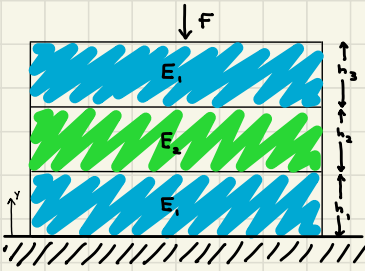
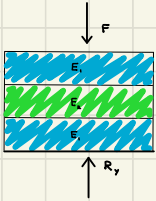


Compression

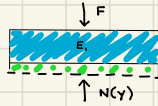


Equilibrium



$$\sum F_y = 0 = R_y - F$$

$$R_y = F$$



$$\sum F_y = 0 = N(y) - F$$

$$N(y) = F$$

Compatibility

$$\epsilon_{1yy} = \frac{\delta_1}{h_1}$$

$$\epsilon_{2yy} = \frac{\delta_2}{h_2}$$

$$\epsilon_{nyy} = \frac{\delta_n}{h_n}$$

$$\delta = \delta_1 + \delta_2 + \dots + \delta_n$$

Constitutive

$$\sigma_{yy} = E_1 \epsilon_{1yy}$$

$$\sigma_{yy} = E_2 \epsilon_{2yy}$$

$$N(y) = A \sigma_{yy}$$

$$\sigma_{yy} = \frac{F}{A}$$

assuming constant A

$$A E_i \epsilon_{iyy} = F = A E_i \frac{\delta_i}{h_i} \Rightarrow \delta_i = \frac{F h_i}{A E_i}$$

$$\delta = \delta_1 + \delta_2 + \dots + \delta_n \Rightarrow \delta = \frac{F}{A} \sum_{i=1}^n \frac{h_i}{E_i}$$

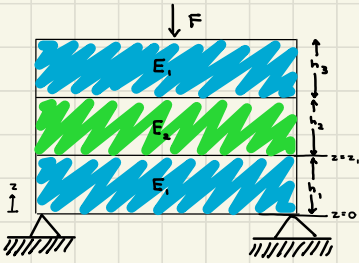
$$\sigma = E_{eff} \epsilon \Rightarrow \frac{F}{A} = E_{eff} \frac{\delta}{\sum h_i}$$

$$\underbrace{\frac{F}{A}}_{\sigma_{eff}} \quad \underbrace{\frac{\delta}{\sum h_i}}_{\epsilon_{eff}} = E_{eff}$$

$$\frac{\sum h_i}{\sum h_i / E_i} = E_{eff}$$

$$E_{eff} = \frac{E_1 \sum_{i=1}^n h_i}{\sum_{i=1,2,\dots} h_i E_i + \sum_{i=2,3,4,\dots} h_i E_i}$$

Bending



Constitutive

$$\sigma_i = E_i \varepsilon_i$$

Compatibility

$$\varepsilon_i = \kappa y = \frac{1}{\rho} y \quad \rightarrow \text{distance from neutral axis}$$

(+) strain on outside (-) y

Finding the neutral axis

Stress along neutral axis = 0

$z_c \rightarrow$ neutral axis

$$\int \sigma_{xx} dA = 0 = F$$

$$\int_{A_i} -E_i \kappa (z_i - z_c) dA = 0 = \int_{z_{i-1}}^{z_i} E_i (z - z_c) dz = 0$$

$$\Rightarrow \sum_i \frac{1}{2} E_i (z_i^2 - z_{i-1}^2) - \sum_i E_i z_c (z_i - z_{i-1}) = 0$$

$$\Rightarrow z_c = \frac{\sum_{i=1}^n E_i (z_i^2 - z_{i-1}^2)}{2 \sum_{i=1}^n E_i (z_i - z_{i-1})}$$

position of neutral axis from bottom layer $z=0$

Moment Balance

for each cross section

$$\int_{A_i} \sigma_y A = \sum_i \int_{A_i} \sigma (z - z_c) dA + M = 0$$

$$= \sum_i \int_{A_i} \sigma (z - z_c) d(b(z - z_c)) + M = 0$$

\rightarrow width of cross section assumed uniform

$$-M = \sum_i \int_{A_i} \sigma (z - z_c) d(b(z - z_c))$$

$$+M = \int b \sum_i \int_{A_i} -E_i \kappa (z - z_c) (z - z_c) dA$$

$$M = \sum_i \frac{1}{3} E_i (z - z_c)^3 \Big|_{z_{i-1}}^{z_i} b \kappa$$

$$M = E_{eff} I \kappa$$

$$\sum_i \frac{1}{3} E_i \left[(z - z_c)^3 - (z_{i-1} - z_c)^3 \right] = \frac{1}{12} E_{eff} h^3$$

I

$$\frac{1}{12} h^3 E_{eff} = \sum_{i=1}^n E_i - I_i$$

Need dimensionless

divide by h^3 and z_n
 \rightarrow total thickness

$$\frac{z_c}{z_n} = \frac{\sum_{i=1}^n E_i \left(\frac{z_i}{z_n} - \frac{z_{i-1}}{z_n} \right)}{2 \sum_{i=1}^n E_i \left(\frac{z_i}{z_n} - \frac{z_{i-1}}{z_n} \right)}$$

Plugging in you get

$$E_{eff} = 4 \sum_{i=1}^n E_i \left[\left(\frac{z_i}{z_n} - \frac{z_c}{z_n} \right)^3 - \left(\frac{z_{i-1}}{z_n} - \frac{z_c}{z_n} \right)^3 \right]$$

as one beam with one E_{eff}

$$-\int E_{eff} \kappa y b dy + M = 0$$

$$\frac{1}{12} E_{eff} h^3 = M$$

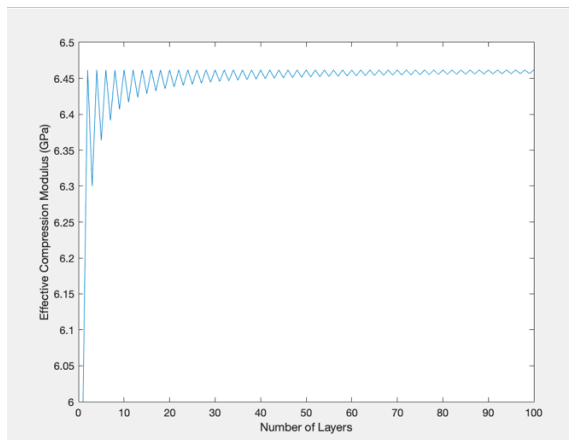
set the two equal

```

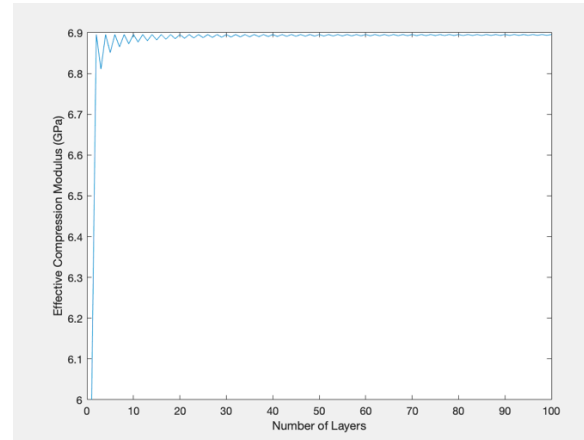
1 % Effective Compression Modulus
2
3 % Known individual modulus (in GPa)
4 E1 = 6;
5 E2 = 7;
6 % Given number of layers
7 n = 10;
8 % Given thicknesses for n layers
9 t1 = 1;
10 t2 = 6;
11 % Create the layers of given thicknesses
12 h = ones(n, 1);
13 for j = 1:length(h)
14     if rem(j,2)== 0
15         h(j) = h(j)*t2;
16     else
17         h(j) = h(j)*t1;
18     end
19 end
20
21 % The effective modulus for n number of layers is calculated below
22 Eeff_n = (E1*E2*sum(h))/((sum(h(1:2:end))*E2)+(sum(h(2:2:end))*E1));
23
24 % Plot the effective modulus
25 Eeffs = [E1]; %starts with just one layer with known modulus
26 for i = 2:n
27     Eeffs(i) = (E1*E2*sum(h(1:i)))/((sum(h(1:2:i))*E2)+(sum(h(2:2:i))*E1));
28 end
29 plot(1:n,Eeffs);
30
31 xlabel('Number of Layers');
32 ylabel('Effective Compression Modulus (GPa)');

```

n = 100; t1 = 1; t2=1

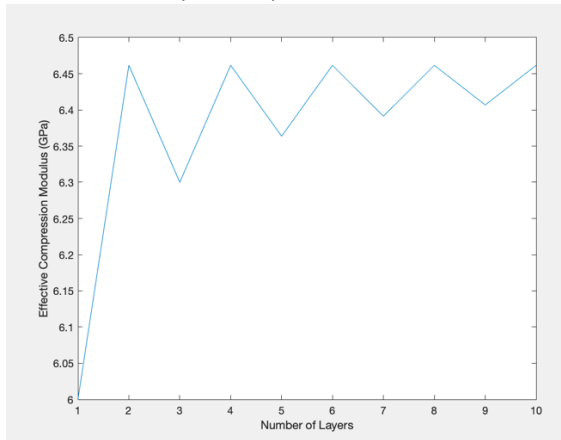


n = 100; t1 = 1; t2 = 100

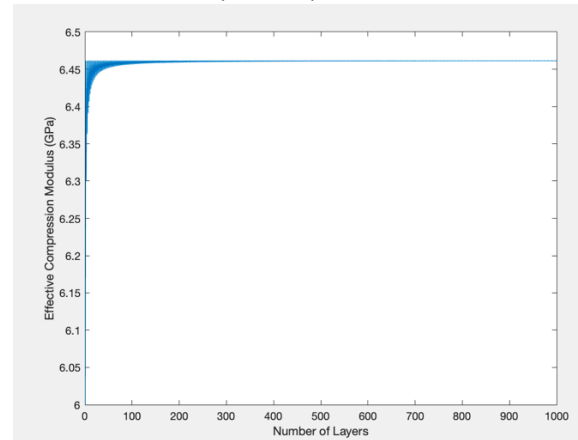


If one material is much thicker than the other, the effective modulus will converge on an effective modulus much faster than if the thicknesses are similar. The effective modulus will also settle at a value closer to the modulus of the thicker material, depending on the ratio between the two thicknesses.

$n = 10; t_1 = 1; t_2 = 1$



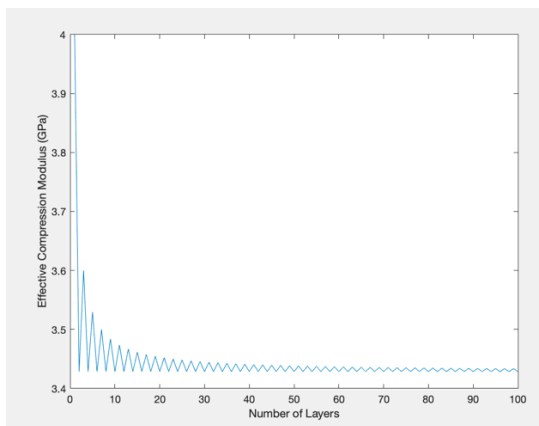
$n = 1000; t_1 = 1; t_2 = 1$



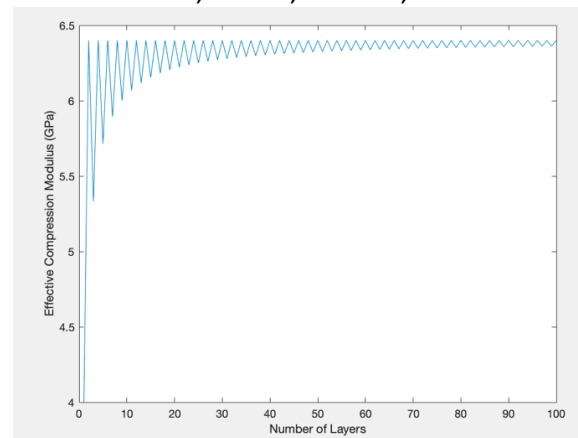
As n increases, the effective modulus begins to converge on a value, while at lower n , it will vary slightly from layer to layer.

The above two cases were done with $E_1 = 6$ and $E_2 = 7$ GPa.

$n = 100; E_1 = 4; E_2 = 3; t_1 = t_2 = 1$



$n = 100; E_1 = 4; E_2 = 16; t_1 = t_2 = 1$



As the ratio between the two individual modulus values increases, the effective modulus will vary more from layer to layer, so the amplitude of the oscillations is larger on the graph.