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Individual Deliverable #1

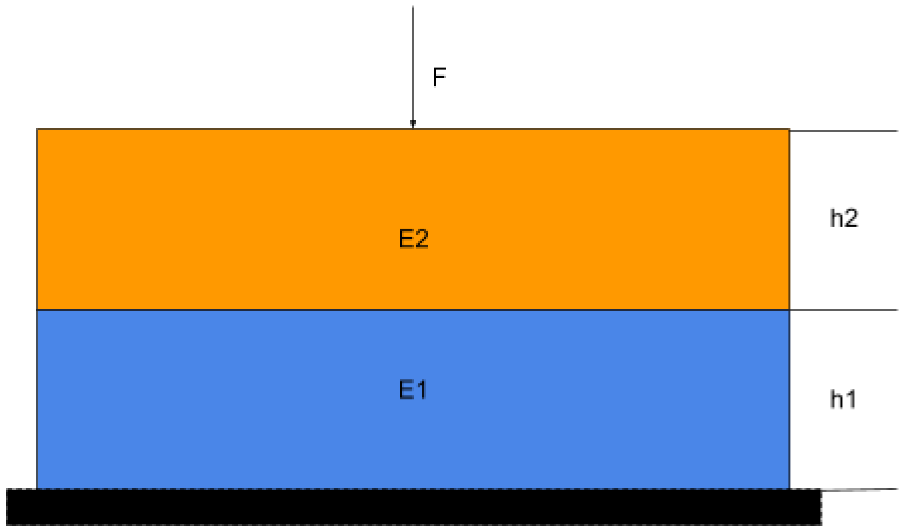
Literature Review:

This project focuses on the midsole of athletic shoes. This component is usually made up of thermoplastic urethane (TPU), ethyl vinyl acetate (EVA), or both [1]. EVA is the most widely used midsole material for shoes [2]. This is because EVA is lightweight, resists compression set, forms easily into molds, and is readily available in many different colors [2]. TPU, which makes up Adidas' Boost midsole material, is popular because of its similar properties. However, it differs slightly from EVA in that it is slightly heavier, yet more long lasting and resistant to abrasion and long-term deformation [3].

These foams do not respond to material deformations in a perfectly linearly elastic way. More complex measurements of a foams material properties are required to model the behavior of a foam in compression. These properties are: E_0 (the modulus of the base polymer), φ (a term used to describe the matrix geometry of a polymer), and ψ (the volume fraction of polymer in the foam). Given these features, one can model the material response a foam has to a given load-compression specification[4].

The summation of these foam types, their material compositions, and their responses to compression is the core of athletic shoe design. These factors all combine to produce different effects that athletes feel. The most impactful metric of a shoe is its energy return. In order to have a high energy return, a shoe must provide a balance between compliance and resilience. It must be able to stretch to store energy, and also return that stored energy[5]. However, as all athletes do not run in the same manner, it is not necessarily true that the highest possible energy returning shoe is ideal for every athlete. Different styles of running, as well as differences in foot and leg anatomy make it essential for athletes to find a shoe that compliments their individual characteristics [3]. This validates the objective of this project, as we set out to create a model that will help deliver an athlete their ideal shoe.

Effective Compression:



$$N(y) = -F$$
$$R_y = F$$

2 Layers:

Constitutive Relationships:

$$\sigma_{yy} = E_1 * \varepsilon_{1yy}$$
$$\sigma_{yy} = E_2 * \varepsilon_{2yy}$$

$$E_1 * A * \delta_1 = -F * h_1$$
$$\delta_1 = (-F*h_1)/(A*E_1)$$

$$E_2 * A * \delta_2 = -F * h_2$$
$$\delta_2 = (-F*h_2)/(A*E_2)$$

Compatibility:

$$\varepsilon_{1yy} = \delta_1/ h_1$$
$$\varepsilon_{2yy} = \delta_2/ h_2$$

$$\delta = \delta_1 + \delta_2$$

$$\delta = (-F/A) * (h_1/E_1 + h_2/E_2)$$

$$-F/A = E_{eff} * \delta/(h_1 + h_2)$$

$$\mathbf{E_{eff} = (E_1E_2(h_1+h_2))/(E_2h_1+E_1h_2)}$$
$$\mathbf{E_{eff}/E_1 = (1+h_1/h_2)/(E_1/E_2 + h_1/h_2)}$$

N Layers:

$$\mathbf{E_{eff}} = \frac{\sum_{i=1}^{N} \mathbf{h_i} \;*\; \mathbf{E_1 E_2}}{\sum_{j=1\ (odd)}^{N} \mathbf{E_2 h_j} + \sum_{k=2\ (even)}^{N} \mathbf{E_1 h_k}}$$

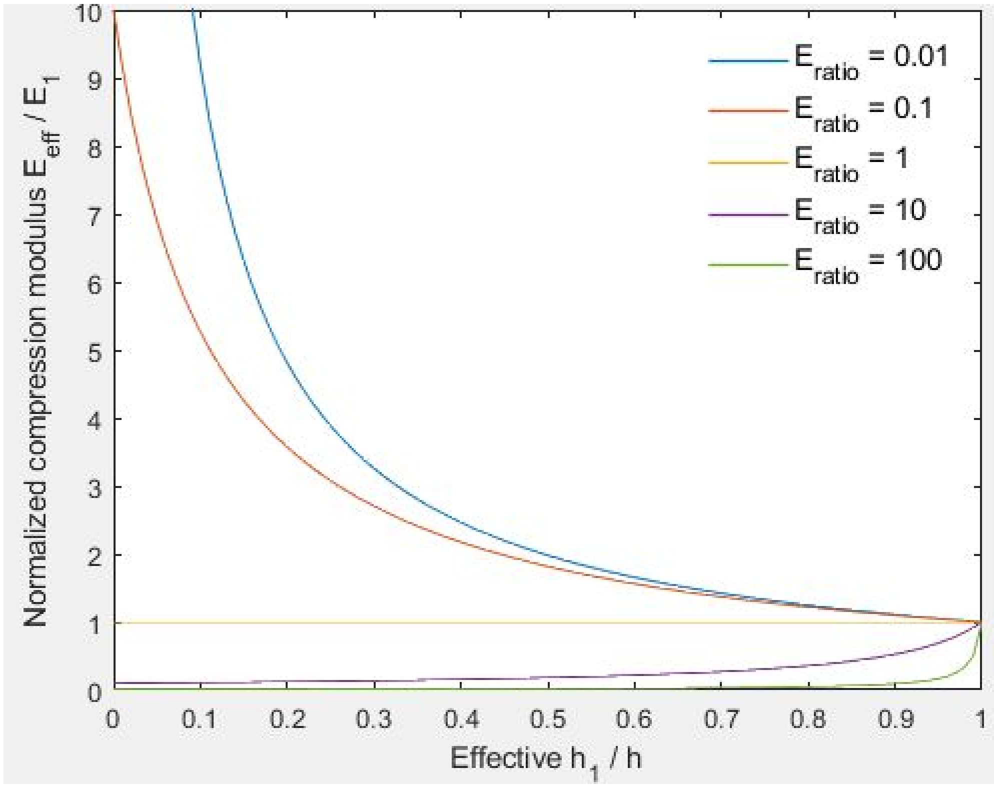
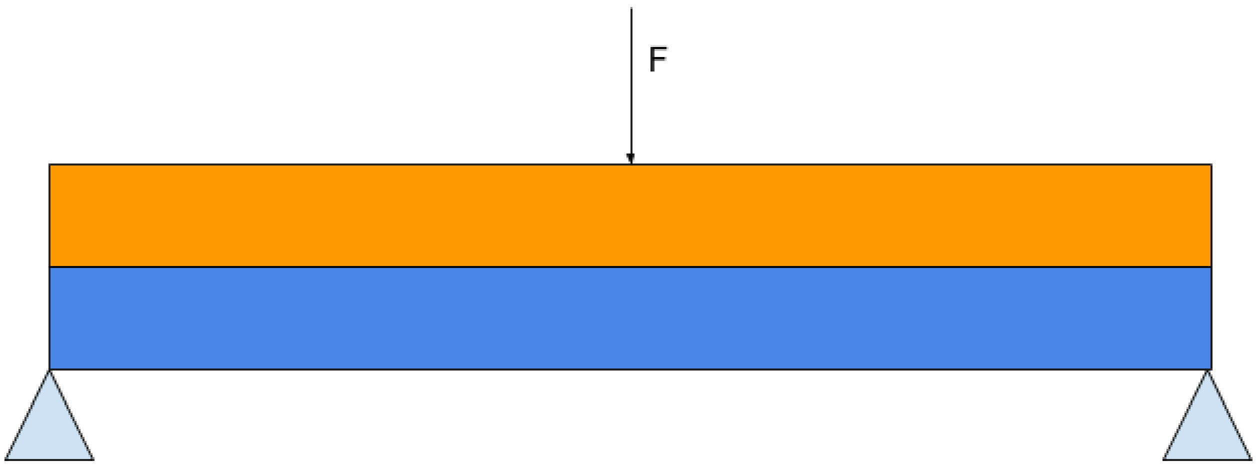


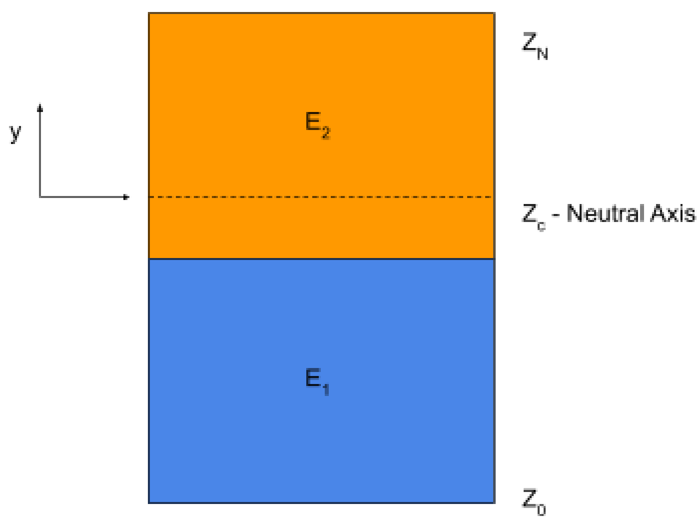
Figure 1: Normalized Compression modulus vs. Material height Ratios at various ratios between E_1 and E_2

Analysis of Graph:

- This graph was obtained using the Normalized Compression Modulus found in the derivation above. Different ratios between E_1 and E_2 were used to highlight the relationship between the normalized compression modulus and the height ratio between layers.
- From the graph, it is clear that when there is a very low ratio of E_1/E_2, a small change in h_1 creates a very large change in the effective compression of the composite, and vice versa. This means that adding even small layers of a material with a compression modulus much lower than the other material causes a large change in the over response of a material. On the other hand, adding small layers of a material with a high young's modulus has relatively low impact on the composite material's response.
- There does not appear to be any suspicious data within this graph. The only concern is the manufacturing feasibility of a midsole at the very limits of the height ratios, as it would be difficult to mass produce such a thin sheet of material

<u>Effective Bending:</u>





Cross-section of beam

Constitutive:

$$\sigma = E\varepsilon$$

Geometric:

$$\varepsilon = Ky$$

$\varepsilon = Ky = -K(h-Z_c)$ where h is height of the total stack

$$\sigma = E\varepsilon = -EK(h-Z_c)$$

$\sum_{i=1}^{N} \int_{hi-1}^{hi} E_i\,(h-h_c)\,dz = 0$ ---> to find position of neutral axis

$$\mathbf{Z_c} = \frac{\sum_{i=1}^{N} \mathbf{E_i\,(h_i^2 - h_{i-1}^2)}}{\mathbf{2^* \sum_{i=1}^{N} E_i\,(h_i - h_{i-1})}} \text{---> position of neutral axis}$$

Finding equivalent bending modulus with this neutral axis:

$$E_{eff} * (h^3/12) = -EK(h-Z_c)$$

$$E_{eff} * (h^3/12) = \sum_{i=1}^{N} E_i/3 *[(h_i-h_c)^3 - (h_{i-1}-h_c)^3]$$

$$E_{eff} * (h^3/12) = \sum_{i=1}^{N} E_i - I$$

After dividing Z_c by Z_n (total stack height):

$$\mathbf{E_{eff}/E_1 = 4 * \sum_{i=1}^{N} E_i\,[(Z_i/Z_n - Z_c/Z_n)^3 - (Z_{i-1}/Z_n - Z_c/Z_n)^3]}$$

MATLAB Code:

```
clc
clear
close all
%COMPRESSION
% This script is for the calculation of effective compression modulus B (dimensionless)

% Step 1: defination of variables
h1_eff = 0:0.01:1;                          % overall thickness of material 1
h1_eff = h1_eff';
[row, column] = size(h1_eff);
h2_eff = ones([row, column]) - h1_eff;              % overall thickness of material 2

E_ratio = [0.01; 0.1; 1; 10; 100];                  % modulus ratio of materials 1/2
B_bar = zeros(row, length(E_ratio));

% Step 2: calculation of effective compression modulus B
for i = 1:5                                 % index for E_ratio

   B_bar(:,i) = 1./(h1_eff + E_ratio(i)*h2_eff);       % dimensionless effective compression modulus B

end

% Step 3: plot results
% Plot the influence of h1_eff on B_bar
figure(1)
for i = 1:5
   plot(h1_eff, B_bar(:,i), 'LineWidth',1);
   hold on

end
hold off

% Figure setup
xlim([0,1]);
ylim([0,10]);
xlabel('Effective h_{1} / h','FontSize',11);
ylabel('Normalized compression modulus E_{eff} / E_{1}','FontSize',11);
legend({'E_{ratio} = 0.01','E_{ratio} = 0.1','E_{ratio} = 1','E_{ratio} = 10','E_{ratio} = 100'},'FontSize',11);



%BENDING
clc
clear
close all
z_n = 1;                   %stack height
```

```
n = 5;                    %number of layers
z_vec = 0:(z_n/n):z_n;
z_num = zeros(n);
z_den = zeros(n);
E_ratio = [0.01; 0.1; 1; 10; 100];

for i = [2:length(z_vec)]
   if mod(i,2) == 0          %even
      z_num(i) = E_ratio*((z_vec(i)^2/z_n^2)-(z_vec(i-1)^2/z_n^2));
      z_den(i) = E_ratio*((z_vec(i)/z_n)-(z_vec(i-1)/z_n));
   else                      %odd
      z_num(i) = ((z_vec(i)^2/z_n^2)-(z_vec(i-1)^2/z_n^2));
      z_den(i) = ((z_vec(i)/z_n)-(z_vec(i-1)/z_n));
   end
end
z_num
z_den
zc = sum(z_num)/(2*sum(z_den)) %location of neutral axis




E = zeros(n,1);
for i = [2:length(z_vec)]
   if mod(i,2) == 0                 % for even items
      E(i) = E_ratio*(((z_vec(i)/z_n)-(zc/z_n))^3-((z_vec(i-1)/z_n)-(zc/z_n))^3);
   else                             % for odd items
      E(i) = (((z_vec(i)/z_n)-(zc/z_n))^3-((z_vec(i-1)/z_n)-(zc/z_n))^3);
   end
end
E

E_eff = 4*sum(E)
```

Works Cited

[1]AAPSM. (n.d.). Footwear - Running Shoe Anatomy.

[2]Shoe Factory. (2019, September 24). Shoe Materials: EVA Midsoles.

[3]National Physical Therapy. (n.d.). Smart Running Discussion.

[4]Rusch, K.C. (1969), Load–compression behavior of flexible foams. J. Appl. Polym. Sci., 13: 2297-2311.

[5]Maldarelli, C. (2019, March 18). Can this new running shoe make novice runners faster?