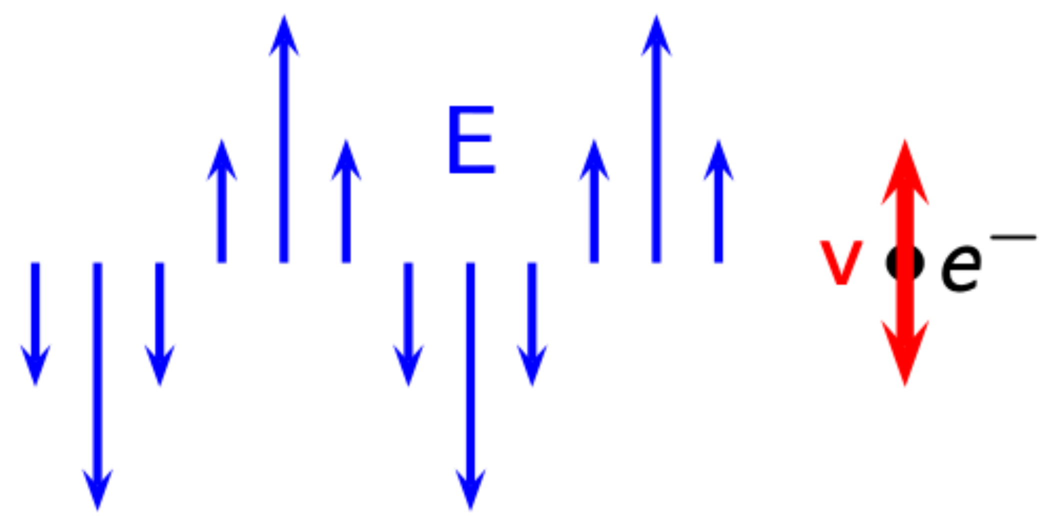
Thomson Scatter Summary

- Thomson scatter from electrons is a fundamental physical process
- Radar cross section of one electron is a constant independent of wavelength ($\sim 10^{-28}\ \mathrm{m}^2$)
- Scatter from ions is negligible
- Even though one electron has a tiny cross section, scatter can still be detectable from a whole volume of electrons

Thomson Scatter from One Electron

Incident wave:

$$\mathbf{E} = \hat{z} E_0 e^{j\omega t - j\mathbf{k}_0 \cdot \mathbf{r}}$$



Motion of the electron:

$$j\omega m_e \mathbf{v} = -e\mathbf{E} \rightarrow \mathbf{v} = \frac{je}{\omega m_e} E_0 \hat{z}$$

Effective Hertzian Dipole with $Id\ell \rightarrow e\mathbf{v}$ (also note $\omega / k_0 = c$)

$$\mathbf{E}_{scat} = \frac{-\eta_0 e^2}{4\pi r m_e c} E_0 \sin\theta e^{j\omega t - j\mathbf{k}_0 \cdot \mathbf{r}} \hat{\theta} = -\frac{r_e}{r} E_0 e^{j\omega t - j\mathbf{k}_0 r} \hat{\theta}$$

Where the classical electron radius is

$$r_e = \frac{\eta_0 e^2}{4\pi m_e c} = \frac{e^2}{4\pi \epsilon_0 m_e c^2} \approx 2.818 \times 10^{-15} \text{ m}$$

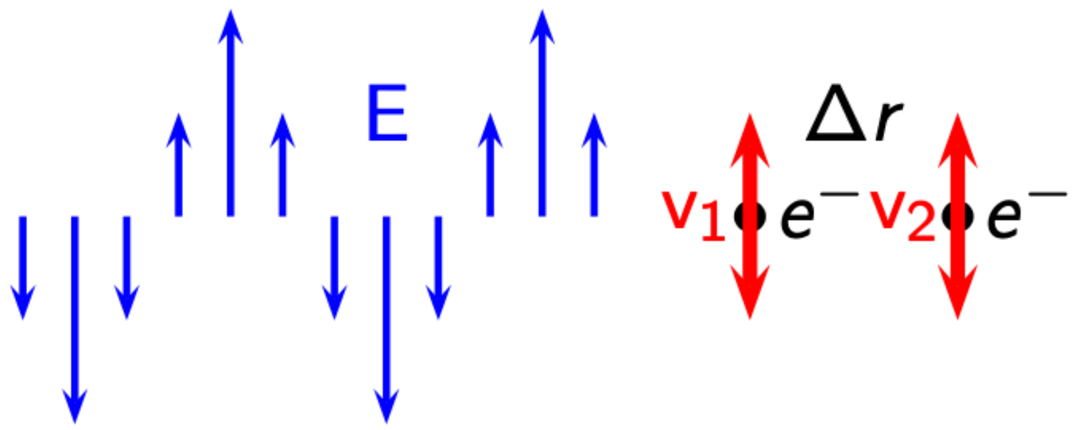
For backscatter $\theta = 90°$, so the radar cross section of one electron is

$$\sigma = 4\pi r_e^2 \approx 10^{-28} \text{ m}^2 \qquad (\sim 0.9979 \times 10^{-28} \text{m}^2)$$

Bragg Scatter Summary

- Scatter from targets spaced by the Bragg wavelength ($\lambda/2$) add constructively
- Scatter from a large number of electrons samples the Fourier transform of the electron density distribution at the Bragg wavenumber
- Thermal plasmas are naturally full of a whole spectrum of waves
- ISR is Bragg scatter from those thermal waves that match the Bragg wavenumber

Scatter from Two Electrons



Incident on first electron:

$$E_1 = E_0 e^{j\omega t}$$

Scattered from first electron:

$$E_{s1} = -\frac{r_e}{r} E_1 e^{-jk_0 r} = -\frac{r_e}{r} E_0 e^{j\omega t - jk_0 r}$$

Incident on second electron:

$$E_2 = E_0 e^{j\omega t - jk_0 \Delta r}$$

Scattered from second electron:

$$E_{s2} = -\frac{r_e}{r + \Delta r} E_2 e^{-jk_0 (r + \Delta r)} = -\frac{r_e}{r} E_0 e^{j\omega t - jk_0 r - j2k_0 \Delta r}$$

In the far field $\frac{1}{r+\Delta r} \approx \frac{1}{r}$, so the sum of the fields is

$$E_{s1} + E_{s2} = -\frac{r_e}{r} E_0 e^{j\omega t - jk_0 r} \left(1 + e^{-j2k_0 \Delta r}\right)$$

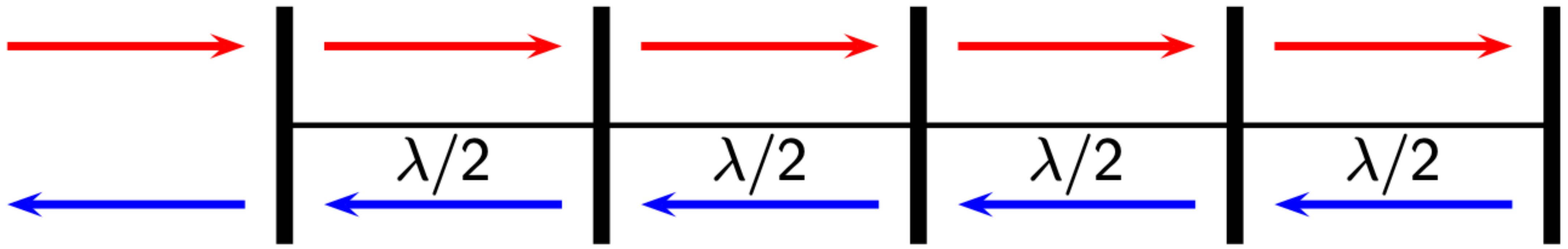
For scatter from two electrons

$$|E_{s1} + E_{s2}|^2 \propto \left|1 + e^{-j2k_0 \Delta r}\right|^2 = 4\cos^2(k_0 \Delta r)$$

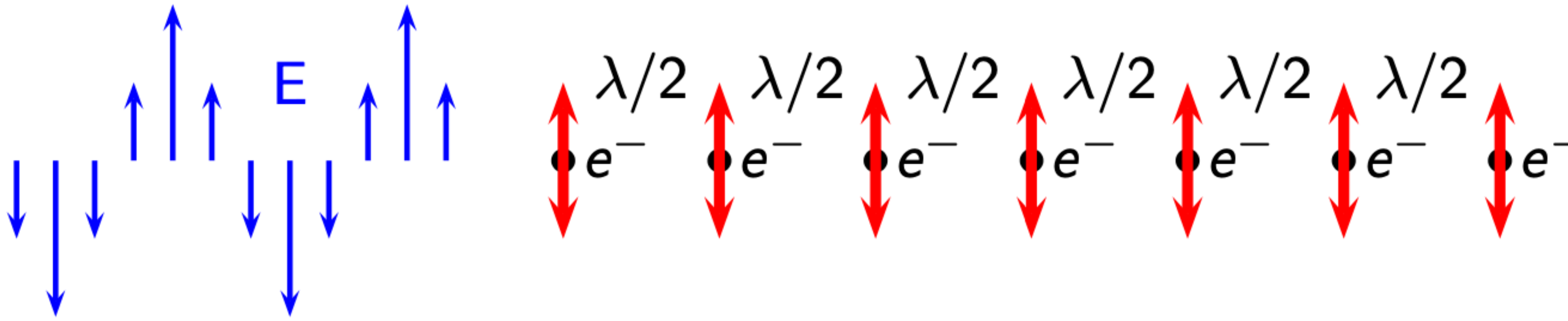
Scatter from Many Electrons

$$E_s = -\frac{r_e}{r} E_0 e^{j\omega t - jkr} \left(\sum_{p=0}^{N-1} e^{-j2\mathrm{k}_0 \cdot \Delta \mathrm{r}_p} \right) = -\frac{r_e}{r} E_0 e^{j\omega t - jk_0 r} \int n_e(\mathrm{r}) e^{-j2\mathrm{k}_0 \cdot \Delta \mathrm{r}_p} d^3 r$$

Stack of reflecting structures



Stack of electrons



The scatter is most sensitive to density structures at the Bragg wavelength.

Total Scattered Power Summary

- In the collective regime $\sigma \neq \sigma_e V N_e$
- Correction terms can be understood using dressed particle theory concepts
- Corrections depend on temperature ratio (T_e/T_i) and Debye length
- ISR typically report both uncorrected N_e (from power) and corrected N_e (from fitted ACFs)
- Dressed particle theory concepts also explain enhanced plasma line observations

- **Non-Collective Limit:** $k^2\lambda_{De}^2 \gg 1$
  Electron line dominates (wide bandwidth)
  $$\sigma = \sigma_e V N_e$$
- **Collective Limit:** $k^2\lambda_{De}^2 \ll 1$
  Ion line dominates (narrow bandwidth)
  $$\sigma = \sigma_e V \frac{N_e}{1 + \frac{T_e}{T_i}}$$

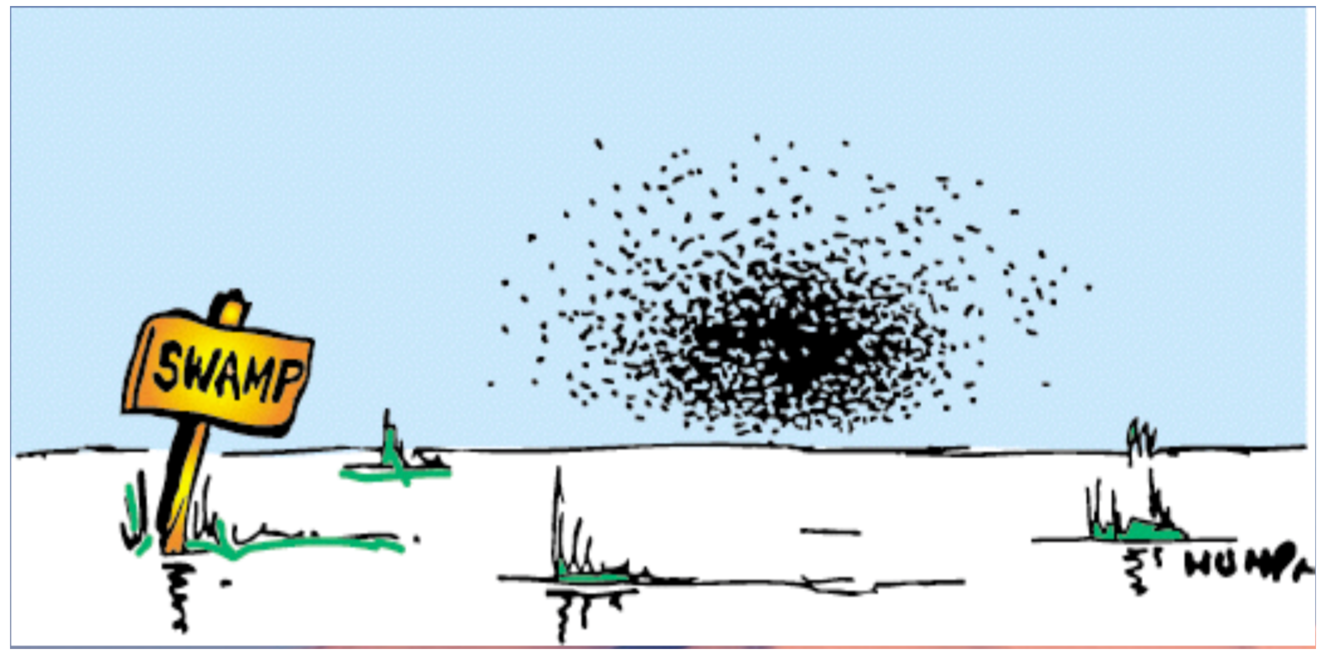
Reporting Electron Density from Ion Line Power

Ion Line Cross Section:

$$\sigma = \sigma_e V \frac{N_e}{2} \zeta$$

Temperature Correction: $$\zeta = \frac{2}{\left(1 + k^2\lambda_{De}^2\right)\left(1 + k^2\lambda_{De}^2 + T_e/T_i\right)}$$

- Uncorrected N_e: Assume $\zeta = 1$.
  - $T_e/T_i = 1$
  - $k^2\lambda_{De}^2 \ll 1$.
- N_e with model: Compute ζ using an empirical model of T_e/T_i as a function of altitude.
- N_e with fits: Compute ζ with T_e and T_i estimated from fitted ACF.



Radio Noise

Nyquist Noise Theorem: $\boxed{P_N = k_B T_{sys} B}$

- A good UHF receiver will have a $T_{sys} \approx 125$ K.
- B is the receiver bandwidth.

Doppler shift from electron thermal motion:

$$\Delta f = \frac{2}{c} f_{\mathrm{Tx}} v \approx \frac{2}{c} f_{\mathrm{Tx}} \sqrt{\frac{k_B T_e}{m_e}}$$

Let's assume we need to capture $B = 4\Delta f$ to get the full spectrum. For $f_{\mathrm{Tx}} = 450$ MHz and $T_e = 1000$ K:

$$B = 1.48 \text{ MHz} \Rightarrow \boxed{P_N = 2.55 \times 10^{-15} \text{ W}}$$

Electrons control bandwidth
(no collective effects)

What if instead the bandwidth is related to the ion motion?

$$v_i = \sqrt{\frac{m_e}{m_i}} v_e \Rightarrow v_i = 5.83 \times 10^{-3} v_e \text{ for } \mathrm{O}^+$$

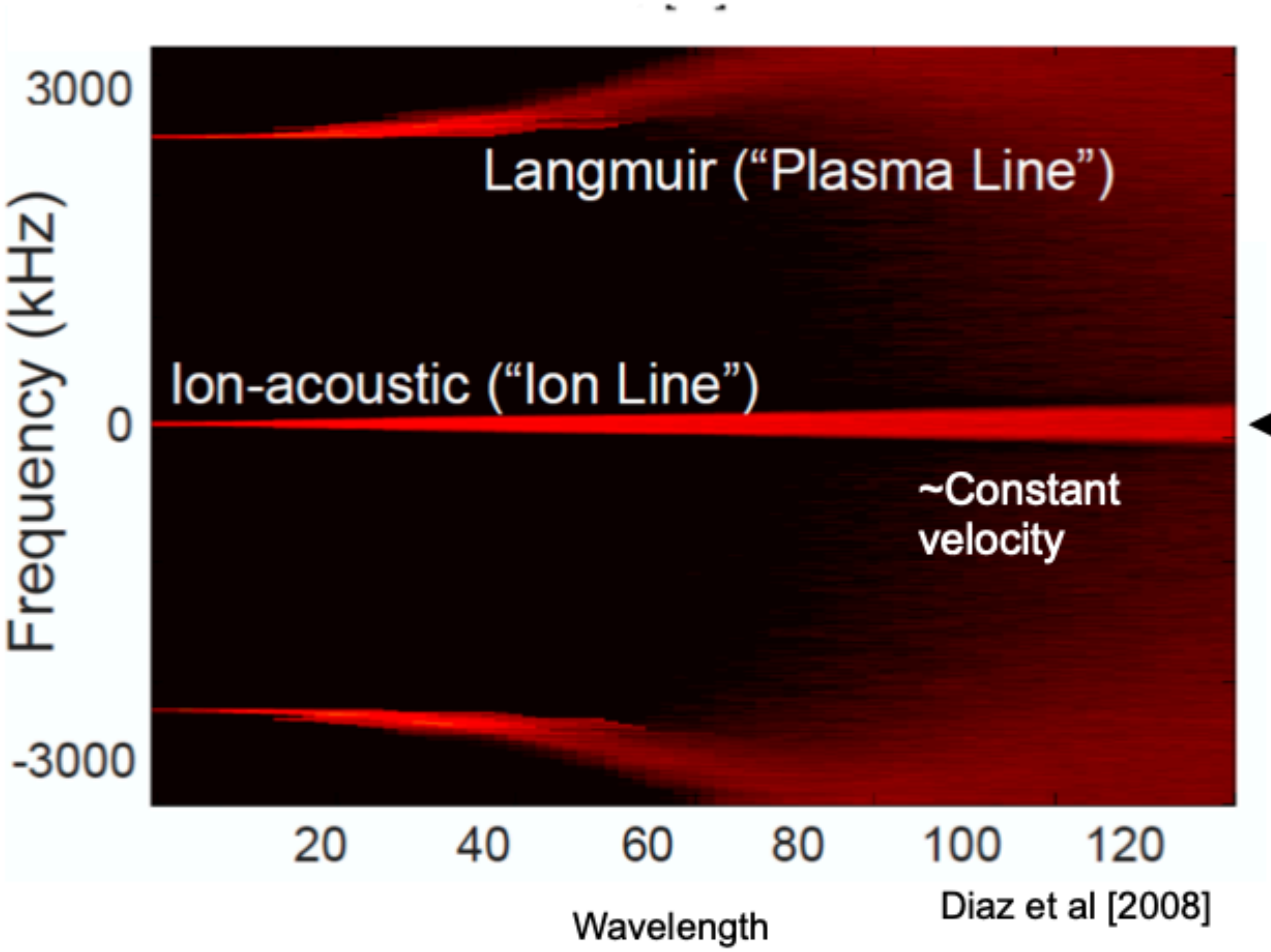
The same numbers would yield

$$B = 8.63 \text{kHz} \Rightarrow \boxed{P_N = 1.48 \times 10^{-17} \text{ W}}$$

Ions control bandwidth
(collective effects)

Spectrum of Density Fluctuations

- Thermal plasmas are filled with ambient density fluctuations.
- The spectra of the ambient fluctuations peak around ω, k pairs that satisfy a dispersion relation for a plasma normal mode.
- An ISR would pick out one slice of this spectrum at $k = k_b$.



$$\frac{\omega}{k} = \sqrt{\frac{k_B T_e + \gamma_i k_B T_i}{m_i}} = V_s$$

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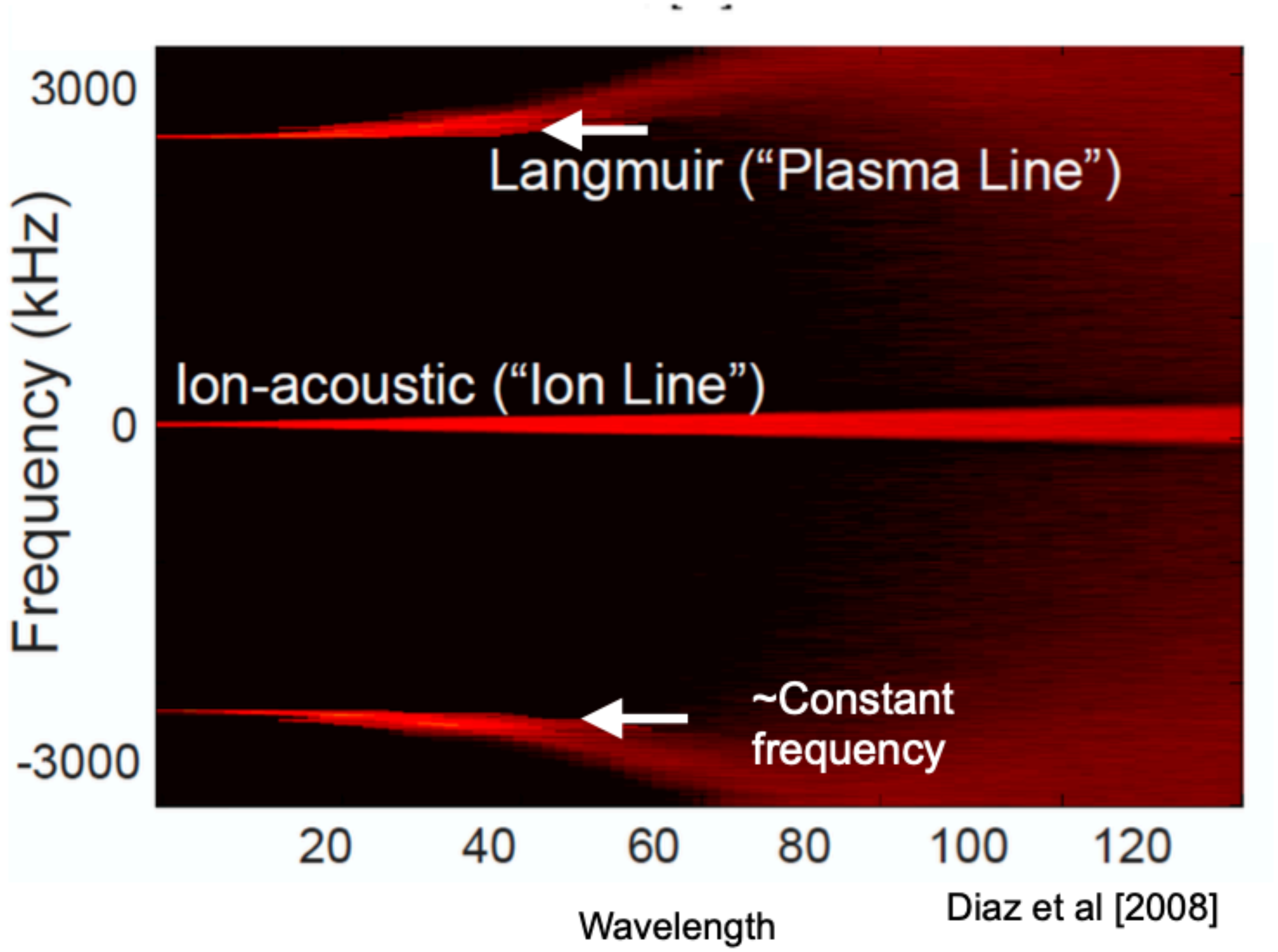
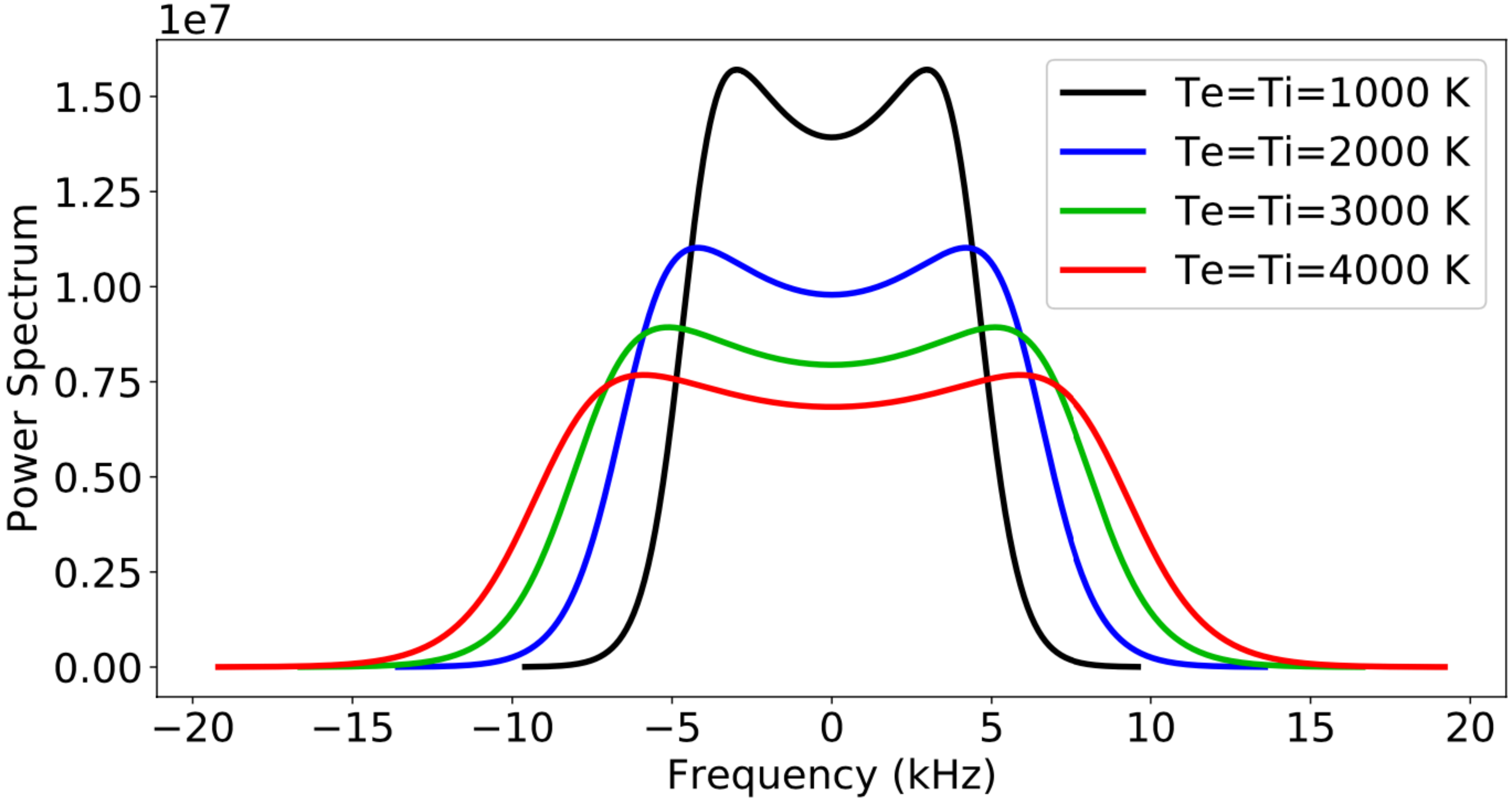


$$\omega^2 = \omega_p^2 + \frac{3}{2}k^2 v_{th}^2$$

$$v_{th}^2 = 2k_B T_e / m_e$$

Temperature Effects ($T_e/T_i = 1$)



$$f = 449.3 \text{ MHz} \quad N_e = 3 \times 10^{11} \text{ m}^{-3} \quad m_i = 16 \text{ amu}$$