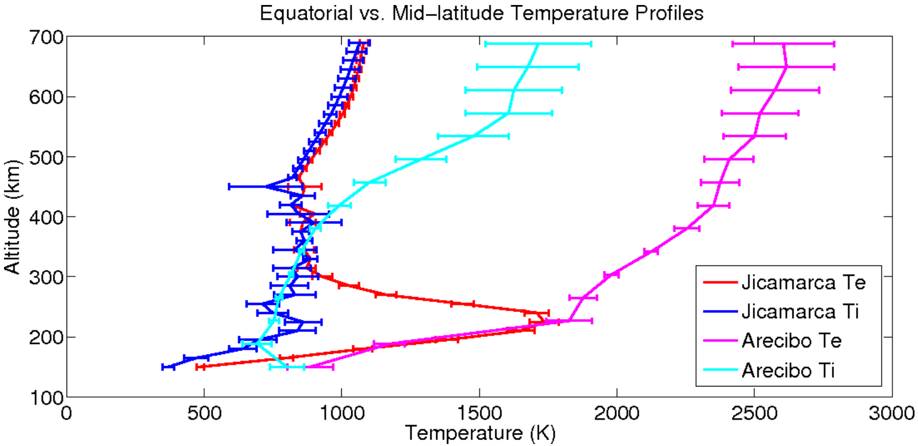
IR Cooling of Thermosphere



Typical Daytime Mid-latitude Temperature Profiles



Temperature Profiles at Different latitudes



Importance of the Magnetic Field

Dynamic Pressure: mnu^2

Thermal Pressure: $nk_B T = mn\left(\sqrt{\frac{k_B T}{m}}\right)^2$

Magnetic Pressure: $\frac{B^2}{2\mu_0} = mn\frac{1}{2}\left(\frac{B}{\sqrt{\mu_0 mn}}\right)^2$

Typical Numbers for the ionosphere:

$$u \approx 100 \text{ m/s Midlatitude Ionosphere}$$

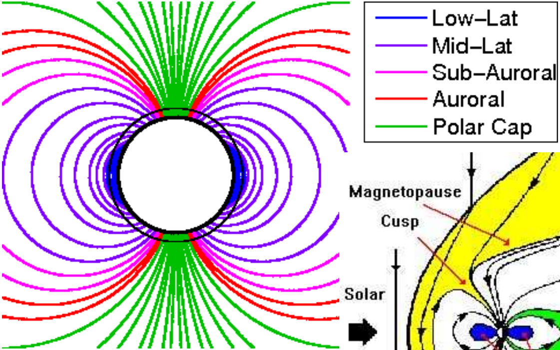
$$u \approx 20 \text{ km/s H}^+ \text{ Outflow in Polar Wind}$$

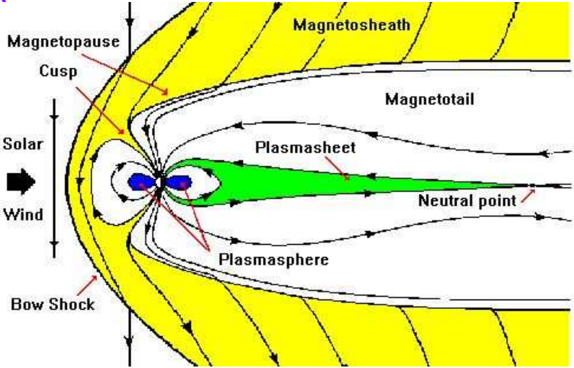
$$T \approx 2000\text{K} \rightarrow \sqrt{\frac{k_B T}{m}} = \begin{cases} 1 \text{ km/s} & \text{for O}^+ \\ 4 \text{ km/s} & \text{for H}^+ \end{cases}$$

$$B \approx 3 \times 10^{-5} \text{ T}, \ n \approx 10^{11} \text{ m}^{-3} \rightarrow \frac{B}{\sqrt{\mu_0 mn}} \approx 500 \text{ km/s}$$

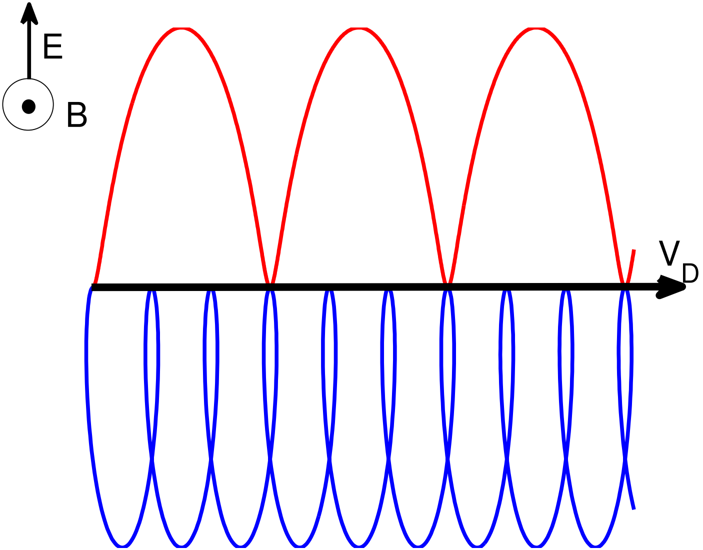
Transport parallel and perpendicular to B are fundamentally different.

Magnetic Structure of the Ionosphere and Magnetosphere





Perpendicular Transport: The $\mathbf{E} \times \mathbf{B}$ Drift



$$v_D = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

Moments of the Boltzmann Equation

Boltzmann Equation:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \left[\frac{e}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) + \mathbf{g}\right] \cdot \frac{\partial f}{\partial \mathbf{v}} = \left.\frac{\delta f}{\delta t}\right|_{\text{collisions}}$$

Continuity Equation: Apply $\int d\mathbf{v}$ to Boltzmann Equation

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial \mathbf{x}} \cdot (n\mathbf{u}) = \frac{\delta n}{\delta t}$$

Momentum Equation: Apply $\int m\mathbf{v} d\mathbf{v}$ to Boltzmann Equation

$$\frac{\partial}{\partial t}(mn\mathbf{u}) + \frac{\partial}{\partial \mathbf{x}} \cdot (mn\mathbf{u}\mathbf{u} + \mathbf{P}) = ne\left(\mathbf{E} + \mathbf{u} \times \mathbf{B}\right) + nm\mathbf{g} + \frac{\delta M}{\delta t}$$

Energy Equation: Apply $\int \frac{1}{2} m v^2 d\mathbf{v}$ to Boltzmann Equation

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial}{\partial \mathbf{x}} \cdot \left[\mathbf{u} \cdot (\epsilon \mathbf{I} + \mathbf{P}) + \mathbf{q}\right] = ne\mathbf{u} \cdot \mathbf{E} + nm\mathbf{u} \cdot \mathbf{g} + \frac{\delta E}{\delta t}$$

Closing the System of Transport Equations

- Easiest way: Assume an isotropic Maxwellian distribution $\rightarrow$ 5-moment approximation

$$\mathbf{P} \rightarrow p\mathbf{I} \qquad \mathbf{q} \rightarrow 0$$

  Energy equation reduces to the adiabatic gas law $\frac{D}{Dt}\left(\frac{p}{n^\gamma}\right) = 0$
- Hard way: Assume more complicated distributions (e.g. Maxwellians times truncated series expansions). The 8-, 10-, 13-, 16-, and 20-moment equations are derived this way.
- Middle Ground: Assume higher moments are small and derive steady state limits of high-moment transport equations

$$\mathbf{q} = -\kappa \cdot \nabla T$$

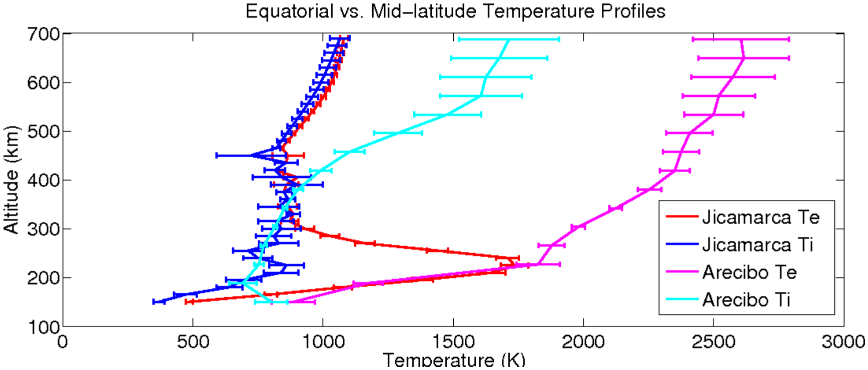
Thermal Conduction

In the F-region $\kappa = \kappa_\parallel \hat{\mathbf{b}}\hat{\mathbf{b}} \rightarrow \mathbf{q} = -\kappa_\parallel \nabla_\parallel T \hat{\mathbf{b}}$

For a fully ionized plasma $\kappa_\parallel = 7.7 \times 10^5 T_e^{5/2} \ \mathrm{eVcm^{-2}s^{-1}K^{-1}}$

Parallel Temperature Equation:

$$\frac{\partial T}{\partial t} + u_\parallel \nabla_\parallel T + \frac{2}{3} T \nabla_\parallel \cdot \mathbf{u} - \frac{2}{3}\frac{1}{nk_B} \nabla_\parallel \cdot \kappa_\parallel \nabla_\parallel T = \frac{2}{3}\frac{1}{nk_B}(Q - L)$$



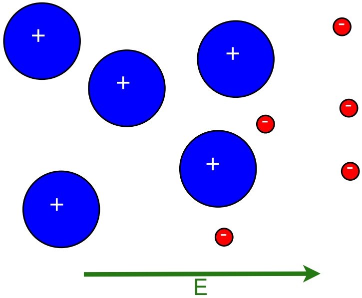
Ambipolar Electric Fields and Ambipolar Diffusion

Steady state parallel electron momentum equation:

$$\cancel{m_e \left[\frac{\partial}{\partial t} \left(n_e u_e \right) + \nabla_\parallel \cdot \left(n_e u_e^2 \right) \right]} = -\nabla_\parallel p_e - n_e e E_\parallel \longrightarrow E_\parallel = -\frac{1}{e n_e} \nabla_\parallel p_e$$

Substitute into parallel ion momentum equation:

$$m_i \left[\frac{\partial}{\partial t} \left(n_i u_i \right) + \nabla_\parallel \cdot \left(n_i u_i^2 \right) \right] = -\nabla_\parallel p_i - \frac{n_i}{n_e} \nabla_\parallel p_e - m_i n_i g_\parallel$$
$$- m_i n_i \sum_j \nu_{ij} \left(u_i - u_j \right)$$



Energetic Electron Transport

Populations of electrons in the ionosphere:

- Thermal: $k_B T_e \sim 0.2\ \mathrm{eV}$
- Photoelectrons: mostly $< 60\ \mathrm{eV}$, peak energy flux at $\sim 20\ \mathrm{eV}$
- Soft Precipitation (e.g. cusp, polar rain): $100 - 1000\ \mathrm{eV}$
- Auroral Precipitation: $> 1\ \mathrm{keV}$

Simplified kinetic equations derived by

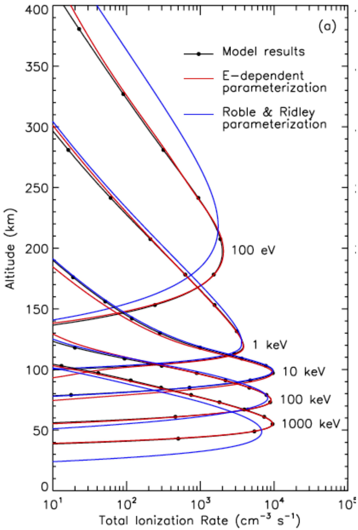
- Assuming suprathermal density $\ll$ thermal density
- Ignoring perpendicular transport
- Assuming gyrotropy (azimuthal symmetry about **B**)
- Assuming steady state ($m_e \to 0$)

Simplest possible form is derived by additionally neglecting $E_{||}$, $\frac{\partial B}{\partial s}$, and Coulomb collisions and assuming isotropic elastic collisions.

$$\mu \frac{\partial \Phi}{\partial s} = q + \sum_n \left\{ -\left[\sigma_{an}(\mathcal{E}) + \sigma_{en}(\mathcal{E})\right] N_n \Phi + \frac{\sigma_{en}(\mathcal{E})}{2} N_n \int_{-1}^{1} \Phi(s, \mathcal{E}, \mu')\, d\mu' \right\}$$

This has the same mathematical form as a radiative transfer equator

Auroral Particle Deposition

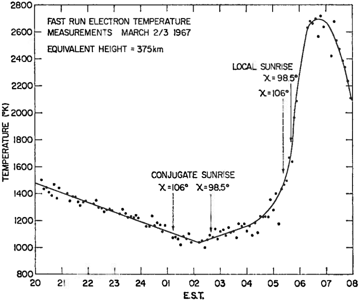


Higher energy particles penetrate deeper into atmosphere.

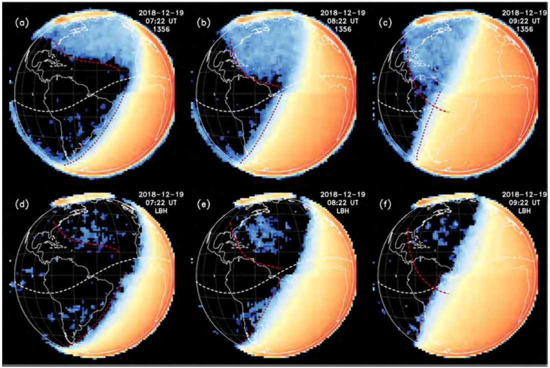
Fang et al. (2008)

Conjugate Photoelectron Transport

Millstone Hill T_e Observations



GOLD Ultraviolet Emissions



Solomon et al. 2020

Fundamentals of Ionospheric Electrodynamics

Electrostatic Limit of Maxwell's Equations:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \cancelto{0}{\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}} \qquad \longrightarrow \nabla \cdot \mathbf{J} = 0$$

$$\nabla \times \mathbf{E} = -\cancelto{0}{\frac{\partial \mathbf{B}}{\partial t}} \qquad \longrightarrow \mathbf{E} = -\nabla \Phi$$

Ohm's Law for the ionosphere:

$$\mathbf{J} = \sigma \cdot \mathbf{E} + \mathbf{J}_0$$

Putting everything together yields a boundary value problem:

$$\nabla \cdot \sigma \cdot \nabla \Phi = \nabla \cdot \mathbf{J}_0$$

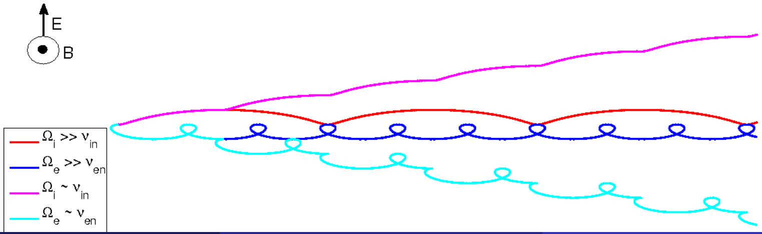
Ohm's Law for the Ionosphere

Steady-state momentum equation for each species (zero neutral wind case):

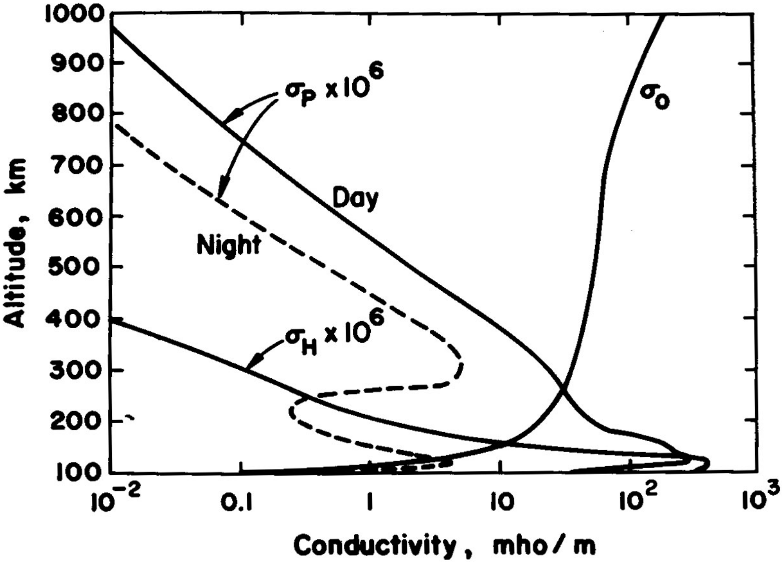
$$0 = n_\alpha q_\alpha (\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B}) - \nu_{\alpha n} m_\alpha n_\alpha \mathbf{u}_\alpha$$

Resulting Ohm's Law:

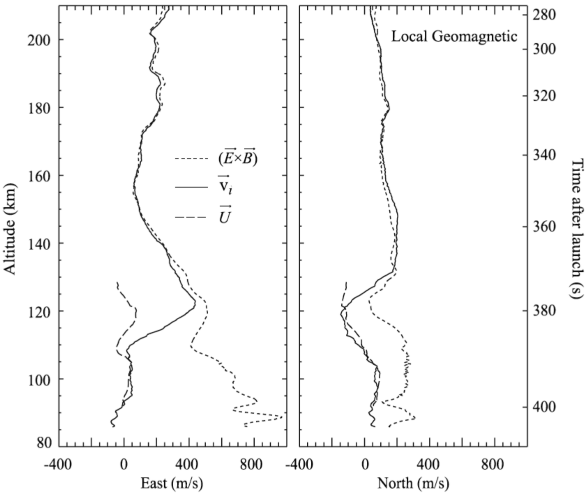
$$\mathbf{J} = \sum_\alpha n_\alpha q_\alpha \mathbf{u}_\alpha \longrightarrow \mathbf{J} = \begin{pmatrix} \sigma_P & -\sigma_H & 0 \\ \sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_0 \end{pmatrix} \cdot \mathbf{E}$$



Conductivity Profiles



Ion Velocity Rotation

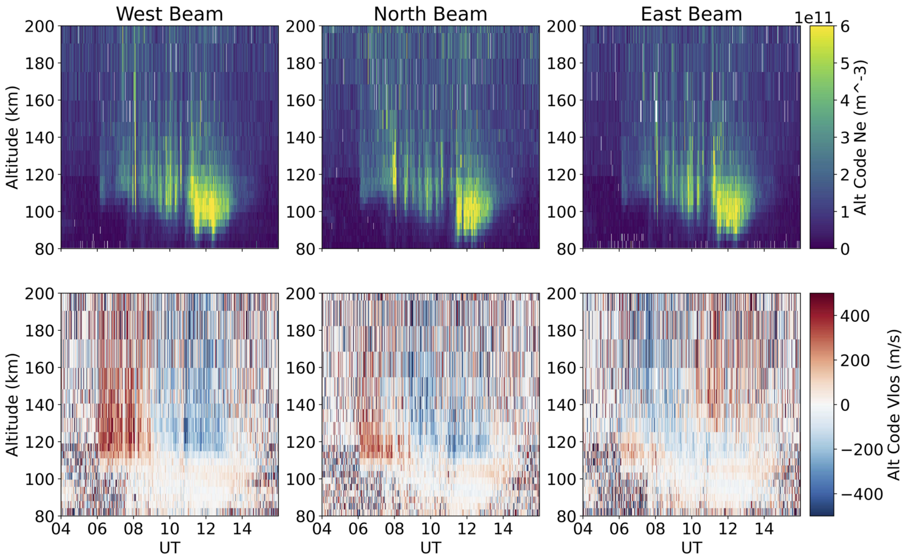


Rocket measurements comparing

- **E** from double-probe
- $\mathbf{v}_i$ from ion imager
- $\mathbf{u}_n$ from TMA chemical release

Sangalli et al. (2009) doi:10.1029/2008JA013757

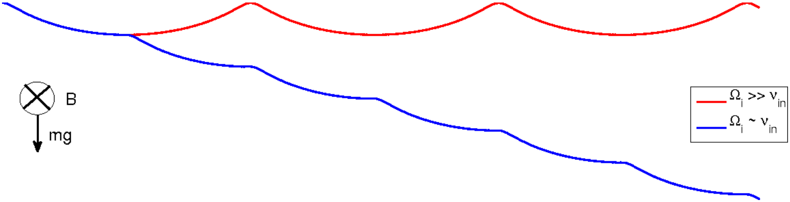
PFISR E-region Velocity Measurements



Other Kinds of Current

Substitute $\mathbf{F}$ for $q_\alpha \mathbf{E}$ in steady state momentum equation.

- Wind drag: $\mathbf{F} = \nu_{\alpha n} m_\alpha \mathbf{u}_n \longrightarrow \mathbf{J} = \sigma \cdot (\mathbf{u}_n \times \mathbf{B})$
- Gravity: $\mathbf{F} = m_\alpha \mathbf{g} \longrightarrow \mathbf{J} = \mathbf{\Gamma} \cdot \mathbf{g}$

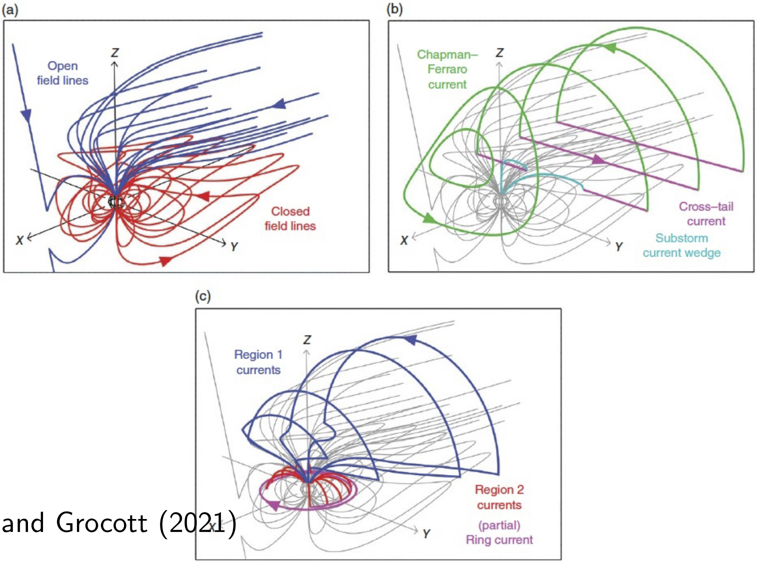


- Pressure Gradients (Diamagnetic Currents):
  $\mathbf{F} = -\frac{1}{n_\alpha} \nabla p_\alpha \longrightarrow \mathbf{J} = \mathbf{D} \cdot \nabla \sum_\alpha p_\alpha$

Complete Dynamo Equation:

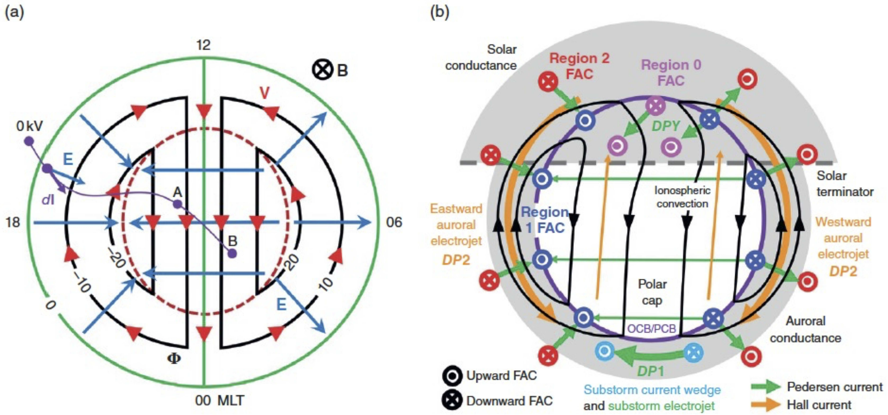
$$\nabla \cdot \sigma \cdot \nabla \Phi = \nabla \cdot \left(\sigma \cdot (\mathbf{u}_n \times \mathbf{B}) + \mathbf{\Gamma} \cdot \mathbf{g} + \mathbf{D} \cdot \nabla \sum_\alpha p_\alpha \right)$$

Current Systems in the Ionosphere and Magnetosphere



Milan and Grocott (2021)

Current Systems in the Ionosphere and Magnetosphere



High Latitude Convection Patterns