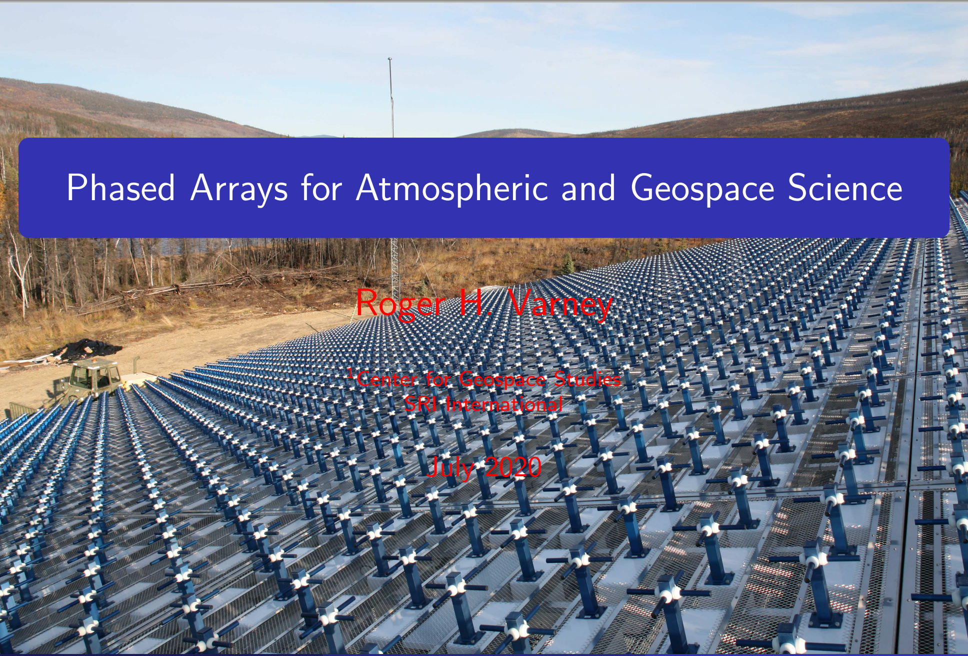

Phased Arrays for Atmospheric and Geospace Science

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Superposition Principle

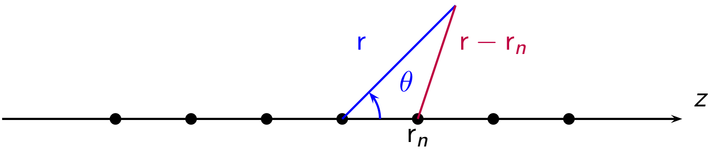
Maxwell's Equations are Linear:

$$J_1 = \frac{1}{\mu_0}\nabla \times (B_1) - \epsilon_0 \frac{\partial}{\partial t}(E_1)$$
$$0 = \nabla \times (E_1) + \frac{\partial}{\partial t}(B_1)$$

$$J_2 = \frac{1}{\mu_0}\nabla \times (B_2) - \epsilon_0 \frac{\partial}{\partial t}(E_2)$$
$$0 = \nabla \times (E_2) + \frac{\partial}{\partial t}(B_2)$$

$$J_1 + J_2 = \frac{1}{\mu_0}\nabla \times (B_1 + B_2) - \epsilon_0 \frac{\partial}{\partial t}(E_1 + E_2)$$
$$0 = \nabla \times (E_1 + E_2) + \frac{\partial}{\partial t}(B_1 + B_2)$$

Superposition Applied to Antenna Arrays



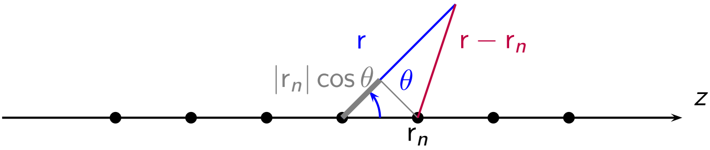
Fields radiated by single element at the origin with applied current I_0:

$$\mathrm{E} = \mathrm{E}_0 I_0 \frac{e^{-jk|\mathrm{r}|}}{|\mathrm{r}|}$$

Fields radiated by entire array:

$$\mathrm{E} = \mathrm{E}_0 \sum_{n=0}^{N-1} I_n \frac{e^{-jk|\mathrm{r}-\mathrm{r}_n|}}{|\mathrm{r}-\mathrm{r}_n|}$$

Far Field Approximation (Fraunhofer Zone)



If r and $\mathrm{r} - \mathrm{r}_n$ are almost parallel lines:

$$\mathrm{r} - \mathrm{r}_n \approx \mathrm{r} - |\mathrm{r}_n| \cos\theta \hat{r}$$

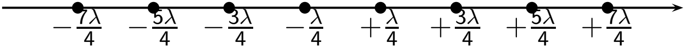
Assume $|\mathrm{r}_n| \ll |\mathrm{r}|$:

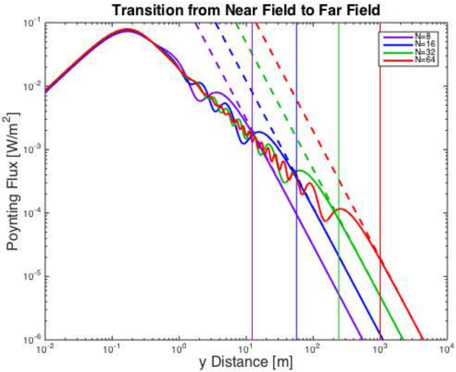
$$|\mathrm{r} - \mathrm{r}_n| \approx |\mathrm{r}| \text{ for demoninator terms}$$

$$-jk\,|\mathrm{r} - \mathrm{r}_n| \approx -jk\,|\mathrm{r}| + jk\,|\mathrm{r}_n| \cos\theta$$

$$\mathrm{E} \approx \underbrace{\mathrm{E}_0 \frac{e^{-jk|\mathrm{r}|}}{|\mathrm{r}|}}_{\text{Element Factor}} \underbrace{\sum_{n=0}^{N-1} I_n e^{jk|\mathrm{r}_n|\cos\theta}}_{\text{Array Factor}}$$

Distance to Far Field: Fresnel Numbers





Fresnel Number:

$$\frac{L^2}{r\lambda} \ll 1 \rightarrow \text{Far Field}$$

$$\frac{L^2}{r\lambda} > 1 \rightarrow \text{Near Field}$$

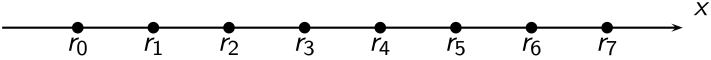
$$L = \text{Array length}$$

$$\lambda = \text{wavelength}$$

AMISR: $L = 32$ m $\lambda = 0.67$ m $L^2/\lambda = 1.5$ km

Arecibo: $L = 305$ m $\lambda = 0.70$ m $L^2/\lambda = 133$ km

1-D Linear Phased Array



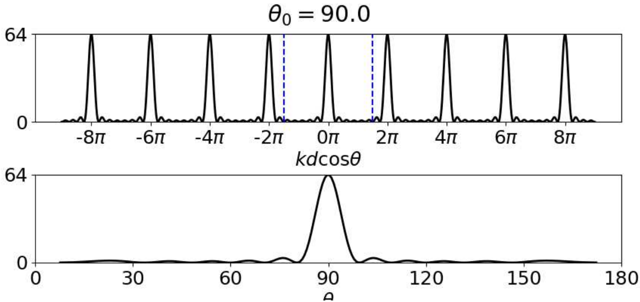
$$|\mathrm{r}_n| = nd \qquad I_n = e^{jn\alpha}$$

Array Factor:

$$\begin{aligned}
F &= \sum_{n=0}^{N-1} e^{jn\alpha} e^{jknd\cos\theta} \\
&= \frac{1 - e^{jN\alpha + jNkd\cos\theta}}{1 - e^{j\alpha + jkd\cos\theta}} \\
&= e^{j\frac{(N-1)}{2}(kd\cos\theta + \alpha)} \frac{\sin\left[\frac{N}{2}(kd\cos\theta + \alpha)\right]}{\sin\left[\frac{1}{2}(kd\cos\theta + \alpha)\right]} \\
|F|^2 &= \frac{\sin^2\left[\frac{N}{2}(kd\cos\theta + \alpha)\right]}{\sin^2\left[\frac{1}{2}(kd\cos\theta + \alpha)\right]}
\end{aligned}$$

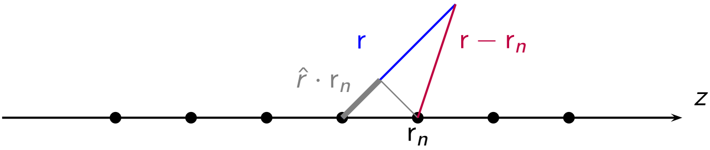
1-D Linear Phased Array Cont.

$$|F|^2 = \frac{\sin^2\left[\frac{N}{2}\left(kd\cos\theta + \alpha\right)\right]}{\sin^2\left[\frac{1}{2}\left(kd\cos\theta + \alpha\right)\right]}$$



Peak appears when $kd\cos\theta = -\alpha \rightarrow \alpha = -kd\cos\theta_0$
Additional peaks could appear when $kd\cos\theta = -\alpha + 2\pi m$ (Grating Lobes)
Visible Region: $0 < \theta < \pi \rightarrow -kd < kd\cos\theta < kd$

Multi-Dimensional Arrays



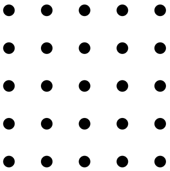
$$-jk\,|\mathrm{r}-\mathrm{r}_n| \approx -jk\,|\mathrm{r}| + jk\,(\hat{r}\cdot\mathrm{r}_n)$$

$$\mathrm{E} \approx \underbrace{\mathrm{E}_0\frac{e^{-jk|\mathrm{r}|}}{|\mathrm{r}|}}_{\text{Element Factor}} \underbrace{\sum_{n=0}^{N-1} I_n e^{jk(\hat{r}\cdot\mathrm{r}_n)}}_{\text{Array Factor}}$$

In spherical coordinates:

$$\hat{r}\cdot\mathrm{r}_n = x_n\cos\phi\sin\theta + y_n\sin\phi\sin\theta + z_n\cos\theta$$

2-D Rectangular Array



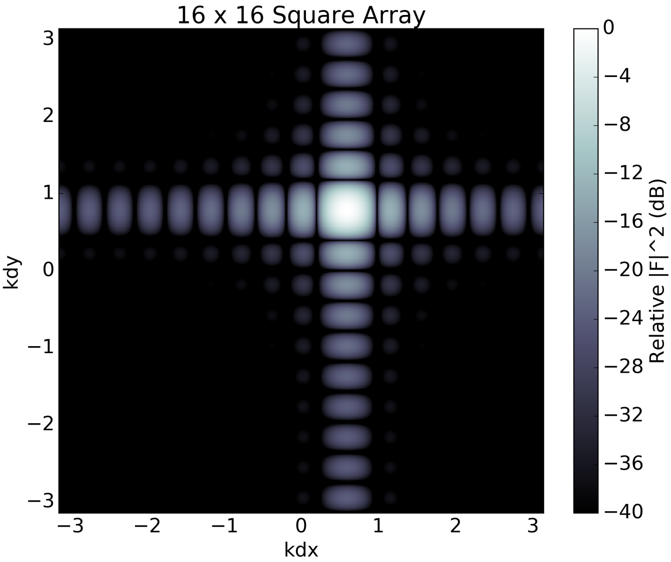
$$\mathrm{r}_{nm} = nd_x\hat{\mathrm{x}} + md_y\hat{\mathrm{y}} \qquad I_{nm} = e^{j(n\alpha + m\beta)}$$

Array Factor:

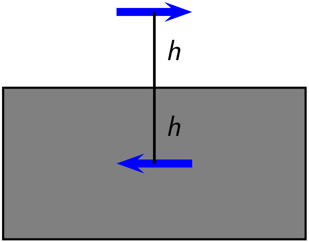
$$|F(\theta,\phi)|^2 = \left| \sum_{n=0}^{N_x-1} \sum_{m=0}^{N_y-1} e^{j(nkd_x \cos\phi \sin\theta + n\alpha + mkd_y \sin\phi \sin\theta + m\beta)} \right|^2$$

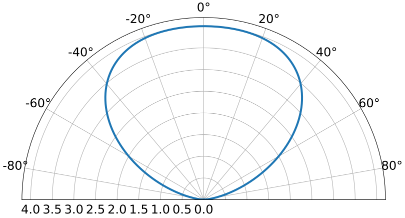
$$= \frac{\sin^2\left[\frac{N_x}{2}\left(kd_x \cos\phi \sin\theta + \alpha\right)\right]}{\sin^2\left[\frac{1}{2}\left(kd_x \cos\phi \sin\theta + \alpha\right)\right]} \frac{\sin^2\left[\frac{N_y}{2}\left(kd_y \sin\phi \sin\theta + \beta\right)\right]}{\sin^2\left[\frac{1}{2}\left(kd_y \sin\phi \sin\theta + \beta\right)\right]}$$

2-D Rectangular Array



Antenna above conducting ground plane





$$F = I_0 e^{jkh\cos\theta} - I_0 e^{-jkh\cos\theta}$$
$$= 2jI_0 \sin\left(kh\cos\theta\right)$$
$$|F|^2 = 4\,|I_0|^2 \sin^2\left(kh\cos\theta\right)$$

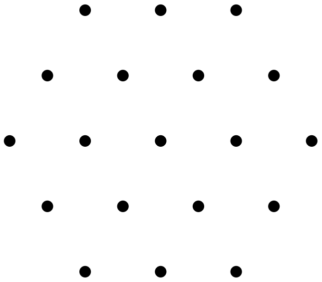
When $h = \frac{\lambda}{4} \rightarrow kh = \frac{\pi}{2}$

$$|F|^2 = 4\,|I_0|^2 \sin^2\left(\frac{\pi}{2}\cos\theta\right)$$

Peaks at $\theta = 0$ (upwards).

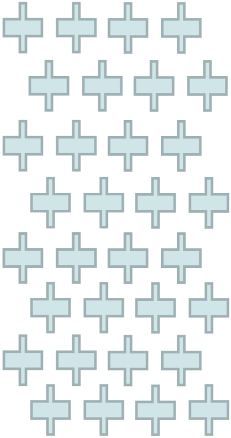
Hexagonal Spacing

Hexagon

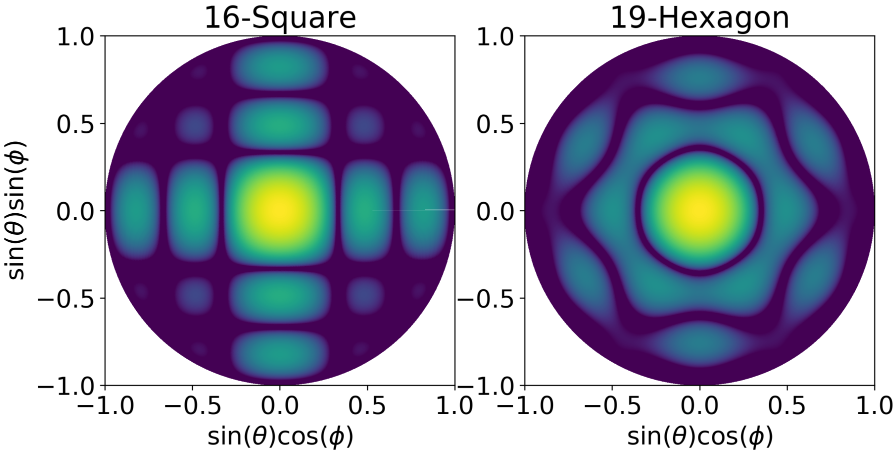


Honeycomb Rectangular Array

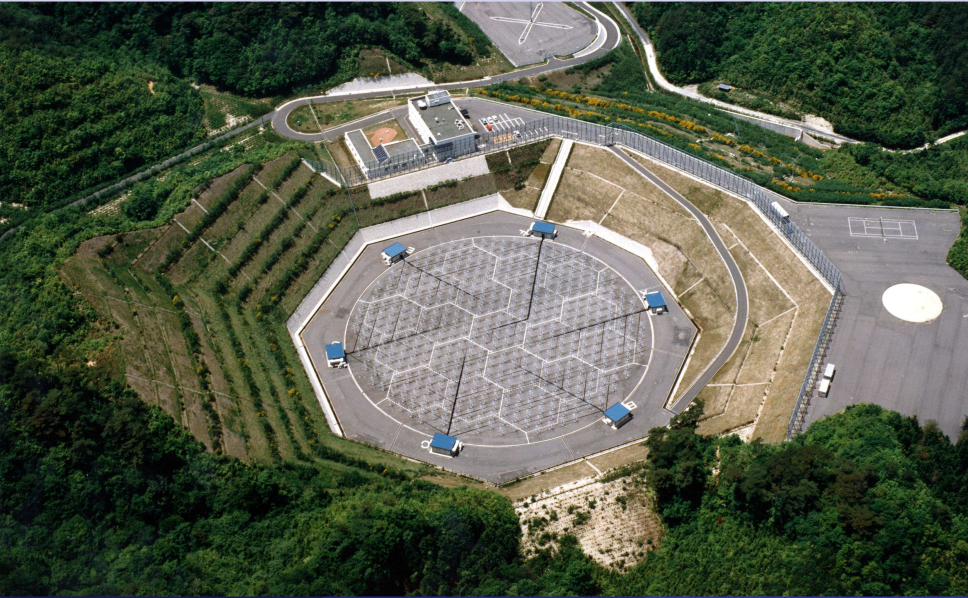
One AMISR panel:



Sibelobe Patterns for Squares vs Hexagons



A “Circular” Array: MU Radar



Introduction to Phased Array Summary

- Phased arrays work due to the superposition principle
- Focusing is achieved in the far field
- Gain pattern is dominated by the array factor
- Gain pattern typically has a main lobe and many smaller side lobes
- Side lobe pattern depends on the shape of the array (rectangular vs "circular")