ISR Theory 1: Short Introduction to Incoherent Scatter

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Thanks to Tuomo Nygrén, Anja Strømme and US colleagues

EISCAT Svalbard radars

Global Network of Incoherent Scatter Radars

Can Measure Physical Properties of the Space Environment
<u>as a function of altitude</u>:

electron density, electron temperature, ion temperature, plasma velocity

Can Infer:

electric field strength, conductivity, current

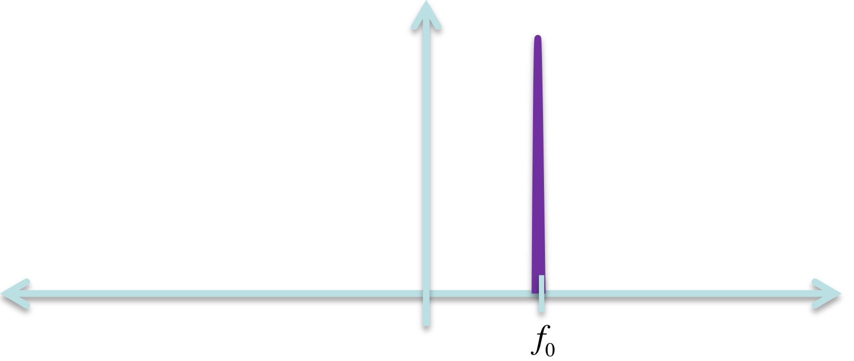
IS radar parameters

IS radar gives us the following plasma parameters:

- Ne (electron density)
- Te, Ti (electron and ion temperatures)
- Vi (ion velocity): either in the beam direction (monostatic radar) or along the bisector (receiver not in the same location as transmitter). For vector velocity, 3 components need to be measured.
- Requires special analysis: ion mass mi, usually taken from a model
- Requires special analysis: ion-neutral collision frequency ν_{in} (typically Te/Ti is assumed equal to 1 in that analysis)
- From **V**i vector in the F-region, electric field **E** can be calculated
- From **V**i vector in the E and F regions, neutral winds **u** in the E-region under some assumptions can be inferred

Doppler radar (total reflection)

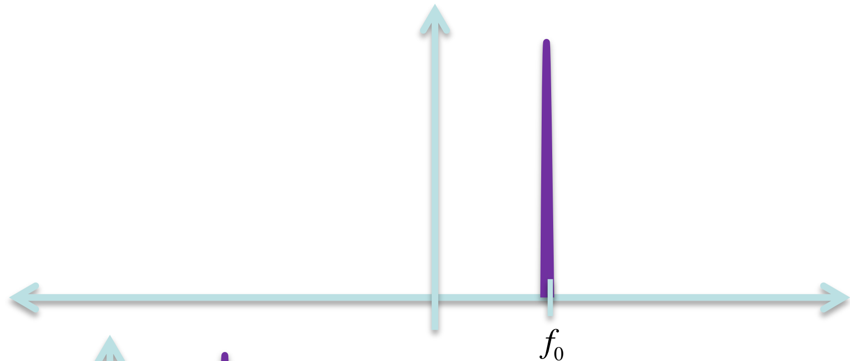
Transmitter frequency f_o



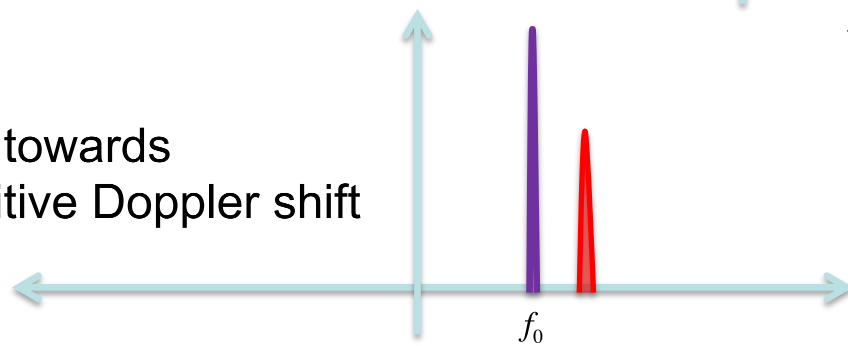
f_0

Doppler radar (total reflection)

Transmitter frequency f_o

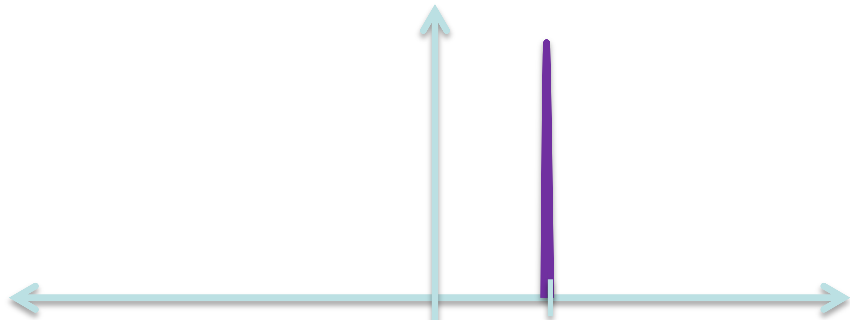


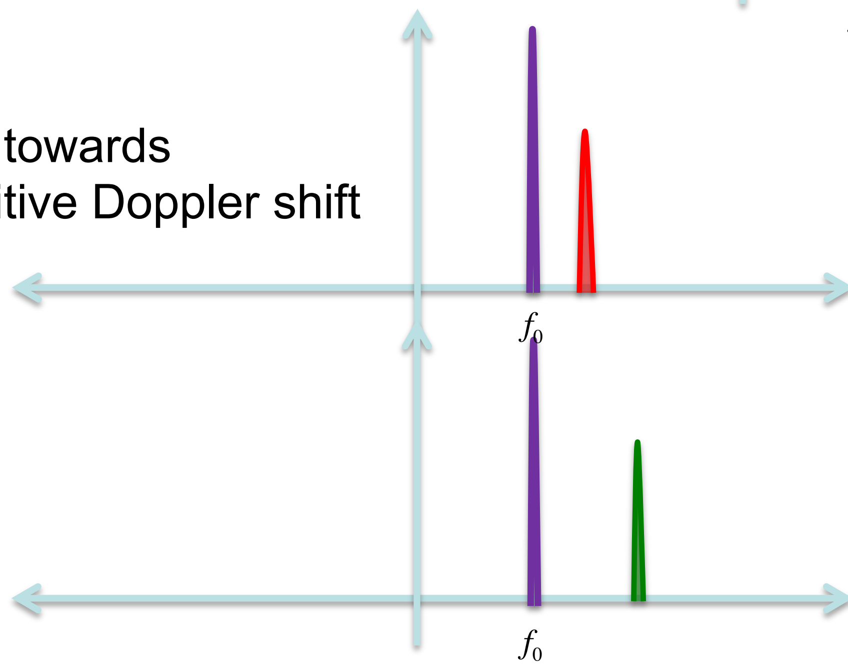
Slow velocity towards
=> Small positive Doppler shift



Doppler radar (total reflection)

Transmitter frequency f_o



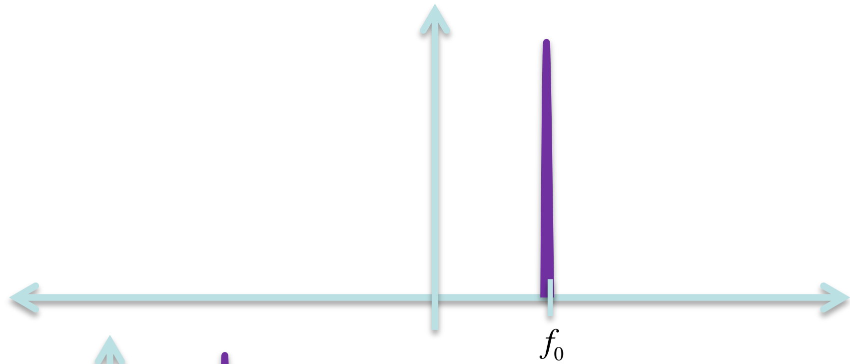


Slow velocity towards
=> Small positive Doppler shift

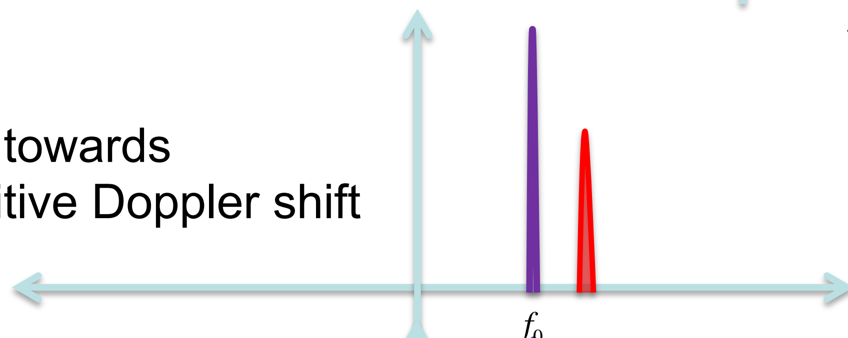
High velocity towards
=> Large positive Doppler shift

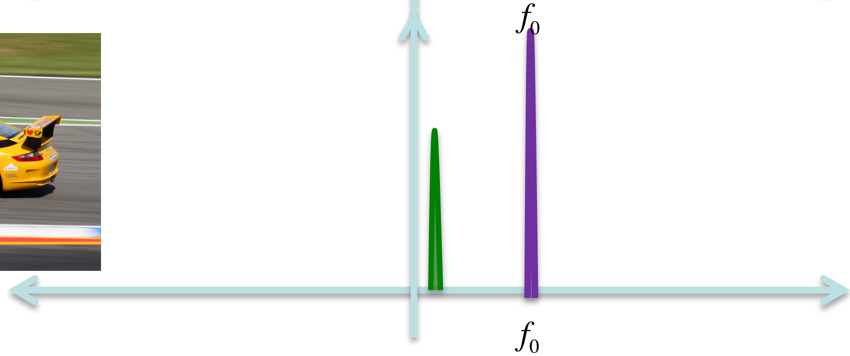
Doppler radar (total reflection)

Transmitter frequency f_o



Slow velocity towards
=> Small positive Doppler shift



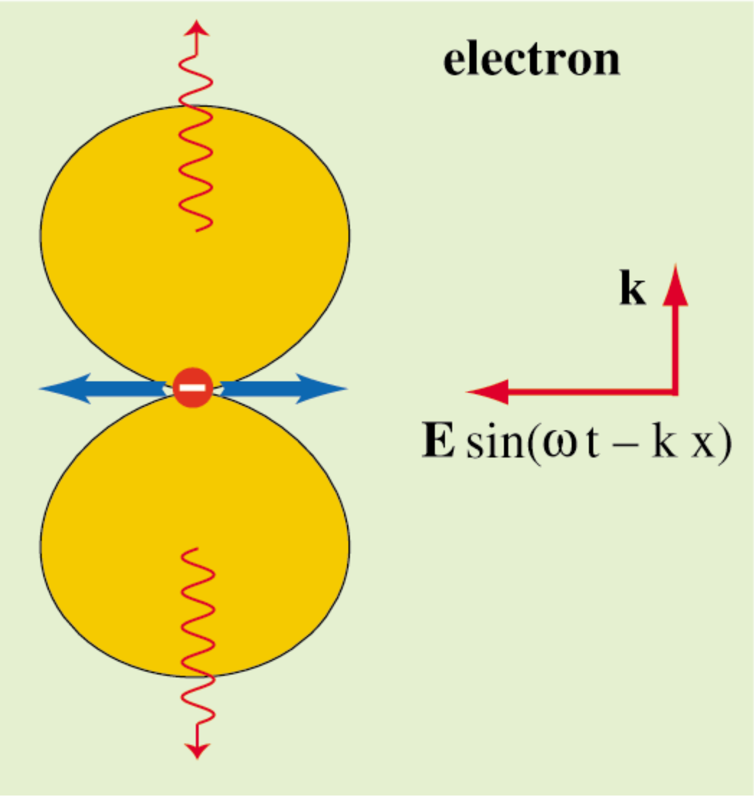


High velocity away
=> Large negative Doppler shift

What is meant by Incoherent Scatter radar (IS)?

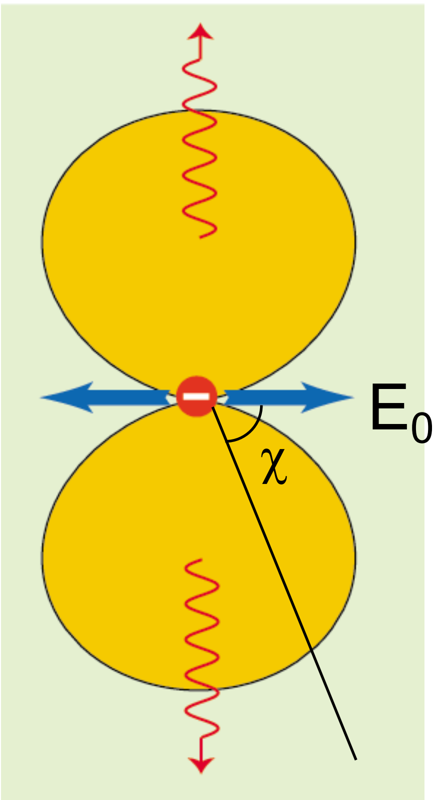
- Scattering (partial reflection), not total reflection
- Incoherent: the term means that phases of the scattered waves are randomly distributed. However, it turns out that the IS signal is produced by quasi-coherent waves, so the original name is somewhat misleading.
- Original idea was that incoherent scattering comes from the random thermal fluctuations of electrons in the ionosphere.

What is IS radar measuring?



1906 J.J. Thomson showed that free electrons are capable of scattering electromagnetic radiation (so called Thomson scattering). The electric field of the incident wave accelerates the charged particle, causing it to oscillate and emit radiation at the same frequency as the incident wave, and thus the wave is scattered.

Thomson scattering



Power scattered by a single electron to a solid angle $d\Omega$ around direction towards angle χ is

$$dP = r_o^2 \sin^2 \chi d\Omega \; S_i \ ,$$

where the intensity of the incident radiation S_i is proportional to the amplitude of electric field E_0 squared $S_i = 1/(2\mu_0 c)E_0^2$, and the classical radius of electron is given by

$$r_o = \frac{e^2}{4\pi\varepsilon_0 m_e c^2} = 2.82 \cdot 10^{-15} \ \mathrm{m} \ .$$

The total power scattered by a single electron is given by

$$P = \frac{8}{3}\pi r_o^2 S_i = \sigma_t S_i \ ,$$

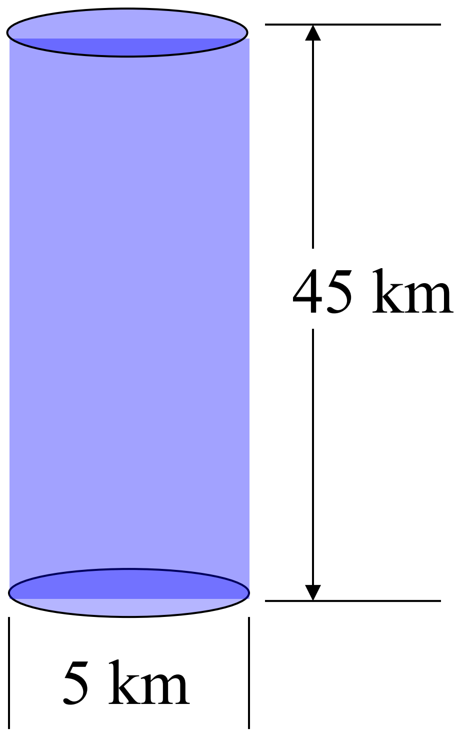
where the cross section of Thomson scattering is given by

$$\sigma_t = \frac{8}{3}\pi r_o^2 = 6.65 \cdot 10^{-29} \ \mathrm{m}^2$$

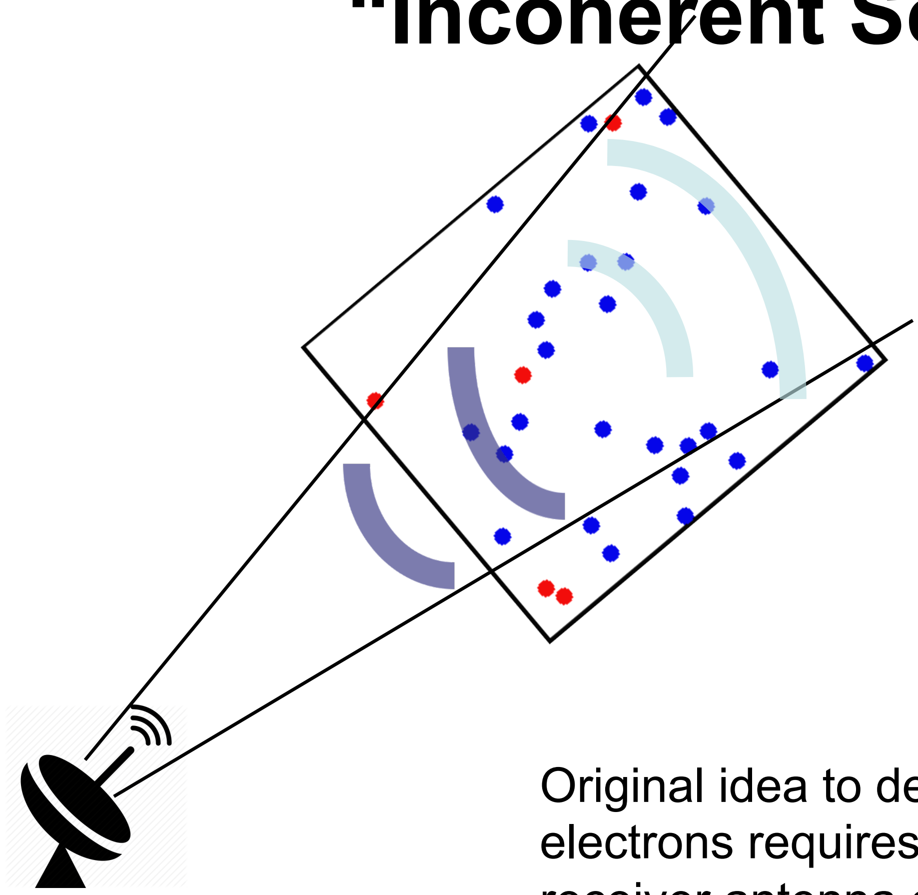
So, for a single electron the cross section is roughly 10^{-28} m^2

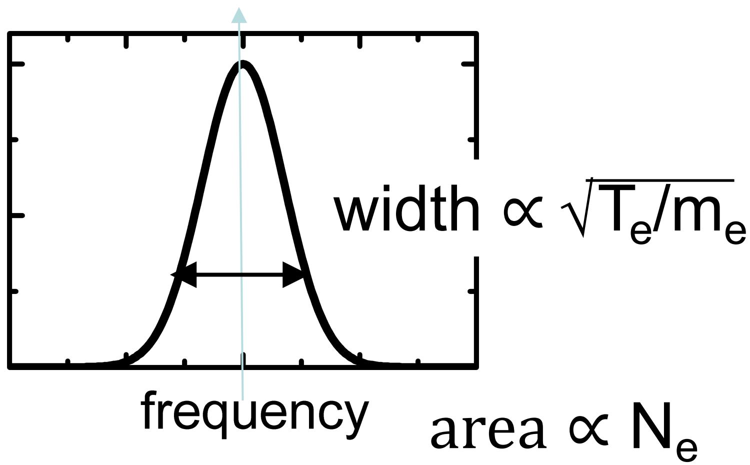
Total cross section estimate

Consider an antenna with a 1^o beam measuring the ionospheric plasma at 300 km range and using a 300 μs (τ) pulse, which corresponds to a length of 45 km (eq. $c\tau/2$).
If the electron density is 10^{12} m^{-3}, the total number of electrons scattering into a given measurement is ~8.8×10^{23}. With the electron Thomson scatter cross section of 10^{-28} m^2, this yields a total cross-section of 88 mm^2 – we need a big radar!



Thermal fluctuating electrons
"Incoherent Scattering"

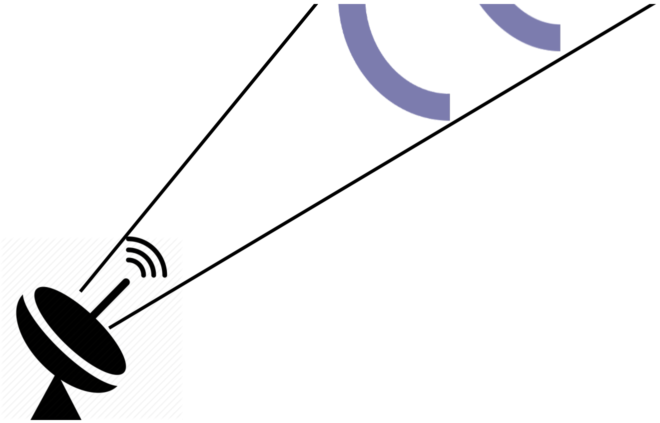




Original idea to detect properties of ionospheric electrons requires large transmitter power and a large receiver antenna due to the weak backscatter expected. The width of the spectrum would correspond to Maxwellian distribution of electron thermal speeds related to sqrt(T_e/m_e) and for a 100 MHz radar this would be 100 kHz (for EISCAT UHF it would be 1 MHz), so very wide.

Thermal fluctuating electrons "Incoherent Scattering"

The calculations by Gordon (1958) indicated that even these faint signals should be observable by a radar with a 1 MW transmitting power, a 300 m antenna and a 100 kHz receiver bandwidth.



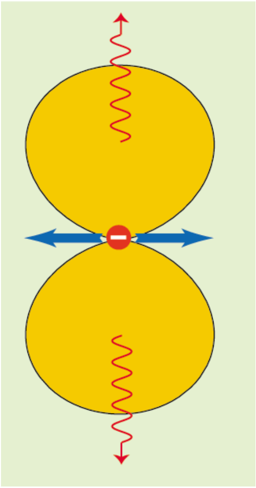
Arecibo radar (Puerto Rico, USA) is built with these specifications

Thermal fluctuating electrons
"Incoherent Scattering"

In 1958, Bowles reported the first actual observations of echoes using a newly constructed high-power transmitter at Long Branch, Illinois. He found that the total scattered power was of the magnitude predicted by Gordon, but the bandwidth was very much smaller and, hence, the scattered power per unit bandwidth very much greater.

- Power was related to electron density Ne, as expected.
- Width of spectrum was much narrower than expected and it corresponded rather the thermal velocities of ions (sqrt(T_i/m_i)) than electrons.

Thermal fluctuating ~~electrons~~ ions "Incoherent Scattering"

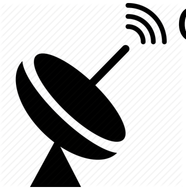


Thomson scatter is the microscopic scattering mechanism, but electrons are not free, since their motions are controlled by the ions via electrostatic forces.

Incoherent scatter is a very weak scattering process, and most of the power that we send, traverses the ionosphere and goes to the space! The peak powers of e.g. the EISCAT radars are typically 1-2 MW, and they transmit pulses (not continuous waves). Only some femtowats (10^{-15} W) are received back.

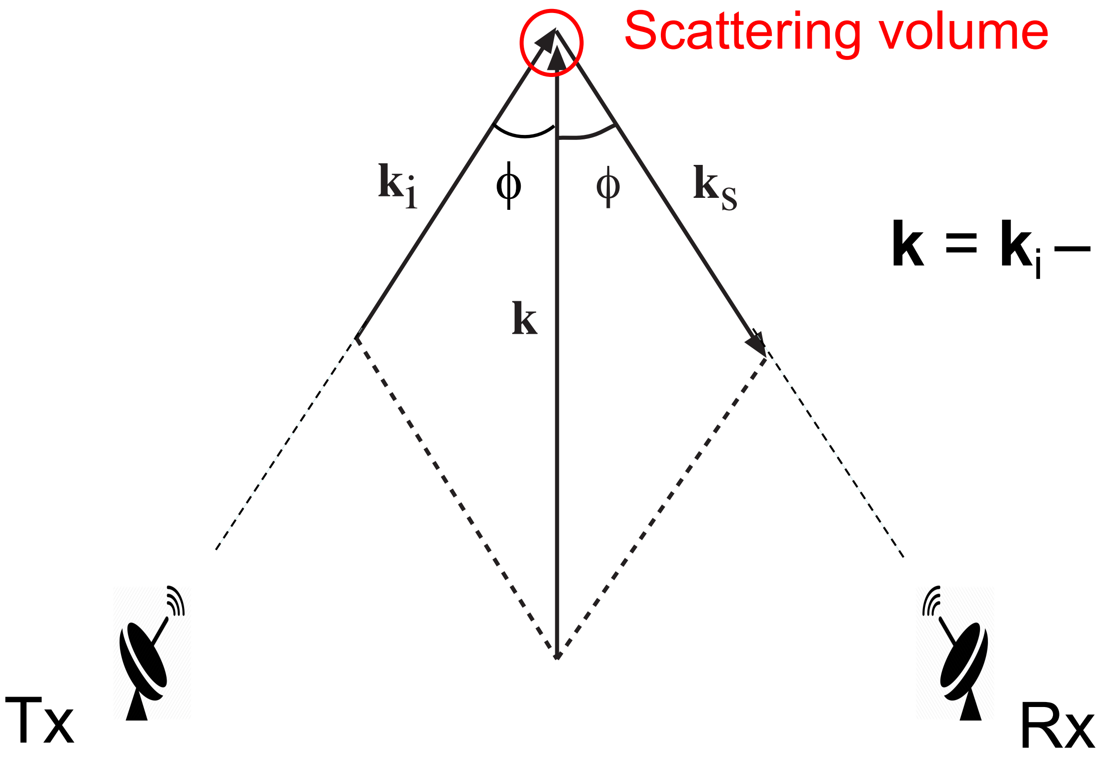
Ion acoustic waves

- The actual scattering takes place from ion-acoustic waves that are all the time generated and attenuated in the plasma. This is a stochastic process and therefore the radar radar pulses are sent and received hundreds (or thousands) of times so that a statistical average is obtained .
- **IS radars observe quasi-coherent scatter from electron density fluctuations that propagate in the plasma as ion acoustic waves.** This gives the IS spectrum that can be analyzed for Ne, Te, Ti, mi and vi. In addition, the spectrum contains contribution from Langmuir waves (so called plasma lines, at different frequencies than ion acoustic waves).



Bragg condition

It can be shown that the quasi-coherent scatter occurs for wave vectors that obey $\mathbf{k}=\mathbf{k}_i-\mathbf{k}_s$. This means that in the case of backscatter (transmitter and receiver are in the same location), the wavelength that gives the quasicoherent backscatter is half of the radar wavelength, $\lambda=\lambda_0/2$. For multistatic case, scattering occurs from wavefronts that propagate along the bisector of the two radar beams.



$$\mathbf{k} = \mathbf{k}_i - \mathbf{k}_s$$

Bragg condition

The wavenumbers for k_i and k_s are same as for the radar with wavenumber k_0 and radar wavelength λ_0, i.e.

$$k_i = k_s = k_o = \frac{2\pi}{\lambda_0} \ .$$

From the figure we can see that

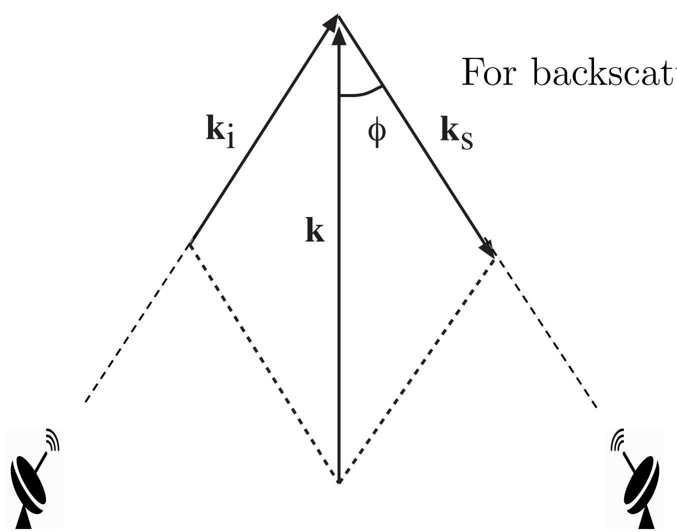
$$k = 2k_0 \cos\phi$$

and then the wavelength of ionospheric fluctuations that produce the scatter is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2k_0\cos\phi} = \frac{\lambda_0}{2\cos\phi}$$

For backscatter, $\phi = 0$ and then

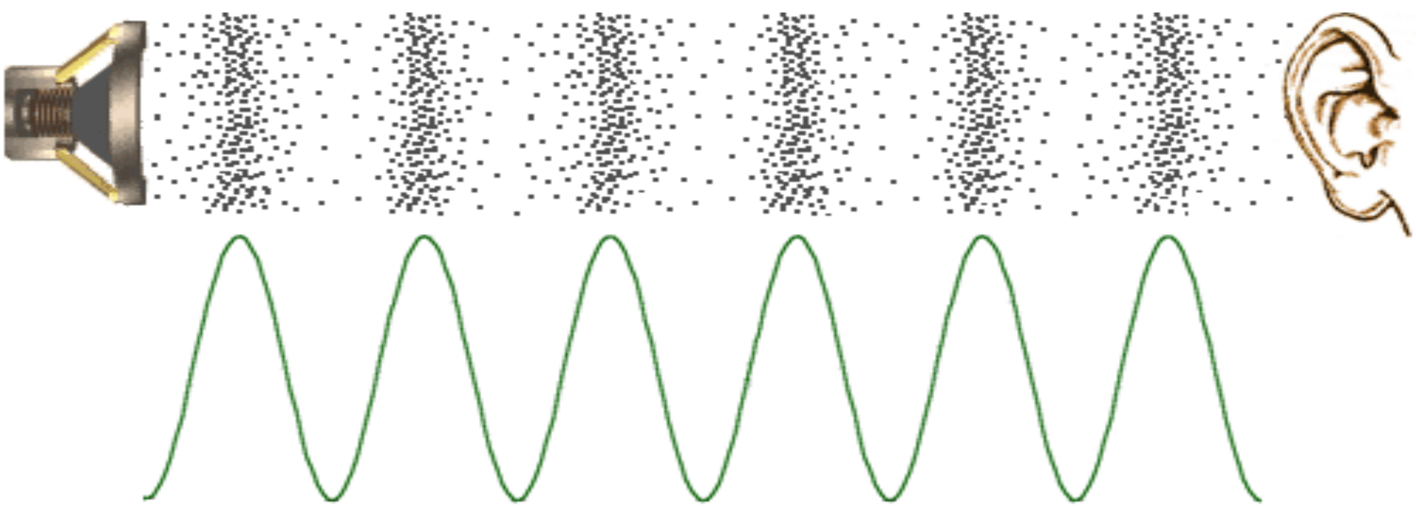
$$\lambda = \frac{\lambda_0}{2}$$



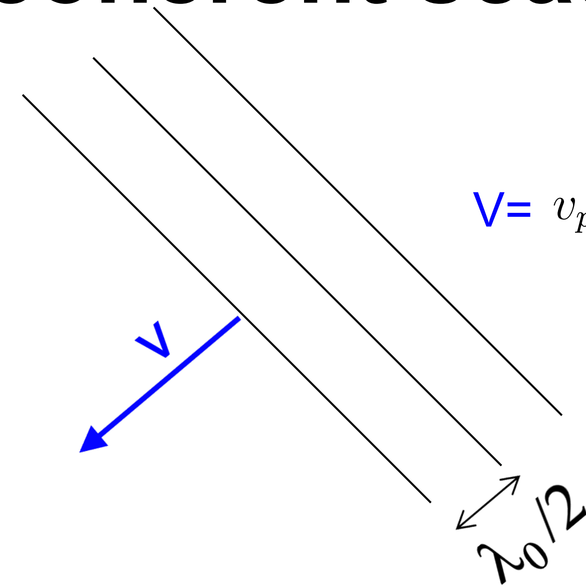
Spectrum of the ion ”lines”

Ion acoustic wave is one type of longitudinal oscillation of the ions and electrons in a plasma, much like acoustic waves traveling in neutral gas. The wave dispersion equation is (adiabatic plasma assumed, in isothermal case factor 3 disappears):

$$v_{ph} = \frac{\omega}{k} = \sqrt{\frac{k_B(3T_i + T_e)}{m_i}}$$

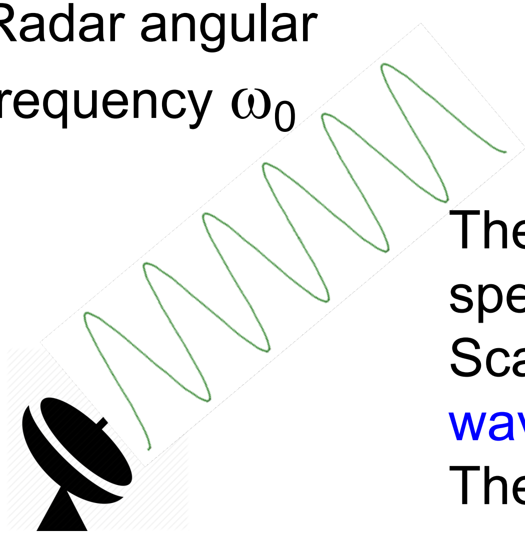


How ion acoustic waves produce incoherent scatter?



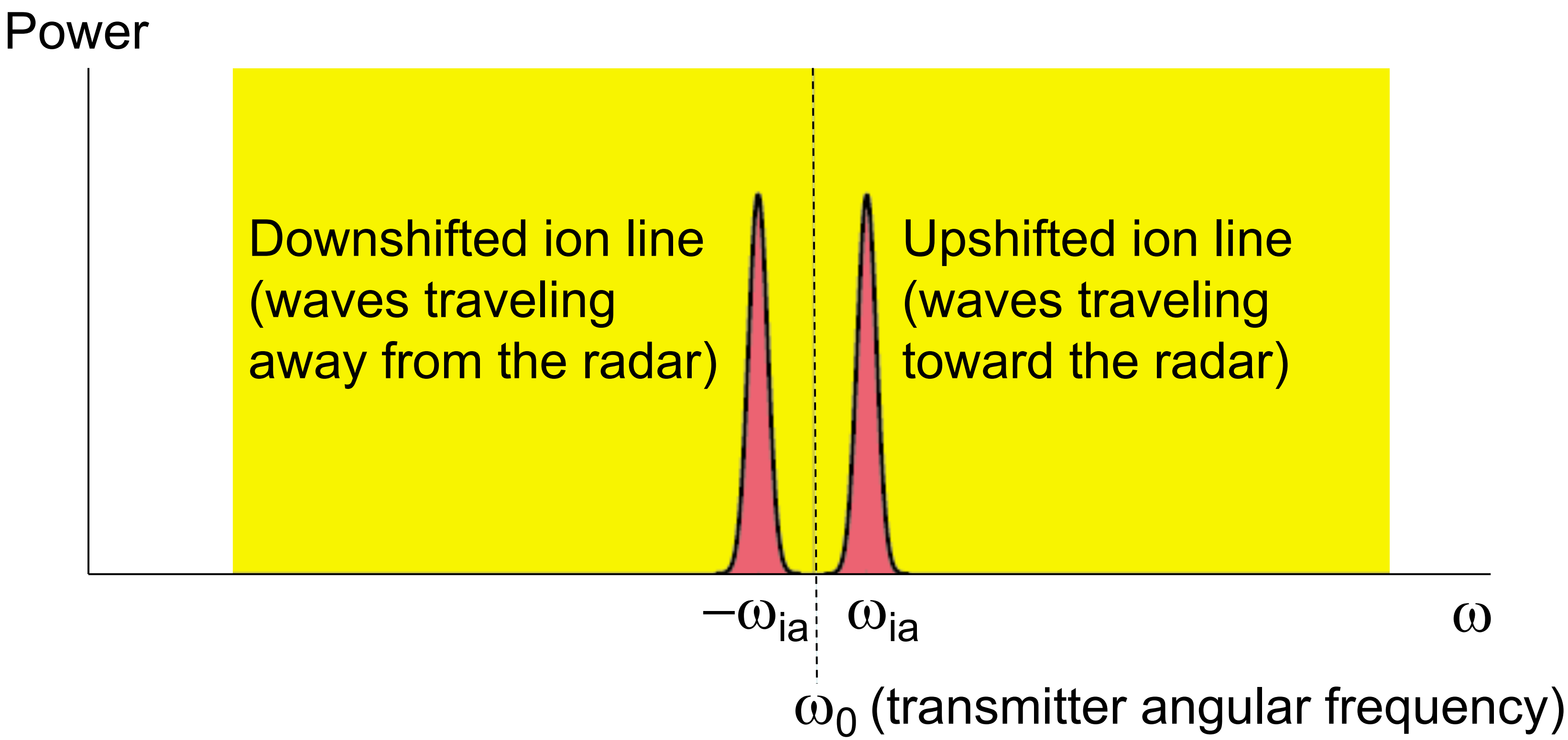
V= $v_{ph} = \frac{\omega}{k} = \sqrt{\frac{k_B(3T_i + T_e)}{m_i}}$

Radar angular frequency ω_0

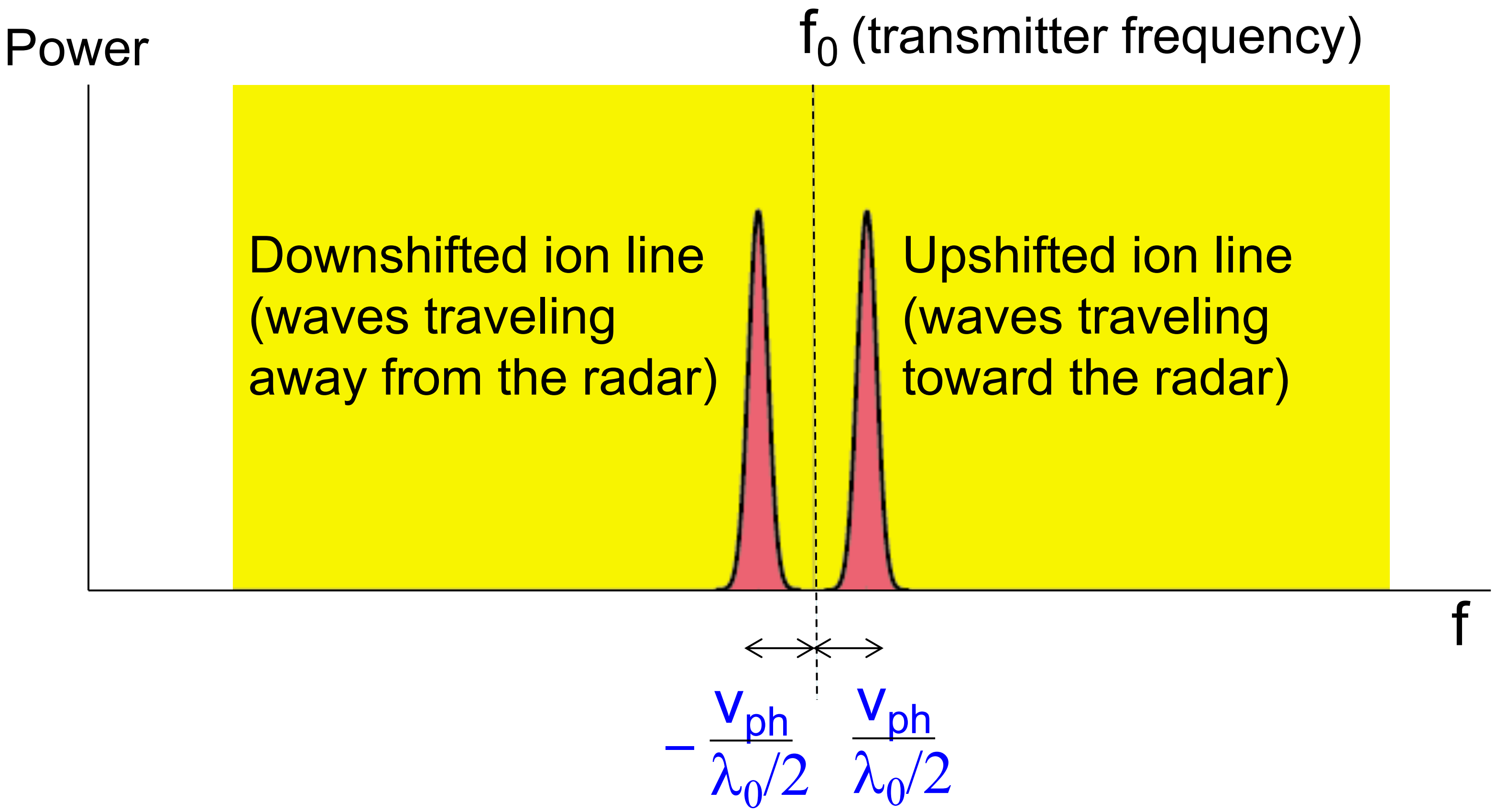


Thermally excited ion-acoustic waves occur over a wide spectrum of wavelengths propagating in all directions. Scattering of radar signal takes place at the moving wave fronts of ion acoustic waves by Bragg scatter. Therefore, the waves experience Doppler shift. Both ion-acoustic waves propagating towards the radar and away from the radar give the quasi-coherent backscatter that is observed by the radar.

Spectrum of the ion "lines"



Spectrum of the ion ”lines”

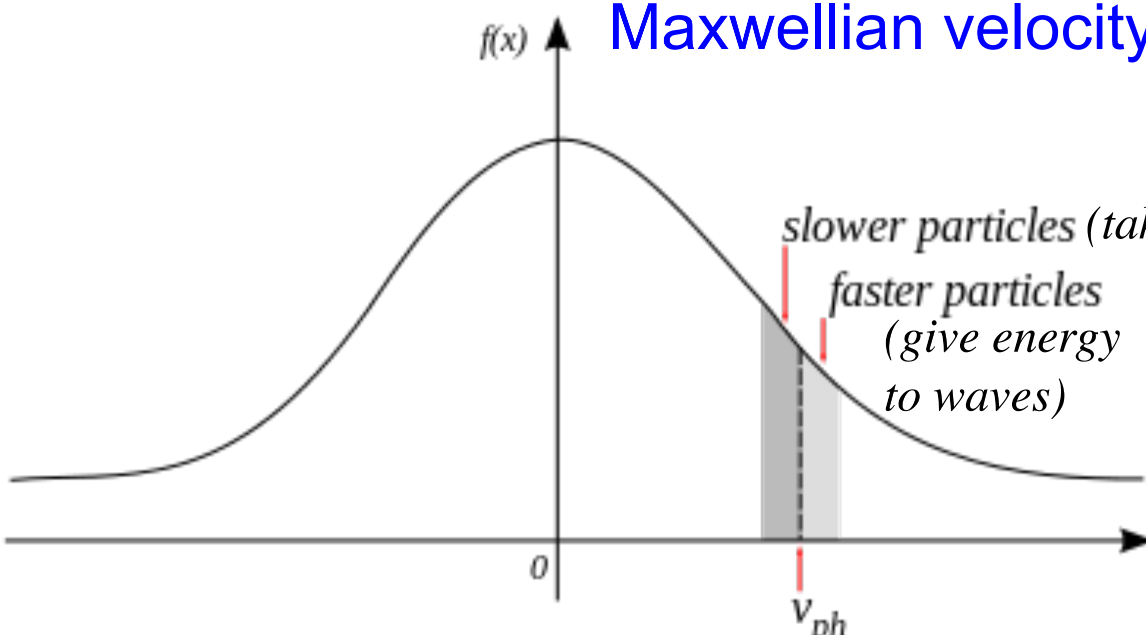


λ_0 is radar wavelength

Landau damping of the ion acoustic waves

When charged particles in a plasma are moving in the same direction as a wave, but at speeds very slightly less than the wave velocity, energy will be transferred from the wave to the particles; the particles will be accelerated and the wave attenuated. If the particles are moving at a speed very slightly greater than the wave, they will feed energy into the wave and the wave will be enhanced.

Maxwellian velocity distribution of ions

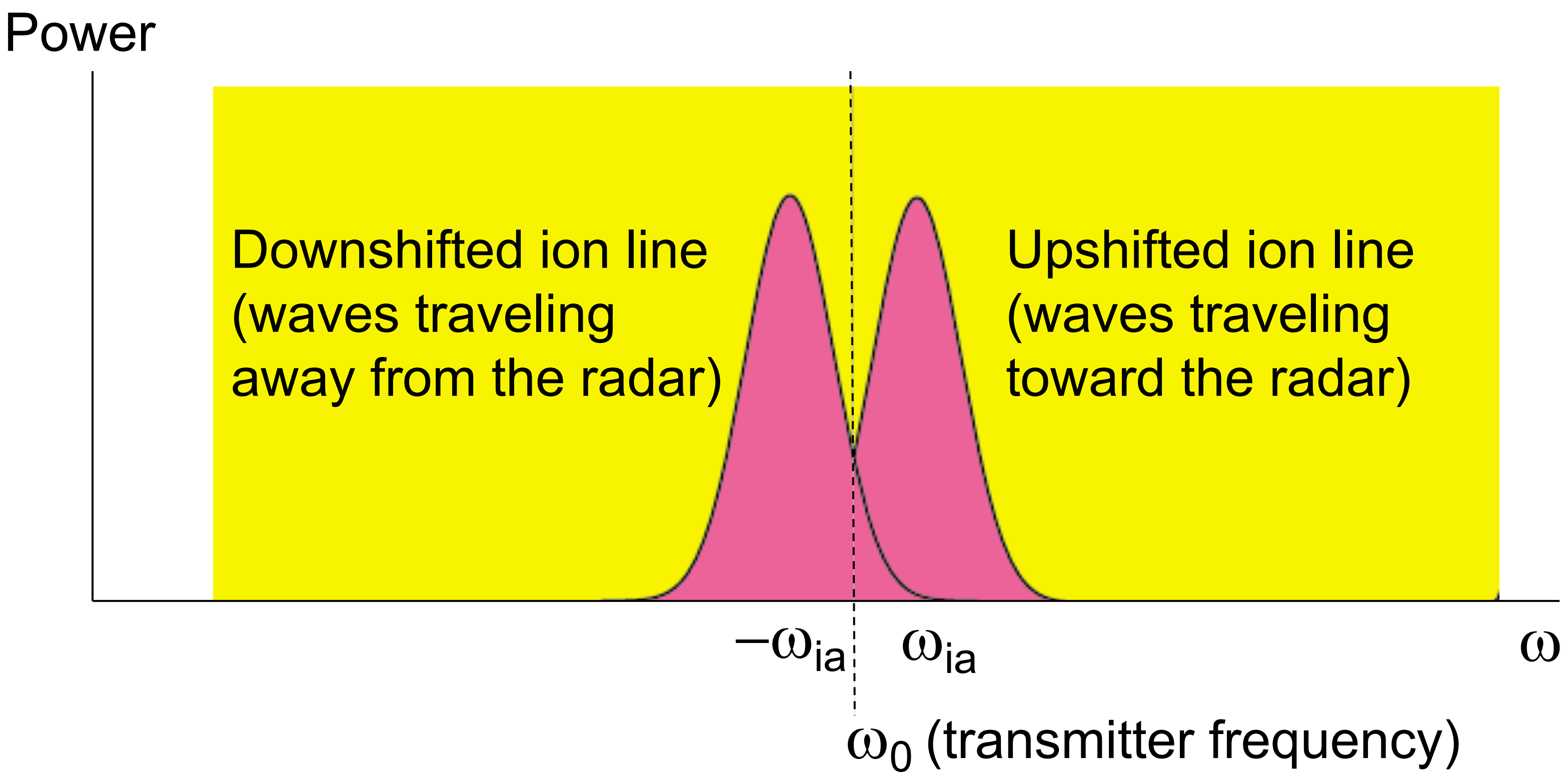


Ion-acoustic wave velocity

More slower than faster particles versus the wave phase velocity

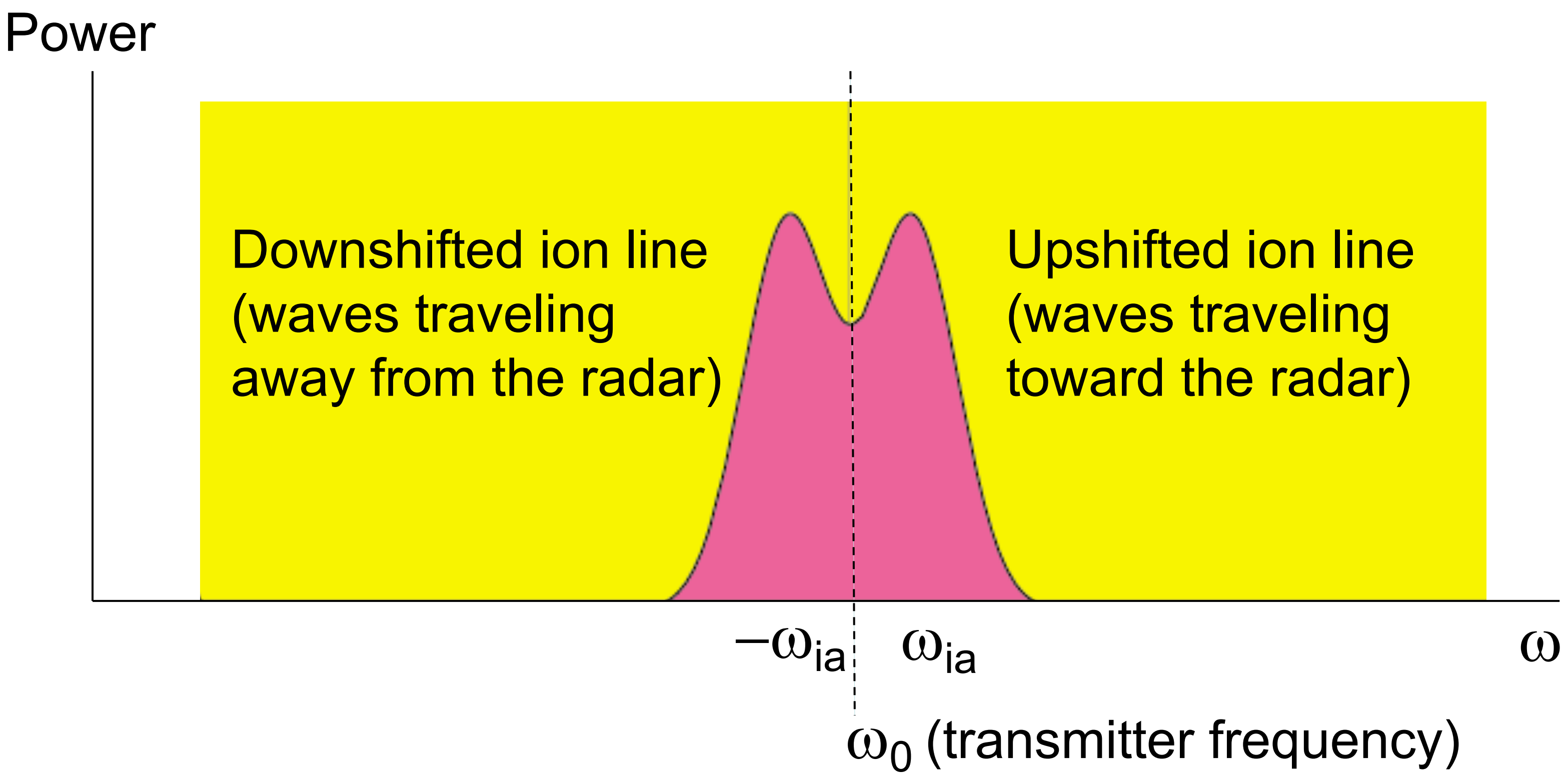
$\Rightarrow$ Wave loses energy and is attenuated

Spectrum of the ion "lines"



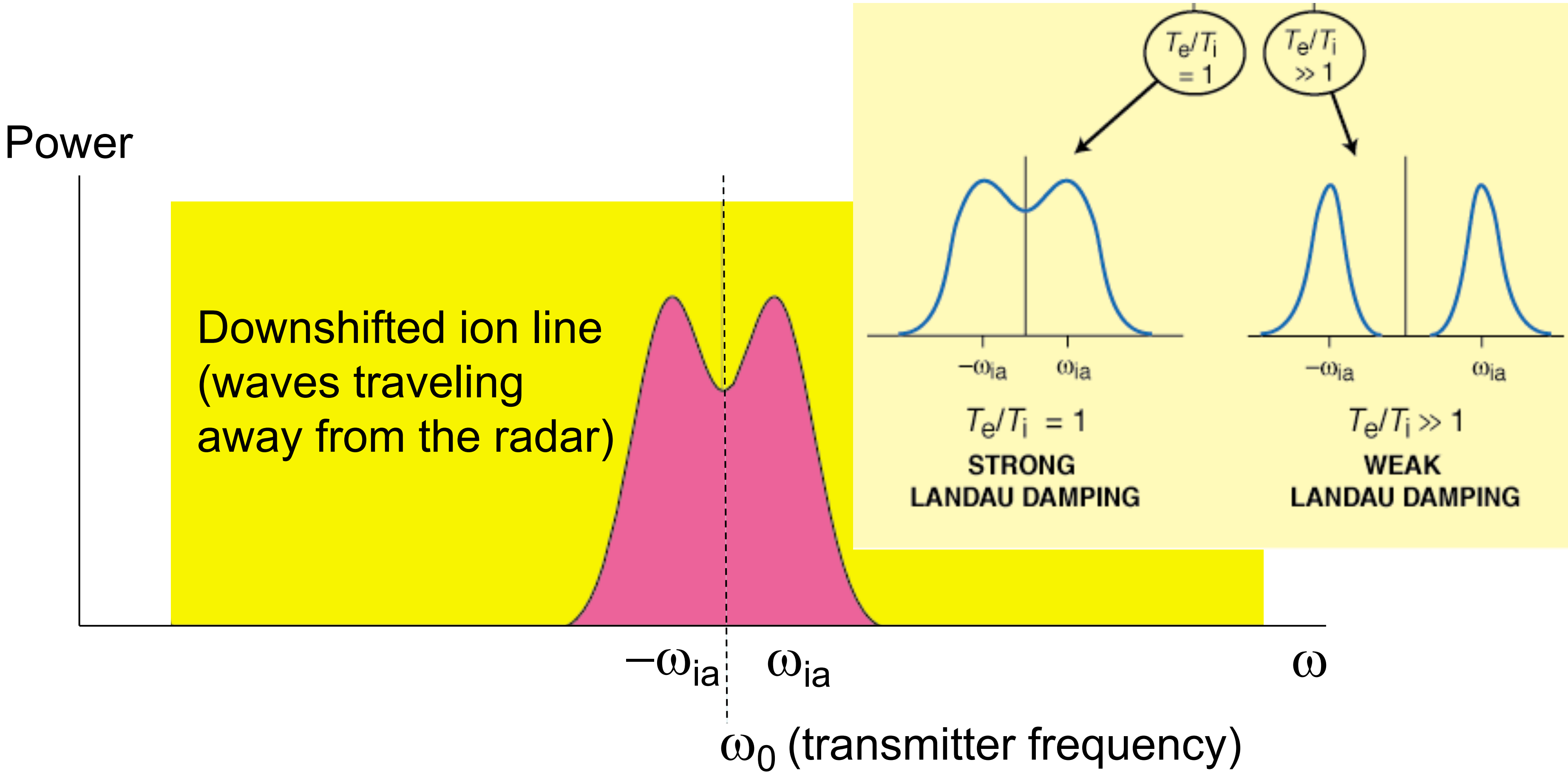
Landau damping broadens the two lines.

Spectrum of the ion "lines"



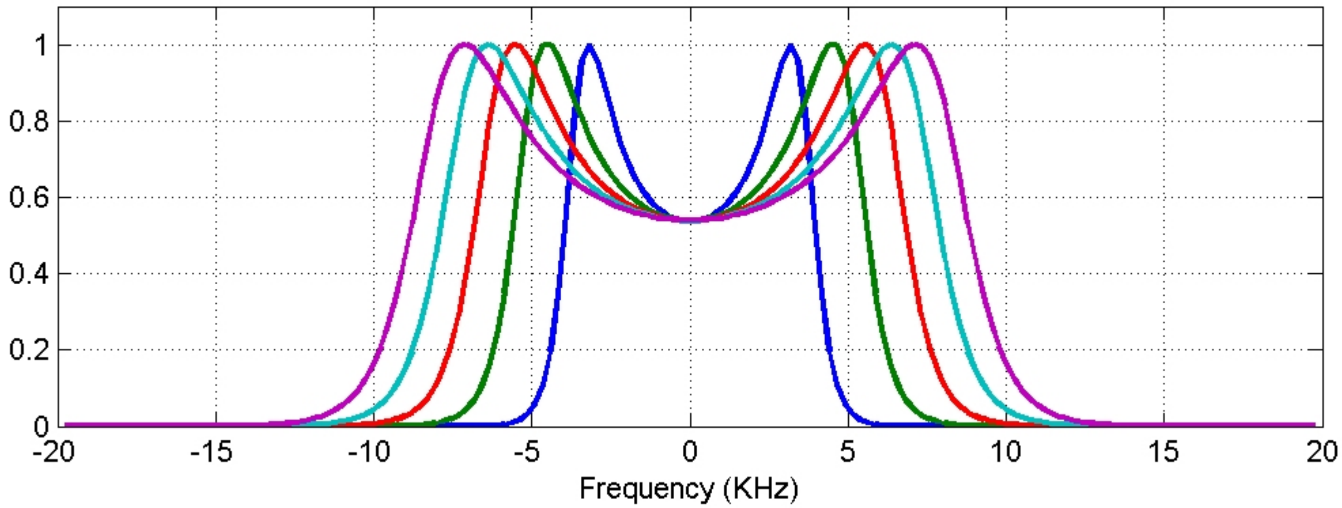
Due to Landau damping, the two ion lines merge into a double-humped spectrum, the ion spectrum or IS spectrum.

Spectrum of the ion ”lines”



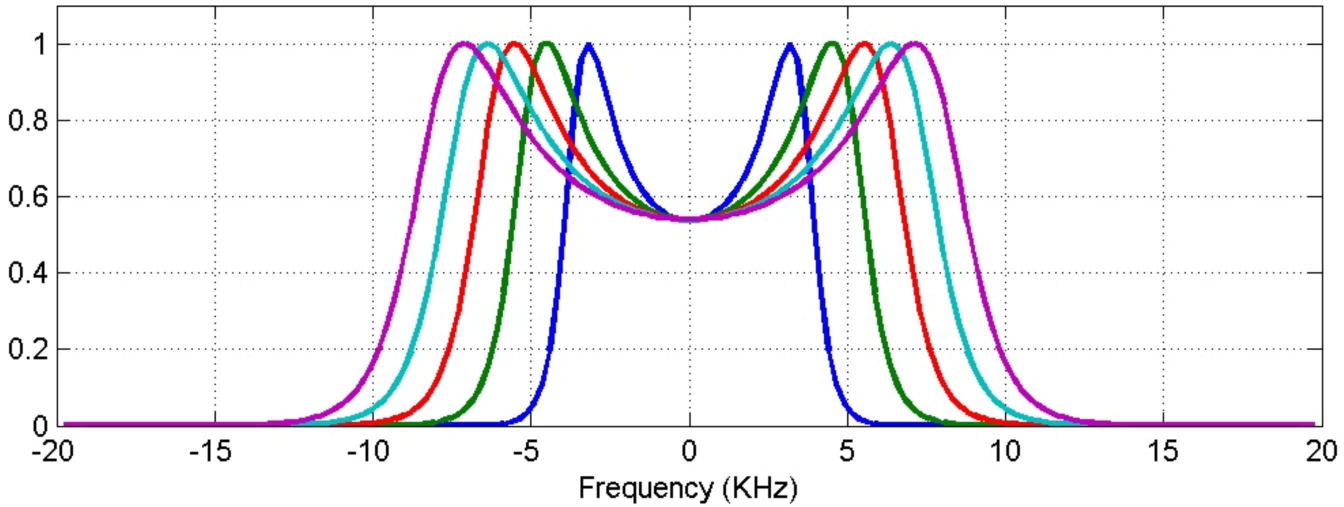
The Landau damping is affected by the electron to ion temperature ratio.

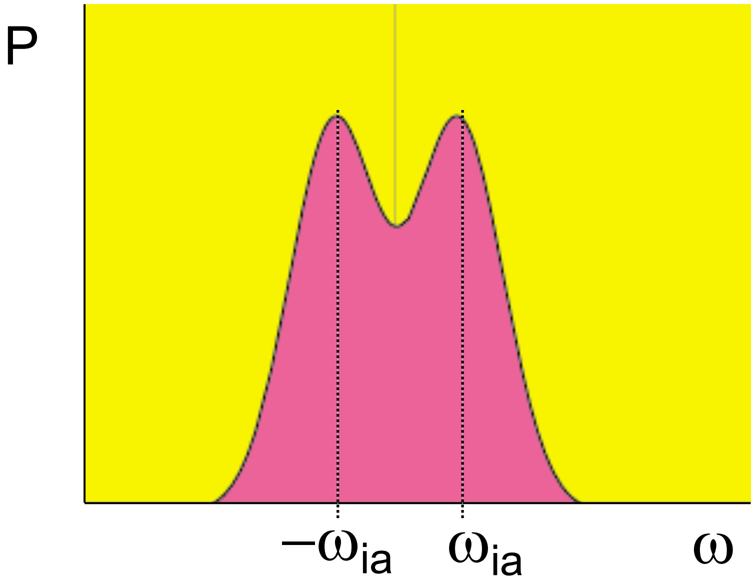
Question (about what you learned so far)



Which plasma parameter(s) are changed to produce these spectra (initial condition is the blue curve)?

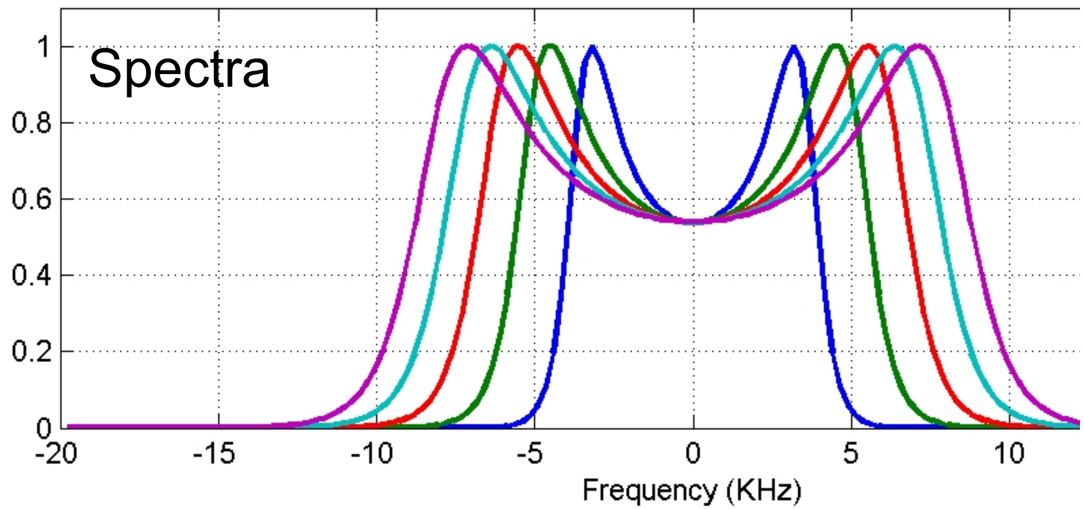
Guideline



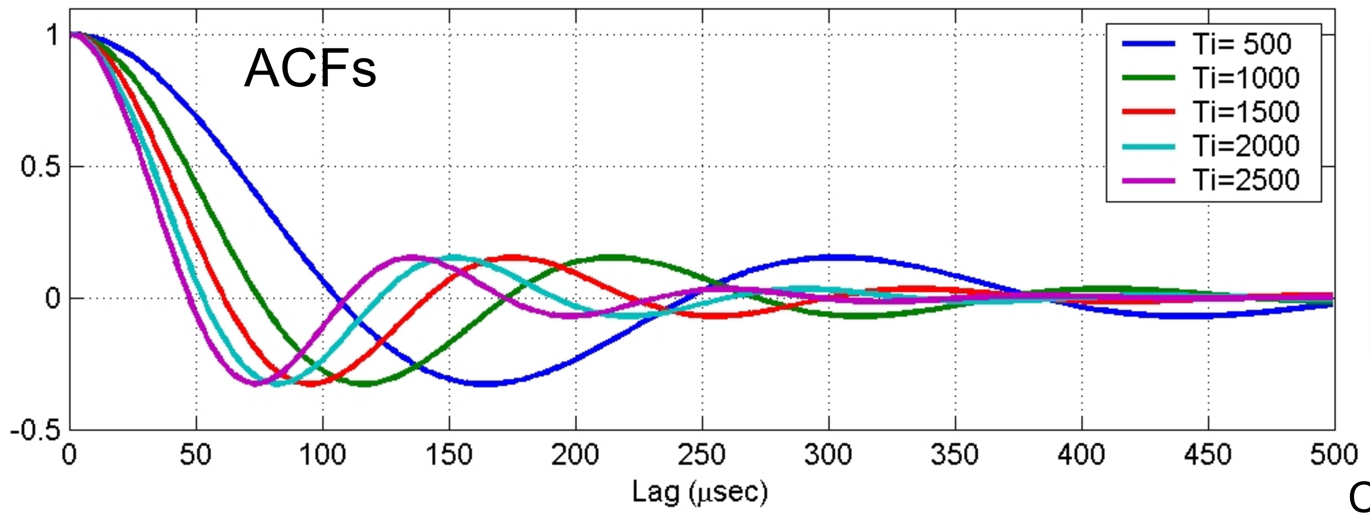


$$\omega_{ia} = v_{ph}k = \sqrt{\frac{k_B(3T_i + T_e)}{m_i}}k$$

Answer



Ti increases: peaks move away from each other to higher frequencies. Since the valley between the peaks is not becoming deeper, Te/Ti must be constant (so Te is increased as well).



Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Te: 2*Ti
Comp: 100% O^{+}
ν_{in}: 10^{-6} KHz

Craig Heinselman

Spectrum of the plasma lines

Another wave mode that can exist in ionospheric plasma are the Langmuir waves. The dispersion equation is:

$$v_{ph} = \frac{\omega}{k} = \omega_p \sqrt{\frac{1}{k^2} + 3\lambda_D^2} \ ,$$

where plasma frequency is

$$\omega_p = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$$

and Debye length

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{n_e e^2}}$$

and k_B is Boltzmann constant. If $\lambda_D << \lambda$, the frequency of Langmuir wave is approximately the plasma frequency.

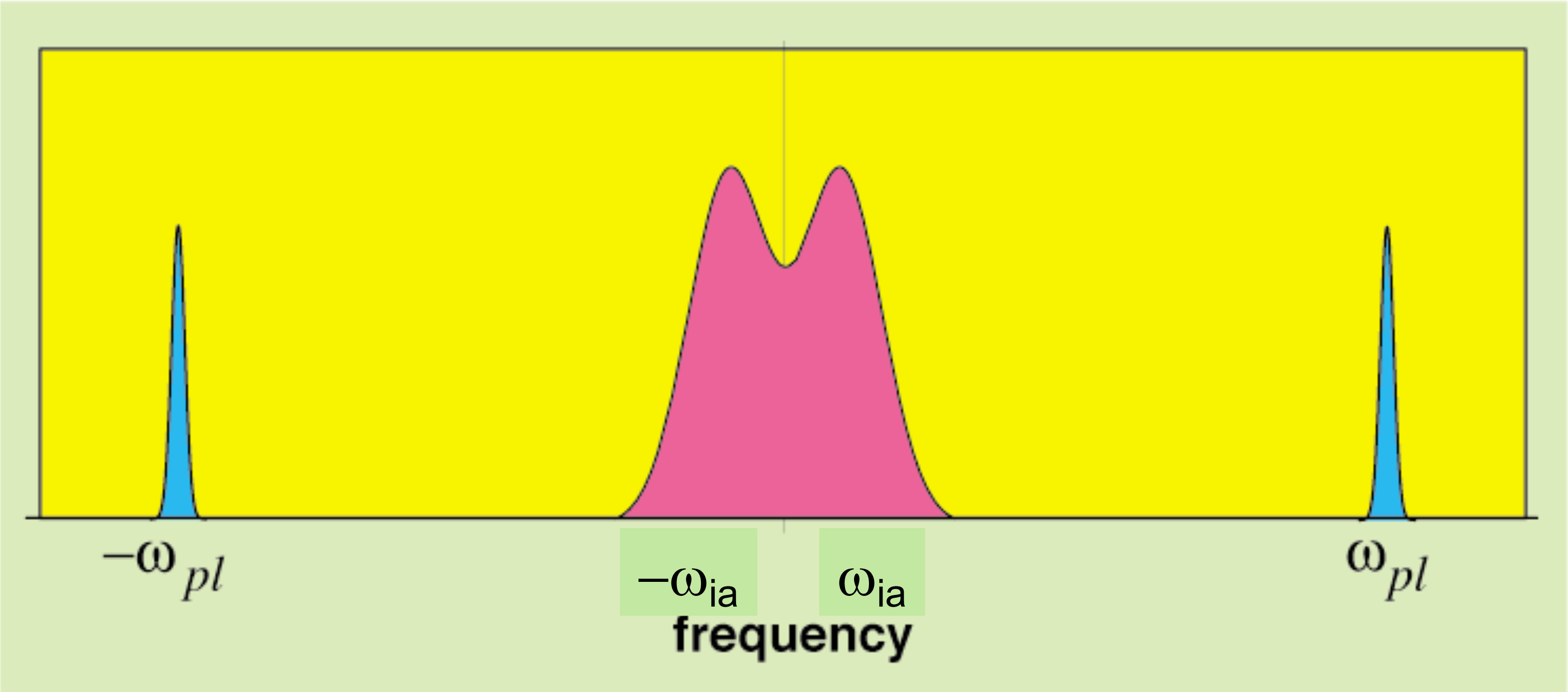
Note that the frequency of plasma waves ("plasma lines") is proprtional to the square root of ambient electron density. Electron density of 10^{12} m^{-3} (very high) corresponds to 9 MHz.

Spectrum of the plasma lines

Plasma waves travel at a far greater velocity than the thermal velocities of the majority of electrons is, so there is very little attenuation (and the plasma lines remain very sharp in frequency).

If there is a influx of suprathermal electrons, such as photoelectrons, and these are travelling at a slightly greater speed than the Langmuir waves, then the plasma waves will be enhanced.

Spectrum of the ion and plasma lines



The ion spectrum is measured to get the plasma parameters: Ne, Te, Ti and vi. The plasma lines can be used e.g. to give additional measurement of Ne.

Spectrum of the ion and plasma lines

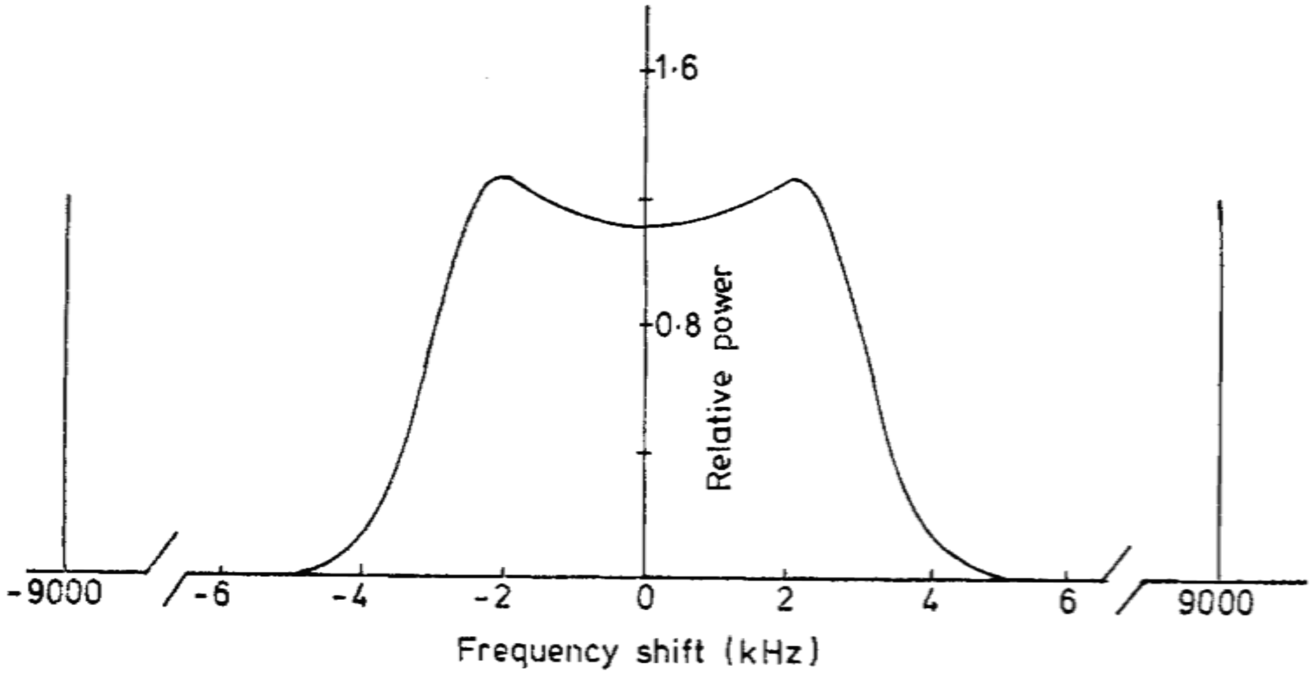


Figure 6. Ion spectrum and plasma lines. $N = 10^{12}\ \mathrm{m}^{-3}$, $T_e = 1000$ K, $T_i = 1000$ K, $\lambda = 1$ m, $M_i = 16$.

Typical IS pectrum width is only a few kHz, and plasma lines occur at frequency of several MHz (exact value depending on electron density).

Power spectrum - ACF

In practice the incoherent scatter spectrum is obtained from the autocorrelation function (ACF) estimate, which is calculated from digital samples of the signal. The autocorrelation function and the power spectral density of a signal make a Fourier transform pair.

$$R_x(t_i, t_j) = \langle x(t_i) x(t_j) \rangle = \langle x_i x_j \rangle$$

Autocorrelation function R_x, t_i and t_j are two instants of time, where x is the random variable and brackets indicate expectation value.

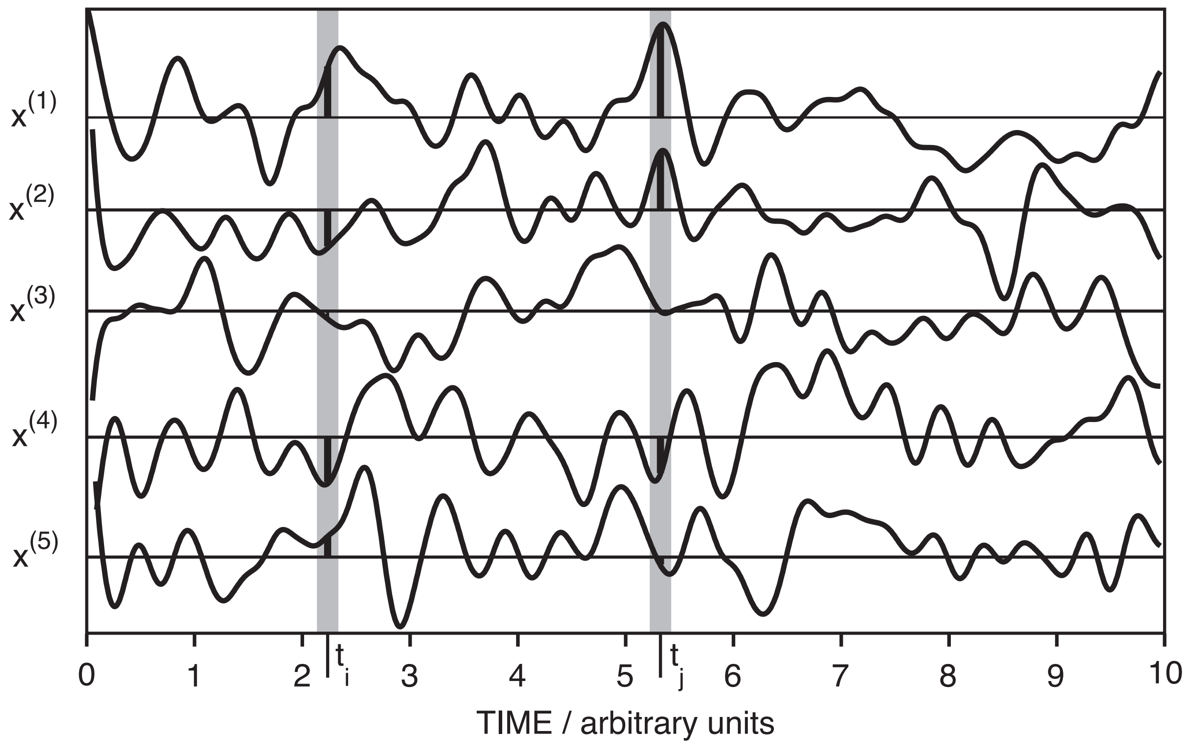
$$R_x(t_i, t_j) = \langle x_i x_j \rangle \approx \frac{1}{N} \sum_{k=1}^{N} x^{(k)}(t_i) x^{(k)}(t_j).$$

If the autocorrelation function R_x depends only on the time difference between t_i and t_j, which is called a lag or delay $\tau = t_i – t_i$, then

$$R_x(\tau) = \langle x(t) x(t - \tau) \rangle$$

ACF for a stochastic process

$$R_x(t_i, t_j) = \langle x_i x_j \rangle \approx \frac{1}{N} \sum_{k=1}^{N} x^{(k)}(t_i) x^{(k)}(t_j).$$

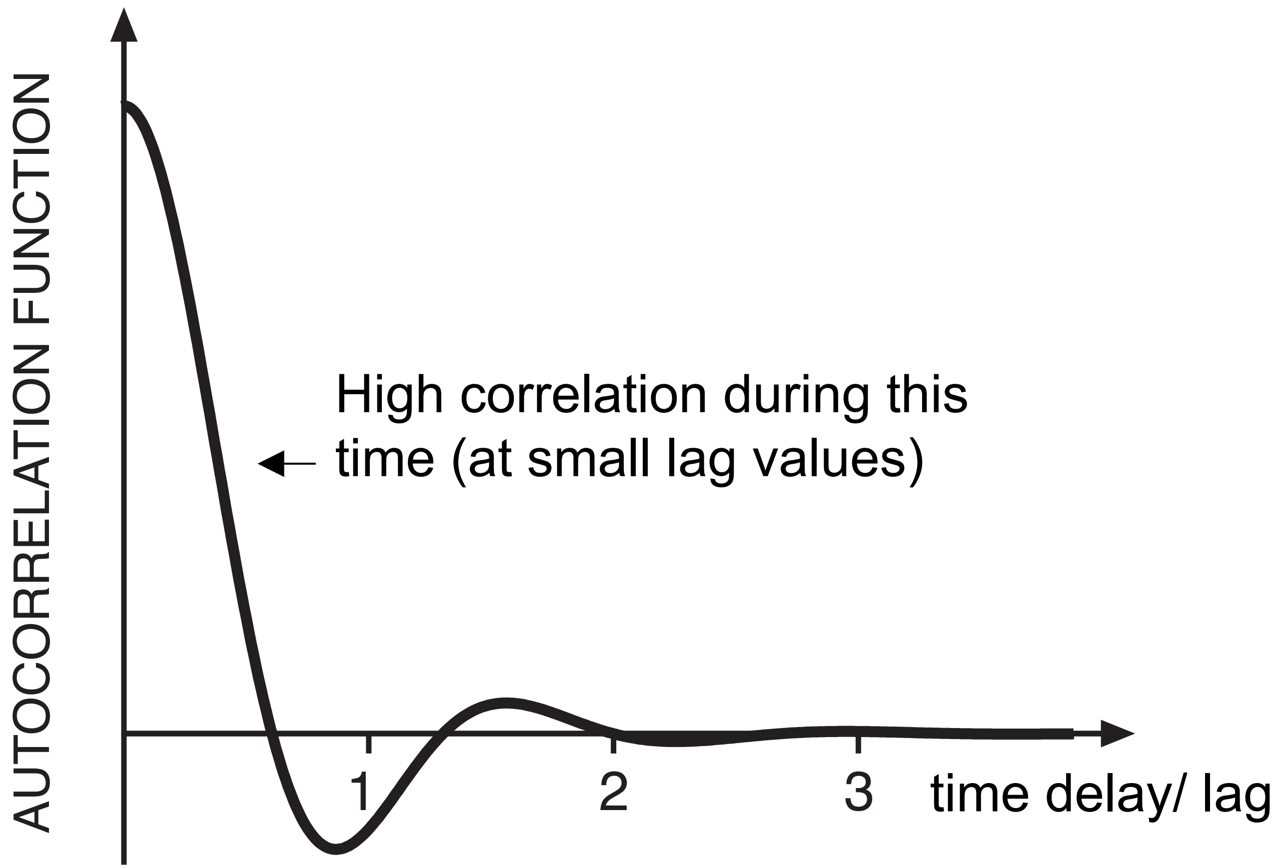


Five realizations of a stochastic process,
which yield statistical properties of the variable.

Tuomo Nygrén

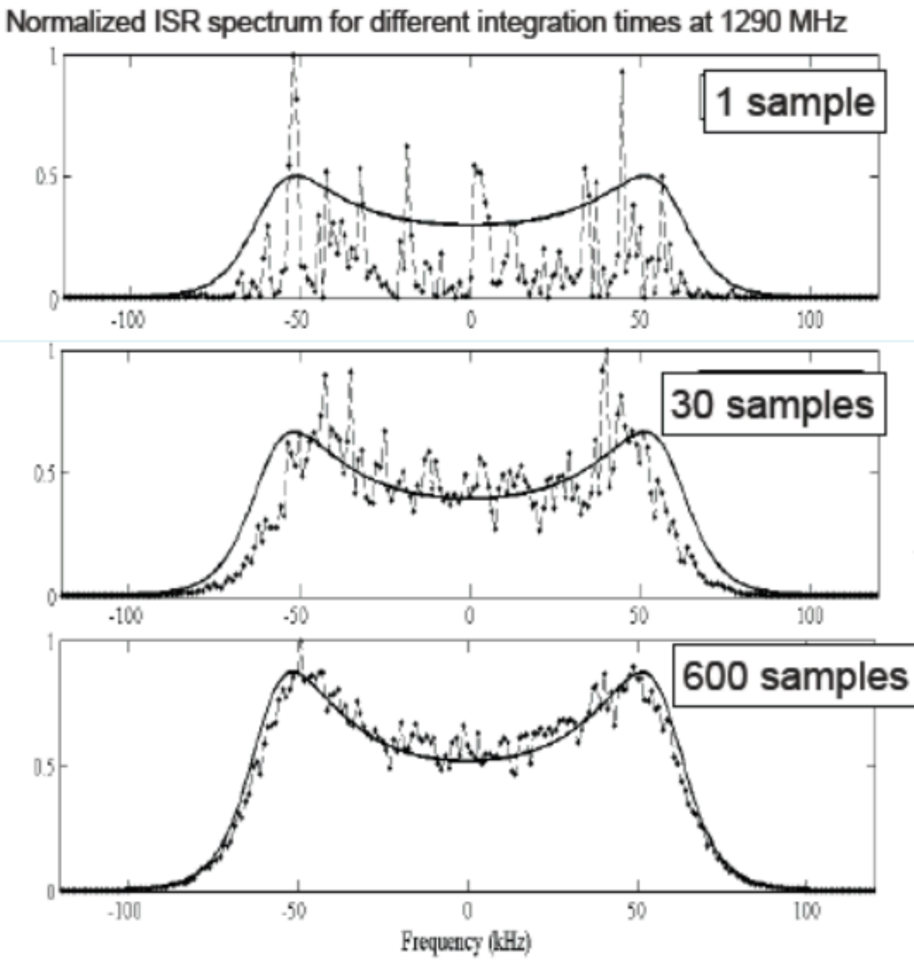
ACF for a stochastic process

ACF calculated from the samples on the previous page. Note tha IS is due to random thermal fluctuations of plasma, and therefore it is a stochastic process.



Tuomo Nygrén

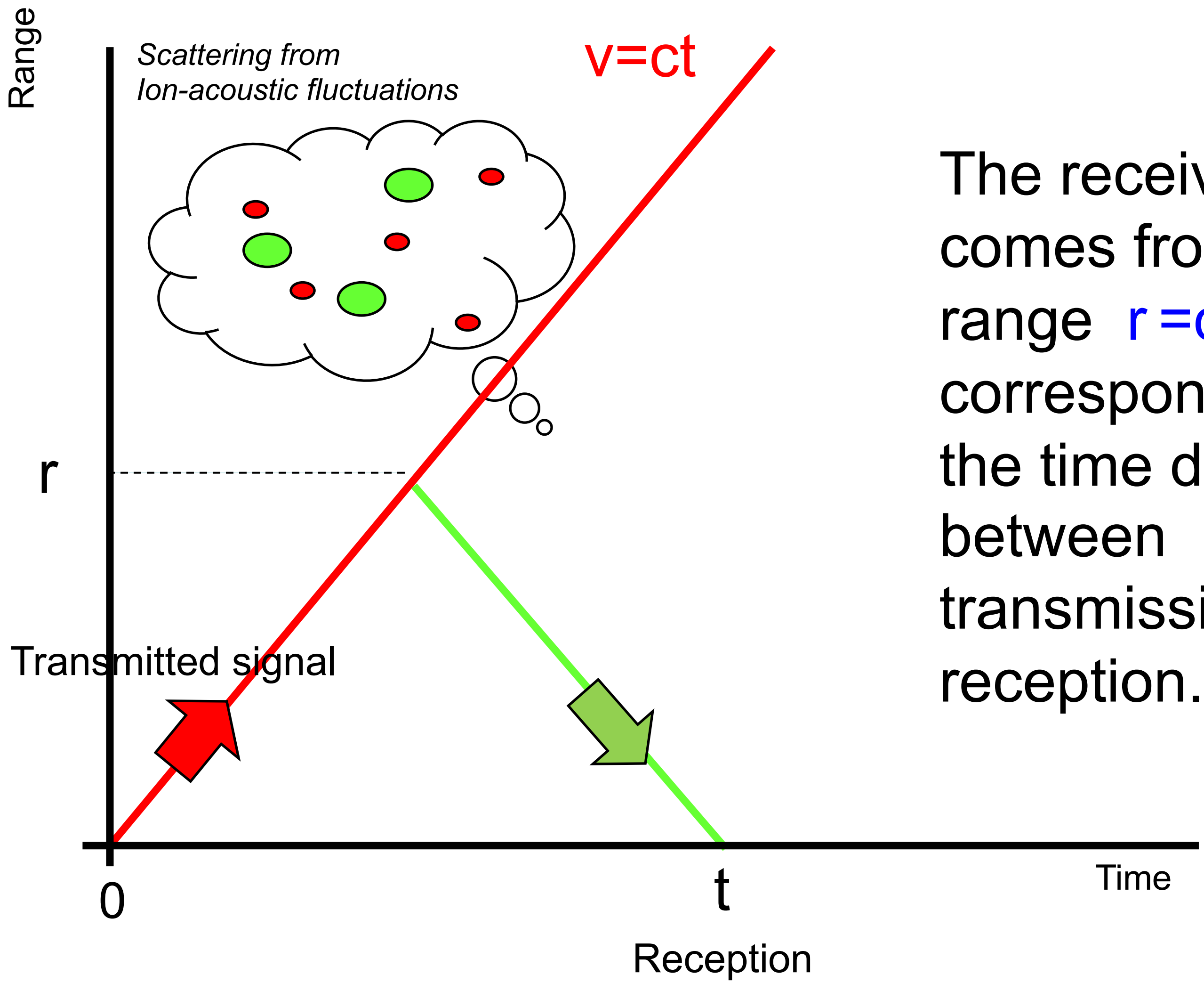
Spectrum in practice



We are seeking to estimate the power spectrum of a Gaussian random process. This requires that we sample and average many independent "realizations" of the process.

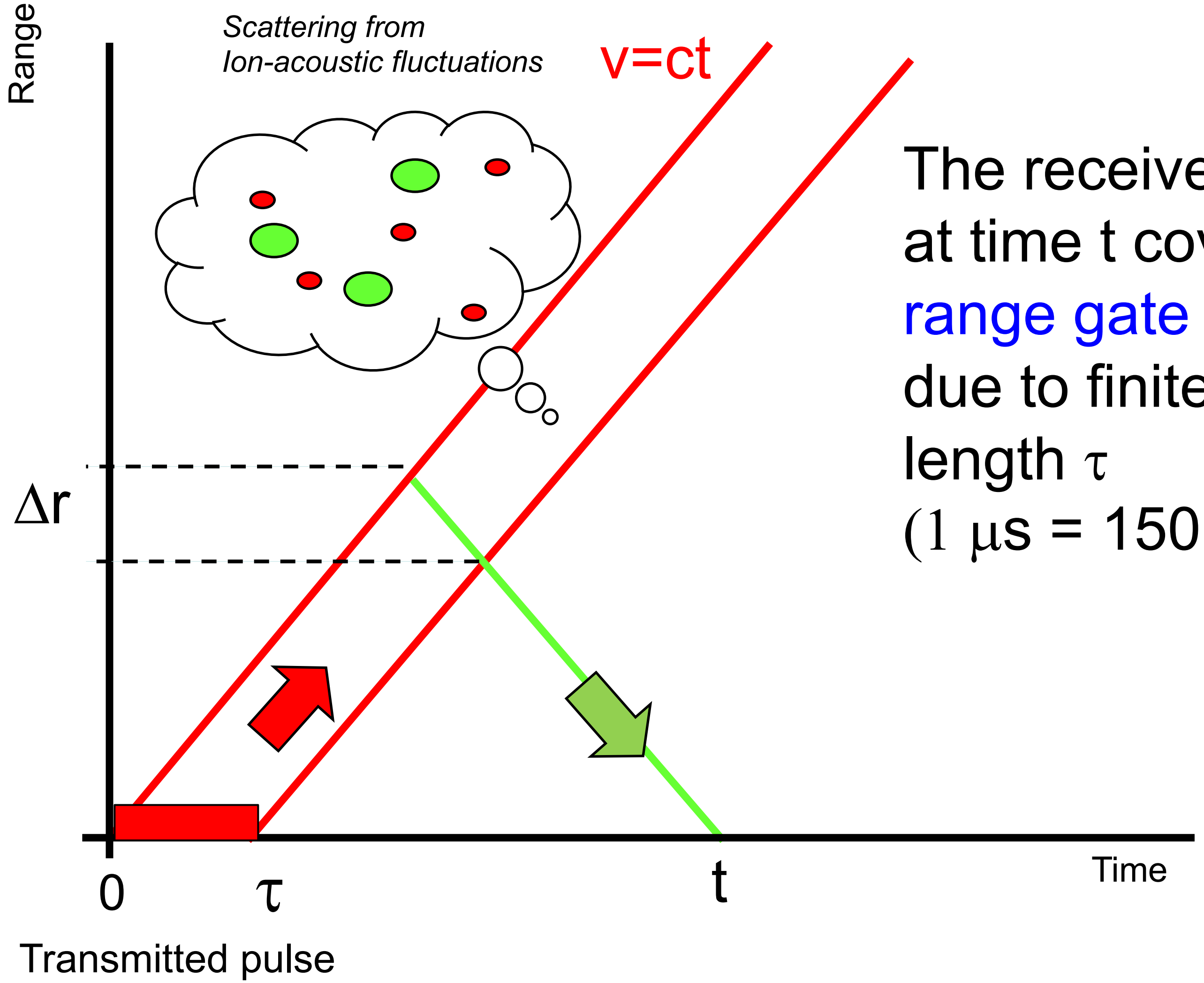
$$\text{Uncertainties} \propto \frac{1}{\sqrt{\text{Number of Samples}}}$$

Range-time diagram for a radar



The received signal comes from a range $r = ct/2$ corresponding to the time delay between transmission and reception.

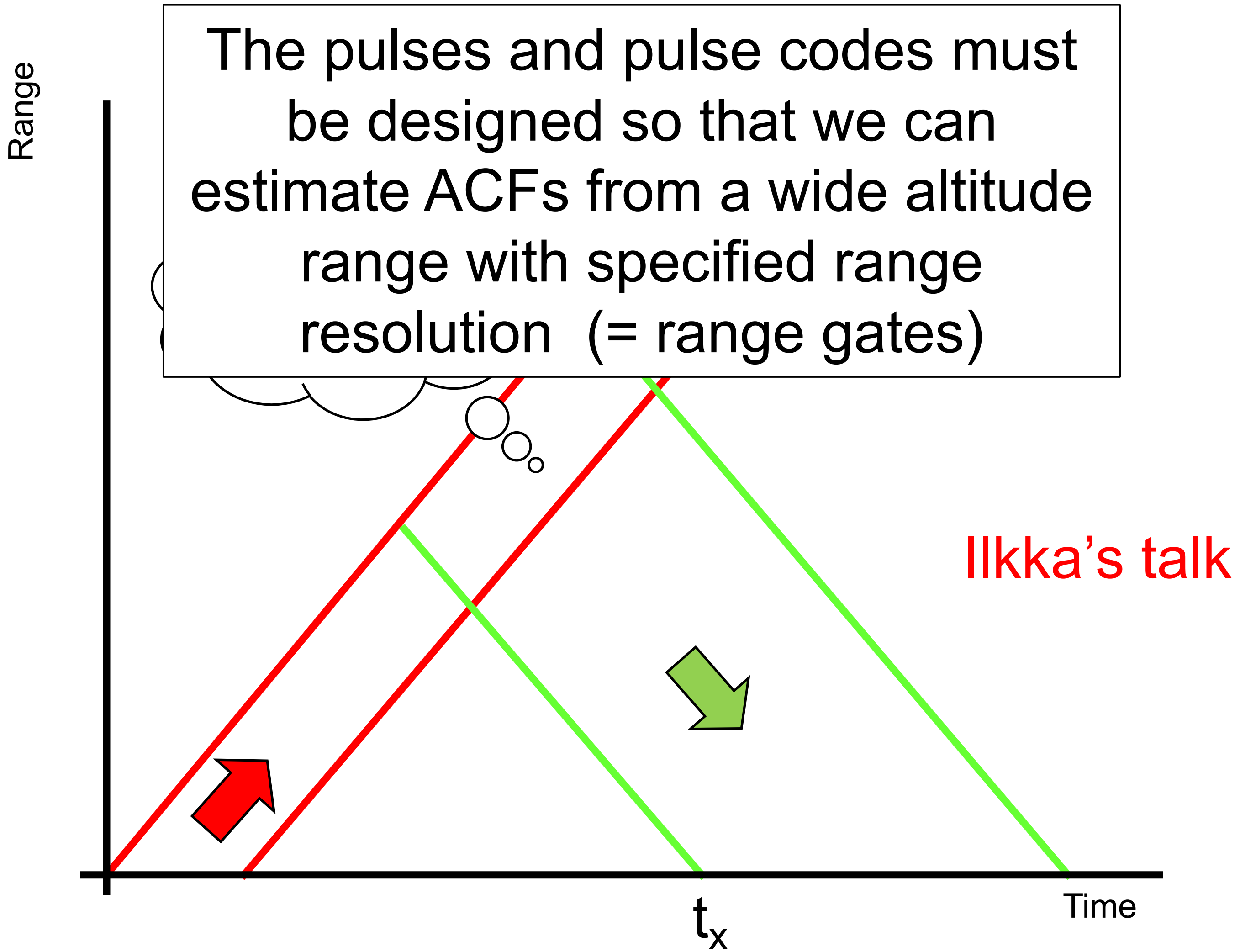
Range-time diagram for a radar



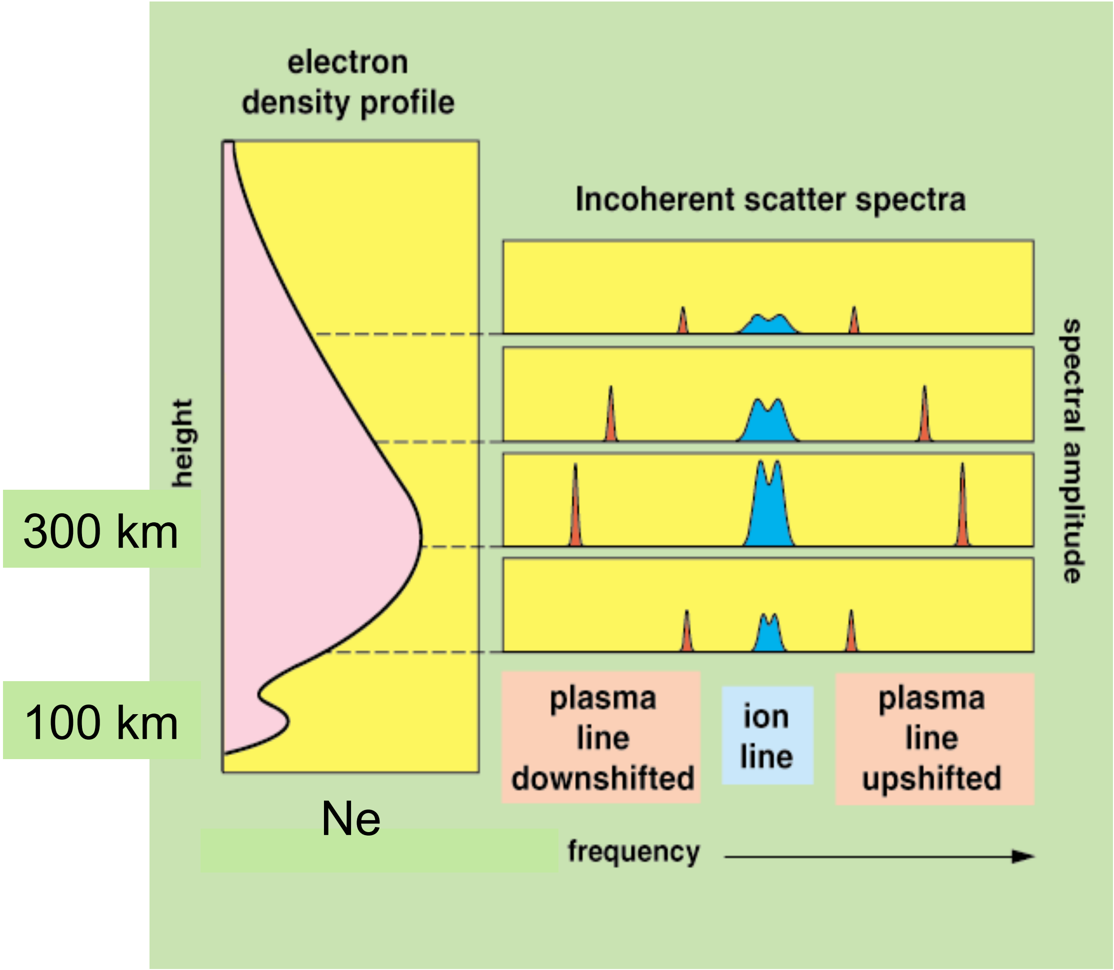
The received signal at time t covers range gate $\Delta r = c\tau/2$ due to finite pulse length τ (1 μs = 150 m)

In addition, reception is not instantenous…

Range-time diagram for a radar



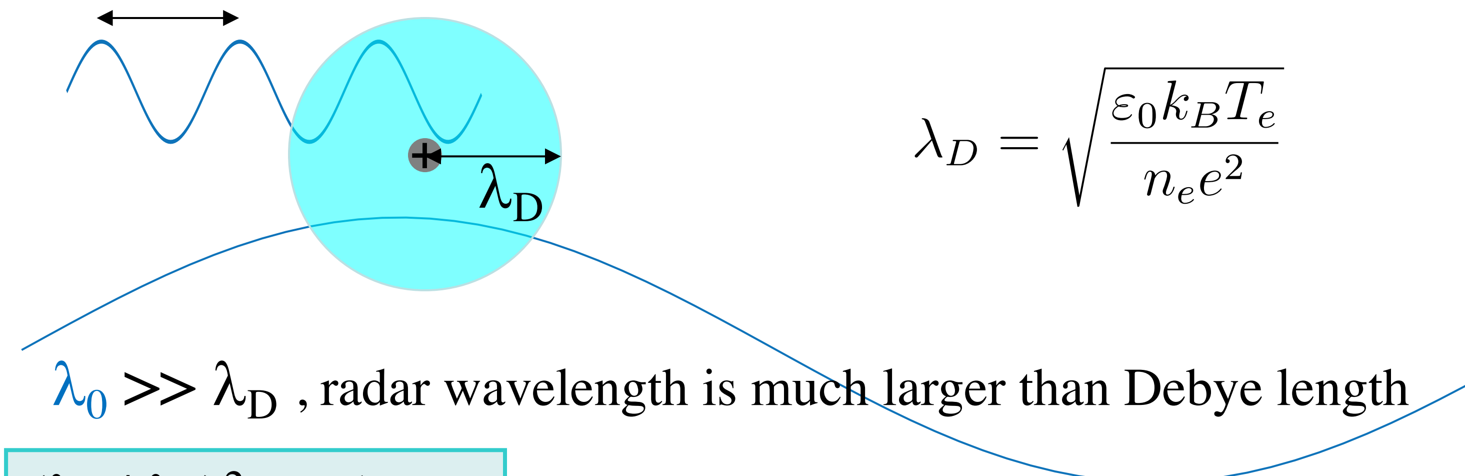
Typical spectra as a function of altitude



Debye length

Debye length determines the distance, outside of which the plasma behaviour is collective (i.e. we don't need to look at the beahaviour of individual ions and electrons). The radar wavelength should larger than the Debye length so that IS from ion-acoustic waves could take place.

$\lambda_0 < \lambda_D$, small radar wavelength



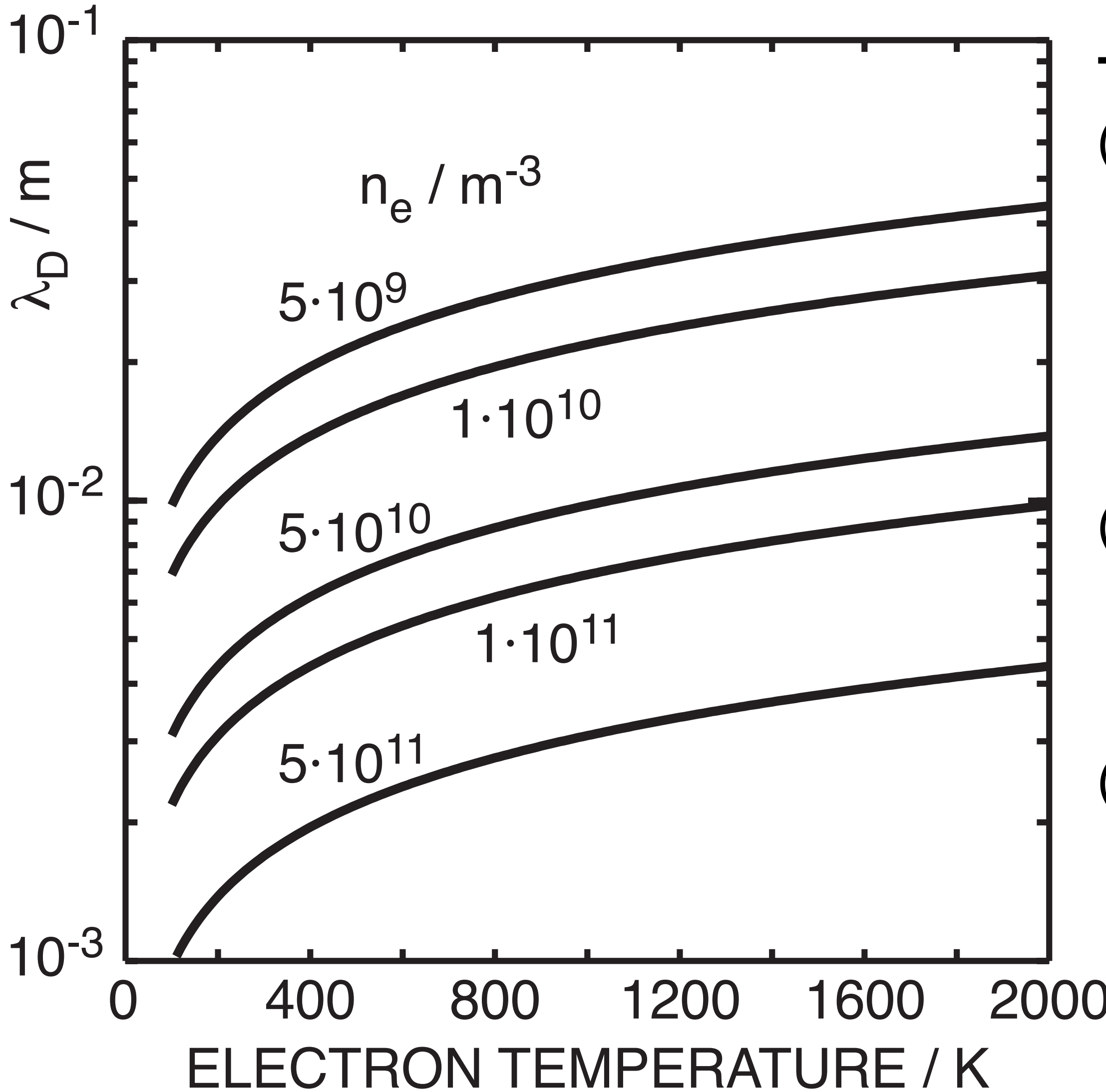
$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{n_e e^2}}$$

$\lambda_0 >> \lambda_D$, radar wavelength is much larger than Debye length

$(\lambda_D / \lambda_0)^2 << 1$

$\Rightarrow (k_0 \lambda_D)^2 << 1$

Debye length task (for tomorrow)



Task: Calculate

(i) Wavelengths that produce scatter for the two EISCAT radars, which have frequencies: UHF: 930 MHz, VHF: 224 MHz

(ii) Debye length that corresponds to the condition $k\lambda_D<1$ for the two radars and

(iii) Estimate from the figure, for which n_e and T_e values the Debye limitation will be violated for the two radars

Radar equation for IS

Electron cross section from plasma theory (not a simple Thomson cross section):

$$\sigma = \frac{4\pi r_0^2}{(1+k^2\lambda_D^2)(1+T_e/T_i+k^2\lambda_D^2)},$$

Where k is the wave number of the ion acoustic wave and r_0 is the classical electron radius

$$r_0 = \frac{e^2}{4\pi\varepsilon_0 m_e c^2} = 2.82\cdot 10^{-15}\,\mathrm{m}$$

Power P_R received from range interval (r, r+Δr):

$$\Delta P_R(r) = \frac{n_e(r)\sigma P_T\lambda^2}{(4\pi)^3 r^2}\left[\int_\Omega G^2(\Omega)d\Omega\right]\Delta r.$$

where P_T is transmitted power, λ wavelength, G antenna gain.

Electron density n_e solved from the equation above:

$$n_e(r) = C\cdot\frac{\Delta P_R(r)}{P_T}\cdot\frac{r^2}{\Delta r}\cdot\frac{(1+k^2\lambda_D^2)(1+T_e/T_i+k^2\lambda_D^2)}{4\pi r_0^2}.$$

Radar equation for IS cont'd

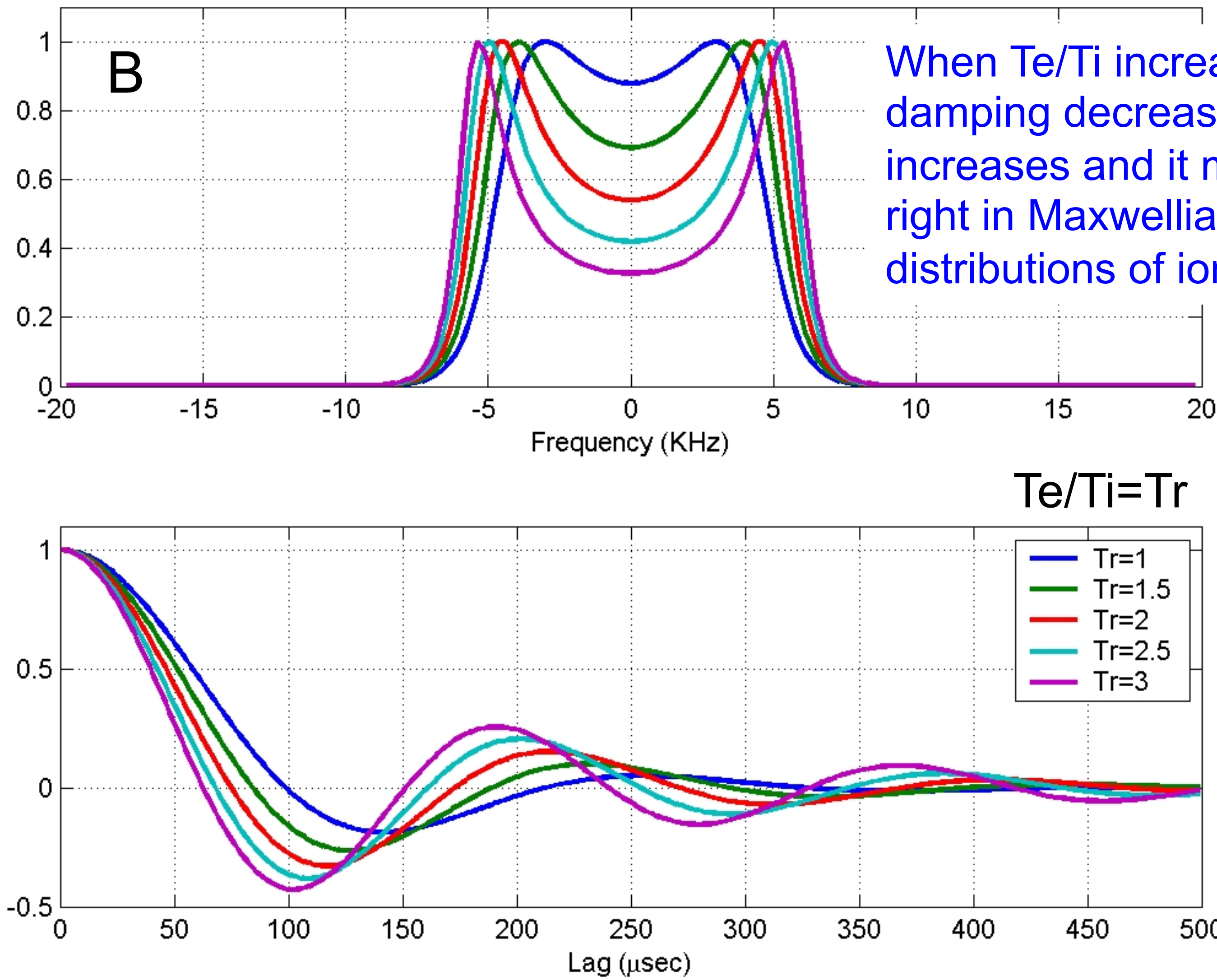
$$n_e(r) = C \cdot \frac{\Delta P_R(r)}{P_T} \cdot \frac{r^2}{\Delta r} \cdot \frac{(1+k^2\lambda_D^2)(1+T_e/T_i+k^2\lambda_D^2)}{4\pi r_0^2}.$$

where C is a constant determined by the antenna gain pattern and the radar wave length. If the Debye correction term $k^2\lambda_D^2 << 1$, then the received power is directly proportional to electron density. If in addition we assume that $T_e/T_i=1$, we get an equation for the so-called raw electron density

$$n_e(r) = C\frac{\Delta P_R(r^2)}{P_t}\frac{r^2}{\Delta r}\frac{1}{2\pi r_0^2}$$

Since this estimate of n_e does not depend on other plasma parameters, it can be estimated from the received power P_R from a specific range gate, i.e. the zero lag of the corresponding ACF.

Dependence of spectra on ionospheric parameters



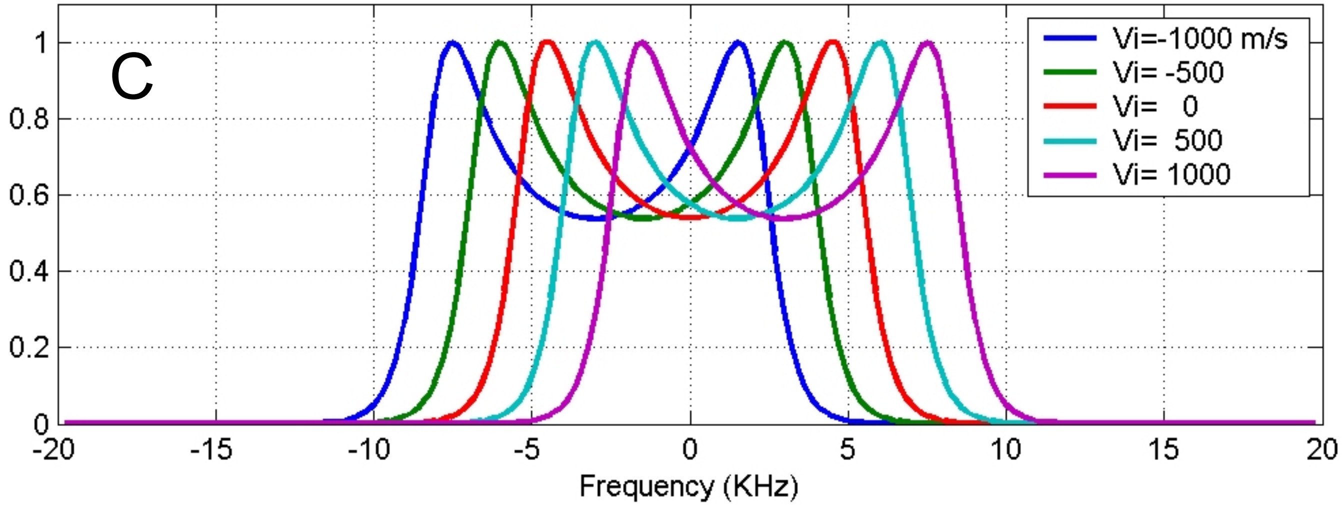
When Te/Ti increases, Landau damping decreases, because v_{ph} increases and it moves to the right in Maxwellian velocity distributions of ions

Te/Ti=Tr

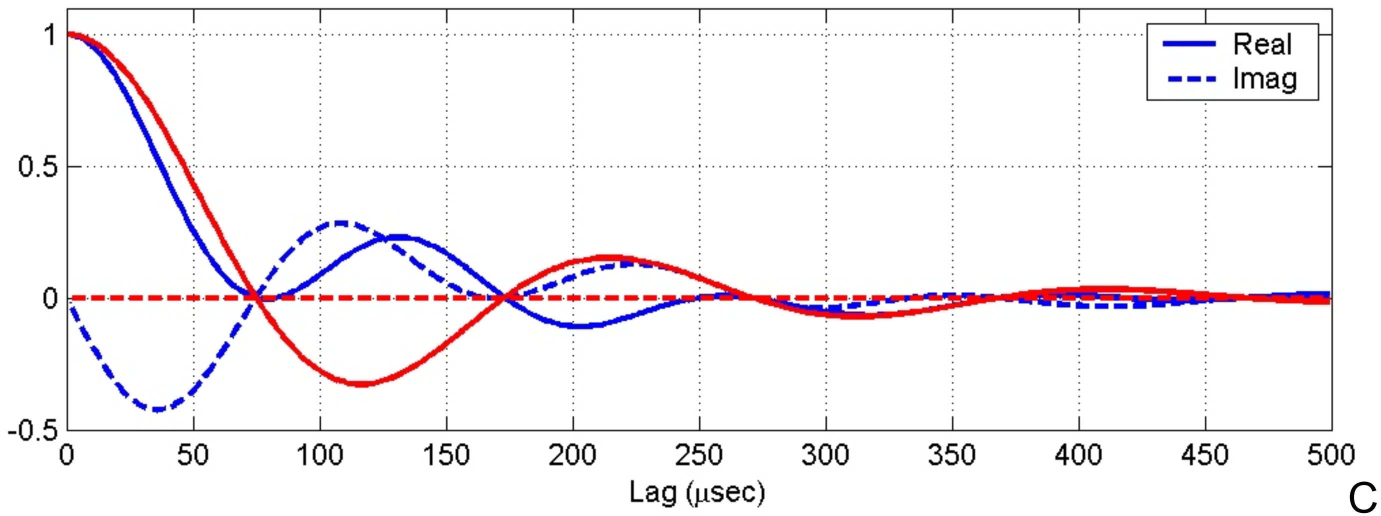
<u>Parameters</u>
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 1000 K
Comp: 100% O^+
ν_{in}: 10^{-6} KHz

Craig Heinselman

Dependence of spectra on ionospheric parameters

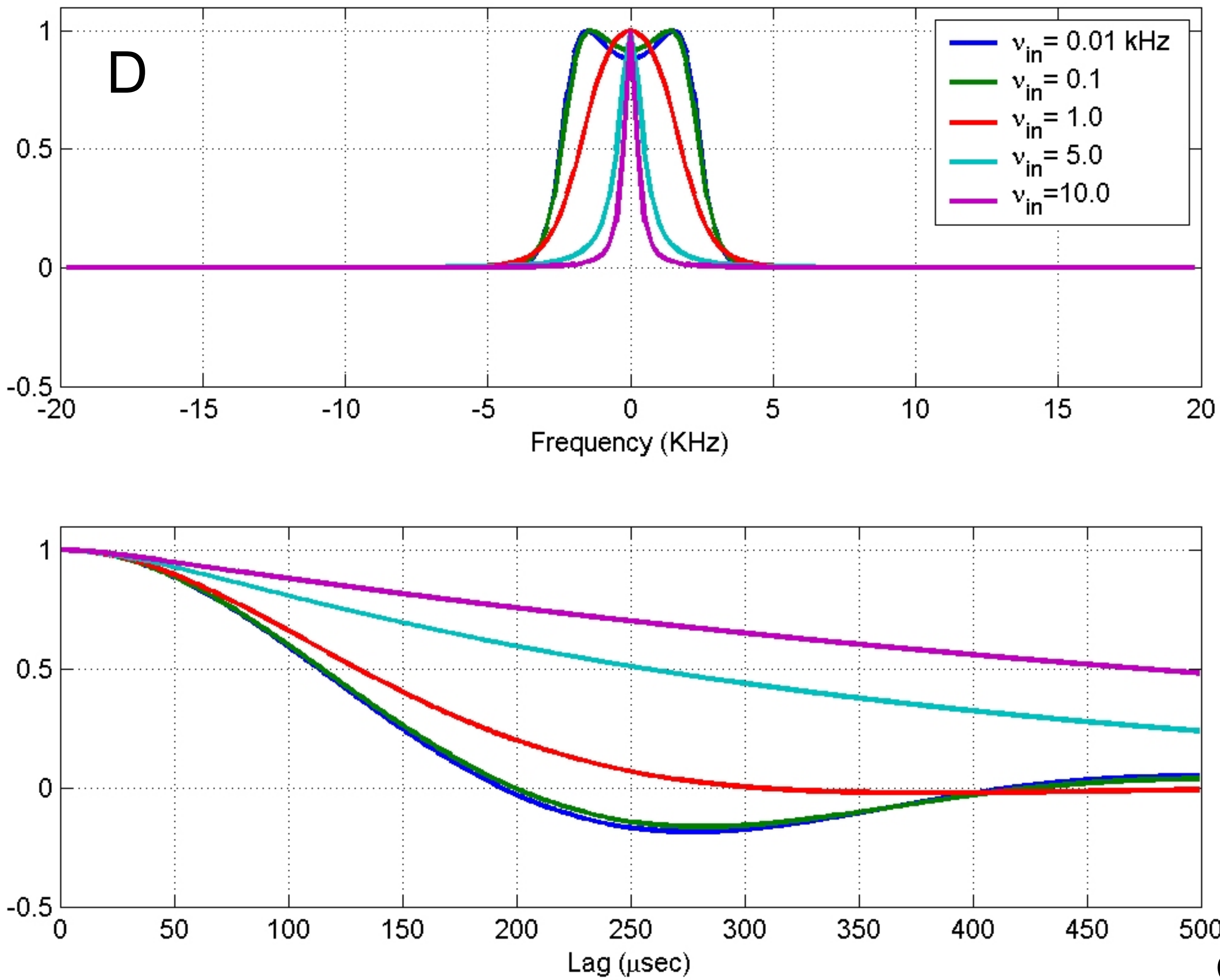


Plasma as a whole moves with vi and produces Doppler shift



Craig Heinselman

Dependence of spectra on ionospheric parameters



Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 500 K
Te: 500 K
Comp: 100% NO^+

In the E and especially D region, ion-neutral collisions start to attenuate the ion-acoustic wave

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