Basic Radar 3.4: ISR ACF Estimation

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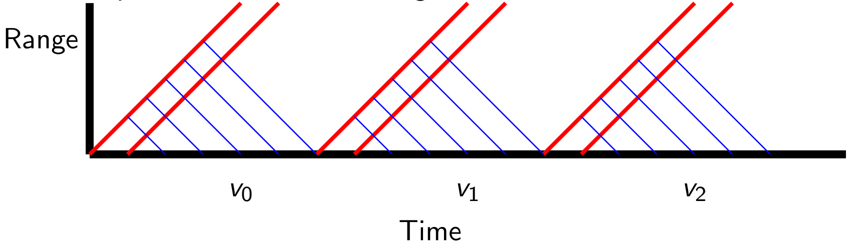
ACF Estimation (Pulse-to-Pulse)

Assume pulses are taken close together and are correlated.



Unbiased Estimator:

$$\hat{R}_\ell = \frac{1}{K-\ell}\sum_{n=\ell}^{K-1} v_n v_{n-\ell}^*$$

$$E\left\{\hat{R}_\ell\right\} = R_\ell$$

Biased Estimator:

$$\tilde{R}_\ell = \frac{1}{K}\sum_{n=\ell}^{K-1} v_n v_{n-\ell}^*$$

$$E\left\{\tilde{R}_\ell\right\} = \frac{K-\ell}{K} R_\ell$$

Zero-padded Periodogram

$$\tilde{S}_n = \left|\sum_{k=0}^{K-1} v_k e^{-2\pi j \frac{nk}{2K}}\right|^2$$

$$\tilde{R}_\ell = \frac{1}{2K}\sum_{n=0}^{2K-1} \tilde{S}_n e^{2\pi j \frac{n\ell}{2K}}$$

Biased ACF estimator equals the iFFT of the zero-padded periodogram.

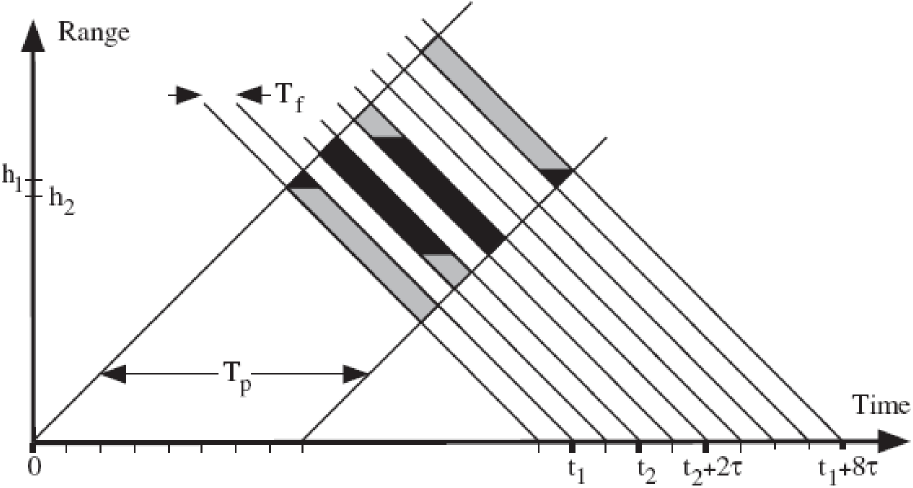
Underspread vs Overspread Targets

- If the IPP is short compared to the correlation time of the signal (inverse bandwidth), pulse-to-pulse processing works great.
- If the IPP is long compared to the correlation time, all pulse-to-pulse lag products give ≈ 0.
- Shortening the IPP is not always an option due to range aliasing.

Terminology:

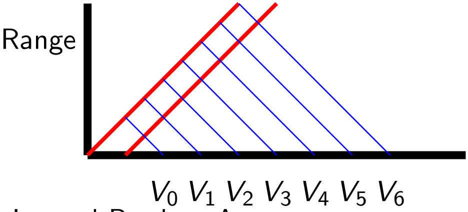
- **Underspread target**: There exists an IPP that is short compared to the correlation time but long enough to avoid range aliasing.
  - D-region ISR
  - Perpendicular to B ISR
  - MST radar
- **Overspread target**: All practical IPP are long compared to the correlation time.
  - Most ISR experiments
  - SuperDARN

Uncoded Long Pulse Experiments



Scattered signals from outside the overlap region do not affect the expected value of a lag product, but they do affect the variance

Lagged Product Arrays (LPA)



Lagged Product Array:

$$L^i_\ell \equiv \left\langle V^*_{i-\left\lfloor \frac{\ell}{2} \right\rfloor} V_{i+\left\lfloor \frac{\ell}{2} \right\rfloor + (\ell \bmod 2)} \right\rangle$$

$$L^i_0 = \langle V^*_i V_i \rangle$$

$$L^i_1 = \langle V^*_i V_{i+1} \rangle$$

$$L^i_2 = \left\langle V^*_{i-1} V_{i+1} \right\rangle$$

$$L^i_3 = \left\langle V^*_{i-1} V_{i+2} \right\rangle$$

etc.

The set of voltage samples from N ranges after a single pulse could be completely characterized by the $N \times N$ covariance matrix. However,

- Covariance matrices are always conjugate symmetric (Hermitian).
- Only samples within 1 pulse length of each other will have non-zero covariance.

For an experiment with 16 samples per pulse length we only need a $N \times 16$ array of covariances.

LPA Estimators

This definition

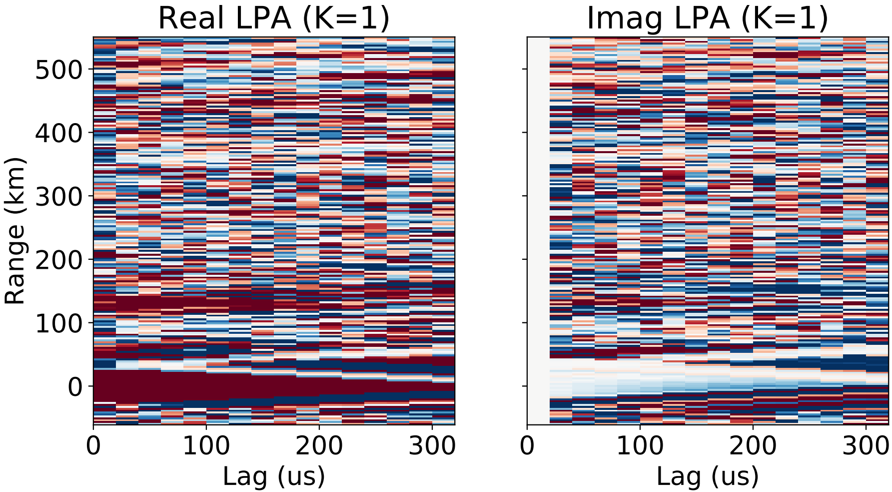
$$L_\ell^i \equiv \left\langle V^*_{i-\left\lfloor \frac{\ell}{2} \right\rfloor} V_{i+\left\lfloor \frac{\ell}{2} \right\rfloor + (\ell \bmod 2)} \right\rangle$$

refers to the *expected value* of the products of samples.
Can be estimated by taking products samples and averaging over many pulses

$$\hat{L}_\ell^i = \frac{1}{K} \sum_{k=0}^{K-1} V^*_{i-\left\lfloor \frac{\ell}{2} \right\rfloor} V_{i+\left\lfloor \frac{\ell}{2} \right\rfloor + (\ell \bmod 2)}$$

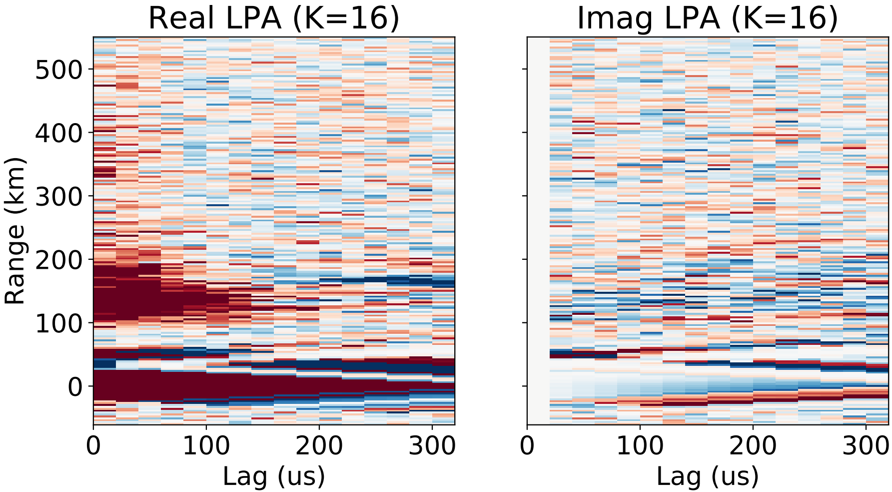
By virtue of the central limit theorem, these estimators will be Gaussian random variables with variances that decrease as $1/K$ (error decreases as $1/\sqrt{K}$).

LPA Estimator from 1 Pulse



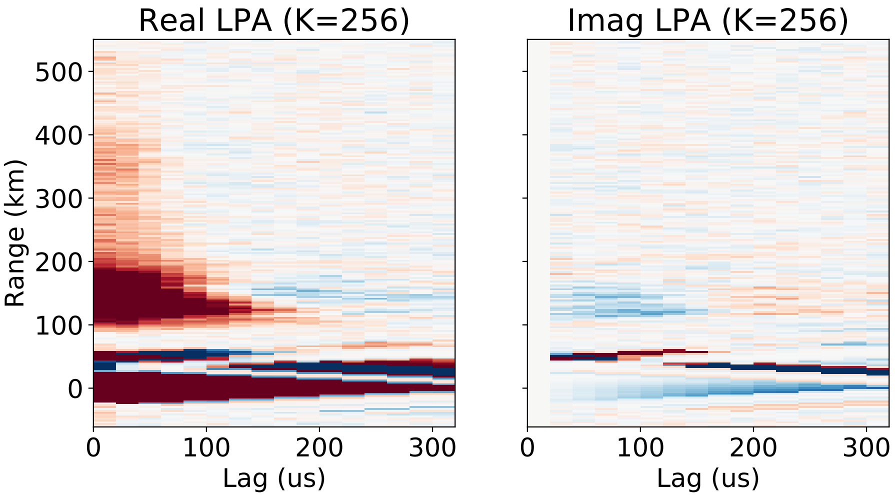
Note the Noise ACF has been subtracted before plotting.

LPA Estimator from 16 Pulses



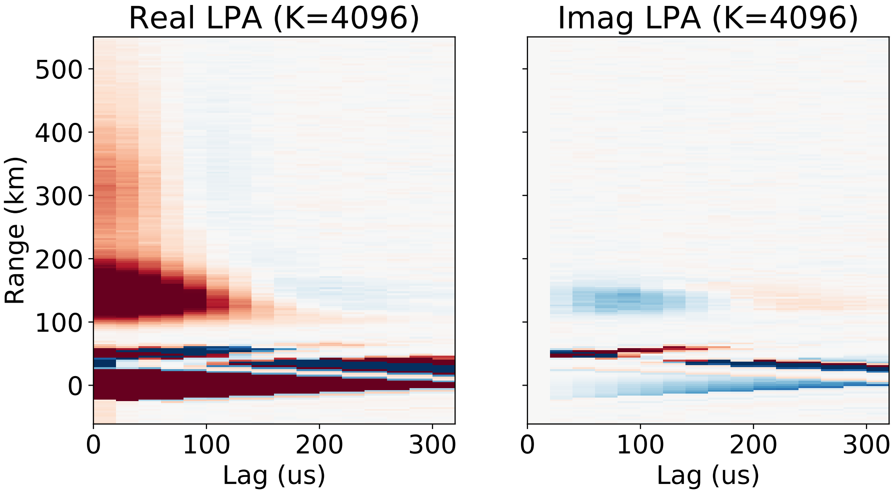
Note the Noise ACF has been subtracted before plotting.

LPA Estimator from 256 Pulses



Note the Noise ACF has been subtracted before plotting.

LPA Estimator from 4096 Pulses



Note the Noise ACF has been subtracted before plotting.

ACF Estimation Summary

- All information in an ISR experiment comes from covariances between voltages sampled at different times.
- For underspread experiments, we can treat one range at a time and take correlations from pulse to pulse.
- For overspread targets we compute lagged product arrays that are related to the 2D ACFs as a function of range and time.
- Estimation requires averaging many pulses together.
- The LPA estimators are the inputs to the fitting algorithms.