Radar Signal Processing: Part 1

Basic Operation of a Pulse-Doppler Radar

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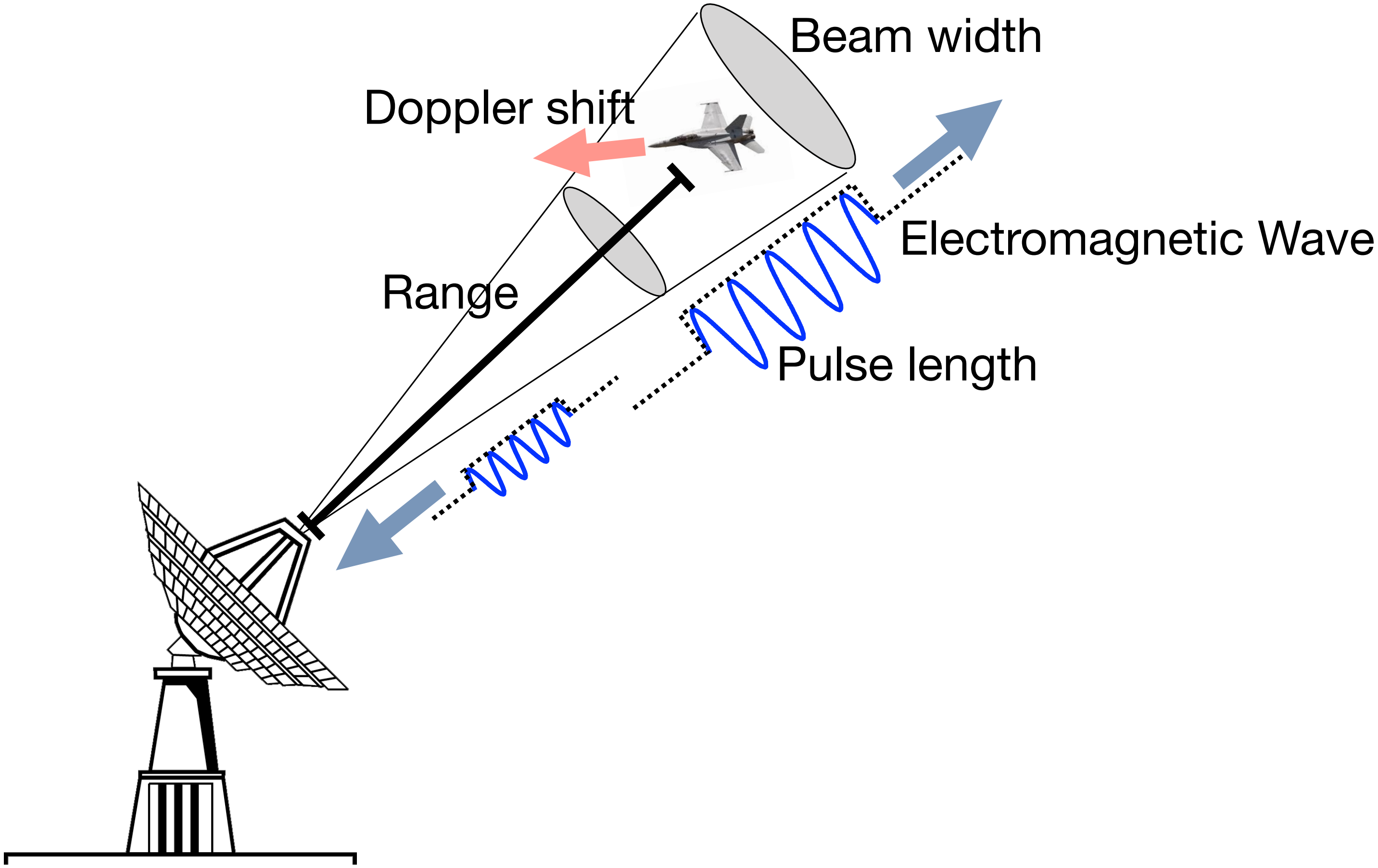
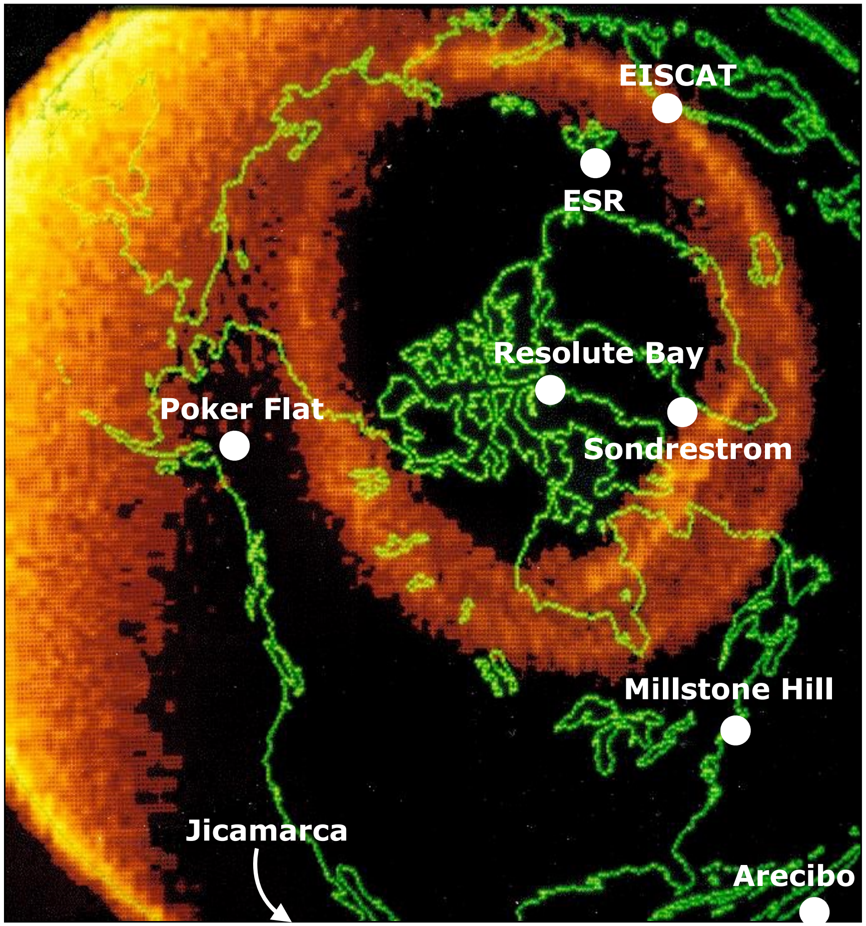
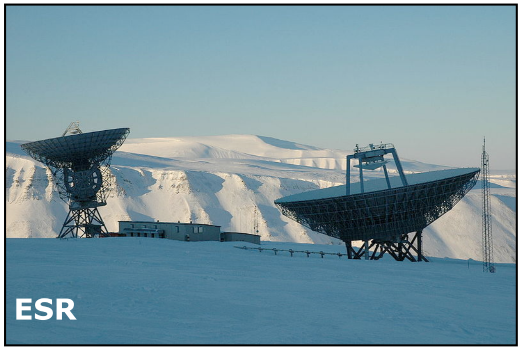
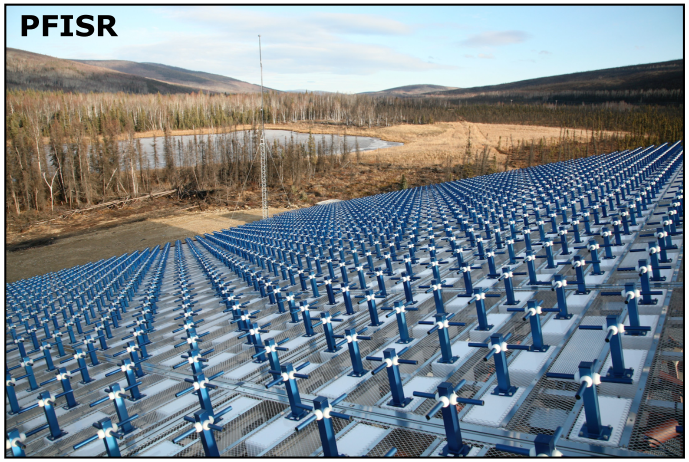
Pulse Doppler Radar

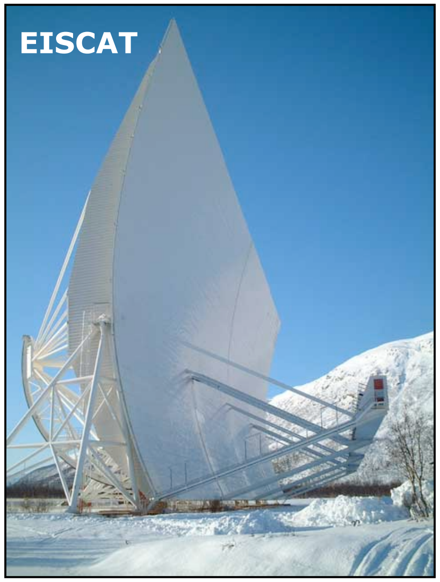


Incoherent Scatter Radar (ISR)









Traveling Waves

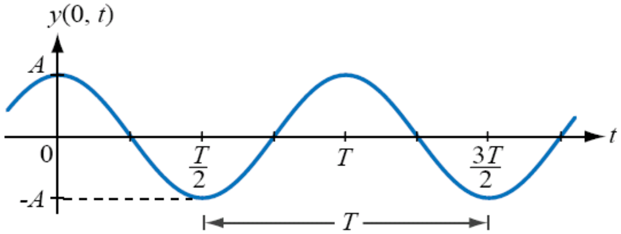
Traveling wave, 1D: $y(x,t) = A\cos(\omega t - kx)$

Angular velocity (radians/s): $\omega = 2\pi f = 2\pi/T$

Wave number (spatial frequency): $k = 2\pi/\lambda$

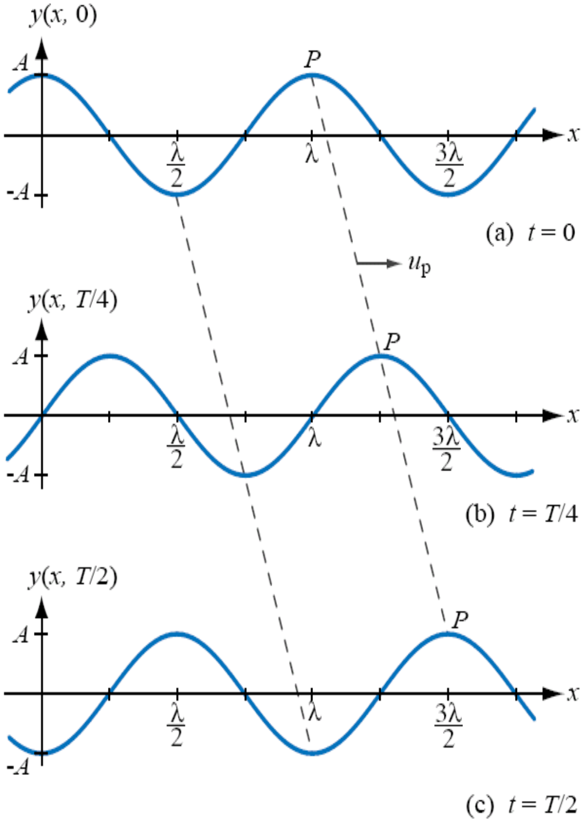
Phase velocity (c in a vacuum): $u_p = \omega/k$

Temporal variation at point in space:



(b) $y(x, t)$ versus t at $x =$

Three snapshots in time:



(a) $t = 0$

(b) $t = T/4$

(c) $t = T/2$

-12

The velocity of a point on the wave is found by setting $\omega t - kx$ = constant. By taking the time derivative we obtain the ***phase velocity,***

$$u_p = \frac{dx}{dt} = \frac{\omega}{k}$$

The functional relationship between ω and k is called a ***dispersion relation.*** It appears ubiquitously in the study of wave phenomena

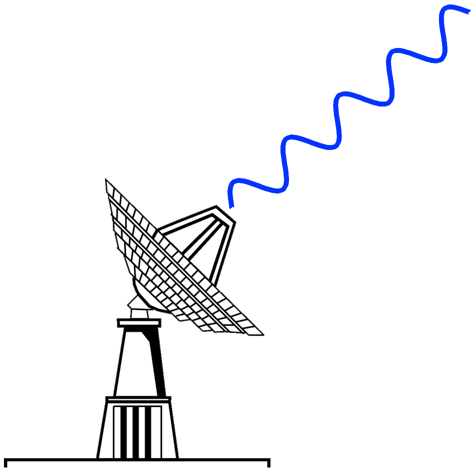
The simplest dispersion relation for an EM wave describes its propagation through free space,

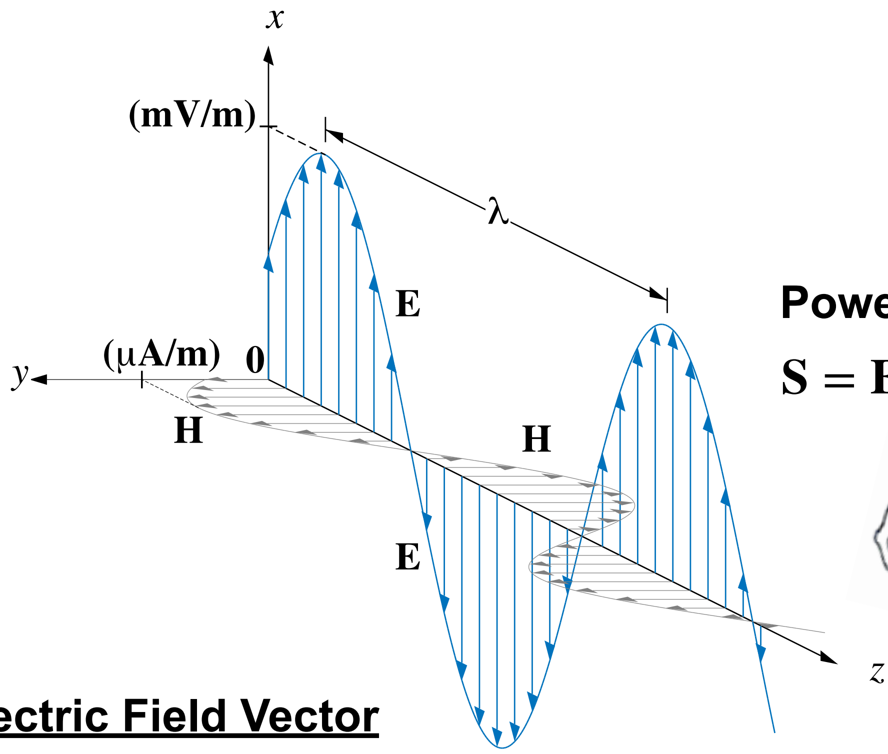
$$\omega = ck$$

where $c = 3 \times 10^8$ m/s. We will encounter more complicated dispersion relations soon!

Transverse Electromagnetic (TEM)

Radars transmit TEM waves and measure the scattered radiation from a target



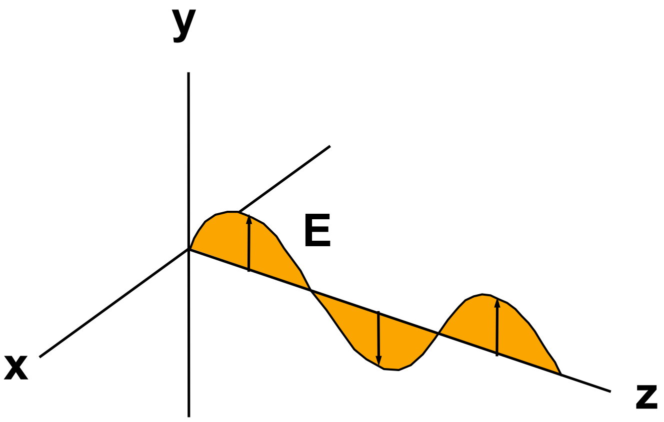


Power flow:

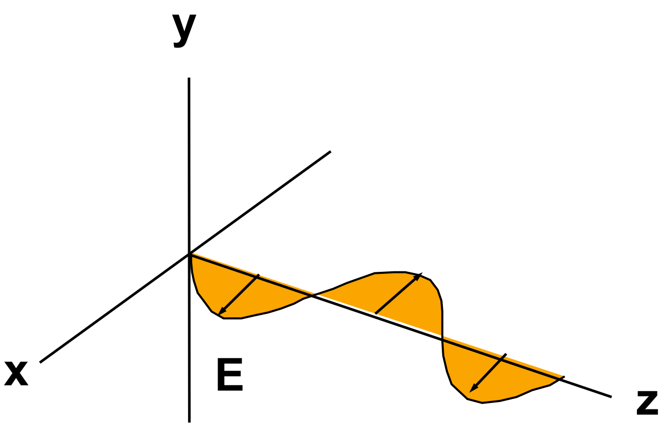
$S = E \times H$ W/m^2

Polarization: Orientation of the Electric Field Vector

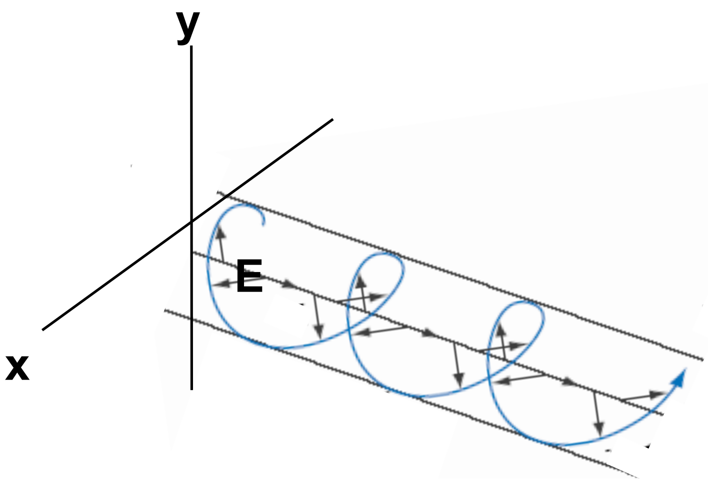
Vertical Polarization



Horizontal Polarization



Circular Polarization

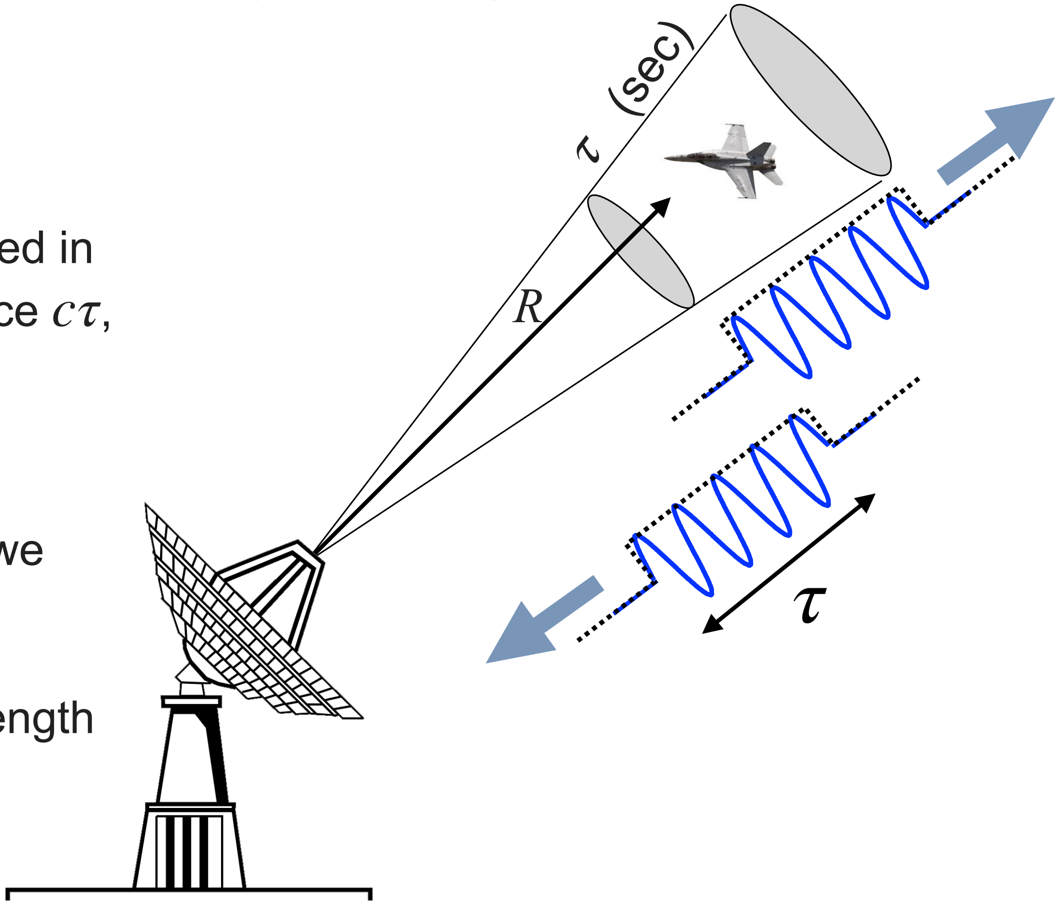


Range

Range R to the target is measured by transmitting a pulse of electromagnetic waves, and measuring the time Δt between transmission and reception of the pulse,

$$R = \frac{c\Delta t}{2}$$

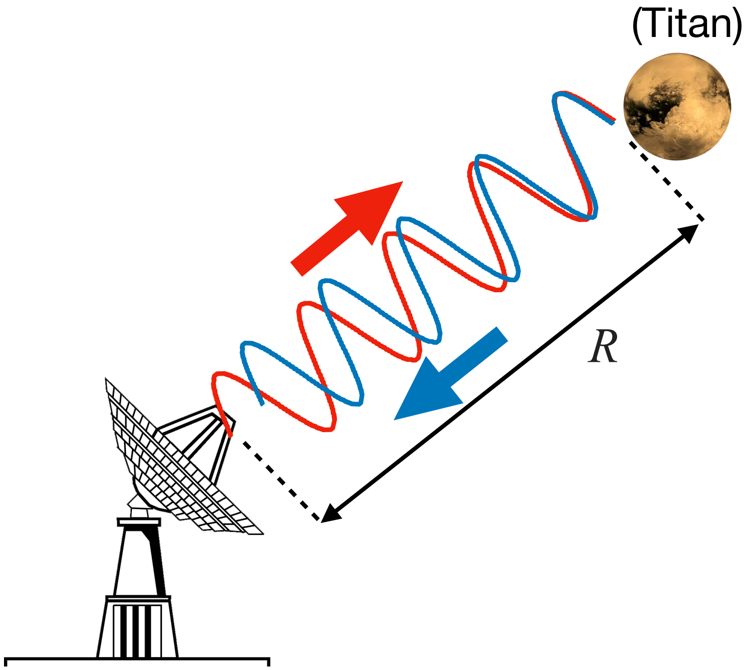
The ***pulse length*** τ is most often expressed in units of time, and corresponds to a distance $c\tau$, where $c = 3 \times 10^8$ m/s



Range resolution depends on how well we can resolve Δt. For the case of a simple on-off pulse, the optimal approach is to match the sampling period to the pulse length (the so-called "matched filter" approach).

Range resolution for a simple on-off pulse ("uncoded pulse") is controlled by τ. Shorter τ yields higher range resolution. But a shorter pulse also carry less total energy, and so the reflected signal is more difficult to discriminate from background noise.

Measuring Velocity



Assume a transmitted signal: $\cos(2\pi f_o t)$

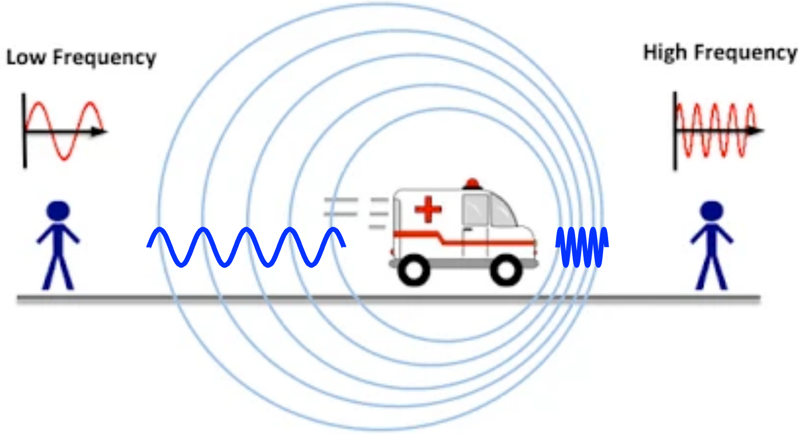
After return from target: $\cos\left[2\pi f_o\left(t+\frac{2R}{c}\right)\right]$

Now let us allow range *R* to vary with time. Let's assume the target moves at a constant velocity, with positive away from the radar and negative toward the radar:

$$R = R_o + v_o t$$

Substituting we obtain:

$$\cos\left[2\pi\left(f_o + \underbrace{f_o\frac{2v_o}{c}}_{-f_D}\right)t + \underbrace{\frac{2\pi f_o R}{c}}_{\text{constant}}\right]$$

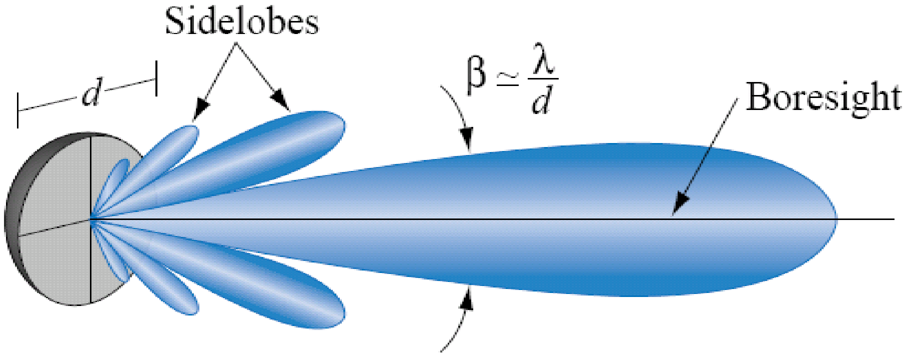


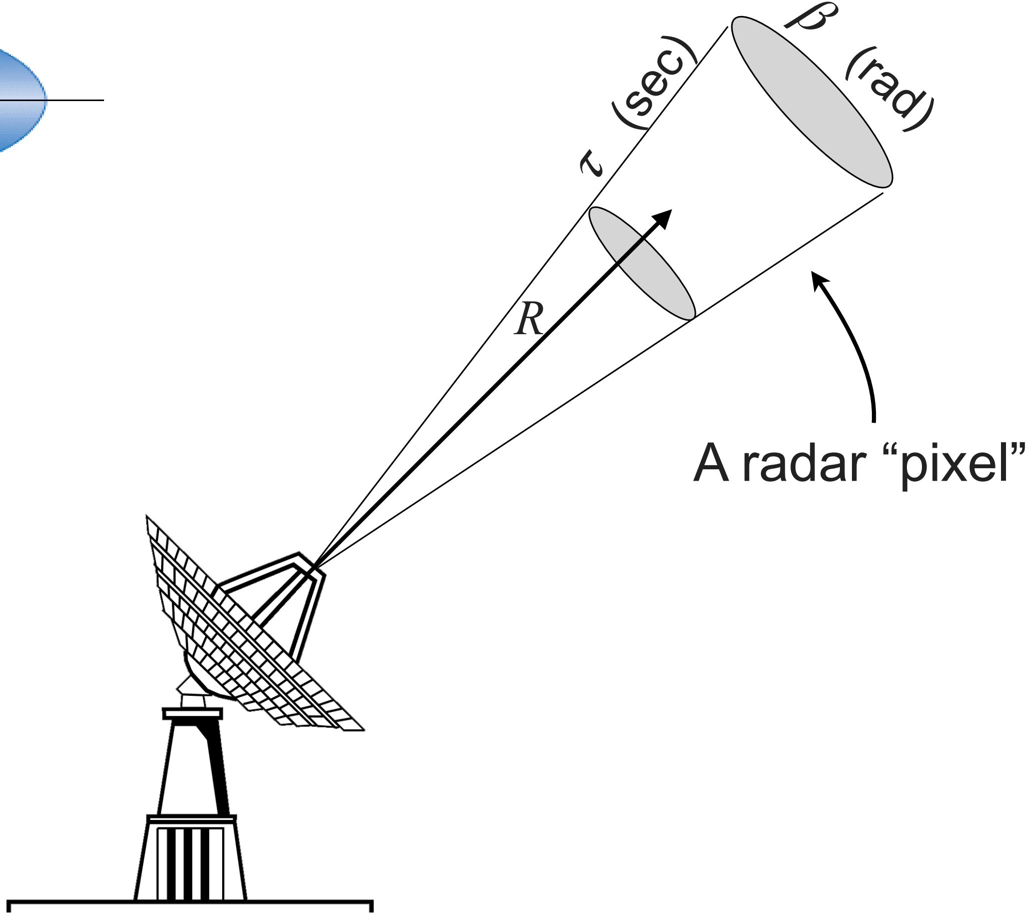
The shift in frequency caused by a moving target is proportional to the *component* of the velocity vector along the radar line of sight:

$$f_D = -\frac{2f_o}{c}v_o$$

How we determine f_D is left for other lectures.

Cross-range resolution (beam width)





The cross-range resolution is usually defined by the angular width of the main lobe of the antenna's power pattern. For a dish antenna this is approximately equal to the ration of the wavelength to the physical diameter,

$$\beta = \frac{\lambda_o}{d} \qquad \text{(radians)}$$

Millstone Hill ISR has a 46-m dish operating at a frequency of 440 MHz, or $\lambda = 0.68$ m, giving a beam width of $\beta \simeq 0.85^\circ$.

Doppler Radar Summary: “Coherent” hard targets

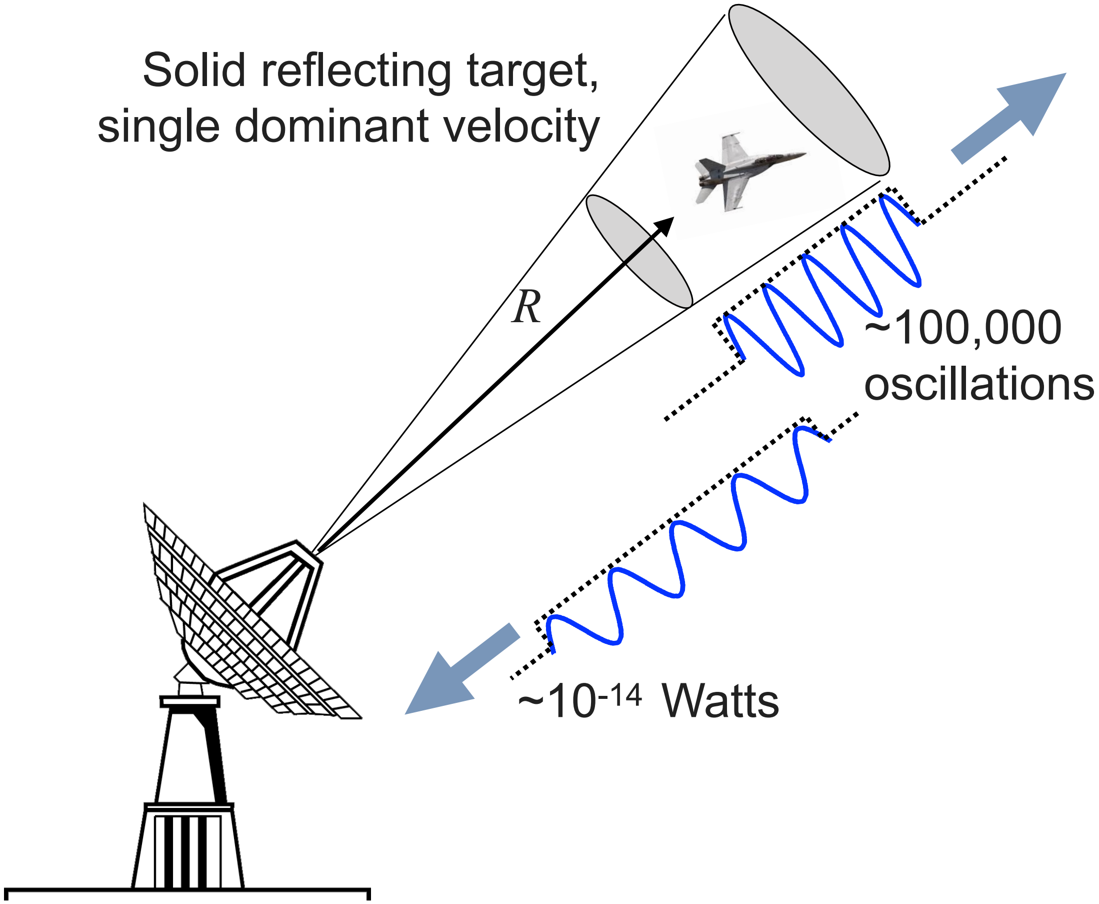
Two key concepts:

Time ⟺ Distance

$$R = -\frac{c\Delta t}{2}$$

Frequency ⟺ Velocity

$$f_D = -\frac{2f_o}{c}v_o$$



A Doppler radar measures backscattered power as a function range and velocity.
Velocity is manifested as a Doppler frequency shift in the received signal.

What happens when we have multiple targets in the radar volume, moving at different velocities?

Doppler Radar Summary: Distributed “Incoherent” Targets

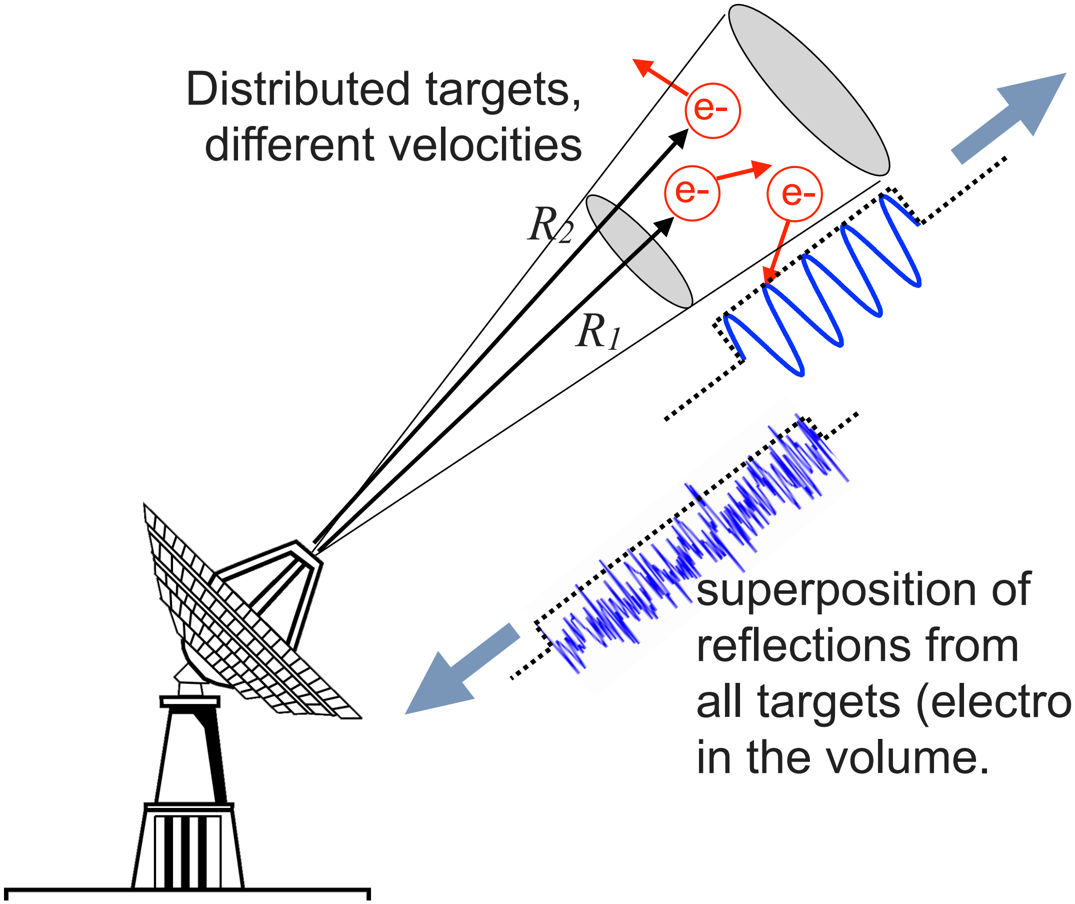
Two key concepts:

Time ⟺ Distance

$$R = -\frac{c\Delta t}{2}$$

Frequency ⟺ Velocity

$$f_D = -\frac{2f_o}{c}v_o$$



Distributed targets, different velocities

superposition of reflections from all targets (electrons) in the volume.

A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

What happens when we have multiple targets in the radar volume, moving at different velocities?

Radar Signal Processing: Part 2

Doppler Spectrum, Bragg Scatter, and the ISR Power Spectrum

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Doppler Radar Summary: "Coherent" hard targets

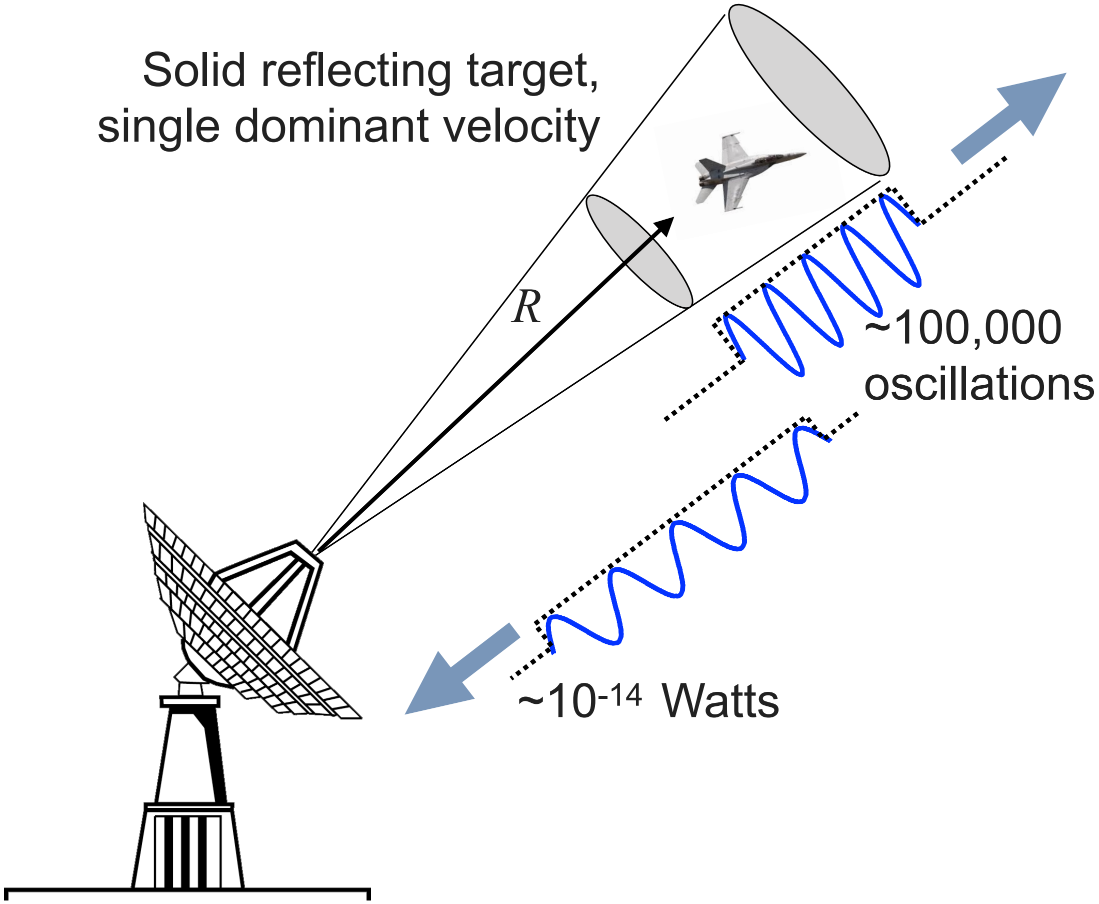
Two key concepts:

Time ⟺ Distance

$$R = -\frac{c\Delta t}{2}$$

Frequency ⟺ Velocity

$$f_D = -\frac{2f_o}{c}v_o$$



A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

What happens when we have multiple targets in the radar volume, moving at different velocities?

Concept of a “Doppler Spectrum”

Superposition of targets moving with different velocities within the radar volume

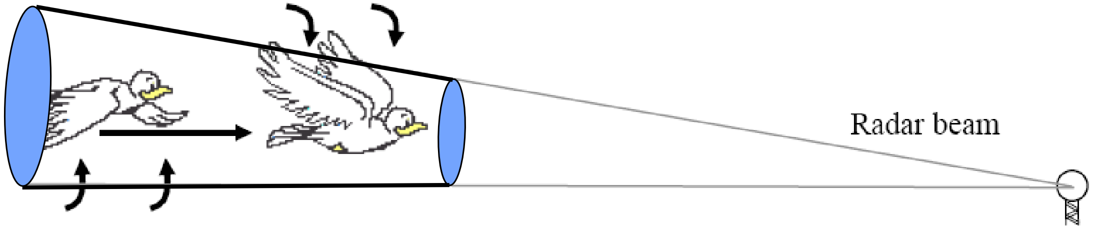
Two key concepts:

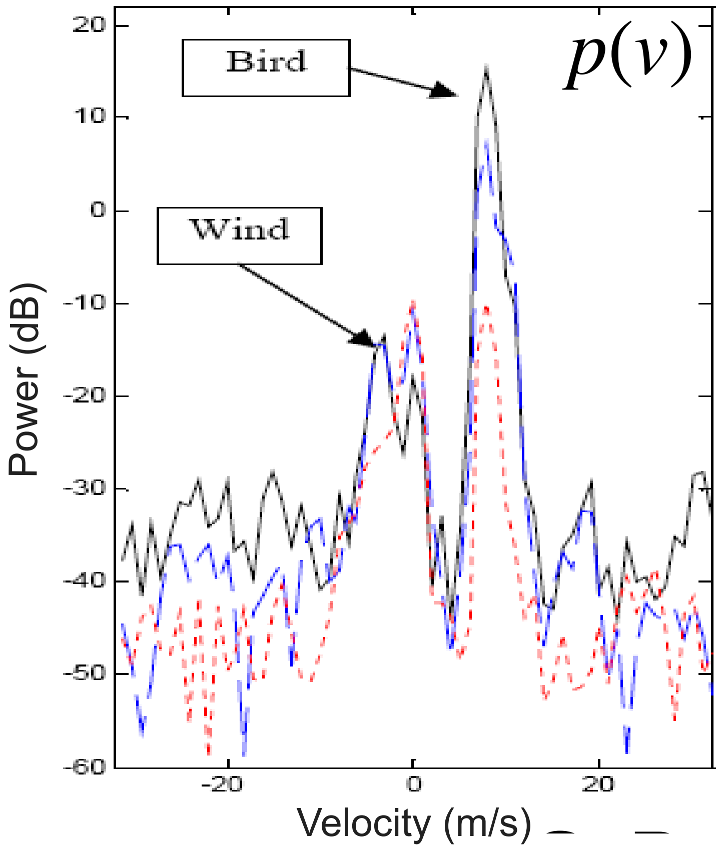
Time ⇔ Distance

$$R = -\frac{c\Delta t}{2}$$

Frequency ⇔ Velocity

$$f_D = -\frac{2f_o}{c}v_o$$





Processing: $p(R, f_D) \rightarrow p(v)$

If there is a distribution of targets withdifferent velocities (e.g., bird, flapping wings,wind) then there is no single Doppler shift but, rather, a Doppler spectrum.

Distributed “beam filling” Target

A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

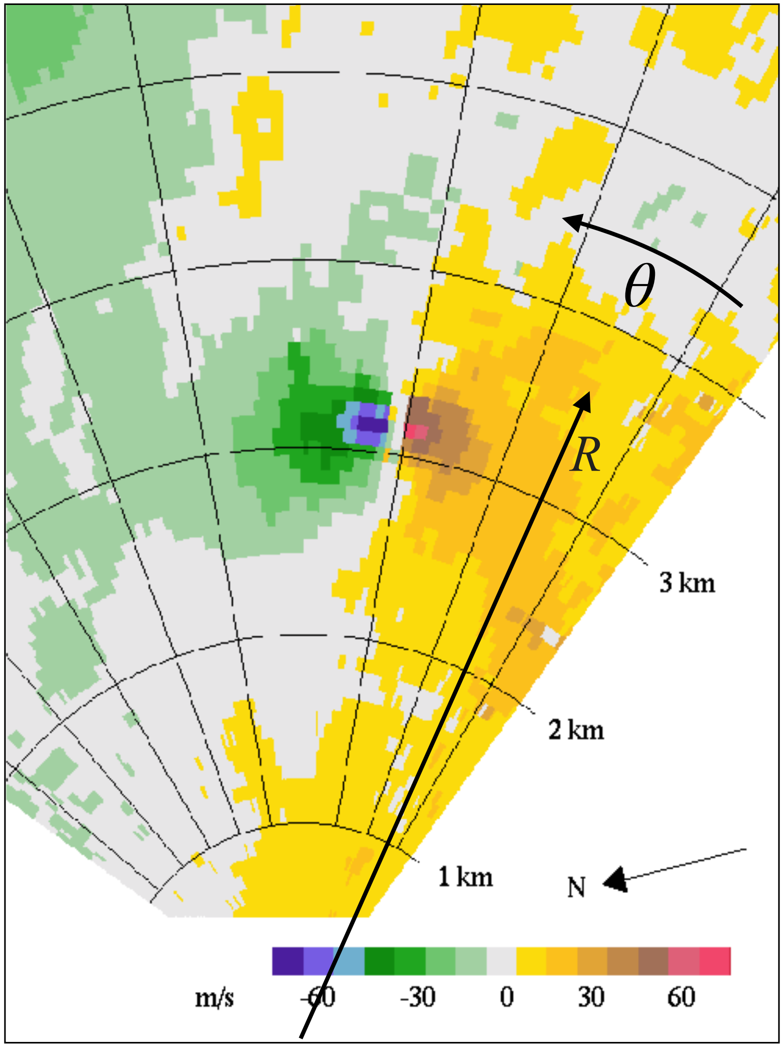
Two key concepts:

Time ⇔ Distance

$$R = -\frac{c\Delta t}{2}$$

Frequency ⇔ Velocity

$$f_D = -\frac{2f_o}{c}v_o$$

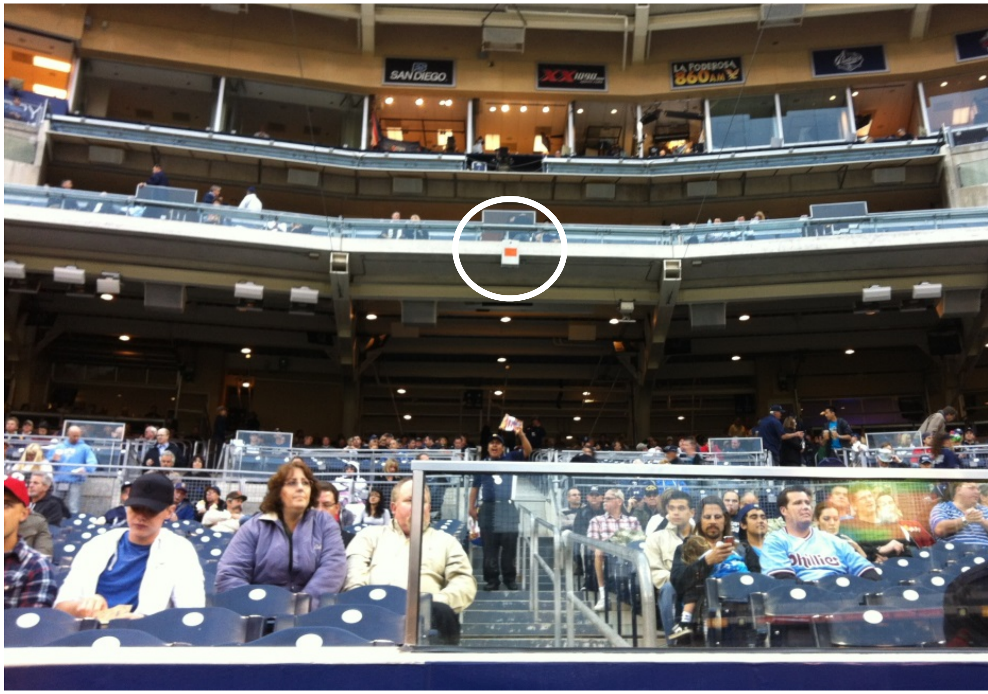


Processing:

$$p(R, f_D, t) \rightarrow f_D(R, t) \rightarrow v(R, \theta)$$

For a beam-filling target (like water droplets in a tornado), the radar can be used to construct insightful images of velocity relative to the radar.

Micro-Doppler Analysis

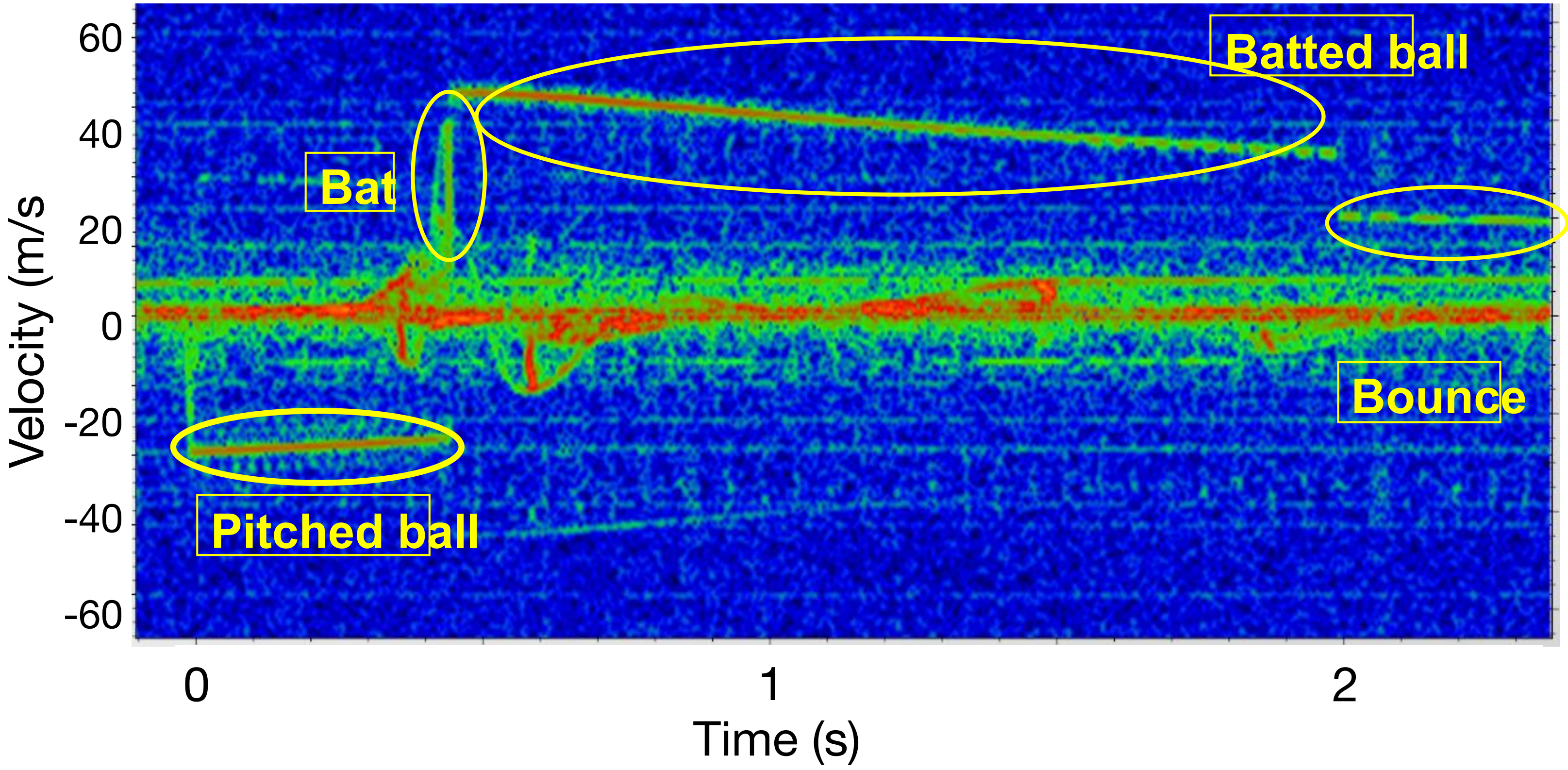


Trackman radar: “continuous wave” (CW) radar: precise Doppler but no range information.

Can identify targets and actions based on Doppler signatures!

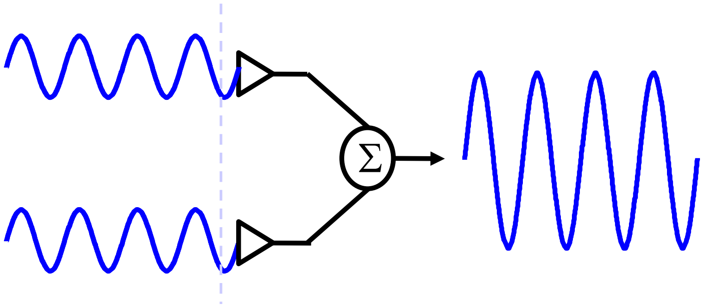
Processing:

$$p(f_D, t) \rightarrow p(v, t)$$

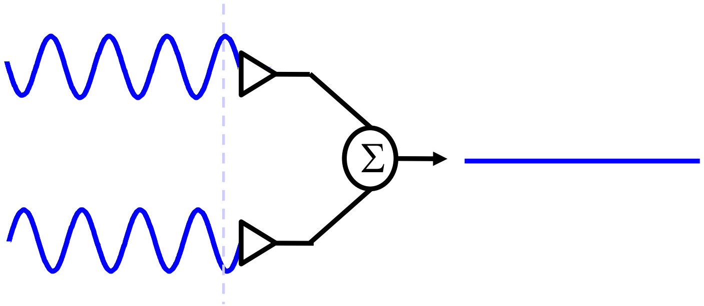


Wave Interference and Bragg Scatter

Consider two waves with the same frequency but different phase.

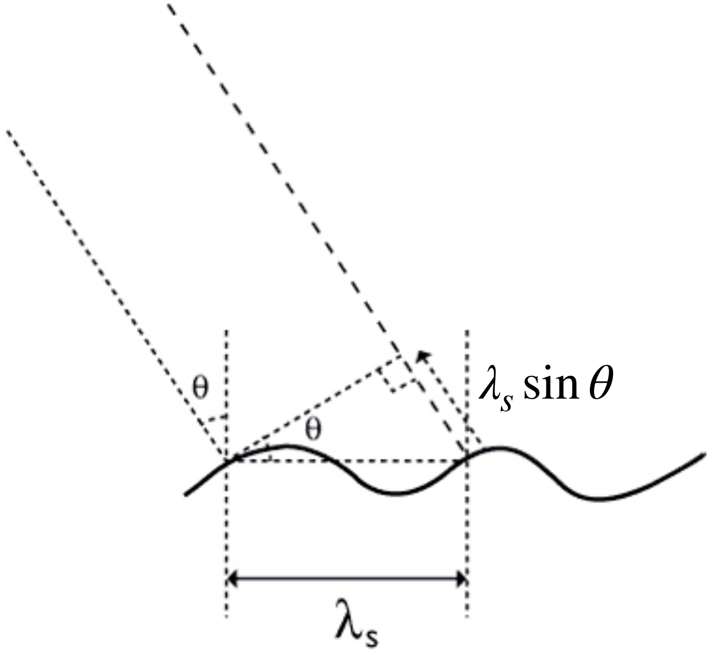


Constructive
(in phase)



Destructive
(180°   out of phase)

Consider a wave along the interface between a dielectric and a conducting (reflective) medium, as depicted below. This is representative of an air-ocean boundary.

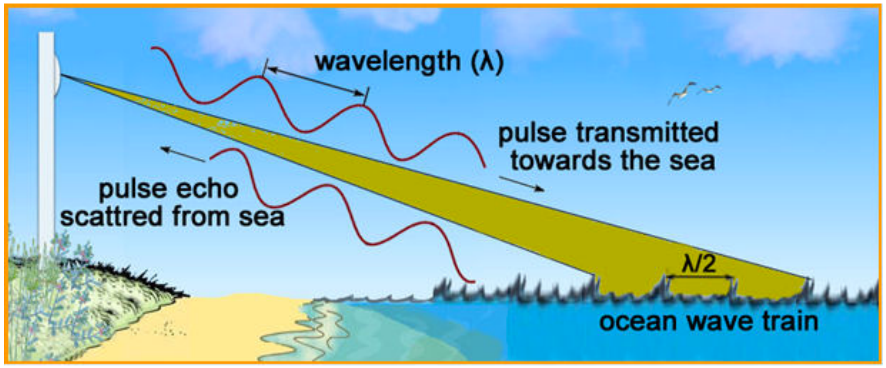


Suppose waves are observed at angle θ using a radar with wavelength λ_o. The condition for maximum constructive interference is

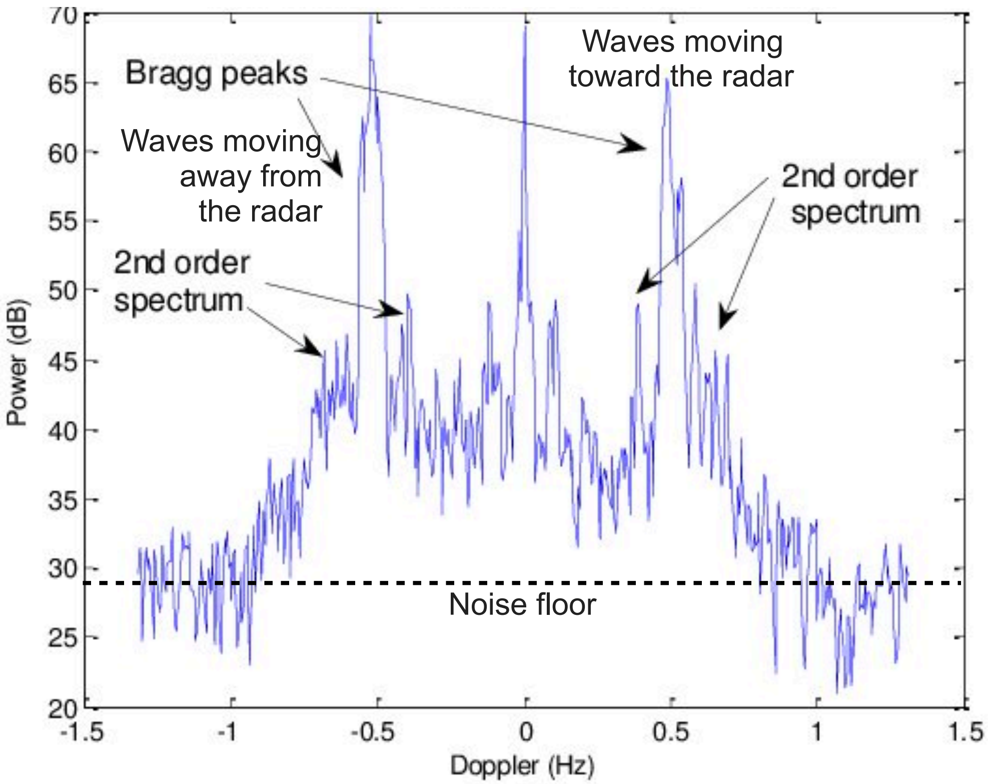
$$n\lambda_0 = 2\lambda_s \sin\theta$$

If $\theta = 90^\circ$ (or if these waves are propagating isotropically), then the Bragg condition is met for $n\lambda_0 = 2\lambda_s$

Doppler spectrum of ocean waves



Backscatter from the ocean at low aspect angle shows peaks in the Doppler spectrum from the subset of waves matching the Bragg condition for the radar (spacing ≃ half the radar wavelength)



Important points:

The target is distributed over the entire radar beam width.
The scattering is from free electrons in the conducting sea water.
The Doppler spectrum has peaks due to Bragg scatter from waves in the medium.
The frequency of the peaks tells us the velocity and direction of the waves.
The height of the peaks tells us something about the amplitude and density of the waves.
The width of the peaks tells us something about the spread in velocity of the waves

Doppler spectrum of the ionosphere

Let's put this all together for the ionosphere. The two predominant longitudinal modes in a thermal plasma:

Ion-acoustic mode:

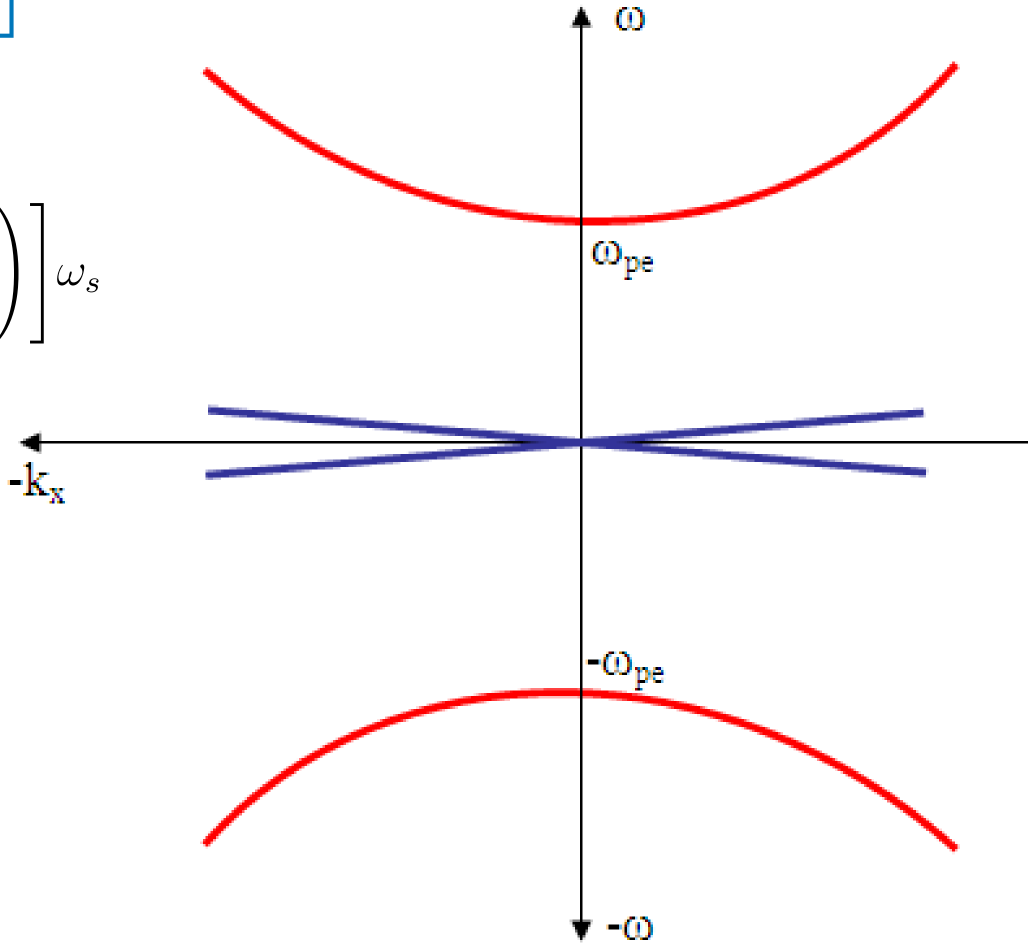
$$\boxed{\omega_s = C_s k \qquad C_s = \sqrt{k_B(T_e + 3T_i)/m_i}}$$

Langmuir mode:

$$\omega_{si} = -\sqrt{\frac{\pi}{8}}\left[\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \left(\frac{T_e}{T_i}\right)^{\frac{3}{2}} \exp\left(-\frac{T_e}{2T_i} - \frac{3}{2}\right)\right]\omega_s$$

$$\omega_L = \sqrt{\omega_{pe}^2 + 3\,k^2\,v_{the}^2} \approx \omega_{pe} + \frac{3}{2}v_{the}\lambda_{De}k^2$$

$$\omega_{Li} \approx -\sqrt{\frac{\pi}{8}}\frac{\omega_{pe}^3}{k^3}\frac{1}{v_{the}^3}exp\left(-\frac{\omega_{pe}^2}{2k^2v_{the}^2} - \frac{3}{2}\right)\omega_L$$



Computer simulation of the ionosphere

Simple rules yield
complex behavior

Particle-in-cell (PIC) simulation:

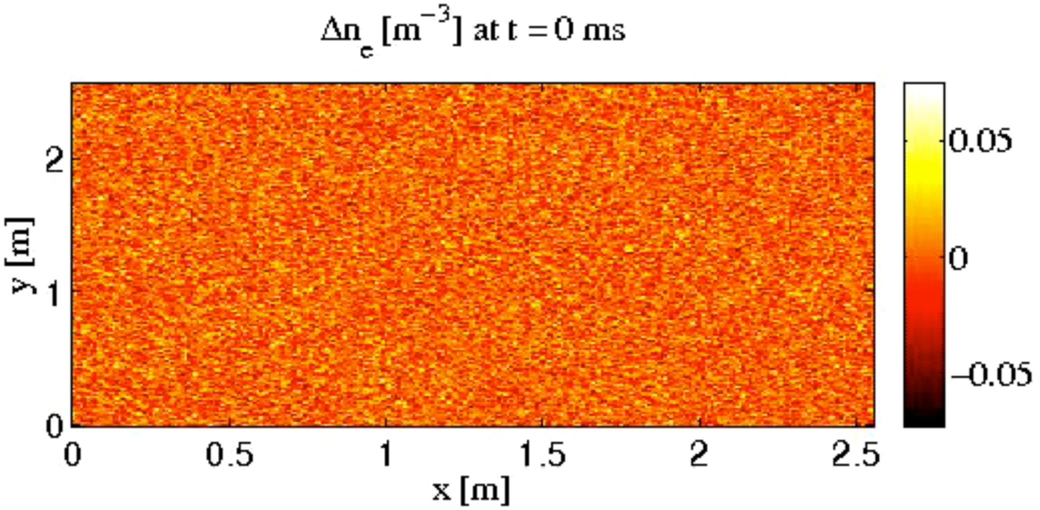
$$\frac{d\,\mathbf{v}_i}{d\,t} = \frac{q_i}{m_i}(\mathbf{E}(\mathbf{x}_i) + \mathbf{v}_i \times \mathbf{B}(\mathbf{x}_i))$$

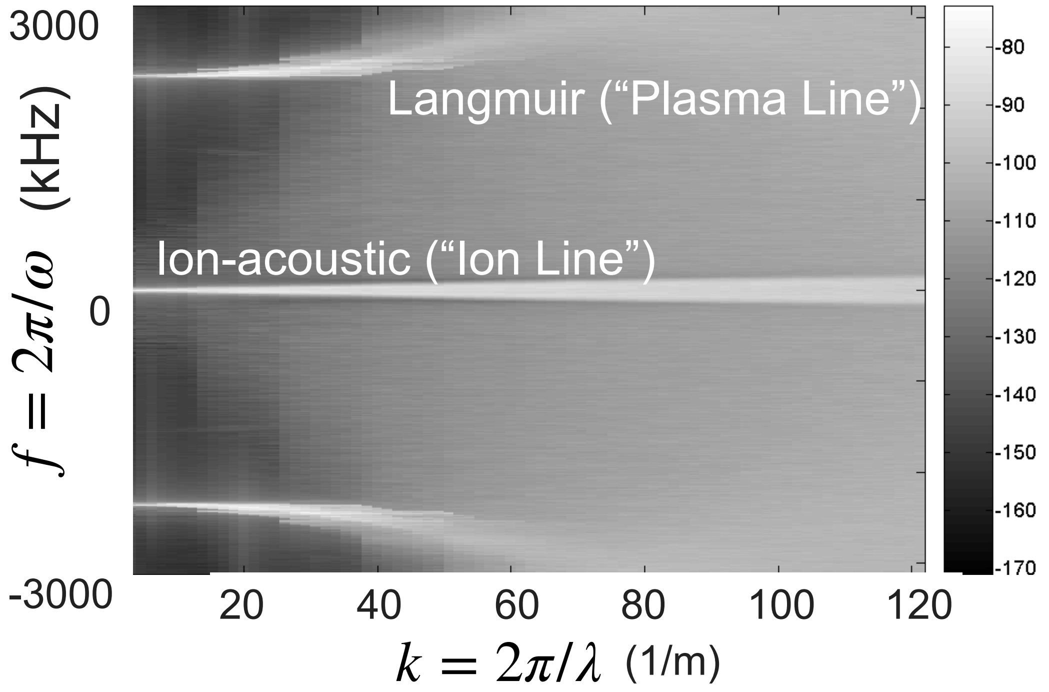
$$\nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

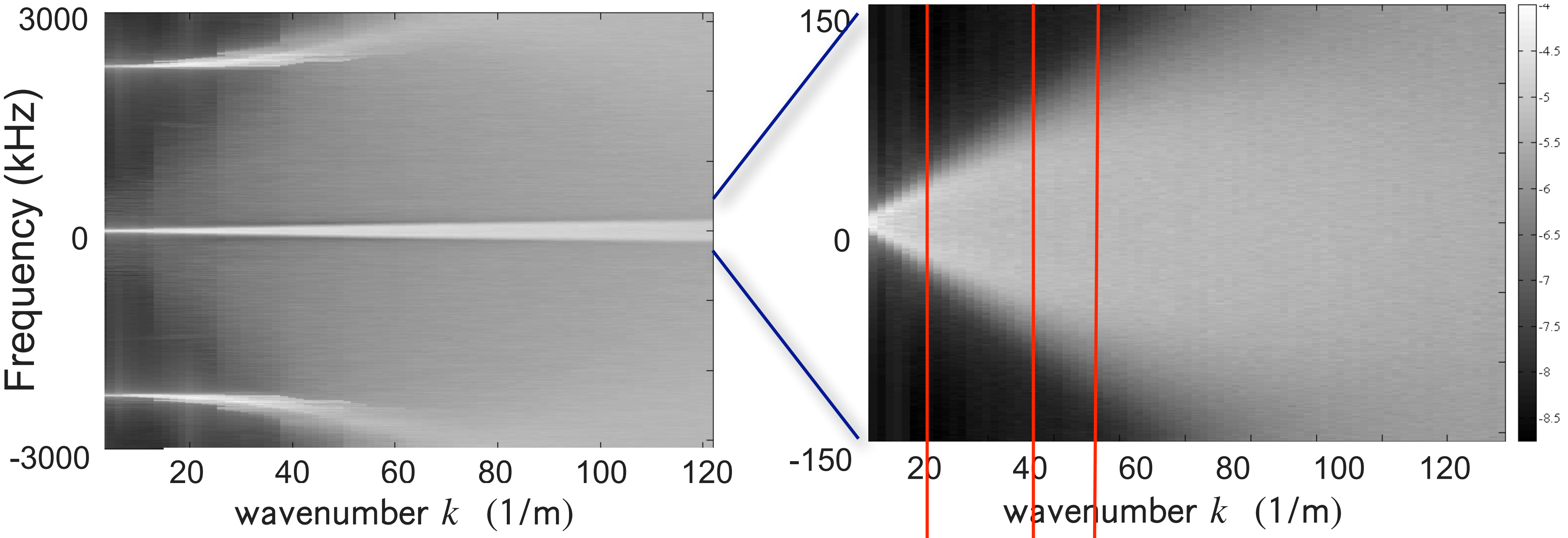
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$





ISR measures a cut through this surface

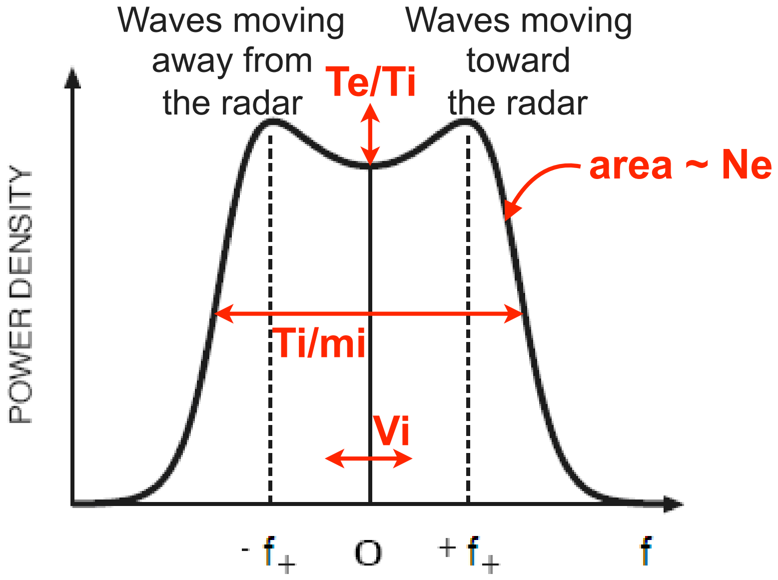


Sondrestrom (1.2 GHz)

EISCAT UHF (930 MHz)

AMISR (450 MHz),
Millstone (440 MHz)

Ion-acoustic “lines” are broadened by Landau damping



Doppler Radar: “Incoherent” Distributed Target

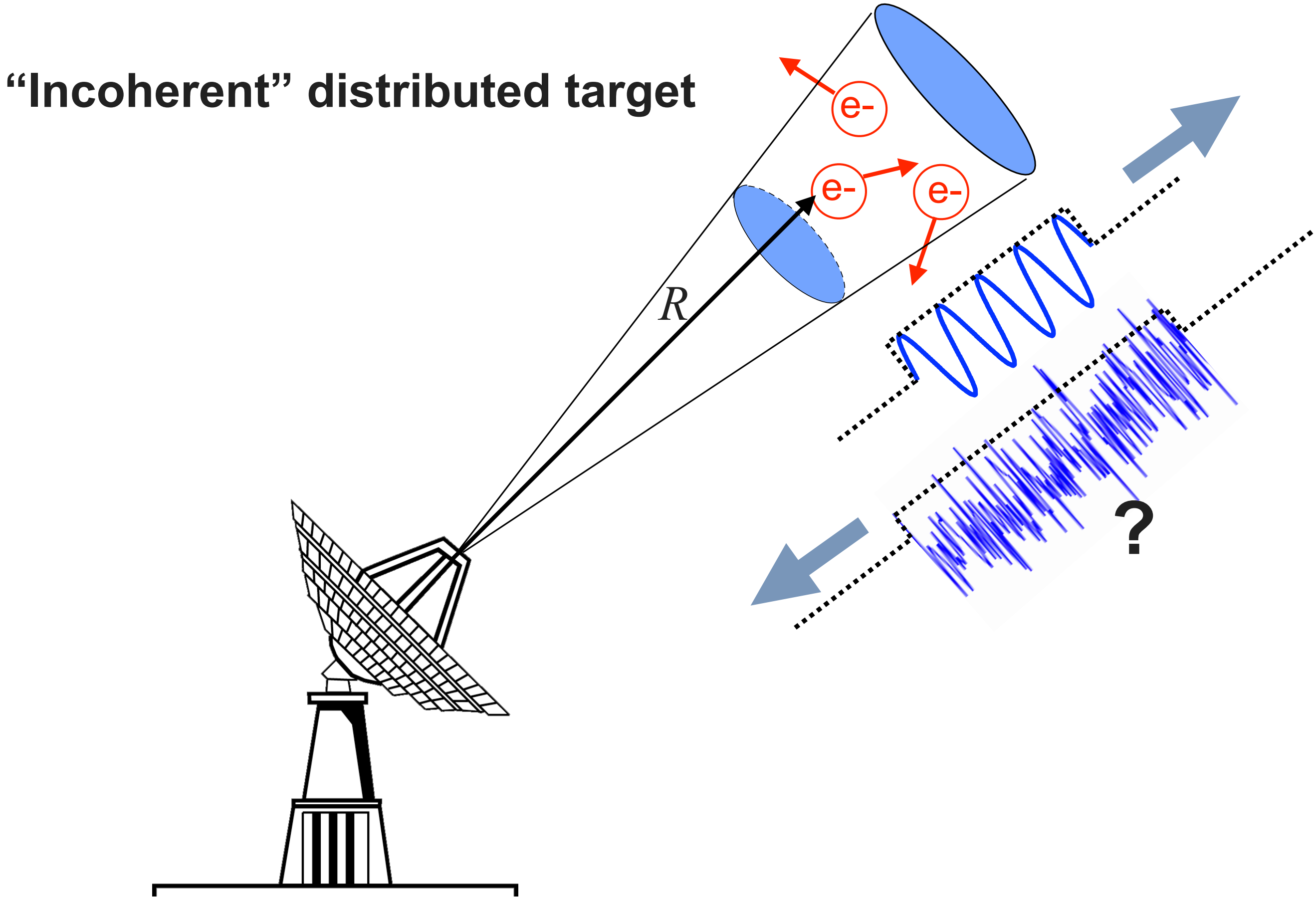
Two key concepts:

Distant ⟺ Time

$$R = -\frac{c\Delta t}{2}$$

Velocity ⟺ Frequency

$$f_D = -\frac{2f_o}{c}v_o$$

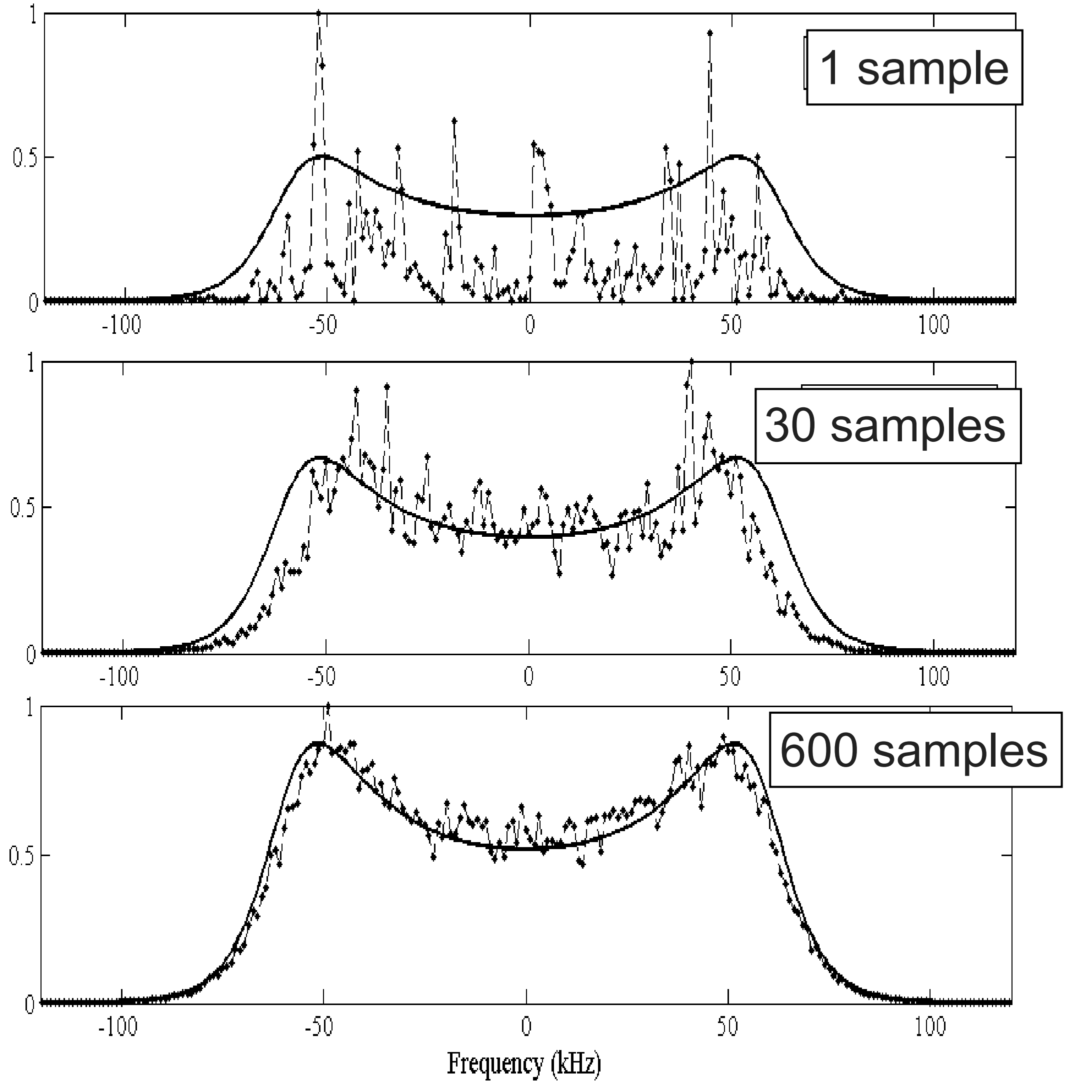


A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

What happens when we have multiple targets in the radar volume, moving at different velocities?

Incoherent Averaging

Normalized ISR spectrum for different integration times at 1290 MHz



We are seeking to estimate the power spectrum of a Gaussian random process. This requires that we sample and average many independent “realizations” of the process.

$$\rho_e \sim \frac{1}{K}\left(1+\frac{1}{SNR}\right)$$

$\rho_e =$ Mean Square Error
$K =$ number of samples
$SNR =$ per-pulse Signal-to-Noise Ratio

Radar Signal Processing: Part 3

I-Q demodulation: Time-domain perspective

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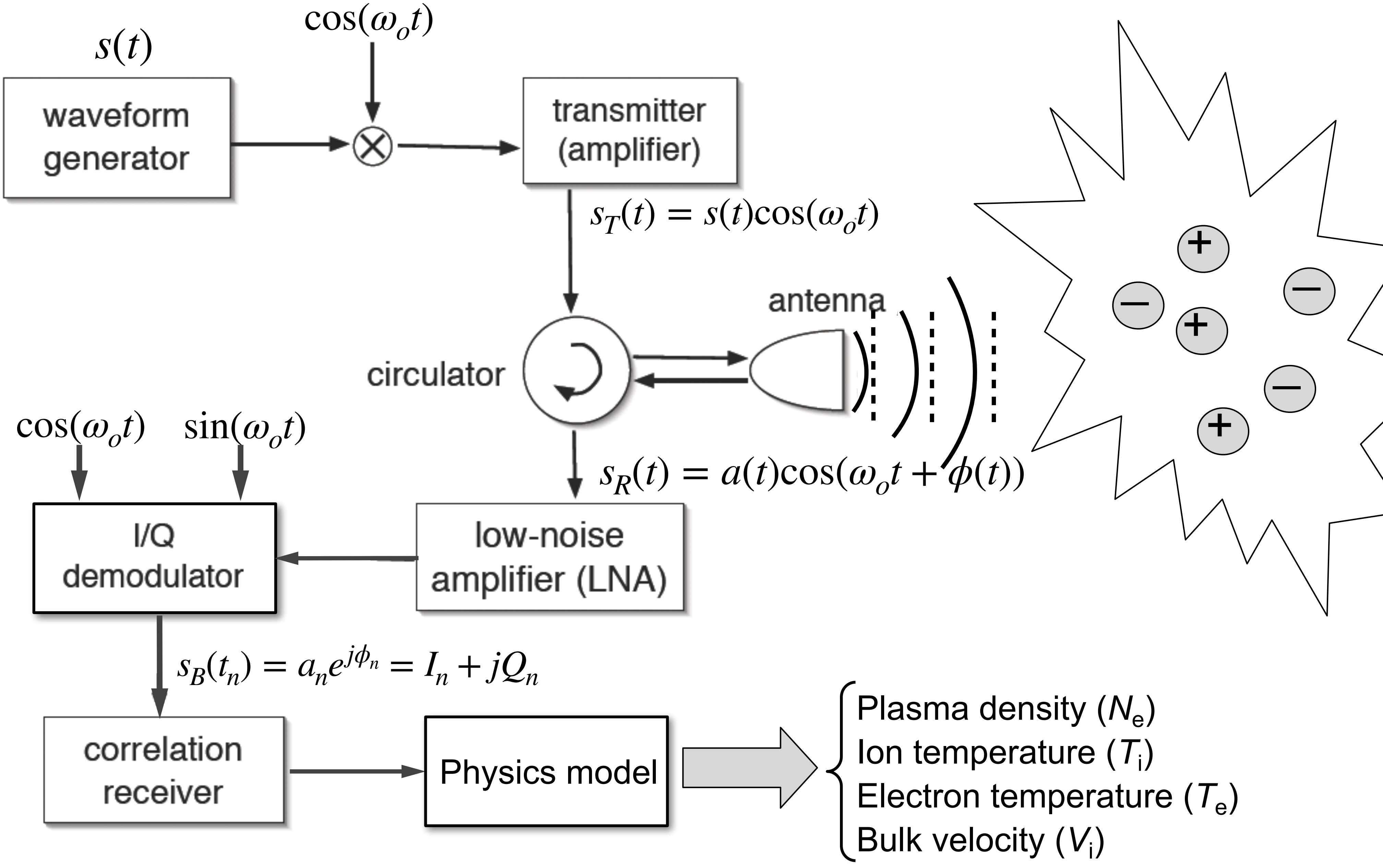
Components of a Pulsed Doppler Radar



$s(t)$

waveform generator

$\cos(\omega_o t)$

transmitter (amplifier)

$s_T(t) = s(t)\cos(\omega_o t)$

circulator

antenna

$s_R(t) = a(t)\cos(\omega_o t + \phi(t))$

$\cos(\omega_o t)$ $\sin(\omega_o t)$

I/Q demodulator

low-noise amplifier (LNA)

$s_B(t_n) = a_n e^{j\phi_n} = I_n + jQ_n$

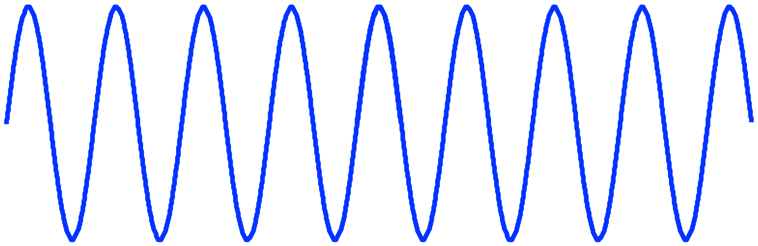
correlation receiver

Physics model

- Plasma density (N_e)
- Ion temperature (T_i)
- Electron temperature (T_e)
- Bulk velocity (V_i)

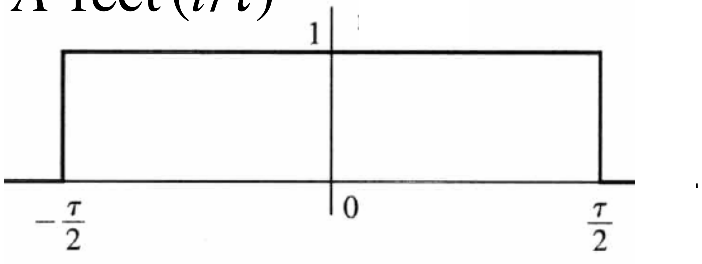
A Simple Radar Pulse

$\cos(\omega_o t)$



X

$s(t) = A\ \mathrm{rect}\,(t/\tau)$



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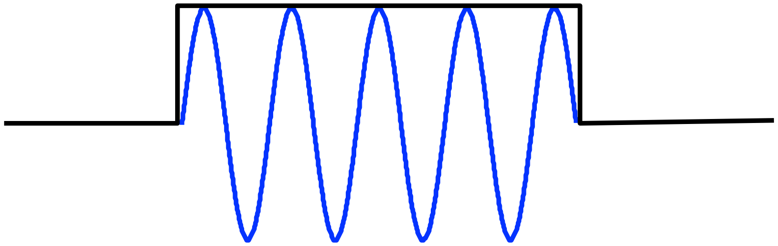
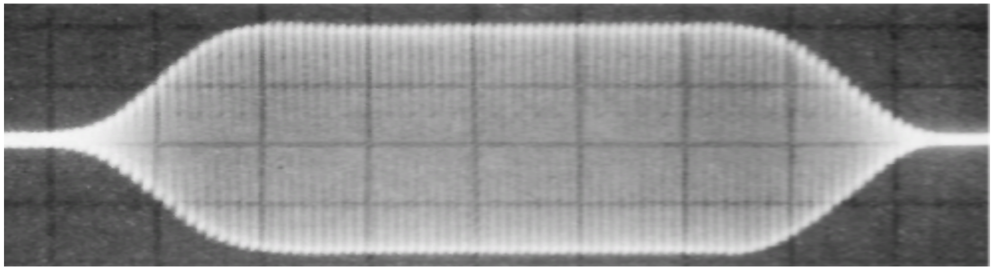
$s_T(t) = s(t)\cos(\omega_o t)$



Waves, modulated by “on-off” action of pulse envelope

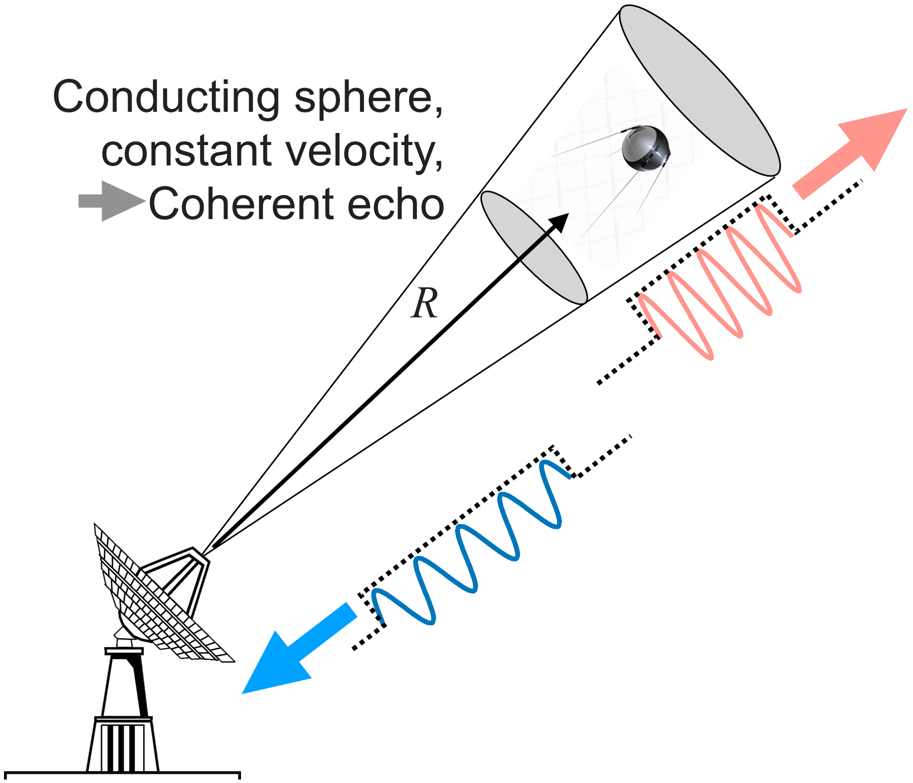


How many cycles are in a typical pulse?

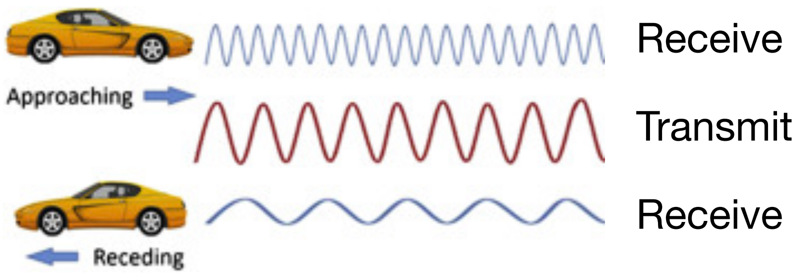
PFISR frequency: 449 MHz
Typical long-pulse length: 480 μs

215,520 cycles!

Measuring Velocity



Conducting sphere,
constant velocity,
→Coherent echo



Assume a transmitted signal: $s(t)\cos(2\pi f_o t)$

After return from target: $a(t)\cos\left[2\pi f_o\left(t+\frac{2R(t)}{c}\right)\right]$

Let's assume target moves with constant velocity with respect to the radar during the measurement,

$$R = R_o + v_o t$$

Substituting we obtain:

$$a(t)\cos\left[\underbrace{2\pi f_o t}_{\omega_o t} + \underbrace{2\pi f_D t + \frac{2\pi f_o R_o}{c}}_{\phi(t)}\right] \qquad f_D = -\frac{2f_o}{c}v_o$$

$$a(t)\cos\left[\omega_o t + \phi(t)\right] \qquad \omega_D = 2\pi f_D = \frac{d\phi}{dt}$$

$f_o \sim 500$ MHz, f_D~ 50 kHz $= 0.0001 f_o$

Two issues:

1) How do we discriminate positive from negative f_D?

2) How do we remove f_o , and just sample $a(t)\cos[\phi(t)]$?

Analytic Signal Model

From Euler's identity

$$re^{j\theta} = (r\cos\theta) + j(r\sin\theta) \qquad j = \sqrt{-1}$$

$$r\cos(\theta) = \Re\{re^{j\theta}\}$$ "real part"

$$r\sin(\theta) = \Im\{re^{j\theta}\}$$ "imaginary part"

Setting $r = a(t)$ and $\theta = \omega_o t + \phi(t)$, we obtain a general complex signal model for radio and radar applications.

$$\boxed{s(t) = a(t)e^{j(\omega_o t+\phi(t))}}$$

PM

AM

Carrier

Or by letting $\omega_d = d\phi/dt \rightarrow \phi(t) = \omega_d t$

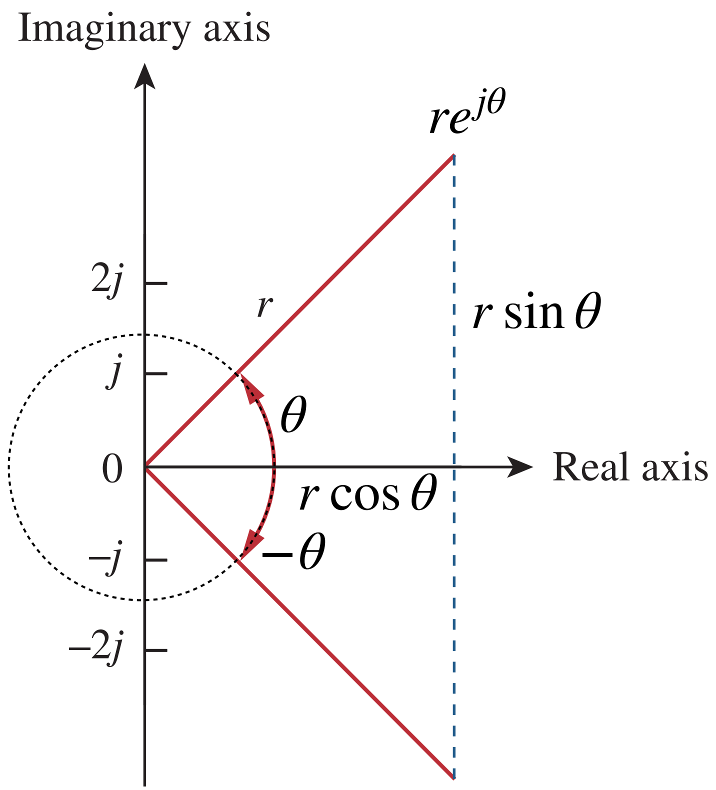
$$s(t) = a(t)e^{j(\omega_o+\omega_d)t}$$

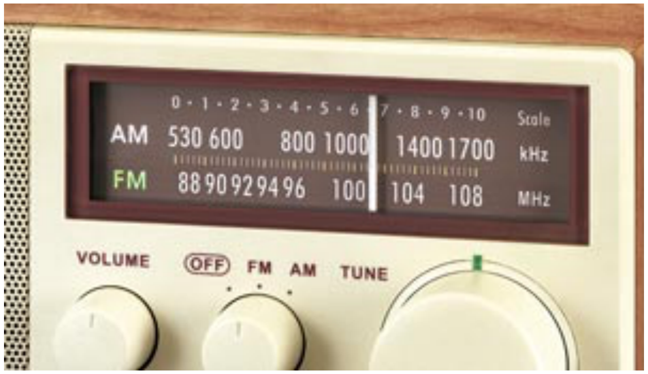
FM

Note that:

$$\Re\{s(t)\} = a(t)\cos(\omega_o t + \phi(t))$$

$$\Im\{s(t)\} = a(t)\sin(\omega_o t + \phi(t))$$

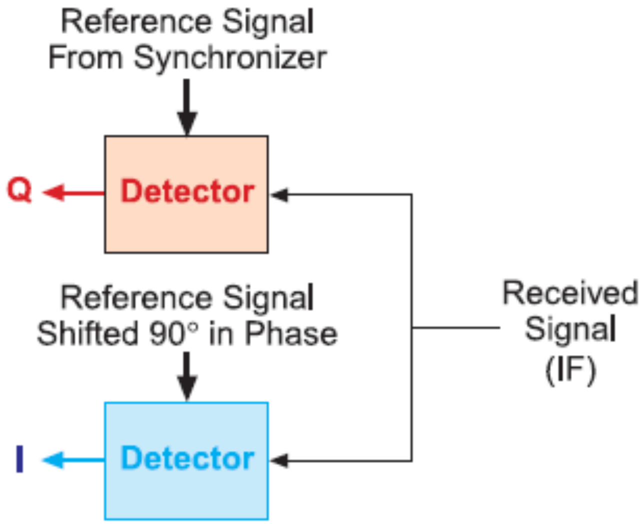




I and Q Demodulation

Consider radar transmission of a simple RF pulse. The reflected signal from the target will be the original pulse with some time varying amplitude and phase applied to it:

$$s_R(t) = a(t)\cos(\omega_o t + \phi(t))$$



Reference Signal From Synchronizer

Q

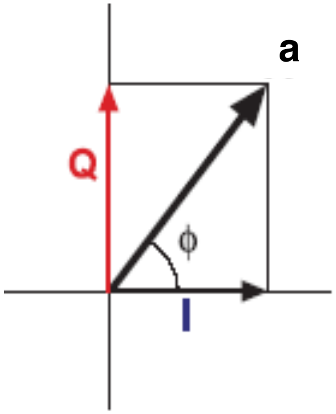
Detector

Reference Signal Shifted 90° in Phase

Received Signal (IF)

I

Detector



We compute the analytic signal by “mixing” with cosine and sine.

Mixing with cosine give the **“in-phase” (I) channel:**

$$\begin{aligned} s_R(t)\cos(\omega_o t) &= a(t)\cos(\omega_o t + \phi(t))\cos(\omega_o t) \\ &= a(t)\frac{1}{2}\left(\underbrace{\cos[2\omega_o t + \phi(t)]}_{\text{filter out}} + \cos[\phi(t)]\right) \end{aligned}$$

Mixing with sine give the **“quadrature” (Q) channel:**

$$\begin{aligned} s_R(t)\sin(\omega_c t) &= a(t)\cos(\omega_o t + \phi(t))\sin(\omega_o t) \\ &= a(t)\frac{1}{2}\left(\underbrace{-\sin[2\omega_o t + \phi(t)]}_{\text{filter out}} + \sin[\phi(t)]\right) \end{aligned}$$

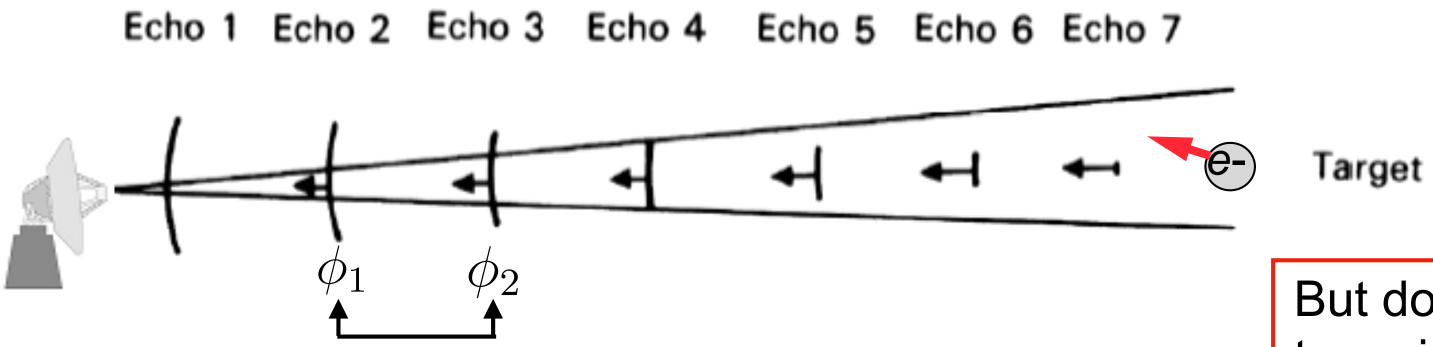
If we include a gain of 2, we retain the original signal energy. Using Euler’s identity we obtain the analytic baseband signal:

$$\boxed{s_B(t) = a(t)e^{j\phi(t)} = a(t)\cos\phi(t) + ja(t)\sin\phi(t) = I + jQ}$$

I/Q demodulation produces a time-series of complex voltage samples (I_n, Q_n) from which we can construct a discrete representation of $s_B(t)$. The Doppler frequency shift is the time rate of change of the phase,$\omega_D = d\phi/dt$.

Doppler Detection: Intuitive Approach

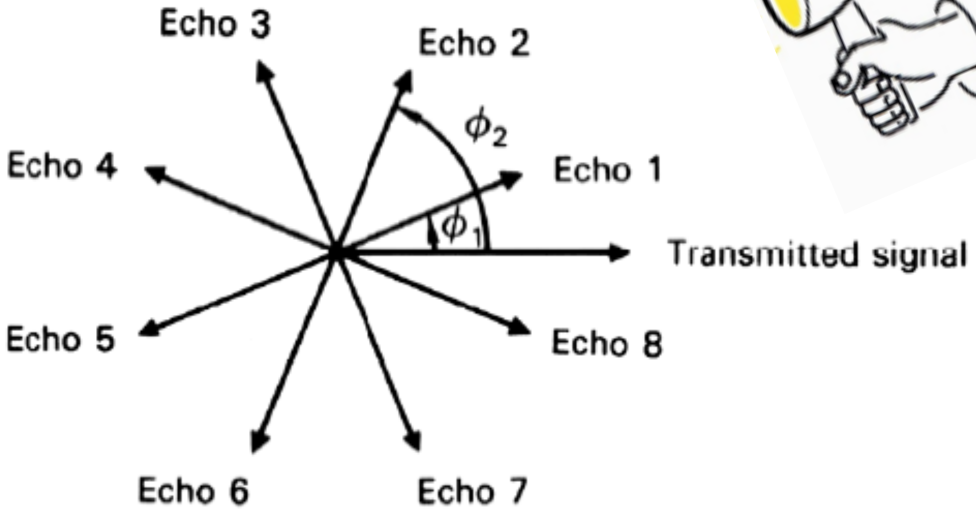
Closing on target – positive Doppler shift

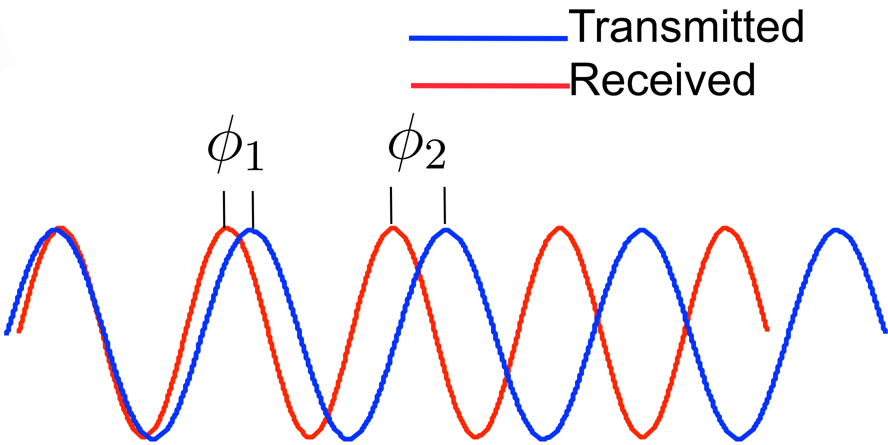


Inter-pulse period (IPP)

But do we expect an electron to maintain a constant velocity between pulses?

Strobe light at ω_o

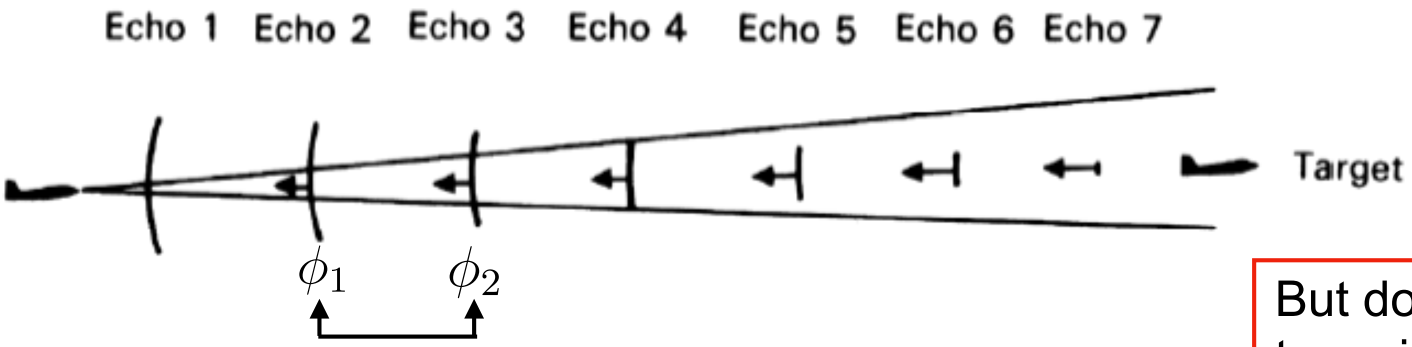




Target's Doppler frequency shows up as a pulse-to-pulse shift in phase.

Doppler Detection: Intuitive Approach

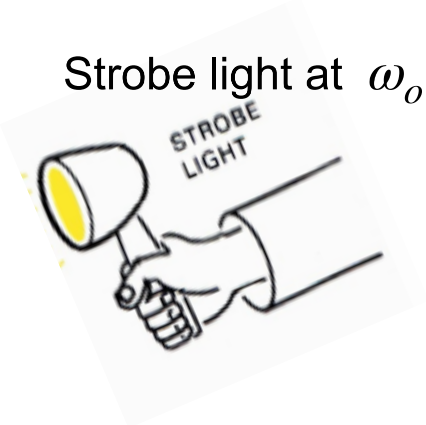
Closing on target – positive Doppler shift

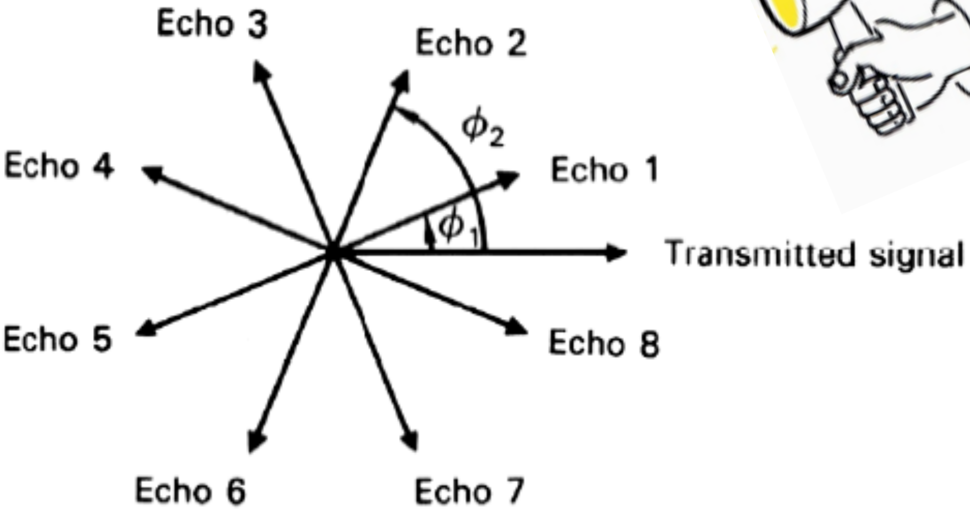


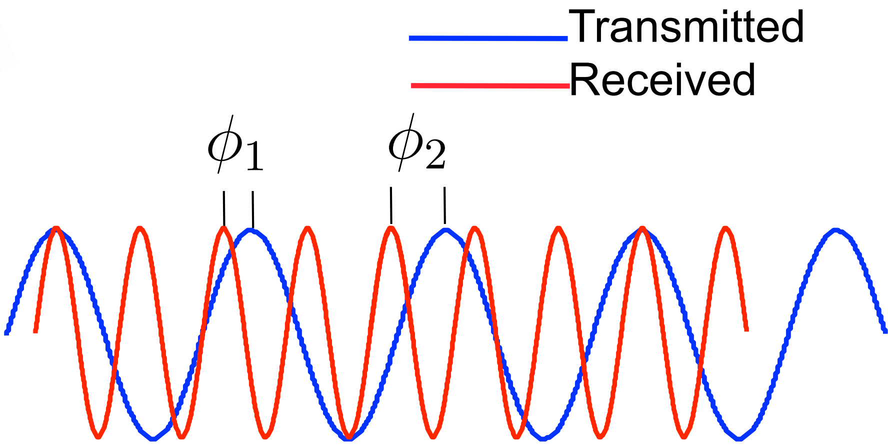
Inter-pulse period (IPP)

But do we expect an electron to maintain a constant velocity between pulses?

Strobe light at ω_o







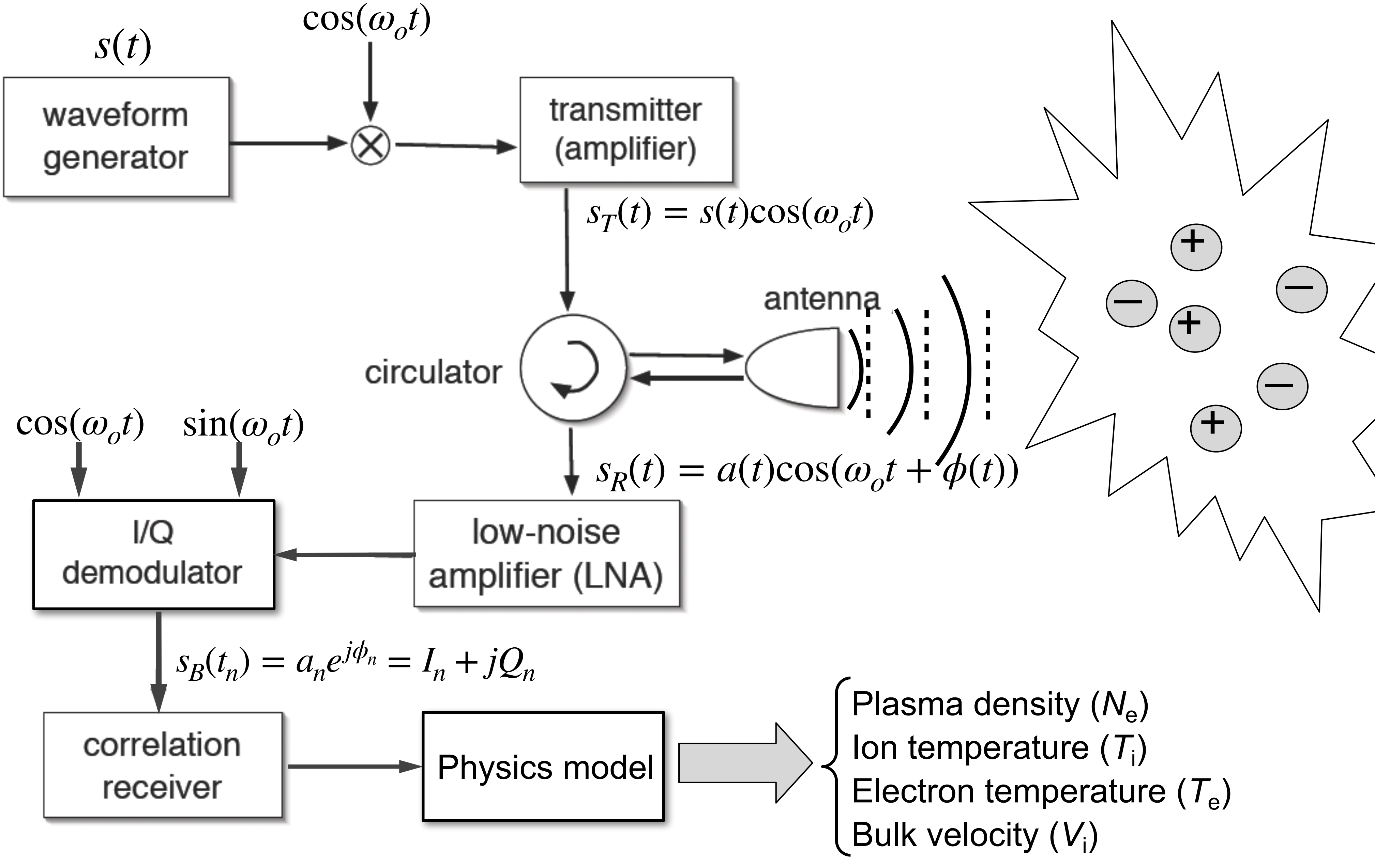
What is the maximum Doppler shift that can be unambiguously measured?

Radar Signal Processing: Part 4

Essential Frequency Domain Concepts for ISR

Josh Semeter
Boston University

Components of a Pulsed Doppler Radar



$s(t)$

waveform generator

$\cos(\omega_o t)$

transmitter (amplifier)

$s_T(t) = s(t)\cos(\omega_o t)$

antenna

circulator

$\cos(\omega_o t)$ $\sin(\omega_o t)$

$s_R(t) = a(t)\cos(\omega_o t + \phi(t))$

I/Q demodulator

low-noise amplifier (LNA)

$s_B(t_n) = a_n e^{j\phi_n} = I_n + jQ_n$

correlation receiver

Physics model

Plasma density (N_e)
Ion temperature (T_i)
Electron temperature (T_e)
Bulk velocity (V_i)

Essential mathematical operations

Fourier Transform: Expresses a function as a weighted sum of complex exponentials.

analysis equation: $F(\omega) = \mathscr{F}\left[f(t)\right] = \int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt$

synthesis equation: $f(t) = \mathscr{F}^{-1}\left[F(\omega)\right] = \dfrac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)e^{j\omega t}d\omega$

$$f(t) \quad \Longleftrightarrow \quad F(\omega)$$

Duality: Comparison of $\mathscr{F}$ and $\mathscr{F}^{-1}$ we obtain

$$F(t) \quad \Longleftrightarrow \quad 2\pi f(-\omega)$$

Convolution: Expresses the action of a linear, time-invariant system on a function.

$$f(t) * g(t) = \int_{-\infty}^{+\infty} f(\tau)g(t-\tau)d\tau$$

$$f(t) * g(t) \quad \Longleftrightarrow \quad F(\omega)G(\omega)$$

$$f(t)g(t) \quad \Longleftrightarrow \quad F(\omega) * G(\omega)$$

Dirac Delta Function $\delta(x)$

A generalized function, or distribution, with the properties

$$\delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases} \qquad \int_{-\infty}^{+\infty} \delta(x)dx = 1$$

Sampling property: From the above it follows that

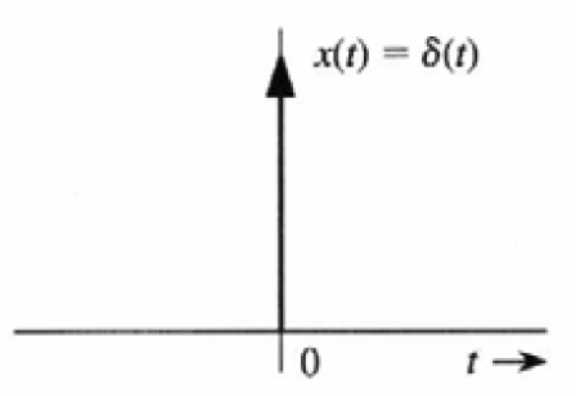
$$f(t_o) = \int_{-\infty}^{+\infty} f(t)\delta(\underbrace{t - t_o}_{})dt$$

argument is zero at $t = t_0$

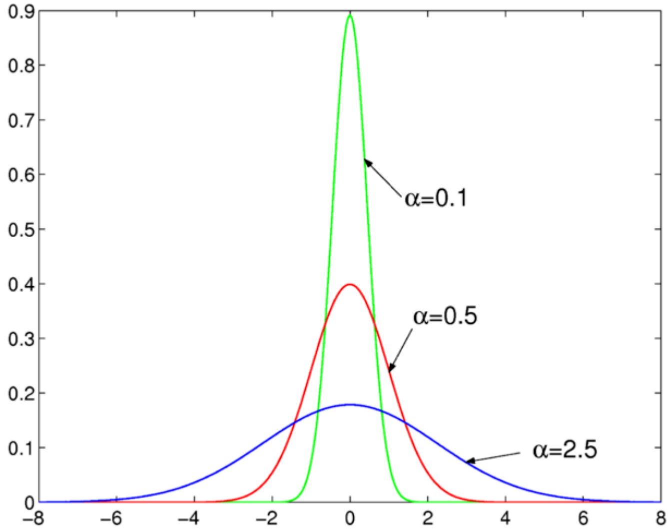
Shift property: Convolution of a function $F(x)$ with $\delta(x - x_o)$ shifts the entire function by x_o. We will use this property to understand mixing. Specifically:

$$F(\omega) * \delta(\omega - \omega_0) = \int_{-\infty}^{+\infty} F(\Omega)\delta(\underbrace{\omega - \omega_0}_{} - \Omega)d\Omega$$

$$= F(\omega - \omega_0)$$



$\delta(t)$ may be expressed as the limit of many functions



$$\delta(t) = \lim_{\alpha \to 0} \frac{1}{\sqrt{4\pi\alpha}} e^{-t^2/(4\alpha)}$$

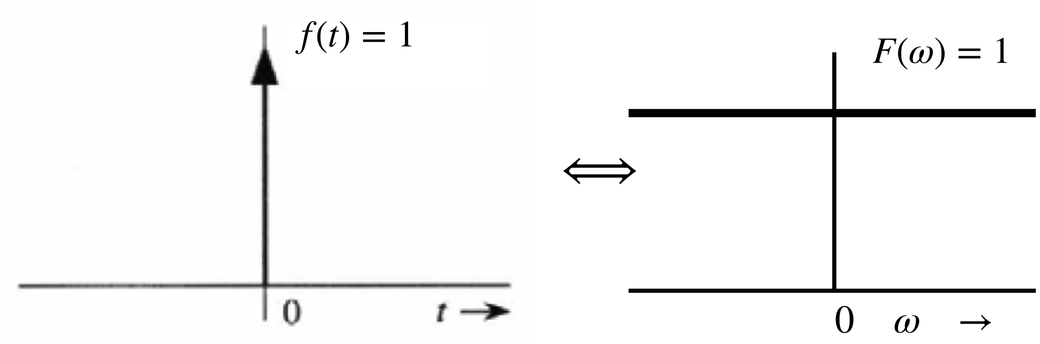
Fourier analysis of harmonic functions

$$\mathscr{F}\left[\delta(t)\right] = \int_{-\infty}^{+\infty} \delta(t) e^{-j\omega t} dt \ = e^{-j0} dt$$

$$\boxed{\mathscr{F}\left[\delta(t)\right] = 1}$$

$$\mathscr{F}\left[\delta(t - t_o)\right] = \int_{-\infty}^{+\infty} \delta(t - t_o) e^{-j\omega t} dt$$

$$\boxed{\mathscr{F}\left[\delta(t - t_o)\right] = e^{-j\omega t_o}}$$



From duality property we can also write,

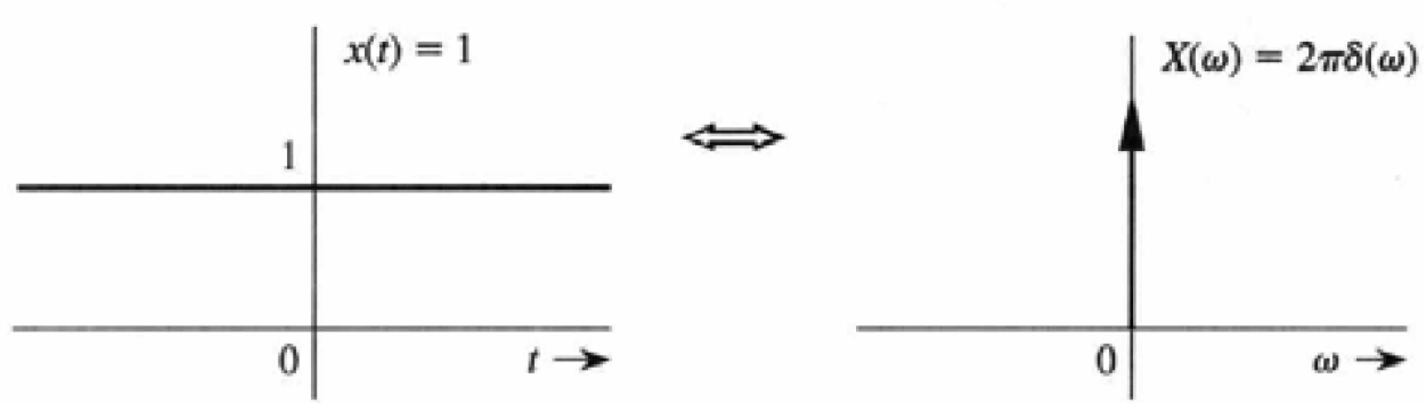
$$\boxed{\mathscr{F}[1] = 2\pi\delta(\omega)} \qquad \boxed{\mathscr{F}\left[e^{j\omega_o t}\right] = 2\pi\delta(\omega - \omega_o)}$$

$$f(t) = \cos(\omega_o t) = \frac{1}{2}\left[e^{j\omega_o t} + e^{-j\omega_o t}\right]$$

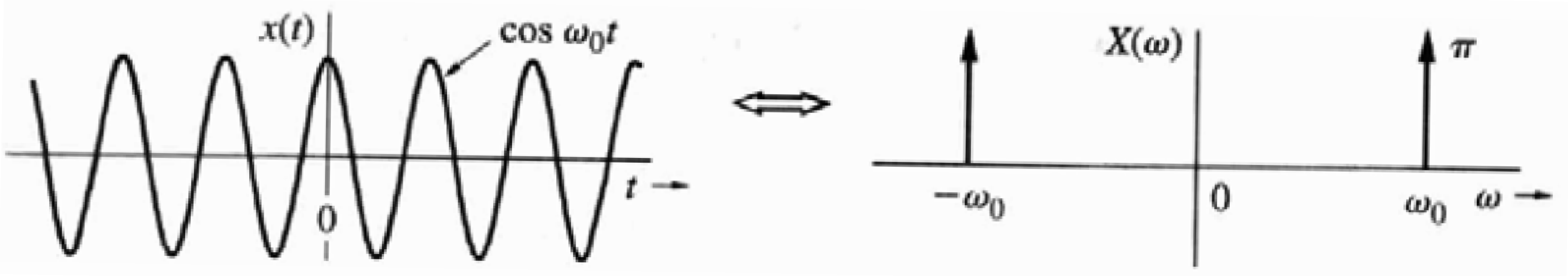
$$F(\omega) = \int_{-\infty}^{+\infty} \frac{1}{2}\left[e^{j\omega_o t} + e^{-j\omega_o t}\right] e^{-j\omega t} dt \ = \frac{1}{2}\left[\int_{-\infty}^{+\infty} e^{j\omega_o t} e^{-j\omega t} dt + \int_{-\infty}^{+\infty} e^{-j\omega_o t} e^{-j\omega t} dt\right]$$

$$\boxed{\mathscr{F}\left[\cos(\omega_o t)\right] = \pi\left[\delta(\omega - \omega_o) + \delta(\omega + \omega_o)\right]}$$

Fourier analysis of harmonic functions



$$\cos \omega_0 t \Longleftrightarrow \pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$$



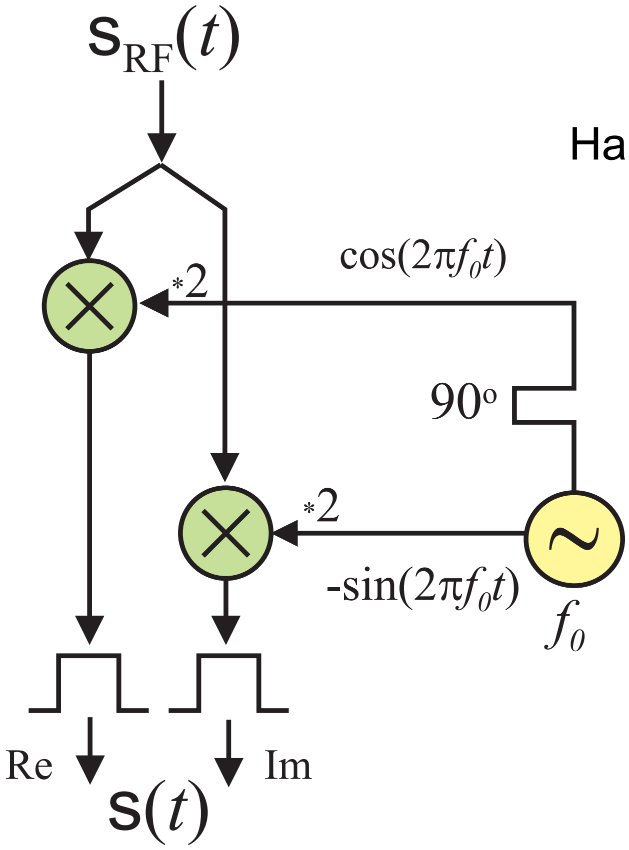
$$\sin \omega_0 t \quad \Longleftrightarrow \quad j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$$

$$e^{j\omega_o t} = \cos(\omega_o t) + j\sin(\omega_o t) \quad \Longleftrightarrow \quad 2\pi\delta(\omega - \omega_0)$$

Summary of tools for I/Q demodulation

Multiplication-convolution: $f(t)g(t) \Longleftrightarrow F(\omega) * G(\omega)$

Frequency shift property: $F(\omega) * \delta(\omega - \omega_0) = F(\omega - \omega_0)$



Harmonic functions:

$$\cos(\omega_o t) \Longleftrightarrow \pi\left[\delta(\omega + \omega_o) + \delta(\omega - \omega_o)\right]$$

$$\sin(\omega_o t) \Longleftrightarrow j\pi\left[\delta(\omega + \omega_o) - \delta(\omega - \omega_o)\right]$$

$$e^{j\omega_o t} \Longleftrightarrow 2\pi\delta(\omega - \omega_o)$$

I/Q Demodulation: Frequency Domain

Transmitted signal:

$$\cos(2\pi f_o t) \quad \Longleftrightarrow$$

Reflected signal from moving target

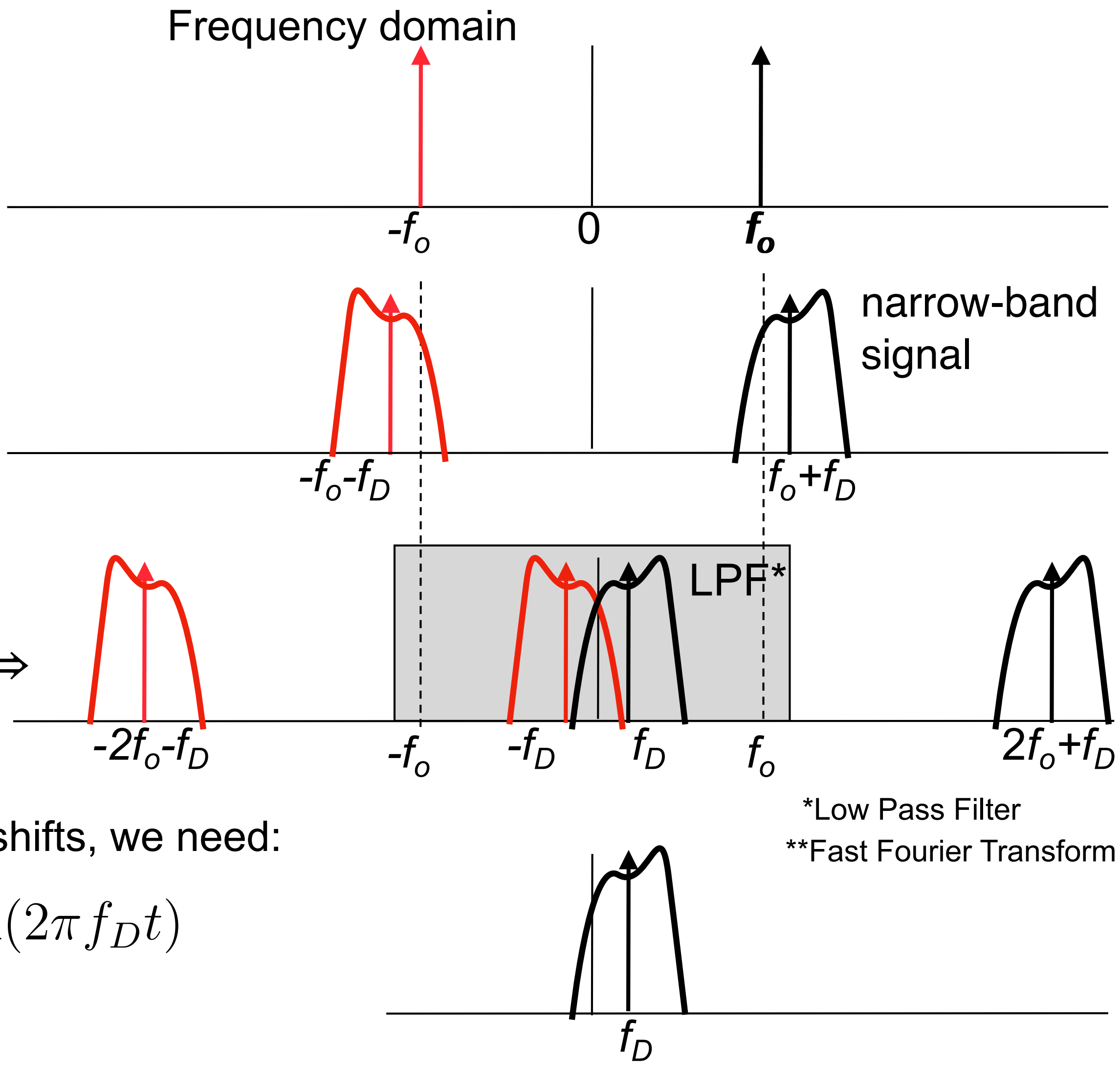
$$\cos(2\pi(f_o + f_D)t) \quad \Longleftrightarrow$$

Mixed (multiplied) with oscillator $\cos(2\pi f_o t)$

$$\frac{1}{2}\cos\left[2\pi(2f_o + f_D)t\right] + \frac{1}{2}\cos[2\pi f_D t] \quad \Longleftrightarrow$$

To resole both positive and negative Doppler shifts, we need:

$$e^{j2\pi f_D t} = \cos(2\pi f_D t) + j\sin(2\pi f_D t)$$



Frequency domain

*Low Pass Filter

**Fast Fourier Transform

We thus need to mix with a second oscillator at same frequency but 90° out of phase (Lecture 3). For a cosine reference, the quadrature function is sine. The two components are called “in phase” (*I*) and “quadrature” (*Q*). Together *I* and *Q* represent discrete samples of the baseband analytic signal,

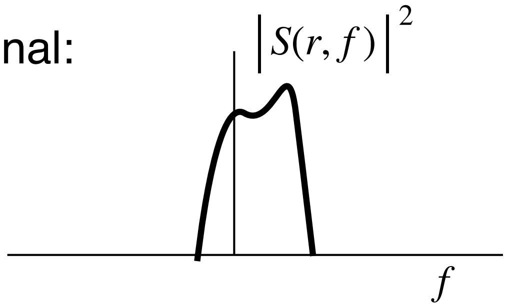
$$s_B(t) = Ae^{2\pi f_D t} = I(t) + jQ(t) \xLeftrightarrow{\text{FFT**}} A\delta(f - f_D)$$

(for a single scatterer)

Correlation and the ISR Spectrum

How do we compute the power spectrum from our complex voltages ?

One approach is to compute Fourier transform of the range-resolved signal:



$$s(r,t) = I(r,t) + Q(r,t) \quad \Longleftrightarrow \quad S(r,f)$$

from which the power spectrum may be represent as $\left|S(r,f)\right|^2$

Based on the stochastic nature of the target, and the way ISR samples the echos, we will take a different approach. We first compute the auto-correlation function (ACF),

$$R_s(r,\tau) = \frac{\left\langle s(r,t)\overline{s(r,t+\tau)}\right\rangle}{\left\langle \left|s(r,t)\right|^2 \right\rangle}$$

where the angle brackets denote the ensemble average, or the expected value.
The power spectral density is given by the Fourier transform of the R_s

$$R_s(r,\tau) \Longleftrightarrow \left|S(r,f)\right|^2$$ (Wiener-Khinchin theorem)

The discrete representation of $R_s(r,\tau)$ is constructed through appropriate scaling and multiplication of the complex voltage samples $s(r_k, t_n)$.

In the next lecture we will begin to explore methods for constructing the ACF.

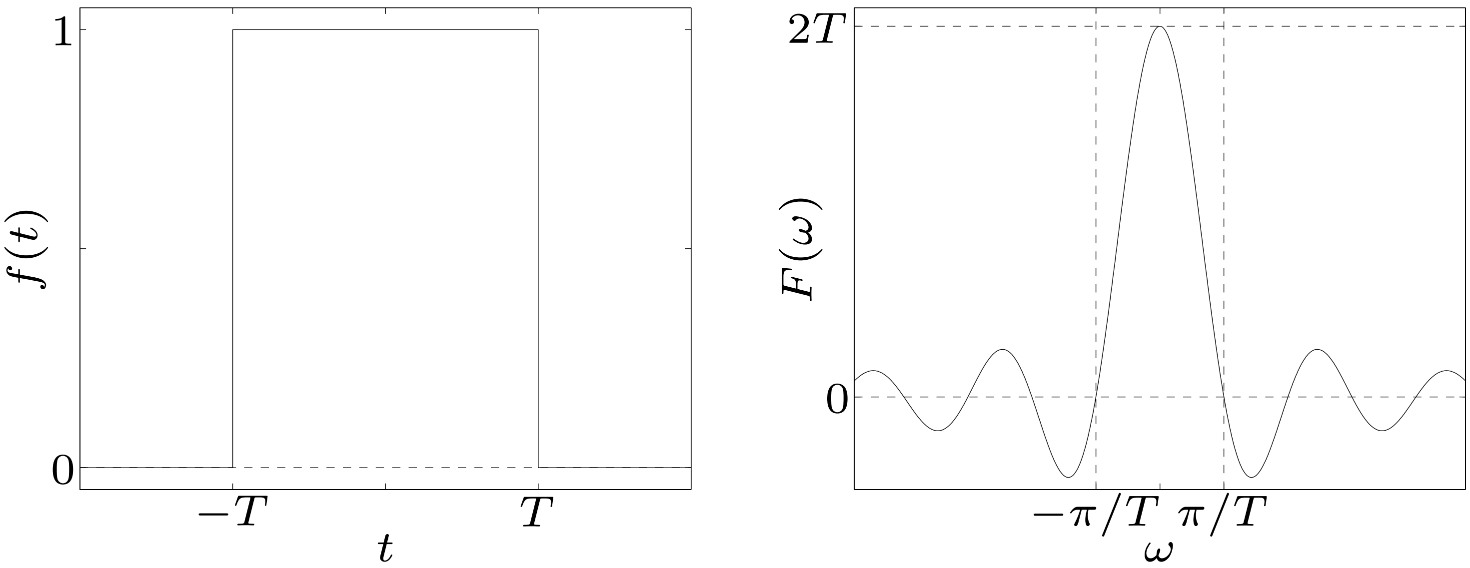
References

- Farley
- DSP textbook
- Beynon/Williams

Gate function

rectangular pulse: $f(t) = \begin{cases} 1 & -T \leq t \leq T \\ 0 & |t| > T \end{cases}$

$$F(\omega) = \int_{-T}^{T} e^{-j\omega t}\, dt = \frac{-1}{j\omega}\left(e^{-j\omega T} - e^{j\omega T}\right) = \frac{2\sin\omega T}{\omega}$$



unit impulse: $f(t) = \delta(t)$

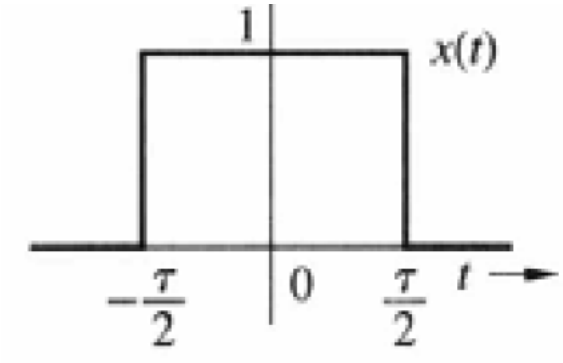
$$F(\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t}\, dt = 1$$

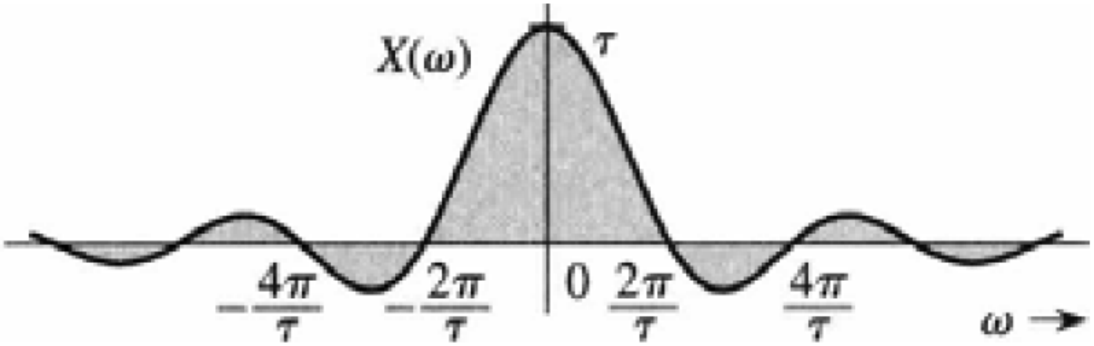
shifted: $f(t) = \delta(t - t_o)$

$F(\omega) =$

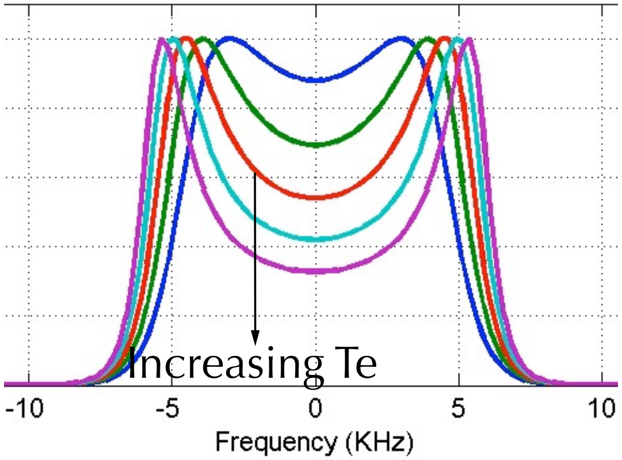
Gate function

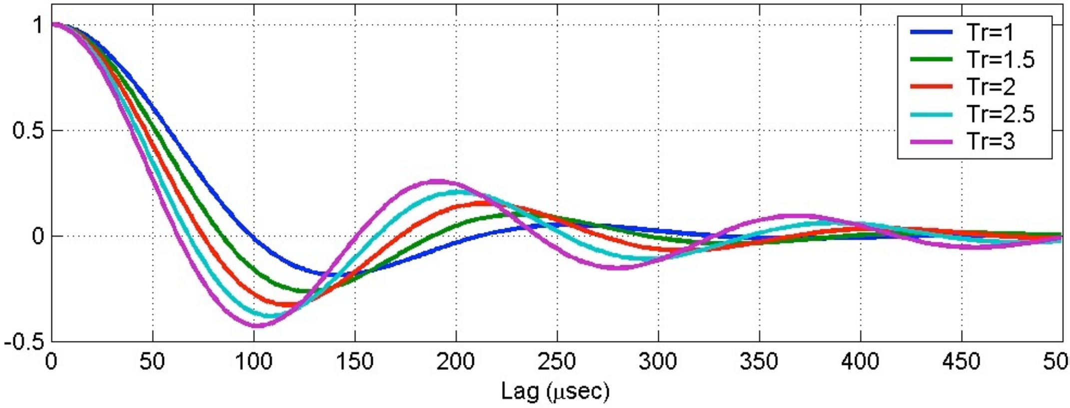
$$\mathrm{rect}(t/\tau) = \begin{cases} 1 & \text{for } -\tau/2 < t < \tau/2 \\ 0 & \text{otherwise} \end{cases} \Longleftrightarrow \tau\ \mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$$





ISR spectrum $\Longleftrightarrow$ Autocorrelation function (ACF)





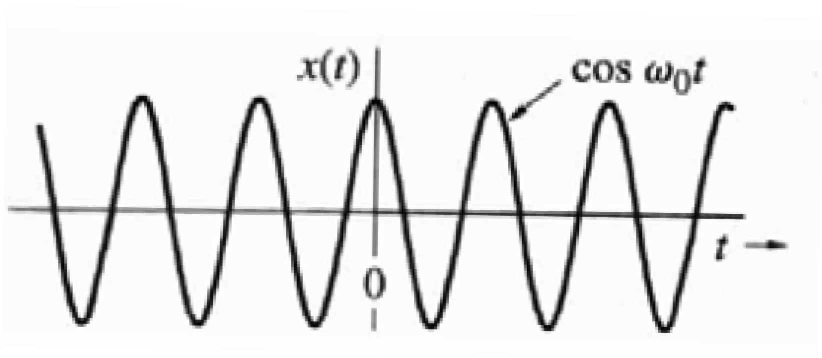
For low Te, the ISR ACF looks like a sinc function. For high Te the ACF becomes more oscillatory and looks more like a cosine (power concentrated at the Doppler frequency corresponding to the ion-acoustic wave speed.

Convolution

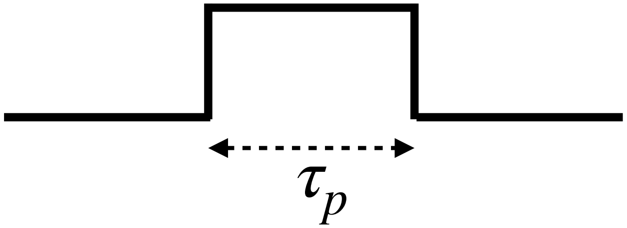
Convolution: Expresses the action of a linear, time-invariant system on a function**.**

$$f(t) * g(t) = \int_{-\infty}^{+\infty} f(\tau)g(t-\tau)d\tau \iff F(\omega)G(\omega)$$

$$F(\omega) * G(\omega) = \int_{-\infty}^{+\infty} F(\omega)G(\omega-\Omega)d\Omega \iff f(t)g(t)$$



X

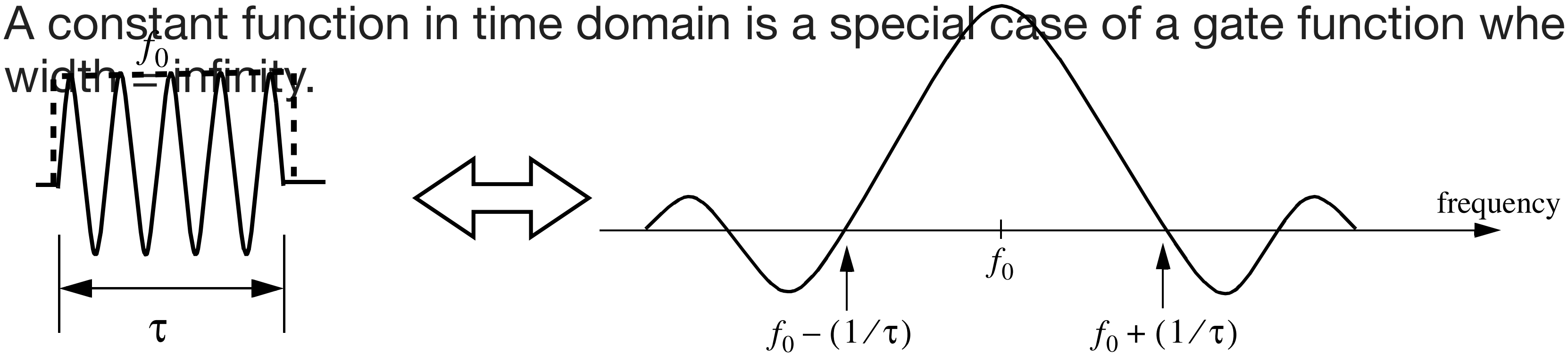


?

Filters, and the gate function

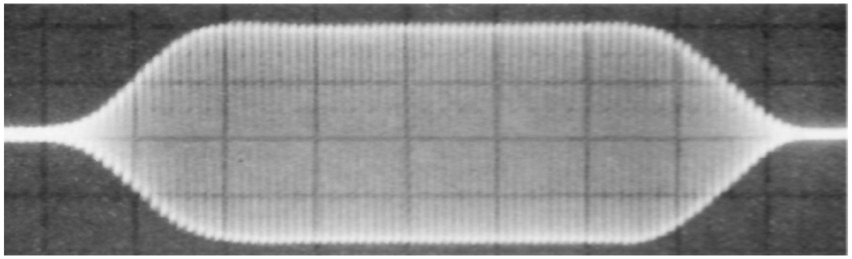
How it all hangs together.

- Duality:
  - Gate function in the time domain represents amplitude modulation
  - Gate function in the frequency domain represents filtering
- Limiting cases:
  - Gate function approaches delta function as width goes to 0 with constant area
  - A constant function in time domain is a special case of harmonic function where frequency = 0.
  - A constant function in time domain is a special case of a gate function where width = infinity.



How many cycles are in a typical ISR pulse?
PFISR frequency: 449 MHz
Typical long-pulse length: 480 μs

⇨ 215,520 cycles!



A linear system

- Convolution property

Correlation

Correlation: A measure of the degree to which two functions look alike at a given offset. If the two functions are the same, we call this the autocorrelation function, or ACF.

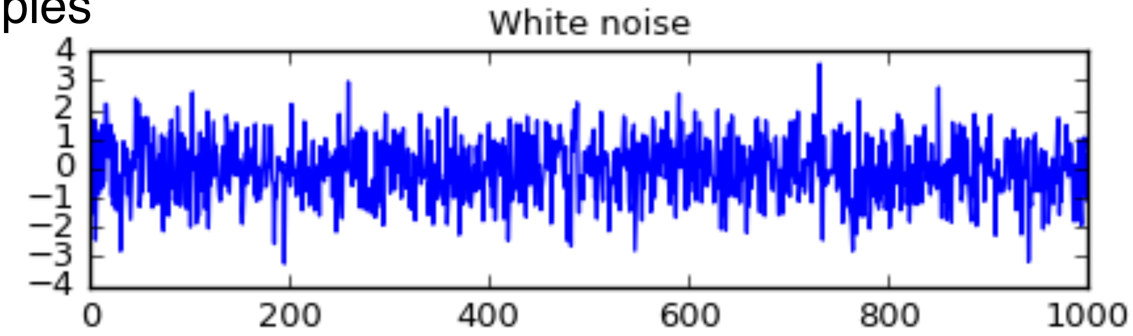
$$R_{ff}(\tau) = \int_{-\infty}^{+\infty} f(t+\tau)\bar{f}(t)dt = f(\tau) * \bar{f}(-\tau)$$

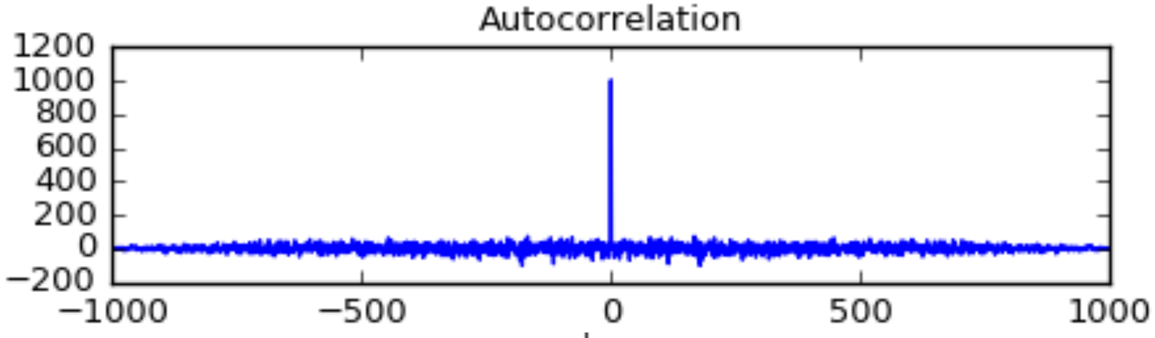
We will be working with discrete samples

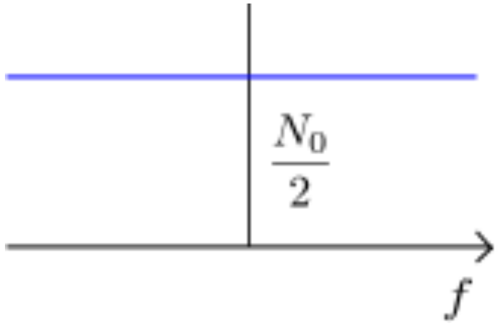
$$R_{ff}(k) = \sum_{n=-\infty}^{+\infty} f(n)\bar{(}f)(n-k)$$

The ‘spectrum’ refers to the power spectrum, which is the Fourier transform of the autocorrelation function

$$R_{ff} \Longleftrightarrow \left| U(\omega) \right|^2$$







I and Q Demodulation: Frequency Domain

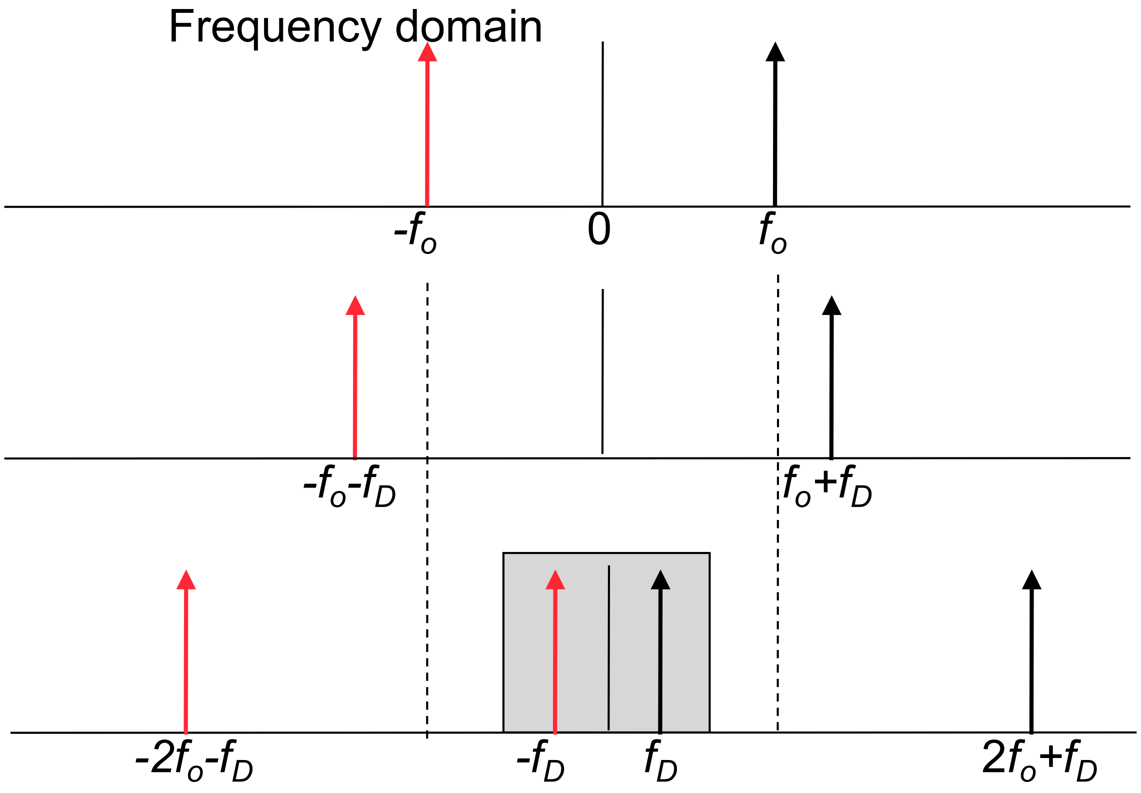
Transmitted signal

$$\cos(2\pi f_o t)$$

Doppler shifted

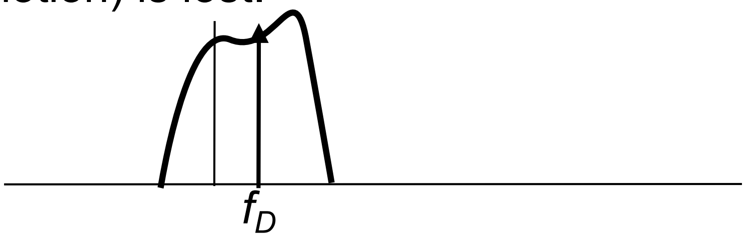
$$\cos(2\pi(f_o + f_D)t)$$

Mixed (multiplied) with carrier $\cos(2\pi f_o t)$



Cosine is even function, so sign of f_D (and, hence, direction of motion) is lost.
What we need instead is:

$$e^{j2\pi f_D t} = \cos(2\pi f_D t) + j\sin(2\pi f_D t)$$



The analytic signal $e^{j2\pi f_D t}$ cannot be measured directly, but the cos and sin components via mixing with two oscillators with same frequency but orthogonal phases. The components are called "in phase" (or *I*) and "in quadrature" (or *Q*):

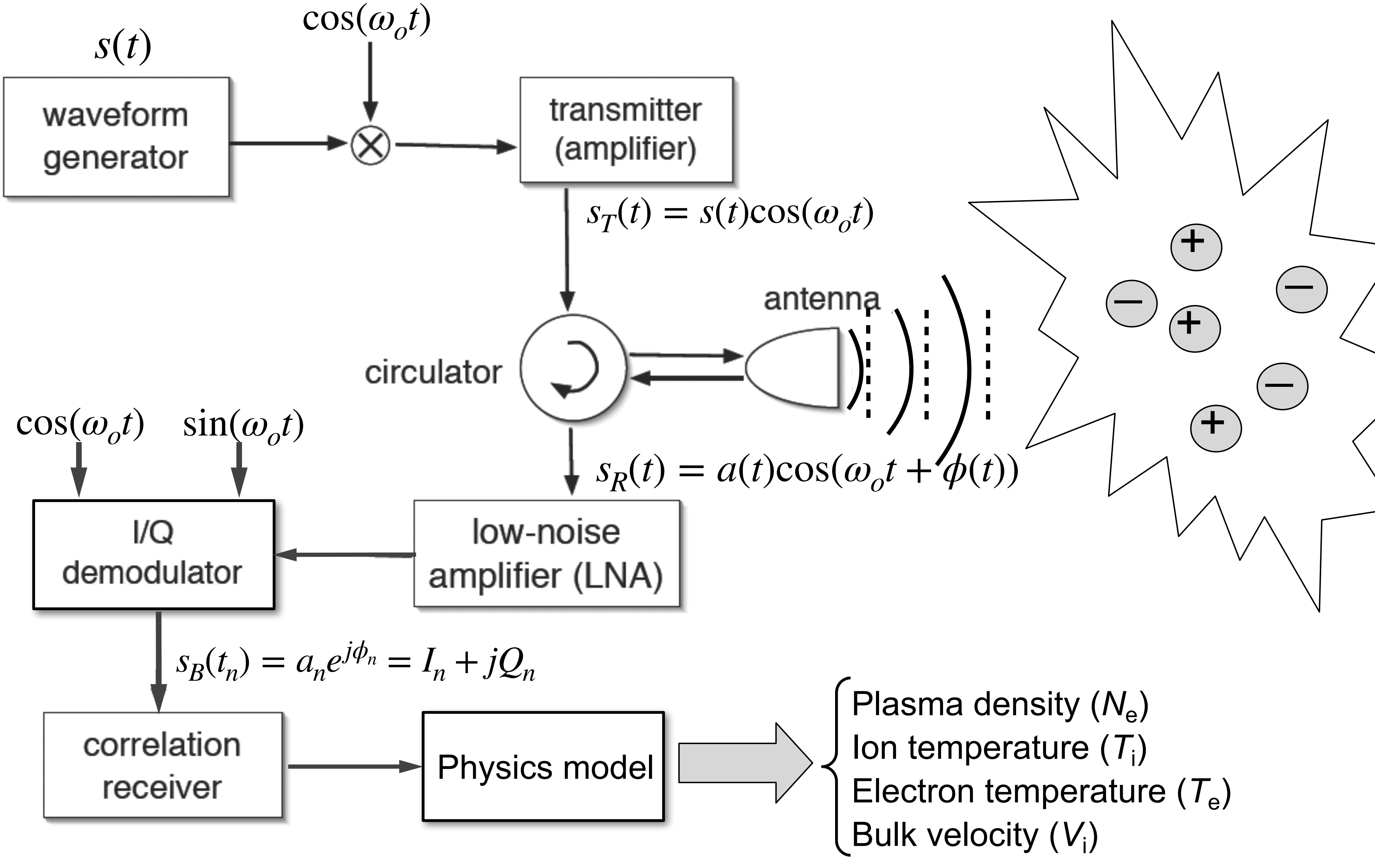
$$Ae^{j2\pi f_D t} = I(t) + jQ(t) \quad \xleftrightarrow{\text{FFT}} \quad A\delta(f_D)$$

(for single scatterer)

Weiner Kinchine Theorem

- The power spectrum has a time domain representation too.
- ACF
- A point in the power spectrum are a measure of power density at a given frequency.
- A point in the ACF is a measure of correlation between the incoming signal and a time-shifted version of it, where time variable is the time shift.
- In ISR processing, it is most common to compute the ACF from the time series of I and Q

Components of a Pulsed Doppler Radar



$s(t)$

waveform generator

$\cos(\omega_o t)$

transmitter (amplifier)

$s_T(t) = s(t)\cos(\omega_o t)$

circulator

antenna

$s_R(t) = a(t)\cos(\omega_o t + \phi(t))$

$\cos(\omega_o t)$ $\sin(\omega_o t)$

I/Q demodulator

low-noise amplifier (LNA)

$s_B(t_n) = a_n e^{j\phi_n} = I_n + jQ_n$

correlation receiver

Physics model

- Plasma density (N_e)
- Ion temperature (T_i)
- Electron temperature (T_e)
- Bulk velocity (V_i)

Doppler power spectrum via FFT

Radar Signal Processing: Part 5

Computing the ACF and Power Spectrum

Josh Semeter
Boston University

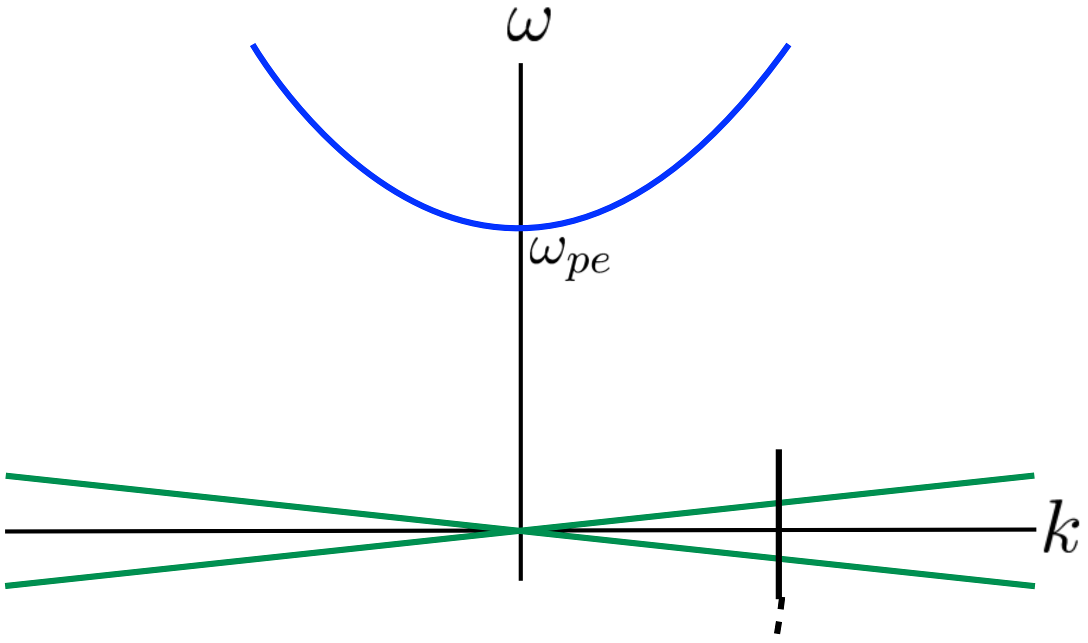
Incoherent Scatter Radar (ISR)

Ion-acoustic

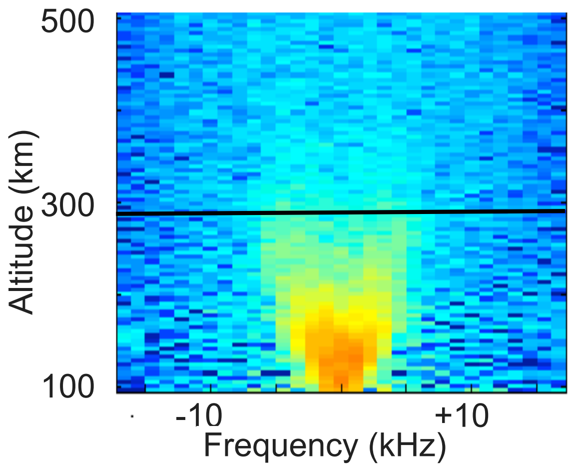
$\omega_s = C_s k \qquad C_s = \sqrt{k_B(T_e + 3T_i)/m_i}$

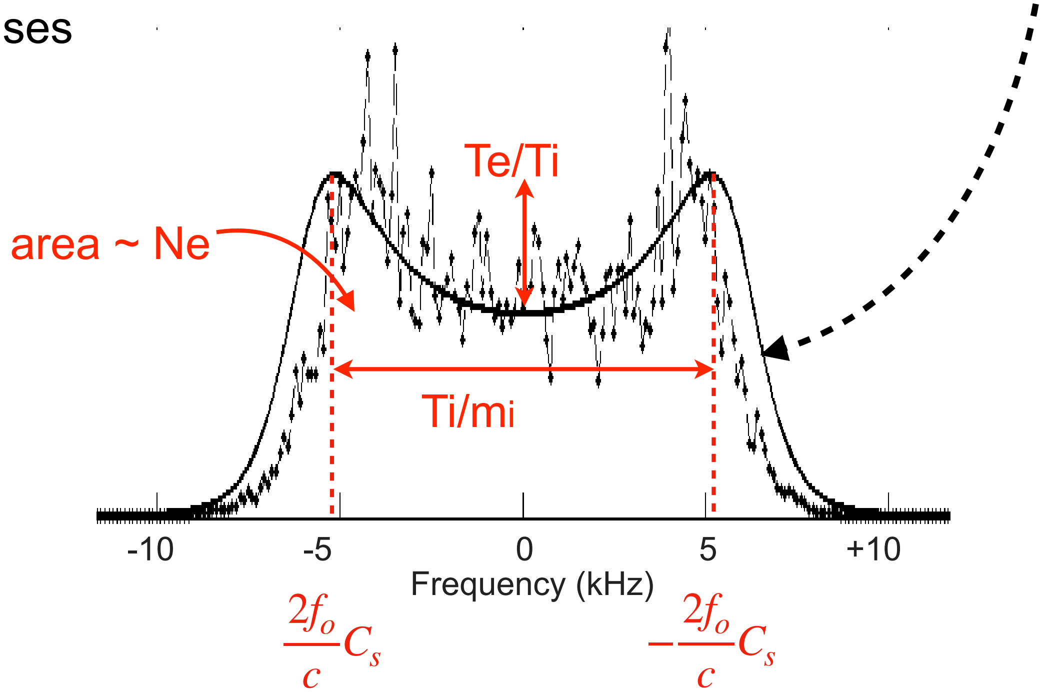
Langmuir

$$\omega_L = \sqrt{\omega_{pe}^2 + 3\,k^2\,v_{the}^2} \approx \omega_{pe} + \frac{3}{2} v_{the} \lambda_{De} k^2$$



Doppler power spectrum, ~50 pulses



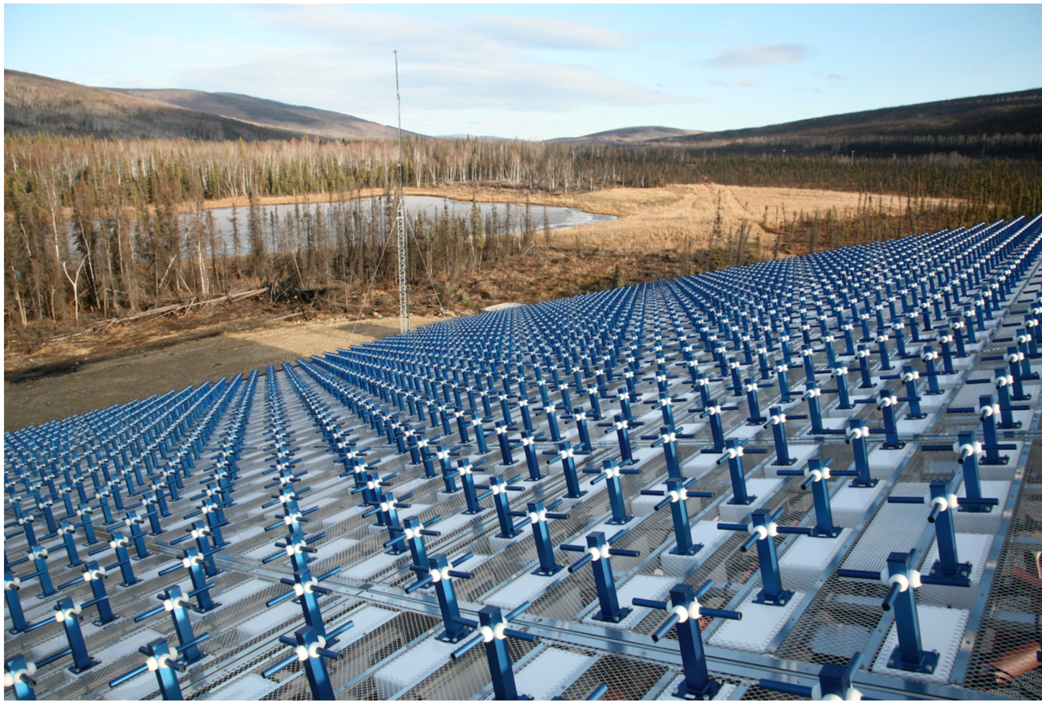


Issues to consider

- connection between how fast we sample, and the maximum doppler velocity that we can observe unambiguously

- This is formalized in something called the Nyquist sampling theorem.
  - To reconstruct a sinusoid, we have to sample it at twice the highest frequency present in our signal.
  - For a sinusoid with frequency f, this means we must sample it at at least $T_s = 2f$.

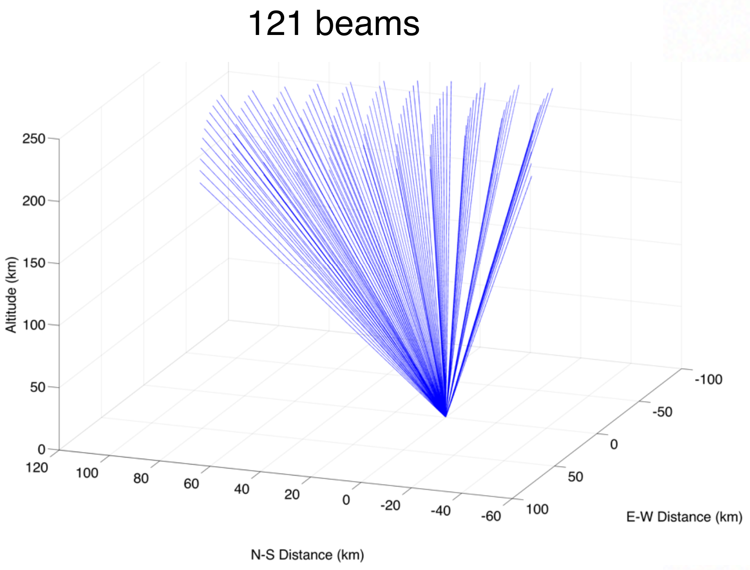
Dish Versus Phased-array

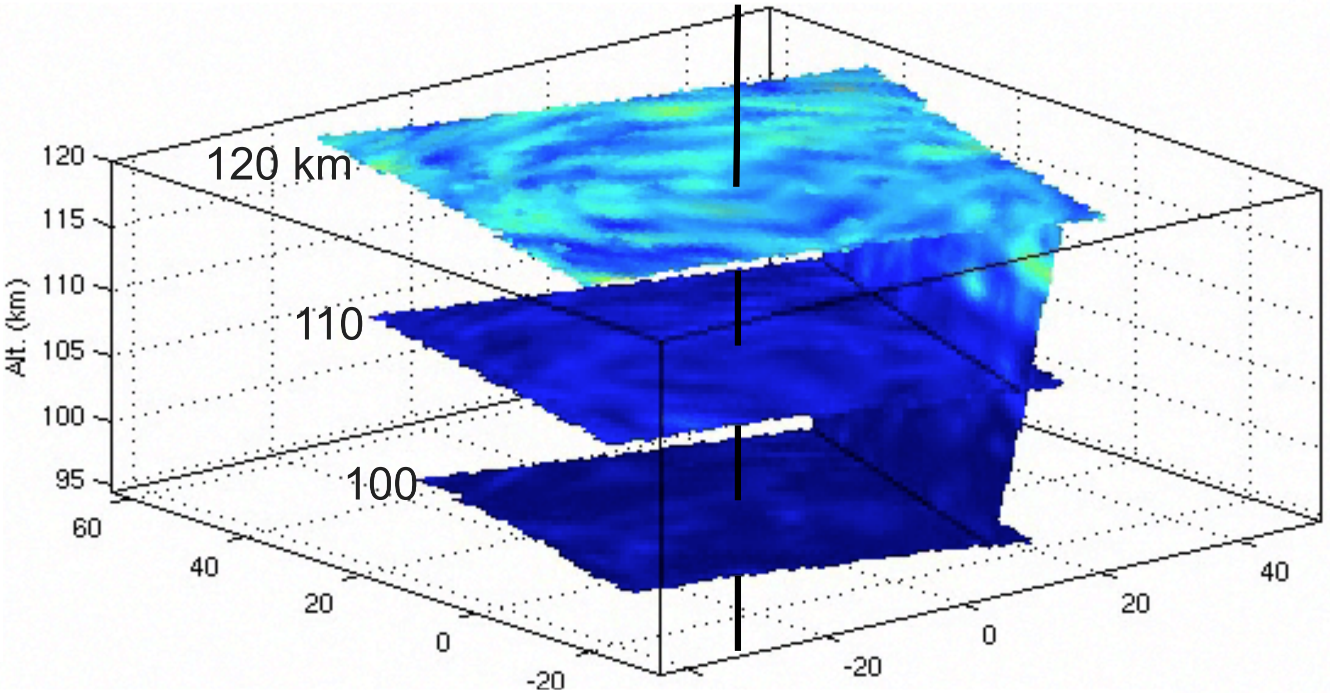


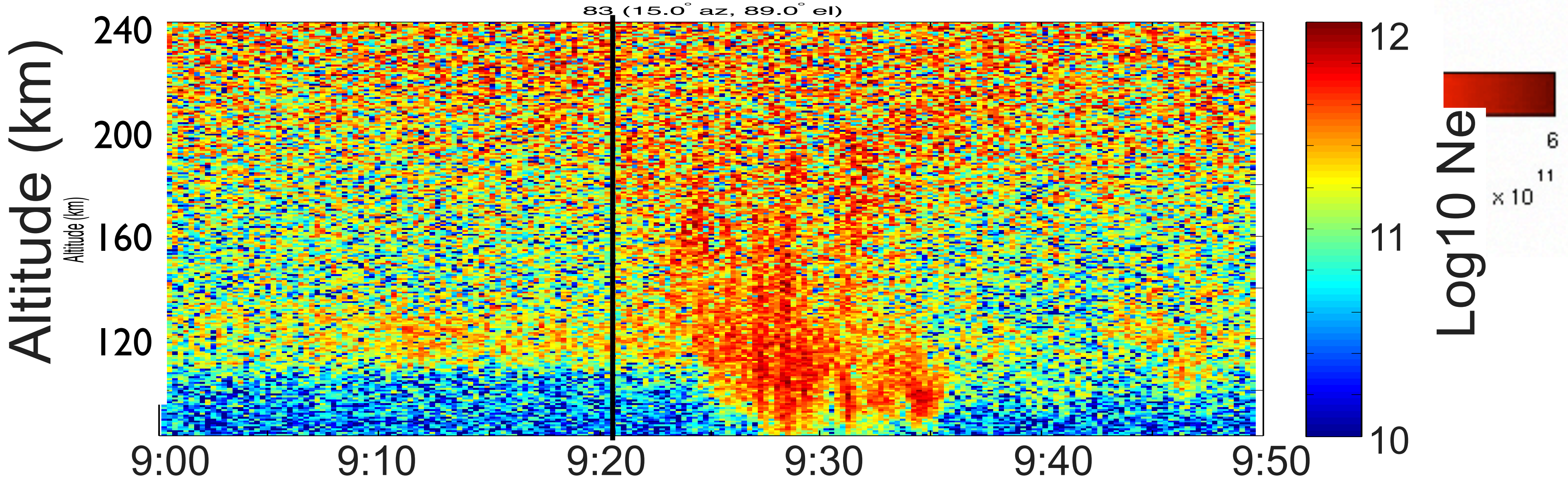
- -FOV: Entire sky
- -Integration at each position before moving
- -Power concentrated at Klystron
- -Significant mechanical complexity

- -FOV: +/- 15 degrees from boresight
- -Integration over all positions simultaneously
- -Power distributed
- -No moving parts

Three-dimensional ionospheric imaging







Semeter et al., JASTP 2009

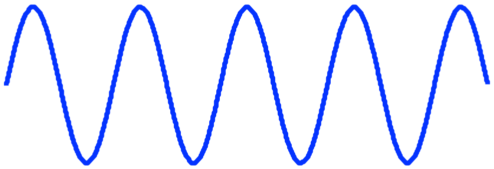
Radar Signal Processing: Part 6

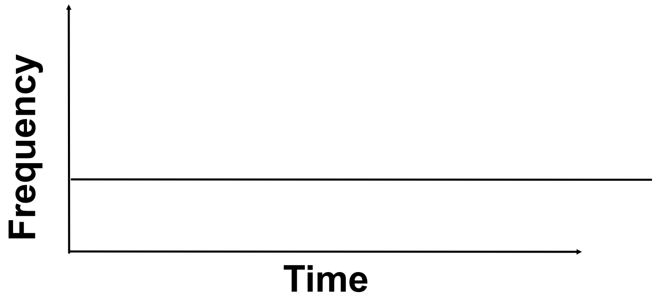
Introduction to Pulse Compression Radar

Josh Semeter
Boston University

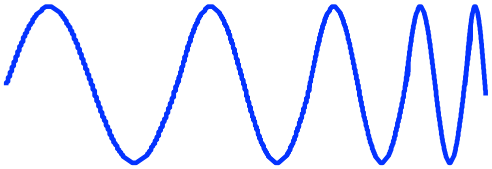
Radar Waveforms (cont'd.)

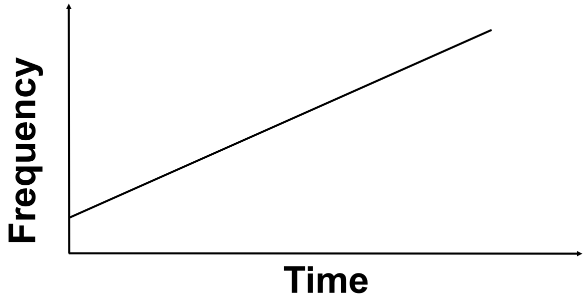
Pulse at single frequency





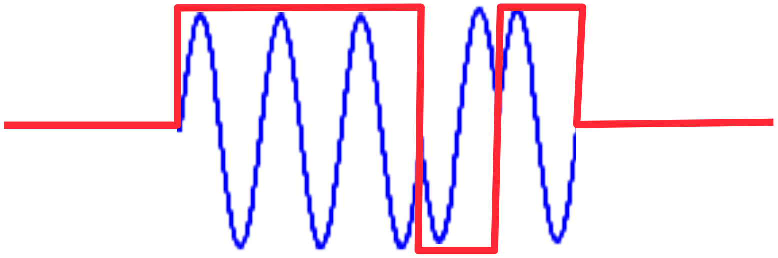
Pulse with changing frequency

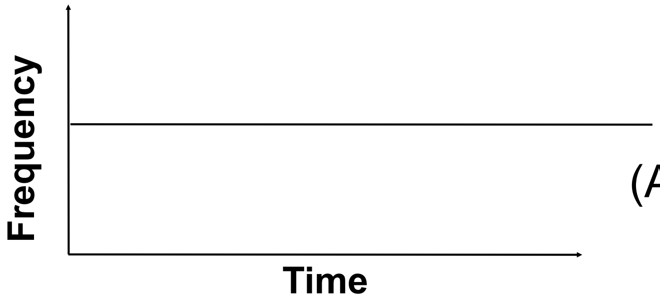




Linear Frequency-Modulated (LFM) Waveform

Pulse at single frequency, but variable phase



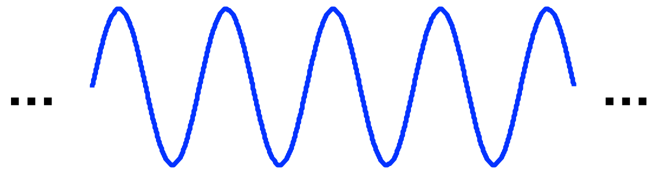


Phase-coded Waveform (Alternating codes Barker Codes)

Radar Waveforms (cont’d.)

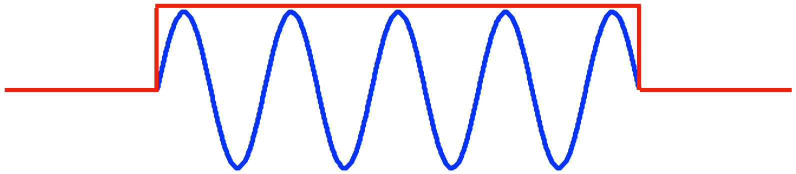
$$s(t) = A(t)\cos\left[2\pi f_o t + \phi(t)\right]$$

Unmodulated RF signal



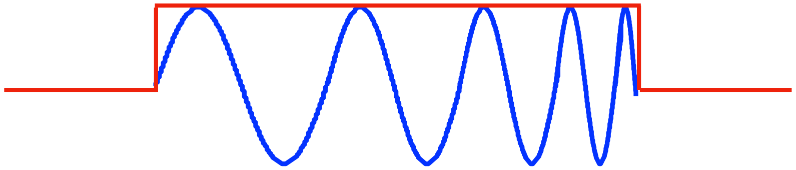
$$s(t) = A_o e^{j2\pi f_o t}$$

RF pulse at a single frequency



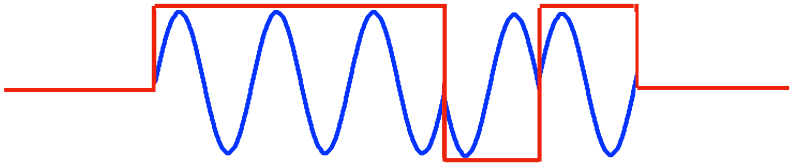
$$s(t) = A(t) e^{j2\pi f_o t}$$

RF Pulse with changing frequency



$$s(t) = A(t) e^{j2\pi (f_o + \Delta f(t)) t}$$

RF Pulse, single frequency, changing phase



$$s(t) = A(t) e^{j2\pi f_o t} e^{j\phi(t)}$$

$$e^{j0} = 1$$

$$e^{j\pi} = -1$$

$$e^{j\pi/2} = j$$

$$e^{-j\pi/2} = -j$$