Summary of QCD

The $SU(N_c)$ QCD Lagrangian without gauge fixing

$$\mathcal{L} = \bar\psi(i\not{D}-m)\psi - \frac{1}{4}G^A_{\mu\nu}G^{\mu\nu A}\,, \qquad G^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - gf^{ABC}A^B_\mu A^C_\nu$$
$$D_\mu = \partial_\mu + igA^A_\mu T^A\,, \qquad [D_\mu, D_\nu] = igG^A_{\mu\nu}T^A\,. \tag{0.1}$$

The equations of motion and Bianchi

$$(i\not{D}-m)\psi = 0\,, \qquad \partial^\mu G^A_{\mu\nu} = gf^{ABC}A^{B\mu}G^C_{\mu\nu} + g\bar\psi\gamma_\nu T^A\psi\,, \qquad \epsilon^{\mu\nu\lambda\sigma}(D_\nu G_{\lambda\sigma})^A = 0. \tag{0.2}$$

Color identites

$$[T^A, T^B] = if^{ABC}T^C\,, \qquad \mathrm{Tr}\big[T^AT^B\big] = T_F\delta^{AB}\,, \qquad \bar T^A = -T^{A*} = -(T^A)^T\,,$$
$$T^AT^A = C_F\mathbf{1}\,, \qquad f^{ACD}f^{BCD} = C_A\delta^{AB}\,, \qquad f^{ABC}T^BT^C = \frac{i}{2}C_AT^A\,,$$
$$T^AT^BT^A = \Big(C_F - \frac{C_A}{2}\Big)T^B\,, \qquad d^{ABC}d^{ABC} = \frac{40}{3}\,, \qquad d^{ABC}d^{A'BC} = \frac{5}{3}\delta^{AA'}\,, \tag{0.3}$$

where $C_F = (N_c^2-1)/(2N_c)$, $C_A = N_c$, $T_F = 1/2$, and $C_F - C_A/2 = -1/(2N_c)$. The color reduction formula and Fierz formula are

$$T^AT^B = \frac{\delta^{AB}}{2N_c}\mathbf{1} + \frac{1}{2}d^{ABC}T^C + \frac{i}{2}f^{ABC}T^C\,, \qquad (T^A)_{ij}(T^A)_{k\ell} = \frac{1}{2}\,\delta_{i\ell}\delta_{kj} - \frac{1}{2N_c}\,\delta_{ij}\delta_{k\ell}\,. \tag{0.4}$$

Feynman gauge rules, fermion, gluon, ghost propagators, and Fermion-gluon vertex

$$\frac{i(\not{p}+m)}{p^2-m^2+i0}\,, \qquad \frac{-ig^{\mu\nu}\delta^{AB}}{k^2+i0}\,, \qquad \frac{i}{k^2+i0}\,, \qquad -ig\gamma^\mu T^A\,. \tag{0.5}$$

Triple gluon and Ghost Feynman rules in covariant gauge for $\{A^A_\mu(k), A^B_\nu(p), A^C_\rho(q)\}$ all with incoming momenta, and $\bar c^A(p)A^B_\mu c^C$ with outgoing momenta p:

$$-gf^{ABC}\big[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu\big]\,, \qquad gf^{ABC}p^\mu\,. \tag{0.6}$$

Triple gluon Feynman rule in bkgnd Field covariant gauge $\mathcal{L}_{gf} = -(D^A_\mu Q^A_\mu)^2/(2\xi)$ for $\{A^A_\mu(k), Q^B_\nu(p), Q^C_\rho(q)\}$ with A^A_μ a bkgnd field:

$$-gf^{ABC}\Big[g^{\mu\nu}\Big(k-p-\frac{q}{\xi}\Big)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}\Big(q-k+\frac{p}{\xi}\Big)^\nu\Big]\,. \tag{0.7}$$

Lorentz gauge:

$$\mathcal{L} = -\frac{(\partial_\mu A^\mu)^2}{2\xi}\,, \qquad D^{\mu\nu}(k) = \frac{-i}{k^2+i0}\Big(g^{\mu\nu} - (1-\xi)\frac{k^\mu k^\nu}{k^2}\Big)\,, \tag{0.8}$$

where Landau gauge is $\xi \to 0$. Coulomb gauge:

$$\vec\nabla\cdot\vec A = 0\,, \qquad D^{\mu\nu}(k) = \frac{-i}{k^2+i0}\bigg(g^{\mu\nu} - \frac{[g^{\nu 0}k^0k^\mu + g^{\mu 0}k^0k^\nu - k^\mu k^\nu]}{\vec k^2}\bigg)\,,$$
$$D^{00}(k) = \frac{i}{\vec k^2 - i0}\,, \qquad D^{ij}(k) = \frac{i}{k^2+i0}\Big(\delta^{ij} - \frac{k^ik^j}{\vec k^2}\Big). \tag{0.9}$$

Running coupling with $\beta_0 = 11C_A/3 - 4T_Fn_f/3 = 11 - 2n_f/3$:

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \frac{\beta_0}{2\pi}\alpha_s(\mu_0)\ln\frac{\mu}{\mu_0}} = \frac{2\pi}{\beta_0\ln\frac{\mu}{\Lambda_{\rm QCD}}}\,, \qquad \frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(\mu_0)} + \frac{\beta_0}{2\pi}\ln\frac{\mu}{\mu_0}\,. \tag{0.10}$$