Introduction to Radar Physics

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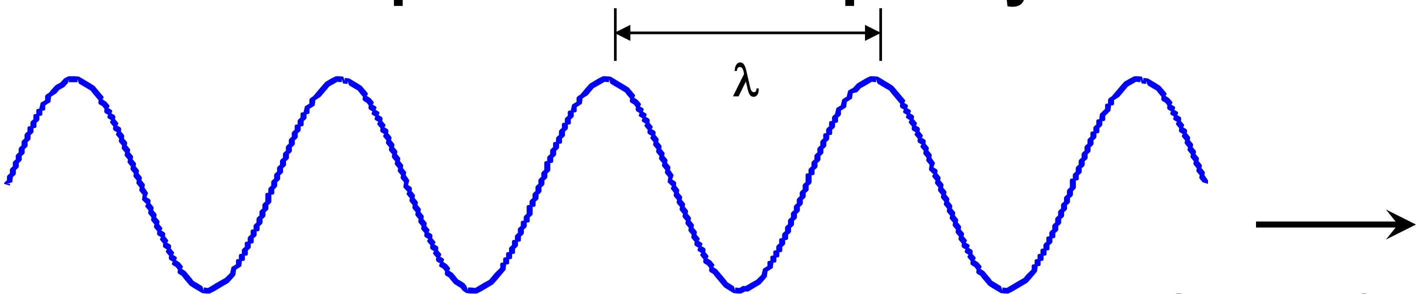
What to know

:

Definition of angular frequency, wave number

Properties of Waves

Relationship Between Frequency and Wavelength



Speed of light, c

$c = 3x10^8$ m/sec

= 300,000,000 m/sec

$$\text{Frequency (1/s)} = \frac{\text{Speed of light (m/s)}}{\text{Wavelength } \lambda \text{ (m)}}$$

Examples:

Frequency	Wavelength
100 MHz	3 m
1 GHz	30 cm
3 GHz	10 cm
10 GHz	3 cm

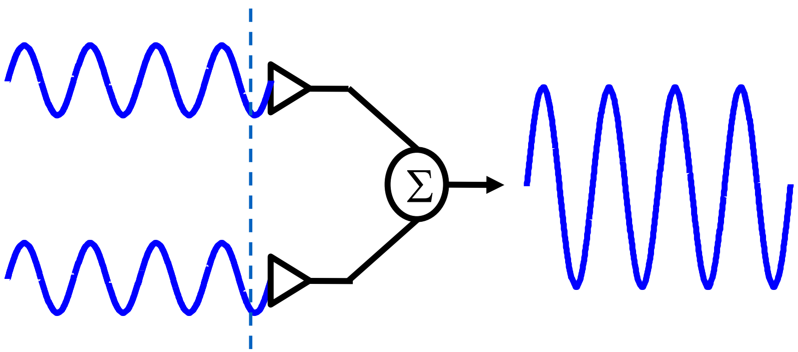
What to know

:

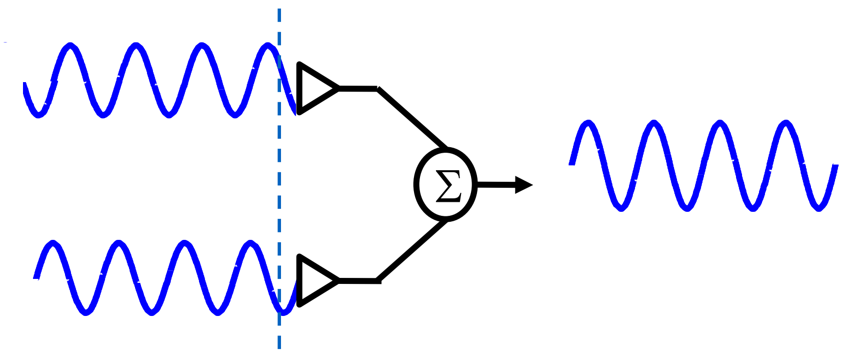
Meaning of constructive and destructive addition

Properties of Waves

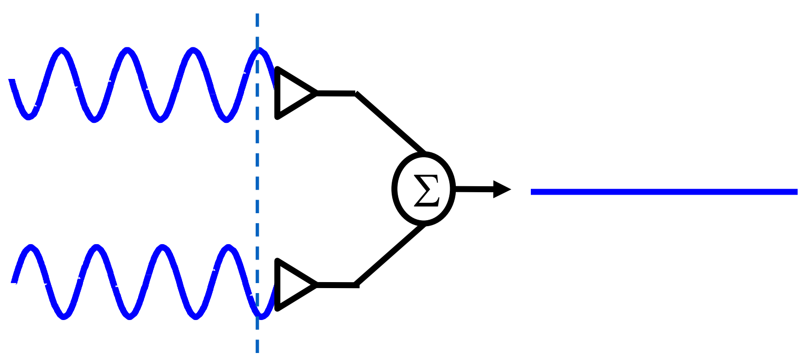
Constructive vs. Destructive Addition



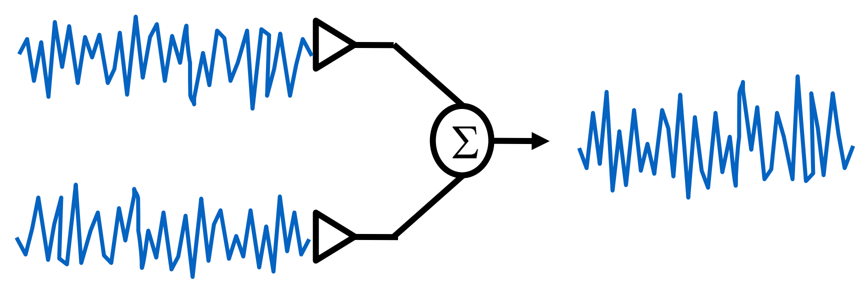
Constructive
(in phase)



Partially Constructive
(somewhat out of phase)

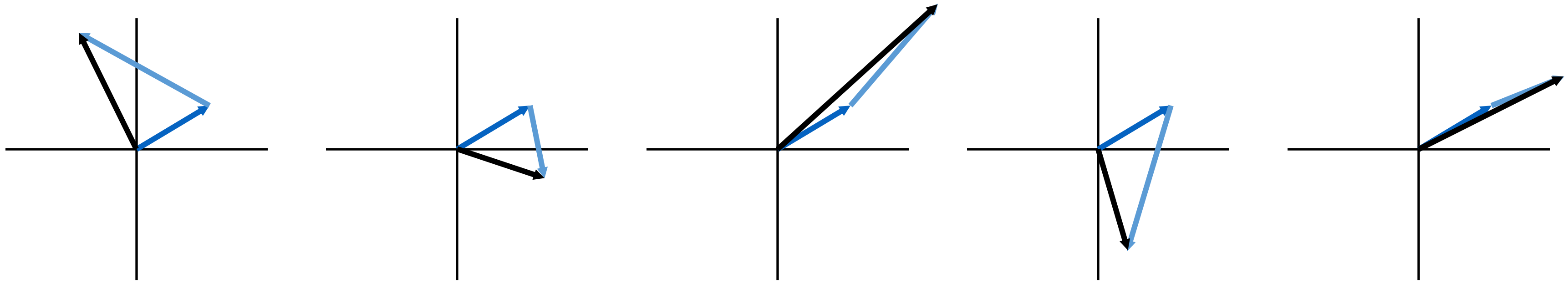


Destructive
(180° out of phase)



Non-coherent signals
(noise)

Coherent Integration

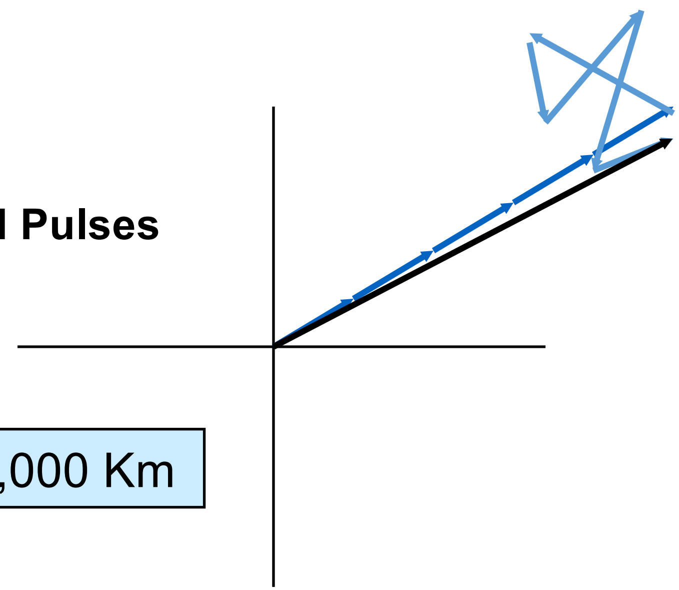


Pulse 1 Pulse 2 Pulse 3 Pulse 4 Pulse 5

- Coherent target returns
- Noise samples at low SNR

- Resultant signal

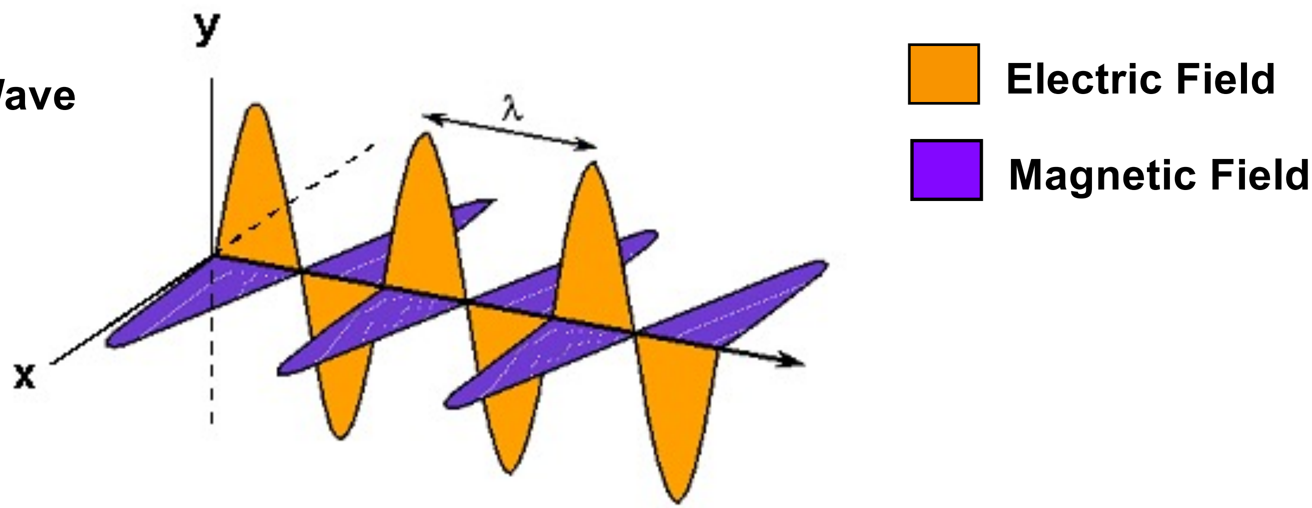
Coherently Integrated Pulses



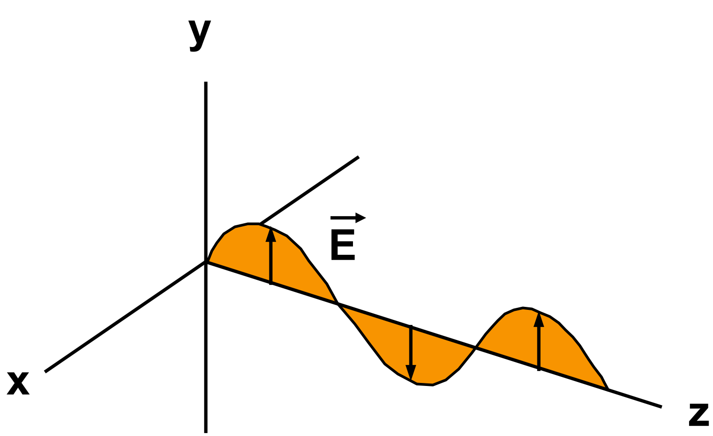
Deep space targets at 30,000 – 40,000 Km

Polarization

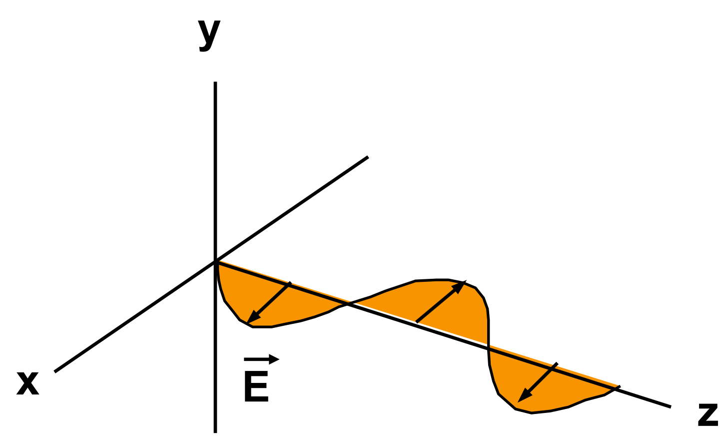
Electromagnetic Wave



Vertical Polarization



Horizontal Polarization

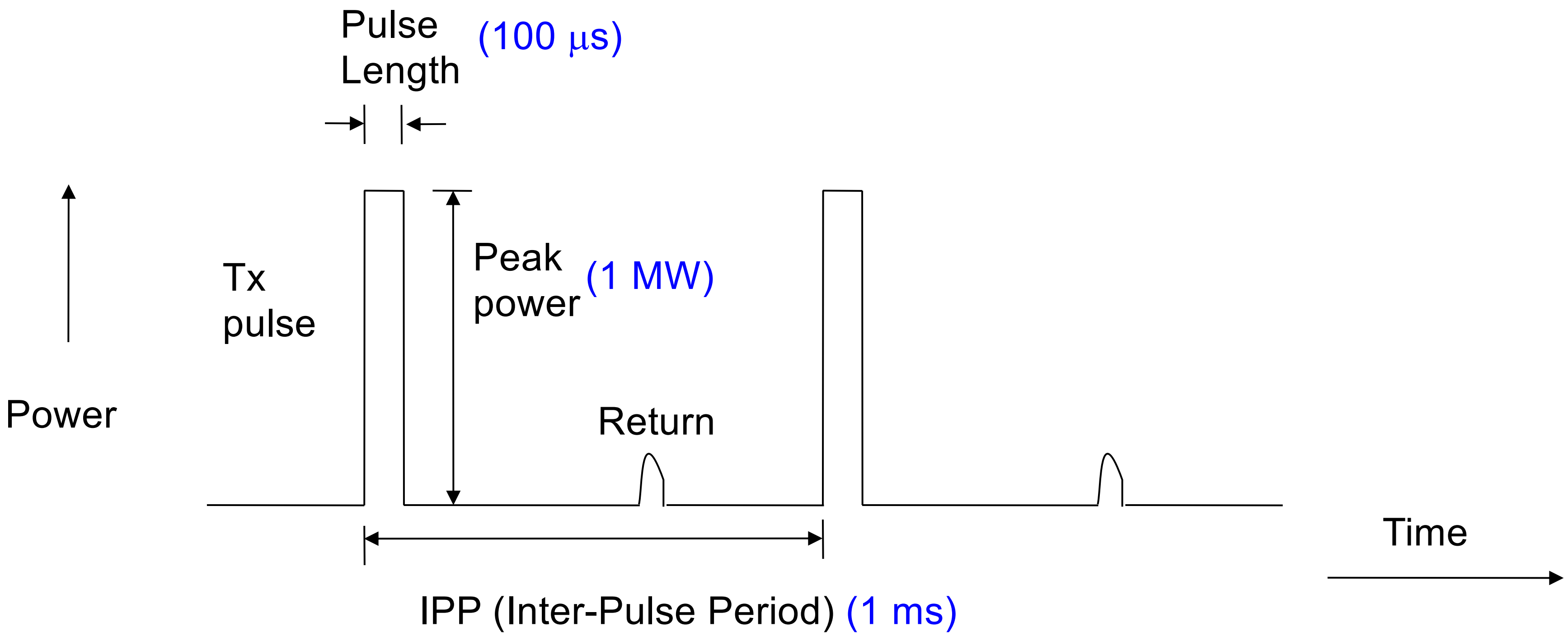


What to know

What does Duty cycle refer to?

What does IPP stand for?

Pulsed Radar



Duty cycle = Pulse Length/IPP (10%)

Average power = Peak power x Duty cycle (100 kW)

PRF (Pulse Repetition Frequency) = 1/IPP (1kHz)

Duty cycle for a CW (continuous wave) radar 100%

What to know

What is the meaning of radar?

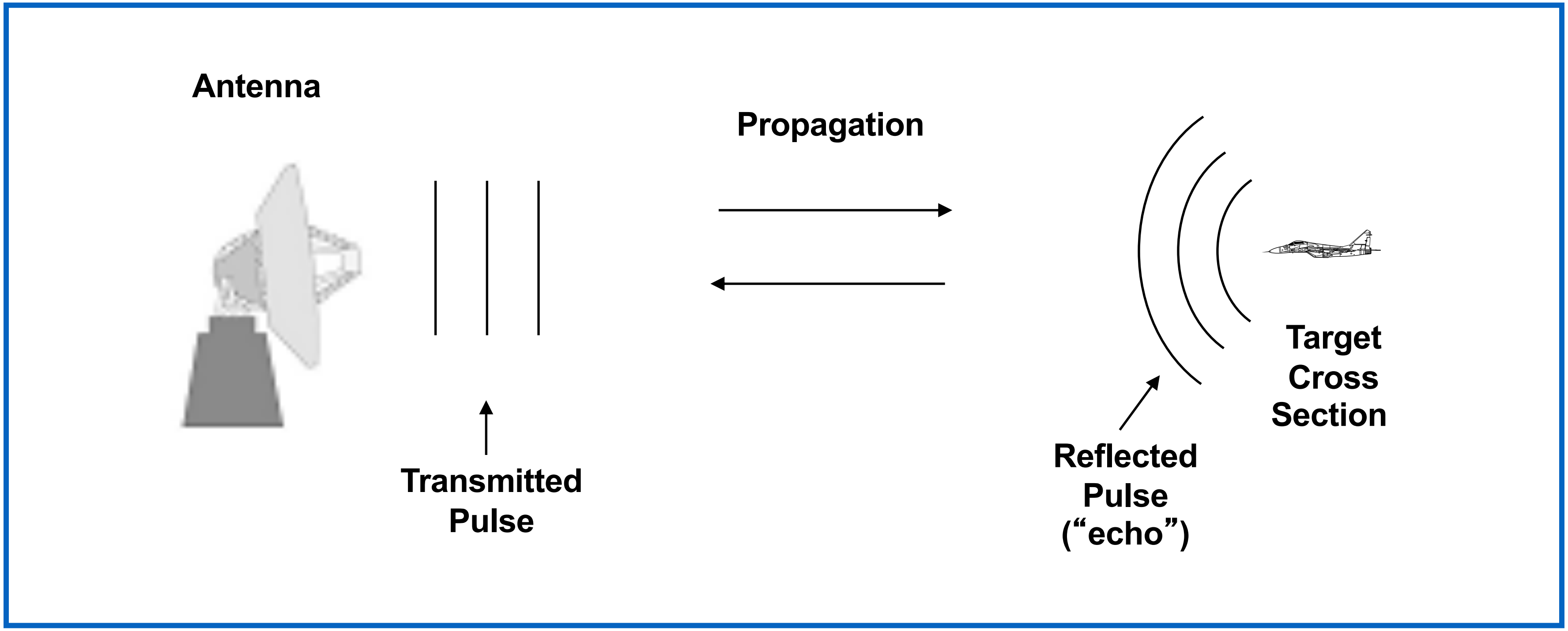
What is the meaning of radar range?

What are the main **parts** of the radar equation?

W
h
a
t

RADAR

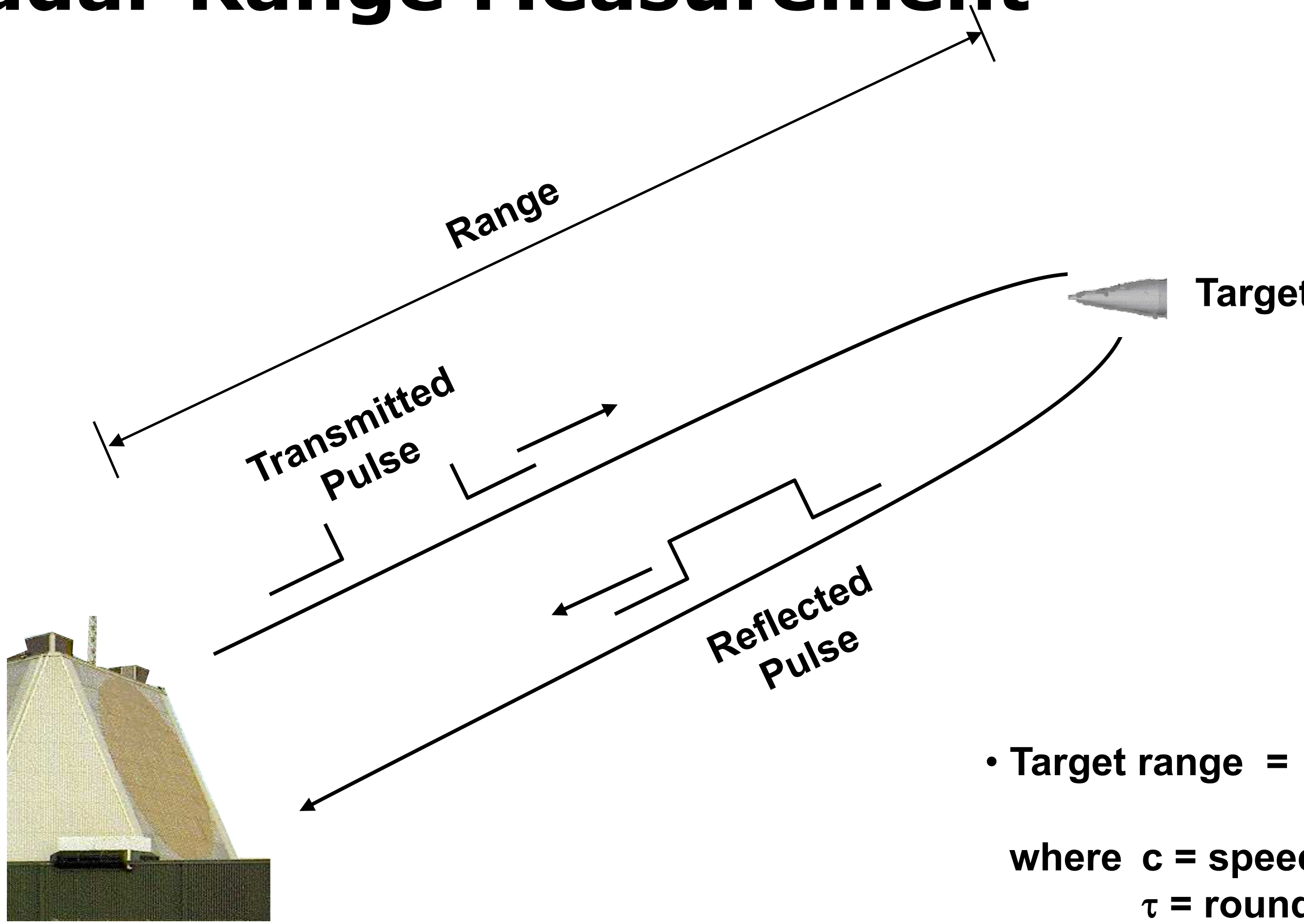
RAdio Detection And Ranging



Radar observables:

- Target range
- Target angles (azimuth & elevation)
- Target size (radar cross section)
- Target speed (Doppler)
- Target features (imaging)

Radar Range Measurement



- Target range $= \dfrac{c\tau}{2}$

  where c = speed of light
  τ = round trip time

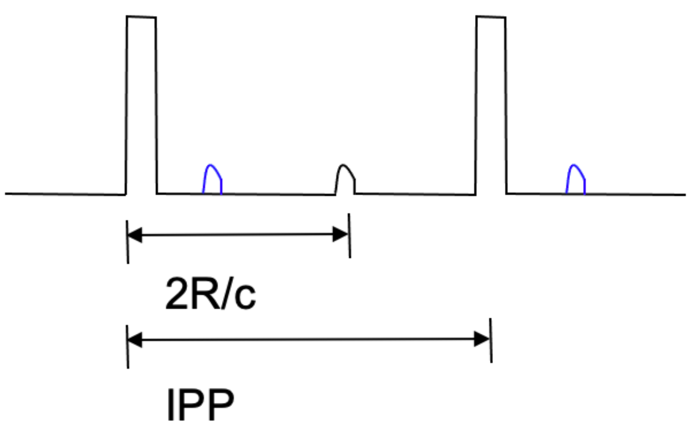
Range Resolution

Or is it ??

Range resolution is set by pulse length

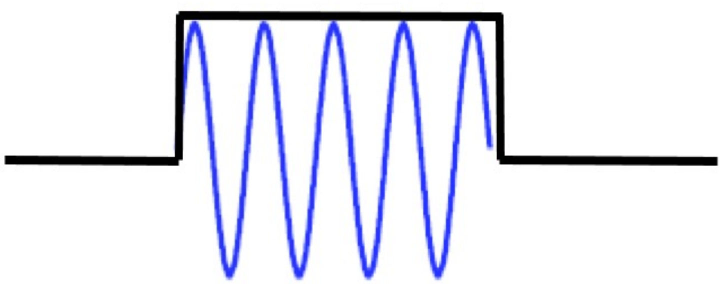
Pulse length = τ_p , Range resolution = $c\tau_p/2$ for a single target.

Maximum unambiguous range



MUR = c*IPP/2

What the radar transmits: Pulses and waves

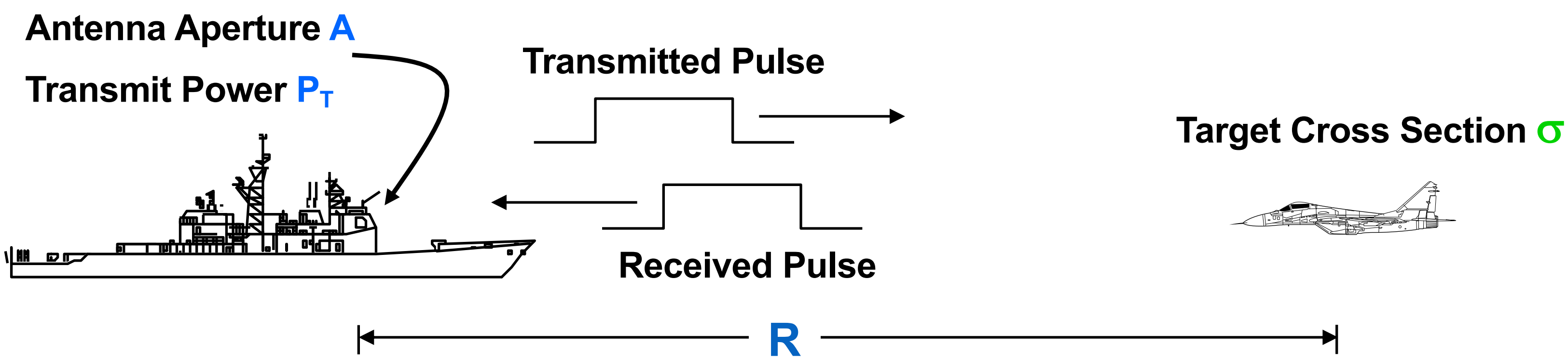


Cycles in a pulse.

PFISR frequency = 449 MHz
Long pulse length = 480 μs
# of cycles = 215520 !

Radar waveforms modulate the waves with on-off sequence

Radar Range Equation



	Transmit Power	Transmit Gain	Spread Factor	Losses	Target RCS	Spread Factor	Receive Aperture	Dwell Time
Received Signal Energy =	$[P_T]$	$\left[\frac{4\pi A}{\lambda^2}\right]$	$\left[\frac{1}{4\pi R^2}\right]$	$\left[\frac{1}{L}\right]$	$[\sigma]$	$\left[\frac{1}{4\pi R^2}\right]$	$[A]$	$[\tau]$

$$\text{Received Signal Energy} = [P_T]\left[\frac{4\pi A}{\lambda^2}\right]\left[\frac{1}{4\pi R^2}\right]\left[\frac{1}{L}\right][\sigma]\left[\frac{1}{4\pi R^2}\right][A][\tau]$$

What to know

Define phase velocity and group velocity

Define refraction and dispersion

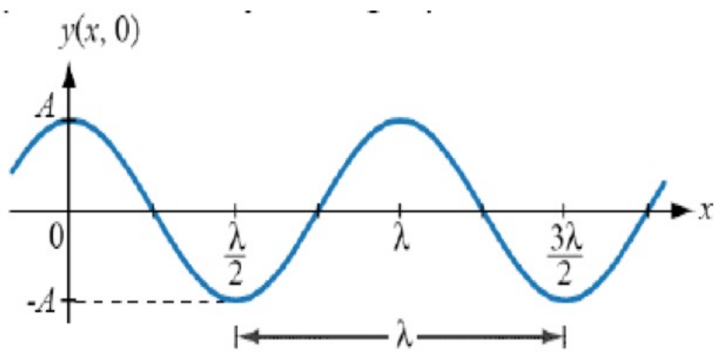
Explain concept of dispersion relation

Radio Waves

$$y(x, t) = A\cos(\omega t - kx + \phi_o)$$

Angular frequency
$\omega = 2\pi f = 2\pi/T$

Wavenumber
$k = 2\pi/\lambda$



(a) y(x, t) versus x at t = 0

Phase velocity defined as $v_{\mathrm{p}} = \frac{\omega}{k}$

The phase velocity **the velocity with which phase fronts propagate in a medium**.

The group velocity of a wave is the **velocity** with which the overall envelope shape of the wave's amplitudes—known as the modulation or envelope of the wave—propagates through space.

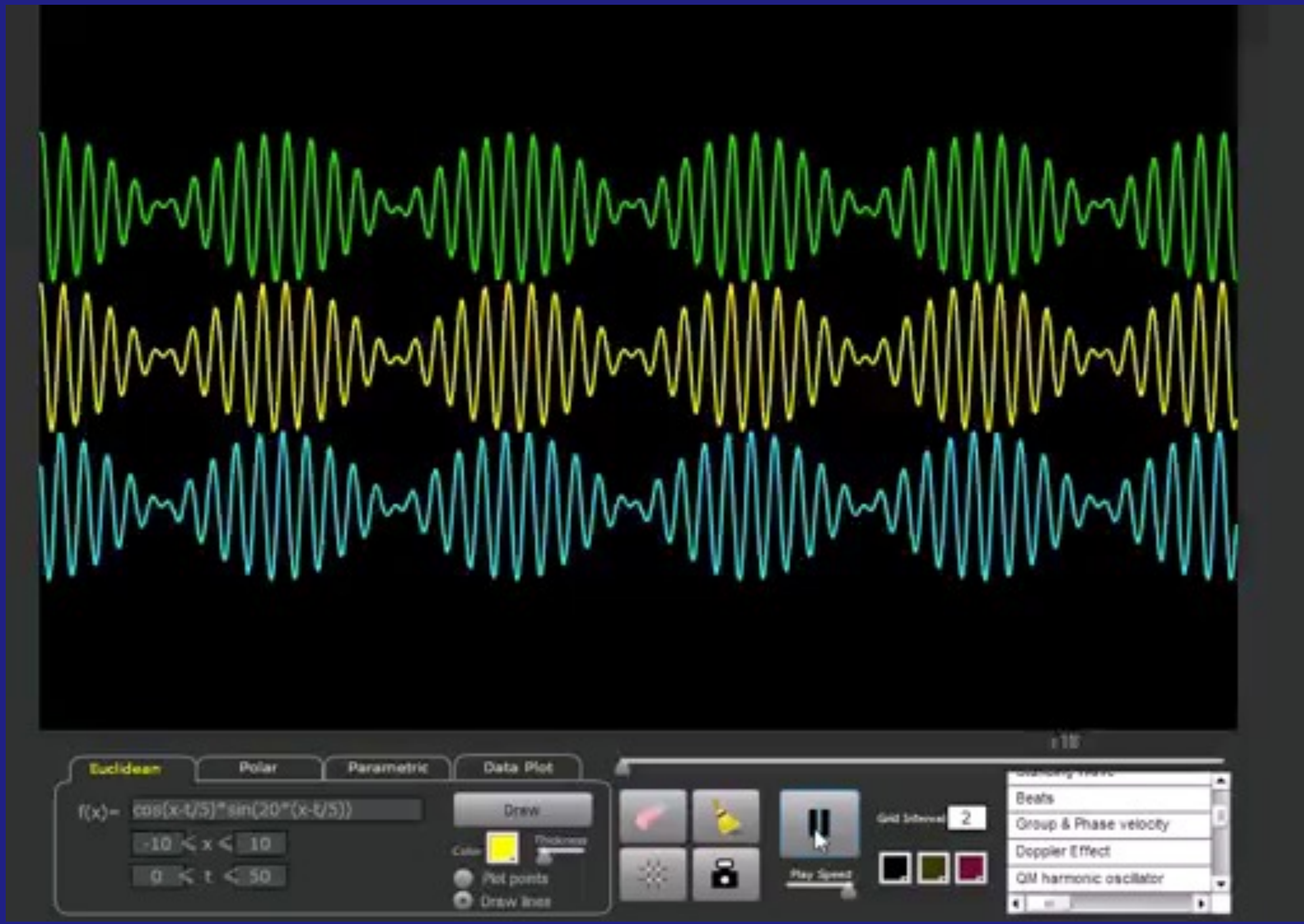
$$v_g \equiv \frac{\partial \omega}{\partial k}$$

Phase Velocity, Group Velocity, Index of Refraction

$$v_p = \frac{\omega}{k}$$

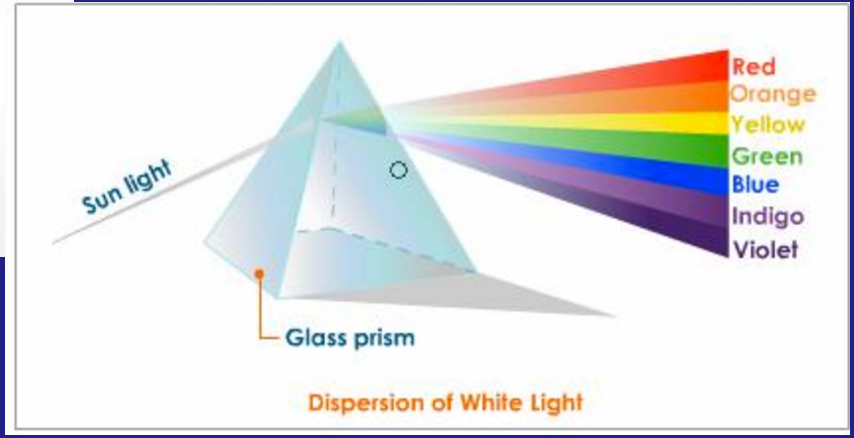
$$v_g \equiv \frac{\partial \omega}{\partial k}$$

$$n = \frac{c}{v_p}.$$



Refraction and Dispersion

$$n = \frac{c}{v_p}.$$



Index of Refraction $n = \frac{c}{v_{\mathrm{p}}}$. in the Ionosphere

$$n^2 = 1 - \frac{X}{1 - iZ - \frac{\frac{1}{2}Y^2 \sin^2\theta}{1-X-iZ} \pm \frac{1}{1-X-iZ}\left(\frac{1}{4}Y^4 \sin^4\theta + Y^2\cos^2\theta(1-X-iZ)^2\right)^{1/2}}$$

where

n is the index of refraction

$$X = \frac{\omega_{pe}^2}{\omega^2} \quad Y = \frac{\omega_c}{\omega} \quad Z = \frac{\nu}{\omega} \quad \omega_{pe} = \left(\frac{Ne^2}{\varepsilon_0 m_e}\right)^{1/2} \quad \omega_c = \frac{e|B|}{m_e}$$

ω = the angular frequency of the radar wave,

$Y_L = Y\cos\theta$, $Y_T = Y\sin\theta$,

θ = angle between the wave vector $\bar{k}$ and $\bar{B}$,

$\bar{k}$ = wave vector of propagating radiation,

$\bar{B}$ = geomagnetic field, N = electron density

e = electronic charge, m_e = electron mass, ν = electron collision frequency

and ε_0 = permittivity constant.

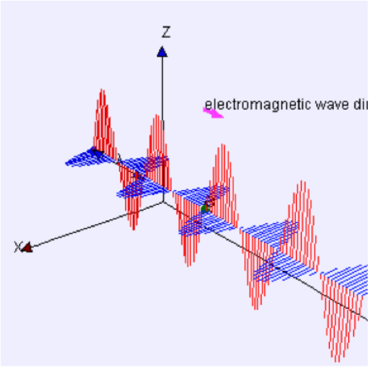
Dispersion relation: the concept

Key concept for wave behavior within a propagation medium.

Describes the relationship between SPATIAL frequency (wavelength)
and TEMPORAL frequency.

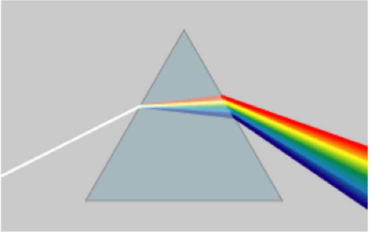
Some wave modes relate wavelength to frequency **linearly,**
but waves in most media have **nonlinear** relation between wavelength and frequency.

Linear dispersion example:
EM radiation propagation through free space
(wavelength / velocity = c)



http://weelookang.blogspot.com/2011/10/ejs-open-source-propagation-of.html

Nonlinear dispersion example:
splitting of light through a prism
(effective speed of light depends on wavelength
due to glass' non-unity index of refraction)



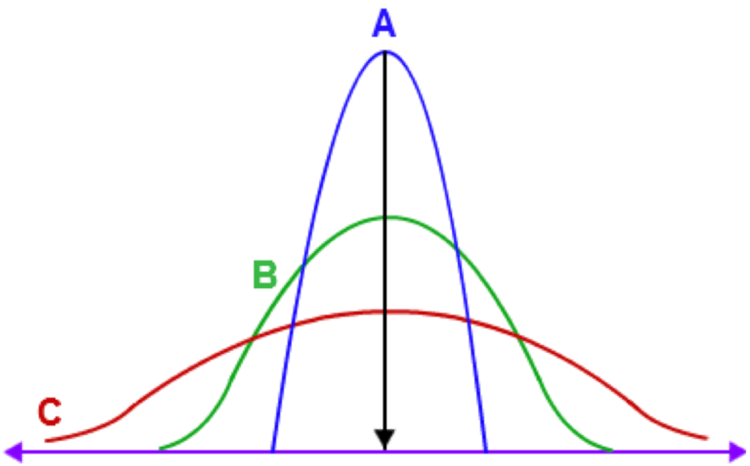
Dispersion relation: the concept

Simple linear case: uniform phase velocity

$$\omega(k) = c\, k$$

Most propagation speeds depend nonlinearly on the wavelength and/or frequency.

NB: for a **nonlinear** dispersion relation, the pulse will typically spread in either spatial frequency or temporal frequency as a function of time.



Example of pulse spreading spatially from time A to B to C.

http://www.mathcaptain.com/statistics/dispersion-statistics.html

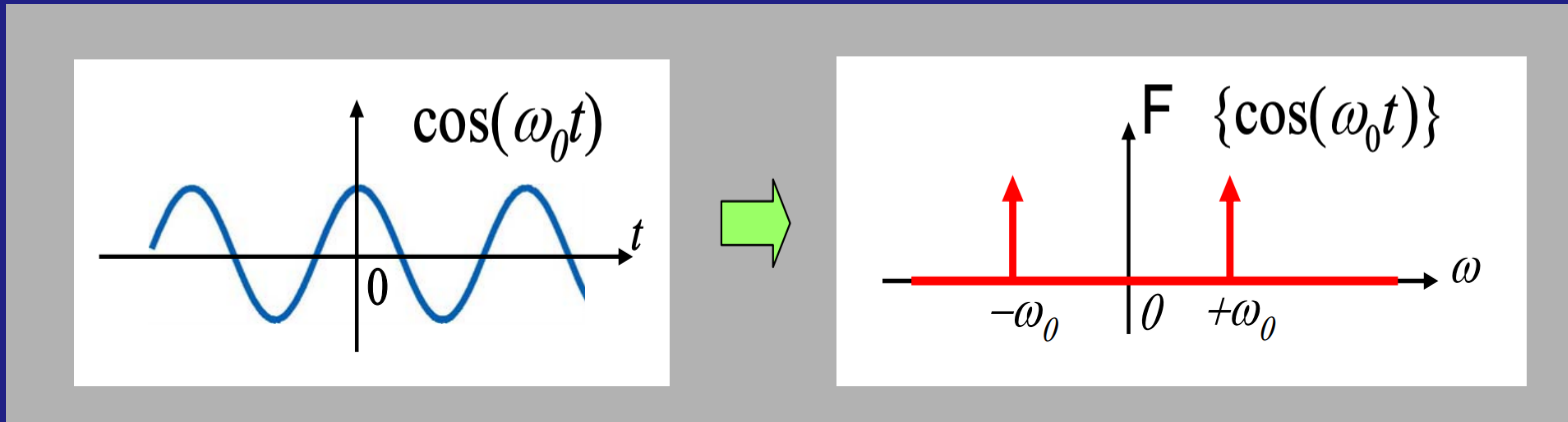
What to know

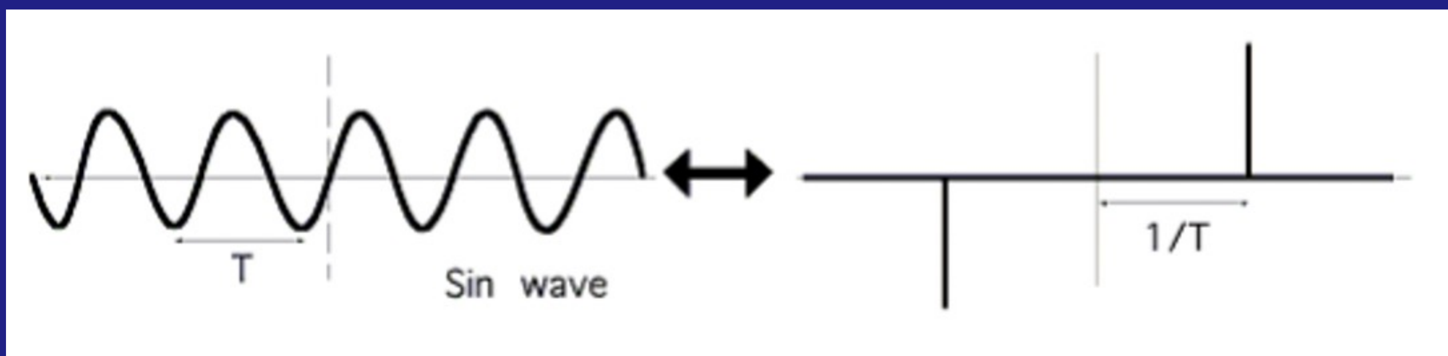
What is the Fourier transform of cosine wave?

What is the Fourier transform of a sine wave?

Write out $e^{i(kx-wt)}$ in the form of sine and cosines

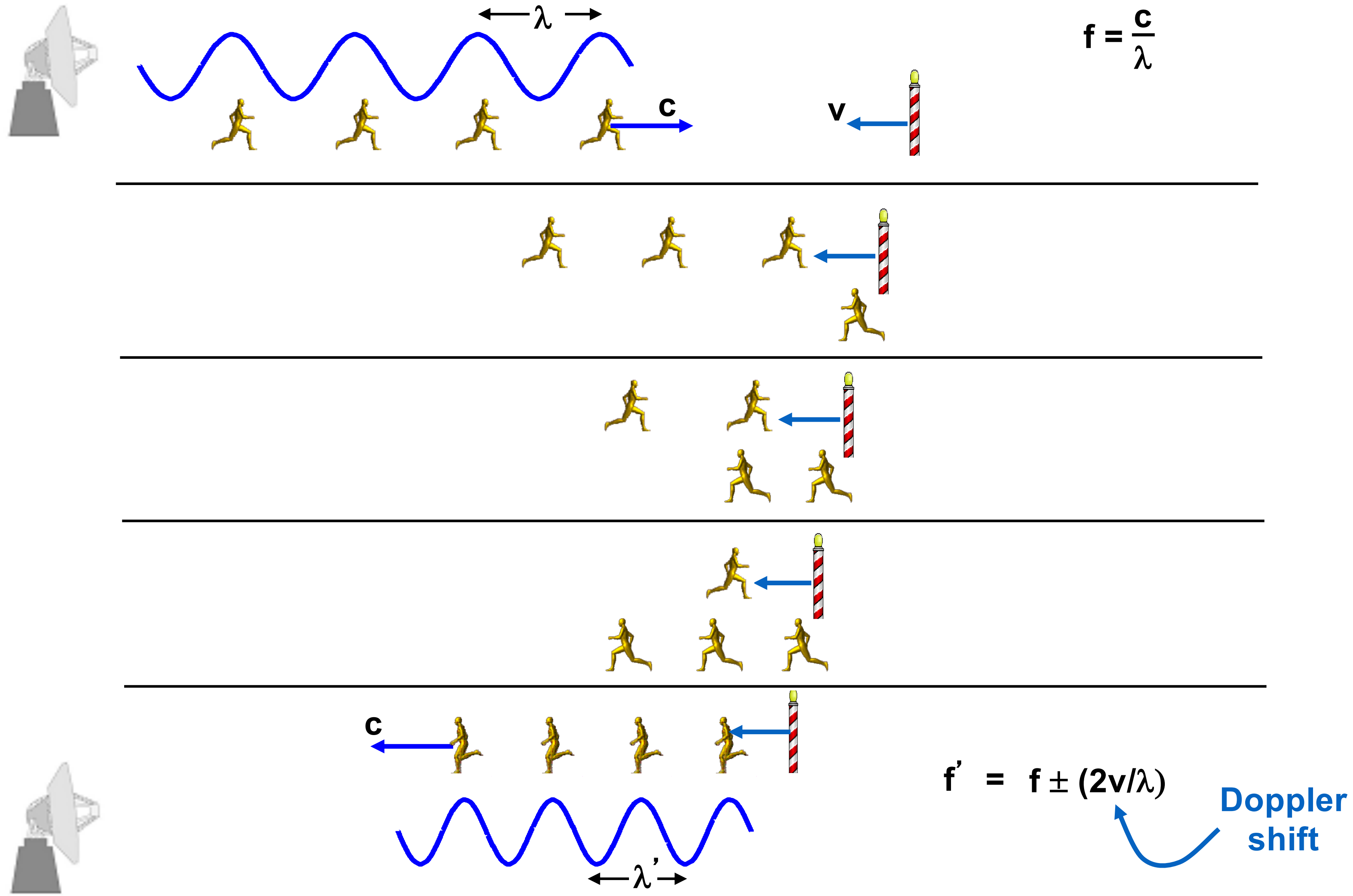
How does one measure the direction of Doppler phase shift?





$$e^{ix} = \cos(x) + i\sin(x)$$

Doppler Shift Concept



$$f = \frac{c}{\lambda}$$

$$f' = f \pm (2v/\lambda)$$

Doppler shift frequency

Tx signal: $\cos(2\pi f_o t)$

Return from a moving target: $\cos[2\pi f_o(t + 2R/c)]$

If target is moving with a constant velocity: $R = R_o + v_o t$

then,

Return: $\cos[2\pi\{(f_o + f_o 2v_o/c)t + 2f_o R_o/c\}]$

Doppler frequency:
$-2f_o v_o/c = -2v_o/\lambda_o$