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2.980 Individual Technical
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Team Adidas Individual Technical #1: Foam Composite Literature Review

Introduction

EVA midsoles are among the most common types in athletic shoes. Made up of corn starch and ethylene-vinyl acetate, it's lightweight, elastic quality makes it an attractive material (LIU). New compositions of midsoles are continuously being created as the athletic shoe industry continues to grow rapidly. Qualities vital in an athletic shoe are comfort and energy return, to optimize both enjoyment when wearing the shoe and opportunity for physical performance. There is an interplay between the effects of material for comfort/injury prevention and material for speed. For athlete's it's important to maximize the energy returned in each step they take while minimizing energy expenturature. Midsoles lose approximately 30% of the energy output [2]. When constructing a shoe, it's important to prioritize reducing this value, finding materials that can accomplish this.

Optimizing Composition in Energy Return, Material Type, and Material Location

Air-sole materials have been shown to result in significantly less oxygen consumption during heel-toe running than shoes with EVA midsoles[2]. This makes foam an attractive material. However, bending of both the midsole and the foot itself lead to energy consumption. The action of compressing the midsole when taking a step and the subsequent bending of it leads to energy absorption. Additionally, there is little energy produced when an athlete jumps or pushes from the ground, as the metatarso-phalangeal (MP joint) absorbs additional energy [2].

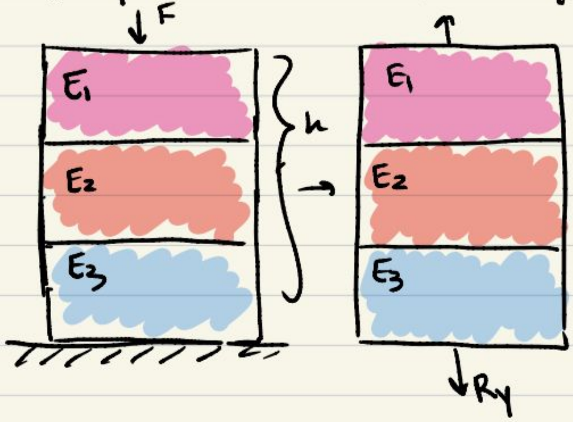
As foam material will be used for this project, it's important to counteract the elasticity of foam leading to this energy consumption with stiffer materials. Creating a foam composite incorporating carbon-fiber plates could be extremely advantageous. Carbon fabric-reinforced laminated composite materials have high specific strength and bending stiffness [3]. Incorporating this material in midsole composites have been shown to increase stiffness, reducing bending of the midsole as well as the resulting flexion in the MP joint [1]. This reduced energy loss during both running and jumping movements [2]. When building a shoe, material location needs to be investigated. Depending on where different materials are located within the midsole system can lead to profound effects on both cushioning and stability [4].

Foam Qualities

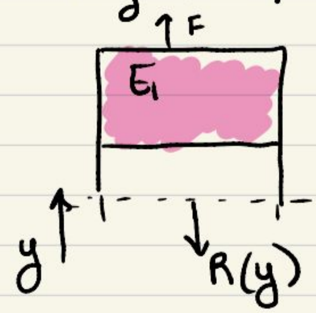
The lightweightness of a midsole is important, as it makes the shoe more aerodynamic, reducing the energy expended by the athlete against external forces, such as gravity and air resistance [2].

The aerated quality of foam contributes to it's lightweightness. The optimal thickness and flexibility of a midsole depends on the purpose of the wearer. For activities such as jogging and walking, a shoe that is flexible is better on a hard surface to conserve energy of the wearer, suggesting that foam would be a great material to use [5]. When it comes to testing the wear of foam for materials, foam exemplifies a low bending moment, but when placed into a shoe, it exhibits a high bending moment. Studies have found that it's important to test the qualities of the material after the composite is constructed, to best understand how all the components will act in a foam composite, as testing our composite will come in to play very soon [5].

compression modulus n-layers



$\Sigma_i F_y = -F + R_y = 0 \rightarrow F = R_y$



constituative: $\sigma_{yy} = E_1 \varepsilon_{yy,1}$

$\sigma_{yy} = E_2 \varepsilon_{yy,2} \longrightarrow \sigma_{yy} = \frac{F}{A} \Rightarrow A E_1 \varepsilon_{yy,1} = A E_2 \varepsilon_{yy,2}$

$\sigma_{yy} = E_3 \varepsilon_{yy,3} \qquad = A E_3 \varepsilon_{yy3} =$

compatability $\varepsilon_1 = \frac{\delta_1}{h_1}$, $\varepsilon_2 = \frac{\delta_2}{h_2}$, $\varepsilon_3 = \frac{\delta_3}{h_3}$ $\Big\} \; \delta = \delta_1 + \delta_2 + \delta_3$

$\hookrightarrow \delta_1 = -\frac{F h_1}{A E_1} \quad \delta_3 = \frac{F h_3}{A E_1}$

$\delta_3 = \frac{F h_2}{A E_1}$

with n layers:

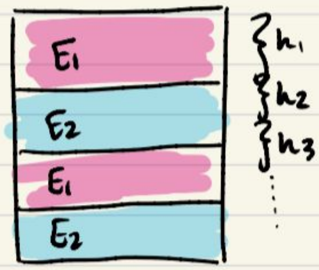
$h = h_1 + h_2 + h_3 + \ldots \rightarrow \sum_{i=1}^{i=n} h_i \qquad \delta = \delta_1 + \delta_2 + \delta + \ldots \delta_n = \sum_{i=1}^{i=n} \delta_i$

$\delta = \frac{-F}{A} \sum_{i=1}^{N} \frac{h_i}{E_i}$

$-F/A = \delta / \Sigma h_i \left(\frac{\Sigma h_i}{\Sigma h_i / E_i} \right)$

$E_{eff} = \frac{\prod_{i=1}^{N} E_i \cdot \sum_{i=1}^{N} h_i}{\sum_{j=1}^{N} \left(\left(\prod_{i=1}^{N} E_i \right) \frac{h_j}{E_j} \right)}$

In our example, we only have E_1, E_2



$$h = h_1 + h_2 + h_3 + \dots h_n = \sum_{i=1}^{N} h_i$$

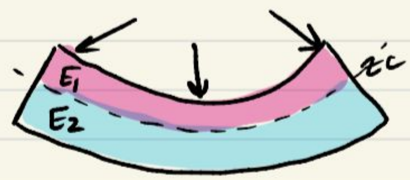
① constituative: $E_i \varepsilon_i = \sigma_i$

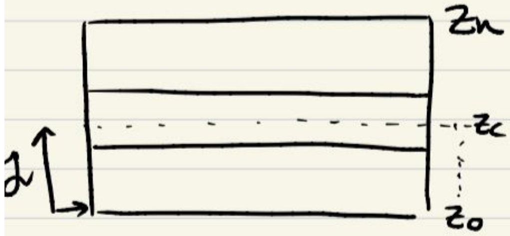
② $\sigma = \sigma_i$, $i = 1, 2, \dots N$

③ compatability $h\varepsilon = \sum_i \varepsilon_i h_i$

$$E_{eff} = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\frac{1}{h}\sum \varepsilon_i h_i} = \frac{1}{\sum E_i \left(\frac{h_i}{h}\right)} \Rightarrow \begin{cases} \sum h_i = h \\ E_i = E_1 \text{ or } E_2 \end{cases}$$

Bending modulus





constituative:

$\sigma = E\varepsilon$

geometric

$\varepsilon = ky$

(defind from neutral axis, as we assume there is no other compression)

$\varepsilon = ky = -k(z - z_c)$

$\sigma = E\varepsilon = -Eky = -Ek(z - z_c)$

$b \sum_i \int_{A_i} -E_i k (z - z_c)\, dA = 0$

$y = z - z_c$

$\delta A = b \cdot$ thickness difference

$b_i \sum_i \int_{z_{i-1}}^{z_i} E_i (z - z_c)\, dz = 0$ can drop b since uniform

$\sum_i \frac{1}{2} E_i (z_i^2 - z_{i-1}^2) - \sum_i E_i z_c (z_i - z_{i-1}) = 0$

$$\Rightarrow \quad z_c = \frac{\sum_{i=1}^{n} E_i (z_i^2 - z_{i-1}^2)}{2 \sum_{i=1}^{n} E_i (z_i - z_{i-1})}$$

position of neutral axis from bottom layer.

2 layers.

$$z_c = \frac{E_1 (z_1^2 - z_0^2) + E_2 (z_2^2 - z_1^2)}{2 \cdot [E_1 (z_1 - z_0) + E_2 (z_2 - z_1)]}$$

Balance of moment: for each cross section, stress σ.

moment contributed by each of cross section. Add in $y \rightarrow$ distance from neutral axis.

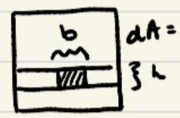
$$\int_{A_i} \sigma y A = \sum_i \int_{A_i} \sigma (z - z_c)\, d(b(z - z_c))^{dA} + M = 0$$

$\rightarrow$ Turn into one beam with material E_{eff}

$-E\kappa(z - z_c)$

$\cdot (z - z_c)$

$$\sum_i \int -E\kappa(z - z_c)\, d(b(z - z_c))$$



$dA = d[b(z - z_c)]$ ($z - z_c$ = y)

differential

$$= -E_{eff}\kappa\, y\, y\, d[by] + M = 0$$

($-E_{eff}\kappa y$ → stress; y → distance; by → area)

① $\frac{1}{3} E_i \kappa (z - z_c)^3 \Big|_{z_{i-1}}^{z_i} b + M = 0$

geometric constant

$$\not{b} \sum_{i=1}^{n} \frac{1}{3} E_i (z - z_c)^3 \Big|_{z_{i-1}}^{z_i} = \not{b}\, E_{eff} \cdot \frac{1}{3} y^3 \Big|_{-h/2}^{h/2}$$

$$= E_{eff} \cdot \frac{1}{3} \cdot \frac{h^3}{8} \cdot 2$$

$$\sum E_i \frac{1}{3}\left[(z_i - z_c)^3 - (z_{i-1} - z_c)^3 \right] = E_{eff} \cdot \frac{1}{3} \cdot \frac{h^3}{8} \cdot 2 = E_{eff} \cdot \frac{1}{12} h^3$$

$\downarrow$

$$\sum_{i=1}^{n} E_i - I_i = E_{eff} \cdot \frac{1}{12} h^3$$

find dimensionless form of z_c:

$$z_c = \frac{\sum_{i=1}^{n} E_i (z_i^2 - z_{i-1})^2}{2 \sum_{i=1}^{n} E (z_i - z_{i-1})} \rightarrow \frac{z_c}{z_n} = \sum_{i=1}^{n} \frac{E_i \left(\frac{z_i^2}{z_n} - \frac{z_{i-1}^2}{z_n}\right)}{2 \sum_{i=1}^{n} E_i \left(\frac{z_i}{z_n} - \frac{z_{i-1}}{z_n}\right)}$$

DIMENSIONLESS

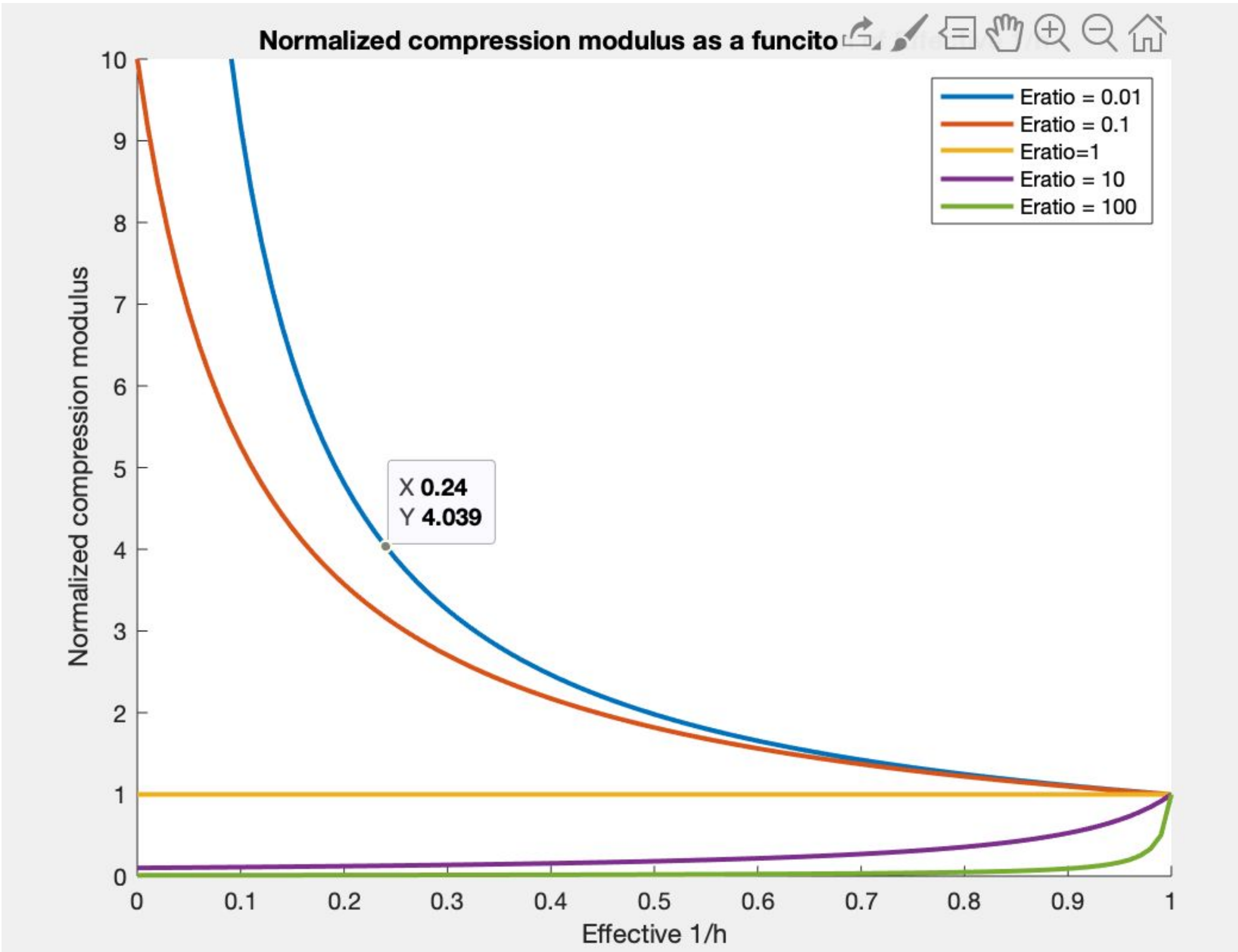
↓

plug into $\frac{z_c}{z_n}$ and solve for $\hat{E}_{eff}$.

$$E_{eff} \cdot \frac{1}{12} = \sum_{i=1}^{n} E_i \left(\frac{1}{3}\right) \left[\left(\frac{z_i}{z_n} - \frac{z_c}{z_n}\right)^3 - \left(\frac{z_{i-1}}{z_n} - \frac{z_c}{z_n}\right)^3 \right]$$

MATLAB Code Evaluation:

Compression:



It's important our equations are dimensionless, so we can use experimental values to reflect a hypothetical situation of compression. This graph demonstrates the effective 1/h, the height, versus the normalized compression modulus. The change of the normalized compression modulus is inversely proportional to the size off the E ratio. For very large E effective ratios, it appears as though there are very small changes in the normalized compression modulus for increasing values of h, suggesting that models with a high E ratio are more resistant to compression. However, for very small E effective ratios, it incremental increases of h result in drastic changes to the normalized compression modulus, possibly suggesting that models with a low E ratio are less resistant to compression.

Bending

I had a difficult time graphically representing my data. The calculation of E_effective relied on what the location of the neutral axis was. As we did not know this value, I tried to represent it as a random variable, generated differently each time the code ran, to result in different values of the calculated E_effective.

MATLAB Code:

```
%% COMPRESSION MODULUS

clc
clear
close all

%initialize values
h_eff = (0: .01: 1); % create a vector of possible h_eff values
h_eff = h_eff'; % invert the row to become a matrix
[r, c] = size(h_eff); % define the size of the h_eff matrix
h2_eff = (ones([r, c]))-h_eff; % create a matrix of h2_eff values
Eratio = [0.01; 0.1; 1; 10; 100]; % define possible values of E_ratio (can do this since Eratio is dimensionless)
B_bar = zeros(r, length(Eratio)); % define initial Matrix

% iterate through the columns of B_bar to update it to contain the relevant
% value
for i = 1:5
    B_bar(:,i) = 1./(h_eff + Eratio(i)*h2_eff);

end

% plot B_bar
figure()
hold on
plot(h_eff, B_bar(:,1:5), "Linewidth", 2)
xlim([0 1])
ylim([0 10])
xlabel("Effective 1/h")
ylabel("Normalized compression modulus")
legend('E{ratio} = 0.01','E{ratio} = 0.1', 'E{ratio}=1', 'E{ratio} = 10', 'E{ratio} = 100')
title("Normalized compression modulus as a function of Effective 1/h")


%% Bending Modulus

% initialize values
N = round(rand()*10); % generate a number for the number of layers
z_ratio = linspace(0, 1, N); % ratio of distance at the neutral axis to the distance in question (represented as a ratio here, but calculated for a final neutral axis location later)
Eratio = [0.01; 0.1; 1; 10; 100]; % possible values of the E ratio
h = ones(size(N)); % an array of ones representing the total length
z = [round(rand(5))*10]; % generate many random numbers for z
E1 = 5; % randomly assign a value to E1 and E2
E2 = 7;
```

```
% initialize expressions
z_numerator = [E1*(z(1)^2)];
z_denominator = [E1*(z(1))];

% calculate what the distance of the neutral axis is
for i = [1:length(z)]
    if mod(i,2) == 0 % check to see if even, and if so, use E2
        z_numerator(i) = E2*(z(i)^2-z(i-1)^2);
        z_denominator(i) = E2*(z(i)-z(i-1));
    if mod(i,2) ~= 0 % check to see if odd, and if so, use E1
        z_numerator(i) = E1*(z(i)^2-z(i-1)^2);
        z_denominator(i) = E1*(z(i)-z(i-1));
    end
    end
end

% Using the information from what is above, calculate the distance to the
% neutral axis
zc = sum(z_numerator)/(2*sum(z_denominator));


E = [E1 * ((z(1)/z(N) - zc/z(N))^3)];

for i = [2:length(z)]
    if mod(i,2) == 0 % calculate for even layers (E2)
        E(i) = E2 * ((z(i)/z(N) - zc/z(N))^3 - (z(i-1)/z(N) - zc/z(N))^3);
    if mod(i, 2) ~=0 % calculate for odd layers (E1)
        E(i) = E1 * ((z(i)/z(N) - zc/z(N))^3 - (z(i-1)/z(N) - zc/z(N))^3);
    end
    end
end

% calculate the effective final values
E_eff_final=4 * sum(E)
```

References:

[1] Gupta, N. “Characterization of Flexural Properties of Syntactic Foam Core Sandwich Composites and Effect of Density Variation.” December 2004.

[2] Stefanyshyn & Nigg. “Energy Aspects Associated with Sports Shoes.” 2000.

[3] Xiong, et. al. “Sandwich Structures with Prismatic and Foam Cores: A Review.” October 2018.

[4] Drougkas, et. al. “Gait-Specific Optimization of Composite Footwear Midsole Systems, Facilitated through Dynamic Finite Element Modelling.” Sept 2018.

[5] Park, et. al. “Effects of Hardness and Thickness of Polyurethane Foam Midsoles on Bending Properties of the Footwear.” 2007. *Fibers and Polymers*.