

# 2.980 Adidas Project: Individual Deliverable

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## Literature Review

### Background

In the past few years, the running world has been shaken up by the introduction of revolutionary shoe designs that have characterized the marathon world record progression. At almost any major marathon, onlookers will most likely find the leading runners in either Adidas' or Nike's flagship racing shoes. In 2014, Adidas' Boost technology brought midsoles to the centerpiece of running technology as Kenya's Dennis Kimetto broke the previous marathon world record in 2:02:57 while wearing Adidas's Adios Boost 2.0 racing flats. Since then, the past four men's marathon world records have also been set in Adidas running shoes<sup>1</sup>. With the 2:00:00 marathon in sight, Nike joined Adidas in the quest to break the seemingly impossible barrier by developing their own project called Breaking2. While highly personalized training regimens and idealized environments aided the effort, Nike's primary development was the Vaporfly 4% shoe that allegedly would help reduce a runner's time by 4%. The key feature of their shoe was the midsole, which for its time, boasted an unusually thick midsole that housed a carbon-fiber shank. Despite failing at the first testing phase, the Breaking2 project eventually succeeded as Eliud Kipchoge shattered the 2:00:00 barrier in 1:59:40 in October 2019. Although the feat did not count for an official world record due to technicalities regarding pacing, it marked a historical moment for running because it revealed the potential power behind midsole technology. With the Tokyo 2020 Olympics just around the corner, controversy surrounding the possible unfair advantage Nike's flagship shoe has kickstarted conversations around regulating its presence in competition. Given the 3-minute drop in marathon world times since the introduction of major midsole redesign in 2014 with Adidas's Adios Boost 2.0, the hot question for running shoe companies that we aim to answer is:

*How can we map design parameters to the foam midsole of shoes?*

### The Foam

Most running shoe foams generally consist of either ethyl vinyl acetate (EVA) or thermoplastic urethane (TPU) and sometimes, they are composite and made up of both. Each of these foams have different properties that lend themselves to certain advantages and disadvantages over the other. Running shoe companies looking to make a lightweight shoe will likely opt for EVA due to its low density as well as its ability to conform easily. Another shoe company might instead use TPU for its durability, but make the sacrifice of dealing with its extra weight. While TPU and EVA are the primary foam components in most running shoe midsoles, they generally are not alone. More recently, there's been a trend of including a stiffer lightweight shank that's usually made of carbon fiber to stiffen the midsole and provide a "springy" effect.

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1. <sup>1</sup> <https://www.cnn.com/2020/01/17/sport/nike-world-athletics-vaporfly-ban-spt-intl/index.html>

As with all materials, analyzing these foams requires us to consider its specific properties as well as its expected behavior. For the scope of our project at this point, we will look at foams through the lens of perfectly linear elastic model. Note that this is not the exact model that predicts a foam's behavior, but for our purposes, will provide a fairly accurate prediction for how the materials will respond under compression/expansion and bending. The next question to answer is:

*What parameters of the foam can we control?*

When designing a shoe, there's a lot of factors we can design around, but we need to identify some key ones in the beginning to start developing a simple model. In this initial report, we will be considering:

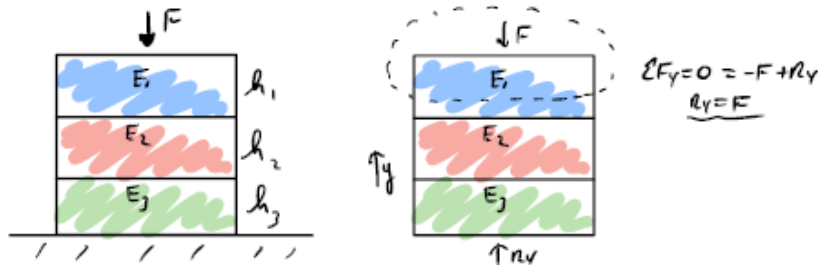
- $E$  Young's Modulus
- $h$  Height of material stack

Note that we will be considering these properties for two different materials for  $n$ -layers. Let's also assume that we will be alternating each layer. Understanding these properties allows us to develop a model for how the foam(s) will respond to compression/expansion and bending forces.

# Technical Analysis

## Stacked Composite Compression Model

### COMPRESSION MODULUS N-LAYERS



### CONSTITUTIVE:

$$\begin{aligned}\sigma_{yy} &= E_1 \epsilon_{yy1} \\ \sigma_{yy} &= E_2 \epsilon_{yy2} \\ \sigma_{yy} &= E_3 \epsilon_{yy3}\end{aligned}$$

$$\Rightarrow \sigma_{yy} = \frac{F}{A} = A E_1 \epsilon_{yy1} = A E_2 \epsilon_{yy2} = A E_3 \epsilon_{yy3} \quad * \text{ SAME STRESS IN ALL LAYERS}$$

### COMPATIBILITY

$$\begin{aligned}\epsilon_1 &= \frac{\delta_1}{h_1} \\ \epsilon_2 &= \frac{\delta_2}{h_2} \\ \epsilon_3 &= \frac{\delta_3}{h_3}\end{aligned} \Rightarrow \text{TOTAL DISPLACEMENT } \delta = \delta_1 + \delta_2 + \delta_3$$

$$\begin{aligned}\delta_1 &= h_1 \left( -\frac{F}{A E_1} \right) \\ \delta_2 &= h_2 \left( -\frac{F}{A E_2} \right) \\ \delta_3 &= h_3 \left( -\frac{F}{A E_3} \right)\end{aligned}$$

PUTTING IT ALL TOGETHER FOR N-LAYERS... LET'S GENERALIZE AND FIND AN Effective

$$h = \sum_{i=1}^N \epsilon h_i \quad \delta = \sum_{i=1}^N \delta_i = \sum_{i=1}^N \frac{\sigma h_i}{E_i} \Rightarrow \delta = \frac{-F}{A} \sum_{i=1}^N \frac{h_i}{E_i} \Rightarrow \frac{-F}{A} = \frac{\delta}{\sum_{i=1}^N \epsilon h_i} \left( \sum_{i=1}^N \epsilon h_i \cdot \frac{E_i}{\epsilon h_i} \right)$$

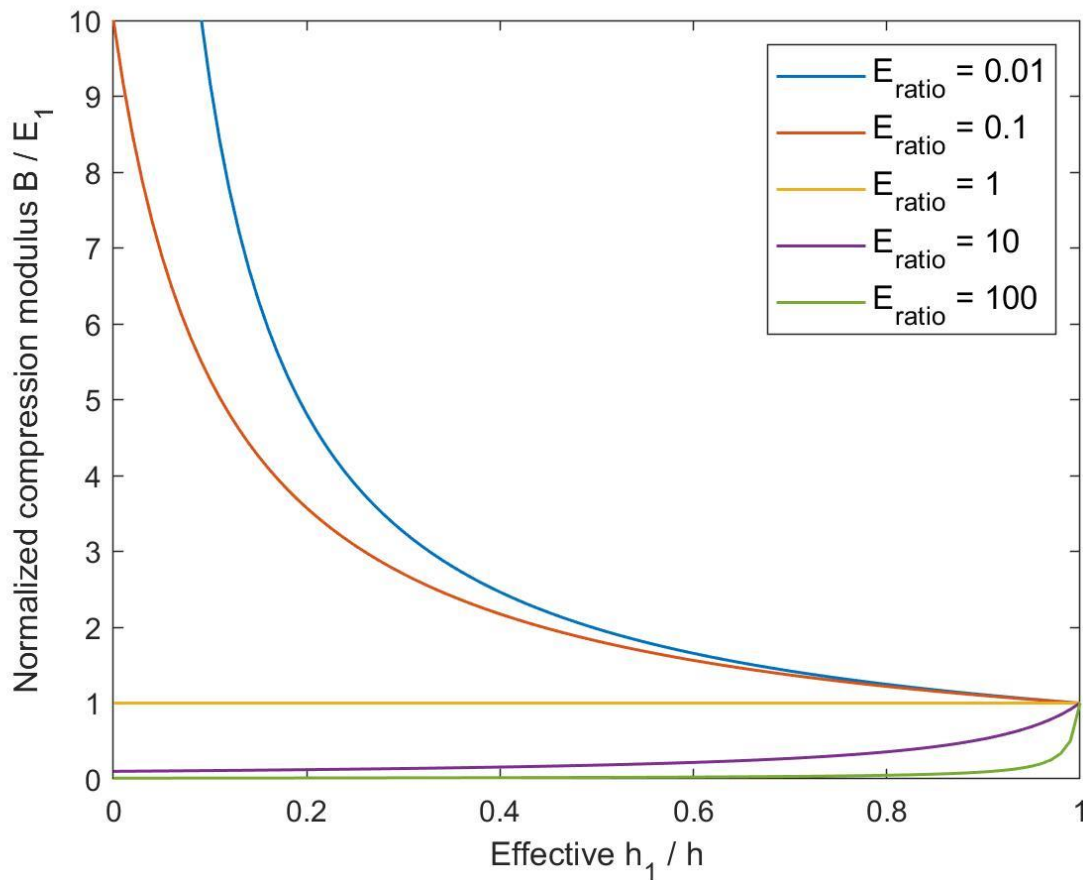
$$E_{\text{eff}} = \frac{\sum_{i=1}^N E_i \cdot \sum_{i=1}^N h_i}{\sum_{i=1}^N \left( \frac{h_i}{E_i} \left( \sum_{i=1}^N E_i \right) \right)}$$

OUT LET'S MAKE IT DIMENSIONLESS FOR MODELLING PURPOSES

→ LASTLY LET'S CONSIDER THAT WE ONLY HAVE 2-LAYERS FOR OUR MODEL

$$E_{\text{eff}} = \frac{\sigma}{\epsilon} = \frac{\sigma \cdot h}{\epsilon \cdot h_i} \Rightarrow$$

$$E_{\text{eff}} = \frac{h}{\sum E_i h_i}$$



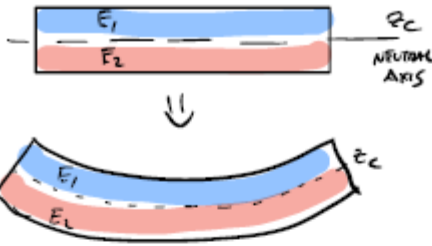
From this graph, we see that the compression modulus is much more skewed by the “softer” material or the material with the lower Young’s modulus. As we can see when  $E_1$  is significantly larger than  $E_2$  which is indicated by an  $E_{\text{ratio}}$  larger than 1, the normalized compression modulus drops dramatically and converges sharply only when the height of material 1 composes nearly all of the stack height (indicated by moving in the positive x-axis direction.)

Alternatively, when  $E_2$  is larger than  $E_1$  (when  $E_{\text{ratio}}$  is less than 1), the normalized compression modulus is skewed in the same but opposite way from the start. Both ends of  $E_{\text{ratio}}$  converge as  $E_1$  begins to make up a majority of the stack height.

Essentially, the takeaway is that the softest material skews the normalized compression modulus. An analogous way to think about this is the compressive force will want to travel through the path of least resistance which is the softer material because it will give way the most.

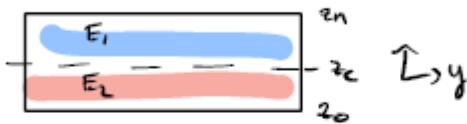
# Effective Bending Modulus

## BENDING MODULUS



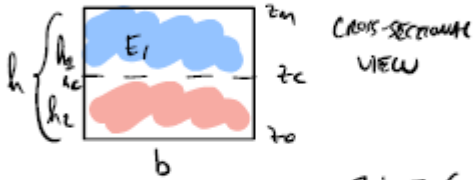
→ GOALS:  
 - LOCATE NEUTRAL AXIS  
 - OBTAIN EFFECTIVE BENDING MODULUS

→ IMPORTANT FORMULAS  
 $\kappa = \frac{1}{\rho}$   
 $\sigma = E\epsilon$      $\epsilon = \kappa y$   
 $\frac{1}{\rho} = \frac{M}{(EI)_{\text{EFF}}}$



$$\epsilon = \kappa y = -\kappa (h - z_c)$$

$$\sigma = -E\kappa (h - z_c)$$



MOMENT OF INERTIA  
 $b \int -E_i \kappa (z - z_c) dA = 0$   
 $\downarrow$   
 $b \int_{z_{i-1}}^{z_i} E_i (z - z_c) dz = 0$     ≠ ASSUME CONSTANT b FOR ALL LAYERS

$$\sum_i \frac{1}{2} E_i (z_i^2 - z_{i-1}^2) - \sum_i E_i z_c (z_i - z_{i-1}) = 0$$

SOLVE FOR  $z_c$ :

$$z_c = \frac{\sum_{i=1}^n E_i (z_i^2 - z_{i-1}^2)}{2 \sum_{i=1}^n E_i (z_i - z_{i-1})}$$

NOW SIMPLIFY FOR 2-LAYER MODEL --

$$z_c = \frac{E_1 (z_1^2 - z_0^2) + E_2 (z_2^2 - z_1^2)}{2 (E_1 (z_1 - z_0) + E_2 (z_2 - z_1))}$$

→ FIND  $E_{\text{EFF}}$

$$I = \frac{bh^3}{12}$$

$$E_{\text{EFF}} I = \kappa y \Rightarrow \frac{E_{\text{EFF}} h^3}{12} = -E\kappa (h - z_c)$$

$$\frac{E_{\text{EFF}} h^3}{12} = \sum_i \frac{E_i}{3} \cdot ((h_i - h_c)^3 - (h_{c,i} - h_c)^3) \Rightarrow \frac{E_{\text{EFF}} h^3}{12} = \sum_{i=1}^n E_i \cdot I_i$$

MAKE DIMENSIONLESS:  
 DIVIDE  $z_c$  BY  $z_n$

$$\frac{E_{\text{EFF}}}{E_1} = 4 \sum_{i=1}^n E_i \left( \left( \frac{z_i}{z_n} - \frac{z_c}{z_n} \right)^3 - \left( \frac{z_{i-1}}{z_n} - \frac{z_c}{z_n} \right)^3 \right)$$

## MATLAB Code:

```
%% EFFECTIVE COMPRESSION MODULUS %%

clc

clear

close all

%initialize values

h_eff = (0: .01: 1); % create a vector of possible h_eff values

h_eff = h_eff'; % invert the row to become a matrix

[r, c] = size(h_eff); % define the size of the h_eff matrix

h2_eff = (ones([r, c]))-h_eff; % create a matrix of h2_eff values

Eratio = [0.01; 0.1; 1; 10; 100]; % define possible values of E_ratio (can do this since Eratio is dimensionless)

B_bar = zeros(r, length(Eratio)); % define initial Matrix

% iterate through the columns of B_bar to update it to contain the relevant
% value

for i = 1:5

    B_bar(:,i) = 1./(h_eff + Eratio(i)*h2_eff);

end

% plot B_bar

figure()

hold on

plot(h_eff, B_bar(:,1:5), "Linewidth", 2)

xlim([0 1])

ylim([0 10])

xlabel("Effective 1/h")

ylabel("Normalized compression modulus")

legend('E{ratio} = 0.01', 'E{ratio} = 0.1', 'E{ratio}=1', 'E{ratio} = 10', 'E{ratio} = 100')

title("Normalized compression modulus as a function of Effective 1/h")
```

## %% EFFECTIVE BENDING MODULUS %%

NOTE: after consulting my team members, we struggled to draw conclusions from the simulation and could not develop graph to visualize the consequences of changing the two parameters we derive from this:  $E_{\text{eff}}$  and  $z_{\text{ratio}}$

```
%% initialize values %%
```

```
N = round(rand()*10); % generate a number for the number of layers%
```

```
z_ratio = linspace(0, 1, N); % using ratio to convey what percentage of stack is material 1 %
```

```
Eratio = [0.01; 0.1; 1; 10; 100]; % possible values of the E ratio
```

```
h = ones(size(N)); % an array of ones representing the total length
```

```
z = [round(rand(5))*10]; % generate many random numbers for z
```

```
E1 = 5; % randomly assign a value to E1 and E2%
```

```
E2 = 7;
```

```
%% create general expressions for z locations %%
```

```
z_numerator = [E1*(z(1)^2)];
```

```
z_denominator = [E1*(z(1))];
```

```
% calculate what the distance of the neutral axis is
```

```
for i = [1:length(z)]
```

```
%%if mod(i,2) == 0 % check to see if even, and if so, use E2 %%
```

```
z_numerator(i) = E2*(z(i)^2-z(i-1)^2);
```

```
z_denominator(i) = E2*(z(i)-z(i-1));
```

```
%% if mod(i,2) ~= 0 % check to see if odd, and if so, use E1 %%
```

```
z_numerator(i) = E1*(z(i)^2-z(i-1)^2);
```

```
z_denominator(i) = E1*(z(i)-z(i-1));
```

```
end
```

```
end
```

```
end
```

```
%% Using the information from what is above, calculate the distance to the neutral axis %%
```

```

zc = sum(z_numerator)/(2*sum(z_denominator));
E = [E1 * ((z(1)/z(N) - zc/z(N))^3)];
for i = [2:length(z)]
%%if mod(i,2) == 0 % calculate for even layers (E2)%%
E(i) = E2 * ((z(i)/z(N) - zc/z(N))^3 - (z(i-1)/z(N) - zc/z(N))^3);
%% if mod(i, 2) ~=0 % calculate for odd layers (E1)%%
E(i) = E1 * ((z(i)/z(N) - zc/z(N))^3 - (z(i-1)/z(N) - zc/z(N))^3);
end
end
end

%% calculate the effective final values %%
E_eff_final=4 * sum(E)

```