ISR Theory: Part 3

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(with thanks to Phil Erickson and Josh Semeter)

Where did we get to so far?

- We talked about radio waves scattered from electrons
- We talked about the concept of Debye length
- For radar wavelength > Debye length we see collective behaviour
- In other words, our radar sees the wave modes in the plasma
- Looking along a beam, we will see waves moving toward and away
- To understand the radar spectrum we need to understand:
    - How fast these waves are moving (dependent on the medium)
    - How these waves are damped
    - What plasma parameter information can be gained from this

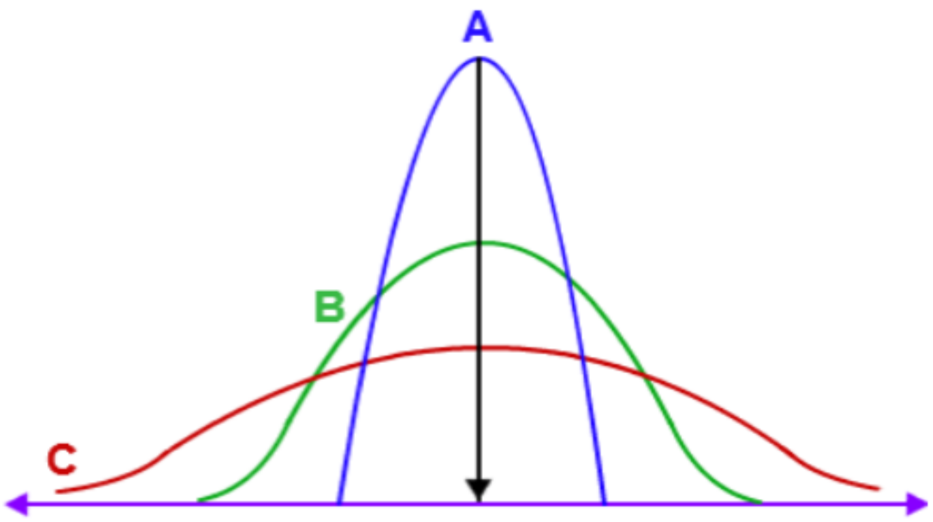
Dispersion relation: the concept

Simple linear case: uniform phase velocity

$$\omega(k) = c\ k$$

Most propagation speeds depend nonlinearly on the wavelength and/or frequency.

NB: for a **nonlinear** dispersion relation, the pulse will typically spread in either spatial frequency or temporal frequency as a function of time.



Example of pulse spreading spatially from time A to B to C.

http://www.mathcaptain.com/statistics/dispersion-statistics.html

Important thermal plasma dispersion relations

$$\epsilon(\omega, \vec{k}) = \text{function}\left(\omega^2/k^2\right)$$

Dielectric constant of the medium

Insert plasma dispersion relation here

We need the full dispersion relation expression.

This is not a plasma waves course so we won't derive it, but the two most important modes are:

1) *Ion-acoustic fluctuations* [sound waves in plasma]

$$\frac{\omega}{k} = \sqrt{\frac{k_B T_e + \gamma_i k_B T_i}{m_i}} = V_s$$

(Note no gamma term for electrons since they are isothermal, but ions are slow and suffer 1D compressions so their gamma term = 3)

NB: ordinary acoustic waves: adiabatic compression / decompression of fluid particles.

Ion-acoustic fluctuations: restoring force = electromagnetic

Important thermal plasma dispersion relations

$$\epsilon(\omega, \vec{k}) = \text{function}\left(\omega^2/k^2\right)$$

Insert plasma dispersion relation here

We need the full dispersion relation expression.

This is not a plasma waves course so we won't derive it, but the two most important modes are:

2) *Langmuir oscillations* (Plasma oscillations):

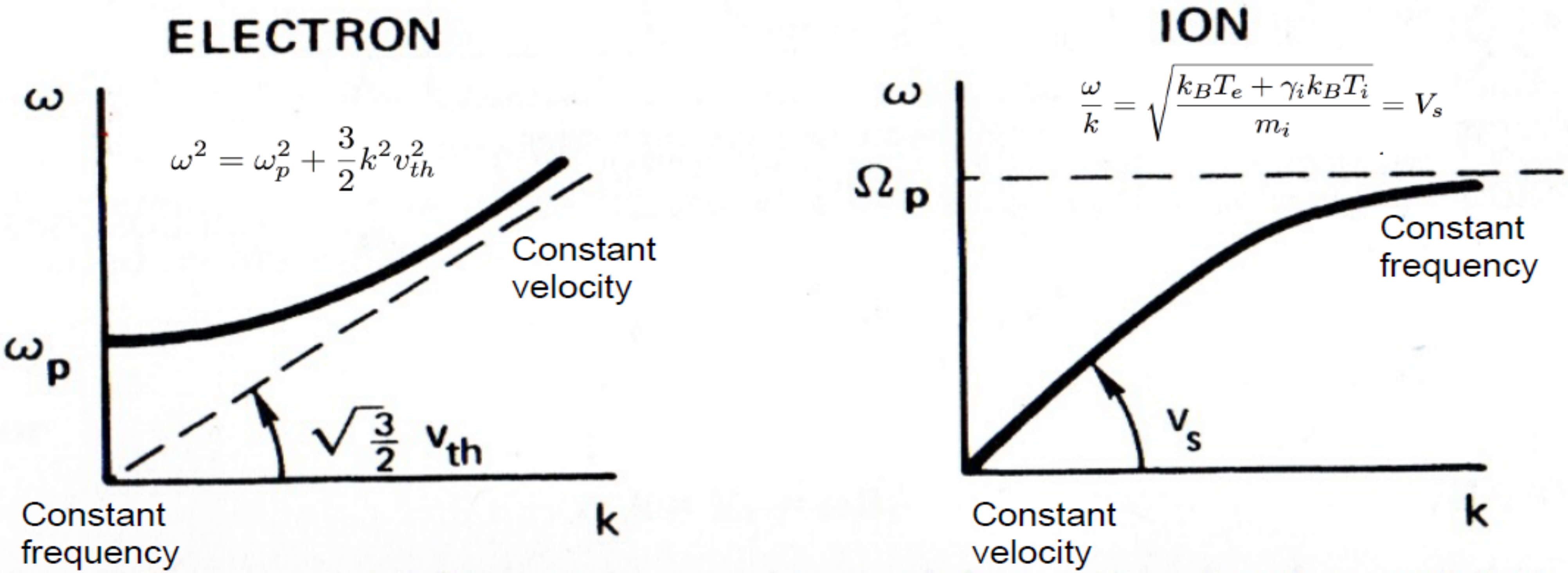
$$\omega^2 = \omega_p^2 + \frac{3}{2}k^2 v_{th}^2 \quad v_{th}^2 = 2k_B T_e/m_e$$

Akin to Brunt-Våisålå oscillations
in fluid (parcel in presence of density gradient) -
here, electrostatic field is restoring force, and
electron pressure gradient transmits information

Irving Langmuir (1881 - 1957)

When Langmuir arrived at the Laboratory, the director, Willis R. Whitney, told him to look around and see if there was anything he would like to "play with." Whitney would often ask him, "Are you having any fun today?" One day, after three years of apparently unproductive research, Langmuir answered, "I'm having a lot of fun, but I really don't know what good this is to the General Electric Company." Whitney replied. "That's not your worry. That's mine."

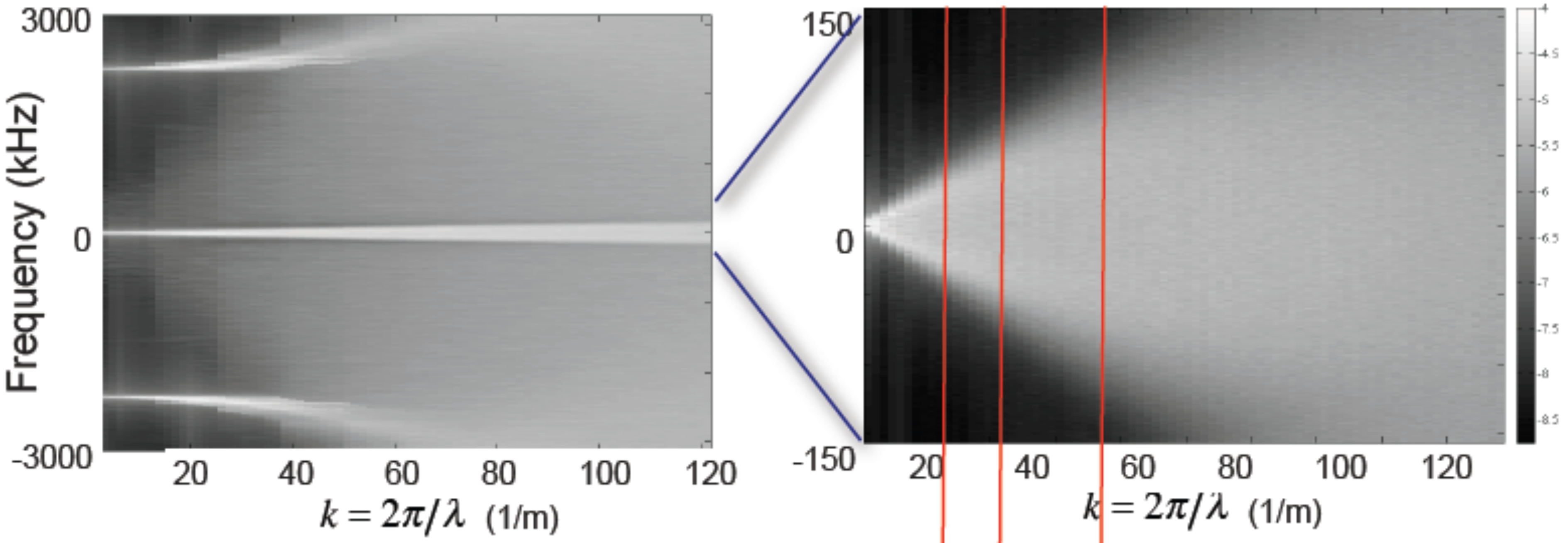
Electron and ion waves: Dispersion relations

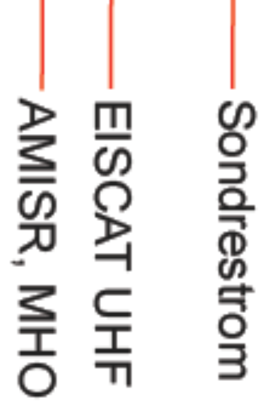


Comparison of the dispersion curves for electron plasma waves and ion acoustic waves.

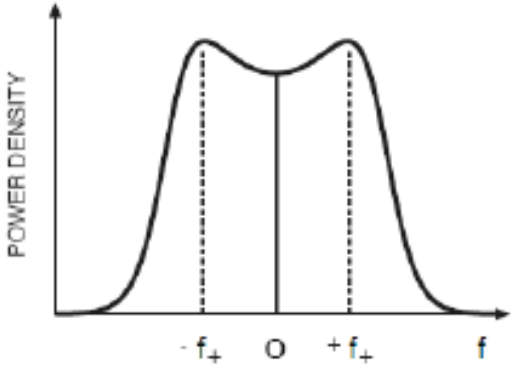
(Chen, Intro to Plasma Physics)

ISR Measures a Cut Through This Surface

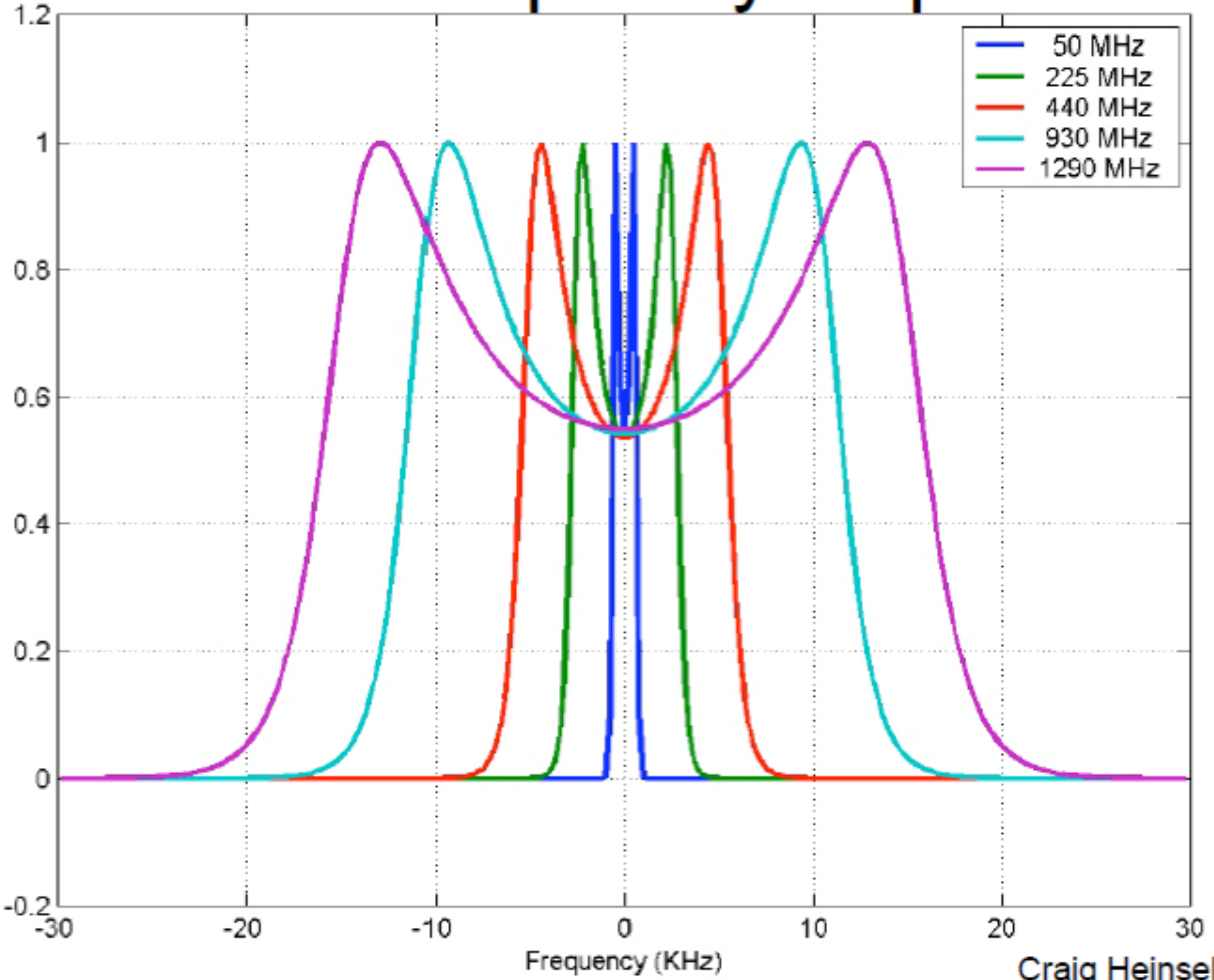




Ion-acoustic “lines” are broadened by Landau damping



Radar Frequency Dependencies



Parameters

Ne: 10^{12} m^{-3}

Ti: 1000 K

Te: 2000 K

Comp: 100% O^{+}

ν_{in}: 10^{-6} KHz

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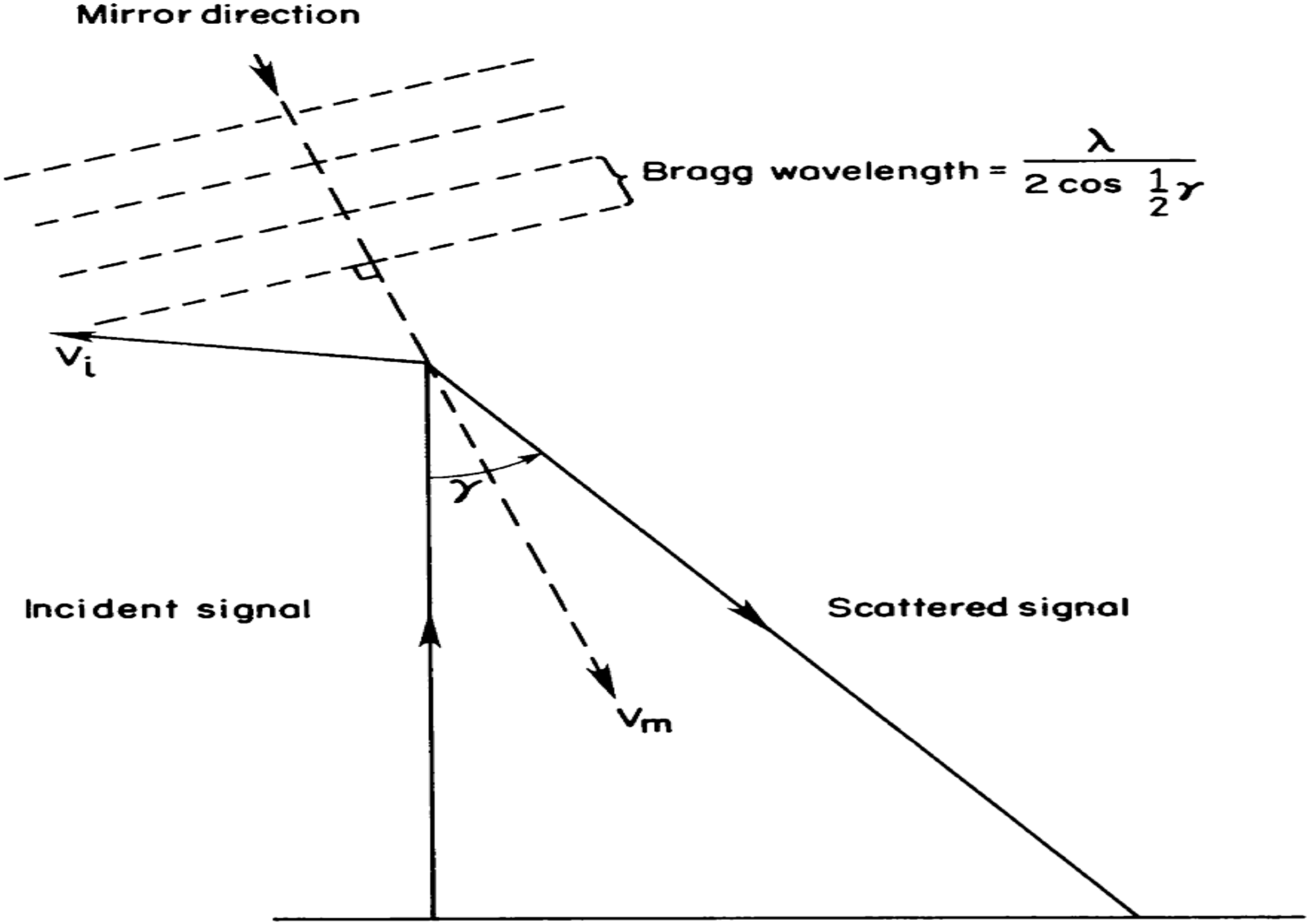
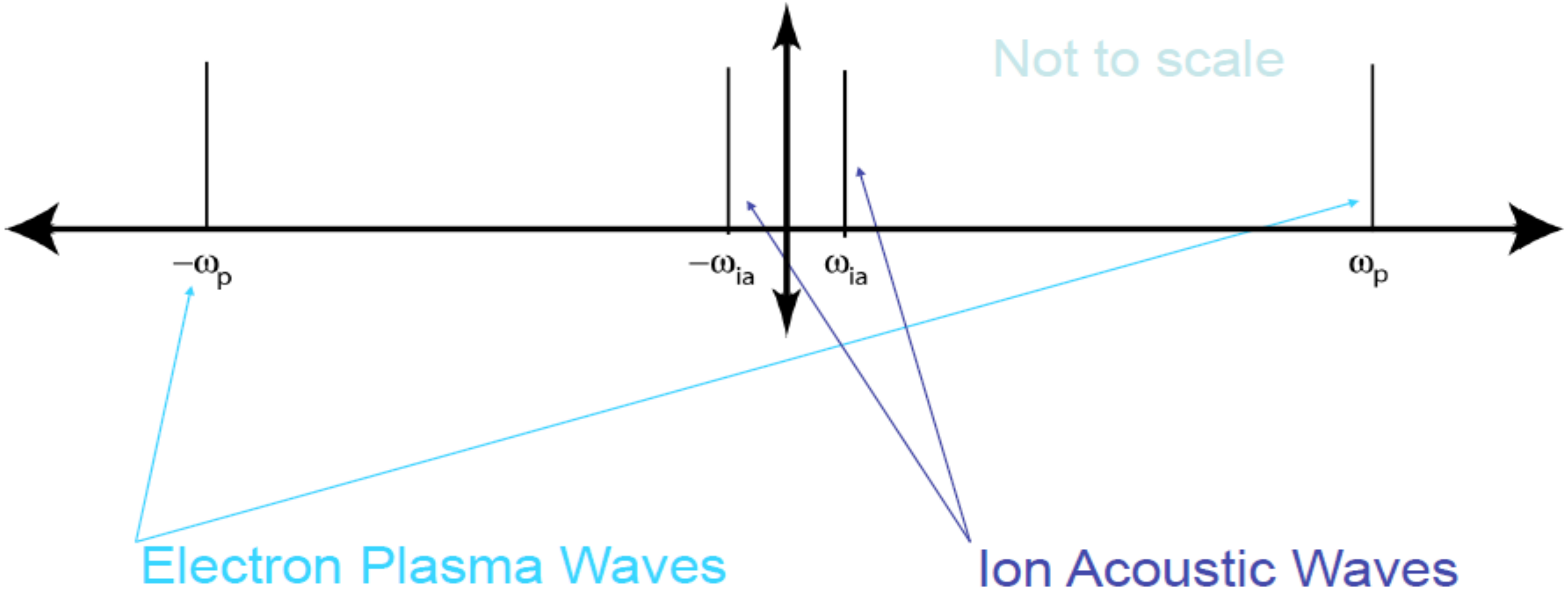
Frequency Ranges for ISR

- Note that ion-acoustic waves exist across a broad range of frequency space
- In principle, we can measure this kind of scatter for every wavelength above (about 5x) the Debye length, for as long as the ion-acoustic wave remains the dominant wave mode
- That second factor puts a long wavelength (low frequency) limit on ISR, because Spread F and other kinds of irregularity start to dominate.
- Our radar frequency determines which component of the frequency spectrum of ion-acoustic waves we pick out to observe
- For radar wavelength λ, the Bragg condition for direct backscatter is satisfied by waves with wavelength $\lambda /2$
- For a general scattering angle γ, the Bragg condition is :
    - $\lambda_b = \lambda / (2 \cos (\gamma/2))$



Mirror direction
Bragg wavelength = $\frac{\lambda}{2\cos\frac{1}{2}\gamma}$
V_i
γ
Incident signal
Scattered signal
V_m

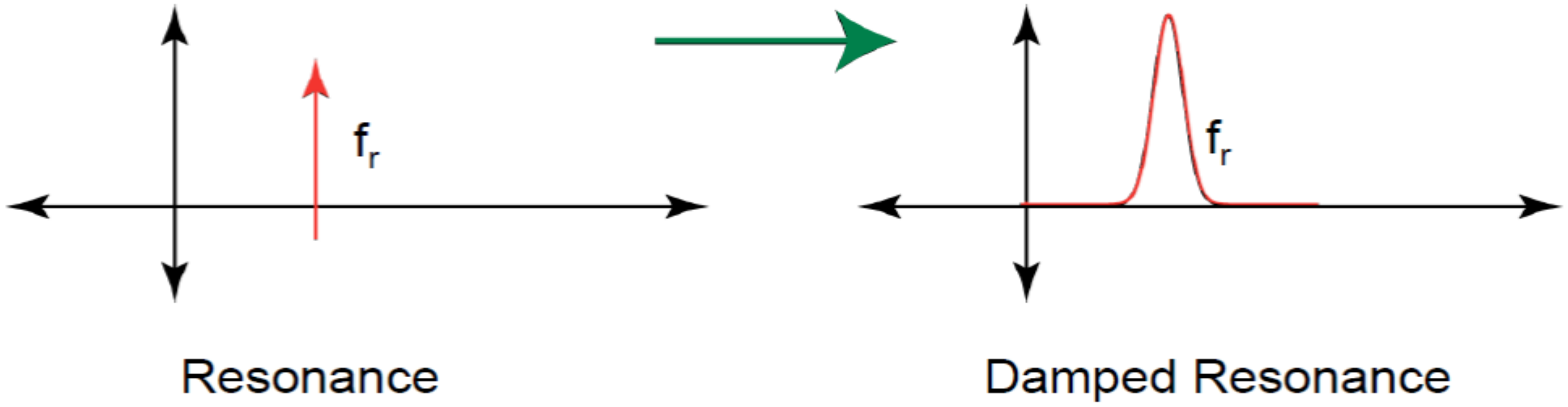
Wave Spectrum



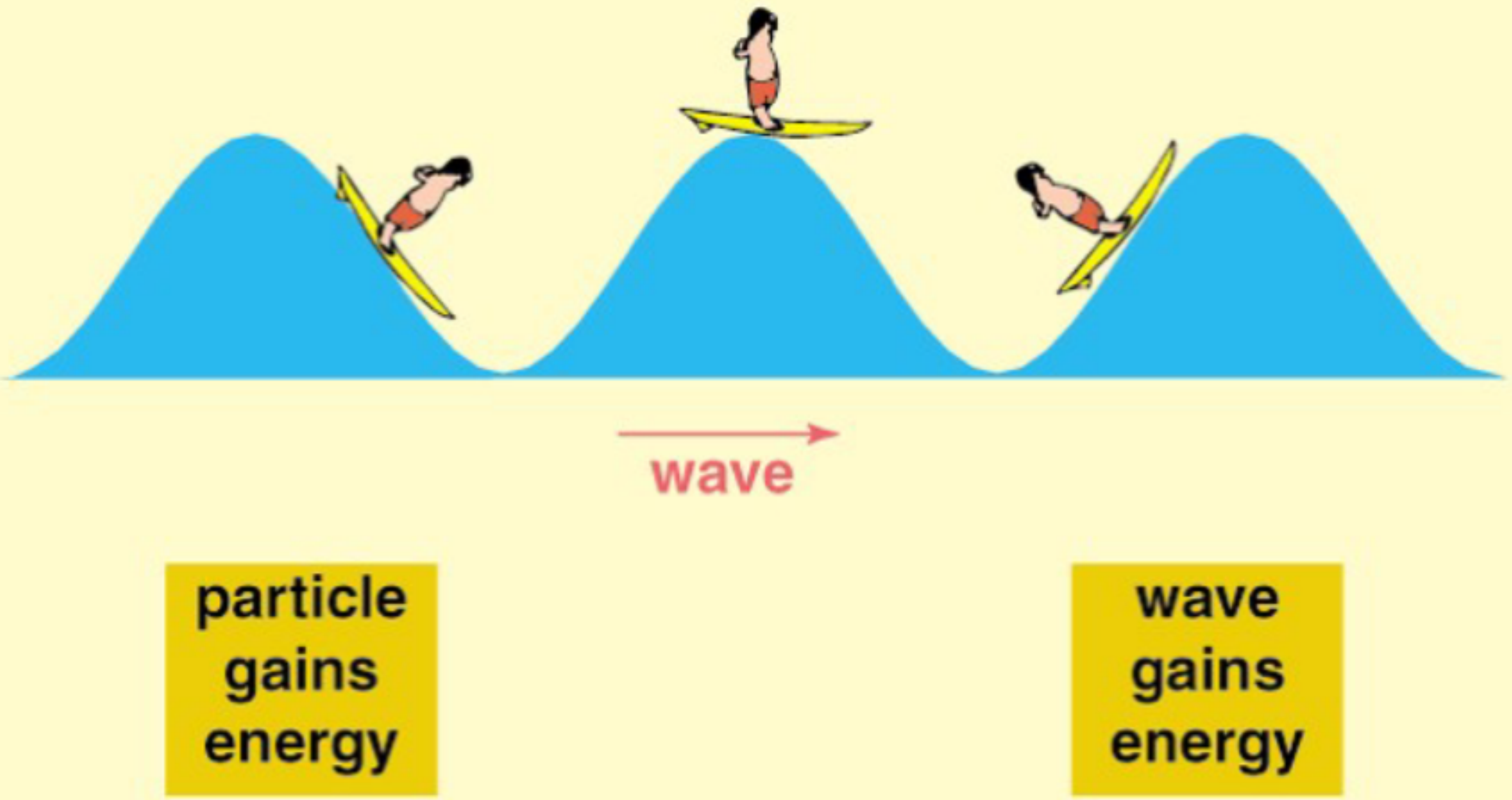
Plasma parameters fluctuate with the waves (density, velocity, etc)

Damped resonance

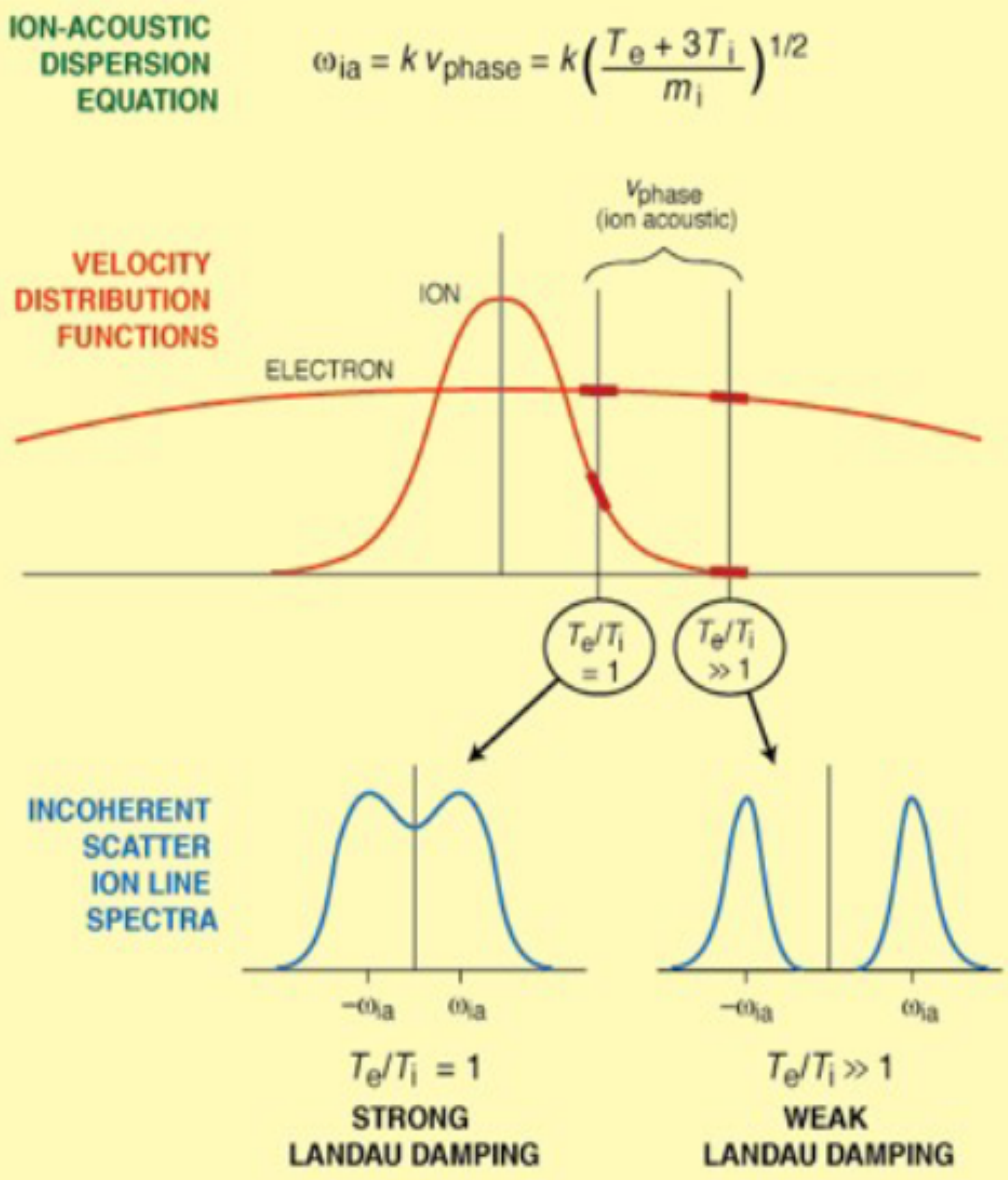
- Waves in a plasma are resonances.
- Damped resonances are not sharp
  - Example – Q of a resonant circuit.
- IS: Thermal ions have motions close to ion-acoustic speed (Landau damping – “surfing”; locked to I-A waves)



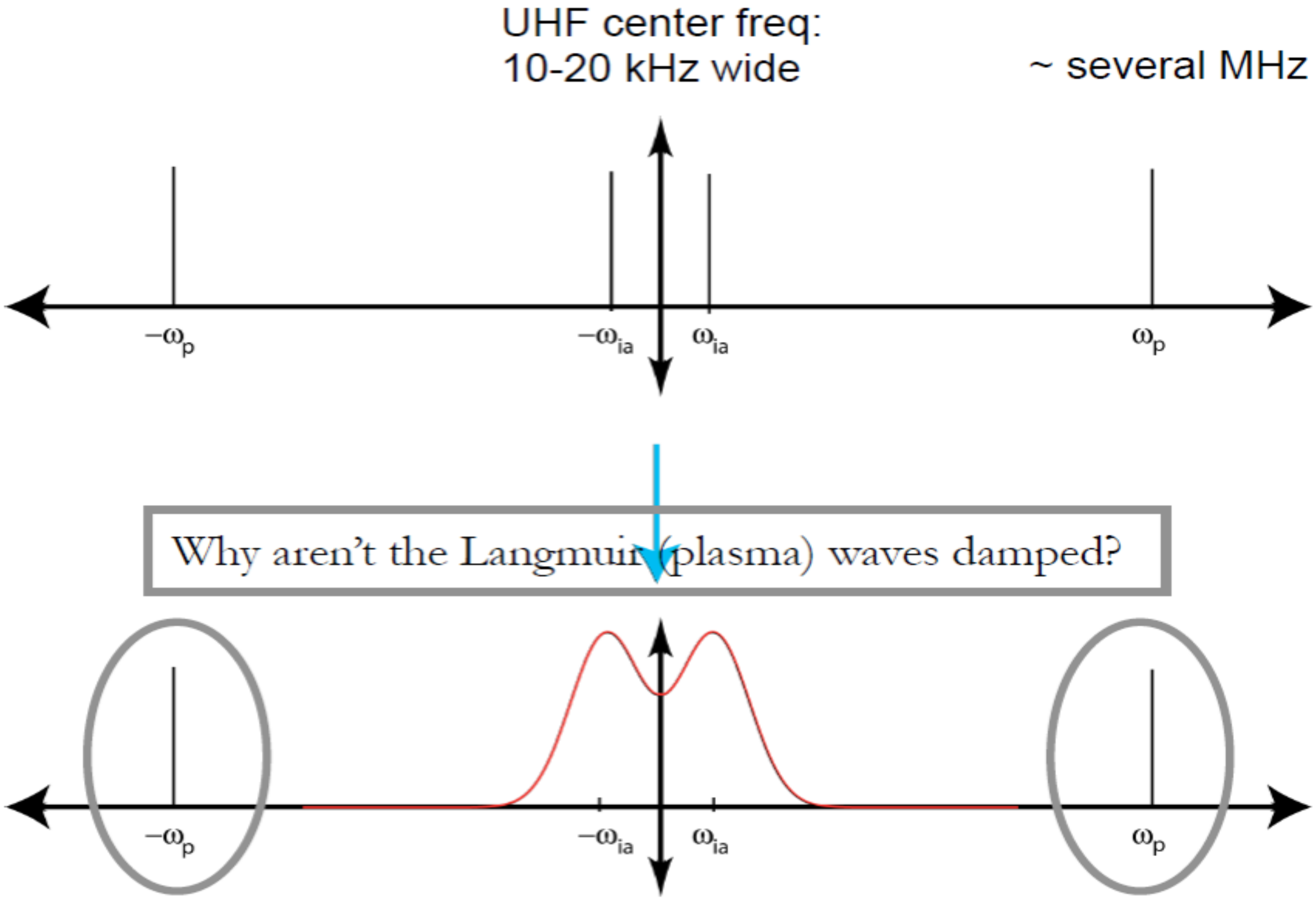
Landau wave-particle interactions



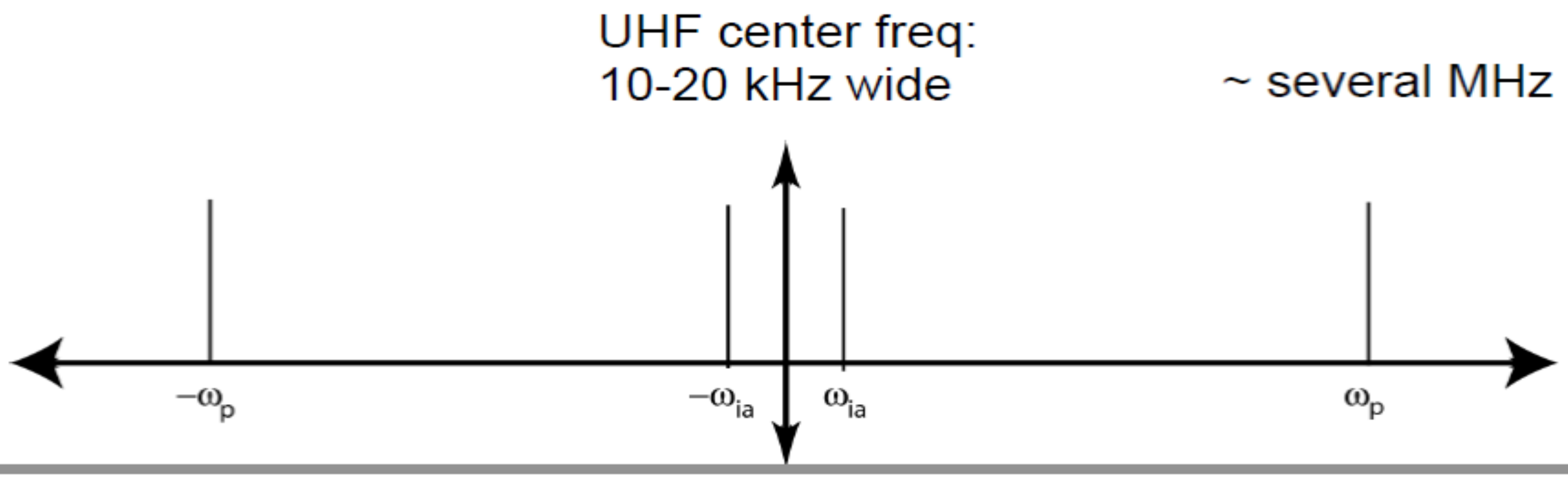
THE EFFECT OF LANDAU DAMPING ON THE INCOHERENT SCATTER ION LINE SPECTRUM



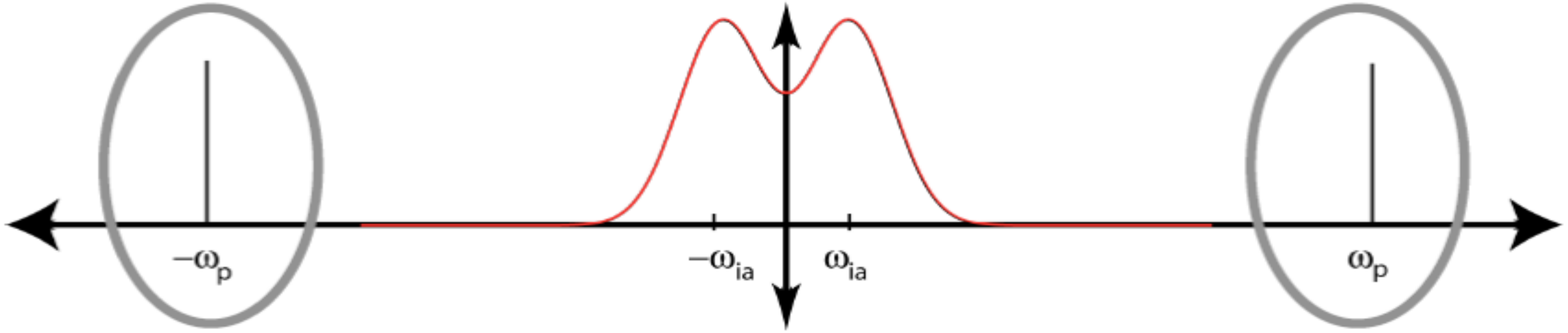
Wave Spectrum (ISR Spectrum)



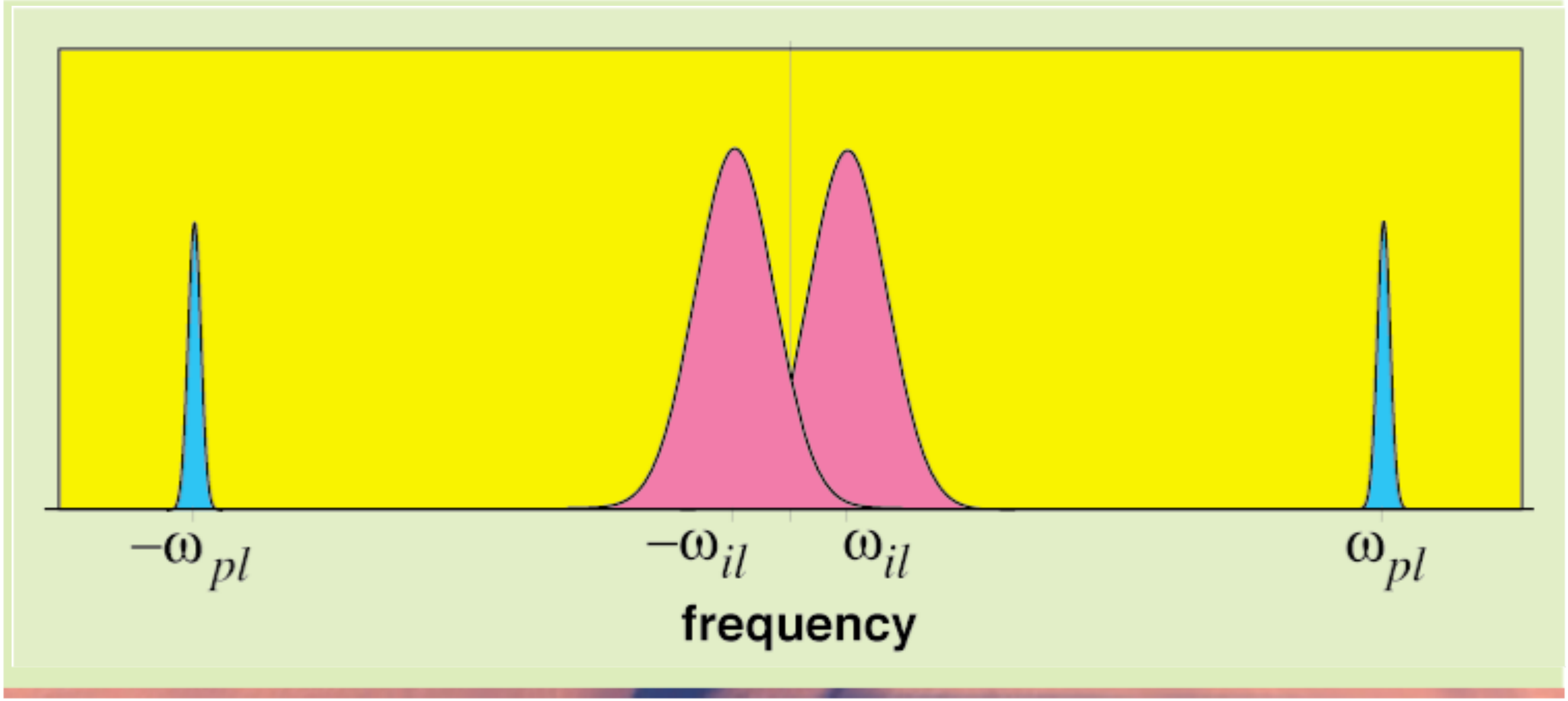
Wave Spectrum (ISR Spectrum)



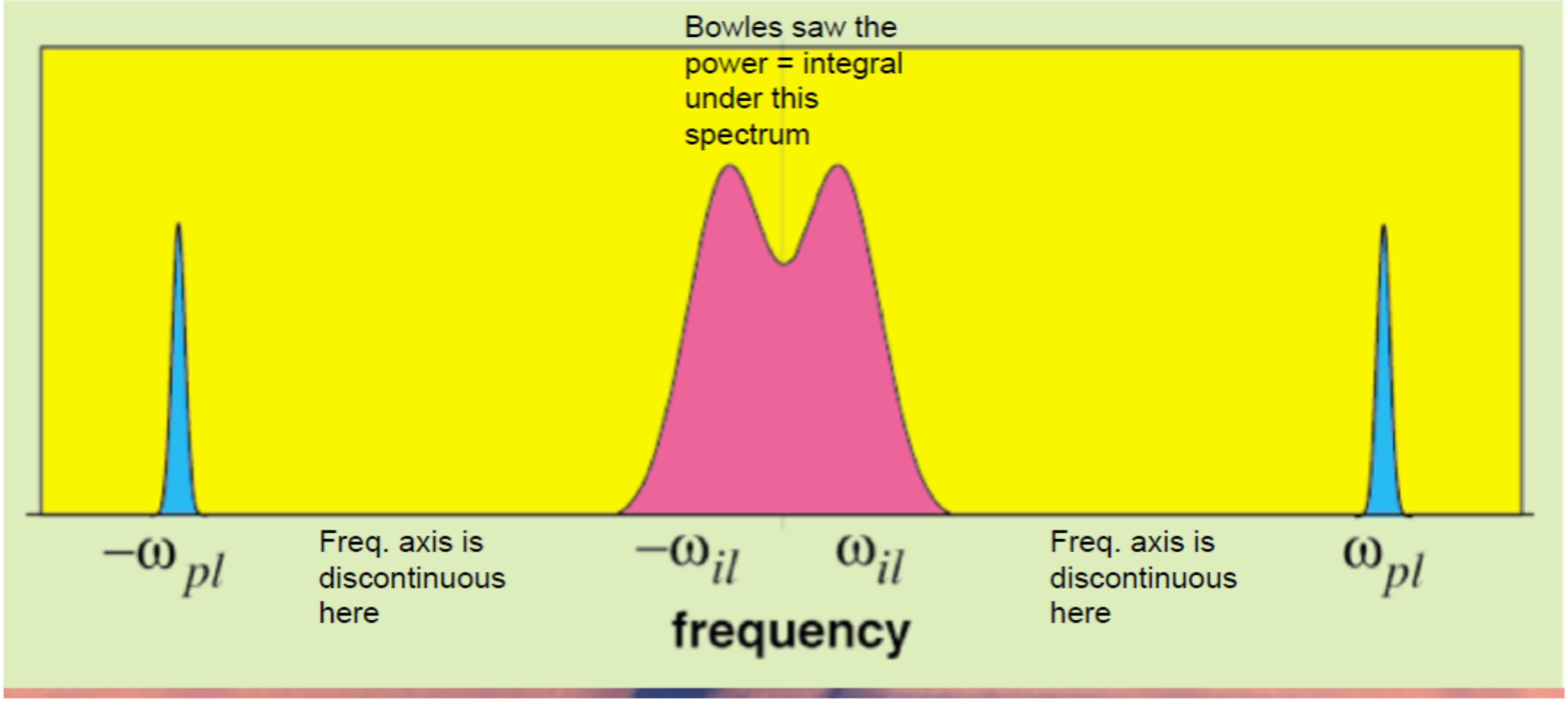
Why aren't the Langmuir (plasma) waves damped?
Electron thermal velocity ~ 125 km/s but plasma wave frequency ~ several MHz –
Not much interaction and not much damping.



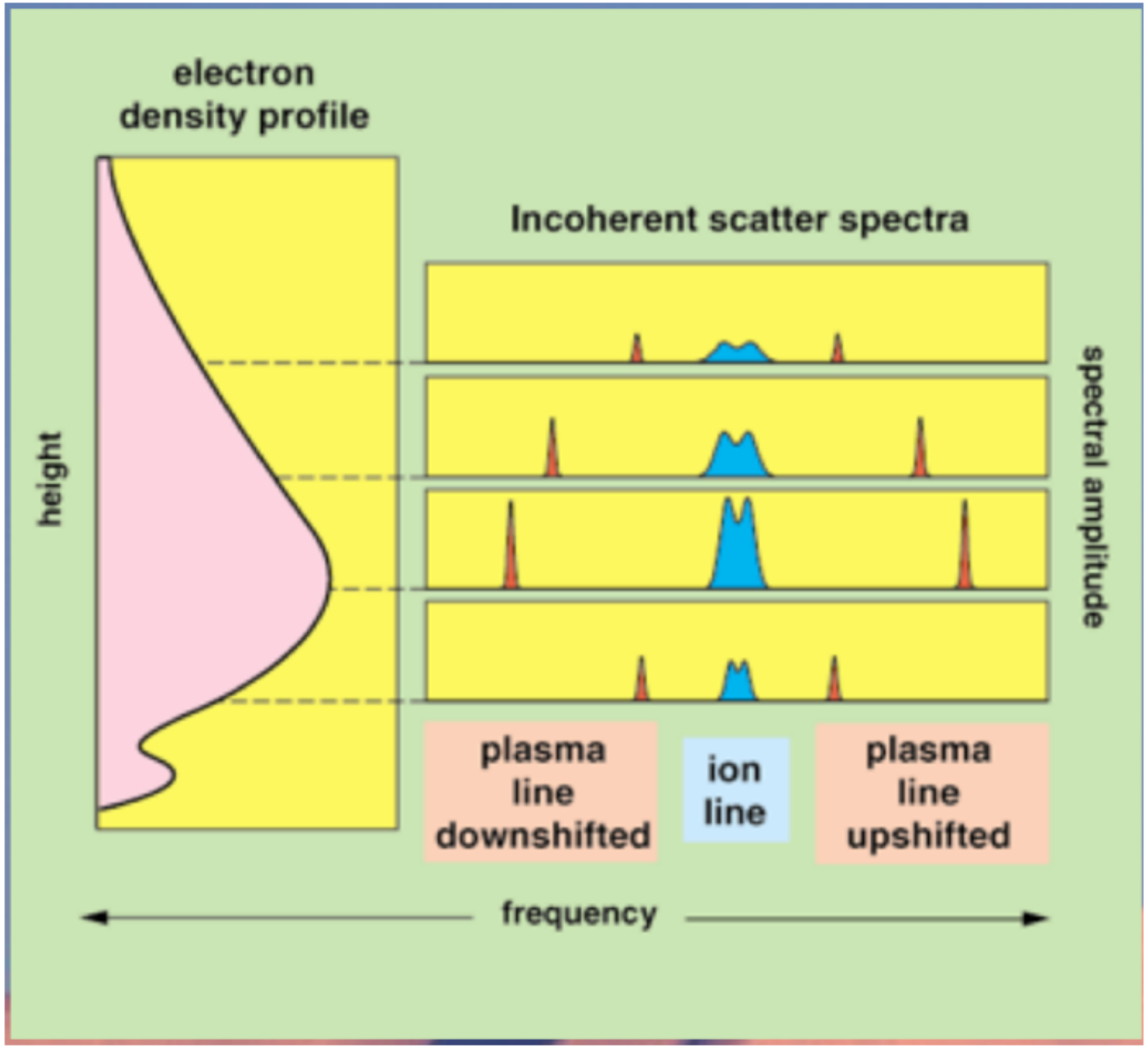
Incoherent Scatter Spectra as seen from a radar

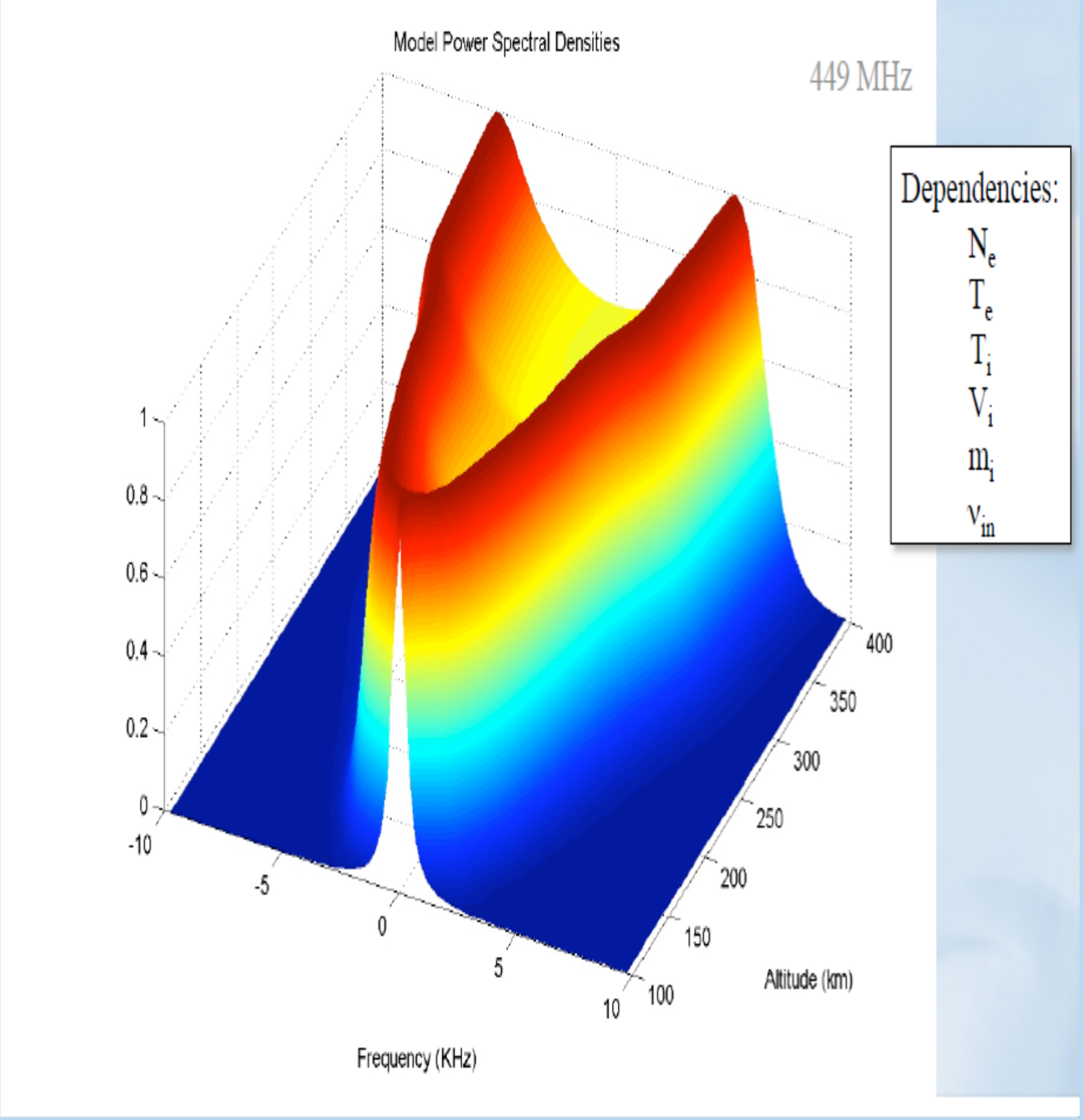


Incoherent Scatter Spectra as seen from a radar

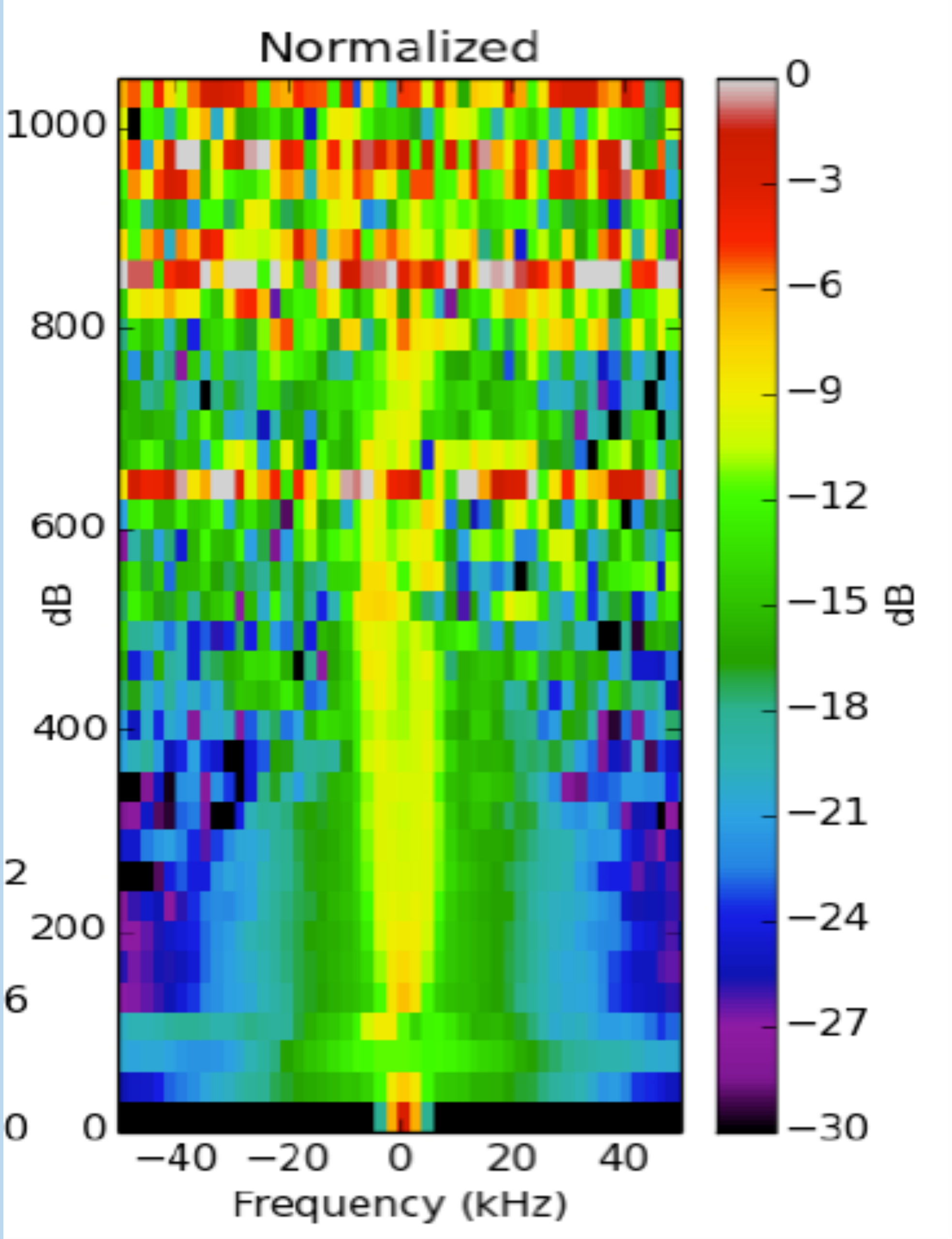


Incoherent Scatter Spectra as seen from a radar





Model Power Spectral Densities
449 MHz
Dependencies:
N_e
T_e
T_i
V_i
m_i
ν_{in}
1
0.8
0.6
0.4
0.2
0
-10
-5
0
5
10
Frequency (KHz)
100
150
200
250
300
350
400
Altitude (km)



Normalized
1000
800
600
400
200
0
dB
−40
−20
0
20
40
Frequency (kHz)
0
−3
−6
−9
−12
−15
−18
−21
−24
−27
−30
dB

ISR in a nutshell

Here's what we measure:

$$SNR = \frac{P_r}{P_n} = \left(\frac{P_t}{4\pi R^2}\right)\left(\frac{\sigma(\omega)}{4\pi R^2}\right)\left(\frac{GA}{KTBN_{sys}}\right)$$

The "radar equation"

$P_r =$ Received power

$P_n =$ Received noise power

$P_t =$ Transmitted power

$\sigma =$ Radar cross section

$G =$ Antenna gain

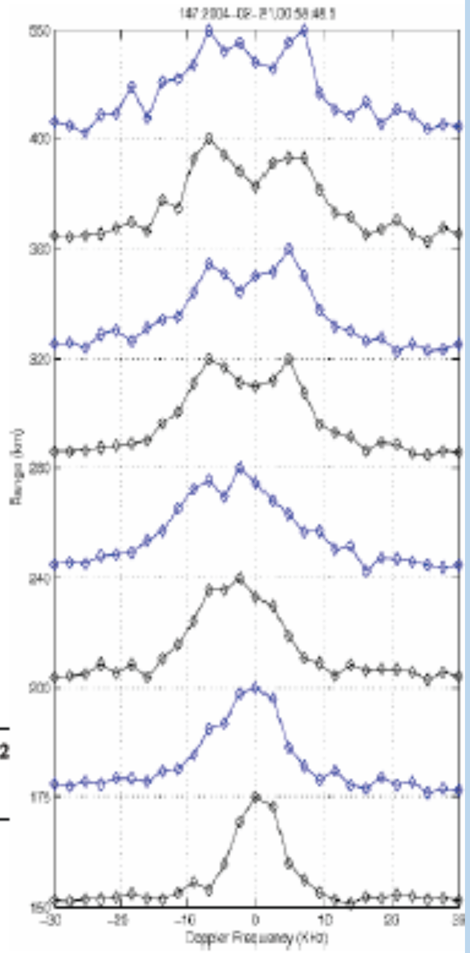
$A =$ Antenna area

$k_B =$ Boltzman's constant

$T =$ Temperature

$B =$ Bandwidth

$N_{sys} =$ System noise temperature



Here's the theory:

$$\sigma(\omega) = \frac{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \sum_1 \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right|^2 \overline{\left|N_e^0(\omega)\right|^2} + \left(\frac{\lambda}{4\pi D_e}\right)^4 \left|F_e(\omega)\right|^2 \sum_i \overline{\left|N_i^0(\omega)\right|^2}}{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \left\{\left(\frac{1}{D_e}\right)^2 \times F_e(\omega) + \sum_i \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right\}\right|^2}$$

$$F_e(\omega) = 1 - \omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\cos(\omega\tau)\,d\tau$$

$$F_i(\omega) = 1 - \omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\cos(\omega\tau)\,d\tau$$

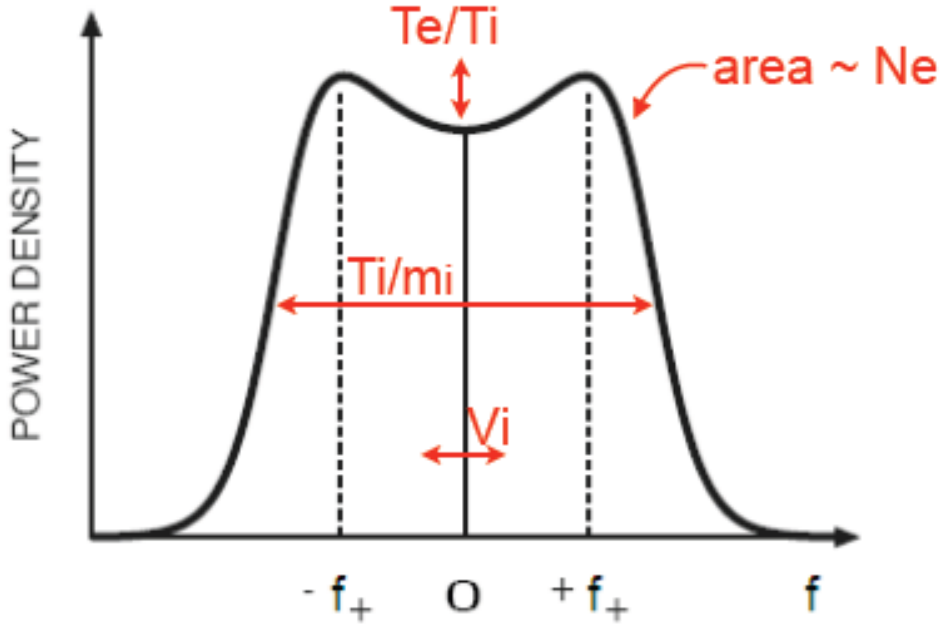
The ISR model

$$\sigma(\omega) = \frac{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \sum_1 \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right|^2 \overline{\left|N_e^0(\omega)\right|^2} + \left(\frac{\lambda}{4\pi D_e}\right)^4 \left|F_e(\omega)\right|^2 \sum_i \overline{\left|N_i^0(\omega)\right|^2}}{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \left\{\left(\frac{1}{D_e}\right)^2 \cdot F_e(\omega) + \sum_i \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right\}\right|^2}$$

where:

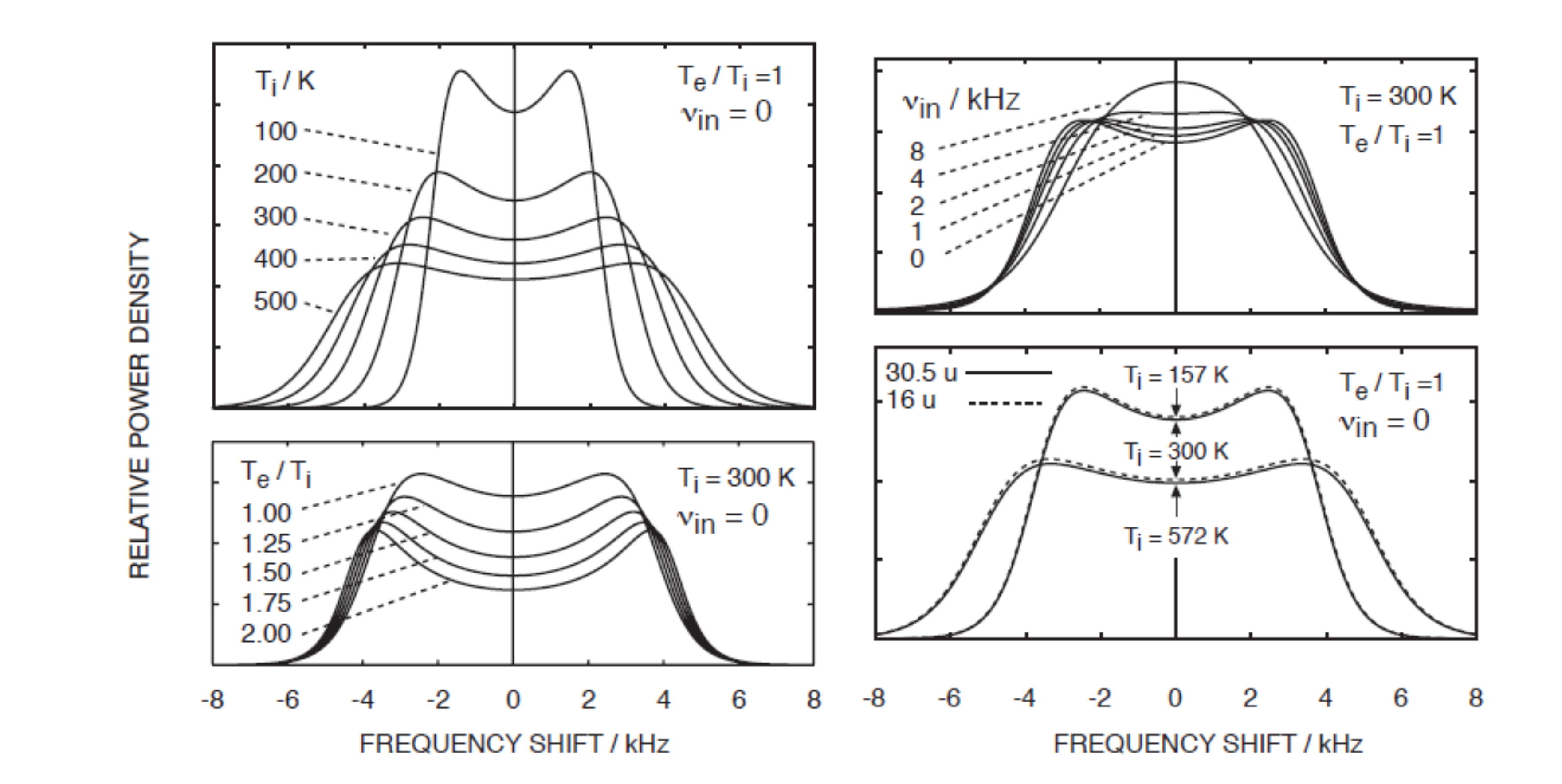
$$F_e(\omega) = 1 - \omega \int_o^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\cos(\omega\tau)\,d\tau$$

$$F_i(\omega) = 1 - \omega \int_o^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\cos(\omega\tau)\,d\tau$$



From Evans, IEEE Transactions, 1969

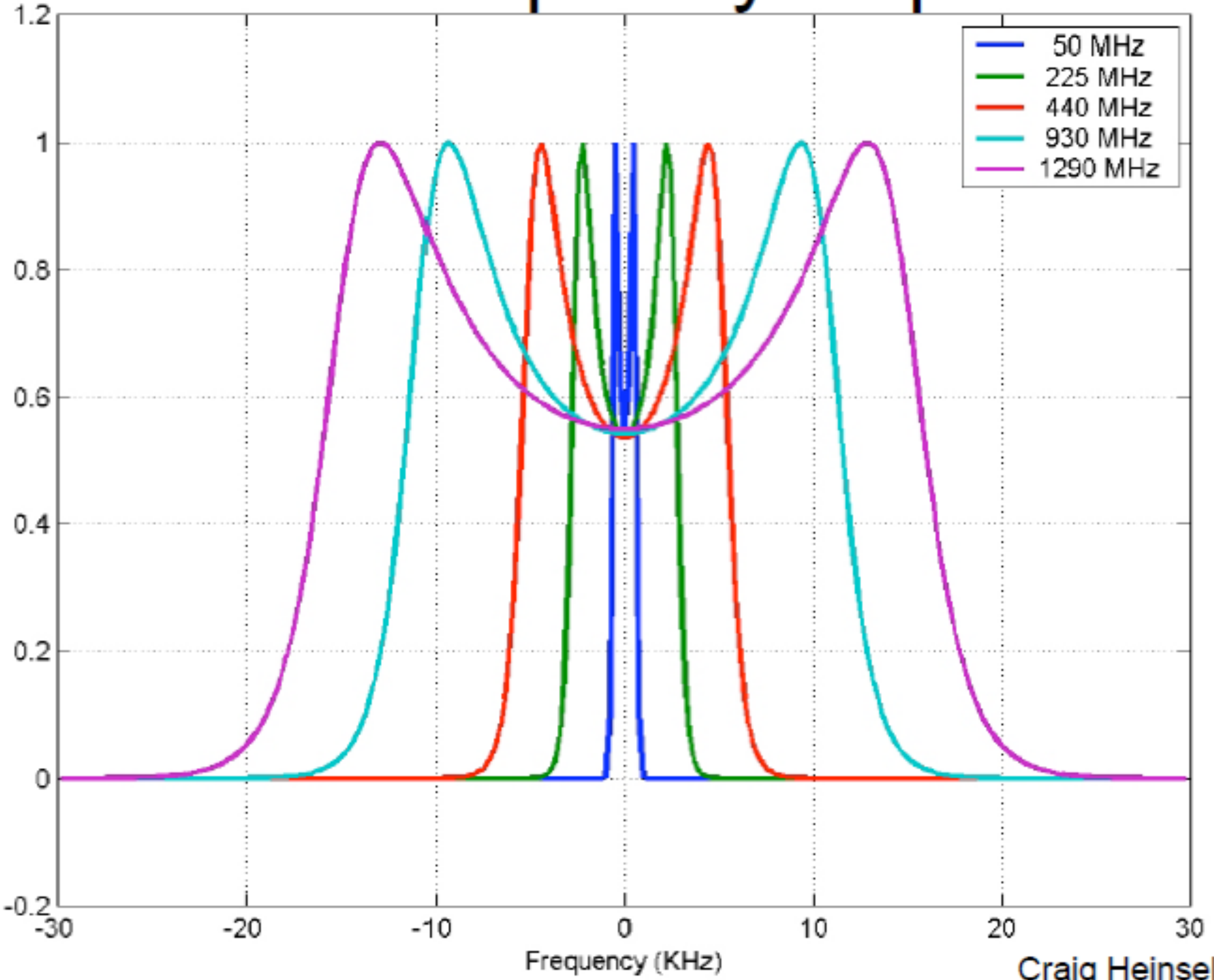
Dependence on Plasma Parameters



To see how incoherent scatter spectrum depends on the plasma parameters, play with the widget at:

http://madrigal.haystack.mit.edu/madrigal/ISR/spectrum/

Radar Frequency Dependencies



Parameters

Ne: 10^{12} m^{-3}

Ti: 1000 K

Te: 2000 K

Comp: 100% O^+

ν_{in}: 10^{-6} KHz

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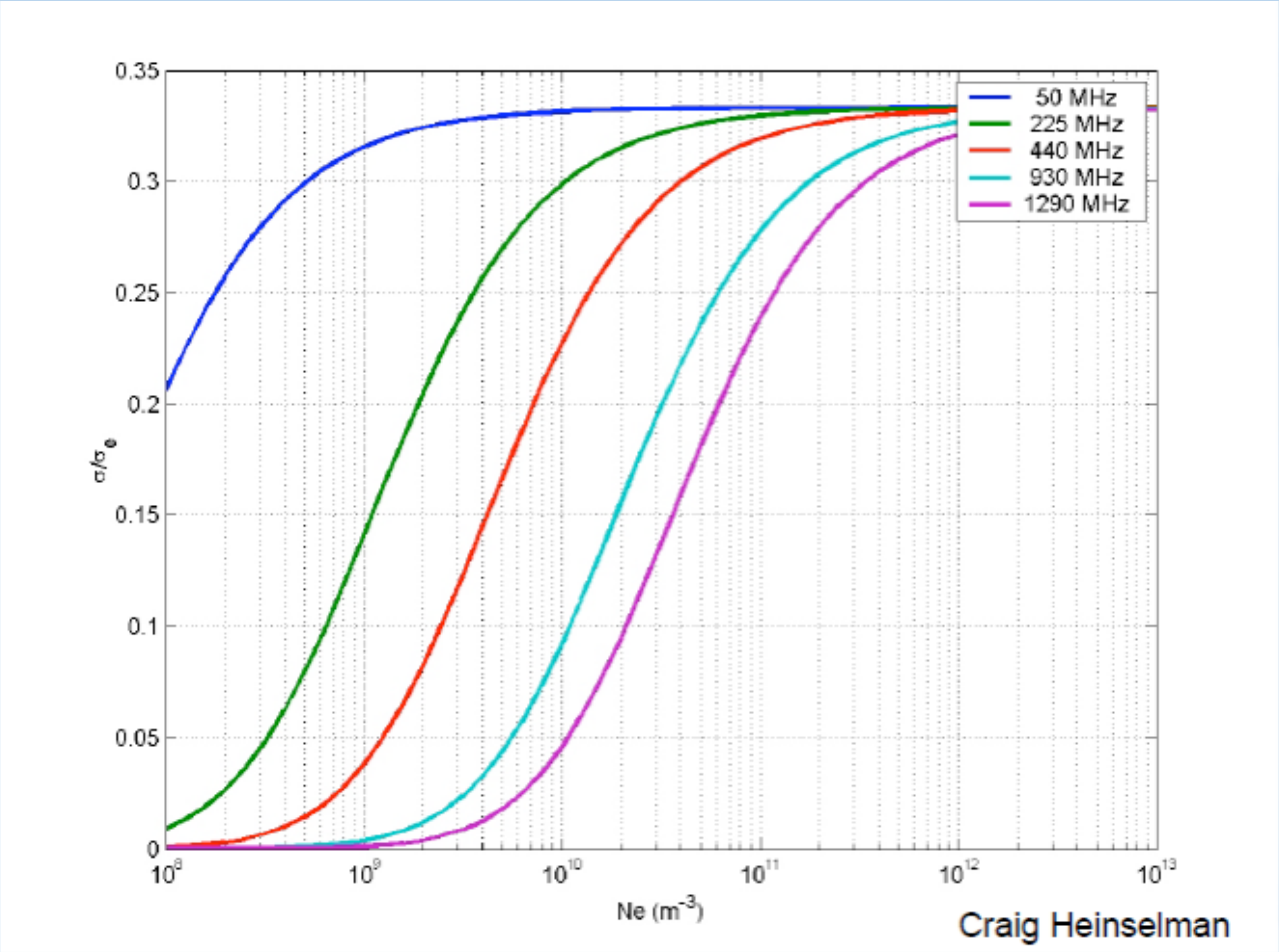
Debye Length Dependencies

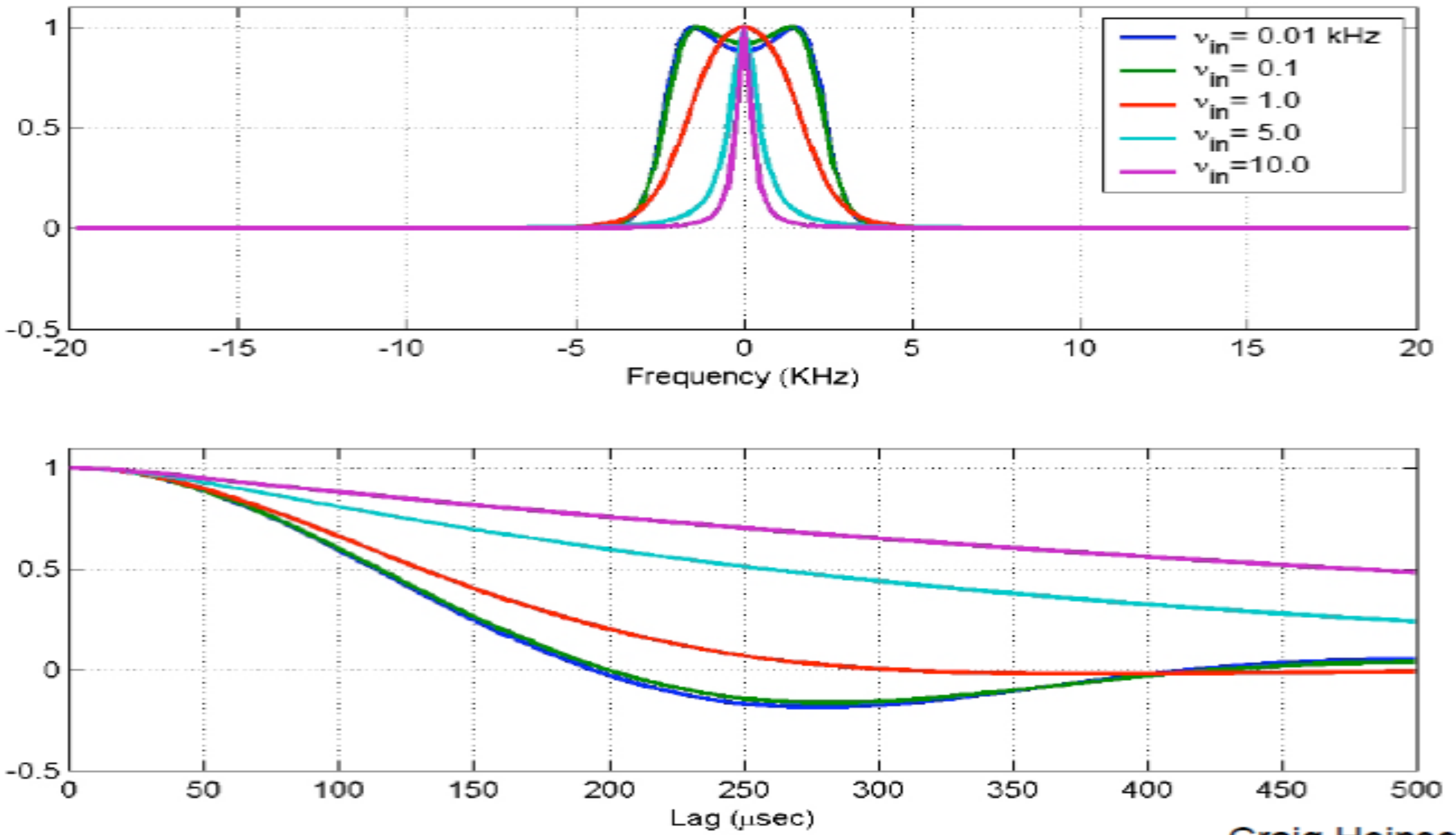


Parameters

Ti: 1000 K

Te: 2000 K

Ion-Neutral Collision Frequency



<u>Parameters</u>
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 500 K
Te: 500 K
Comp: 100% NO^+

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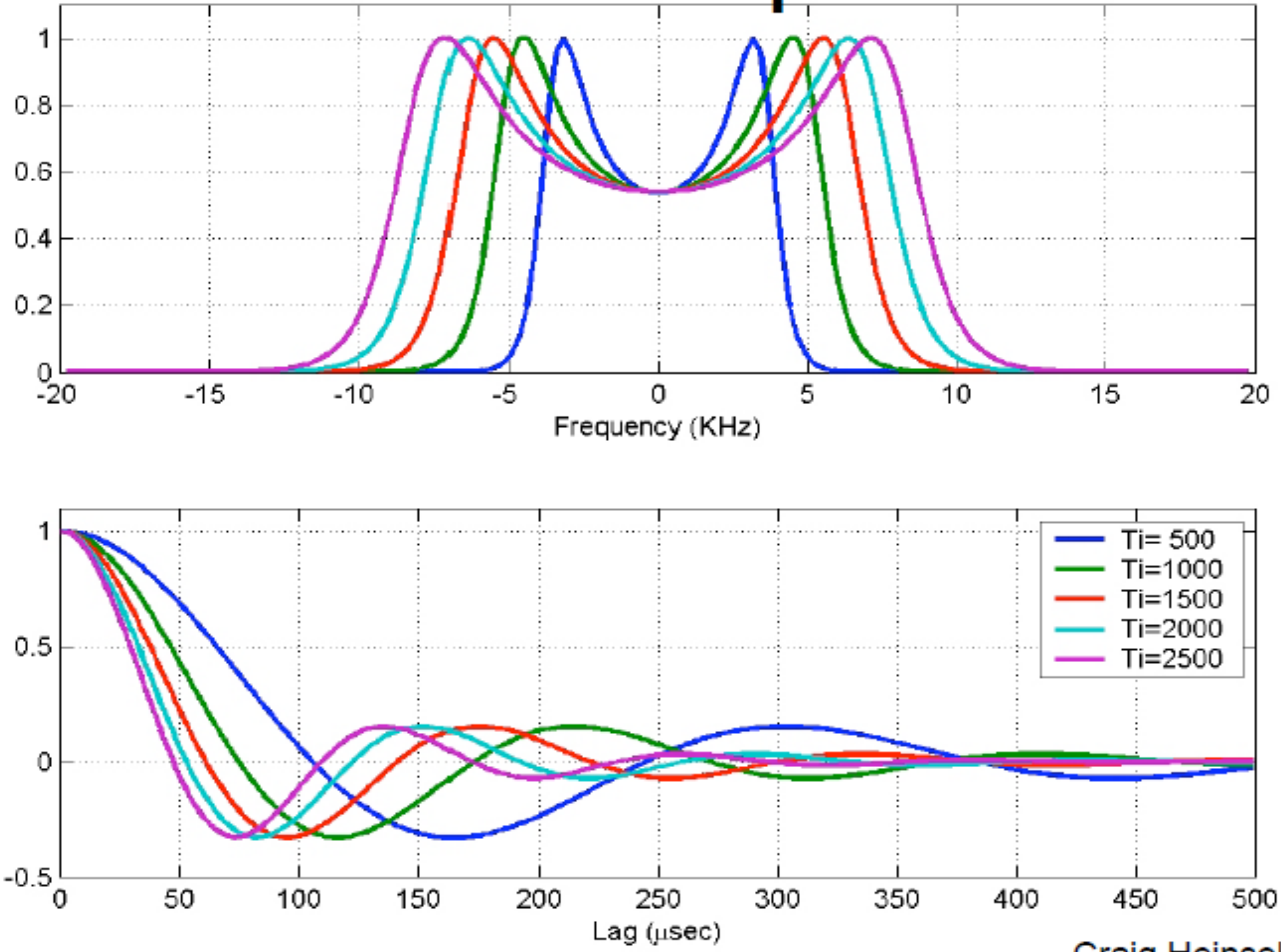
Ion Temperature



Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Te: 2*Ti
Comp: 100% O^{+}
ν_{in}: 10^{-6} KHz

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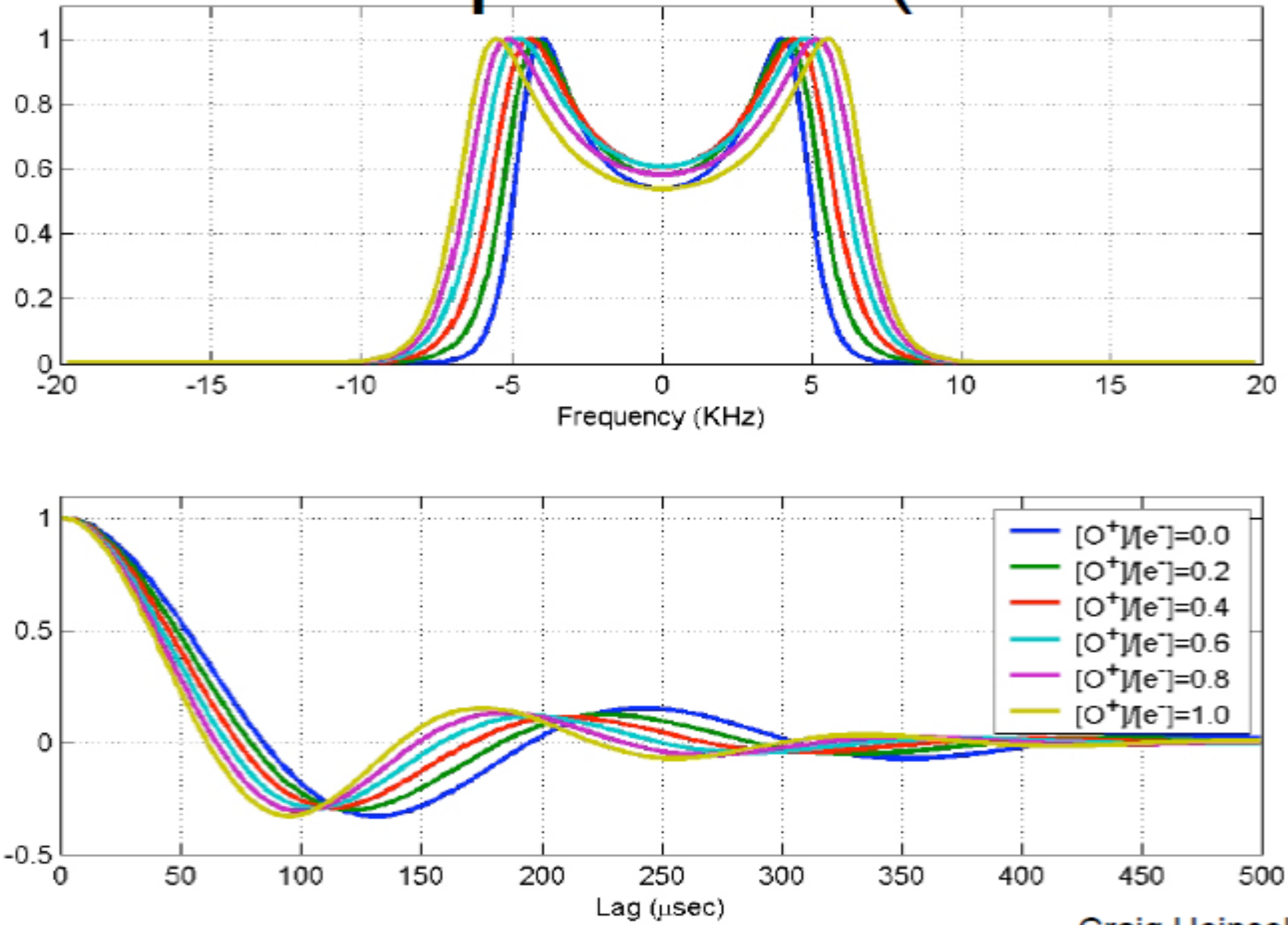
Ion Composition (O^+ vs. NO^+)



<u>Parameters</u>
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 1500 K
Te: 3000 K
ν_{in}: 10^{-6} KHz

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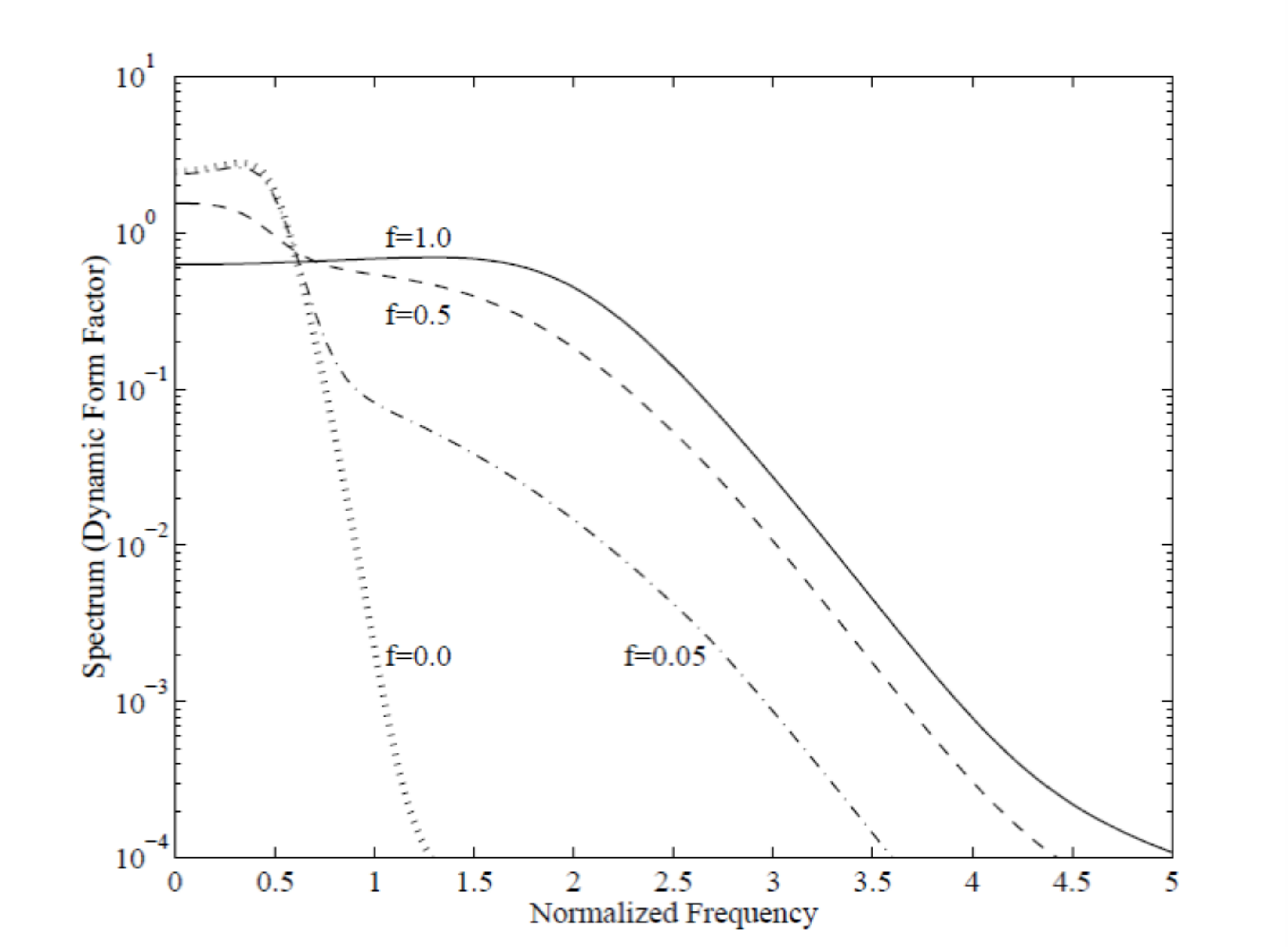
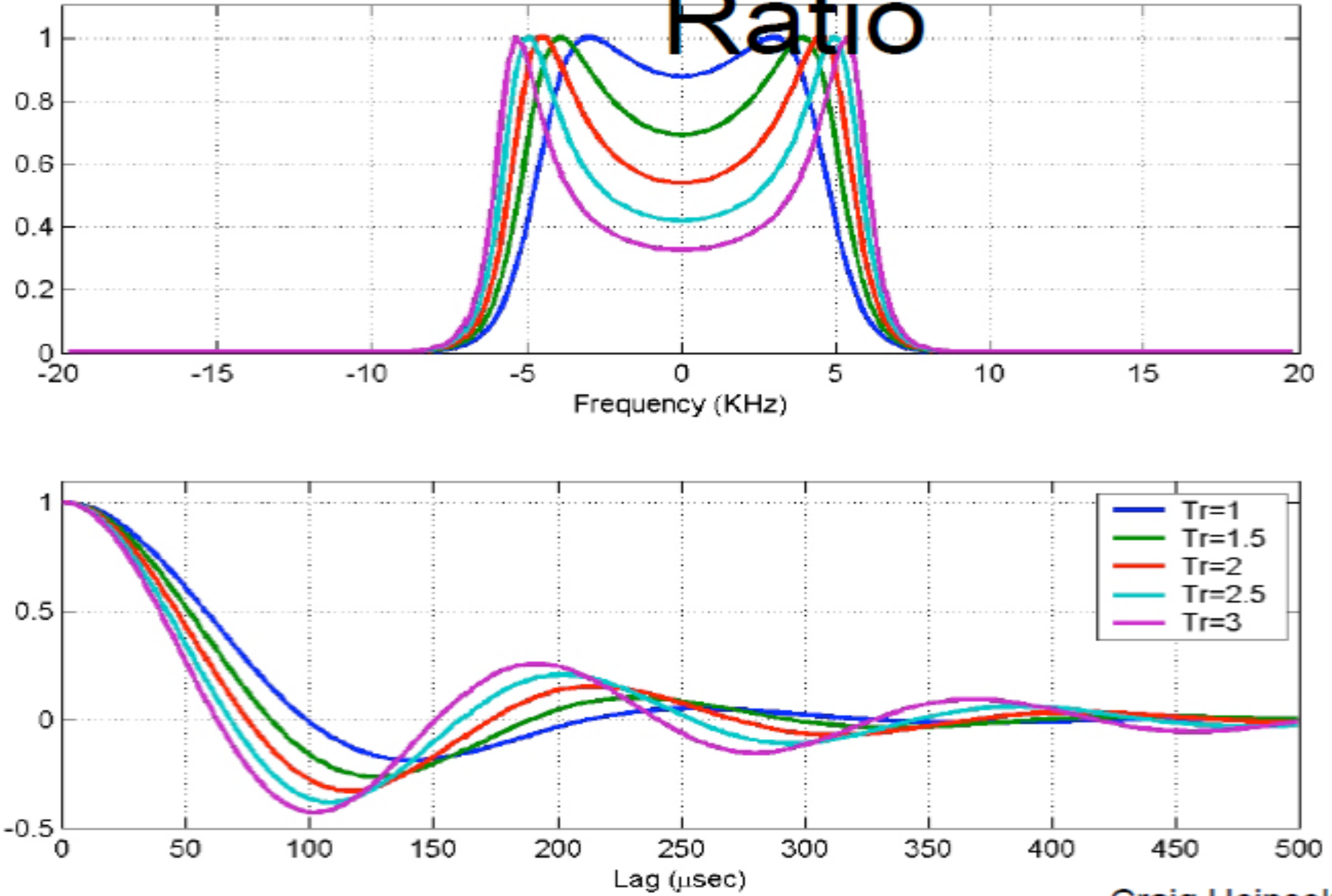
Mixtures of O+ and H+ (topside F-region)



- $f = N_H / (N_H + N_O)$

Electron/Ion Temperature Ratio



Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 1000 K
Comp: 100% O^+
ν_{in}: 10^{-6} KHz

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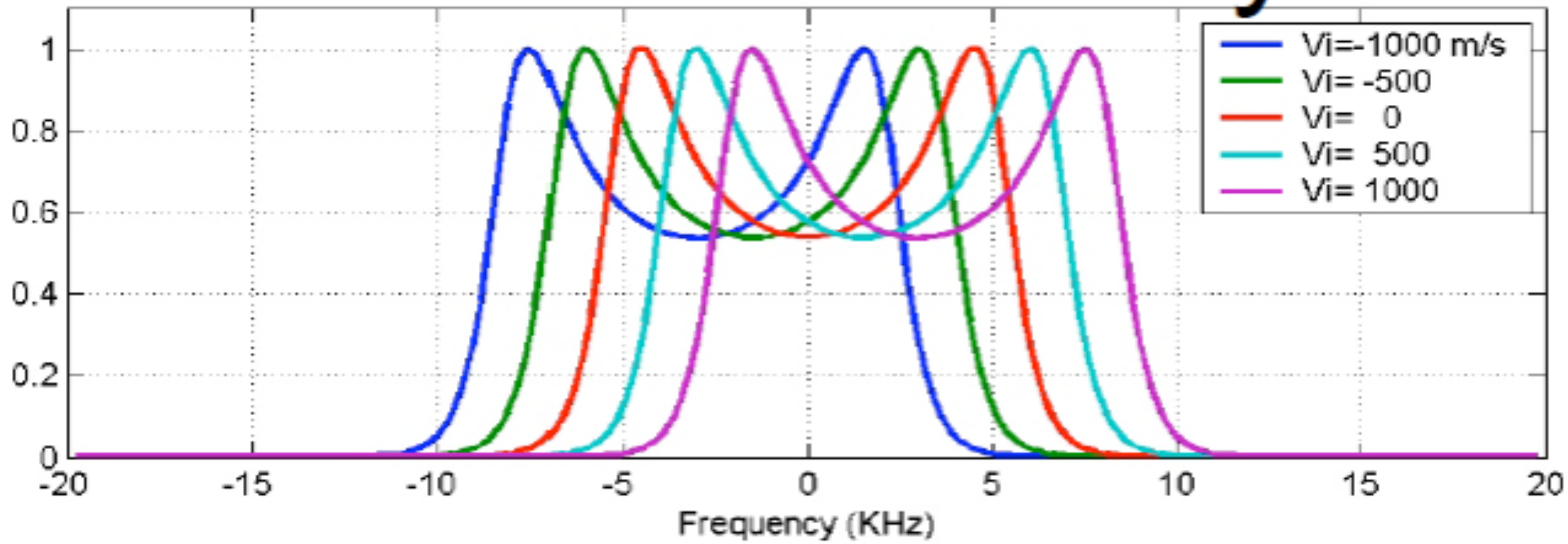
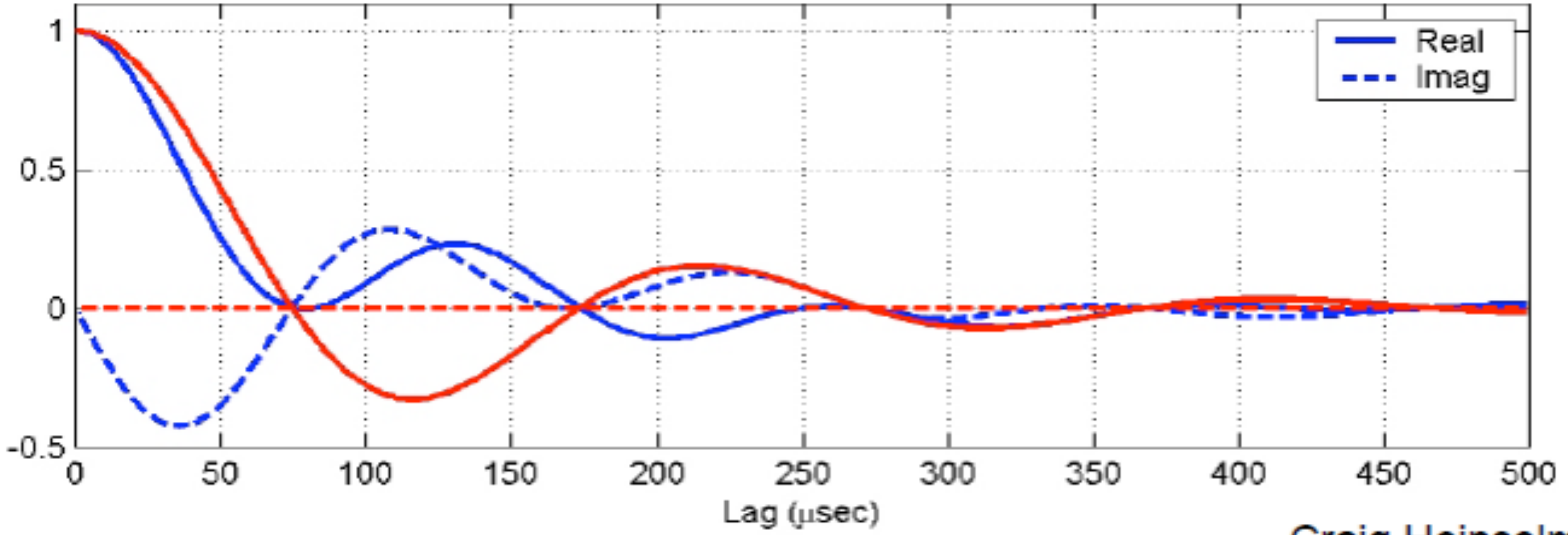
Ion Velocity





Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 1000 K
Te: 2000 K
Comp: 100% O^+
ν_{in}: 10^{-6} KHz

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The plasma autocorrelation function, $r_{xx}(\tau)$

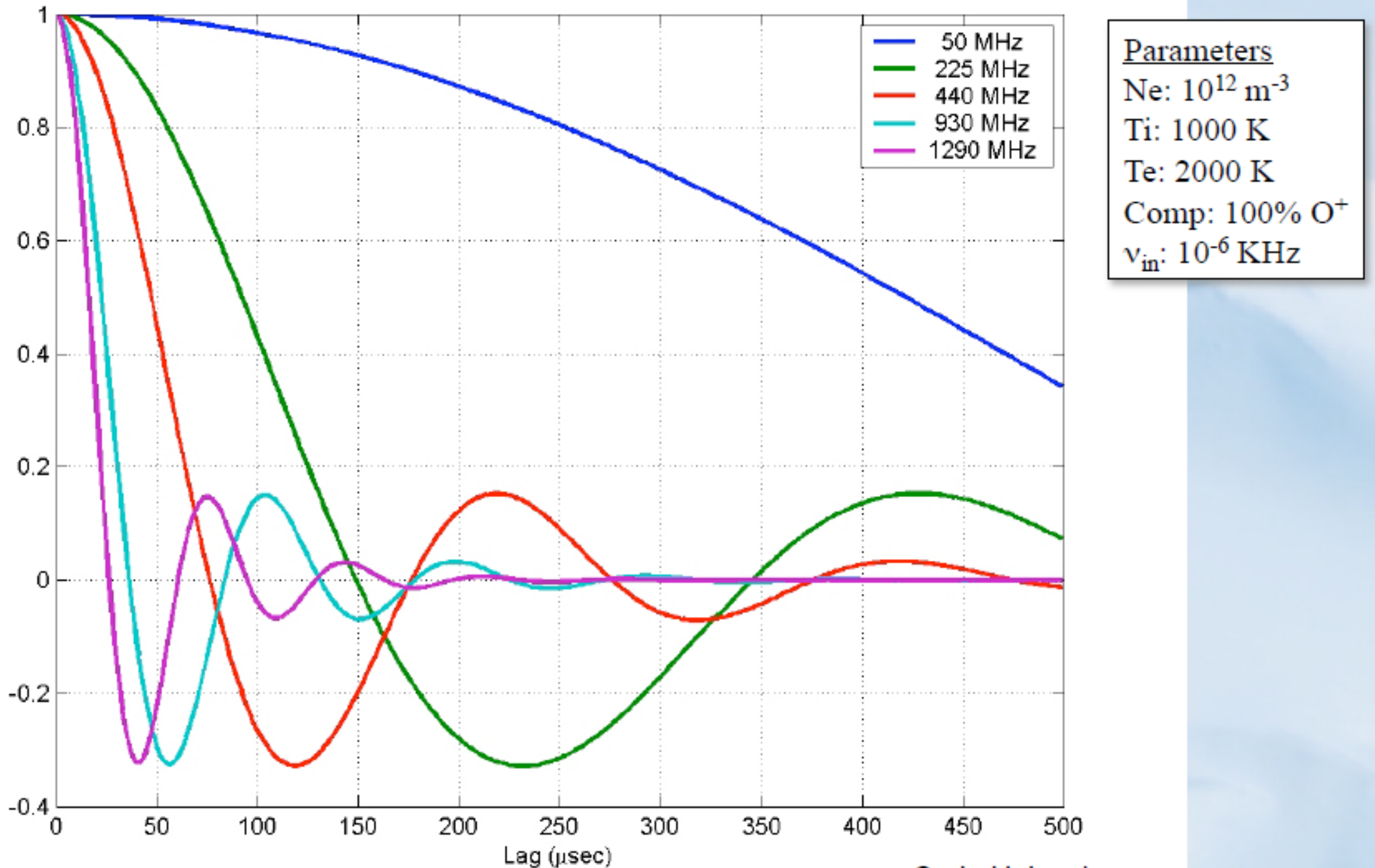
is the Fourier transform of the ion line power spectral density. Using the plasma dispersion relation, we can compute model autocorrelation functions for different combinations of N_e, T_e and T_i

An estimate of the target r_{xx} at *lag time* $n\tau_0$ can be computed from the time series of complex amplitude samples, $s(t)$, output from the receiver:

$$r_{xx}(n\tau_0) = s\,(t)\,s^*(t + n\tau_0)$$

Intuitively, it may appear natural to continue sampling at a given range for so long that the ACF has decayed almost to zero. To see if that helps at all, let us first look at how the different plasma parameters influence the ACF at different lag times :

Dependence of the ACF on frequency



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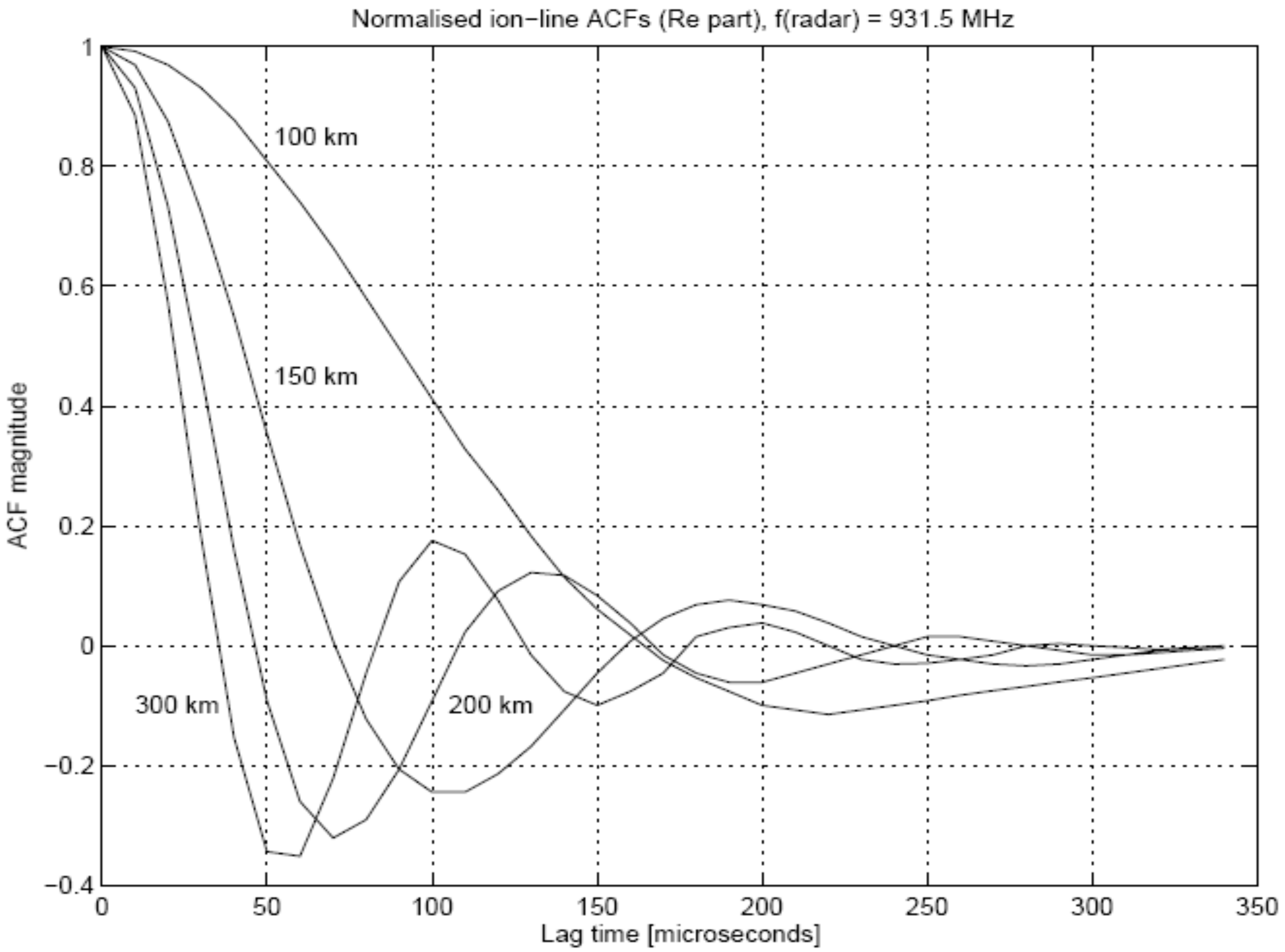
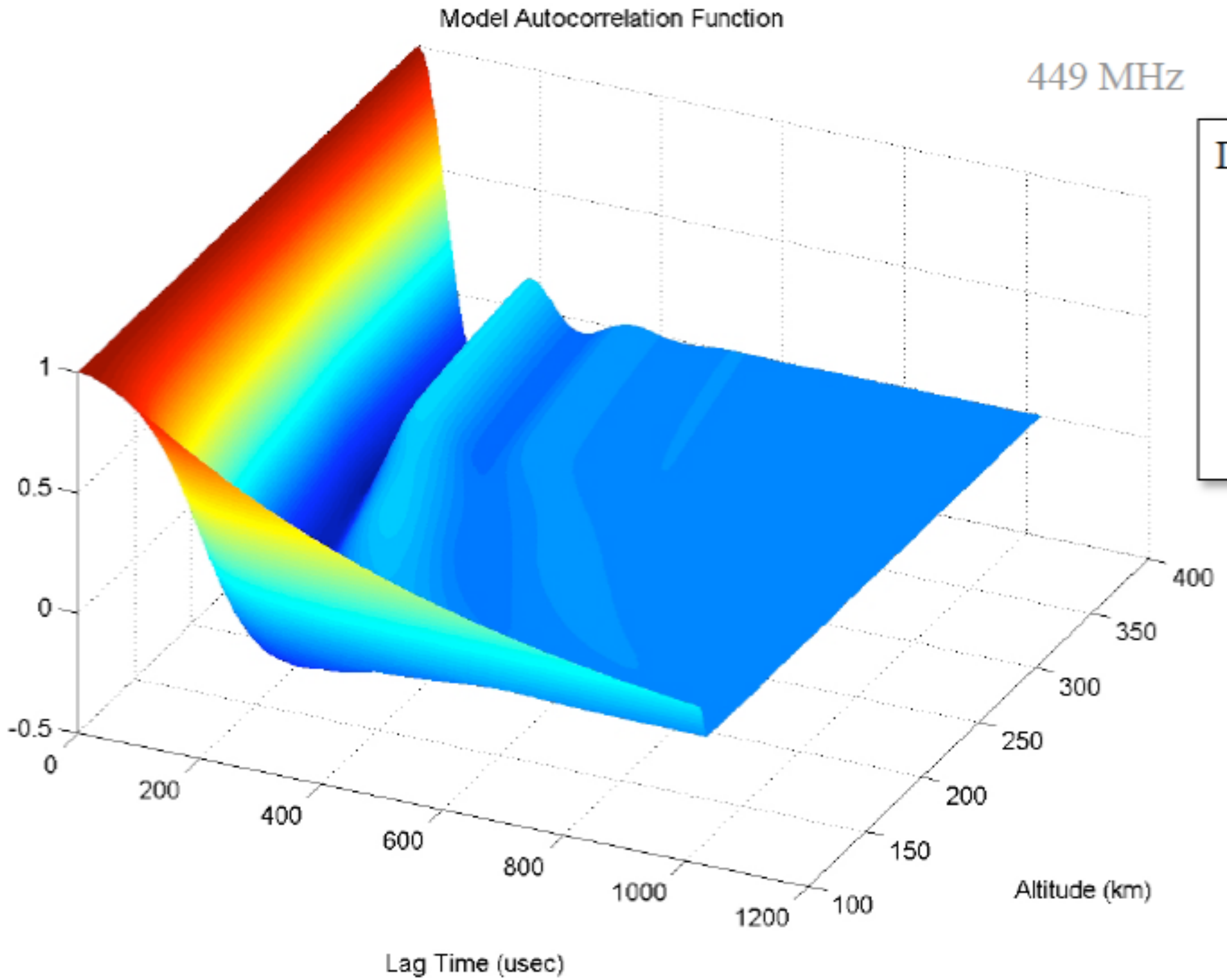
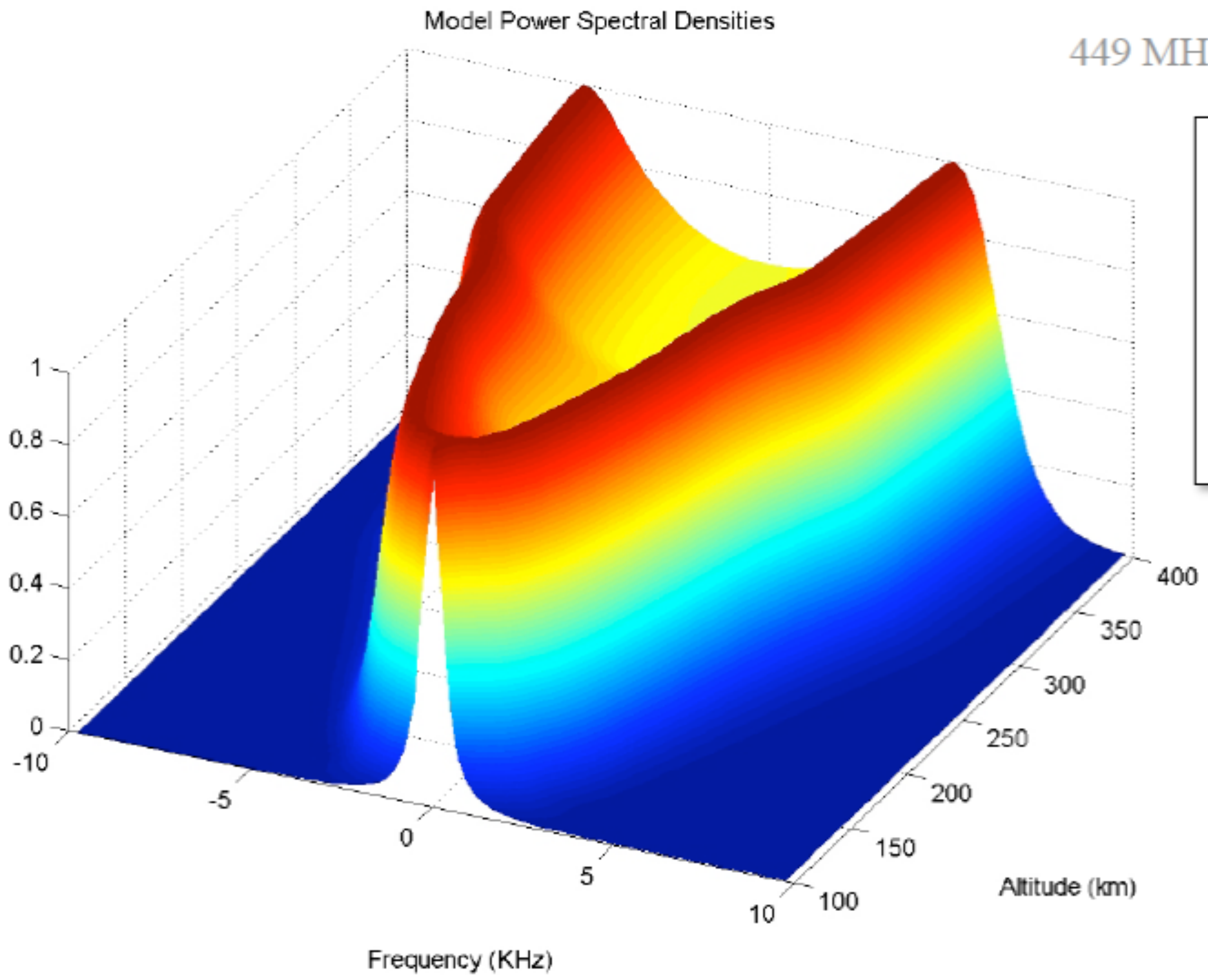


Normalised ion−line ACFs (Re part), f(radar) = 931.5 MHz
ACF magnitude
Lag time [microseconds]
100 km
150 km
200 km
300 km
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
0
50
100
150
200
250
300
350



Dependencies:

N_e
T_e
T_i
V_i
m_i
ν_{in}



449 MHz

Dependencies:

N_e

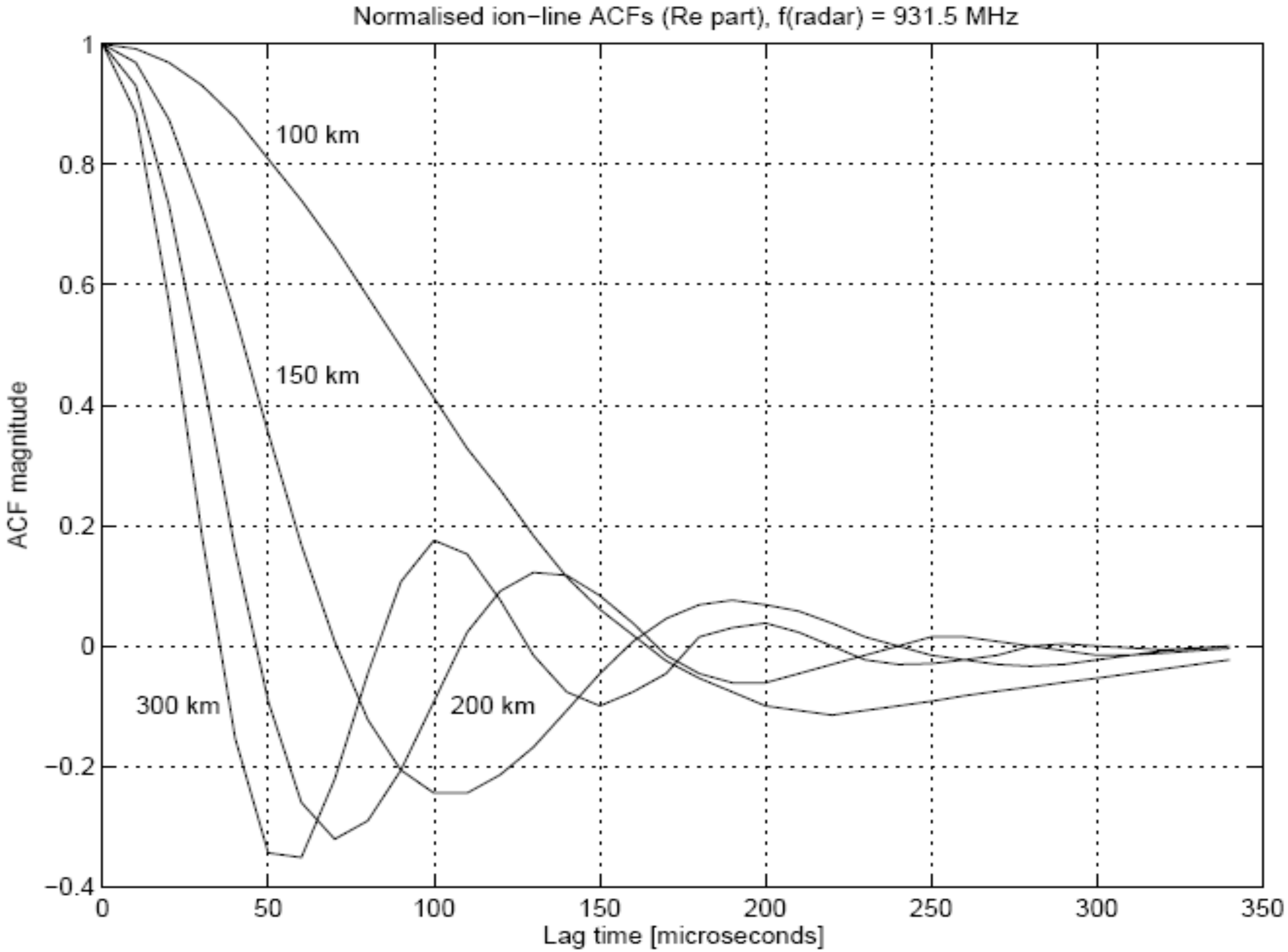
T_e

T_i

V_i

m_i

ν_{in}



Normalised ion−line ACFs (Re part), f(radar) = 931.5 MHz
ACF magnitude
Lag time [microseconds]
100 km
150 km
200 km
300 km
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
0
50
100
150
200
250
300
350

Partial derivatives of the plasma dispersion function:

$\partial r_{xx}(\tau) \ / \ \partial Ne$

$\partial r_{xx}(\tau) \ / \ \partial T_i$

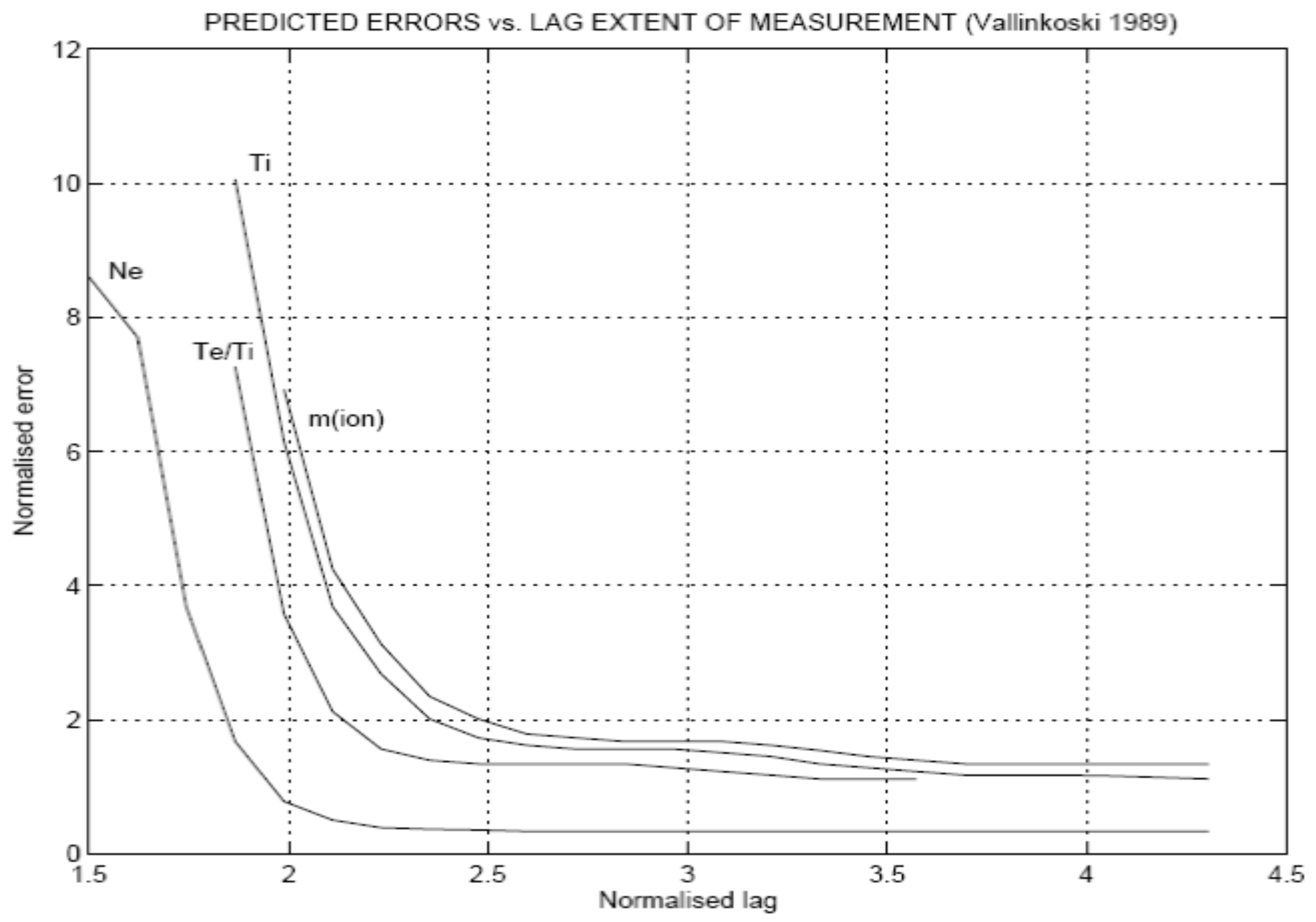
$\partial r_{xx}(\tau) \ / \ \partial (T_e/T_i)$

$\partial r_{xx}(\tau) \ / \ \partial m_i$

$\partial r_{xx}(\tau) \ / \ \partial \nu_{in}$

are shown in terms of τ/τ_0 , where τ_0 , the *plasma correlation time*, is the time to the first zero crossing of the ACF of a undamped ion-acoustic wave with wavelength = $\Lambda = ½ \lambda_{radar}$

NOTE: $\partial r_{xx}(\tau) \ / \ \partial T_i$ and $\partial r_{xx}(\tau) \ / \ \partial m_i$ are almost linearly dependent...

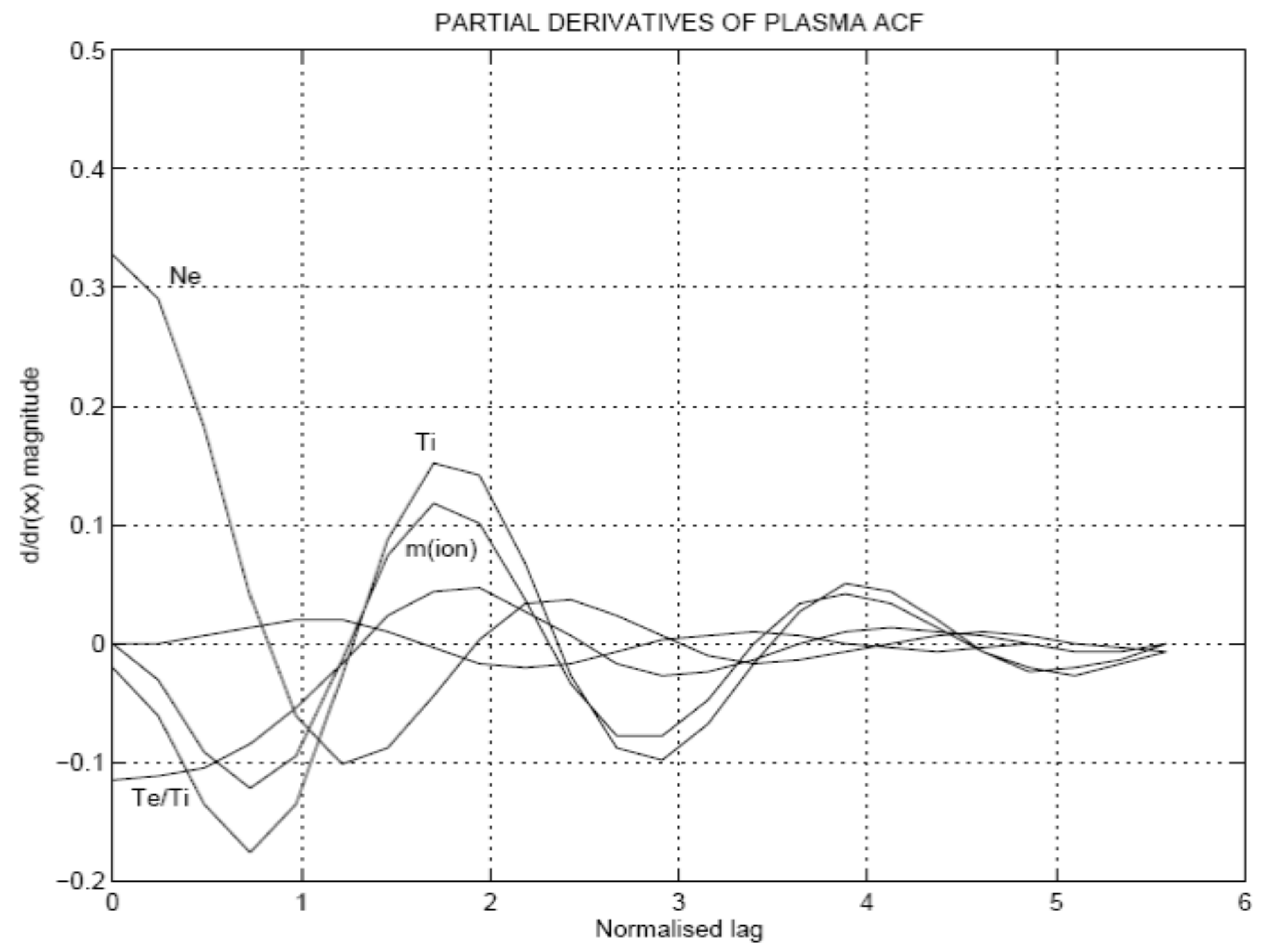

PREDICTED ERRORS vs. LAG EXTENT OF MEASUREMENT (Vallinkoski 1989)
Normalised error
Normalised lag
Ti
Ne
Te/Ti
m(ion)
0
2
4
6
8
10
12
1.5
2
2.5
3
3.5
4
4.5

ACF estimate extent and errors

The next figure (from *Vallinkoski 1989*) shows how the errors of the different plasma parameters behave as functions of lag extent when measurement data are fitted to a five-parameter plasma model.

Comparing this to the previous figure , we see that *as the lag extent is increased to the point where the partial derivative of a given parameter goes through a complete cycle*, the error in that parameter suddenly drops dramatically.

If one is satisfied with slightly less than ultimate accuracy, extending the measurement to τ/τ_0 = 2.5 should be sufficient. By about τ/τ_0 = 3.5, all errors have settled down to their asymptotic value.


PARTIAL DERIVATIVES OF PLASMA ACF
d/dr(xx) magnitude
Normalised lag
Ne
Ti
m(ion)
Te/Ti
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
0
1
2
3
4
5
6

A note on plasma correlation times

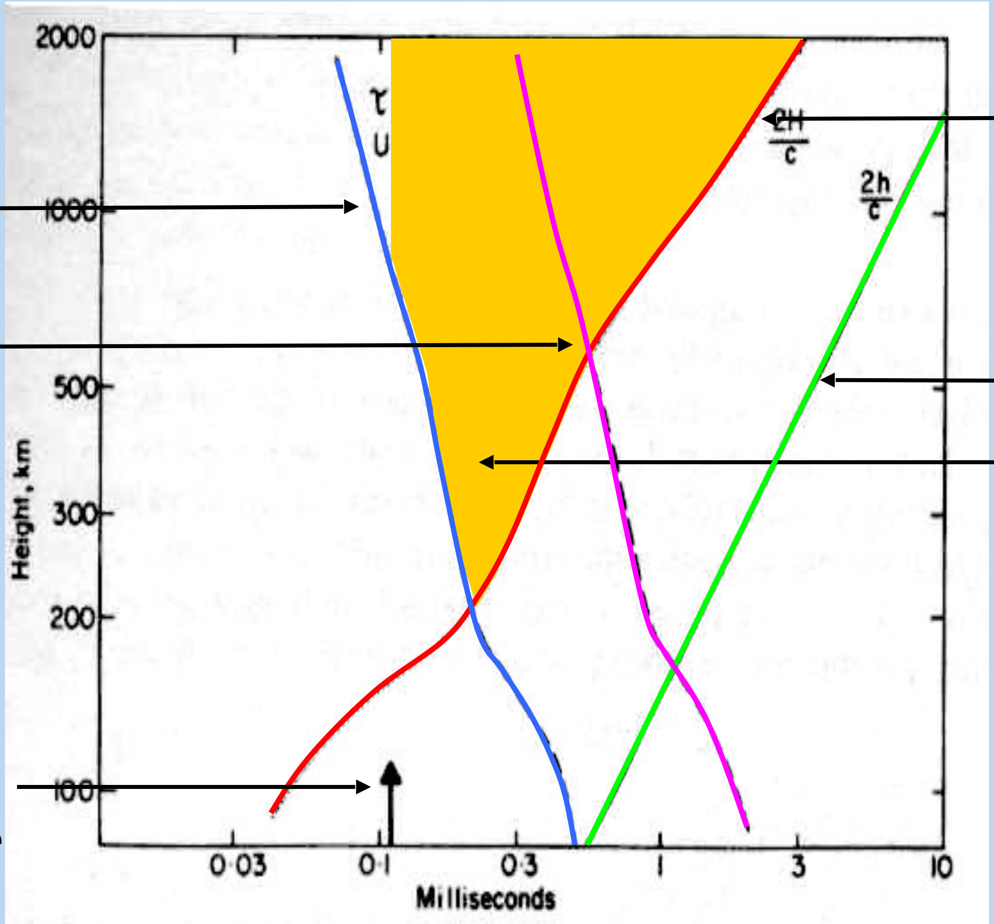
- Ideally, we would measure the full extent of the ACF up to the full temporal limit of the correlation
- This can be challenging for various reasons (e.g. range resolution, pulse coding)
- Nonetheless, we generally need to measure to (something like) the third crossover to get good plasma parameter estimates
    - We also need to sample the ACF sufficiently frequently!
- Correlation times depend on radar frequency
    - Lower frequency means longer correlation times and narrower spectra
    - Higher frequency means wider spectra and shorter ACFs (shorter correlation times)
    - At a given frequency, spectra get wider with altitude (shorter correlation times)
- This can create interesting issues for experiment design

Constraining factors for an incoherent scatter radar experiment

ionospheric correlation time, τ_1 for UHF

ionospheric correlation time, τ_1 for VHF

Minimum pulse length obtainable from transmitter



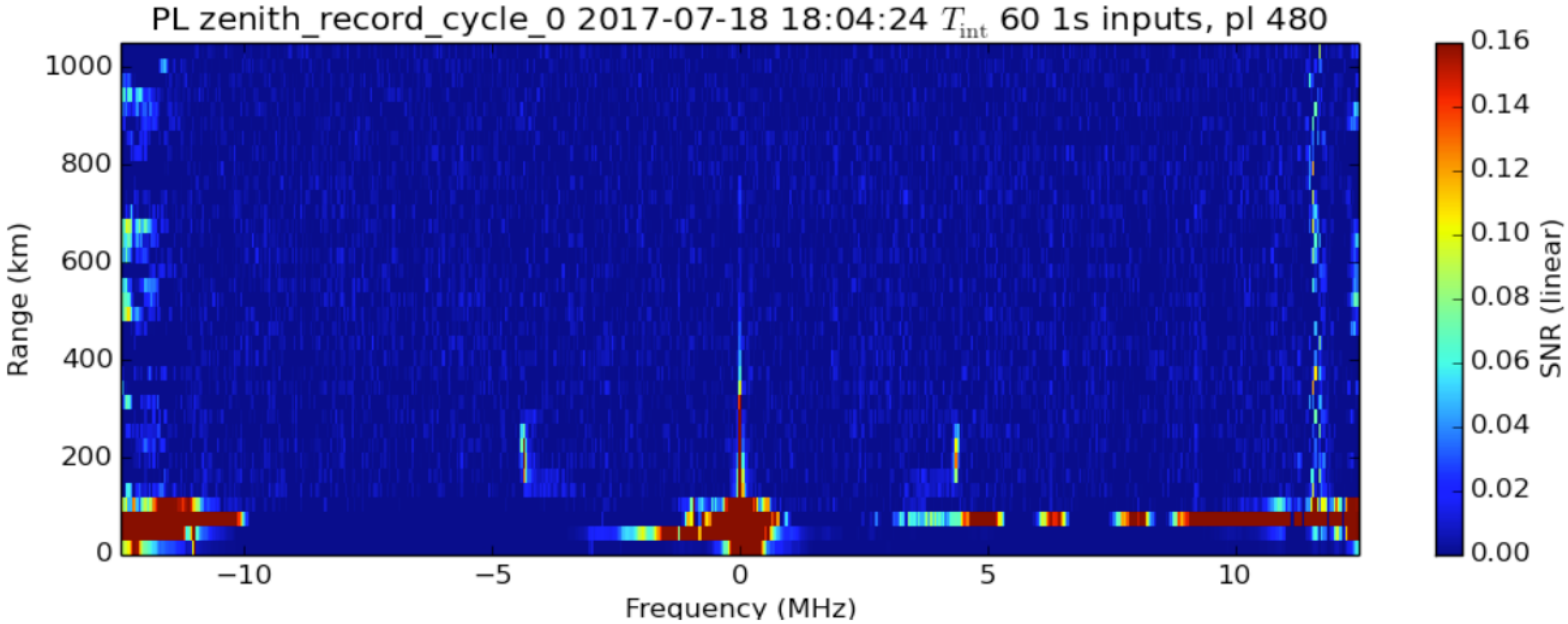
Pulse length giving resolution equal to the scale-height.

Time of flight for radar pulse

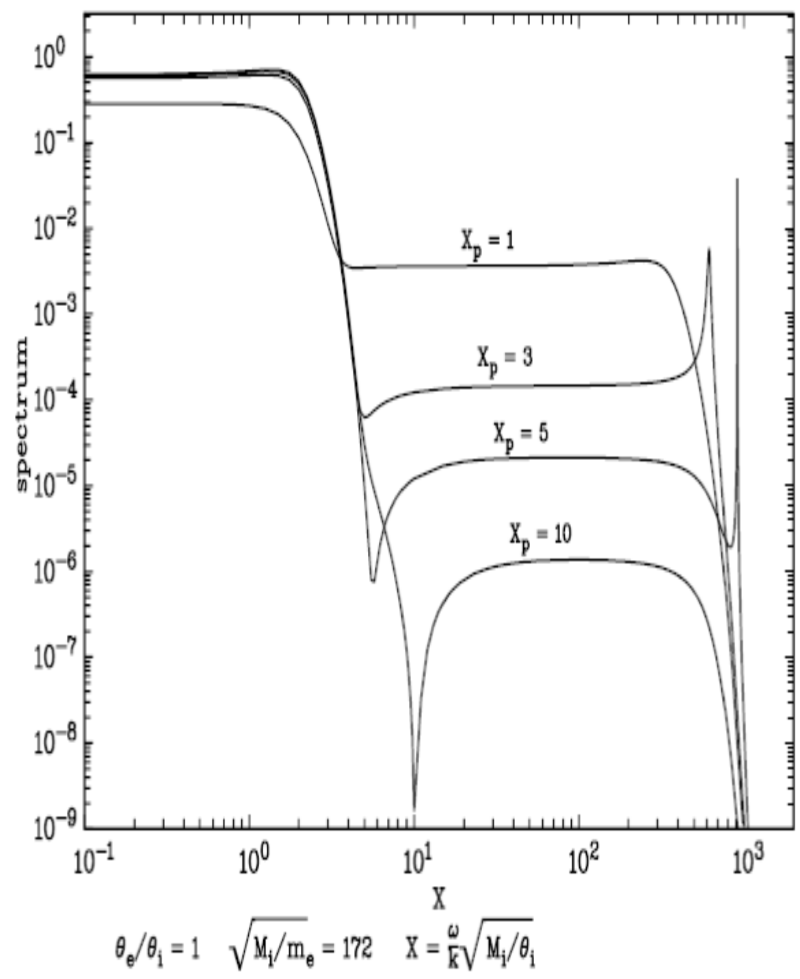
Possible values for UHF experiment

Some constraining factors for incoherent scatter experiments, shown as functions of height for typical ionospheric conditions.

Plasma Line Spectra: Millstone Hill (60 sec integration)



Plasma Lines (from Langmuir Waves) in Incoherent Scatter



Neglecting magnetic field effects, to first order:

$\omega^2 = \omega_p^2(1 + 3k^2\lambda_D^2)$

- ω = Langmuir wave frequency
- ω_p = plasma frequency
- k = wavenumber
- λ_D = Debye length

And for ISR to work, $k^2\lambda_D^2$ must be small, so $\omega \sim \omega_p$

If ion lines and plasma lines can both be measured, the radar is effectively self calibrating

This makes plasma line measurements very useful!

Unfortunately, the plasma line is normally weak for thermal plasmas

Enhanced Plasma Lines

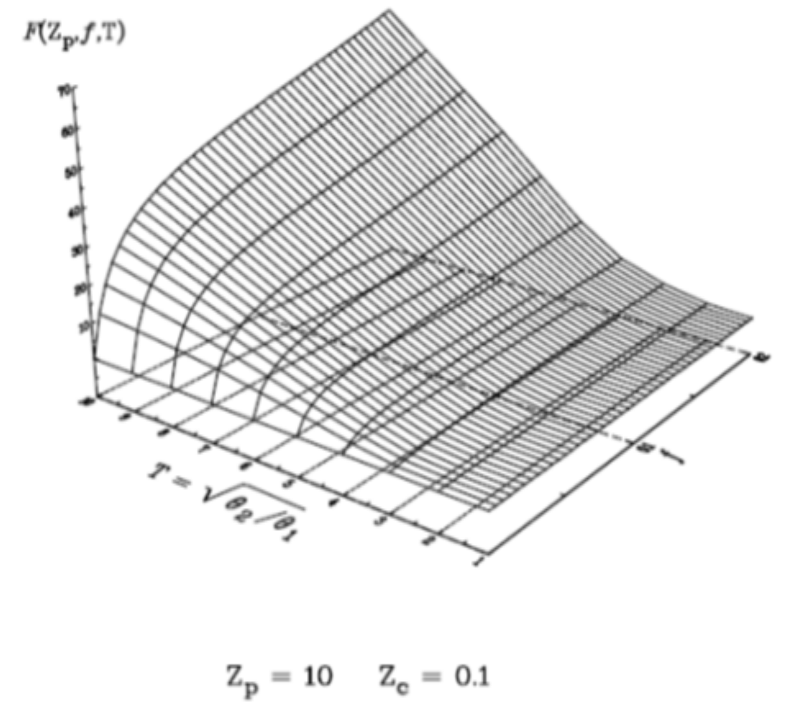


Figure 10. Plasma line enhancement factor plotted against the fraction of photoelectrons f and $T=\sqrt{\theta_2/\theta_1}$ where θ_2 is the temperature of the photoelectrons and θ_1 the temperature of the background.

The power in the plasma line is proportional to $(k\lambda_D)^2$, which, as we have seen, is small

However, plasma lines can be significantly enhanced if there is a source of free energy to increase the amplitude of the plasma waves.

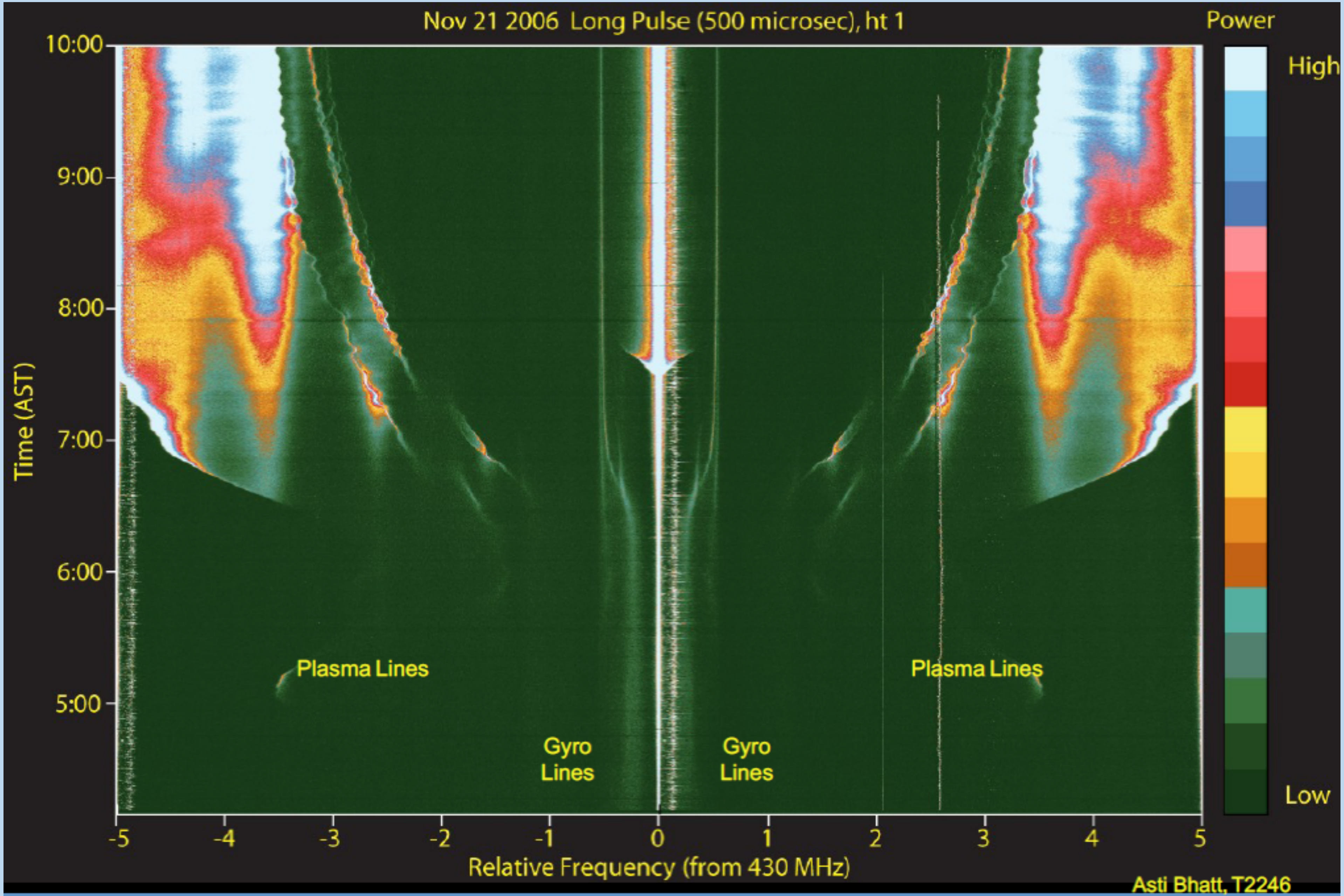
Potential sources are suprathermal electrons arising from:

- Photoelectrons created during ionisation by solar EUV
- Energetic high-latitude precipitation (aurora)

Most ISRs only observe plasma lines when they are enhanced in this way.

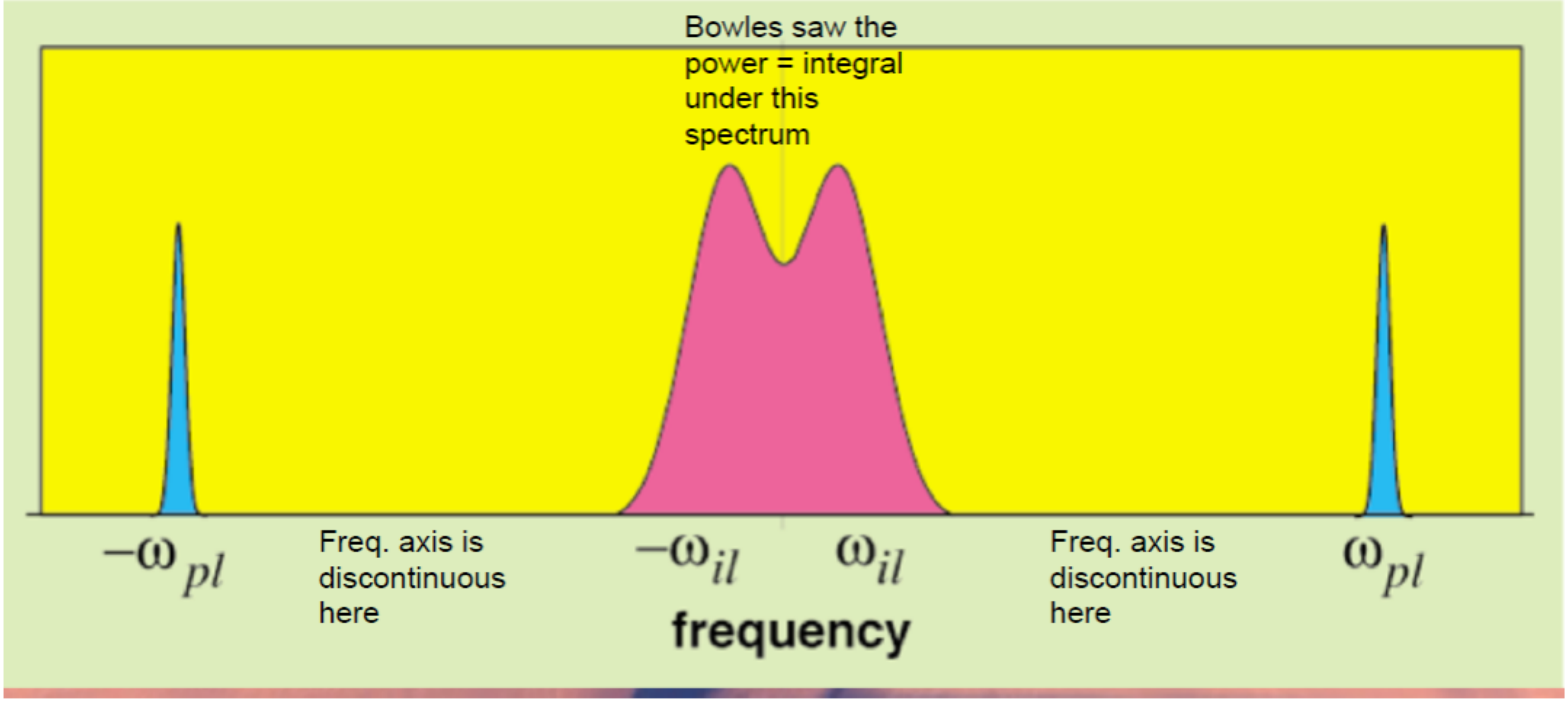
Gyrolines and Magnetic field effects

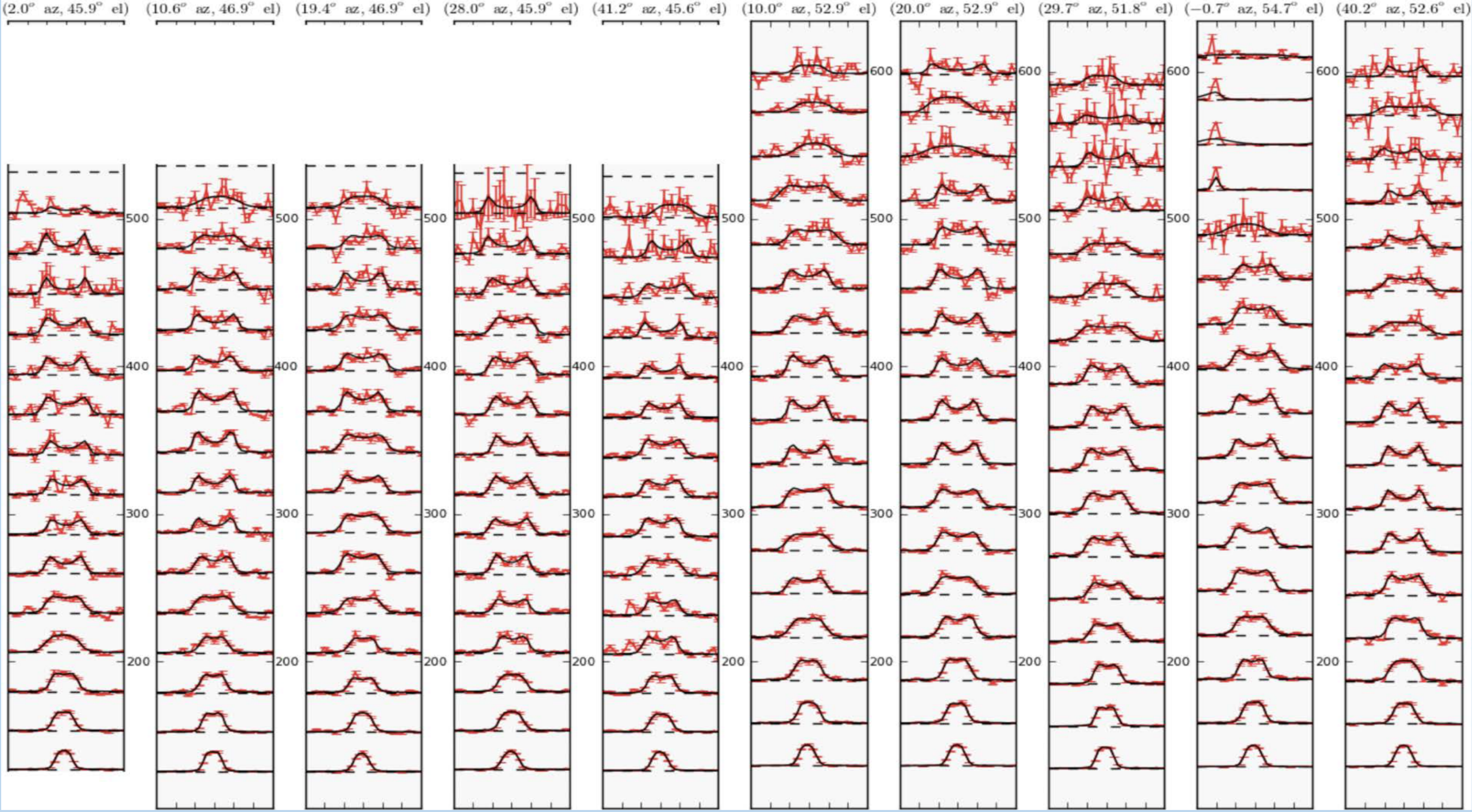
- There are a third class of weak spectral feature arising from the influence of the magnetic field, called “gyrolines”.
- Gyroline frequencies are lower than the plasma line, but significantly above the ion line frequency
- These are another wave-driven mode, conventionally associated with the high-frequency end of the whistler mode spectrum
- Gyrolines are most often observed in the middle to lower ionosphere (E-region and bottomside F-region < 200km) at mid latitudes
- At higher altitudes, higher Te leads to increased cyclotron damping
- At higher latitudes (greater dip angle) wave frequency approaches the electron gyrofrequency



Arecibo Sensitivity: The 305 m dish, 2.5 MW of power, and Tsys of about 80 K (condition dependent) provide high time resolution on even weak features such as the gyro line. The data above are centered on the E region. The strong plasma line after sunrise is "leakage" from the low F region. The complicated behavior of the gyro line is probably due to multiple layers, but is not completely understood.!

Incoherent Scatter Spectra as seen from a radar





(2.0° az, 45.9° el)
(10.6° az, 46.9° el)
(19.4° az, 46.9° el)
(28.0° az, 45.9° el)
(41.2° az, 45.6° el)
(10.0° az, 52.9° el)
(20.0° az, 52.9° el)
(29.7° az, 51.8° el)
(−0.7° az, 54.7° el)
(40.2° az, 52.6° el)
600
500
400
300
200

Anomalous Spectra in Incoherent Scatter

There are (at least) four main types of non-thermal spectra

(a) Non-Maxwellians

- The plasma velocity distribution becomes non-thermal, changing the characteristics of the Landau damping
- This happens mainly in O^+-O dominated regimes under large electric field conditions
- Spectral shape then depends on field strength and aspect angle.

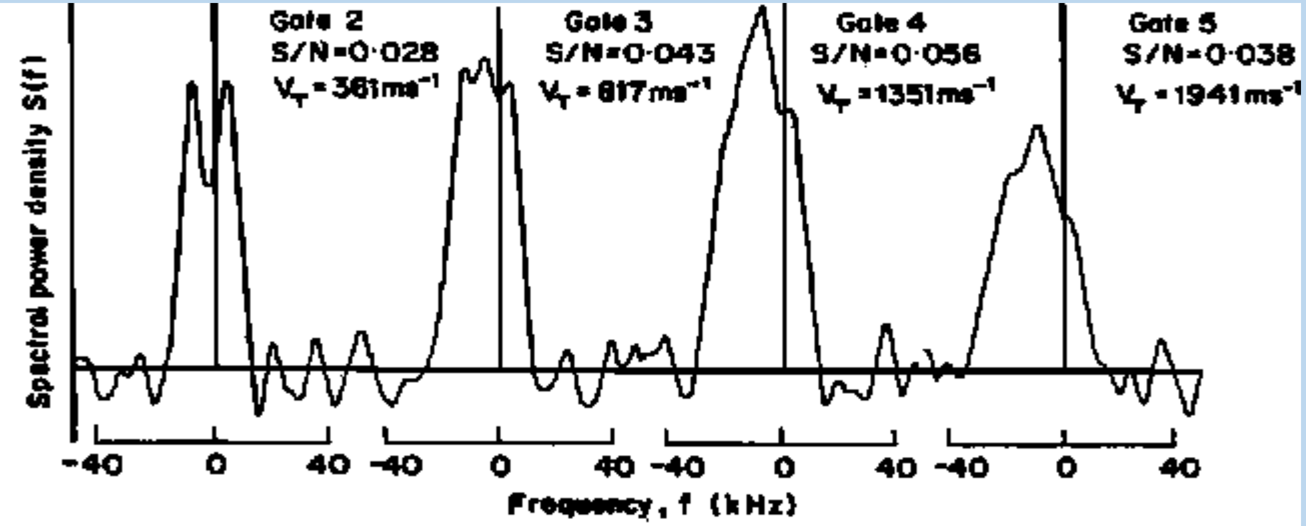


Fig. 1. Signal spectra for the integration

Incoherent Scatter Radar Remote Sensing: Summary

IS Radar Remote Sensing Capabilities

Parameters sensed
(Full altitude profiles):

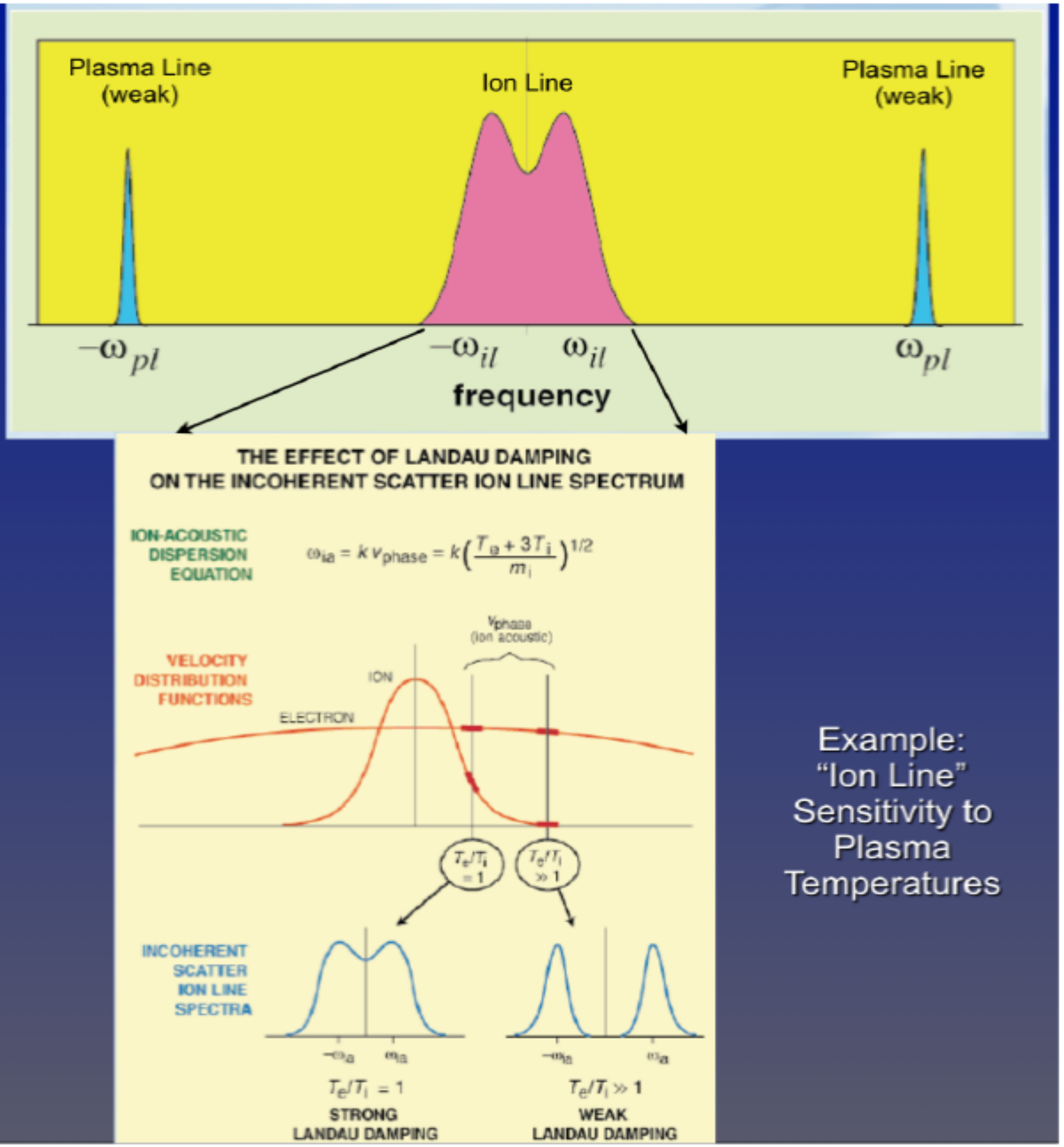
Basic

- Electron density
- Electron temperature
- Ion temperature
- Ion composition
- LOS Velocity

Derived

- Neutral winds
- Neutral temperature
- Vector velocity

More parameters possible

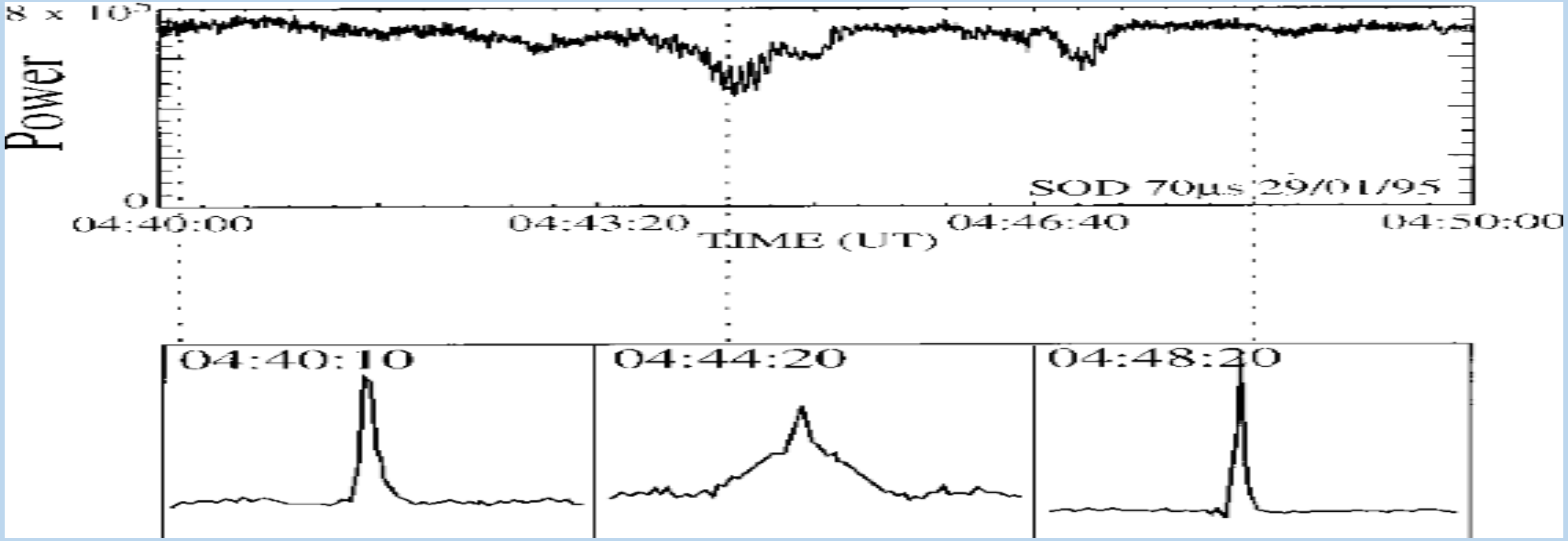


Example:
"Ion Line"
Sensitivity to
Plasma
Temperatures

Anomalous Spectra in Incoherent Scatter

(b) Coherent Scatter Spectra

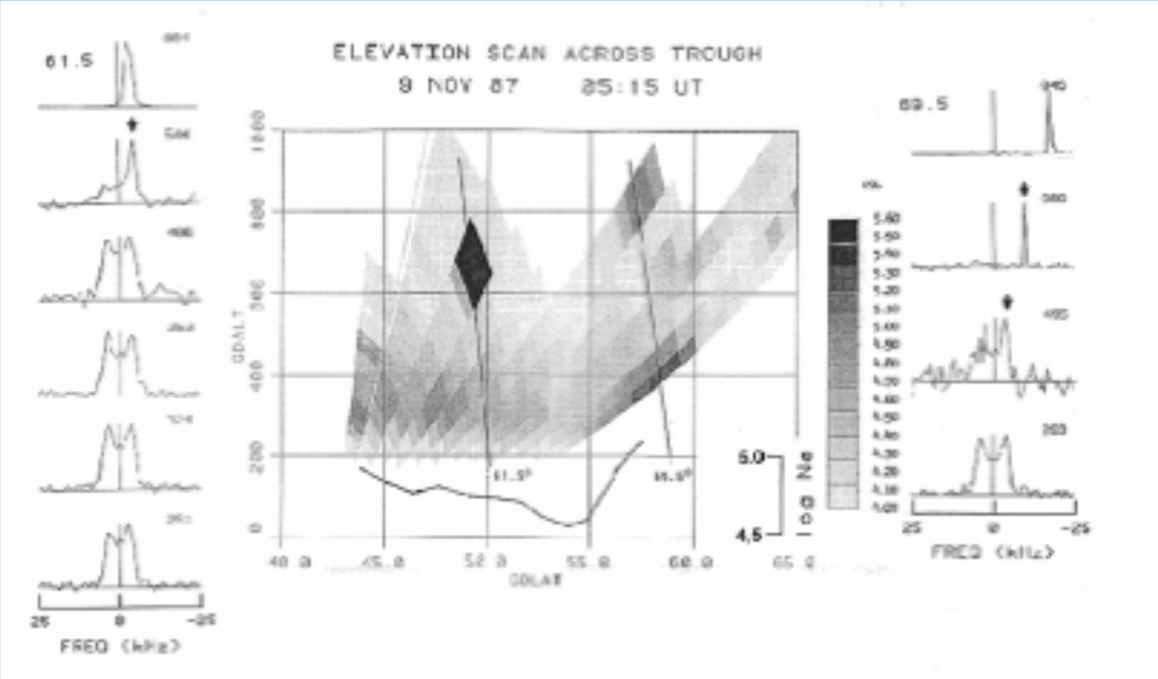
- The scattering wave is no longer a thermal ion-acoustic wave, but larger-amplitude plasma turbulence at the same frequency
- This happens under large electric field conditions with observing directions near perpendicular to the magnetic field
- Spectral shape again depends on field strength and aspect angle.

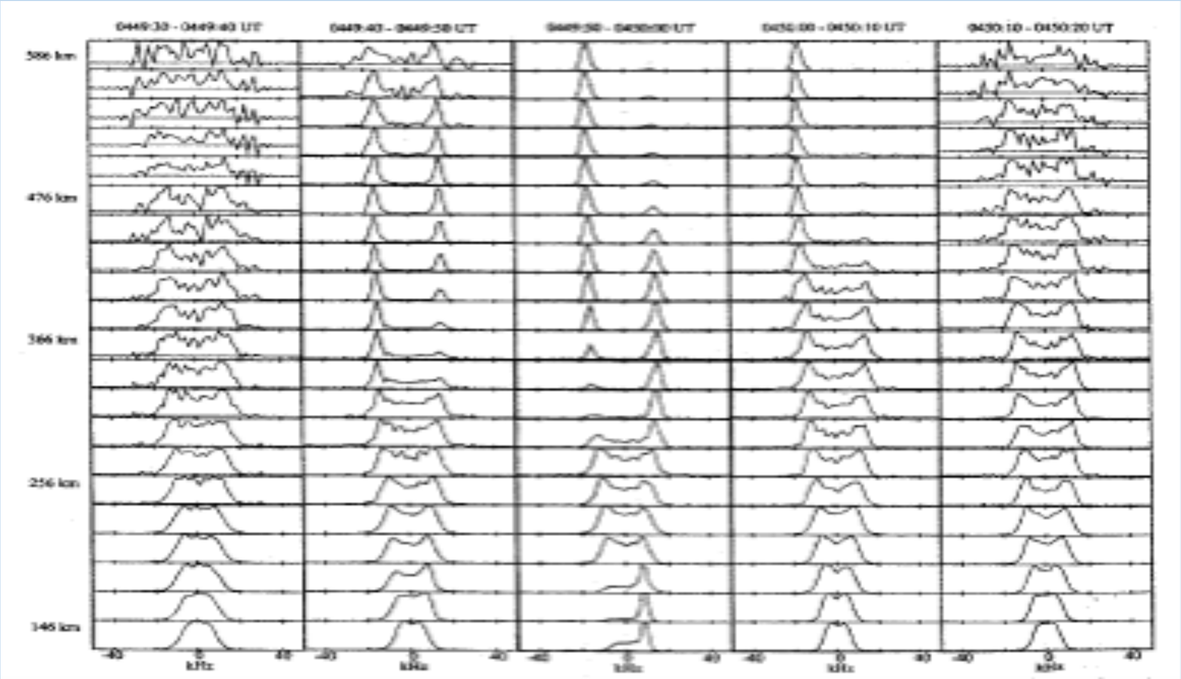


Anomalous Spectra in Incoherent Scatter

(c) NEIALs (Naturally Enhanced Ion Acoustic Spectra)

- The scattering medium is ion-acoustic waves, but these are enhanced due to coupling from energetic particles or other kinds of wave mode
- These are mainly seen close to field-aligned under active auroral conditions
- Spectra are generally strongly asymmetric.

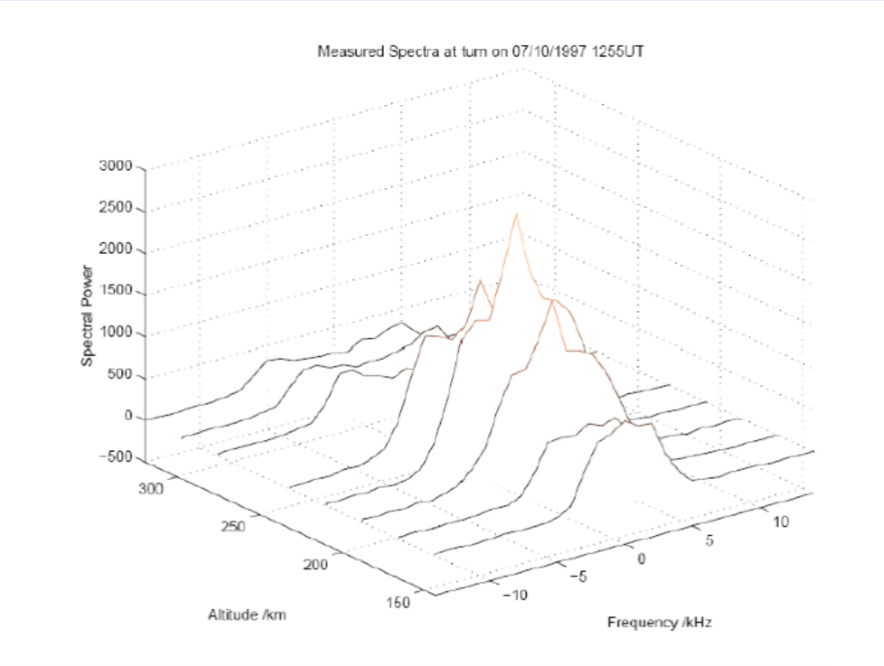


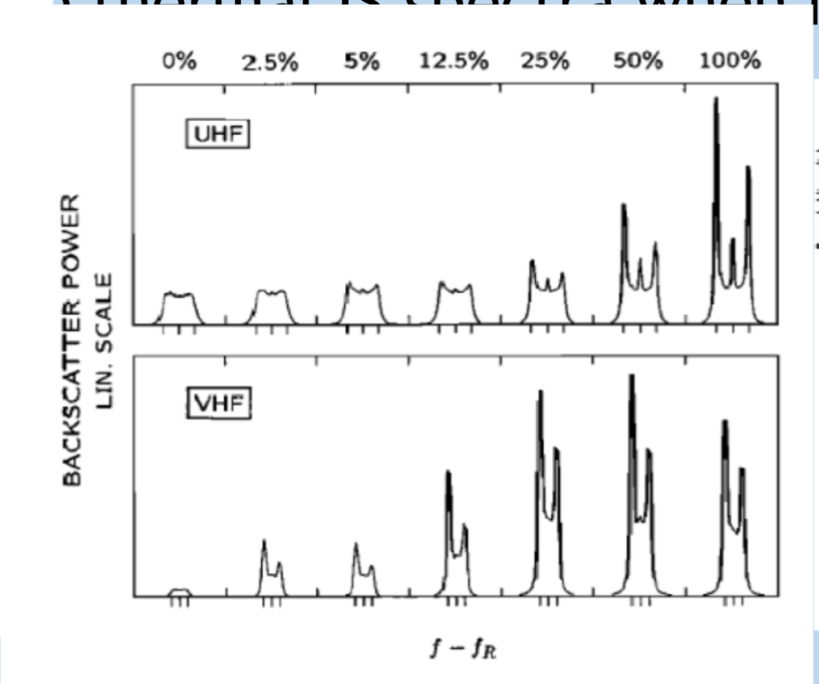


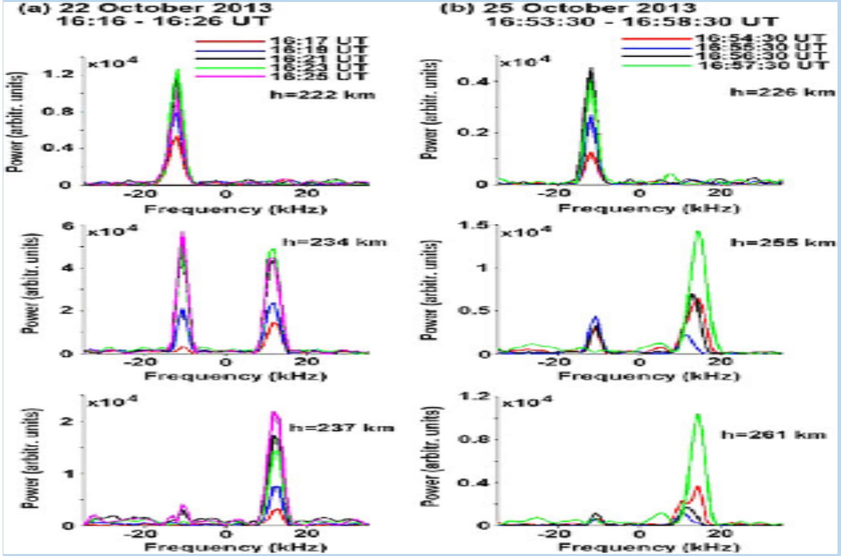
Anomalous Spectra in Incoherent Scatter

(d) Anomalous spectra during HF heating

- The scattering waves are driven by energy coupling from the HF pump wave
- Various effects depending on power, direction and frequency of heater beam
- thermal IS spectra when heating is turned off







Summary

- What we measure with radars is determined by those wave modes of the plasma which match the Bragg condition for our radar wavelength
- Spectral shape is determined by the dispersion relation of these waves, taking into account the relationship of the wave velocity to the thermal velocity (Landau damping)
- Changing plasma parameters changes the spectrum – some of these changes are fairly ambiguous.
- Changing radar wavelength also changes the spectrum for given set of plasma parameters

Summary (2)

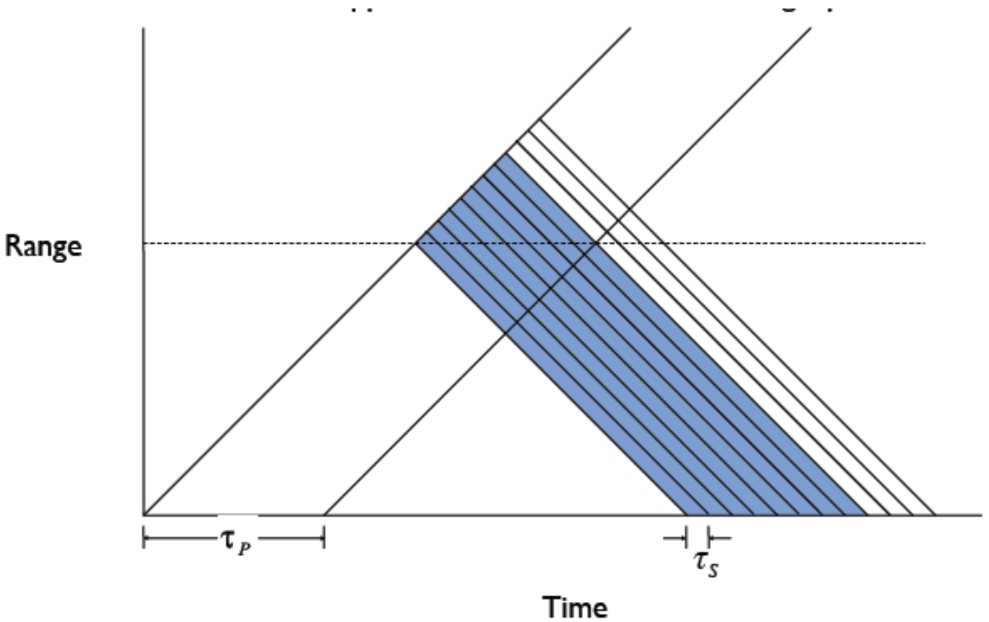
- To derive plasma parameters from the ion line, we generally fit in the time domain (to the ACF)
- Ideally we measure the whole decorrelation time of the plasma, but in principle at least to 2.5 τ_0
- This has to be reconciled with appropriate spatial sampling
- Needs different codes for different altitudes
- Plasma lines can be very useful (e.g. for calibration) but can be hard to observe
- There are several ways of generating non-thermal ISR spectra. In these situations our fitting theory doesn't apply.

Computing the ACF

In the correlator:

$R_{i,j} = \sum(X_iX_j + Y_iY_j)$

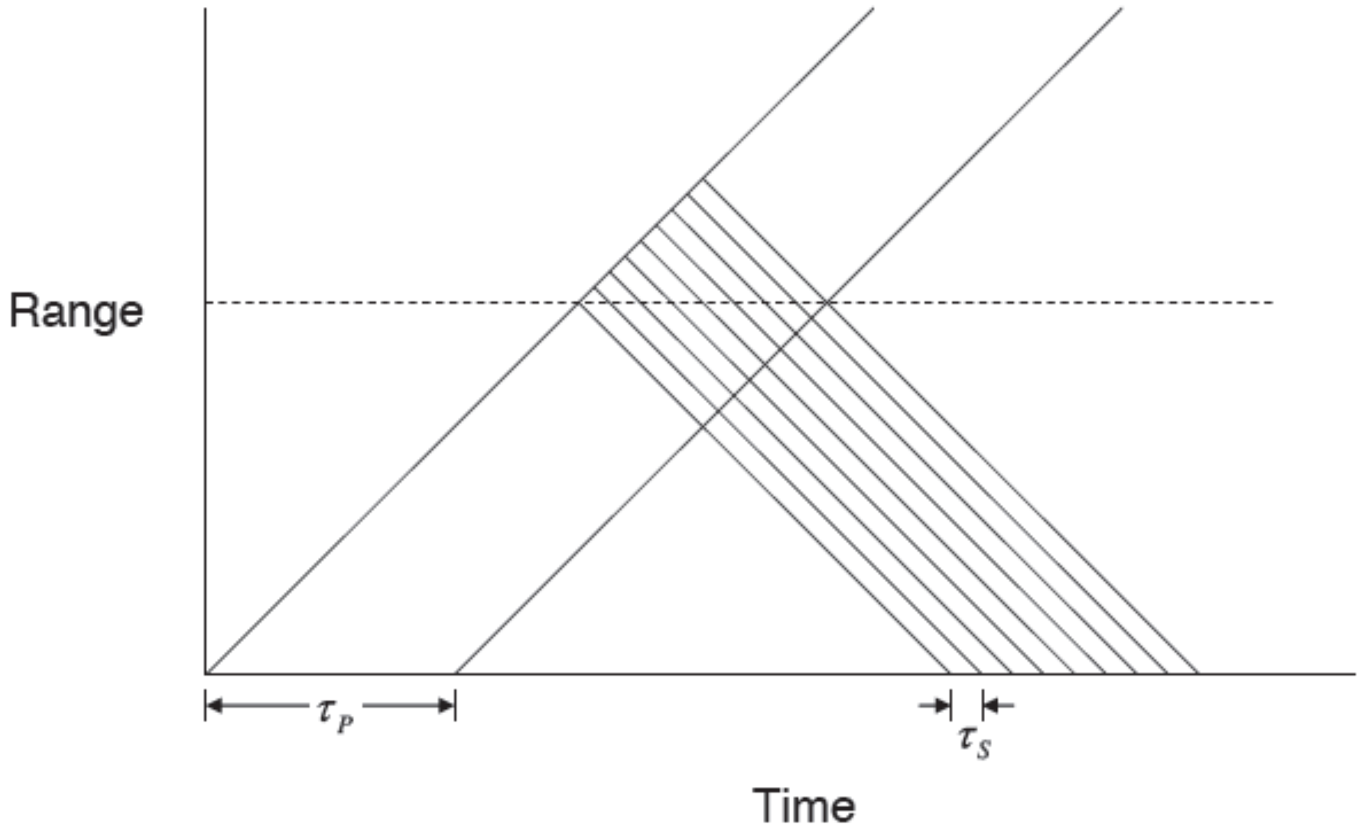
$I_{i,j} = \sum(X_iY_j - Y_iX_j)$



For lag 0: i = j (imaginary part identically zero)

For lag 1: j-i=1, and so on…

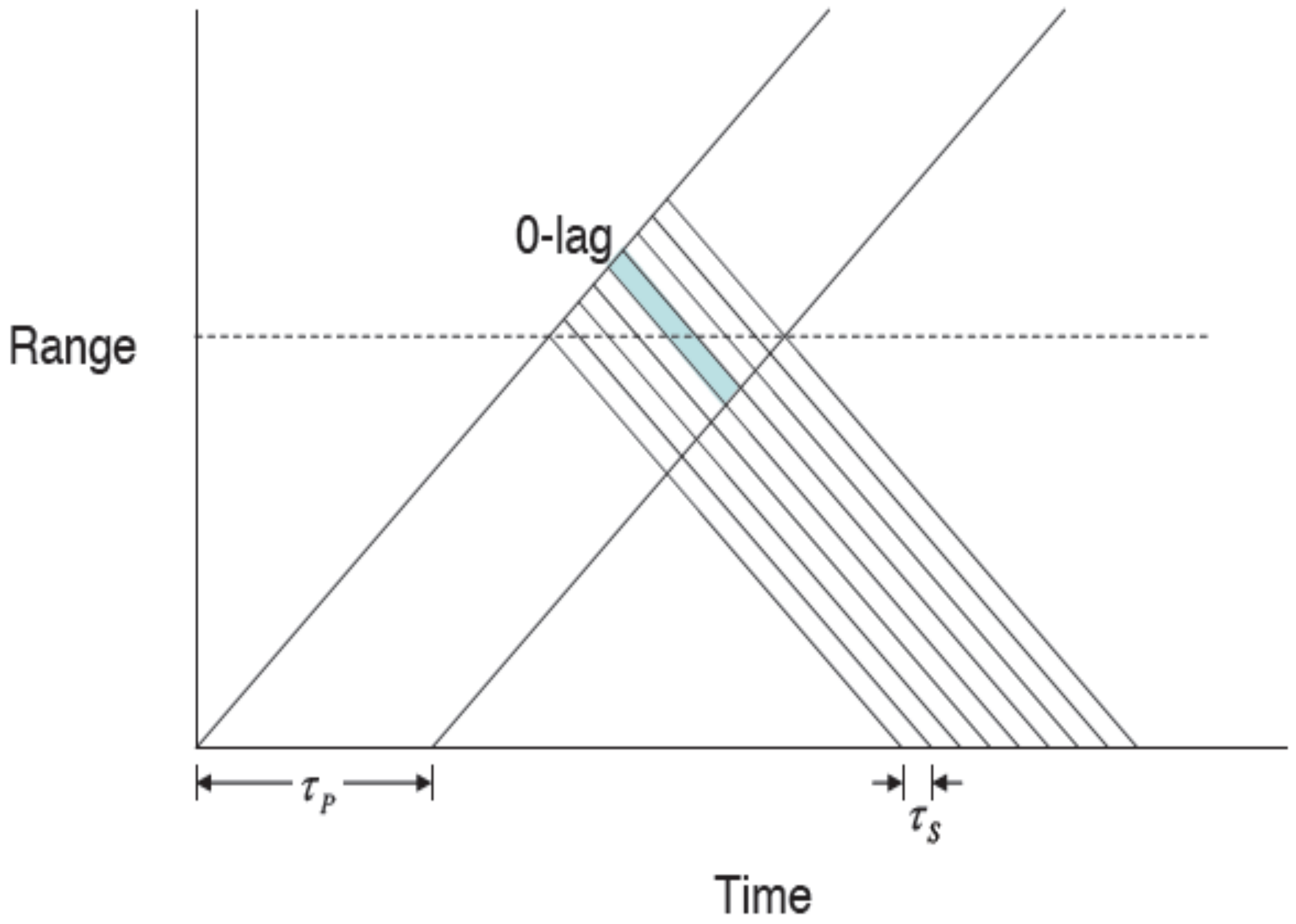
Computing the ACF



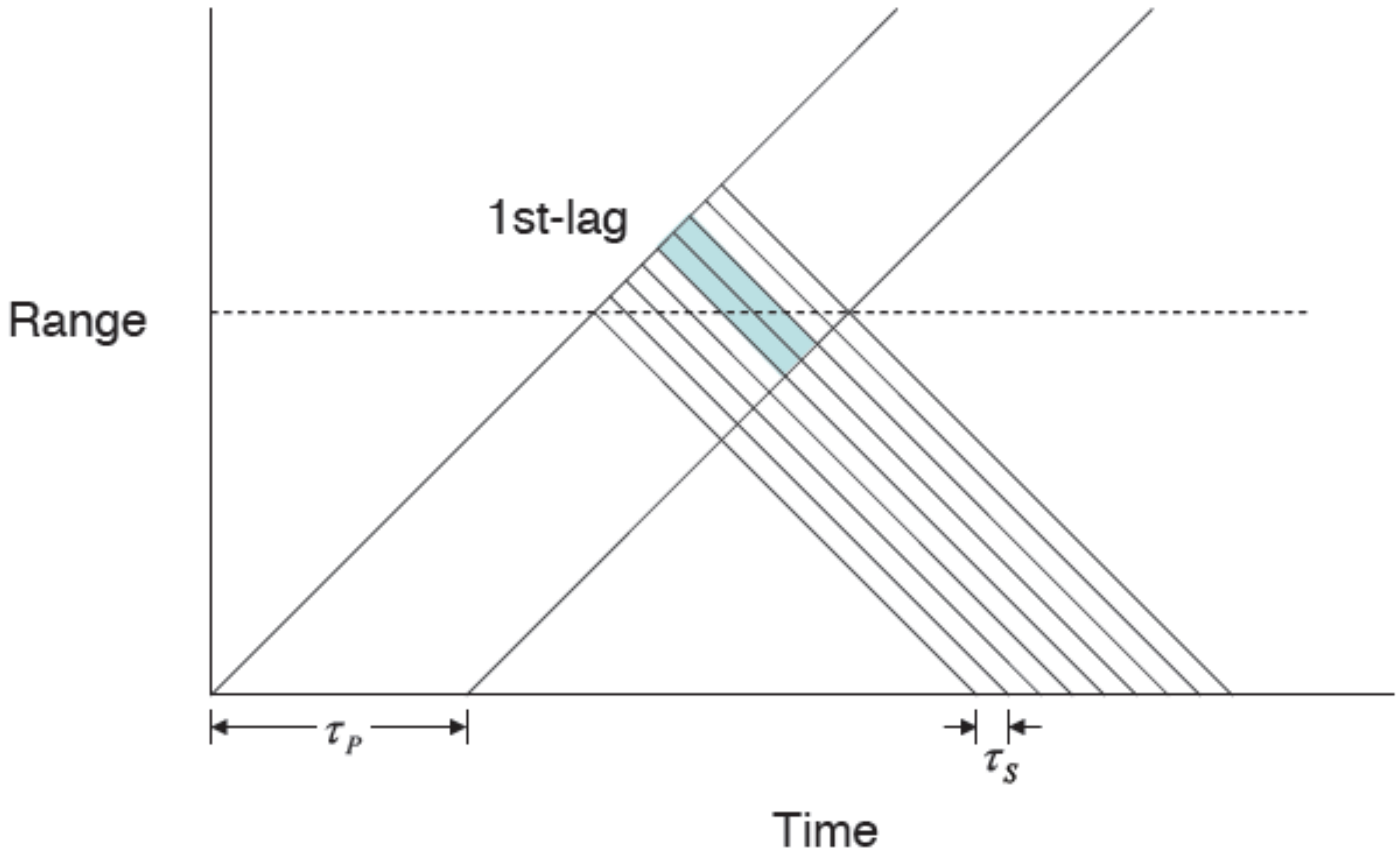
τ_P = Length of RF pulse

τ_s = Sample Period (typically ~ 1/10 pulse length)

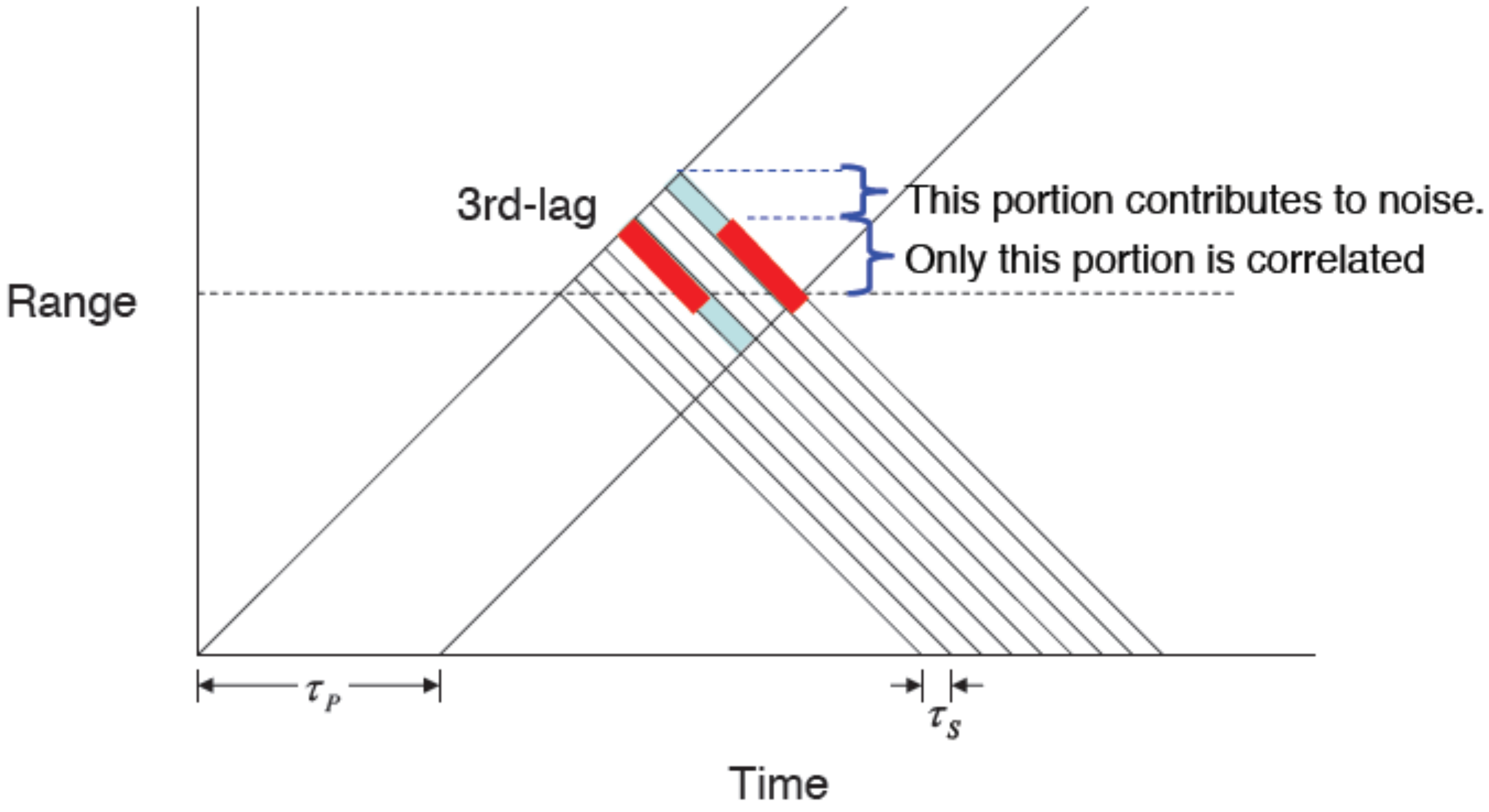
Computing the ACF



Computing the ACF



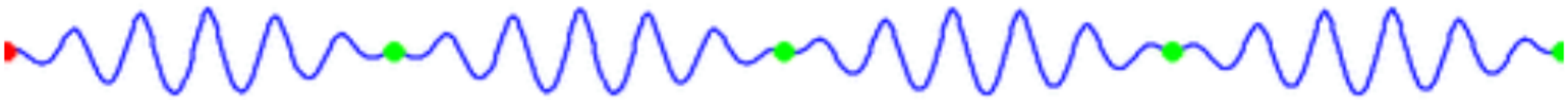
Computing the ACF



Dispersion relation: the concept

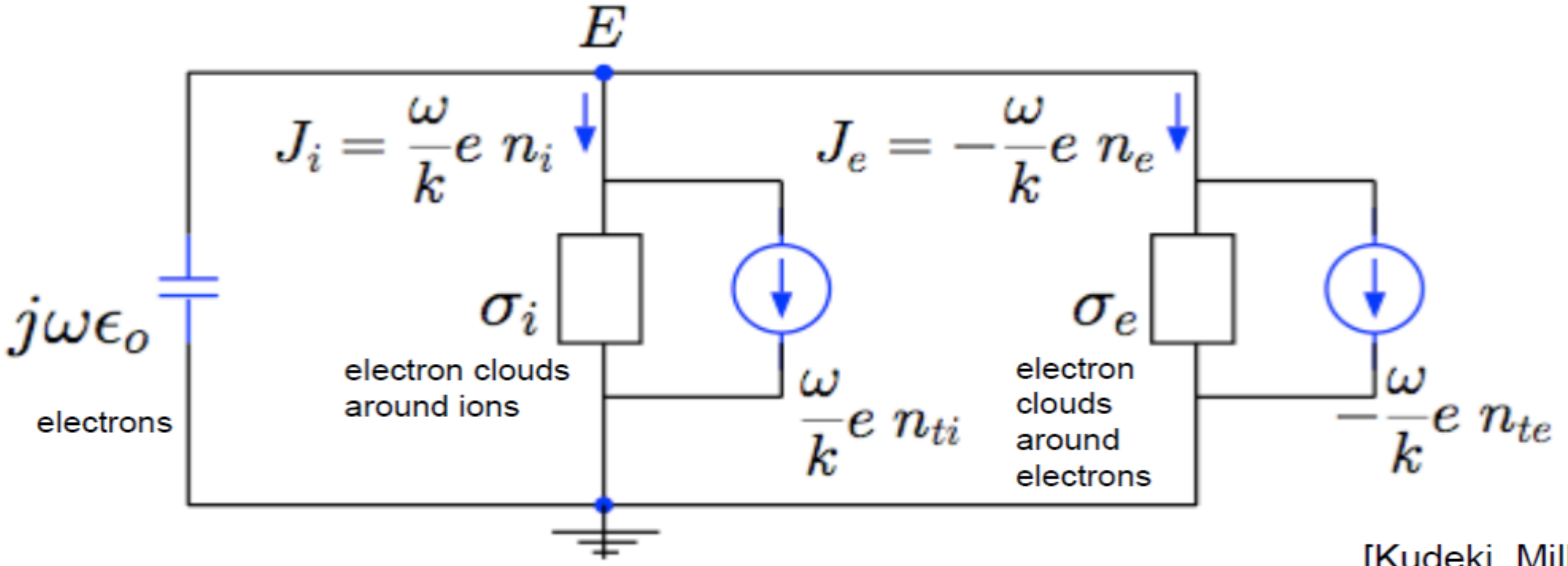
Linear dispersion in a transverse wave with 2 frequencies:
Note that phase (red) velocity = group (green) velocity

Nonlinear dispersion in a transverse wave with 2 frequencies:
Note that phase (red) velocity is **faster** here than group (green) velocity



Incoherent Scattering Model

$$\sigma_0(\omega_o + \omega)d\omega = N_0 r_e^2 \,\mathrm{Re}\left\{ \frac{y_e(y_i + jk^2\lambda_{de}^2)}{y_e + y_i + jk^2\lambda_{de}^2} \frac{d\omega}{\pi\omega} \right\}$$



[Kudeki, Milla]

Incoherent Scattering Model

$$\sigma_0(\omega_o + \omega)d\omega = N_0 r_e^2 \, \mathrm{Re}\left\{ \frac{y_e(y_i + jk^2\lambda_{de}^2)}{y_e + y_i + jk^2\lambda_{de}^2} \frac{d\omega}{\pi\omega} \right\}$$

- Short wavelength limit ($k^2\lambda_{de}^2 >> 1$): pure e^- scatter
- Long wavelength limit: RHS $\rightarrow y_e y_i/(y_e + y_i)$: damped ion-acoustic resonances
- Near plasma frequency: $y_e + y_i + jk^2\lambda_{de}^2 \rightarrow 0$: plasma lines

Incoherent Scattering Model

Spectral response can be evaluated using these frameworks for:

- Thermal inequality $T_e \neq T_i$: decreases Landau damping
- Ion-neutral collisions ν_{in}: narrows spectrum
- Background magnetic field B_0: makes electrons heavier

$$m_e \longrightarrow m_e^* = \frac{m_e}{\cos^2 \alpha}$$

Also, ion gyro-resonance (mass-dependent).

Incoherent Scattering Model

- Ion mixtures: $\frac{T_e}{T_i} y_i \longrightarrow \sum_j \frac{T_e}{T_j} \frac{N_j}{N_0} y_j(m_j, T_j)$
- Unequal ion temperatures
- Particle drifts: $\omega \longrightarrow \omega - \vec{k} \cdot \vec{v}_{de}$
- Plasma line measurements ($[e^-], T_e, v_{\|}$)
- Photoelectron heating, non-Maxwellian plasmas
- Faraday rotation effects (equator, low TX freq)

Things can get hairy. For example, magnetic field evaluation requires Gordeyev integral:

$$\int e^{j(\theta - j\phi)t - \frac{\sin^2\alpha}{\phi^2}\sin^2(\frac{\phi t}{2}) - \frac{t^2}{4}\cos^2\alpha} dt$$

Plasma dispersion relations

$$\epsilon(\omega, \vec{k}) = \text{function}\,(\omega^2/k^2)$$

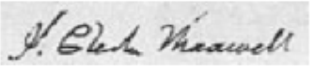
Dielectric constant of the medium — Insert plasma dispersion relation here

The physics of the medium is described by the dielectric constant (related to plasma conductivities)

in free space:
H = B D = E

Gauss' Law (electric field around charges)	$\nabla \cdot \mathbf{D} = \rho_f$
Gauss' Law for magnetism (no magnetic monopoles)	$\nabla \cdot \mathbf{B} = 0$
Faraday's Law (electric field around a changing magnetic field)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
Ampere's Law (magnetic field circulation around electric charges)	$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$

Maxwell's correction (displacement current)



(circles = places where dielectric constant shows up in Gauss, Ampere)