Introduction to Radar Signal Processing

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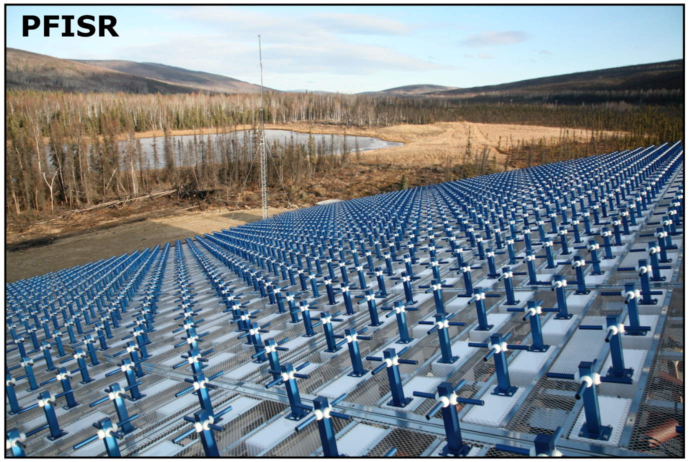
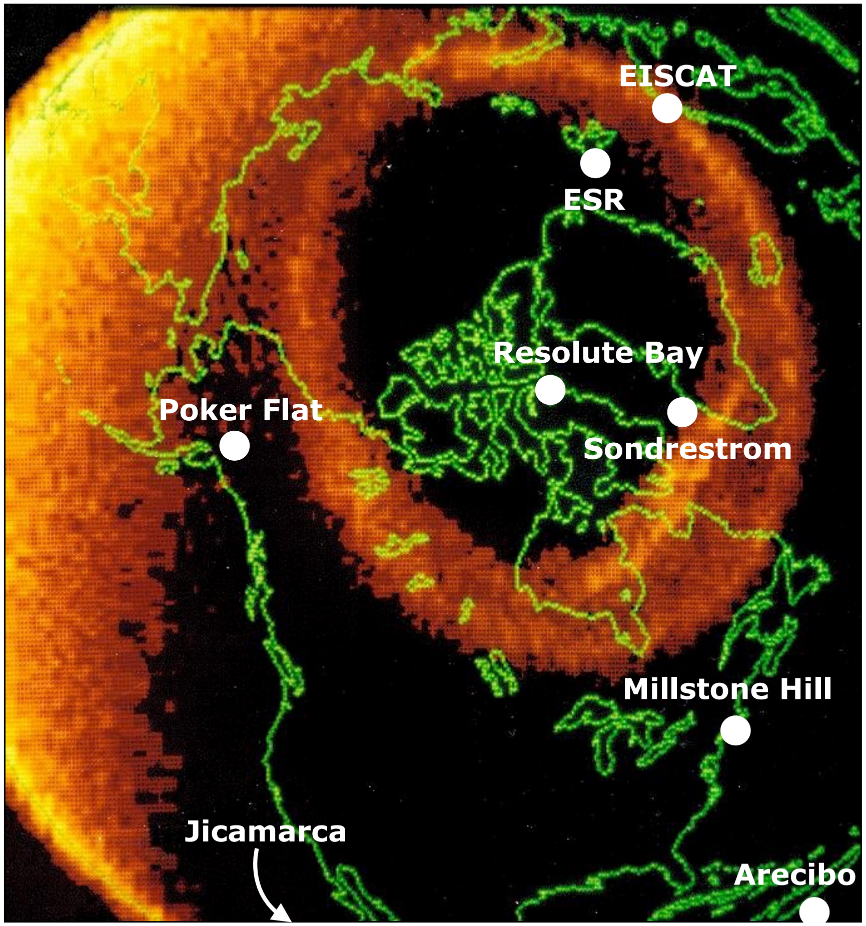
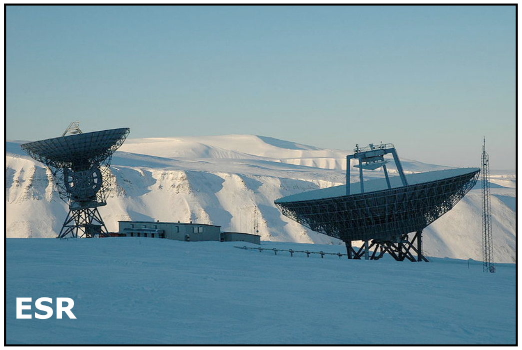
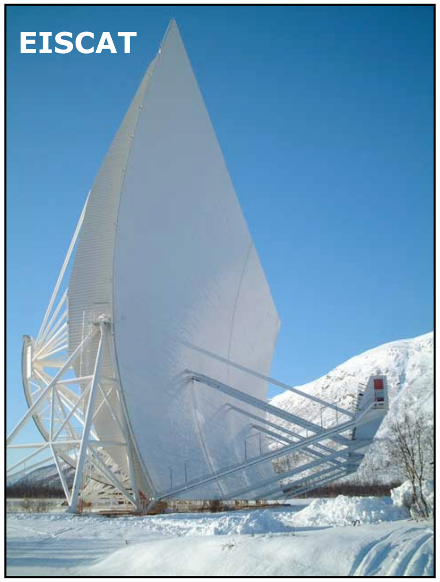
Incoherent Scatter Radar (ISR)

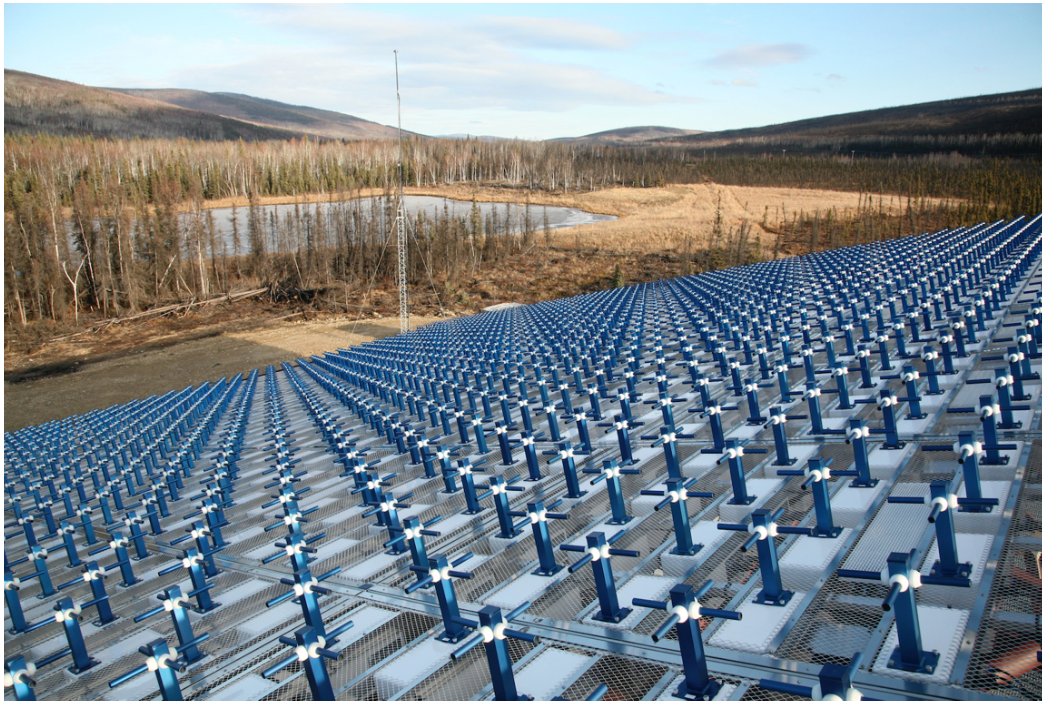








Dish Versus Phased-array

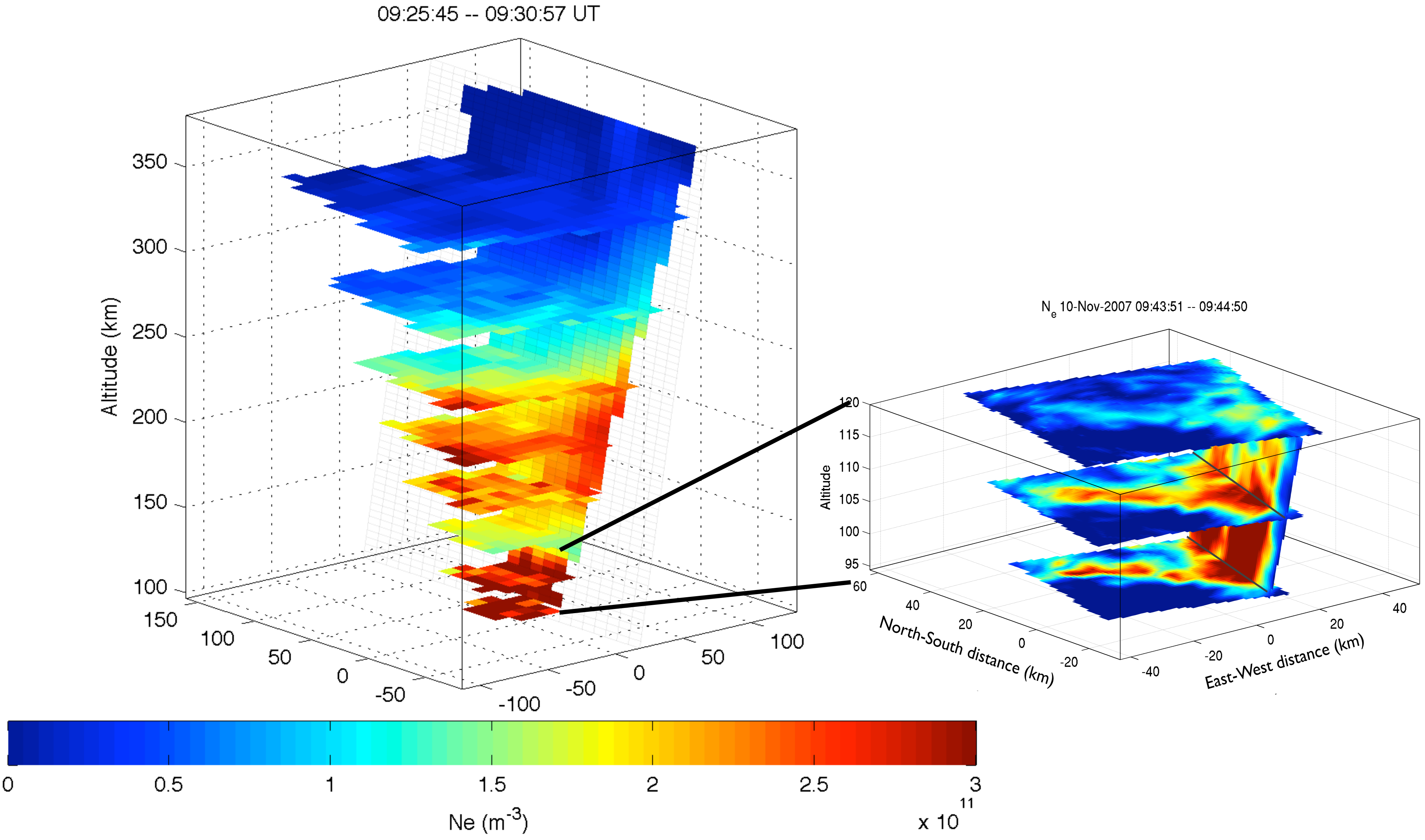


- -FOV: Entire sky
- -Integration at each position before moving
- -Power concentrated at Klystron
- -Significant mechanical complexity

- -FOV: +/- 15 degrees from boresight
- -Integration over all positions simultaneously
- -Power distributed
- -No moving parts

Uncoded 480us pulse
5-min integration

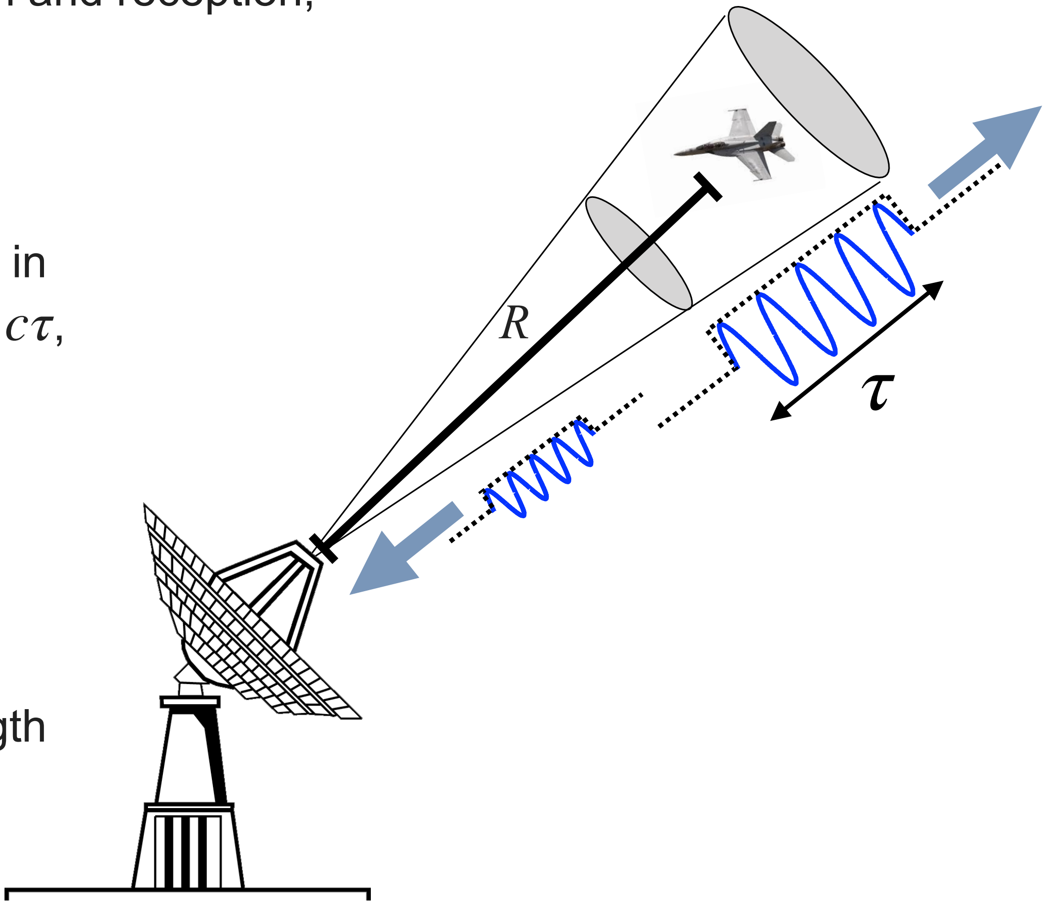
13-baud Barker coded pulse
15-s integration



Range

Range R to the target is measured by transmitting a pulse of electromagnetic waves, and measuring the time Δt between transmission and reception,

$$R = \frac{c\Delta t}{2}$$

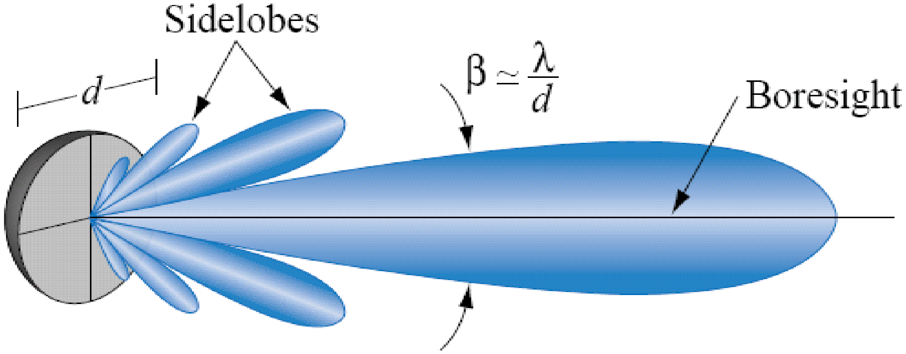


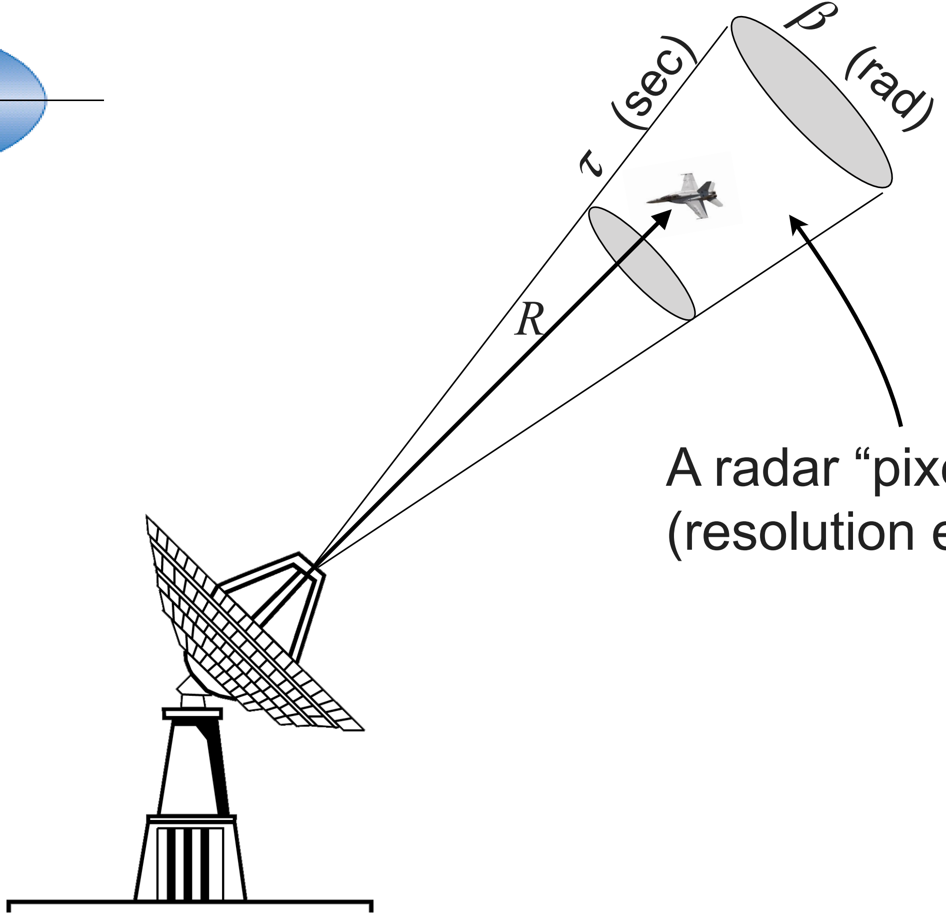
The ***pulse length*** τ is most often expressed in units of time, and corresponds to a distance $c\tau$, where $c = 3 \times 10^8$ m/s.

Range resolution depends on how well we can resolve Δt. For the case of a simple on-off pulse, the optimal approach is to match the sampling period to the pulse length (the so-called "matched filter" approach).

Range resolution for a simple on-off pulse ("uncoded pulse") is controlled by τ. Shorter τ yields higher range resolution. But a shorter pulse also carry less total energy, and so the reflected signal is more difficult to discriminate from background noise.

Cross-range resolution (beam width)





A radar “pixel”
(resolution element)

The cross-range resolution is usually defined by the angular width of the main lobe of the antenna's power pattern. For a dish antenna this is approximately equal to the ration of the wavelength to the physical diameter,

$$\beta = \frac{\lambda_o}{d} \qquad \text{(radians)}$$

Millstone Hill ISR has a 46-m dish operating at a frequency of 440 MHz, or $\lambda = 0.68$ m, giving a beam width of $\beta \simeq 0.85^\circ$.

Concept of a “Doppler Spectrum”

Superposition of targets moving with different velocities within the radar volume

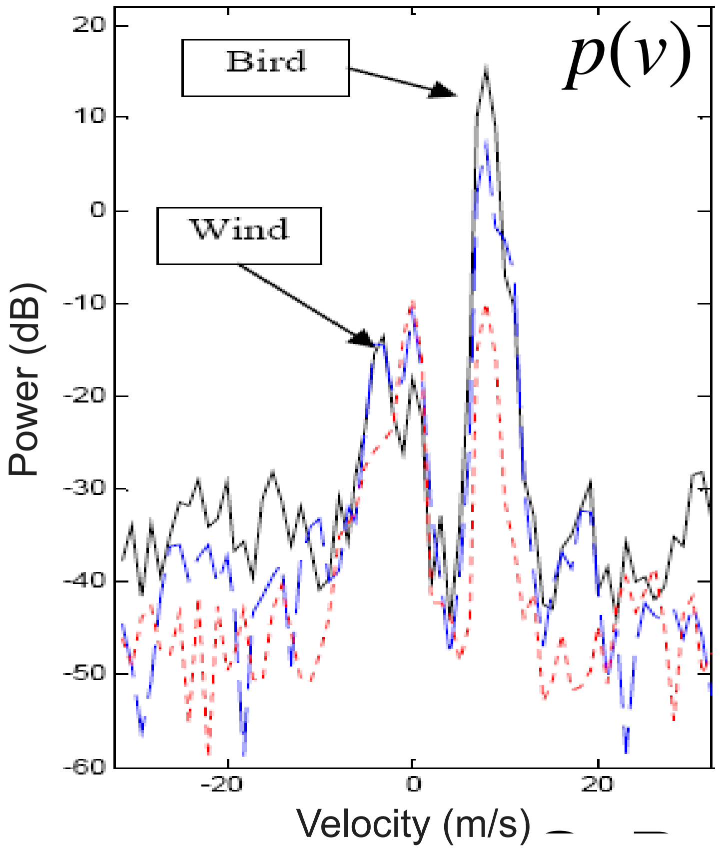
Two key concepts:

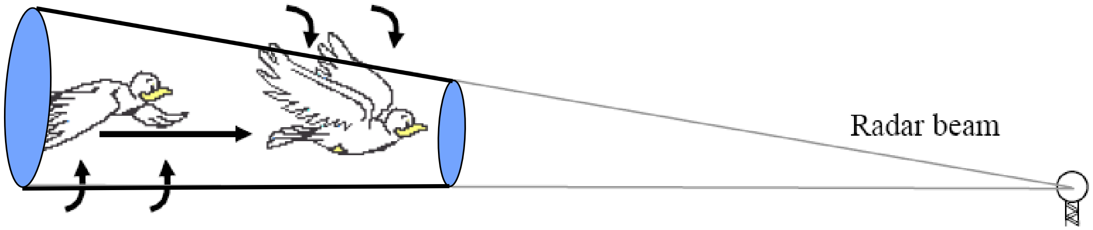
Time ⇔ Distance

$$R = \frac{c\Delta t}{2}$$

Frequency ⇔ Velocity

$$f_D = -\frac{2f_o}{c}v_o$$





Processing: $p(R, f_D) \rightarrow p(R, v)$

If there is a distribution of targets withdifferent velocities (e.g., bird, flapping wings,wind) then there is no single Doppler shift but, rather, a Doppler spectrum.

Distributed “beam filling” Target

A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

Two key concepts:

Time ⇔ Distance

$$R = \frac{c\Delta t}{2}$$

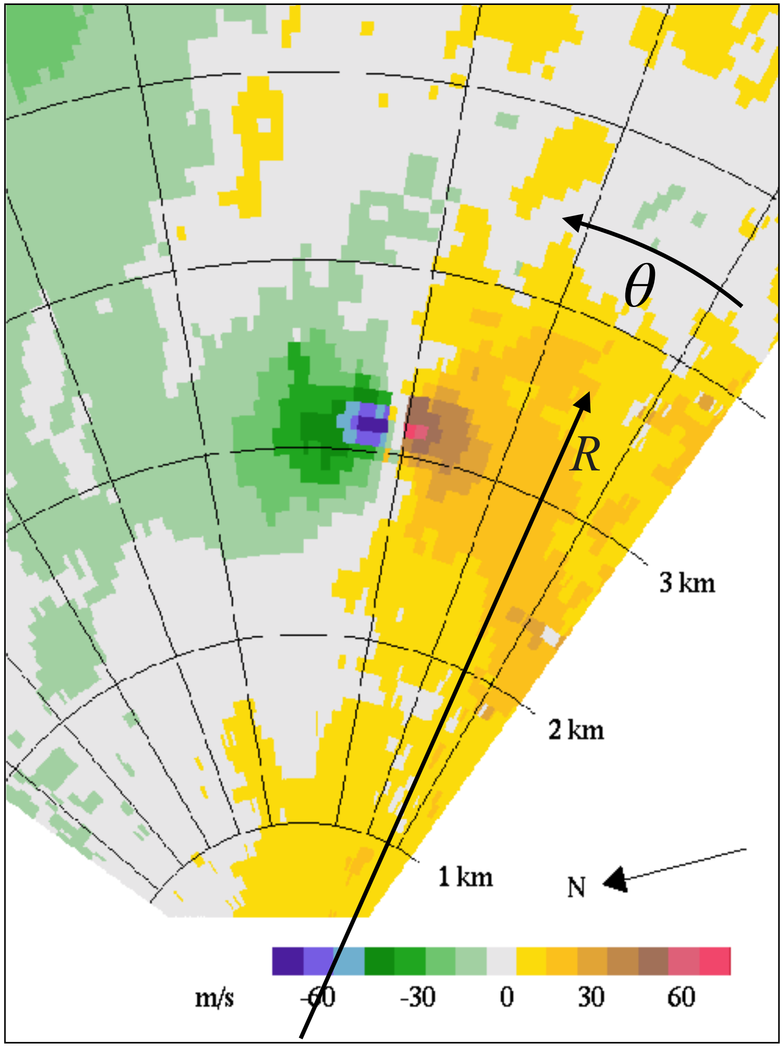
Frequency ⇔ Velocity

$$f_D = -\frac{2f_o}{c}v_o$$

Processing:

$$p(R, f_D, t) \longrightarrow f_D(R, t) \longrightarrow v(R, \theta)$$

For a beam-filling target (like water droplets in a tornado), the radar can be used to construct insightful images of velocity relative to the radar.



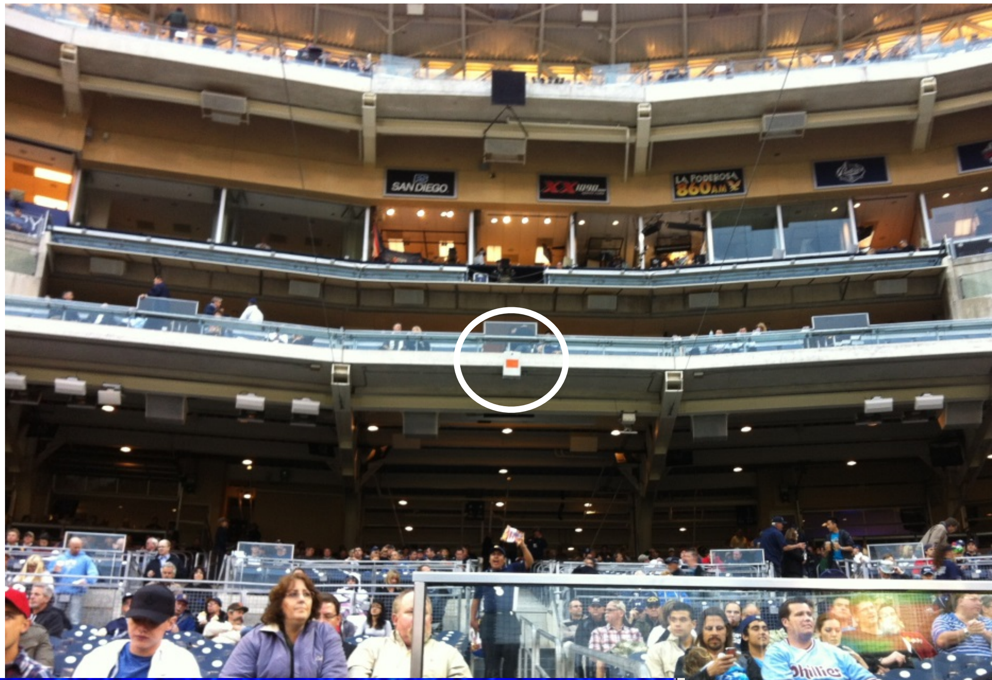
Micro-Doppler Analysis

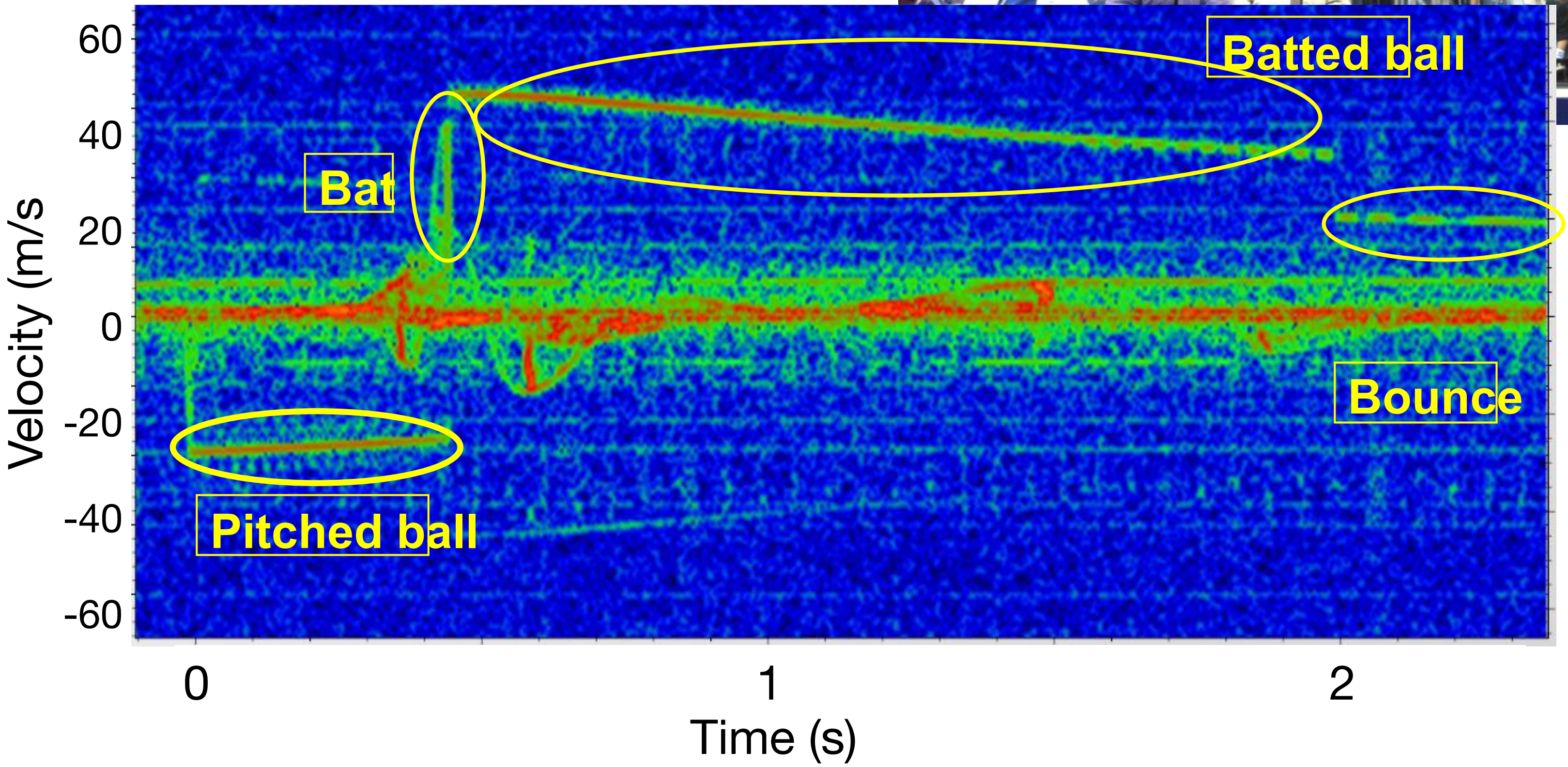
Trackman radar: "continuous wave" (CW) radar: precise Doppler but no range information.

Can identify targets and actions based on Doppler signatures!

Processing:

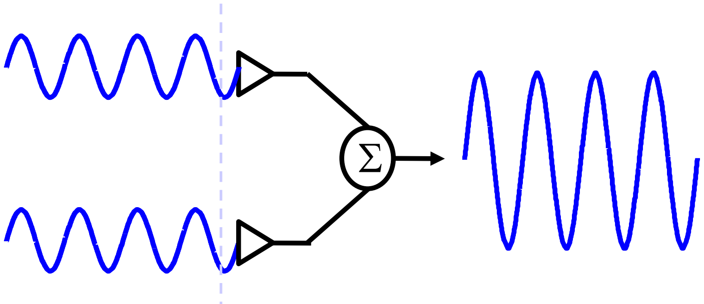
$$p(f_D, t) \longrightarrow p(v, t)$$



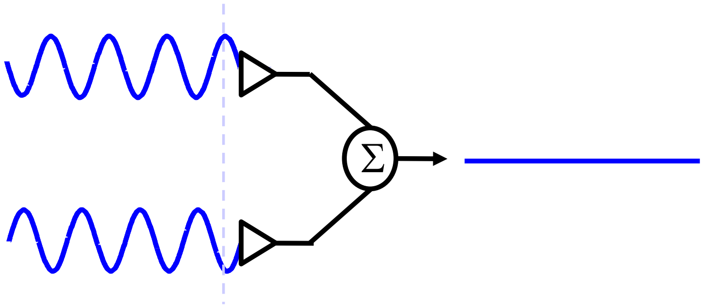


Wave interference and Bragg Scatter

Consider two waves with the same frequency but different phase.

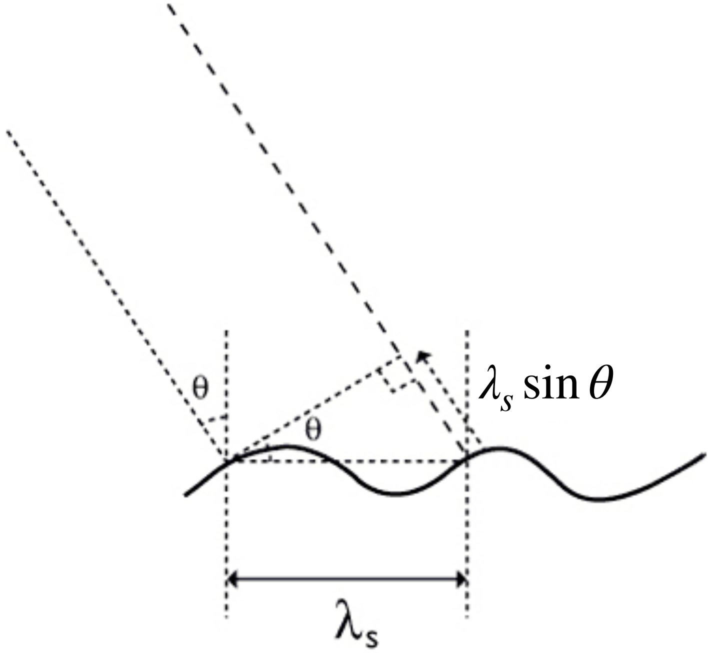


Constructive
(in phase)



Destructive
(180°   out of phase)

Consider a wave along the interface between a dielectric and a conducting (reflective) medium, as depicted below. This is representative of an air-ocean boundary.

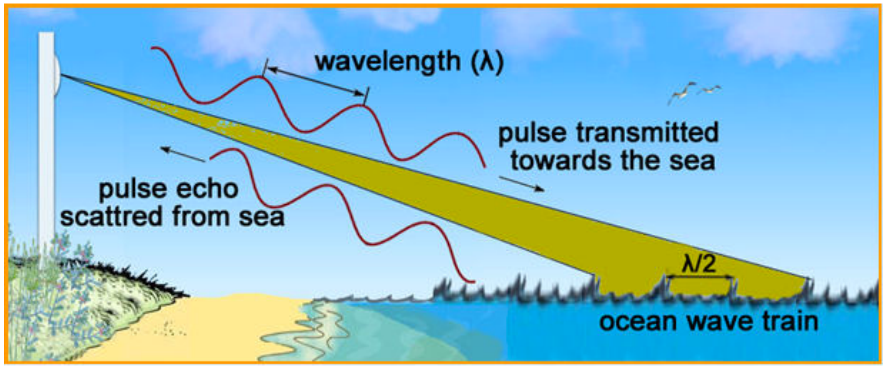


Suppose waves are observed at angle θ using a radar with wavelength λ_o. The condition for maximum constructive interference is

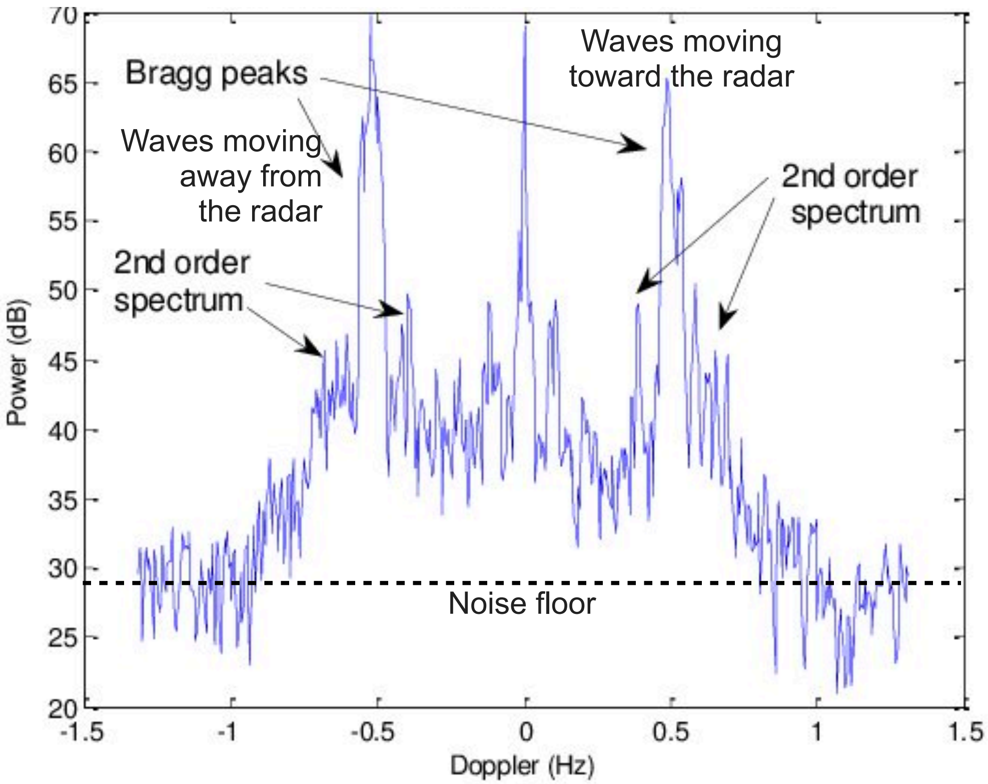
$$n\lambda_0 = 2\lambda_s \sin\theta$$

If $\theta = 90^\circ$ (or if these waves are propagating isotropically), then the Bragg condition is met for $n\lambda_0 = 2\lambda_s$

Doppler spectrum of ocean waves



Backscatter from the ocean at low aspect angle shows peaks in the Doppler spectrum from the subset of waves matching the Bragg condition for the radar (spacing ≃ half the radar wavelength)



Important points:

The target is distributed over the entire radar beam width.

The scattering is from free electrons in the conducting sea water.

The Doppler spectrum has peaks due to Bragg scatter from waves in the medium.

The frequency of the peaks tells us the velocity and direction of the waves.

The height of the peaks tells us something about the amplitude and density of the waves.

The width of the peaks tells us something about the spread in velocity of the waves

Doppler spectrum of the ionosphere

Let's put this all together for the ionosphere. The two predominant longitudinal modes in a thermal plasma:

Ion-acoustic mode:

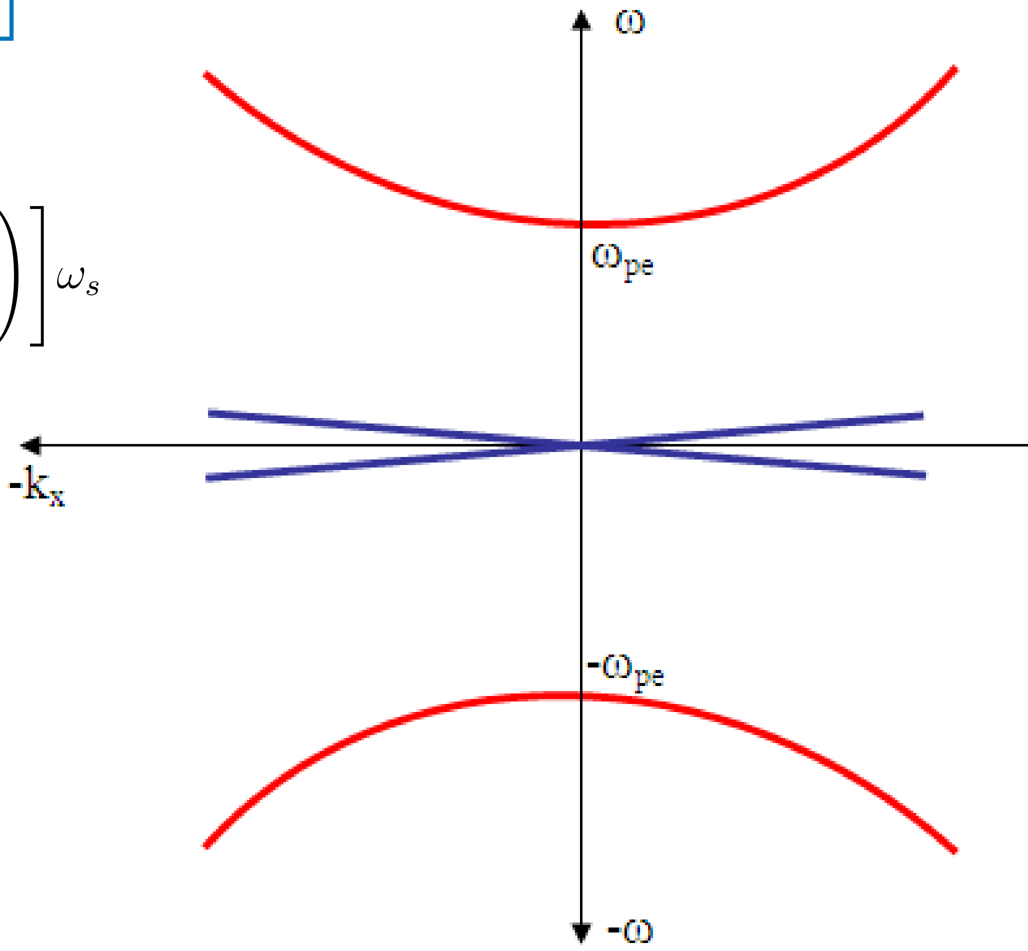
$$\boxed{\omega_s = C_s k \qquad\qquad C_s = \sqrt{k_B(T_e + 3T_i)/m_i}}$$

Langmuir mode:

$$\omega_{si} = -\sqrt{\frac{\pi}{8}}\left[\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \left(\frac{T_e}{T_i}\right)^{\frac{3}{2}} \exp\left(-\frac{T_e}{2T_i} - \frac{3}{2}\right)\right]\omega_s$$

$$\omega_L = \sqrt{\omega_{pe}^2 + 3\,k^2\,v_{the}^2} \approx \omega_{pe} + \frac{3}{2} v_{the}\lambda_{De}k^2$$

$$\omega_{Li} \approx -\sqrt{\frac{\pi}{8}}\frac{\omega_{pe}^3}{k^3}\frac{1}{v_{the}^3} exp\left(-\frac{\omega_{pe}^2}{2k^2v_{the}^2} - \frac{3}{2}\right)\omega_L$$



Computer simulation of the ionosphere

Simple rules yield complex behavior

Particle-in-cell (PIC) simulation:

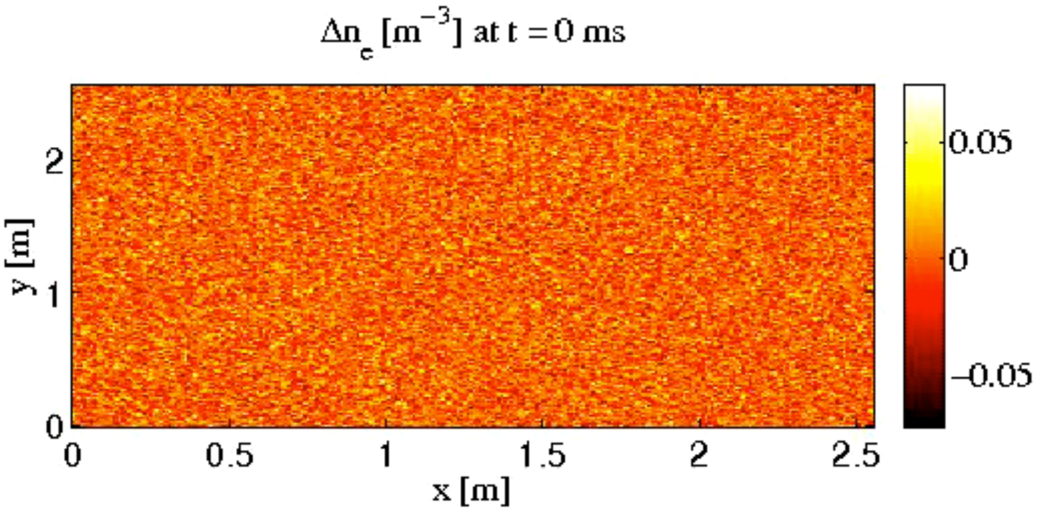
$$\frac{d\,\mathbf{v}_i}{d\,t} = \frac{q_i}{m_i}(\mathbf{E}(\mathbf{x}_i) + \mathbf{v}_i \times \mathbf{B}(\mathbf{x}_i))$$

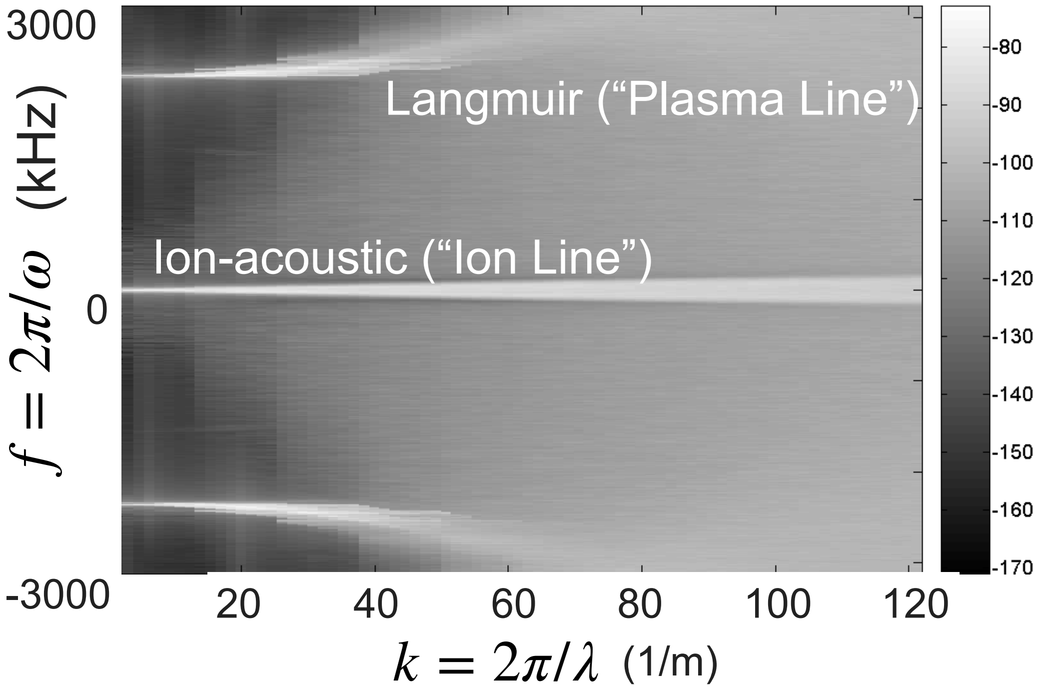
$$\nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

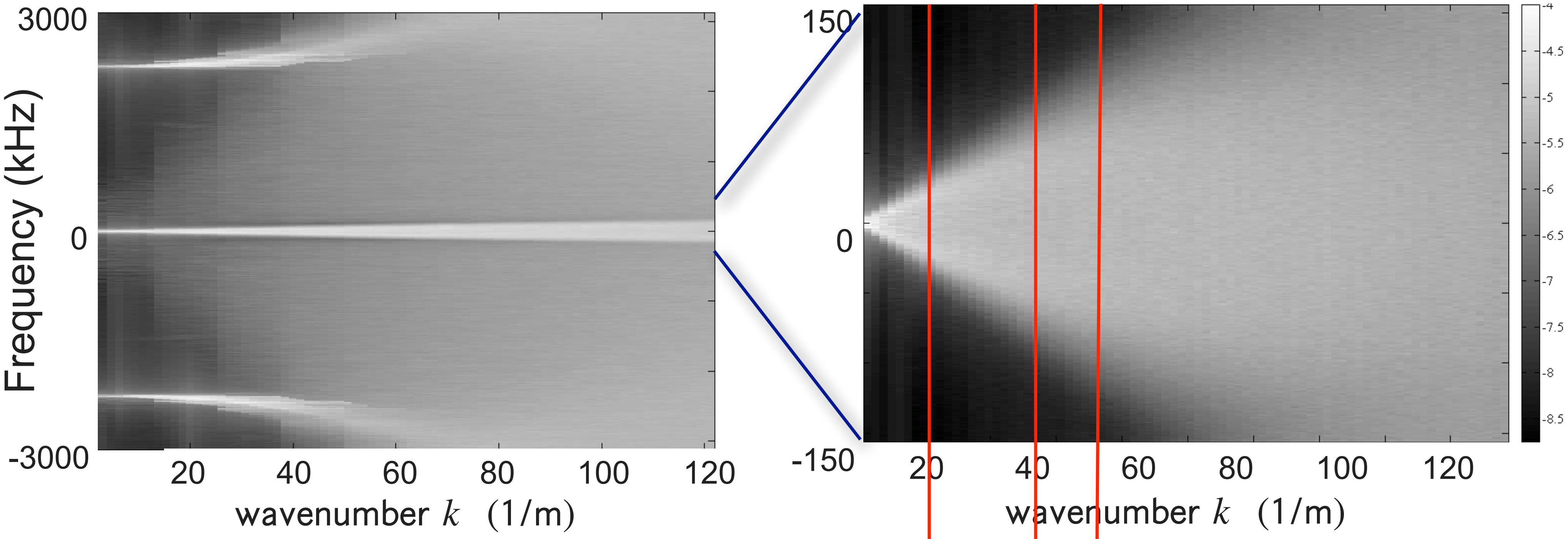
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$





ISR measures a cut through this surface

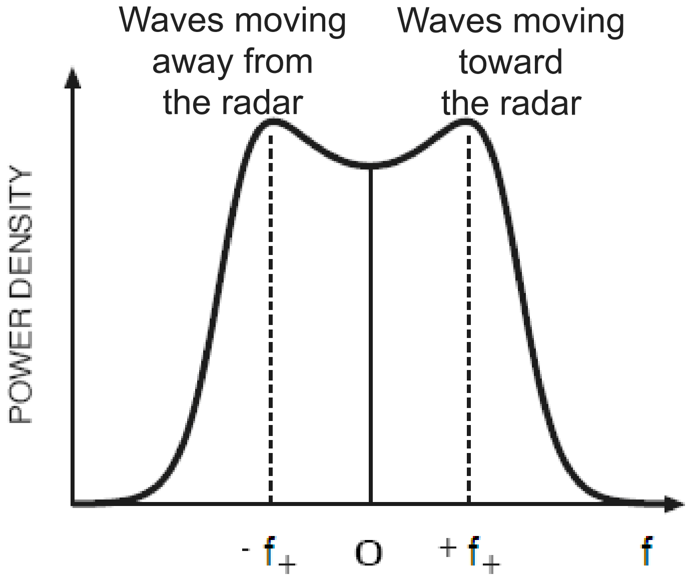


Sondrestrom (1.2 GHz)

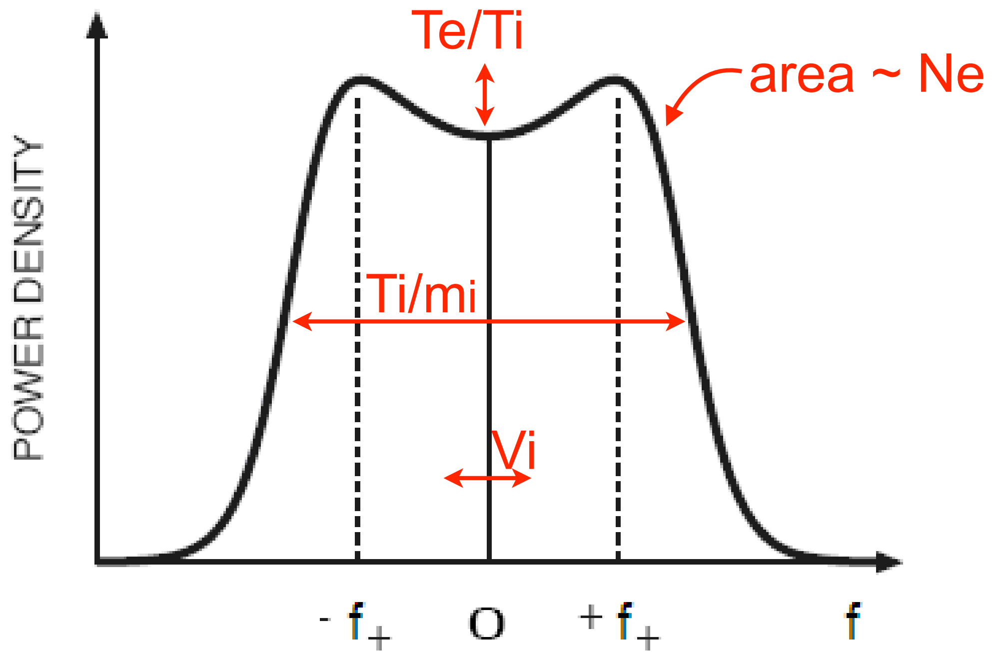
EISCAT UHF (930 MHz)

AMISR (450 MHz),
Millstone (440 MHz)

Ion-acoustic “lines” are broadened by Landau damping



Standard parameters found by fitting the measured Ion-acoustic power spectrum (ion-acoustic line)



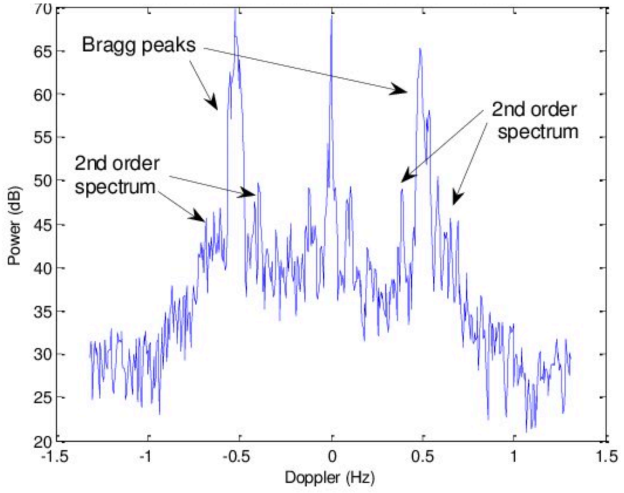
Ion temperature (Ti) to ion mass (mi) ratio from the width of the spectra

Electron to ion temperature ratio (Te/Ti) from “peak_to_valley” ratio

Electron (= ion) density from total area (corrected for temperatures)

Line-of-sight ion velocity (Vi) from the Doppler shift

…recall the ocean wave Doppler spectrum



Doppler Radar Summary: Distributed “Incoherent” Targets

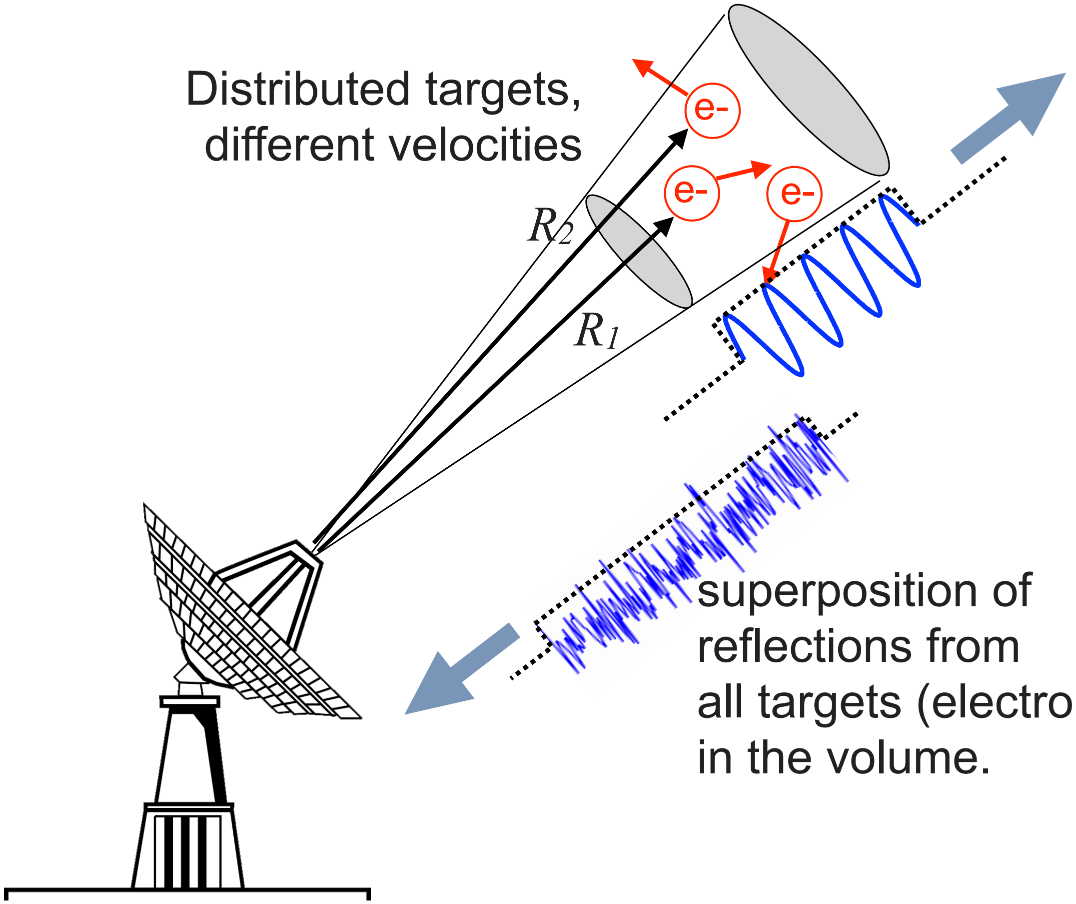
Two key concepts:

Time ⟺ Distance

$$R = \frac{c\Delta t}{2}$$

Frequency ⟺ Velocity

$$f_D = -\frac{2f_o}{c}v_o$$



Distributed targets, different velocities

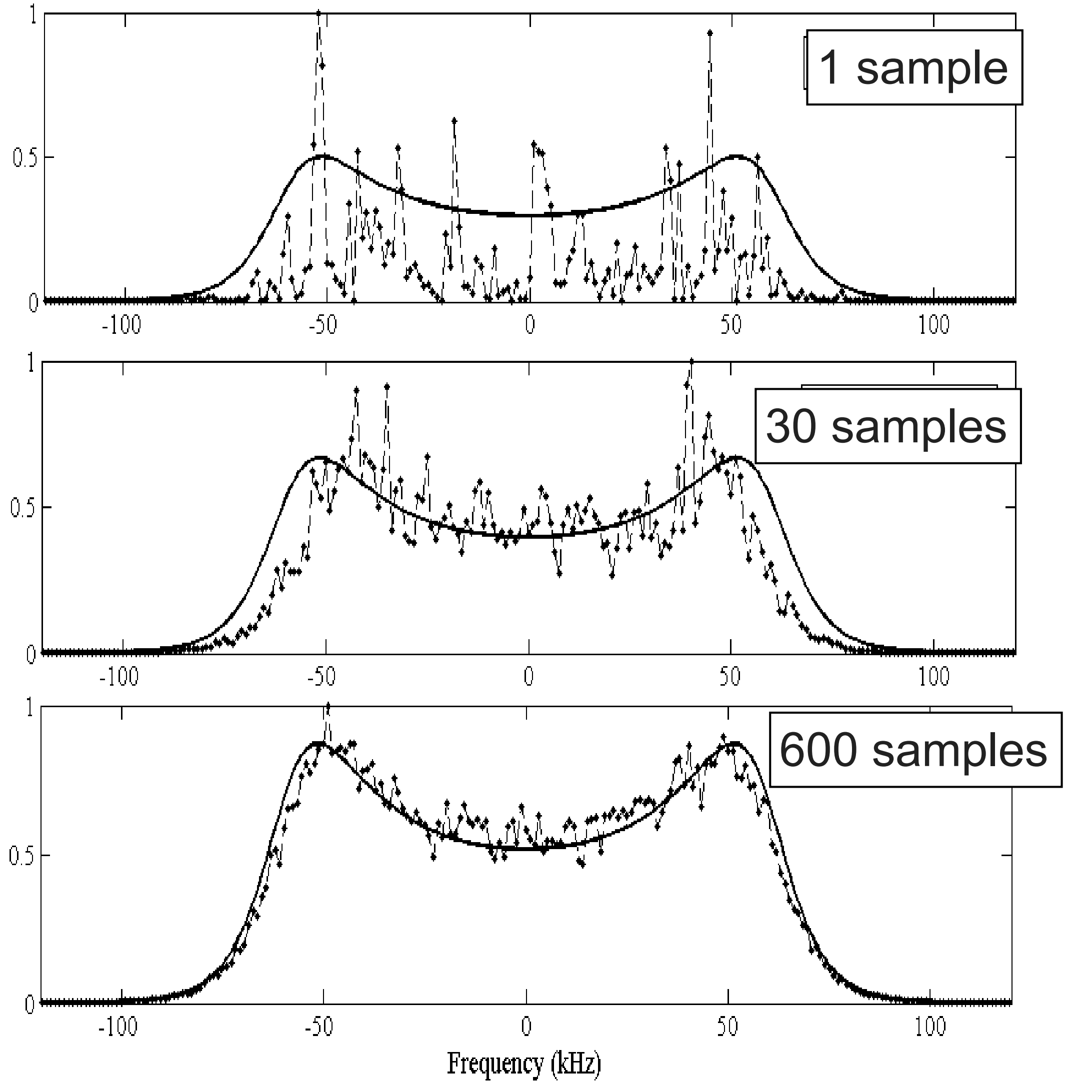
superposition of reflections from all targets (electrons) in the volume.

A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

What happens when we have multiple targets in the radar volume, moving at different velocities?

Incoherent Averaging

Normalized ISR spectrum for different integration times at 1290 MHz

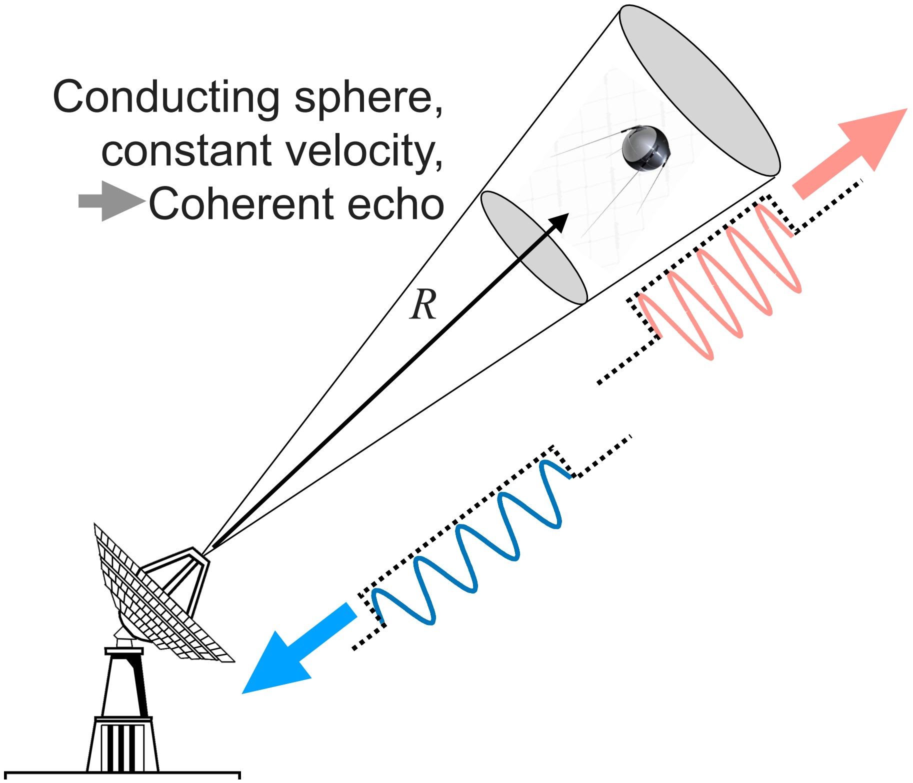


We are seeking to estimate the power spectrum of a Gaussian random process. This requires that we sample and average many independent “realizations” of the process.

$$\rho_e \sim \frac{1}{K}\left(1+\frac{1}{SNR}\right)^2$$

$\rho_e =$ Mean Square Error
$K =$ number of samples
$SNR =$ per-pulse Signal-to-Noise Ratio

Measuring Velocity



Conducting sphere,
constant velocity,
Coherent echo

Assume a transmitted signal: $s(t)\cos(2\pi f_o t)$

After return from target: $a(t)\cos\left[2\pi f_o\left(t-\frac{2R(t)}{c}\right)\right]$

Let's assume target moves with constant velocity with respect to the radar during the measurement,

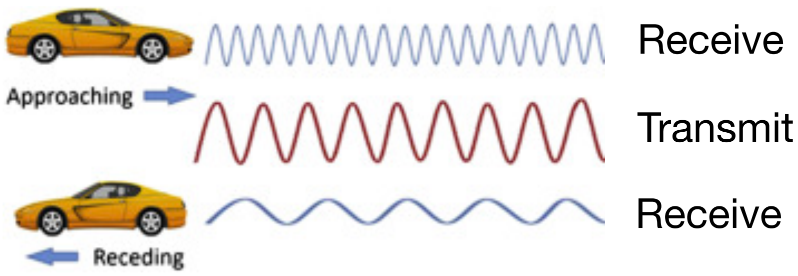
$$R = R_o + v_o t$$

Substituting we obtain:

$$a(t)\cos\left[\underbrace{2\pi f_o t}_{\omega_o}\underbrace{-\,2\pi f_D t - \frac{4\pi f_o R_o}{c}}_{\phi(t)}\right] \qquad f_D = -\frac{2f_o}{c}v_o$$

$$a(t)\cos\left[\omega_o t + \phi(t)\right] \qquad \omega_D = 2\pi f_D = -\frac{d\phi}{dt}$$

$f_o \sim 500$ MHz, f_D~ 50 kHz $= 0.0001 f_o$



Two issues:

1) How do we discriminate positive from negative f_D?

2) How do we remove f_o , and just sample $a(t)$ and $\phi(t)$?

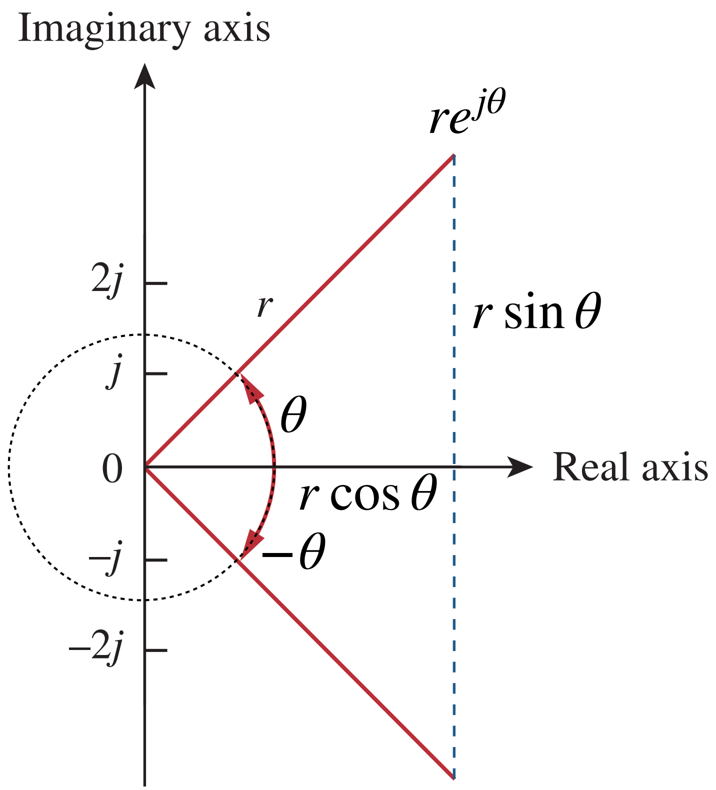
Analytic Signal Model

From Euler's identity

$$re^{j\theta} = (r\cos\theta) + j(r\sin\theta) \qquad j = \sqrt{-1}$$

$$r\cos(\theta) = \Re\{re^{j\theta}\}$$ “real part”

$$r\sin(\theta) = \Im\{re^{j\theta}\}$$ “imaginary part”



Setting $r = a(t)$ and $\theta = \omega_o t + \phi(t)$, we obtain a general complex signal model for radio and radar applications.

$$\boxed{s(t) = a(t)e^{j(\omega_o t + \phi(t))}}$$

PM

AM

Carrier

Or by letting $\omega_d = -\,d\phi/dt \rightarrow \phi(t) = -\,\omega_d t$

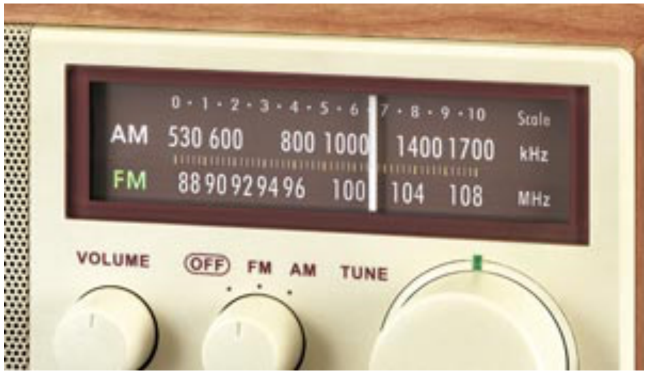
$$s(t) = a(t)e^{j(\omega_o - \omega_d)t}$$

FM

Now through Euler’s identity :

$$\Re\{s(t)\} = a(t)\cos(\omega_o t + \phi(t))$$

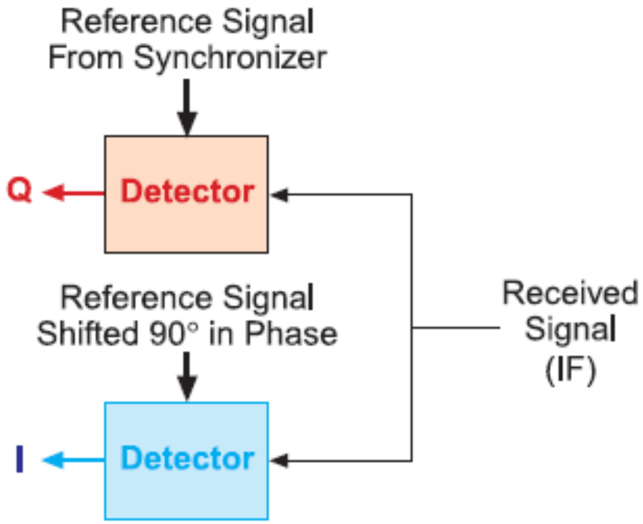
$$\Im\{s(t)\} = a(t)\sin(\omega_o t + \phi(t))$$

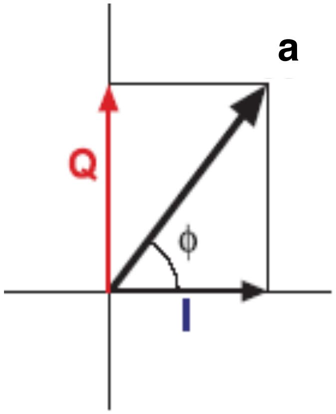


I and Q Demodulation

Consider radar transmission of a simple RF pulse. The reflected signal from the target will be the original pulse with some time varying amplitude and phase applied to it:

$$s_R(t) = a(t)\cos(\omega_o t + \phi(t))$$





We compute the analytic signal by “mixing” with cosine and sine.

Mixing with $\cos(\omega_0 t)$ gives the **“in-phase” (I) channel:**

$$s_R(t)\cos(\omega_o t) = a(t)\cos(\omega_o t + \phi(t))[\cos(\omega_o t)]$$
$$= a(t)\frac{1}{2}\left(\underbrace{\cos[2\omega_o t + \phi(t)]}_{\text{filter out}} + \cos[\phi(t)]\right)$$

Mixing with $\sin(\omega_0 t)$ gives the **“quadrature” (Q) channel:**

$$s_R(t)[\sin(\omega_o t)] = a(t)\cos(\omega_o t + \phi(t))[\sin(\omega_o t)]$$
$$= a(t)\frac{1}{2}\left(\underbrace{\sin[2\omega_o t + \phi(t)]}_{\text{filter out}} + \sin[\phi(t)]\right)$$

If we include a gain of 2, we retain the original signal energy. Using Euler’s identity we obtain the analytic baseband signal:

$$\boxed{s_B(t) = a(t)e^{j\phi(t)} = a(t)\cos\phi(t) + ja(t)\sin\phi(t) = I + jQ}$$

I/Q demodulation produces a time-series of complex voltage samples (I_n, Q_n) from which we can construct a discrete representation of $s_B(t)$. The Doppler frequency shift is the time rate of change of the phase, $\omega_D = -d\phi/dt$.

I/Q Demodulation: Frequency Domain

Transmitted signal:

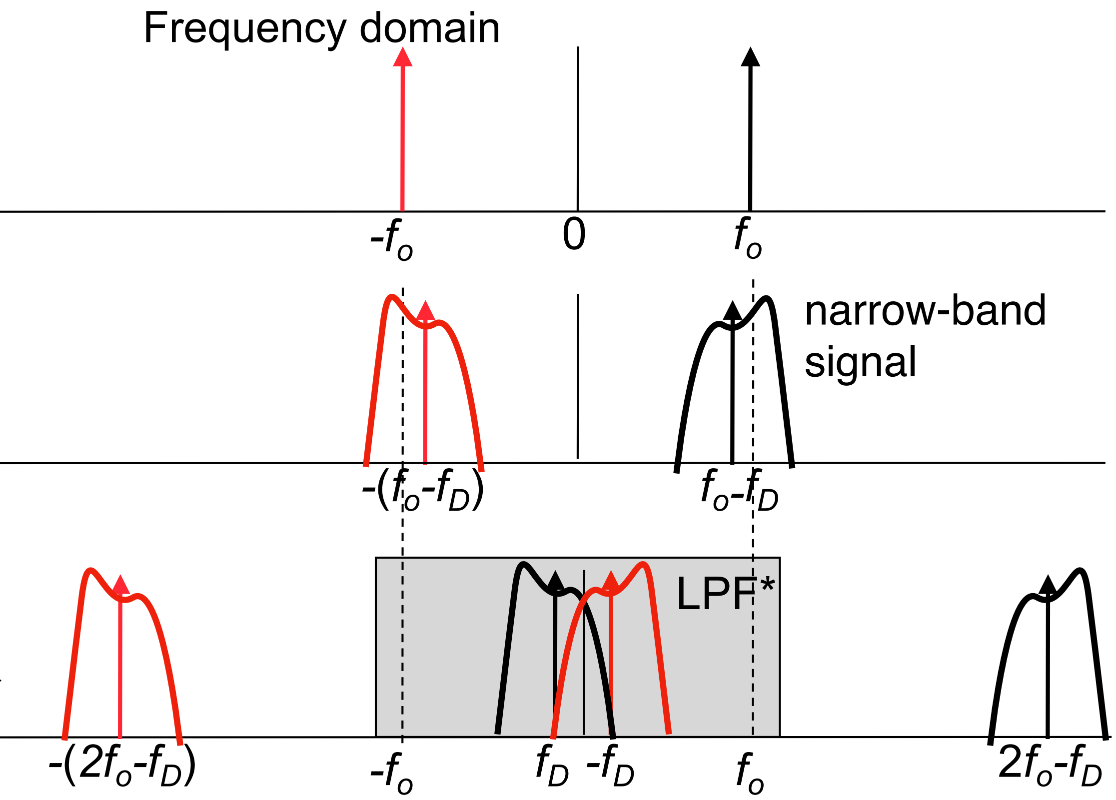
$$\cos(2\pi f_o t) \iff$$

Reflected signal from moving target

$$\cos(2\pi(f_o - f_D)t) \iff$$

Mixed (multiplied) with oscillator $\cos(2\pi f_o t)$

$$\frac{1}{2}\cos\left[2\pi(2f_o - f_D)t\right] + \frac{1}{2}\cos[2\pi f_D t] \iff$$

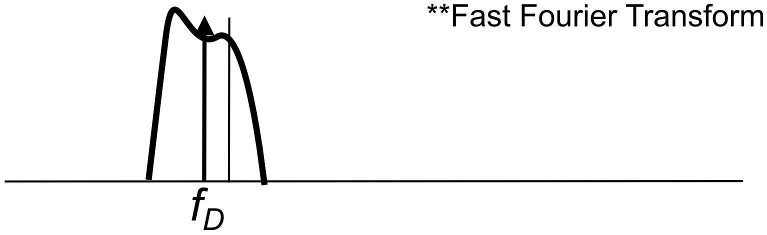


*Low Pass Filter

**Fast Fourier Transform

To resolve both positive and negative Doppler shifts, we need:

$$e^{j2\pi f_D t} = \cos(2\pi f_D t) + j\sin(2\pi f_D t)$$

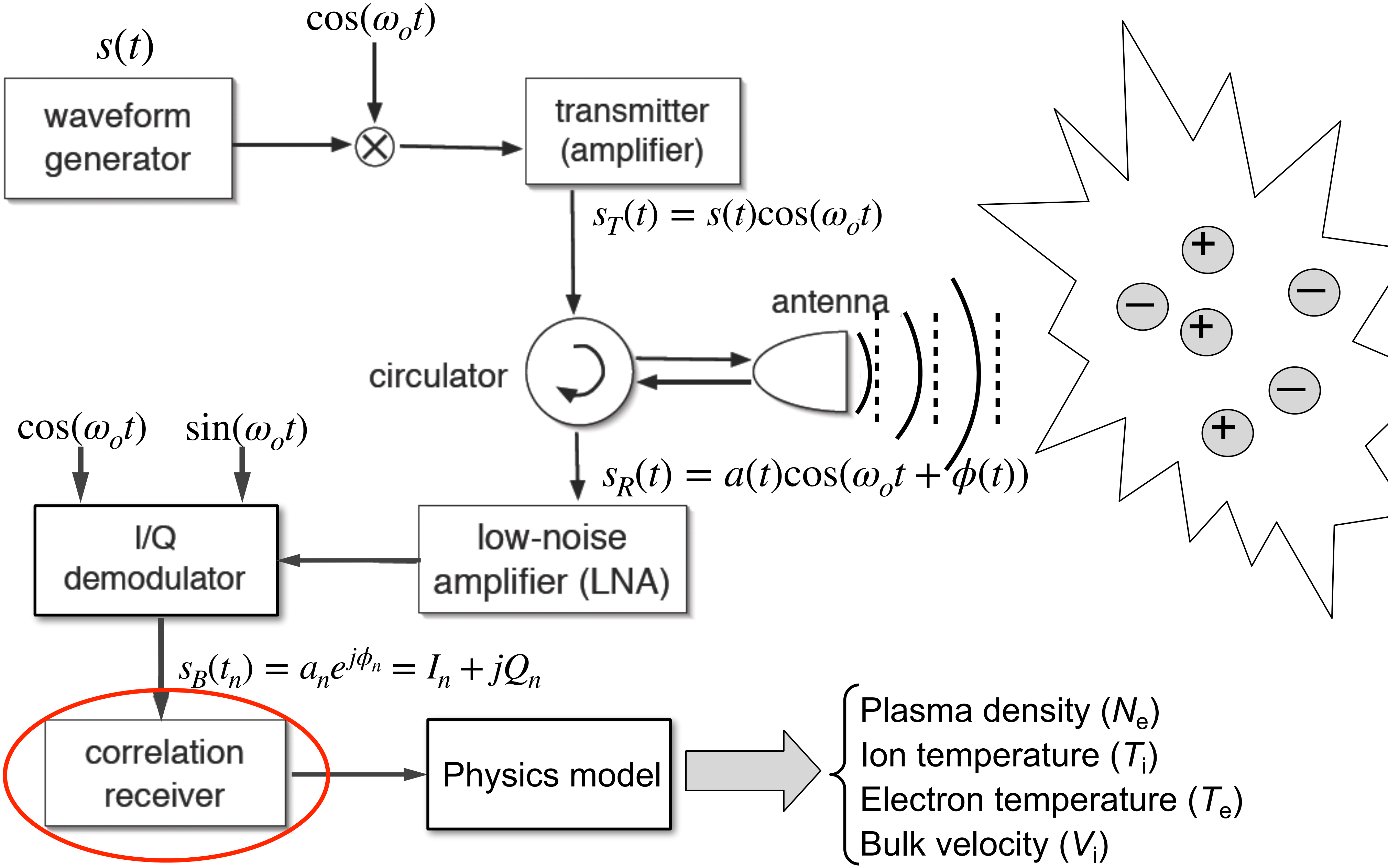


We thus need to mix with a second oscillator at same frequency but 90° out of phase (Lecture 3). For a cosine reference, the quadrature function is sine. The two components are called “in phase” (*I*) and “quadrature” (*Q*). Together *I* and *Q* represent discrete samples of the baseband analytic signal,

$$s_B(t) = Ae^{2\pi f_D t} = I(t) + jQ(t) \xleftrightarrow{\text{FFT**}} A\delta(f - f_D)$$

(for a single scatterer)

Components of a Pulsed Doppler Radar



$s(t)$

waveform generator

$\cos(\omega_o t)$

transmitter (amplifier)

$s_T(t) = s(t)\cos(\omega_o t)$

circulator

antenna

$s_R(t) = a(t)\cos(\omega_o t + \phi(t))$

$\cos(\omega_o t)$ $\sin(\omega_o t)$

I/Q demodulator

low-noise amplifier (LNA)

$s_B(t_n) = a_n e^{j\phi_n} = I_n + jQ_n$

correlation receiver

Physics model

Plasma density (N_e)

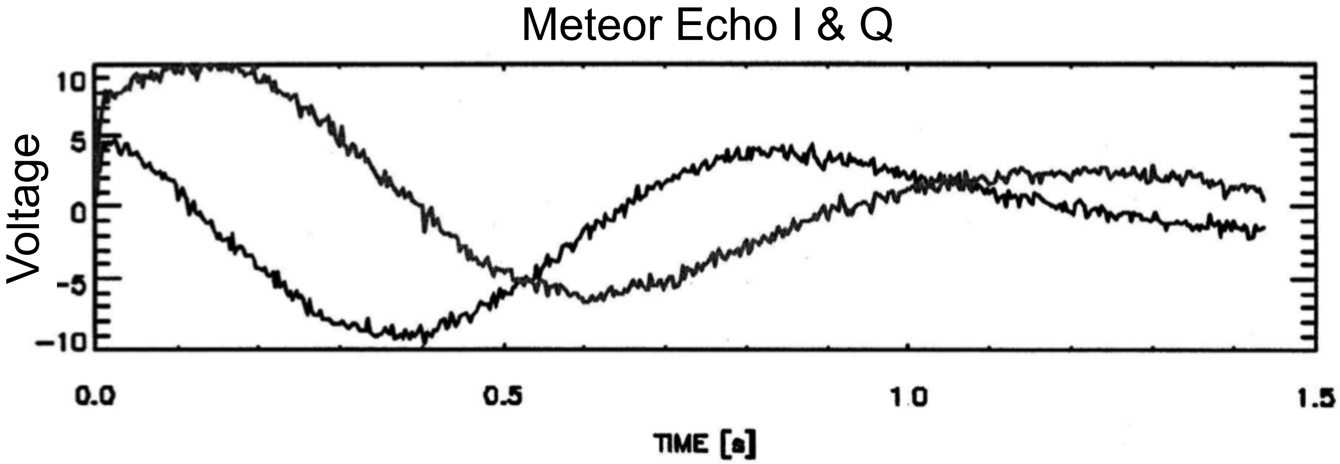
Ion temperature (T_i)

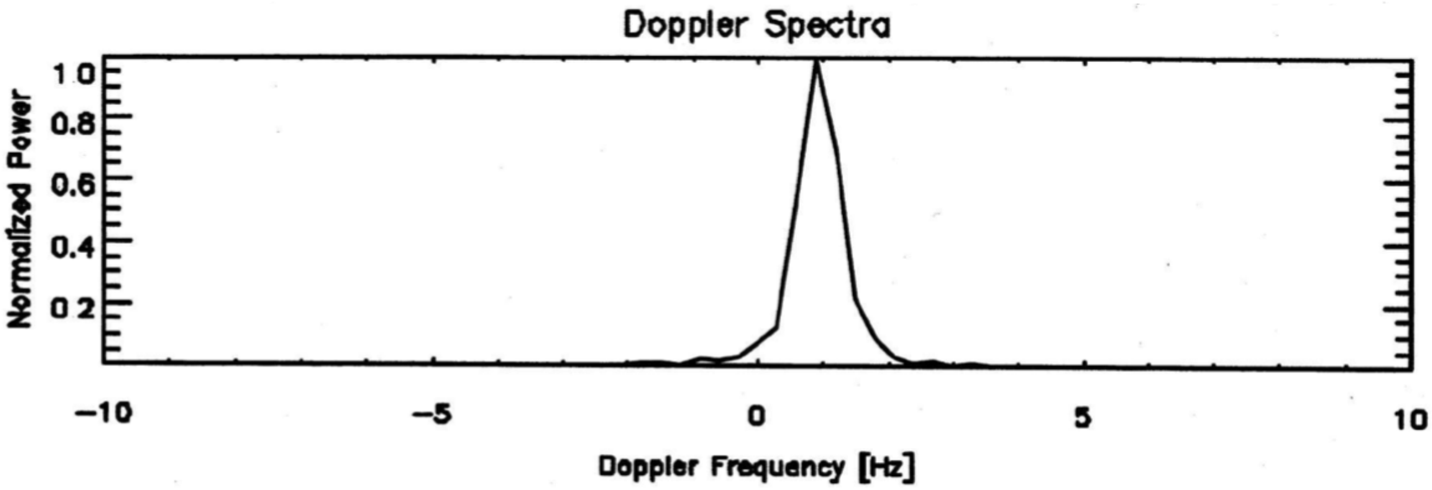
Electron temperature (T_e)

Bulk velocity (V_i)

Meteor Radar Example

- Coherent target (meteor ionization trail), with slowly changing velocity
- Find trail velocity (hence, neutral wind velocity along radar line of sight) by computing I and Q from many pulses, and taking the Fourier Transform (FFT).
- Velocity and reflected power are found from the peak in the power spectrum





Does this strategy work for ISR?

1) The Poker Flat ISR operates at 450 MHz. The echo from the ionosphere is produced by reflection from ion sound waves. A typical phase speed for these waves is 3 km/s. What is the Doppler frequency shift in Hz caused by reflection off these waves? This represents the approximate width of the Doppler spectrum.

2) The Nyquist theorem states that we must sample a signal at a rate of at least twice its highest frequency to fully recover it. For problem 1, this so-called “Nyquist rate” is about 20 kHz, meaning we need samples of I and Q from the target at a rate of 20kHz. What is the maximum target range at which we can obtain independent samples of I and Q at this rate? How does this compare with the altitude of the ionosphere?

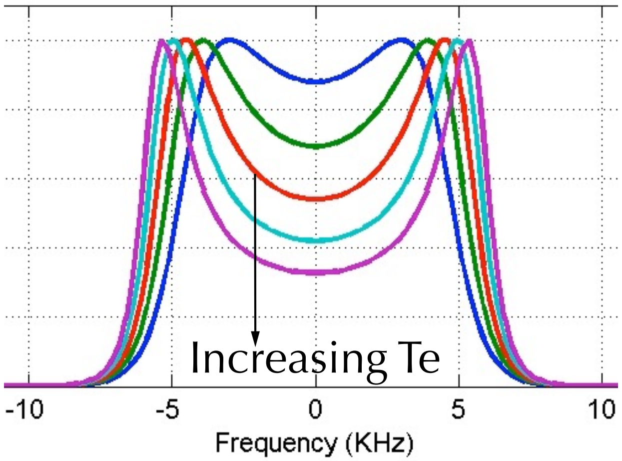
Does this strategy work for ISR?

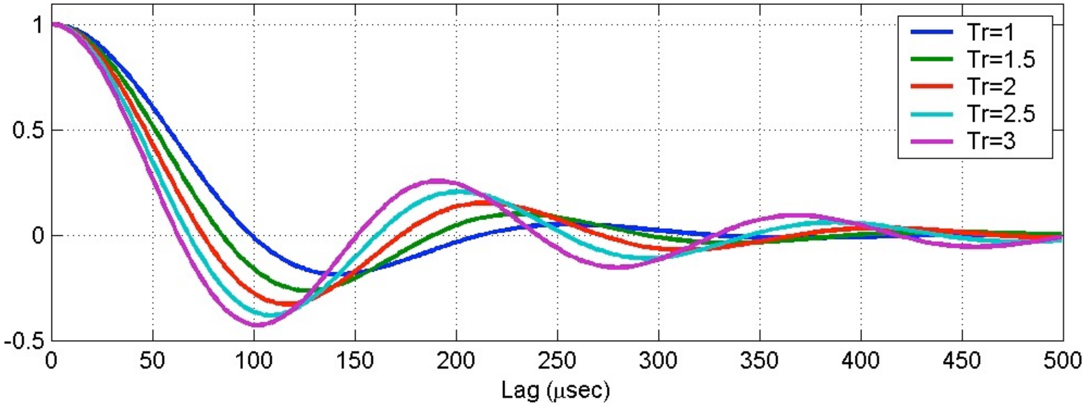
Doppler width at 450 MHz: 10 kHz
de-correlation time (zero crossing): ~1/10kHz = 0.1 ms
Inter-pulse period (IPP) to reach 500 km: *2R/c* = 3ms

Plasma has de-correlated by the time we send the next pulse.

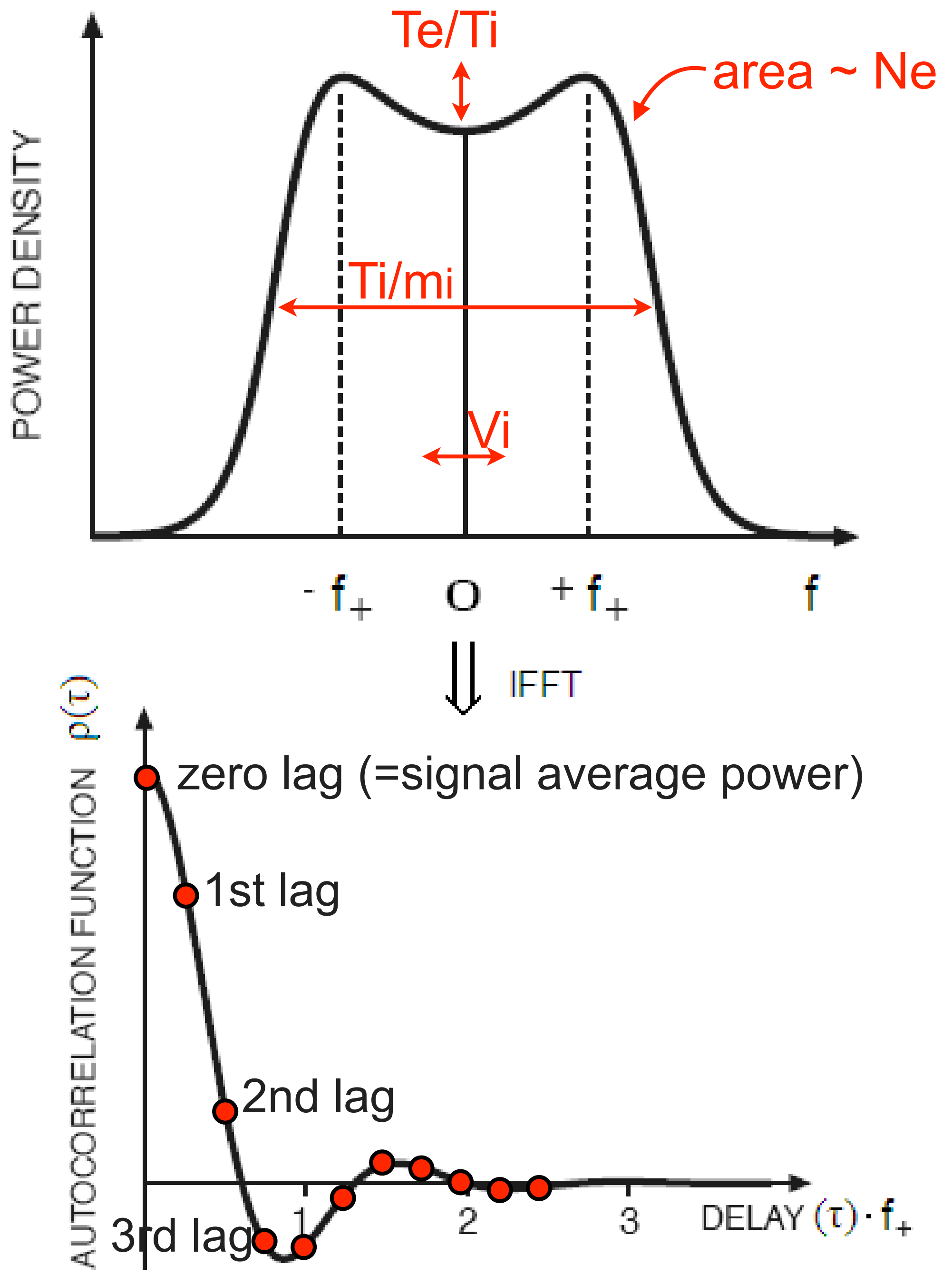
Stated alternately, the Doppler frequency shift of the plasma is much higher than the maximum unambiguous Doppler shift measurable for the pulse-repetition frequency.

ISR spectrum ⟺ Autocorrelation function (ACF)





Autocorrelation function and power spectrum



Ion temperature (Ti) to ion mass (mi) ratio from the width of the spectra

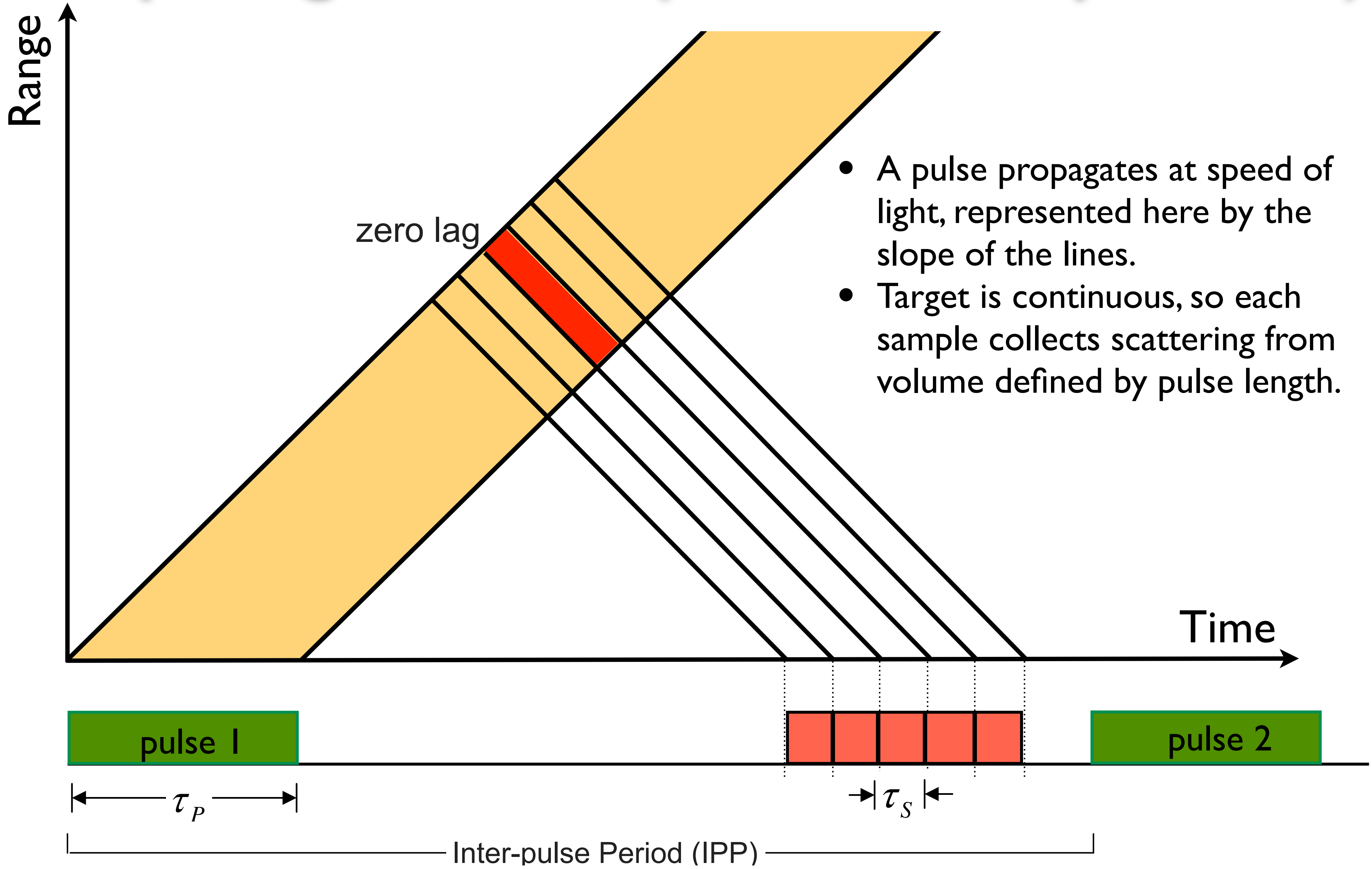
Electron to ion temperature ratio (Te/Ti) from “peak-to-valley” ratio

Electron (= ion) density from total area (corrected for temperatures)

Line-of-sight ion velocity (Vi) from bulk Doppler shift

Our goal is to sample lags with sufficient fidelity to provide meaningful estimates of plasma parameters

Computing the ACF (and, hence, spectrum)



- A pulse propagates at speed of light, represented here by the slope of the lines.
- Target is continuous, so each sample collects scattering from volume defined by pulse length.

τ_P = Length of RF pulse

τ_s = Sample Period (typically ~ 1/10 pulse length)

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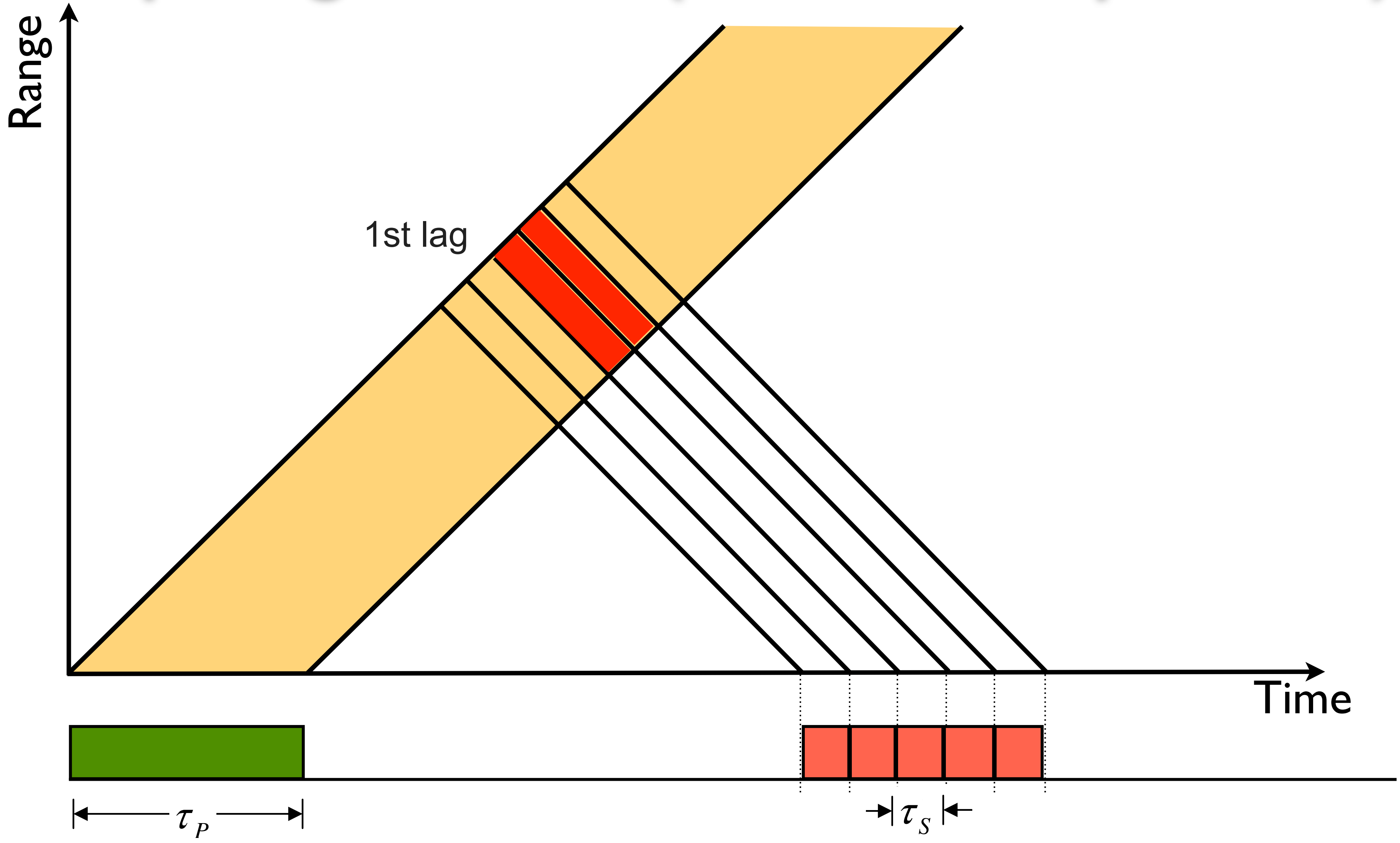

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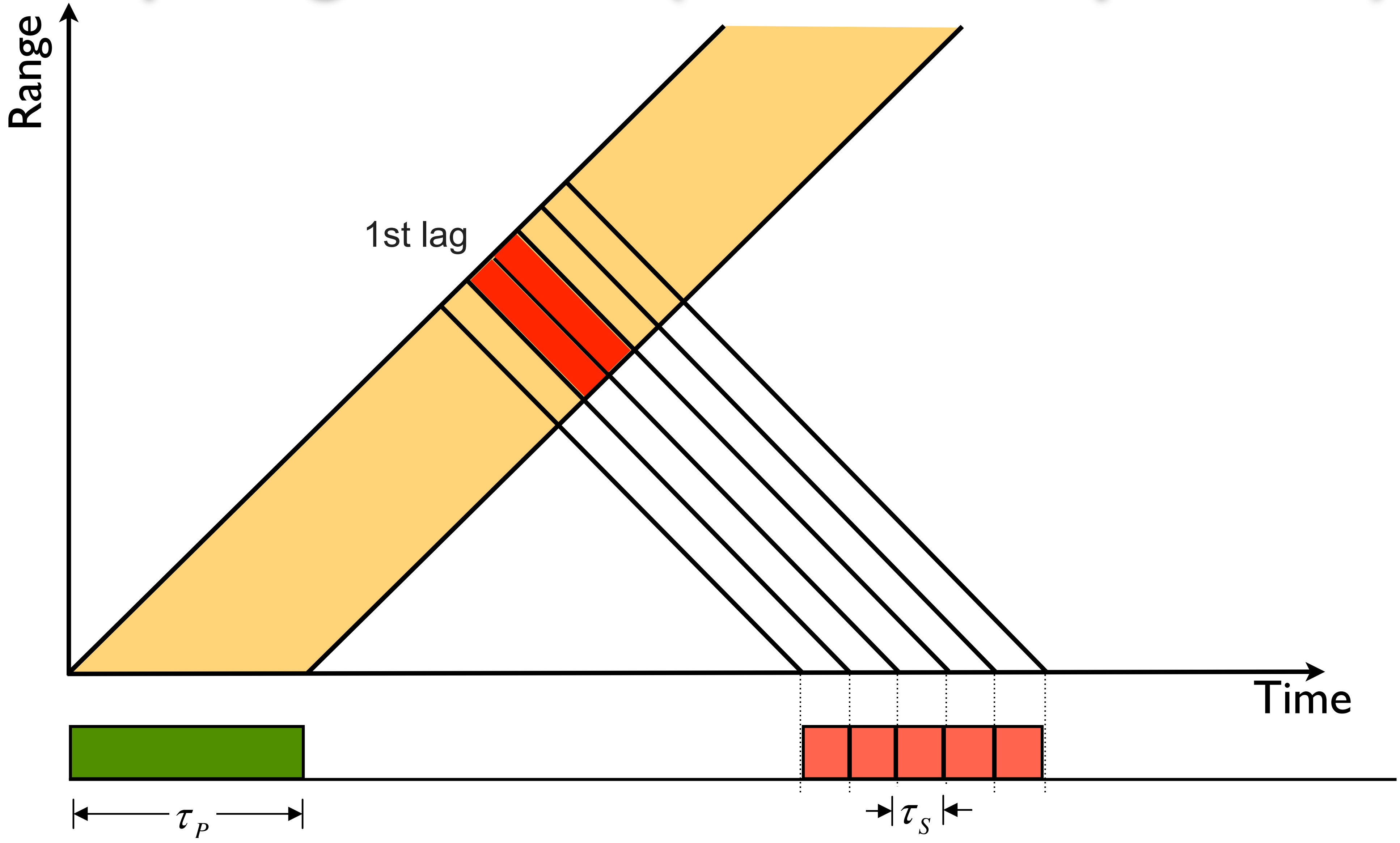

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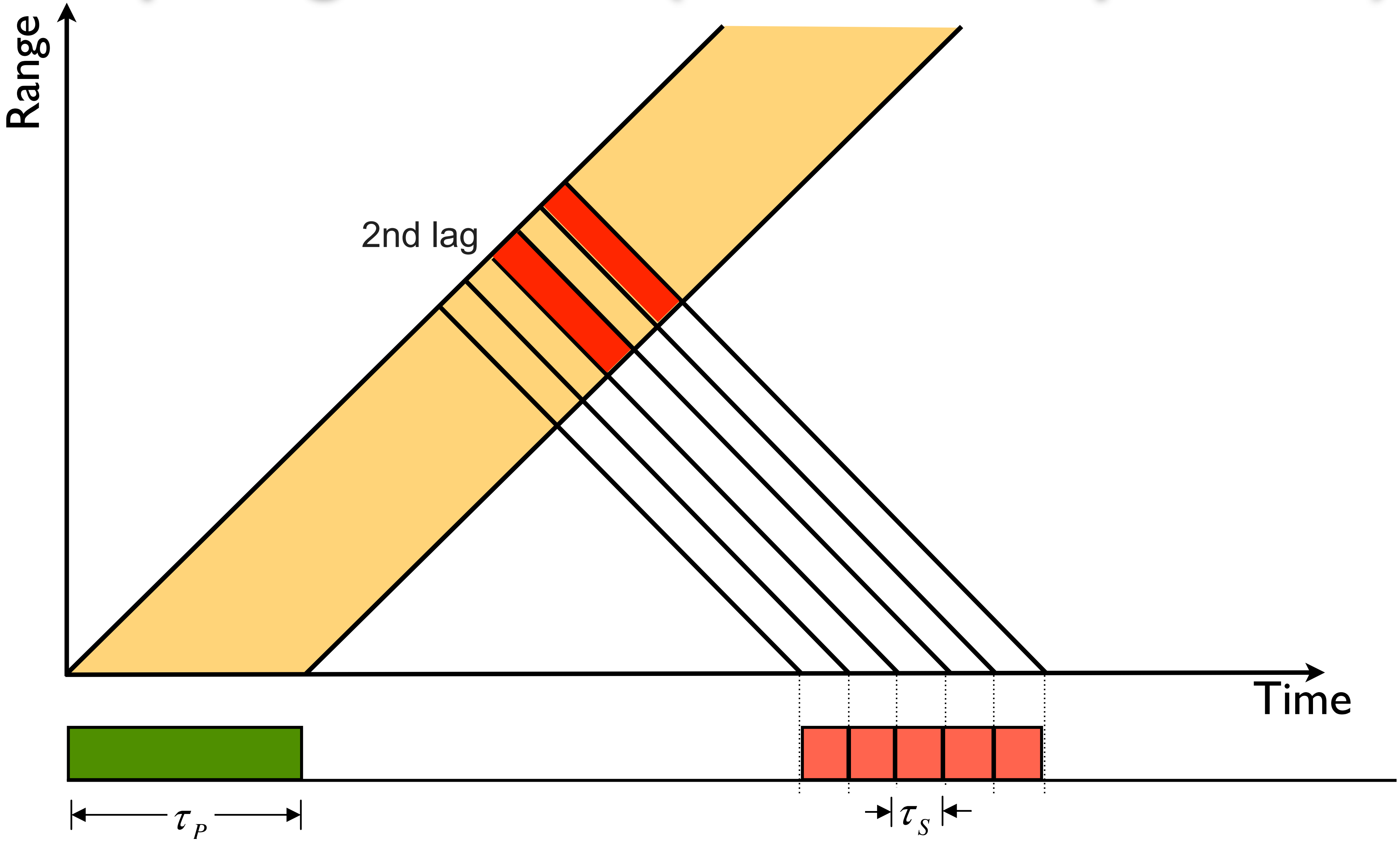

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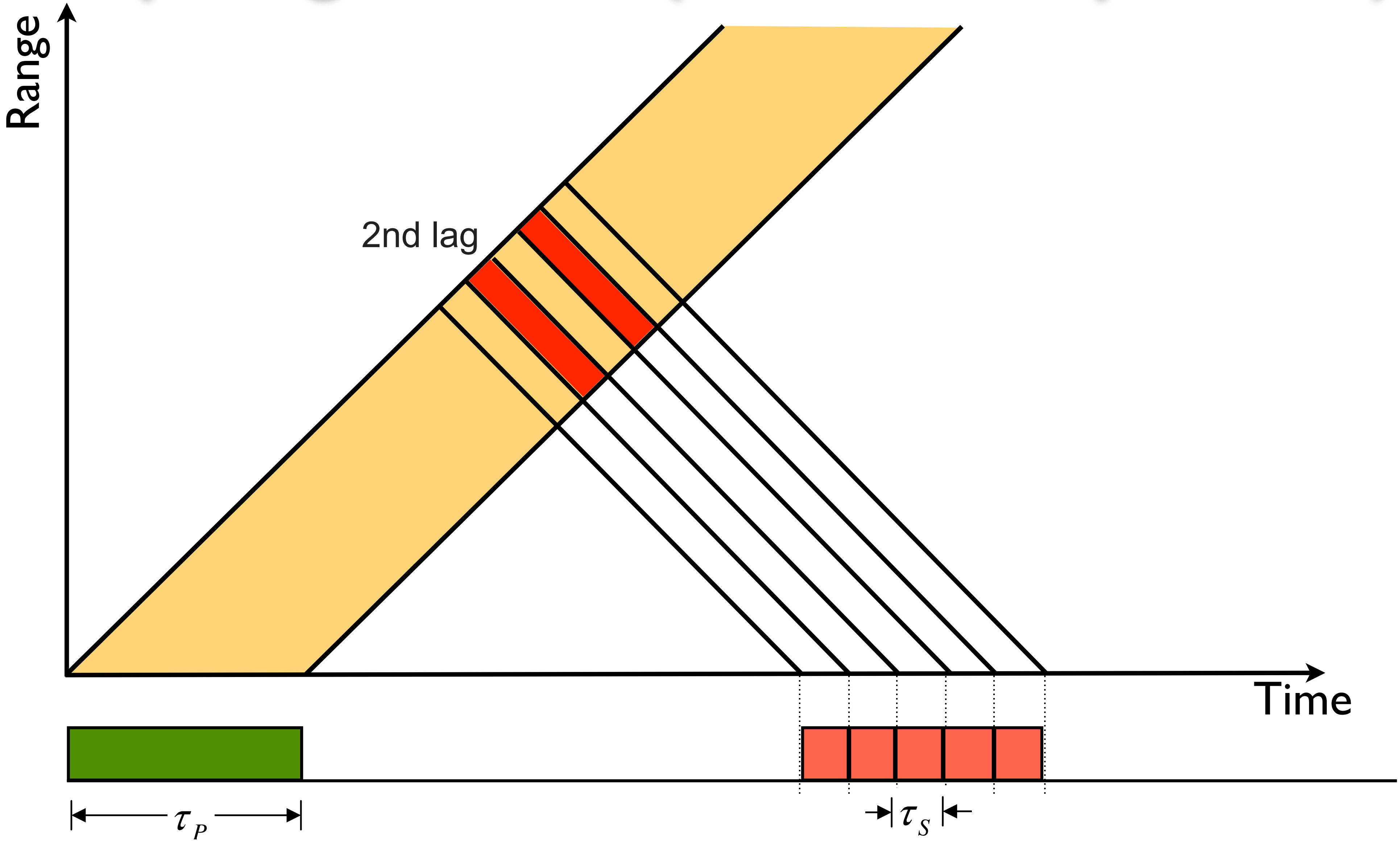

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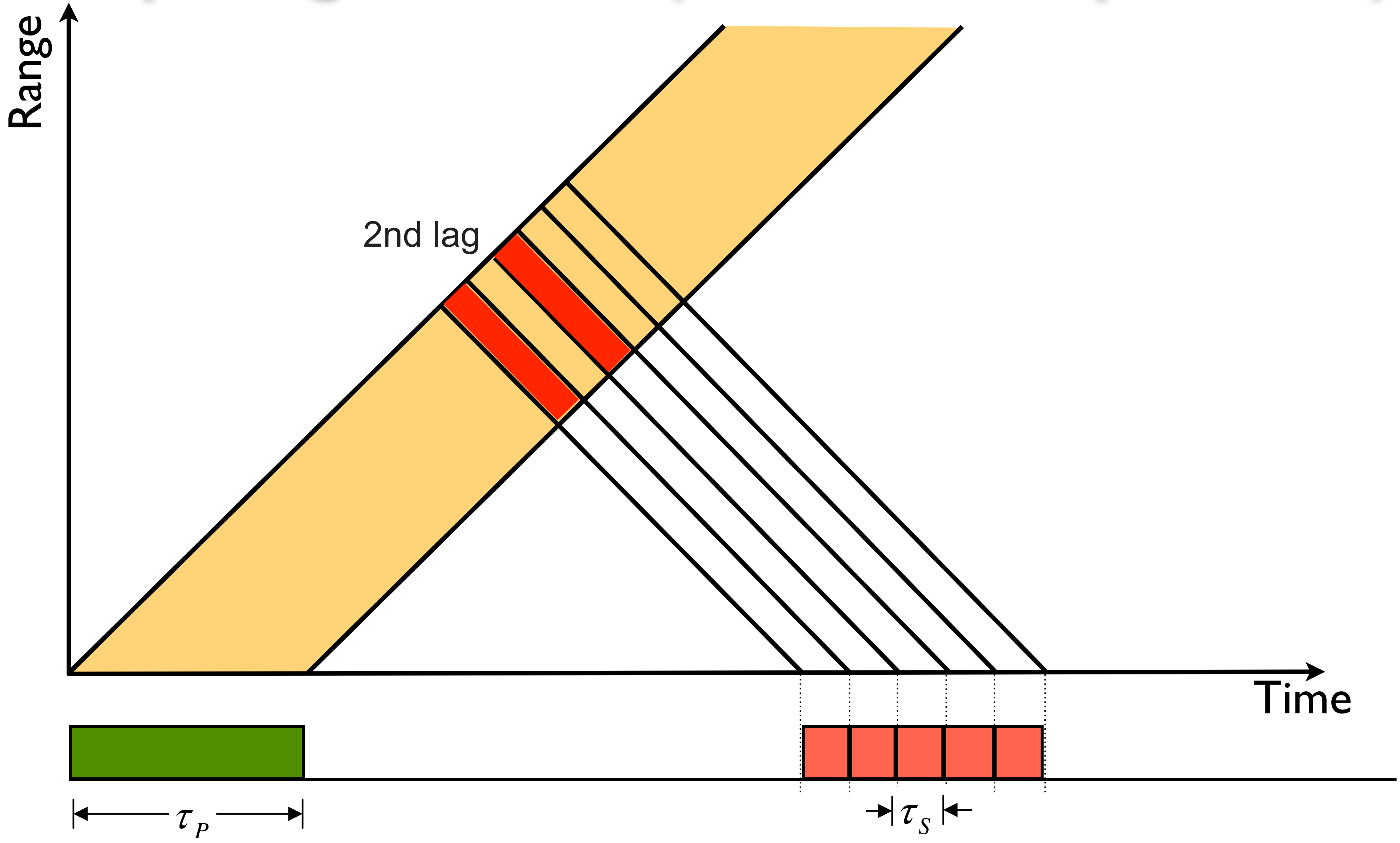

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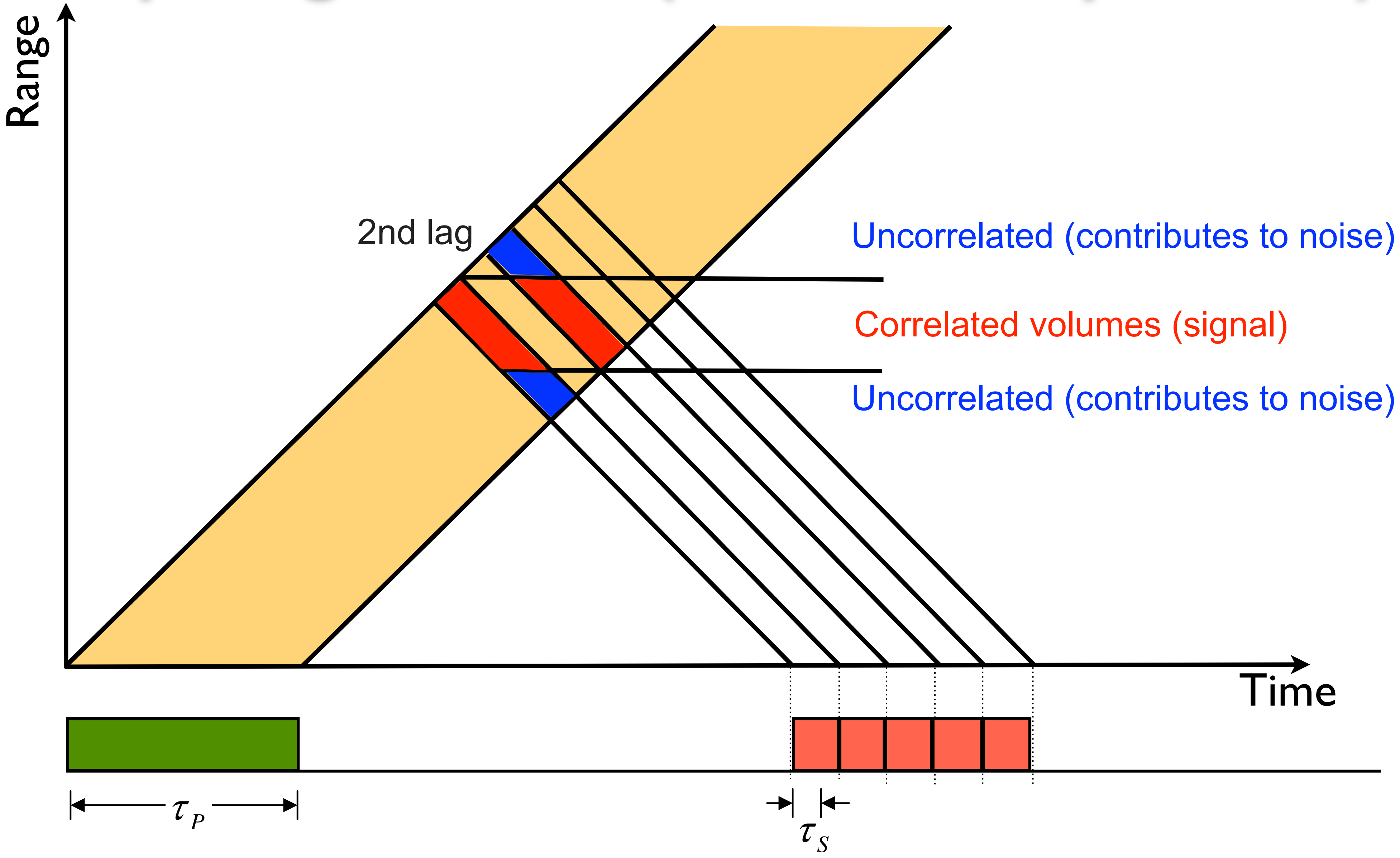

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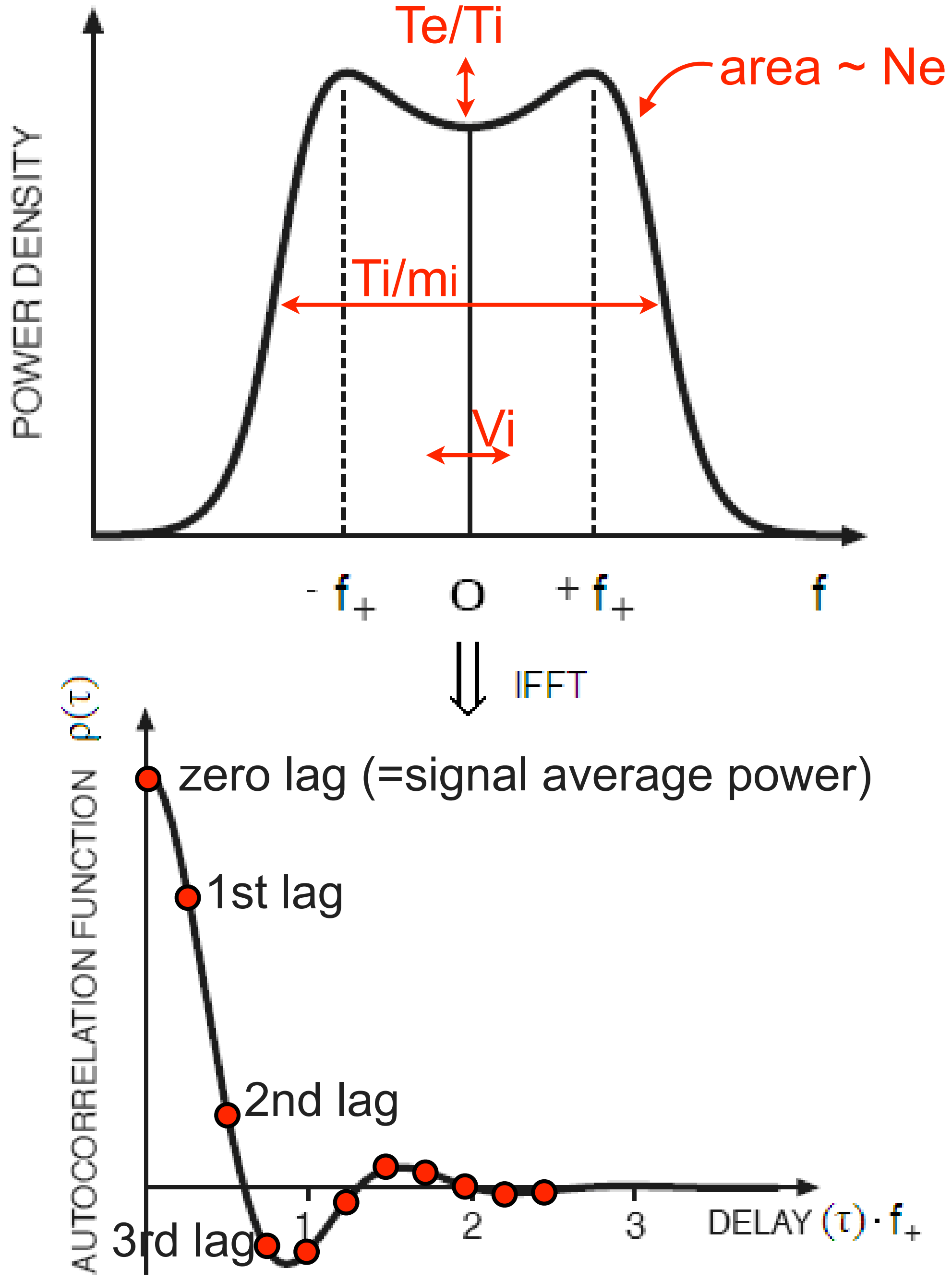

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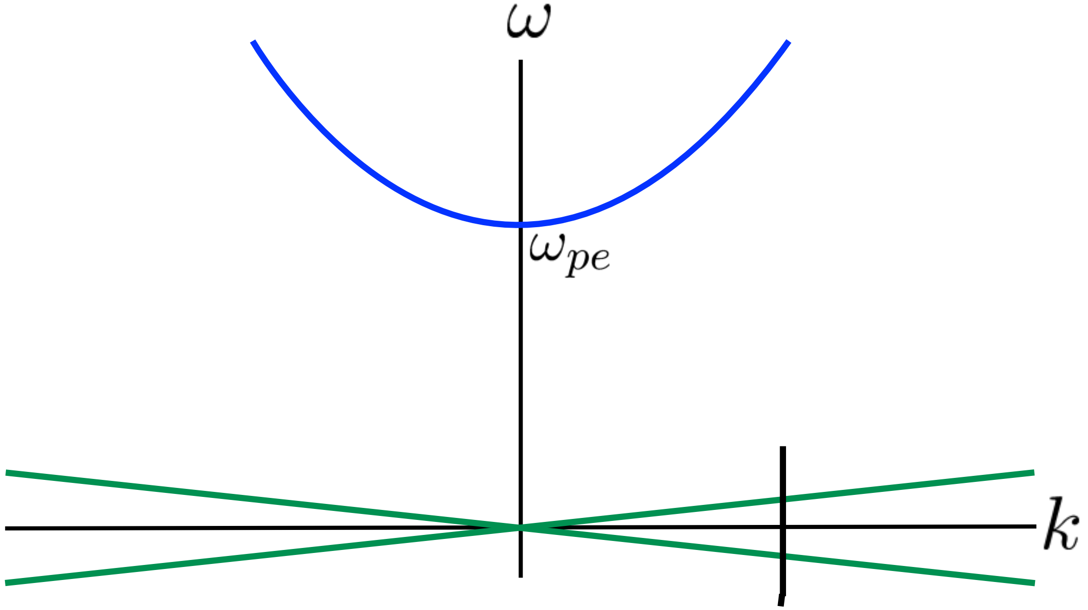
Incoherent Scatter Radar (ISR)

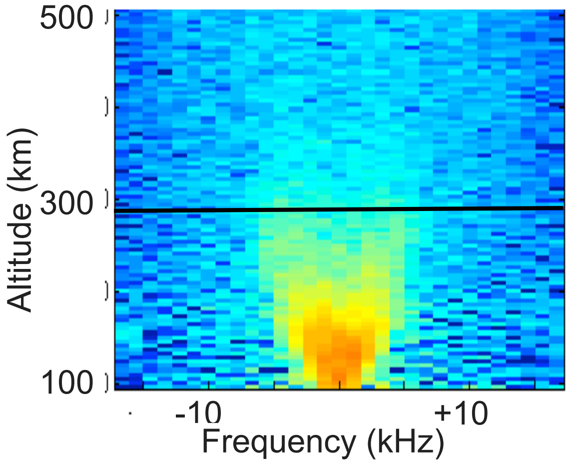
Ion-acoustic

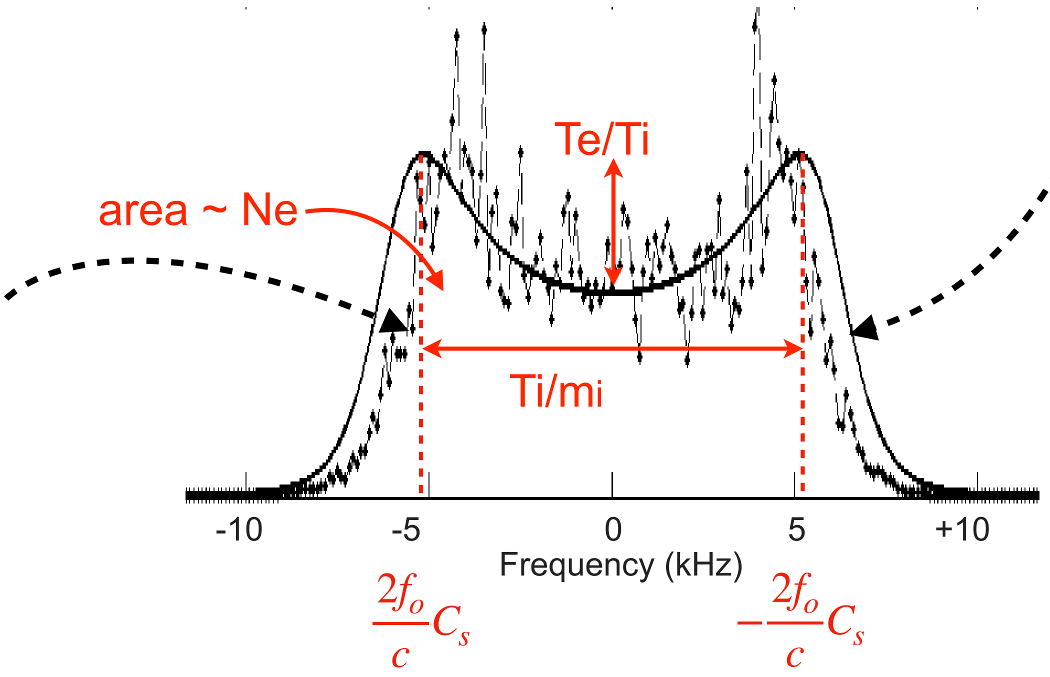
$$\omega_s = C_s k \qquad C_s = \sqrt{k_B(T_e + 3T_i)/m_i}$$

Langmuir

$$\omega_L = \sqrt{\omega_{pe}^2 + 3\,k^2\,v_{the}^2} \approx \omega_{pe} + \frac{3}{2} v_{the} \lambda_{De} k^2$$



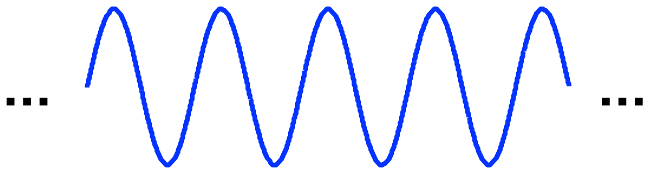




Radar Waveforms (cont'd.)

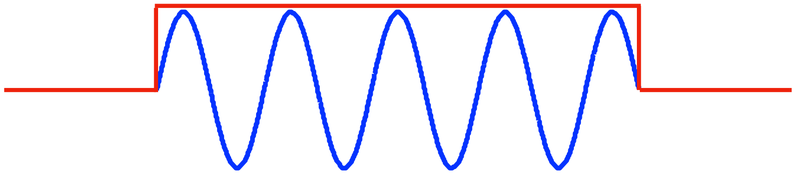
$$s(t) = A(t)\cos\left[2\pi f_o t + \phi(t)\right]$$

Unmodulated RF signal



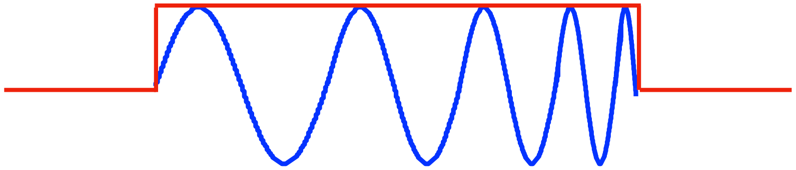
$$s(t) = A_o e^{j2\pi f_o t}$$

RF pulse at a single frequency



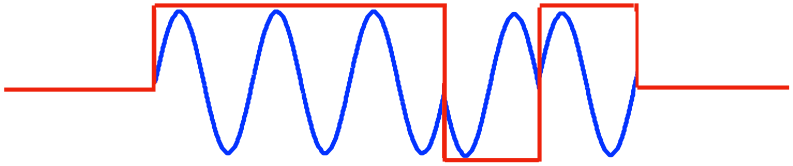
$$s(t) = A(t) e^{j2\pi f_o t}$$

RF Pulse with changing frequency



$$s(t) = A(t) e^{j2\pi(f_o + \Delta f(t)) t}$$

RF Pulse, single frequency, changing phase



$$s(t) = A(t) e^{j2\pi f_o t} e^{j\phi(t)}$$

$$e^{j0} = 1$$

$$e^{j\pi} = -1$$

$$e^{j\pi/2} = j$$

$$e^{-j\pi/2} = -j$$

Pulse Compression

Range resolution is controlled by the length of the transmitted pulse. The optimal detection strategy involves correlating the received signal with a replica of the signal we transmitted (called “Matched Filtering”). In the script given, the two vectors of 1’s represent identical uncoded radar pulses. Running the script plots the so-called “range ambiguity function” for the pulse, which is computed by correlating the pulse with itself. The origin represents the target location, but there is also received power at ranges other than 0, hence there is “range ambiguity” associated with any single detection. Try the following:

a) First let’s try a shorter pulse. Replace pulse2 with the following
pulse2 = [0,0,0,0,1,1,1,1,1,0,0,0,0]
This represents a pulse that is 40% shorter. Rerun the script. What effects do you see compared to the original pulse?

b) We’d like to retain the full energy of the pulse, so instead let’s try flipping the sign of one of the elements of pulse 2. Try randomly changing a few 1’s to -1’s
What effects do you see compared to the original pulse?

Pulse Compression

c) Let’s instead change some signs in a strategic fashion. Replace pulse2 with the following coded version of the pulse,
pulse2 = [1,1,1,1,1,-1,-1,1,1,-1,1,-1,1]
Each element represents a “bit” or “baud” whose signe we can control. This code is called a “13-baud Barker code.” The sign changes can be implemented in hardware by flipping the phase of the transmitted signal 180 degrees for these bauds. Rerun the script now. What have we achieved with this code?

d) The range ambiguity is generally defined as the “full-width at half-maximum” (FWHM) of the main peak of the matched filter output. Compare the range ambiguity of the uncoded and coded pulses based on this definition. The ratio of these quantities is referred to as the “pulse compression ratio”. What costs have we paid for the improved in range resolution from pulse coding?