Radar 2: basic signal processing and radar coding

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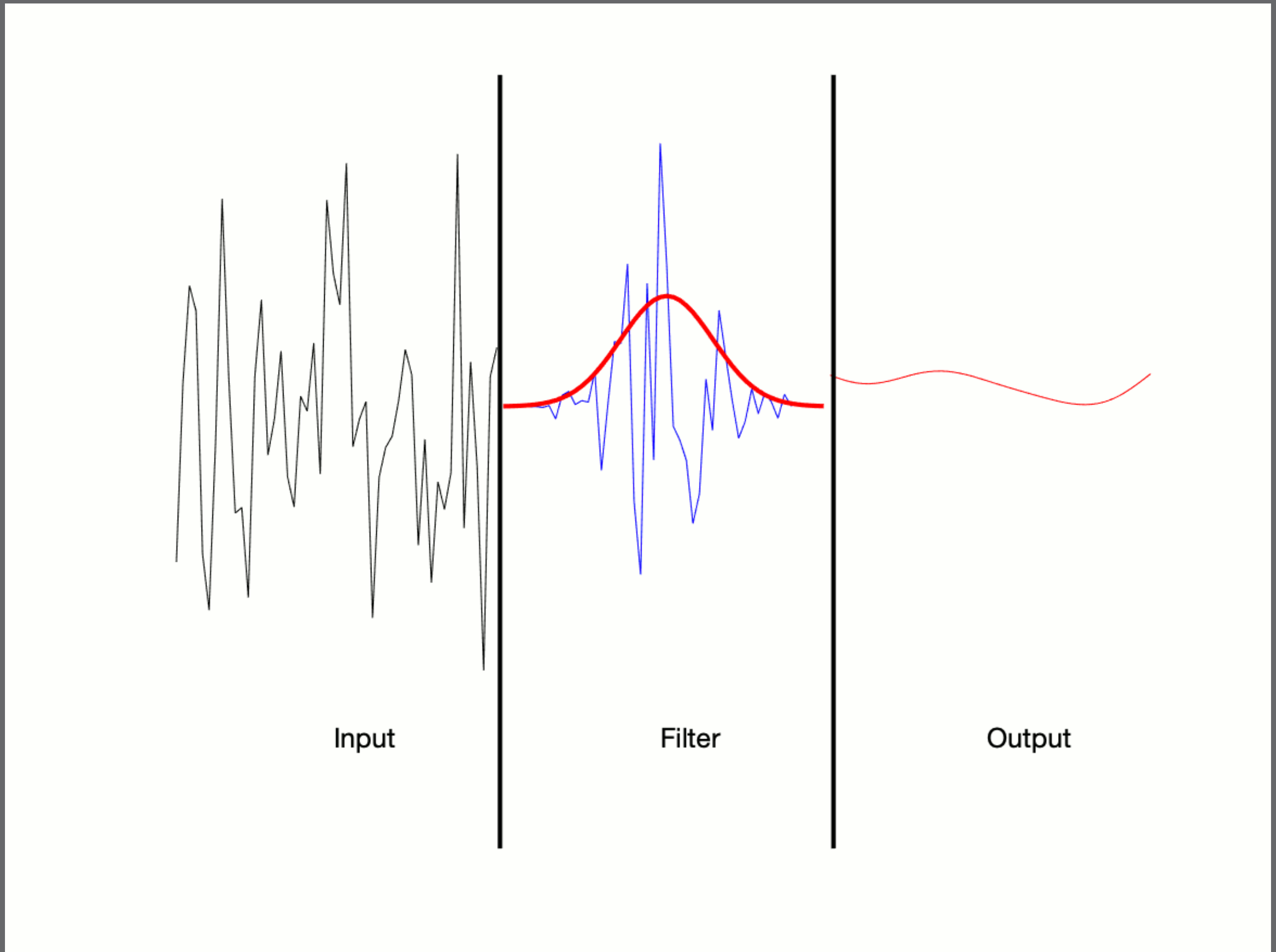
Outline

- Some useful mathematics
- Baseband signal
- Working principle of a pulsed radar
- Range-time diagram
- Range and Doppler aliasing
- Underspread and overspread targets
- Radar coding

Mathematical tools

- **Euler's identity**
  - $e^{i\emptyset} = \cos\emptyset + i\sin\emptyset$
- **Generic signal with amplitude and phase modulation**
  - $z(t) = A(t)e^{i(\omega_0 t + \emptyset(t))}$
- **Fourier transform**
  - $Z(\nu) = \mathcal{F}\{z(t)\} = \int_{-\infty}^{\infty} z(t)e^{-i\omega t}dt$
  - $z(t) = \mathcal{F}^{-1}\{Z(\nu)\} = \int_{-\infty}^{\infty} Z(\nu)e^{i\omega t}d\nu$
  - $\omega = 2\pi\nu$
- **Convolution**
  - $f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$
  - $\mathcal{F}\{f(t) * g(t)\} = F(\nu)G(\nu)$
- **Correlation**
  - $f(t) \circ g(t) = \int_{-\infty}^{\infty} f^*(\tau)g(\tau+t)d\tau$
  - $\mathcal{F}\{f(t) \circ g(t)\} = F^*(\nu)G(\nu)$
- **Autocorrelation**
  - $f(t) \circ f(t) = f(t) * f^*(-t)$
  - $\mathcal{F}\{f(t) \circ f(t)\} = |F(\nu)|^2$

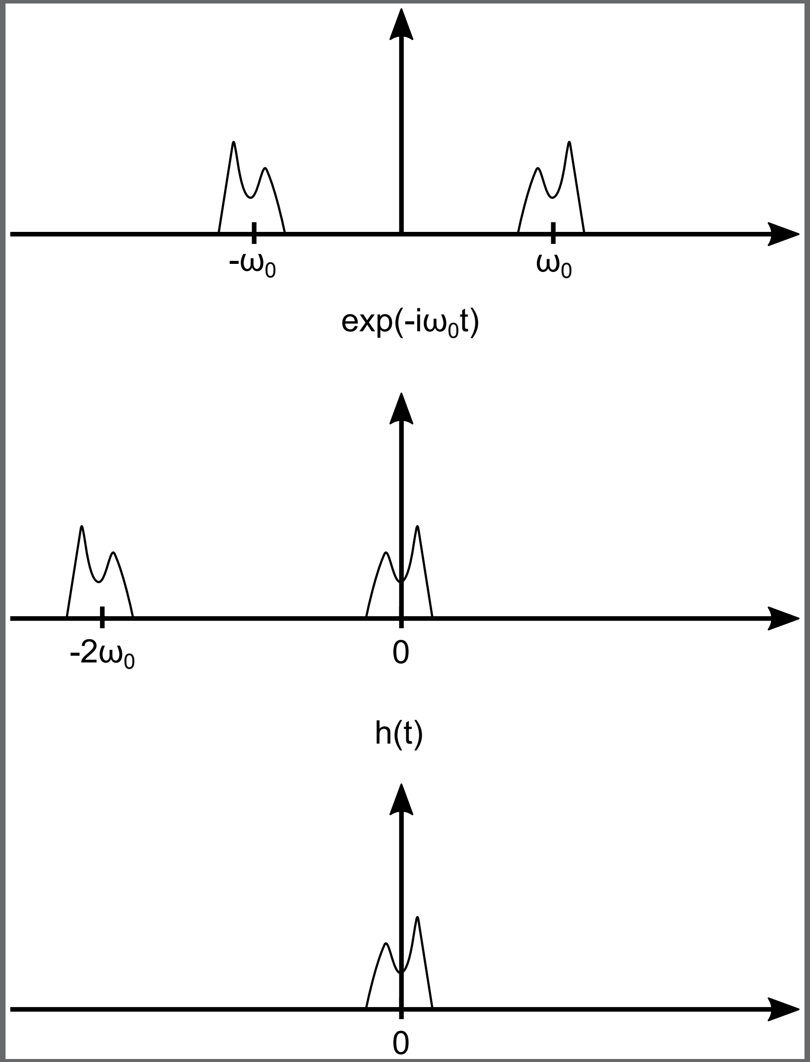
The convolution integral and filtering



- A linear filter has an impulse response $h(t)$
- When the filter input signal is $z_i(t)$, the output signal $z_o(t)$ is the convolution

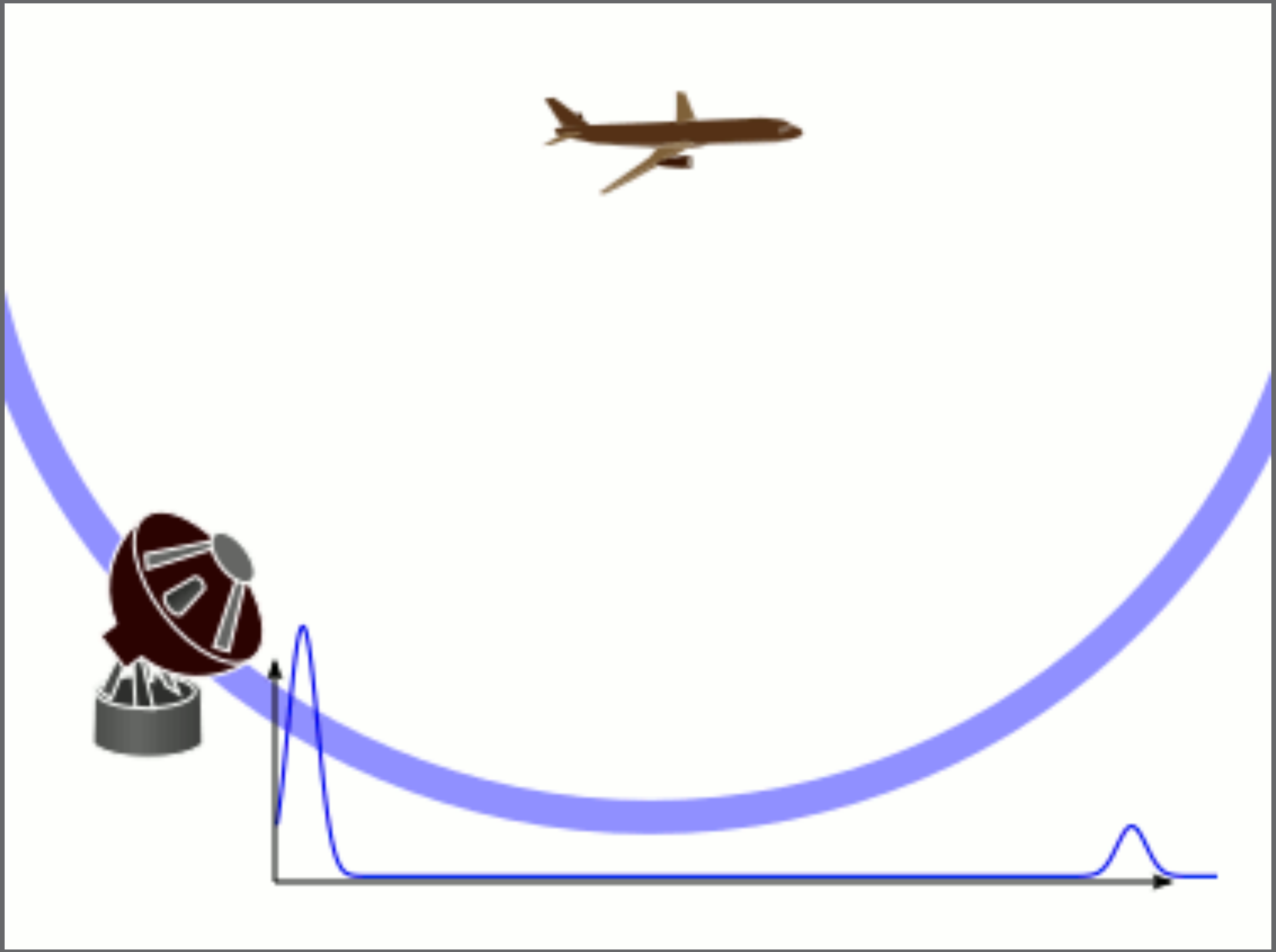
$$z_o(t) = h(t) * z_i(t) = \int_{-\infty}^{\infty} h(\tau)\boldsymbol{z_i}(t-\tau)d\tau$$

Baseband signal

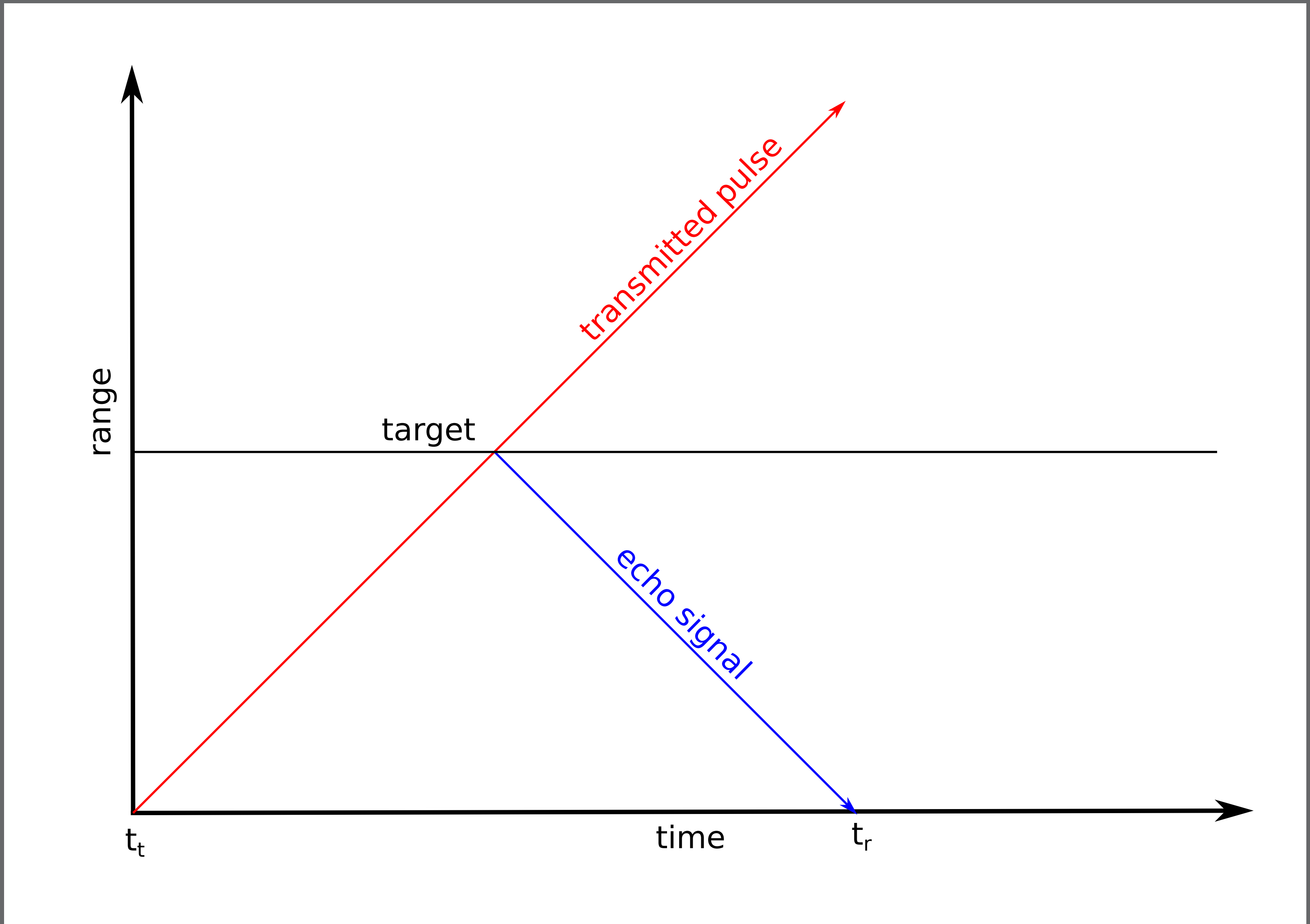


- Spectrum of the original signal $z(t)$ is centered around ω_0 and $-\omega_0$
- Multiplication with $e^{-i\omega_0 t}$ shifts both spectrum peaks by $-\omega_0$
- When the shifted signal is filtered with a sufficient filter $h(t)$, the peak centered around $-\omega_0$ is removed
- Nyquist sampling theorem:
- A signal with cut-off frequency f_m can be perfectly reconstructed from samples taken at frequency f_s if $f_s \geq 2f_m$

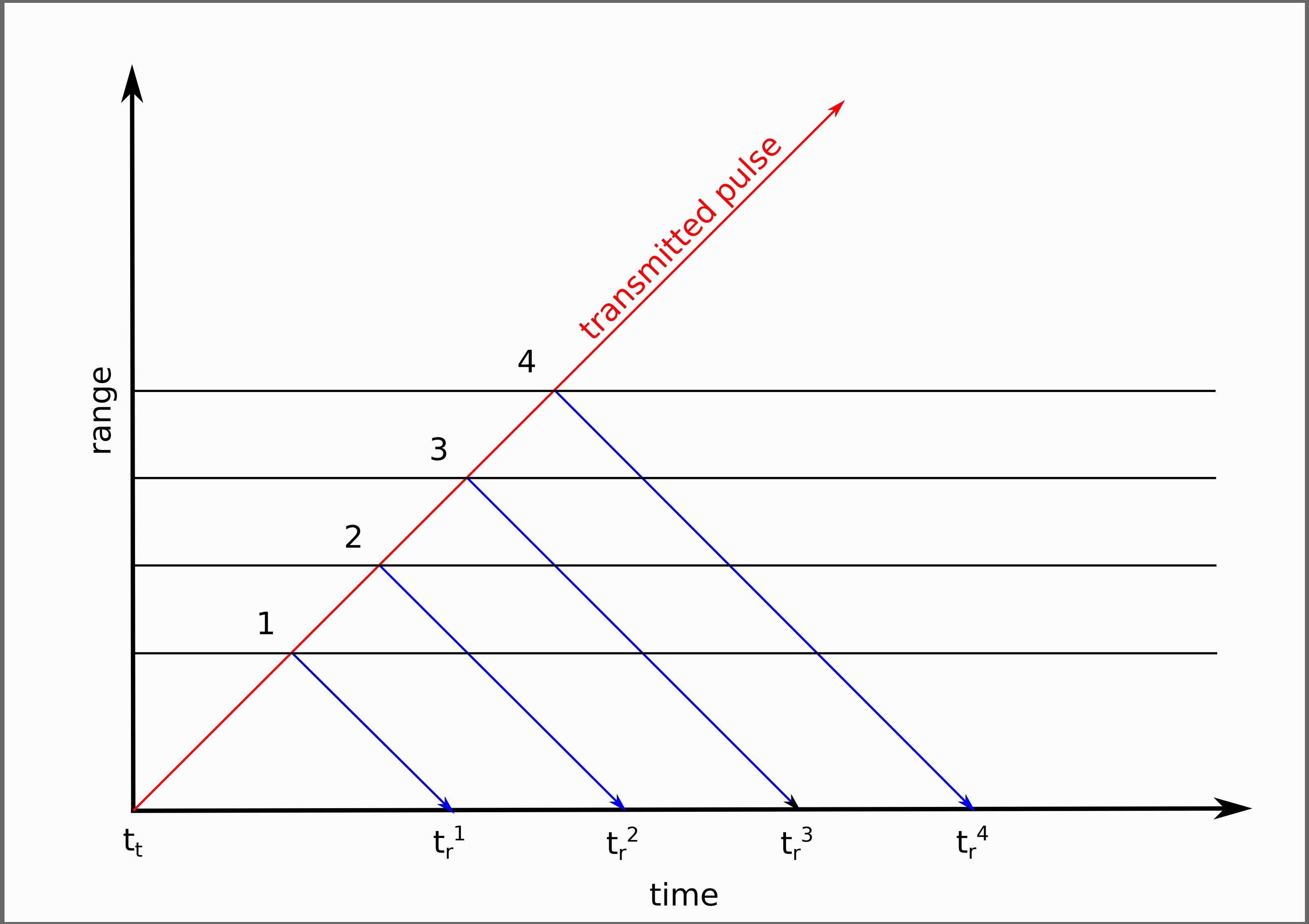
Radar



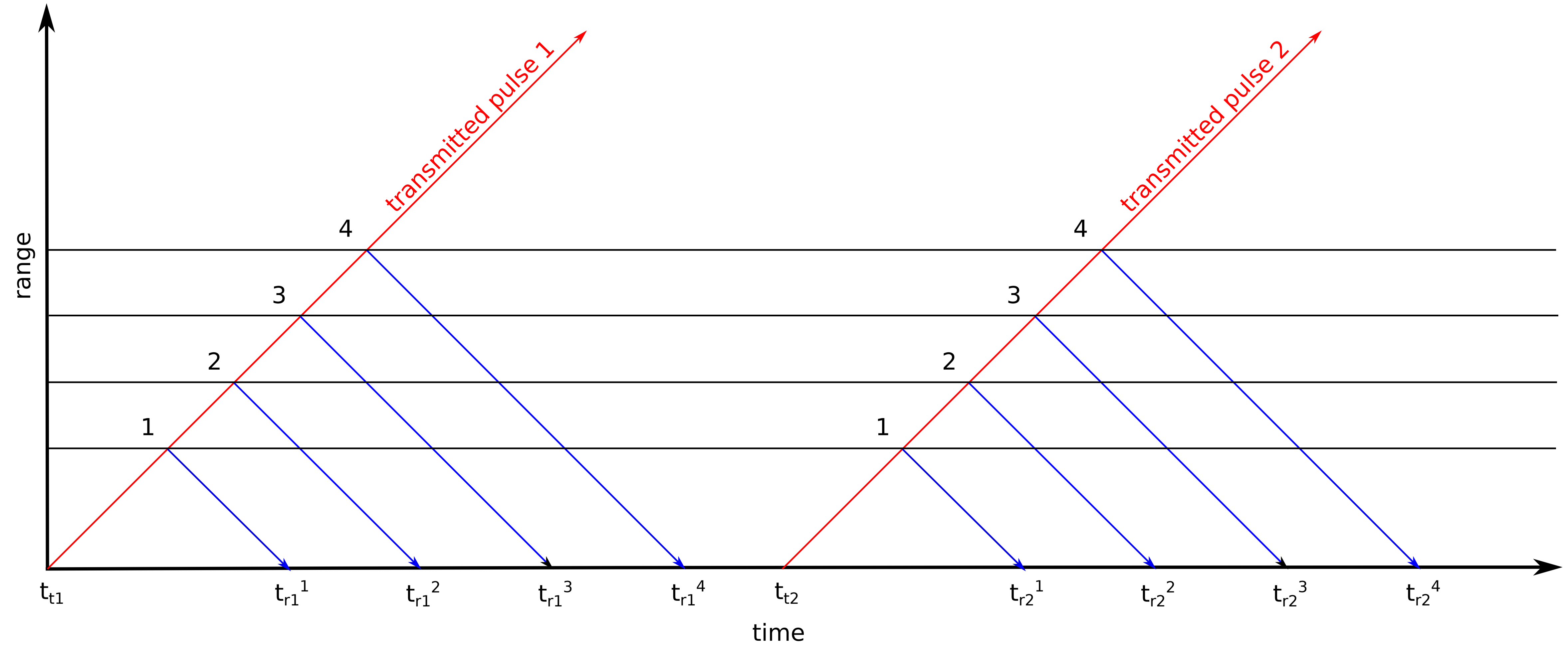
Range-time diagram



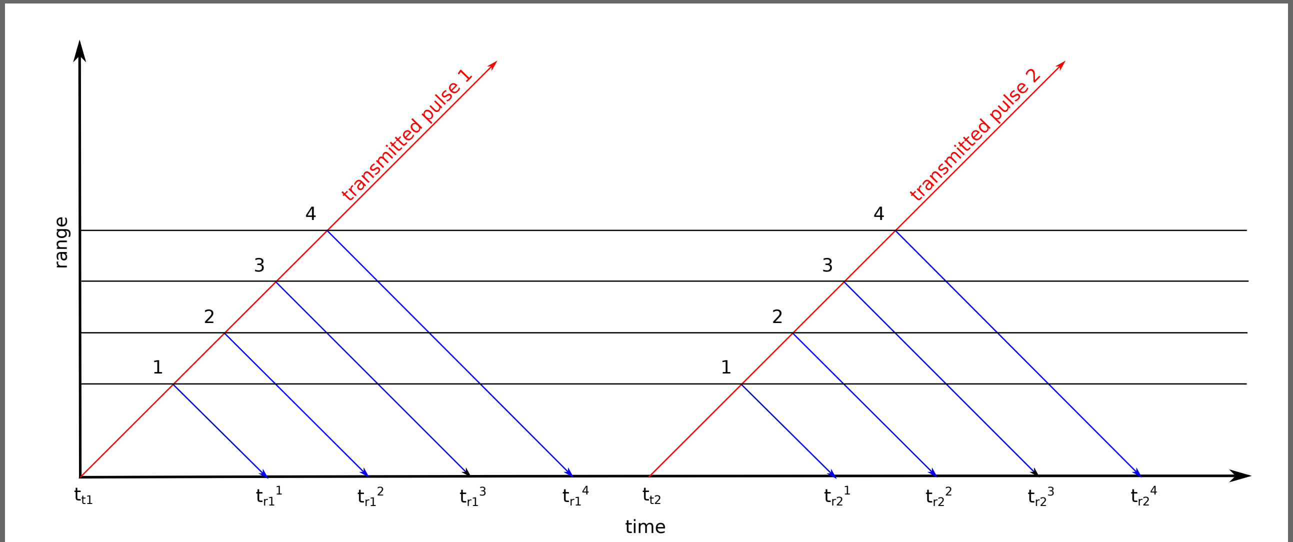
Range-time diagram: 1 short pulse, 4 targets



Range-time diagram: multiple short pulses, 4 targets

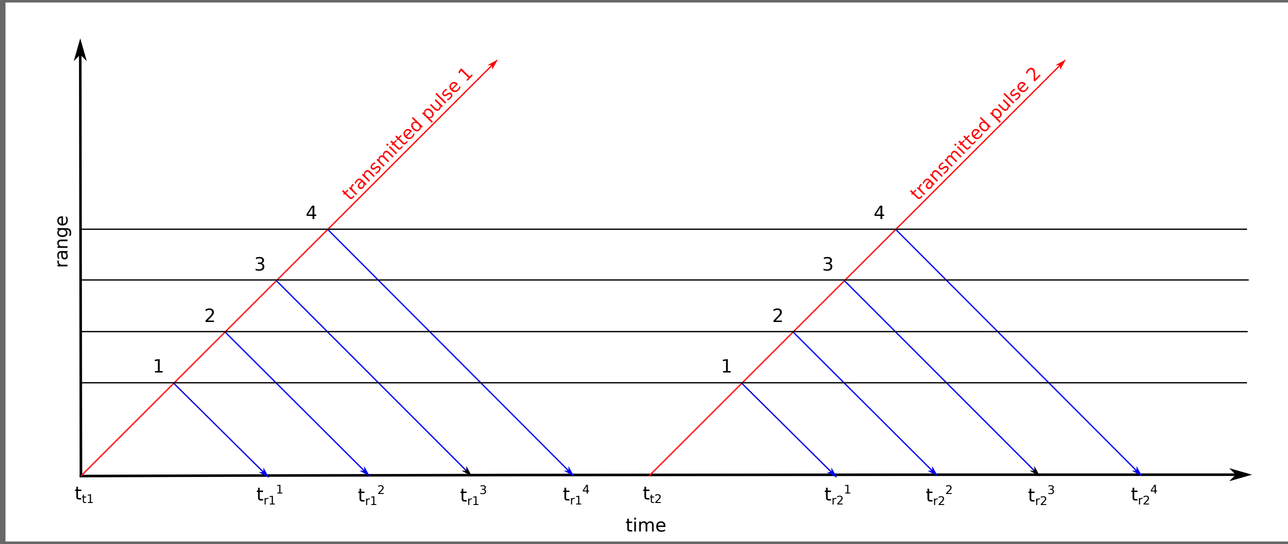


Simple spectrum estimation



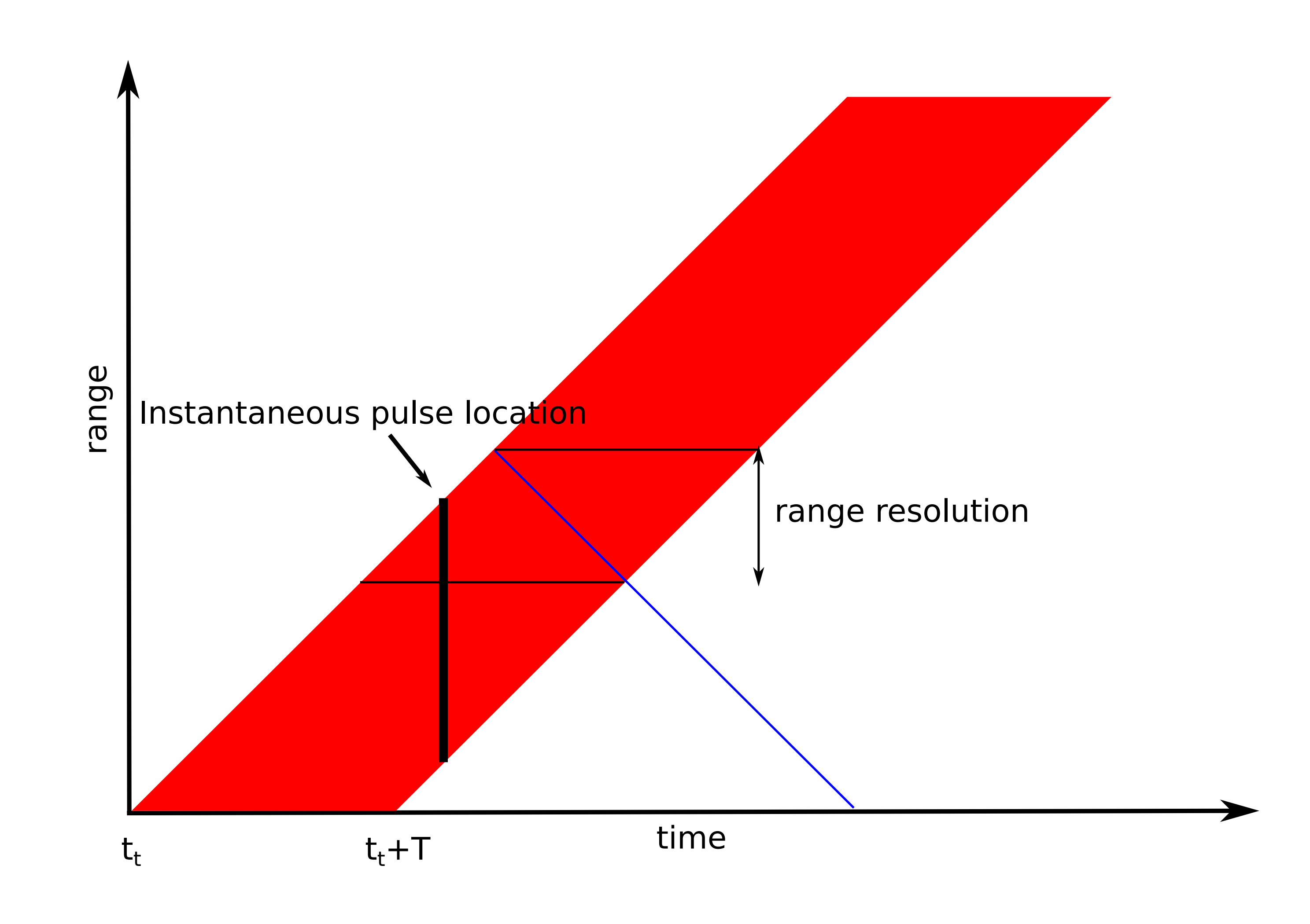
- Short pulses transmitted at regular intervals $\Delta t_t = t_{t2} - t_{t1}$
- We get one echo sample per pulse from each altitude
- Sampling frequency $f_s = 1/\Delta t_t$
- Nyquist sampling theorem:
  - A signal with cut-off frequency f_m can be perfectly reconstructed from samples taken at frequency f_s if $f_s \geq 2f_m$

Underspread vs overspread target



- **Signal roundtrip time $S = \frac{2R}{c}$**
- **Width of the target spectrum Δf**
- **Underspread target: $\Delta f < \frac{1}{S}$**
  - The target spectrum can be estimated using echo samples from subsequent transmitted pulses (pulse-to-pulse correlations)
- **Overspread target: $\Delta f > \frac{1}{S}$**
  - We must collect several echo samples from each range per pulse (intra-pulse correlations)

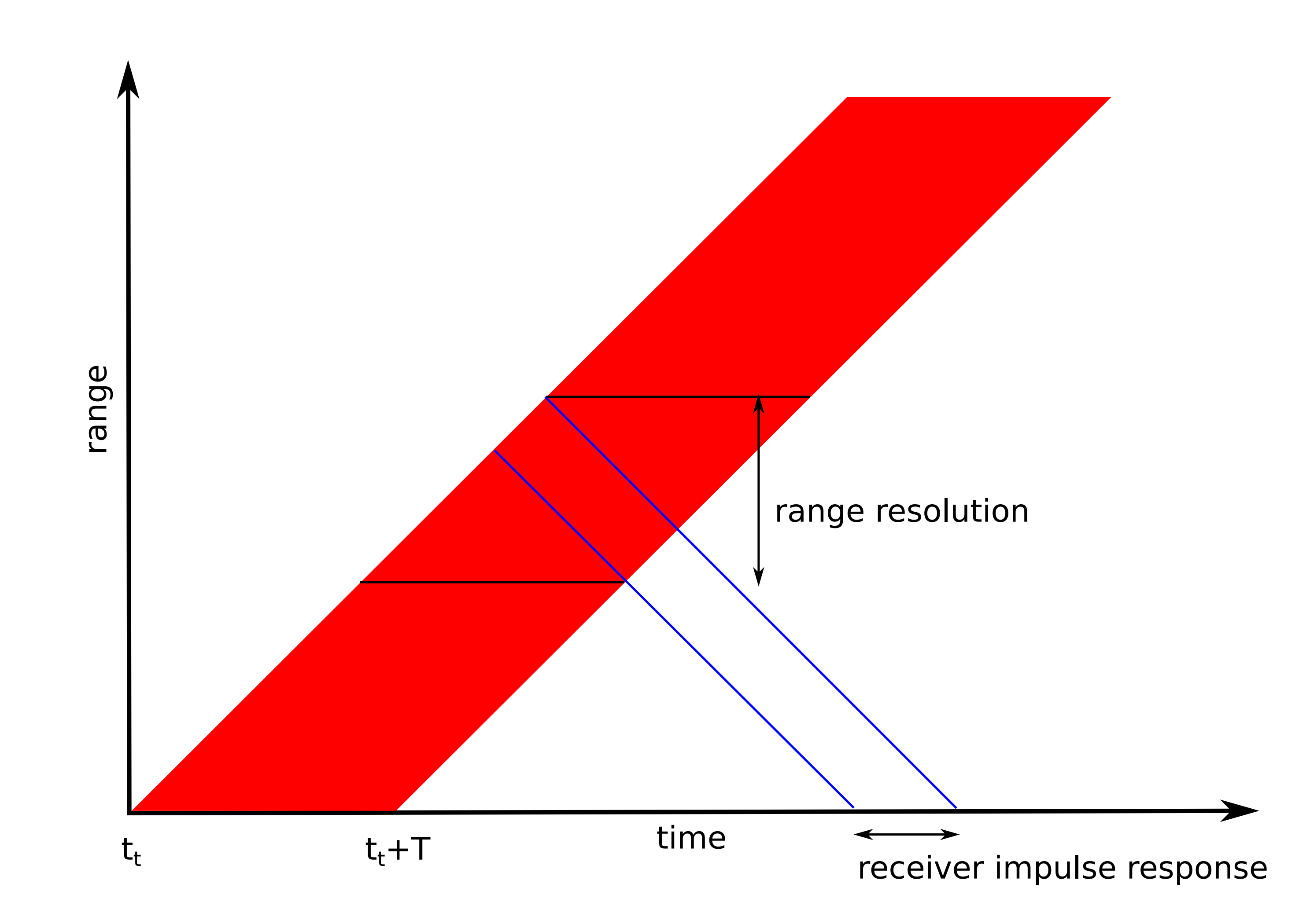
Pulse length vs range resolution



- Echo from a pulse of duration T come from a volume whos length is

$$\Delta S = \frac{c}{2}T$$

Pulse length vs range resolution



- In reality, the received signal $z_r(t)$ is a convolution of the echo signal $z(t)$ and the receiver impulse response $h(t)$

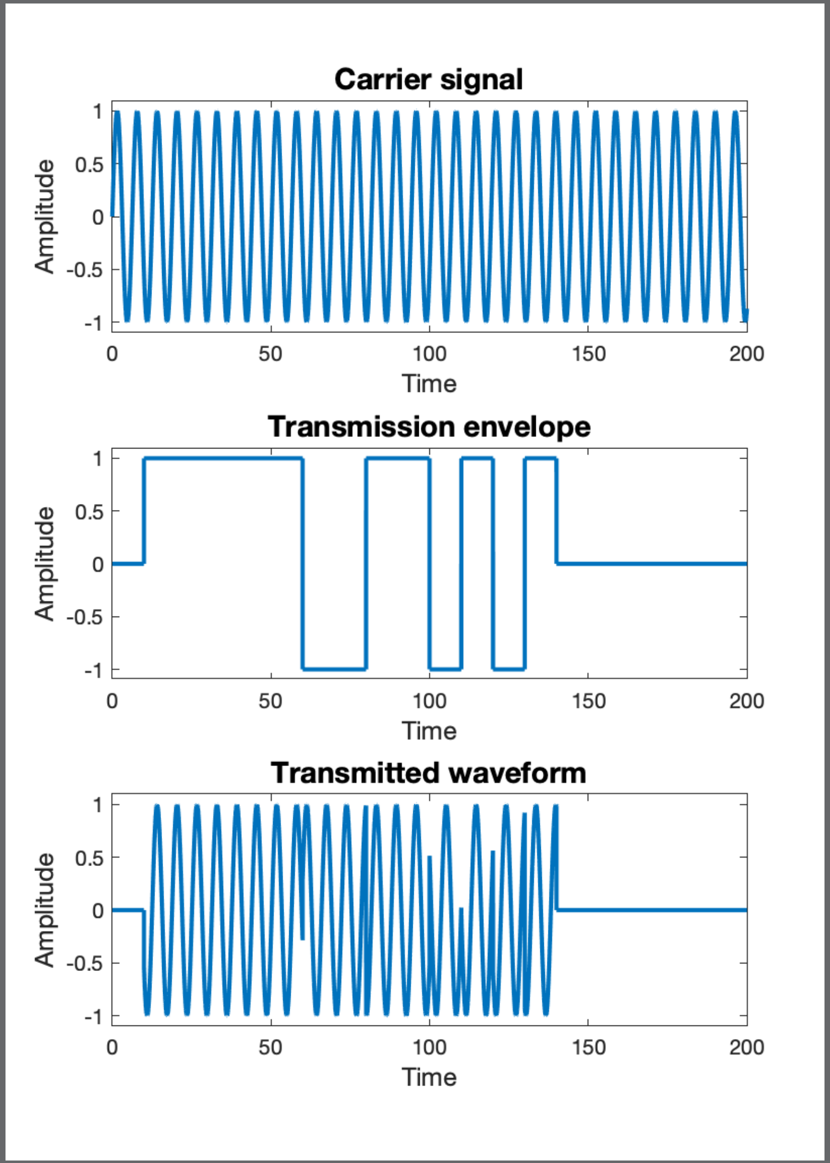
$$z_r(t) = \int_{-\infty}^{\infty} h(\tau) z(t-\tau) d\tau$$

- The final range resolution is

$$\Delta S_r = \frac{c}{2}(T + T_h),$$

where T_h is length of the receiver impulse response

Transmission modulation (radar coding)

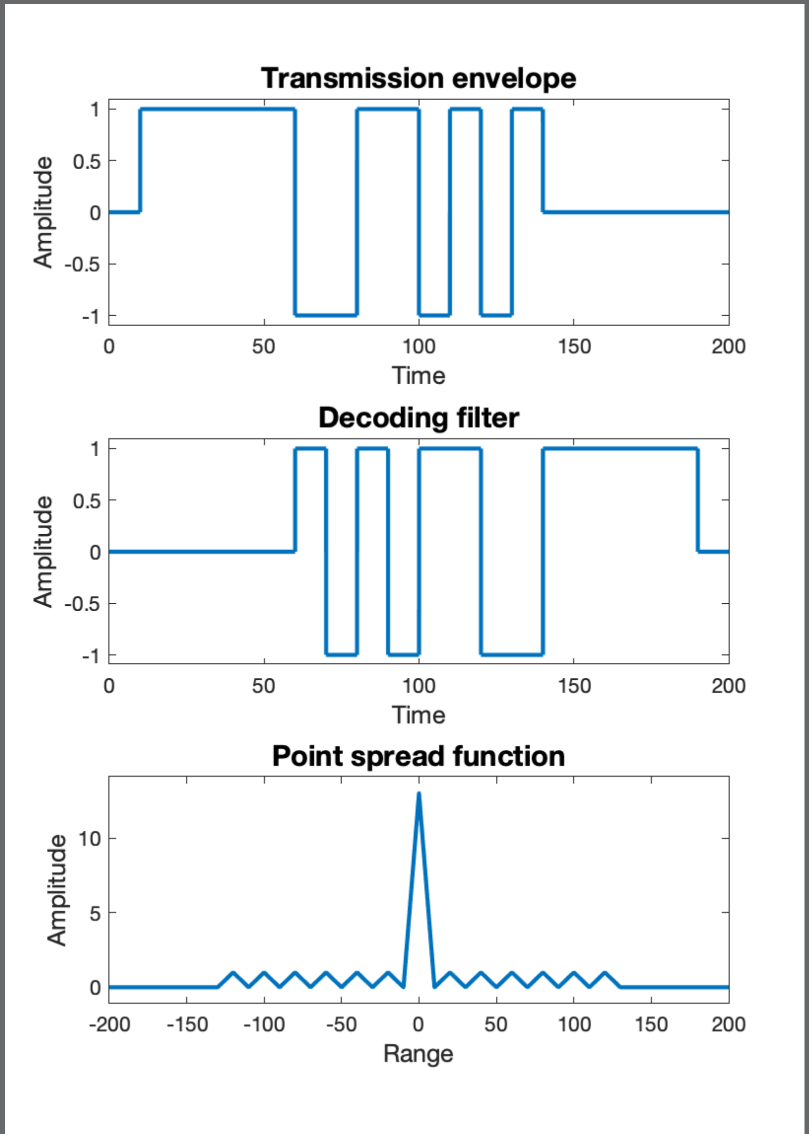


- **Long pulses, which correspond to short pulses with higher peak power after decoding**
- **Carrier signal** $z_c(t) = A_c e^{i\omega_c t}$
- **Transmission envelope** $z_e(t) = A_e(t) e^{i\phi_e(t)}$
- **Transmitted waveform** $z_t(t) = z_c(t) z_e(t)$

- **Binary coding**
- $A_e(t) = \begin{cases} 1, \text{when } 0 < t < T_{pulse} \\ 0, otherwise \end{cases}$
- $\phi_e(t) \in [0, \pi]$

- **From this point on, we will use baseband signals and ignore the carrier frequency**

Pulse compression, coherent target



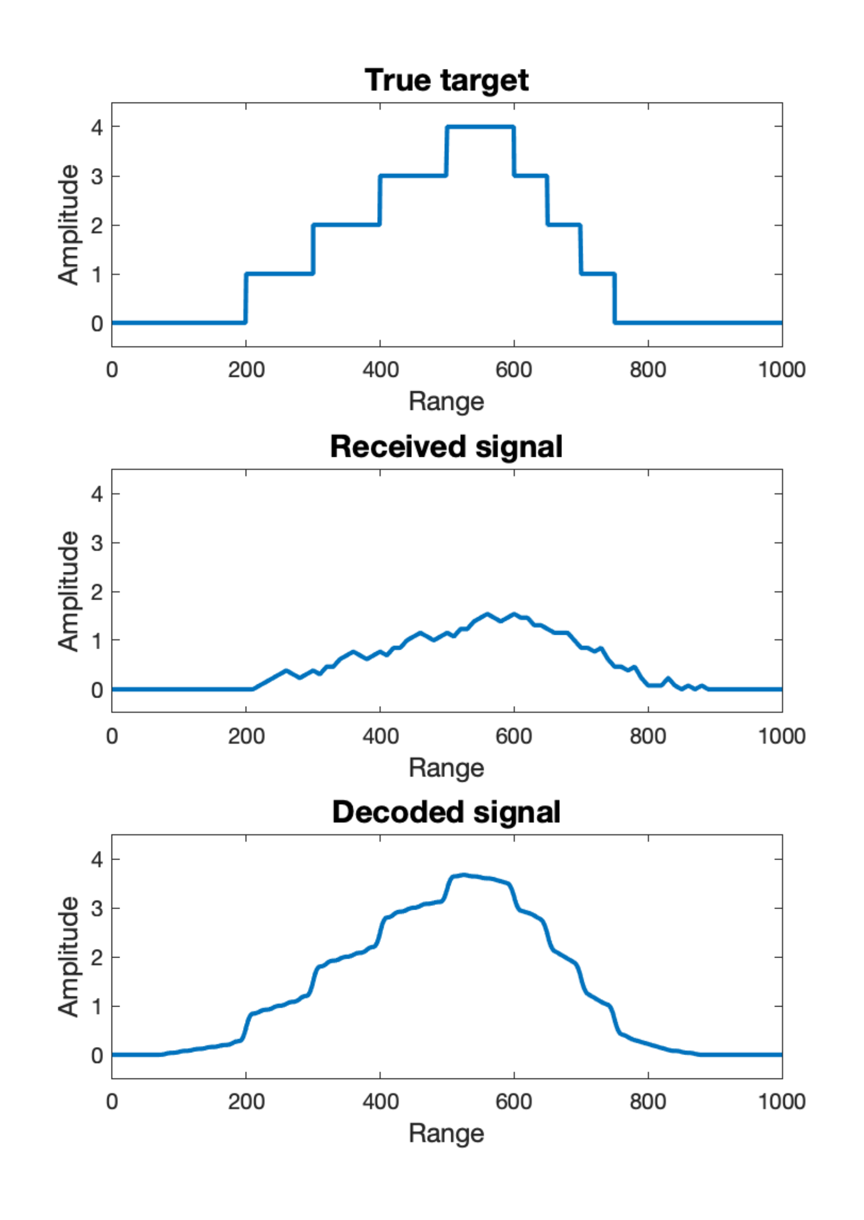
- **Single target at distance r**
  - Signal roundtrip time $S = \frac{2r}{c}$
  - Received echo signal $z_r(t) = A_s z_e(t - S)$
  - The complex constant A_s absorbs the effects of distance, radar cross-section and Doppler shift
- **Matched filter decoding**
  - Convolve the received signal with the decoding filter $z_d(t) = z_e^*(-t)$
  - Decoding filter output

$$z_m(t) = \int_{-\infty}^{\infty} z_r(\tau) z_d(t - \tau) d\tau$$

$$= A_c A_s \int_{-\infty}^{\infty} \boldsymbol{z_e}(\boldsymbol{\tau} - \boldsymbol{S})\, z_e^*(t + \tau) d\tau = A_c A_s \mathcal{R}_{z_e}(t - S)$$

  - $\mathcal{R}_{z_e}(t - S)$ is the autocorrelation function of the transmission envelope z_e (point spread function)

Range-spread target



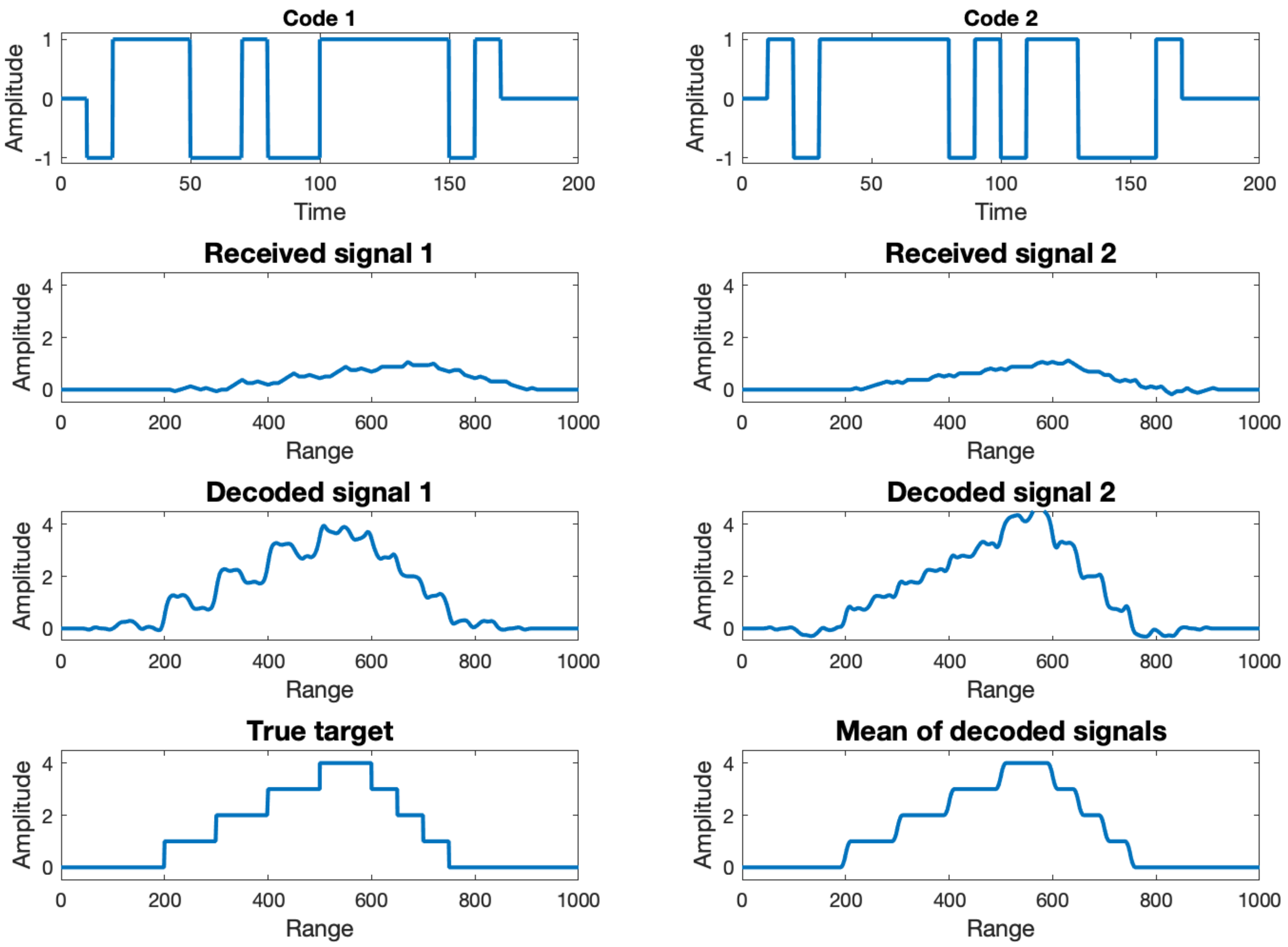
- When the target is not point-like, we get scattering from all ranges. The received signal is a convolution of the transmitted waveform and the target

$$z_r(t) = \int A_s(S) z_e(t-S)\, dS$$

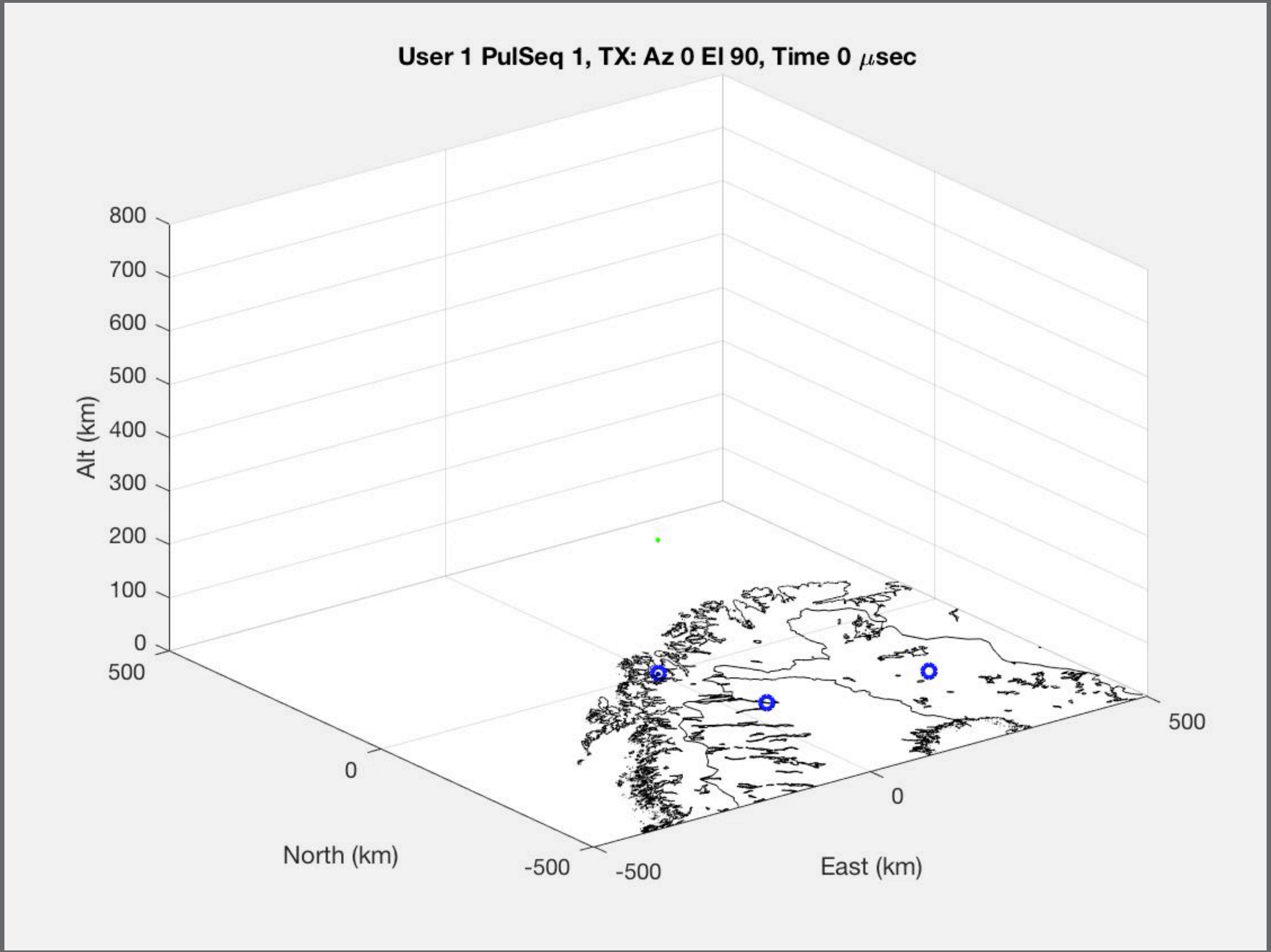
- The decoded signal is a convolution of the target and the point spread function

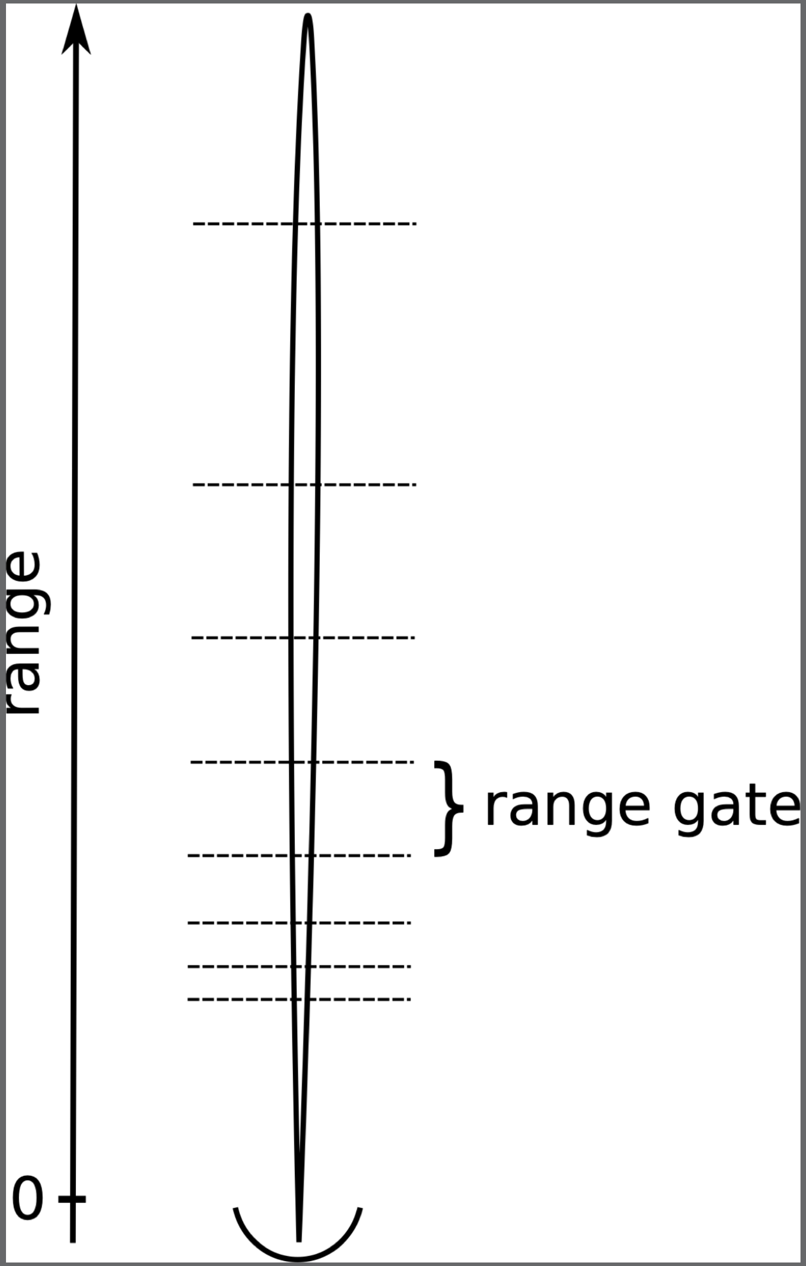
$$z_m(t) = A_c \int A_s(S) \mathcal{R}_{z_e}(t-S)\, dS$$

Complementary codes



EISCAT radar

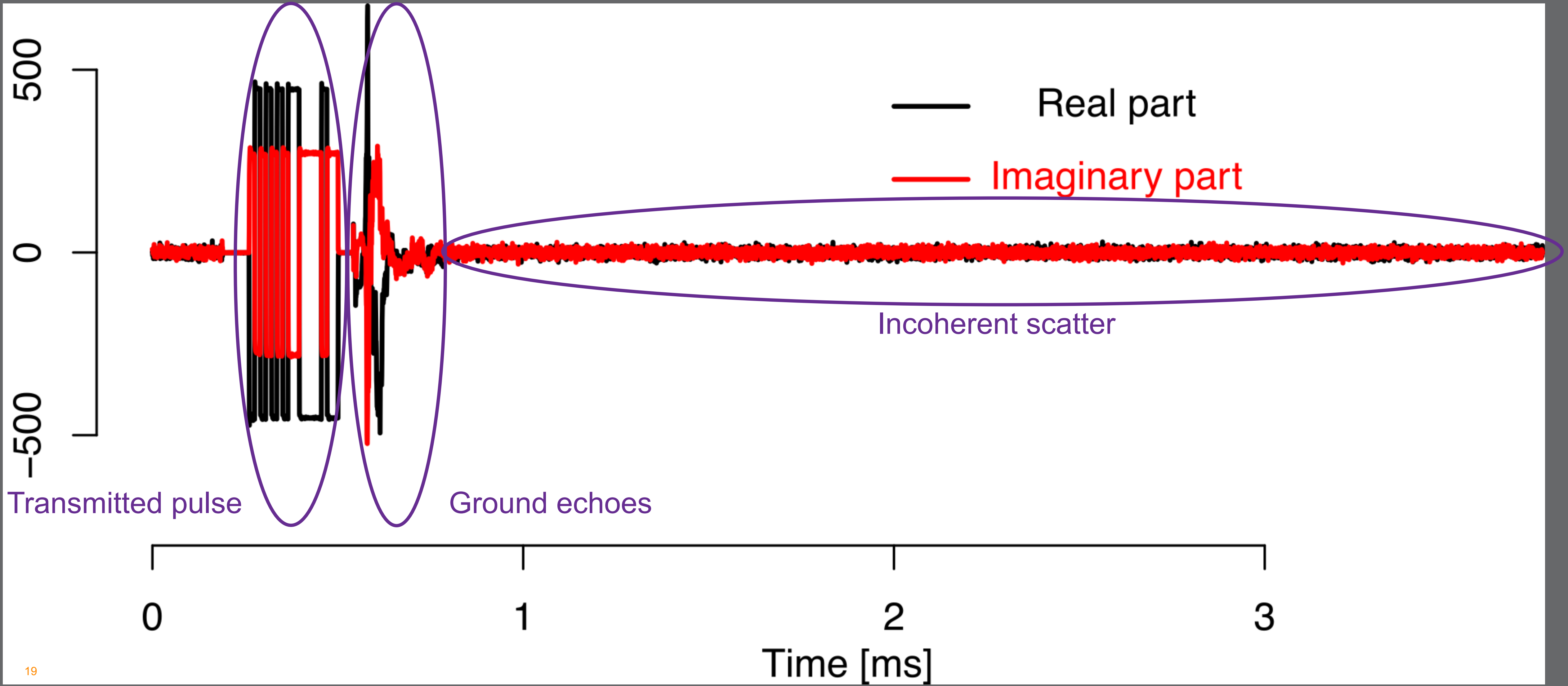


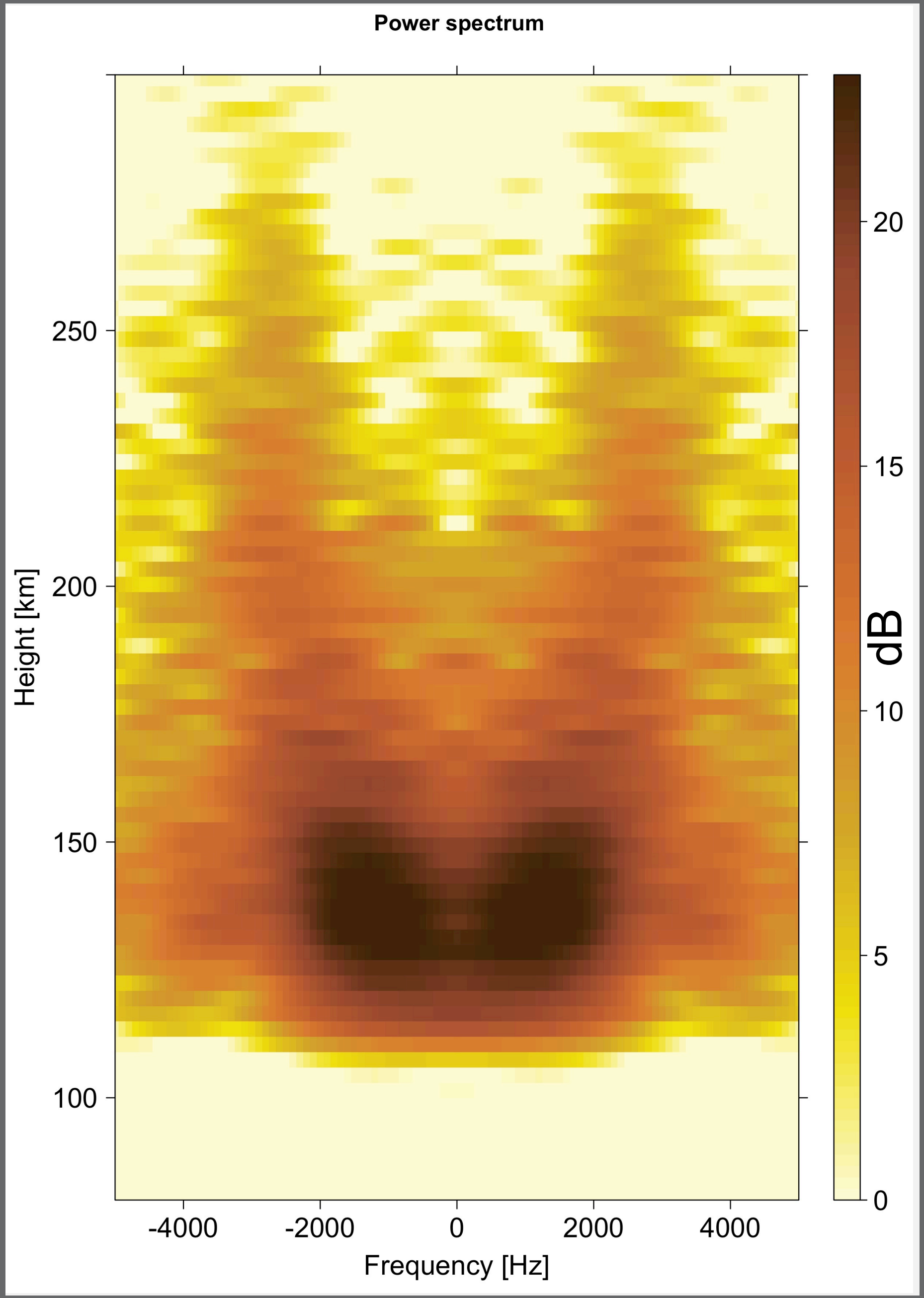


Incoherent scatter radar measurement

- Divide the radar beam into range gates
- Estimate power spectrum of the scattered signal in each gate
- Fit plasma parameters to the observed spectra

Incoherent scatter radar signal

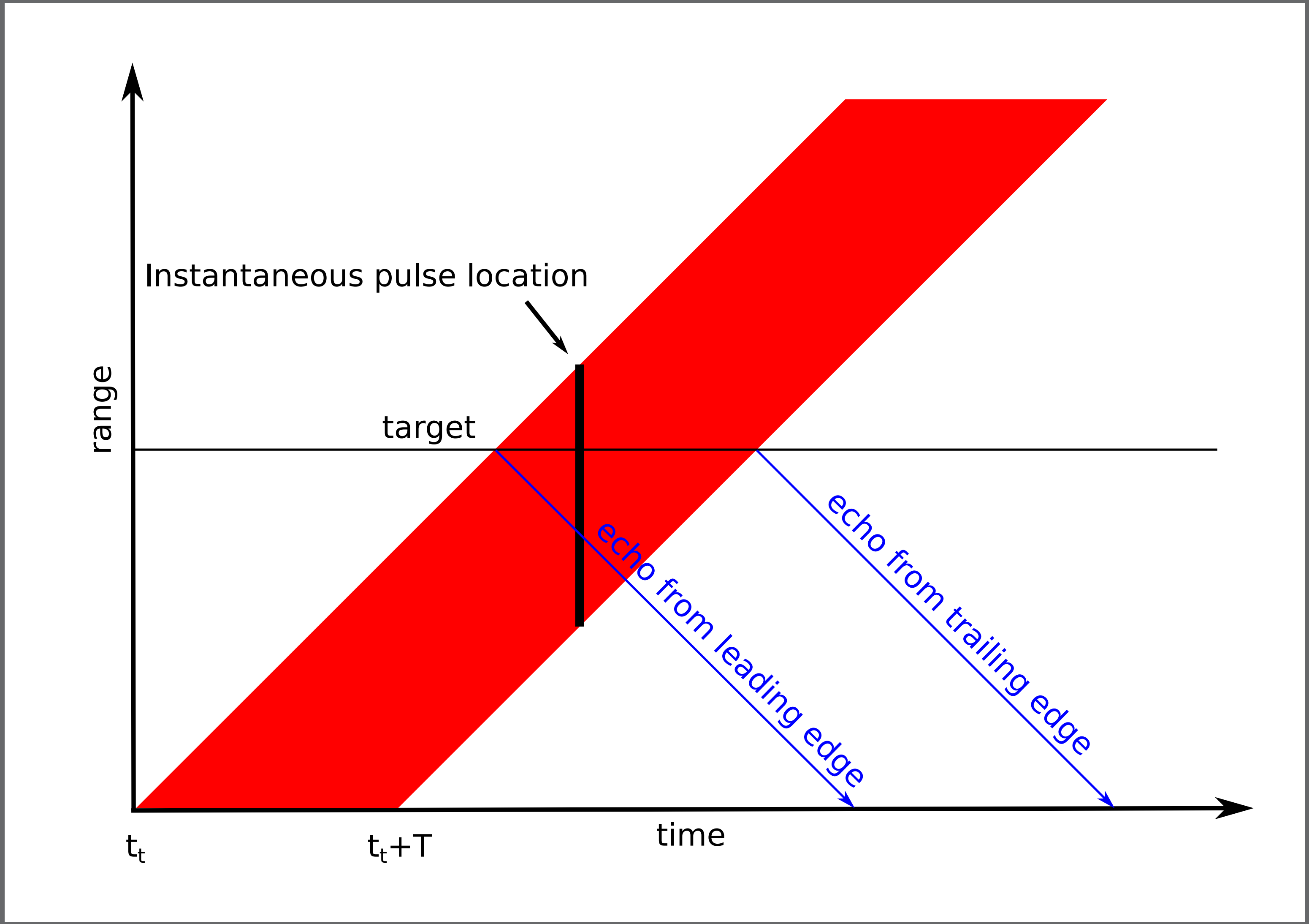




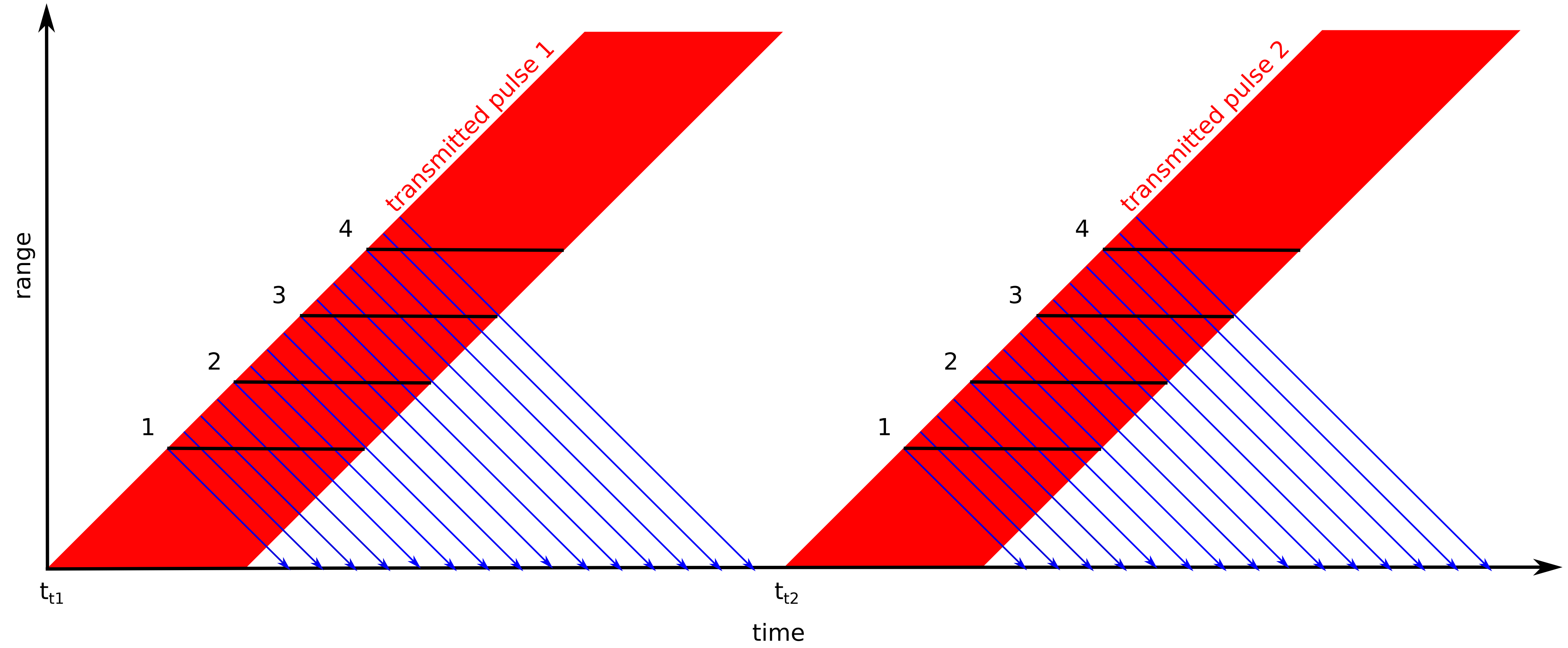
Incoherent scatter spectrum at 224 MHz

- The spectrum is ~10 kHz wide
- We need $f_s \geq 10$ kHz
- Signal roundtrip time to 300 km altitude is 2 ms (0.5 kHz)
- The target is overspread, how to properly sample the spectrum?

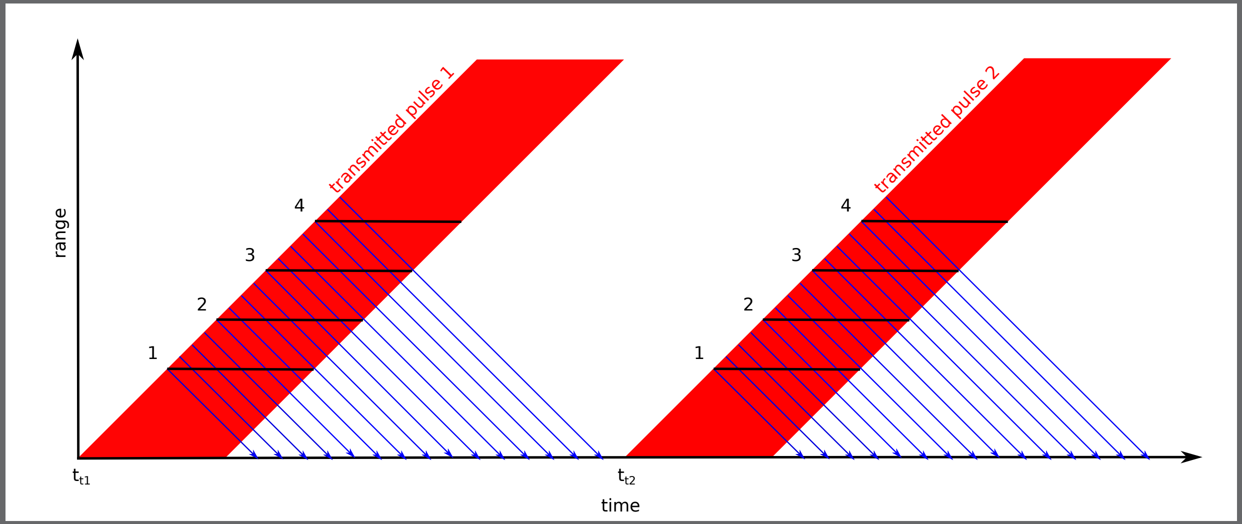
Range-time diagram, pulse length T



Range-time diagram: multiple long pulses, 4 targets

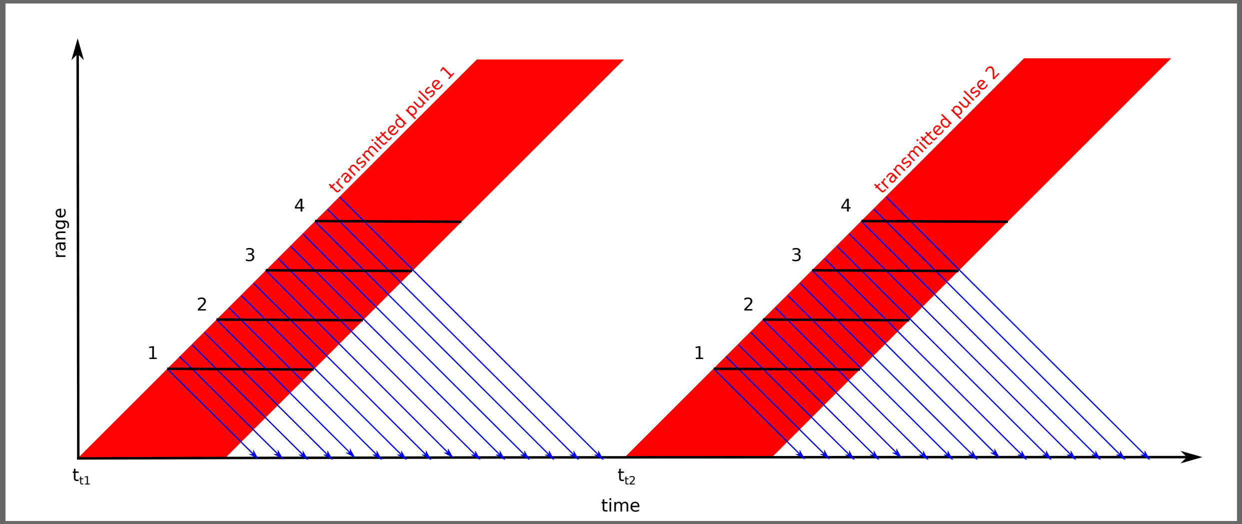


Long pulses



- Enable very high sampling rate
- Each sample has contribution from a wide altitude interval (=several targets)
- We must somehow separate the echoes from different distances
- Phase coding!

Lag profiles and autocorrelation theorem



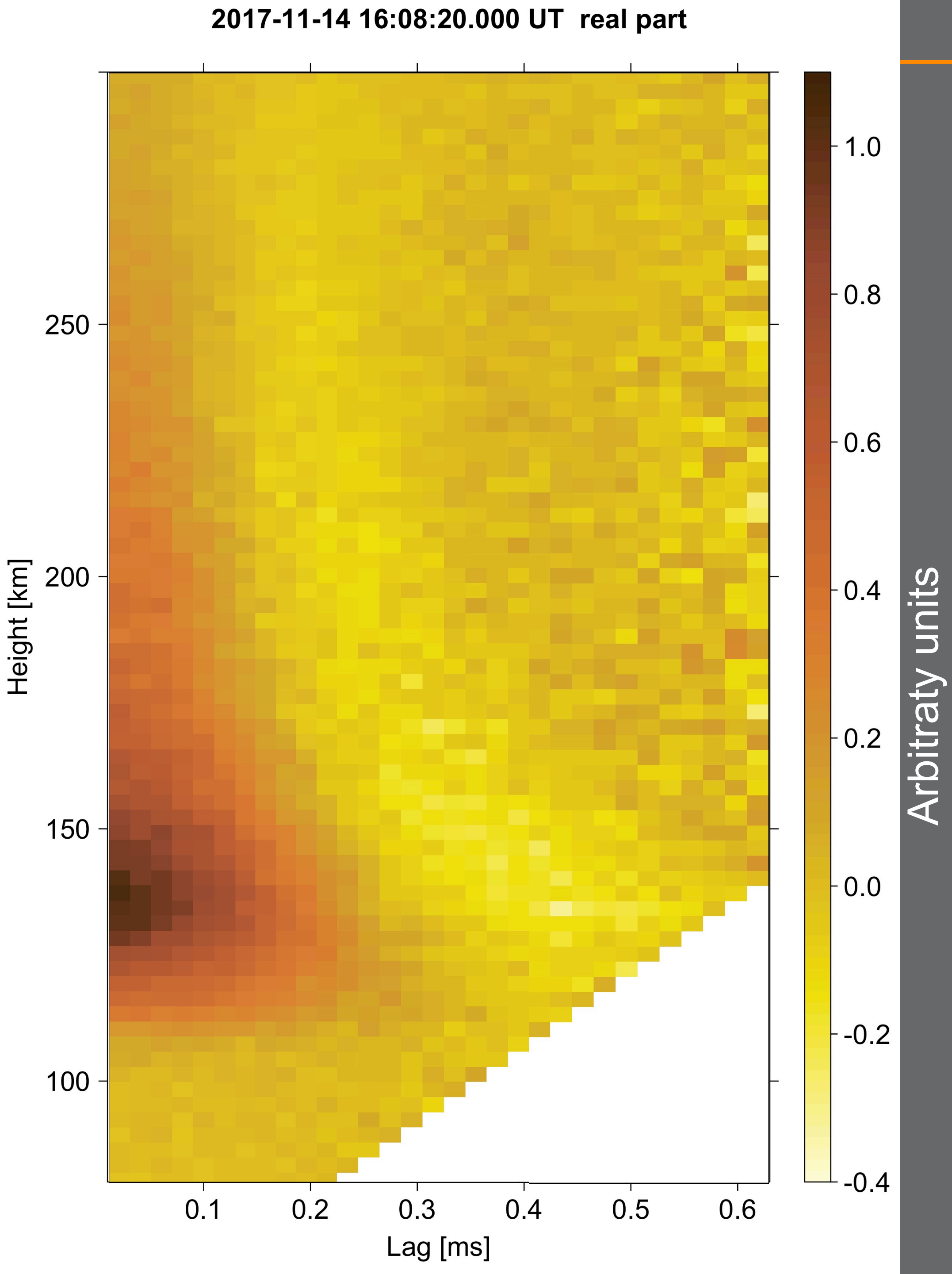
- In the case of incoherent scatter, we cannot decode to resolve the original scattered signal $z(r,t)$

- However, we can resolve the autocorrelation function

$$\mathcal{R}_z(\mathrm{r},\tau) = < z(r,t) z^*(r,t-\tau) >$$

- The power spectrum can be solved with the autocorrelation theorem:

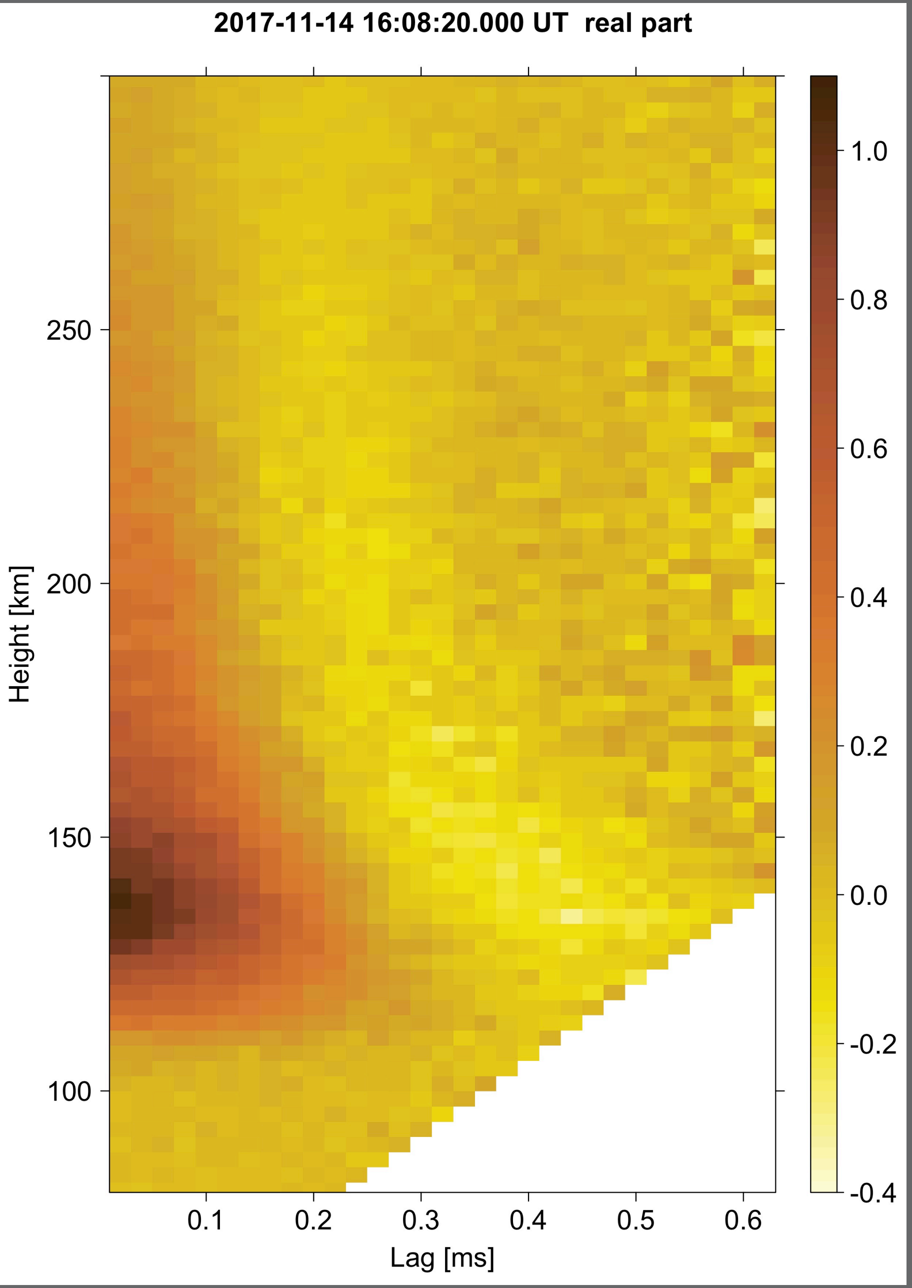
$$S(r,f) = \mathcal{F}\{\mathcal{R}_z(r,\tau)\}$$



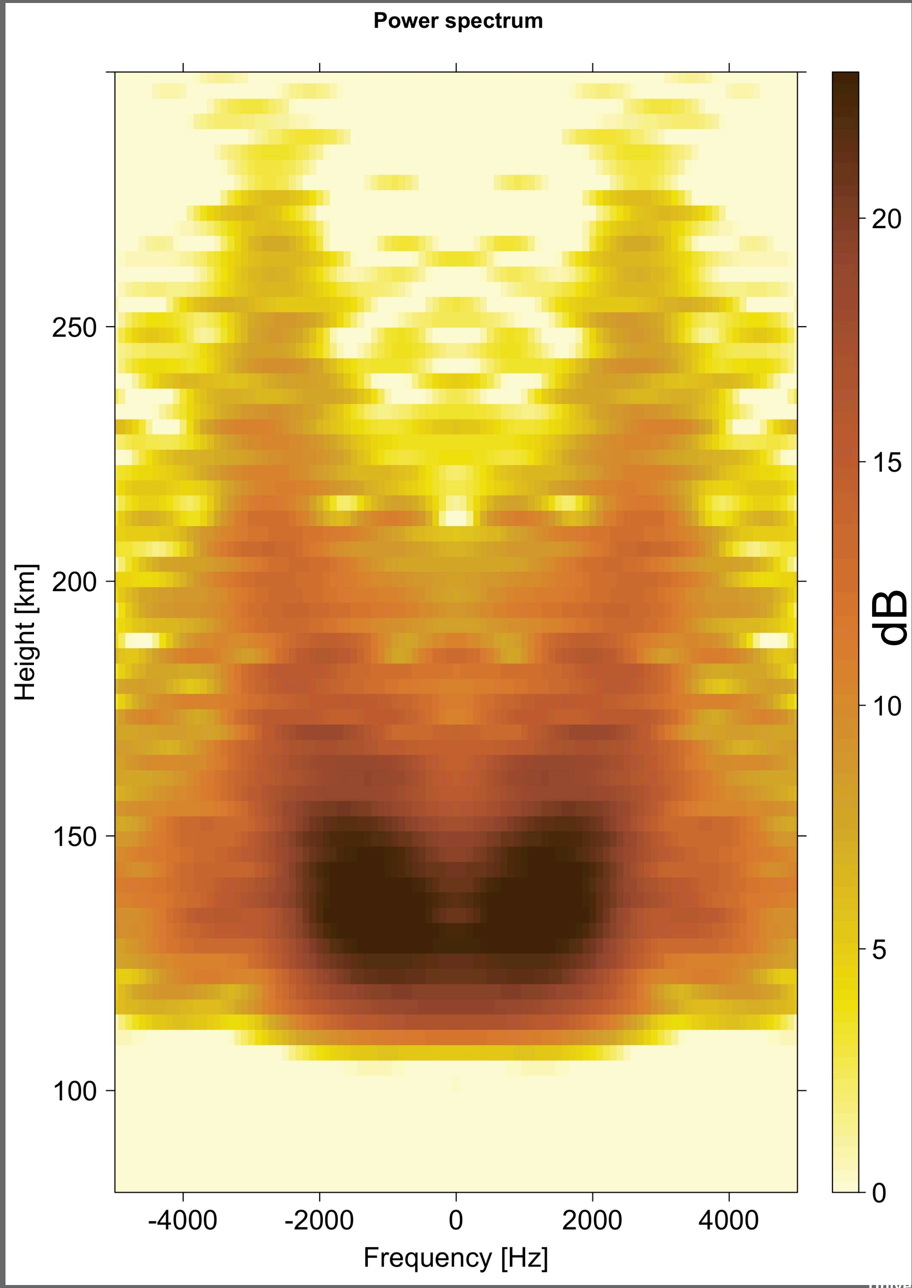
Measuring the autocorrelation function

- Transmit a sequence of long, phase-coded pulses

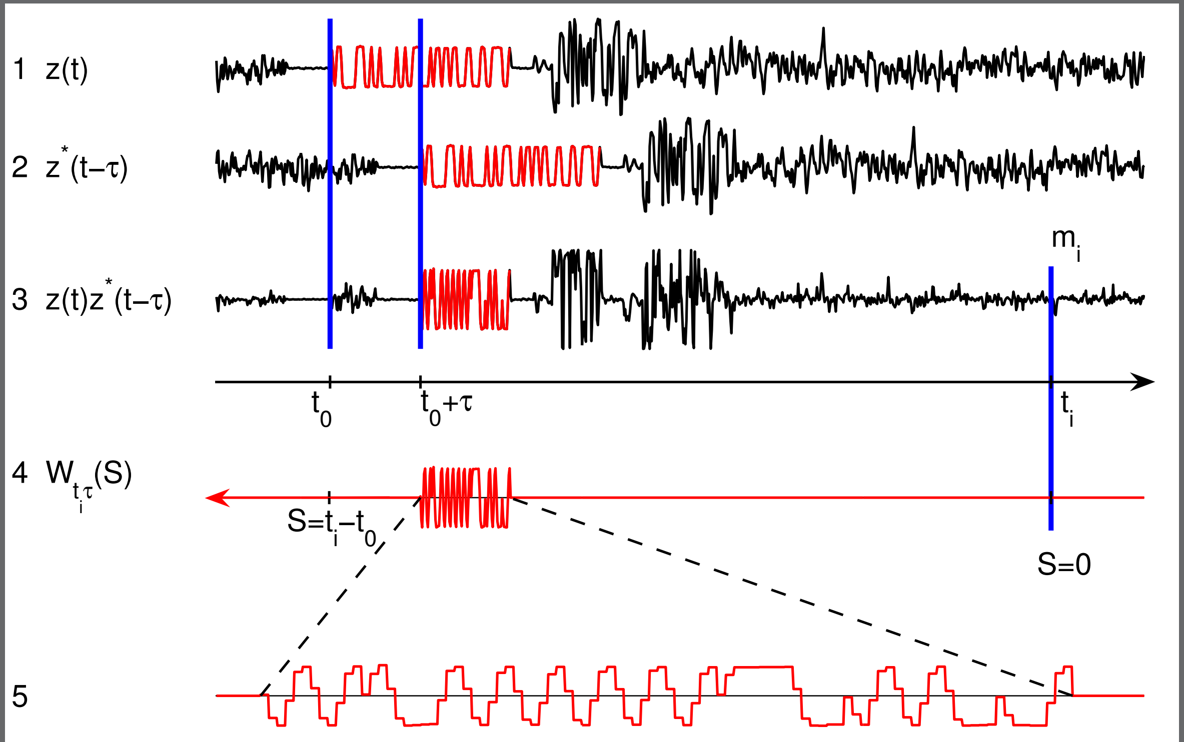
- Resolve ”lag profiles”, range profiles of $\mathcal{R}_z(\mathbf{r},\tau)$ at several different lags τ.


2017-11-14 16:08:20.000 UT real part
Height [km]
250
200
150
100
0.1
0.2
0.3
0.4
0.5
0.6
Lag [ms]
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4

DFT


Power spectrum
Height [km]
250
200
150
100
-4000
-2000
0
2000
4000
Frequency [Hz]
dB
20
15
10
5
0

Pulse compression, incoherent target



- **The target changes in time-scales shorter than the pulse length**
- **Changes in target spectrum are slower**
  - We can decode the autocorrelation function of the scattered signal instead of the signal itself!
- **Analogy to coherent targets**
  - Echo amplitude ⟺ lag profile
  - Transmitted waveform ⟺ Range ambiguity function
- **Lag profile**

$$\mathcal{R}_z(S,\tau) = \langle z(S,t)z^*(S,t-\tau)\rangle$$

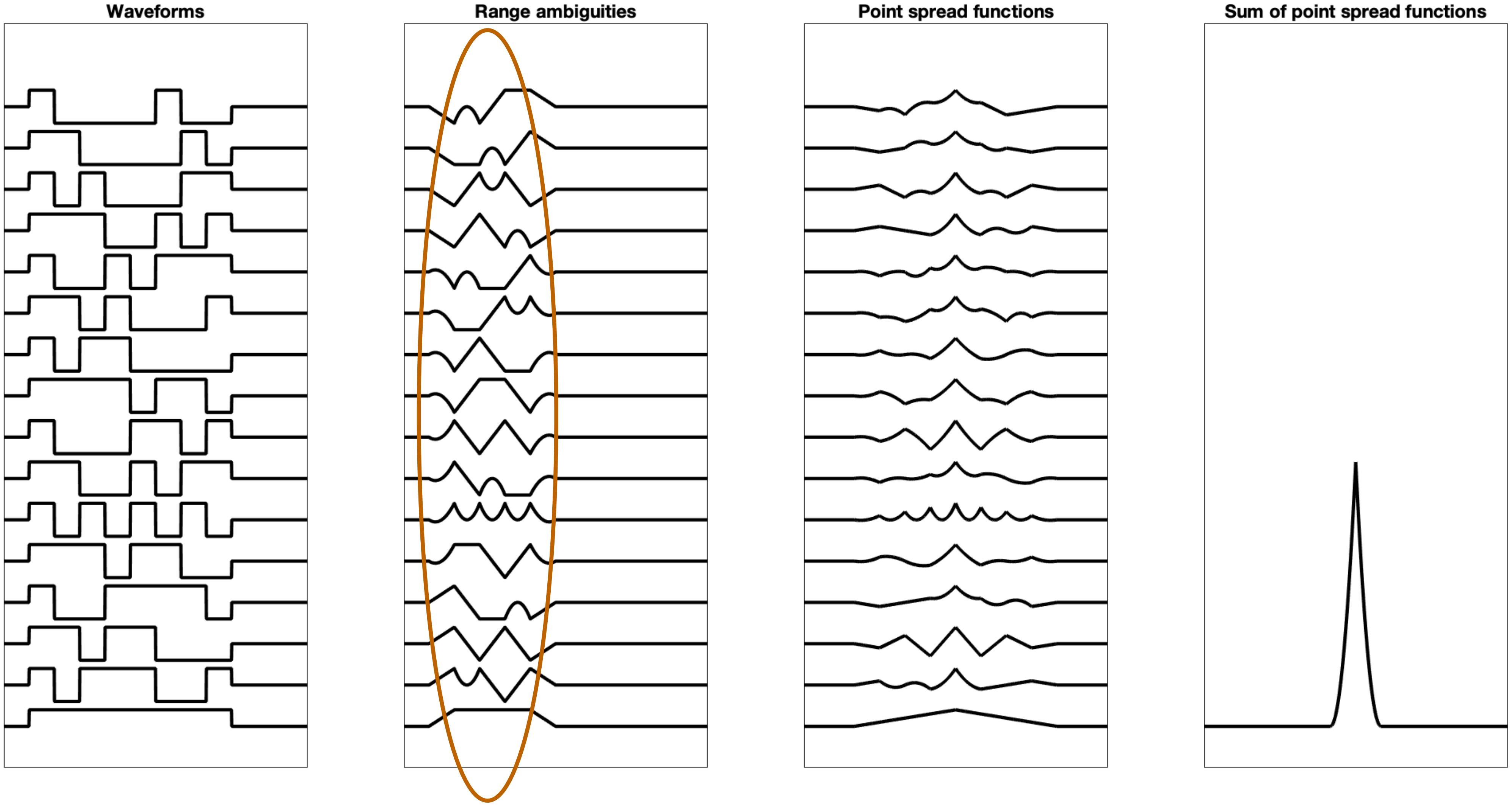
- **Range ambiguity function**

$$W(t,S,\tau) = z_e(t-S)z_e^*(t-\tau-S)$$

Example: alternating codes

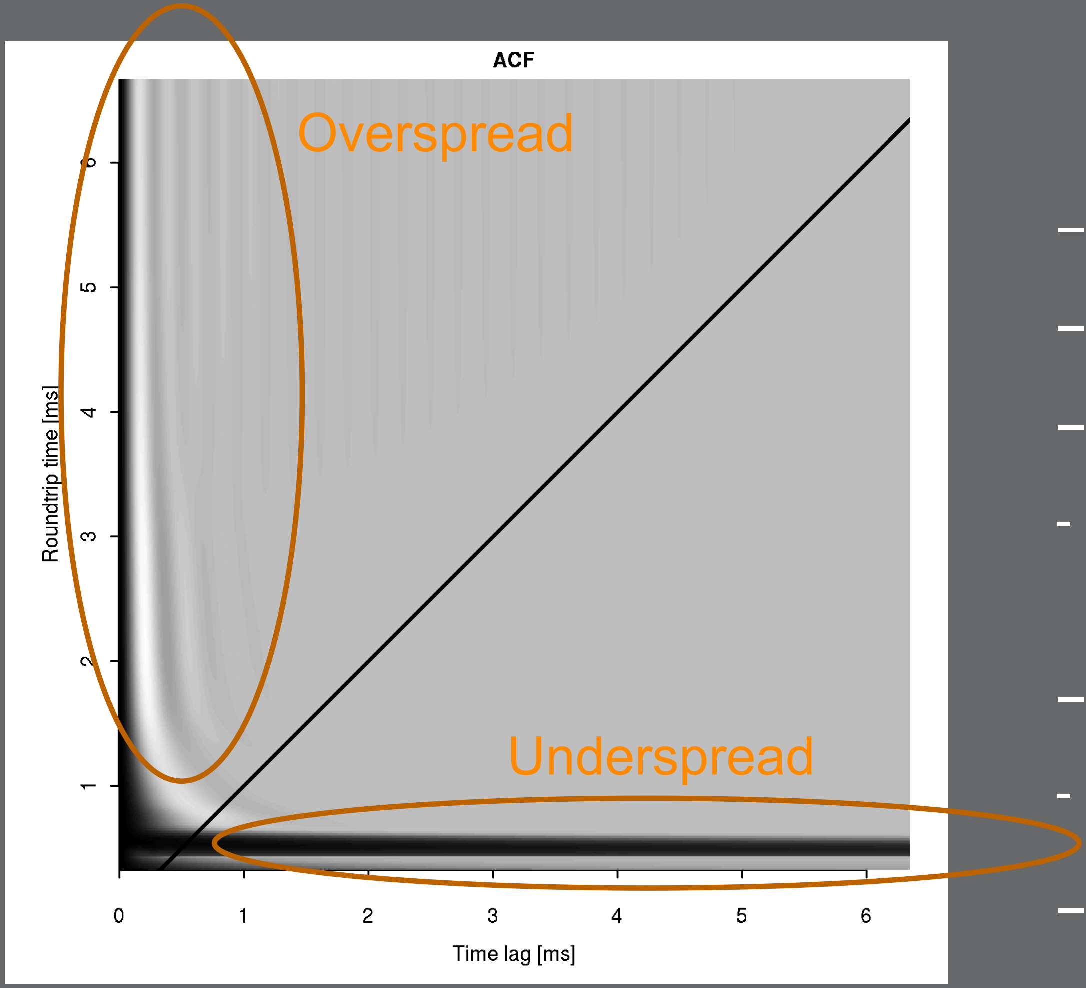
- Sequences of phase-coded pulses
- At each time-lag, the set of range ambiguity functions of the codes forms a complementary code set
- We can decode lag profiles by means of matched filtering!

Example: alternating codes



The funny shapes are caused by the receiver impulse response

Underspread vs overspread target



A normalized ISR lag profile matrix

- Signal roundtrip time $S = \frac{2R}{c}$
- Width of the target spectrum Δf
- Underspread target: $\Delta f < \frac{1}{S}$
  - The target spectrum can be estimated using echo samples from subsequent transmitted pulses (pulse-to-pulse correlations)
- Overspread target: $\Delta f > \frac{1}{S}$
  - We must collect several echo samples from each range per pulse (intra-pulse correlations)
- **The D region is underspread, while the F region is severaly overspread. ISR experiment design is not trivial!**

Thanks for your attention!