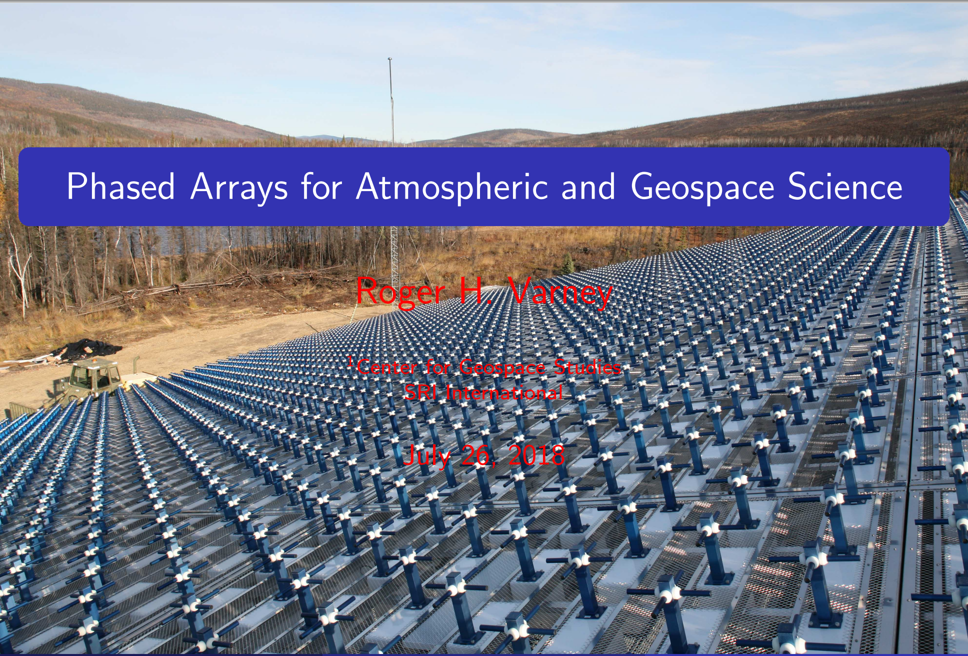

Phased Arrays for Atmospheric and Geospace Science

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Superposition Principle

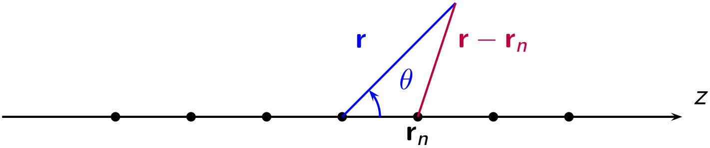
Maxwell's Equations are Linear:

$$\mathbf{J}_1 = \frac{1}{\mu_0}\nabla \times (\mathbf{B}_1) - \epsilon_0 \frac{\partial}{\partial t}(\mathbf{E}_1)$$
$$0 = \nabla \times (\mathbf{E}_1) + \frac{\partial}{\partial t}(\mathbf{B}_1)$$

$$\mathbf{J}_2 = \frac{1}{\mu_0}\nabla \times (\mathbf{B}_2) - \epsilon_0 \frac{\partial}{\partial t}(\mathbf{E}_2)$$
$$0 = \nabla \times (\mathbf{E}_2) + \frac{\partial}{\partial t}(\mathbf{B}_2)$$

$$\mathbf{J}_1 + \mathbf{J}_2 = \frac{1}{\mu_0}\nabla \times (\mathbf{B}_1 + \mathbf{B}_2) - \epsilon_0 \frac{\partial}{\partial t}(\mathbf{E}_1 + \mathbf{E}_2)$$
$$0 = \nabla \times (\mathbf{E}_1 + \mathbf{E}_2) + \frac{\partial}{\partial t}(\mathbf{B}_1 + \mathbf{B}_2)$$

Superposition Applied to Antenna Arrays



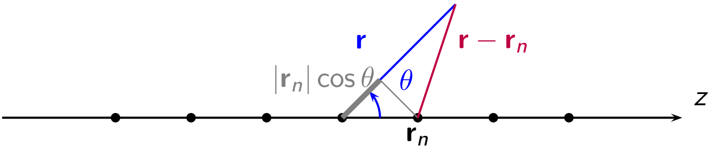
Fields radiated by single element at the origin with applied current I_0:

$$\mathbf{E} = \mathbf{E}_0 I_0 \frac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|}$$

Fields radiated by entire array:

$$\mathbf{E} = \mathbf{E}_0 \sum_{n=0}^{N-1} I_n \frac{e^{-jk|\mathbf{r}-\mathbf{r}_n|}}{|\mathbf{r}-\mathbf{r}_n|}$$

Far Field Approximation (Fraunhofer Zone)



If $\mathbf{r}$ and $\mathbf{r} - \mathbf{r}_n$ are almost parallel lines:

$$\mathbf{r} - \mathbf{r}_n \approx \mathbf{r} - |\mathbf{r}_n| \cos\theta \hat{r}$$

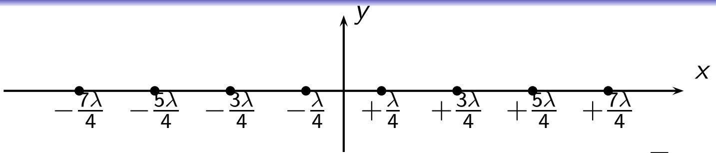
Assume $|\mathbf{r}_n| \ll |\mathbf{r}|$:

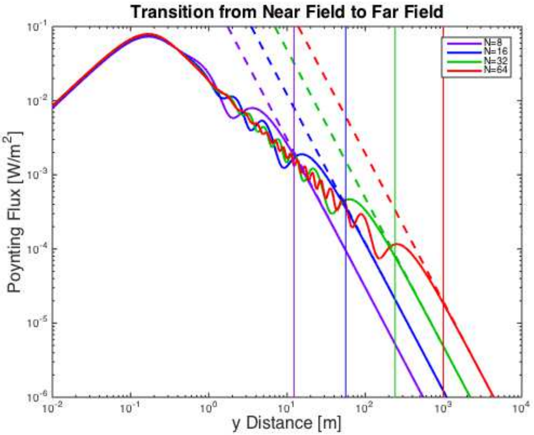
$$|\mathbf{r} - \mathbf{r}_n| \approx |\mathbf{r}| \text{ for demoninator terms}$$

$$-jk|\mathbf{r} - \mathbf{r}_n| \approx -jk|\mathbf{r}| + jk|\mathbf{r}_n| \cos\theta$$

$$\mathbf{E} \approx \underbrace{\mathbf{E}_0 \frac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|}}_{\text{Element Factor}} \underbrace{\sum_{n=0}^{N-1} I_n e^{jk|\mathbf{r}_n| \cos\theta}}_{\text{Array Factor}}$$

Distance to Far Field: Fresnel Numbers





Transition from near to far determined by the **Fresnel Number**:

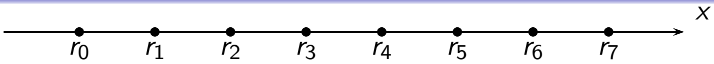
$$\frac{L^2}{r\lambda} \ll 1 \rightarrow \text{Far Field}$$

$$\frac{L^2}{r\lambda} > 1 \rightarrow \text{Near Field}$$

L = Array length

λ = wavelength

1-D Linear Phased Array



$$|\mathbf{r}_n| = nd \qquad I_n = e^{jn\alpha}$$

Array Factor:

$$F = \sum_{n=0}^{N-1} e^{jn\alpha} e^{jknd\cos\theta}$$

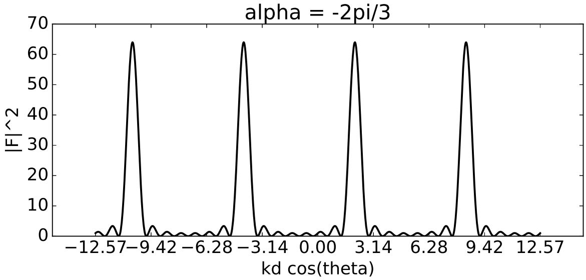
$$= \frac{1 - e^{jN\alpha + jNkd\cos\theta}}{1 - e^{j\alpha + jkd\cos\theta}}$$

$$= e^{j\frac{(N-1)}{2}(kd\cos\theta + \alpha)} \frac{\sin\left[\frac{N}{2}(kd\cos\theta + \alpha)\right]}{\sin\left[\frac{1}{2}(kd\cos\theta + \alpha)\right]}$$

$$|F|^2 = \frac{\sin^2\left[\frac{N}{2}(kd\cos\theta + \alpha)\right]}{\sin^2\left[\frac{1}{2}(kd\cos\theta + \alpha)\right]}$$

1-D Linear Phased Array Cont.

$$|F|^2 = \frac{\sin^2\left[\frac{N}{2}\left(kd\cos\theta + \alpha\right)\right]}{\sin^2\left[\frac{1}{2}\left(kd\cos\theta + \alpha\right)\right]}$$



Peak appears when $kd\cos\theta = -\alpha$

Additional peaks could appear when $kd\cos\theta = -\alpha + 2\pi m$ (Grating Lobes)

Visible Region: $0 < \theta < \pi \rightarrow -kd < kd\cos\theta < kd$

Grating Lobes

- $d < \lambda/2 \rightarrow kd < \pi$: No grating lobes will ever appear
- $\lambda/2 < kd < \lambda \rightarrow \pi < kd < 2\pi$: Grating lobes will only appear at some steering angles
- $d > \lambda \rightarrow kd > 2\pi$: Grating lobes will always appear

Example of linear array with $d = 2\lambda/3$ spacing

Movie

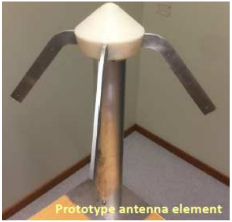
Mutual Coupling

- The true element factor for antennas in an array is different from the same type of antennas in isolation
  - Scattering off of neighboring antennas
  - Inductive coupling involving antenna near-fields

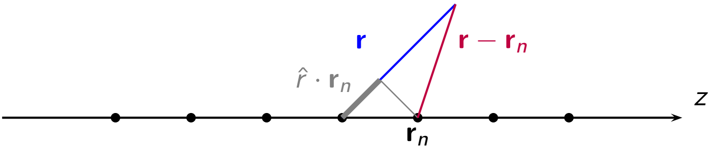
Two possible solutions

- Use a larger antenna separation and live with a limited grating-lobe free steering range.
- Use specially designed antennas to minimize coupling

EISCAT_3D Prototype Drooped Dipole →



Multi-Dimensional Arrays



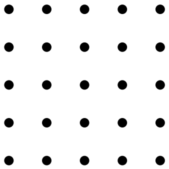
$$-jk\,|\mathbf{r}-\mathbf{r}_n| \approx -jk\,|\mathbf{r}| + jk\,(\hat{r}\cdot\mathbf{r}_n)$$

$$\mathbf{E} \approx \underbrace{\mathbf{E}_0\frac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|}}_{\text{Element Factor}} \underbrace{\sum_{n=0}^{N-1} I_n e^{jk(\hat{r}\cdot\mathbf{r}_n)}}_{\text{Array Factor}}$$

In spherical coordinates:

$$\hat{r}\cdot\mathbf{r}_n = x_n\cos\phi\sin\theta + y_n\sin\phi\sin\theta + z_n\cos\theta$$

2-D Rectangular Array



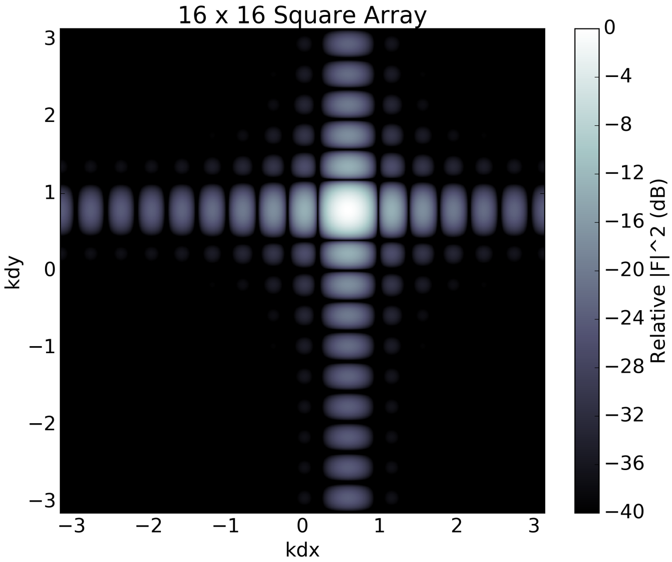
$$\mathbf{r}_{nm} = nd_x\hat{\mathbf{x}} + md_y\hat{\mathbf{y}} \qquad I_{nm} = e^{j(n\alpha+m\beta)}$$

Array Factor:

$$|F(\theta,\phi)|^2 = \left| \sum_{n=0}^{N_x-1} \sum_{m=0}^{N_y-1} e^{j(nkd_x \cos\phi \sin\theta + n\alpha + mkd_y \sin\phi \sin\theta + m\beta)} \right|^2$$

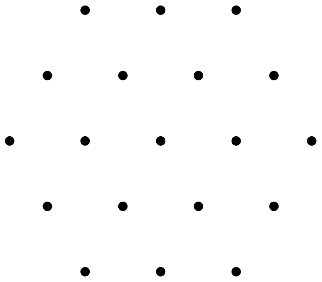
$$= \frac{\sin^2\left[\frac{N_x}{2}\left(kd_x \cos\phi \sin\theta + \alpha\right)\right]}{\sin^2\left[\frac{1}{2}\left(kd_x \cos\phi \sin\theta + \alpha\right)\right]} \frac{\sin^2\left[\frac{N_y}{2}\left(kd_y \sin\phi \sin\theta + \beta\right)\right]}{\sin^2\left[\frac{1}{2}\left(kd_y \sin\phi \sin\theta + \beta\right)\right]}$$

2-D Rectangular Array



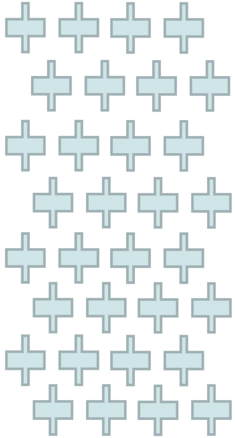
Hexagonal Spacing

Hexagon



Honeycomb Rectangular Array

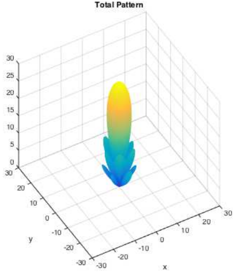
One AMISR panel:



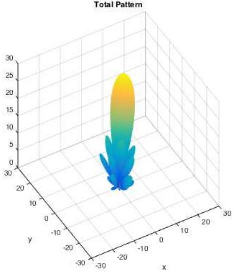
Steering A Hexagonal Array

19-antenna hexagon with $d = 3\lambda/4$.

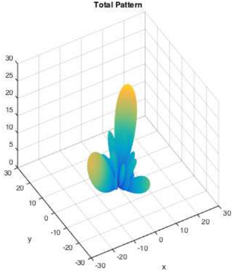
$\theta = 0, \phi = 0$



$\theta = 20^\circ, \phi = 45^\circ$

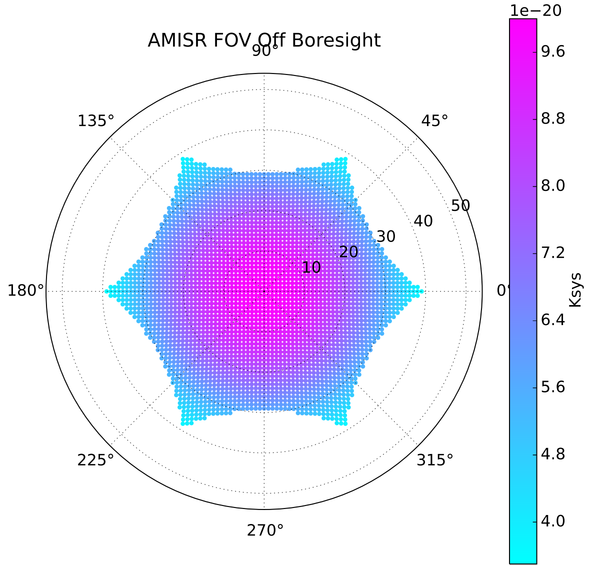


$\theta = 45^\circ, \phi = 45^\circ$



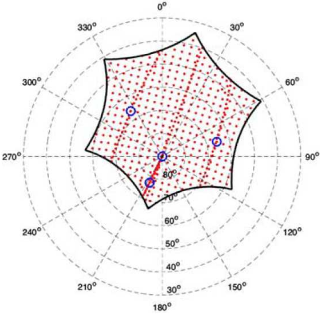
AMISR Antenna Properties

- FOV limited by grating lobe limit $\sim 30^\circ - 40^\circ$
- Antenna gain decreases with steering angle off of boresight
- Antenna works best within $\sim 25^\circ$ off of boresight



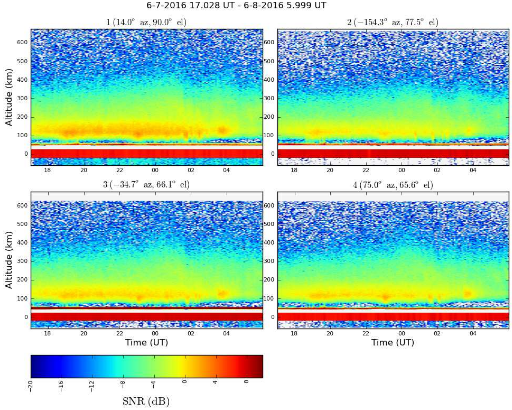
The PFISR Up-B Compromise

IPY Beam Pattern



The Up-B beam is close to the grating lobe limit, and therefore has reduced sensitivity.

Reduced SNR in Up-B (Beam 2)



Passive Phased Arrays: Jicamarca

- One transmitter feeds entire array

Passive Phased Arrays: Jicamarca

Manual Phasing (Jicamarca)

Active Electronically Steerable Phased Arrays

The AMISR UHF System

Antenna Element Unit (AEU)

AMISR AEU = Tx/Rx Unit
- *500 W solid state transmitter*
- *Phasing control*
- *Status monitoring*
- *4096 AEUs/AMISR radar face*

Face

32/panel

AMISR Panel
- *32 Antenna Element Units arranged in hexagonal pattern*
- *3.5 x 2 meters; 19.8 dBi / panel*
- *16 kW peak power per panel*
- *Basic system building block for AMISR*
- *Embedded linux controller*

Panel (with PCU)

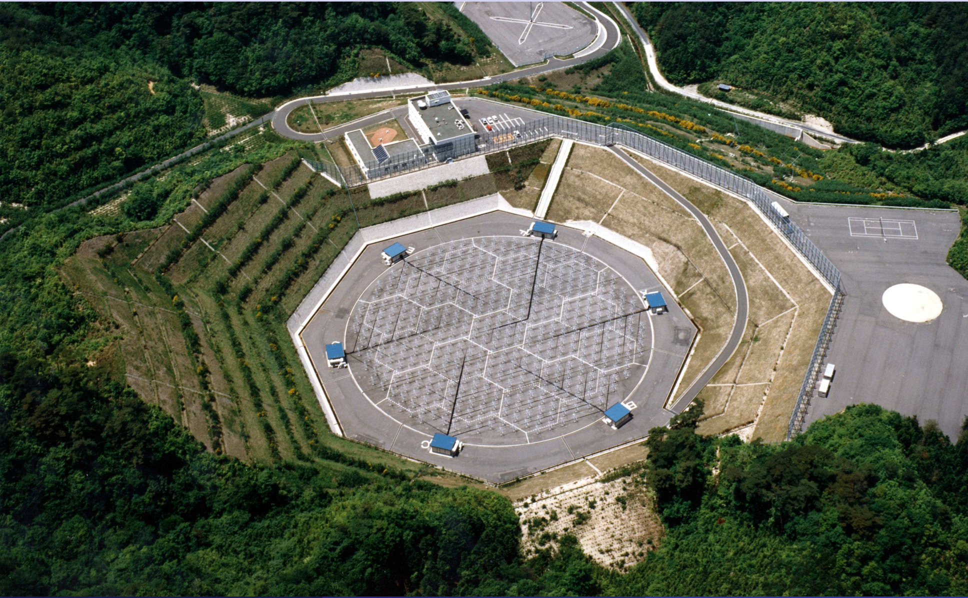
Utility Distribution Unit (UDU)

AMISR Control System (ACS)

AMISR UDU
- *400 Hz JetPower converters*
- *Remote power control units*
- *Fiber distribution system*

AMISR ACS
- *Flexible transmit and receive system*
- *Completely remotely controlled*
- *Experiments run off a scheduler*

MU Radar



MU Radar

MU Radar Cabling

MU Radar Cabling

MU Radar Power Amplifiers

Advanced Modular Incoherent Scatter Radar

Face

DAQ

Group

UDU

- 400 Hz JetPower converters
- Remote power control units
- Fiber distribution system

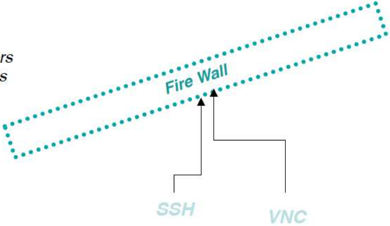
Panel

PC104
PCU

- 32 Antenna Element Units
- Embedded linux controller
- Basic system building block

AEU

- 500 W solid state transmitter
- Phasing control
- Status monitoring

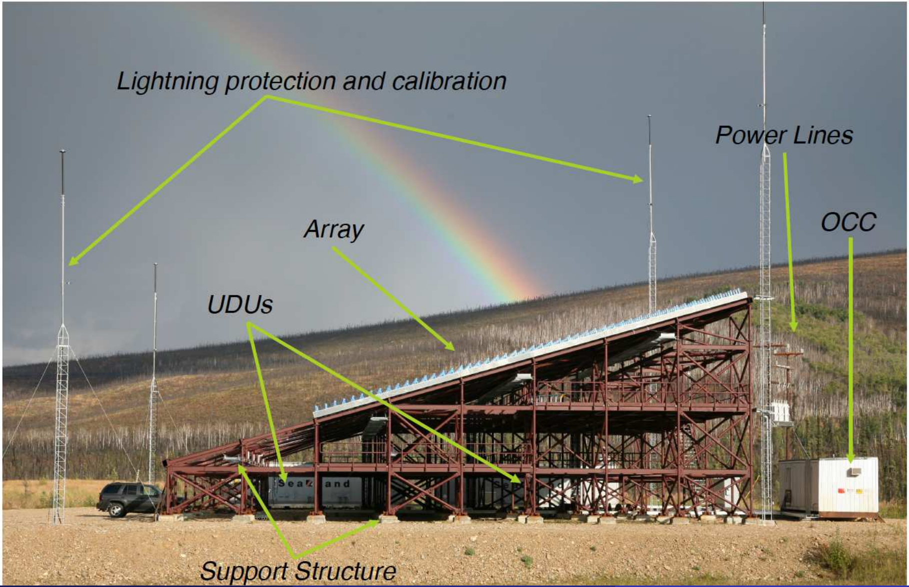


Antenna Element Unit (AEU) Specifications

- Distributed Solid State Power Amplifiers (SSPAs)
- 430-450 MHz instantaneous bandwidth
- 10% Maximum duty cycle
- Minimum PRF interval 500 usec
- Maximum pulsewidth 2 msec
- Passive cooling (no moving parts
- 400 Hz prime power

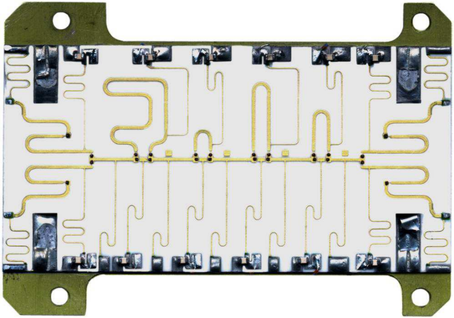
- Crossed dipoles, circular polarization on axis
- Balun built into the antenna support shaft
- Constant impendence over bandwidth and scan angle
- Spacing is hexagonal for efficiency
- Tx/Rx polarizations are opposite and fixed (not measureable)

Poker Flat Incoherent Scatter Radar (PFISR)



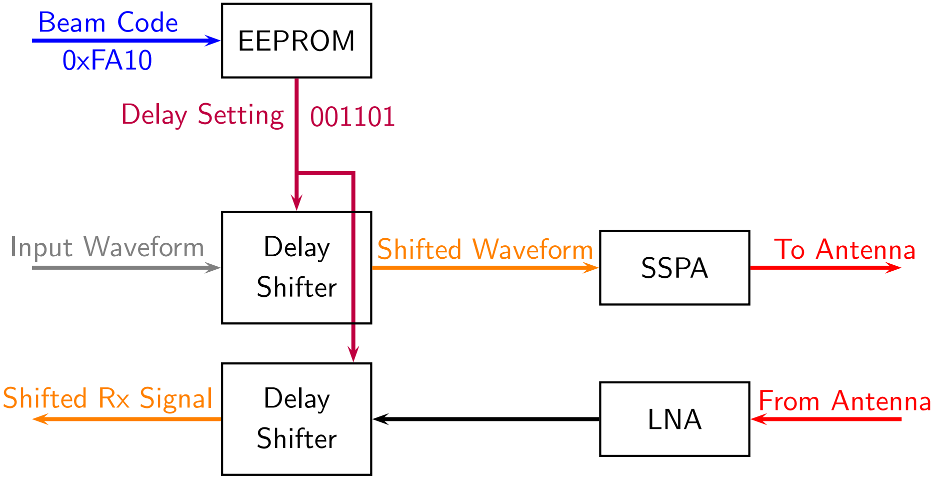
Electronic Steering with Delay Shifters

Example 4-bit delay shifter:

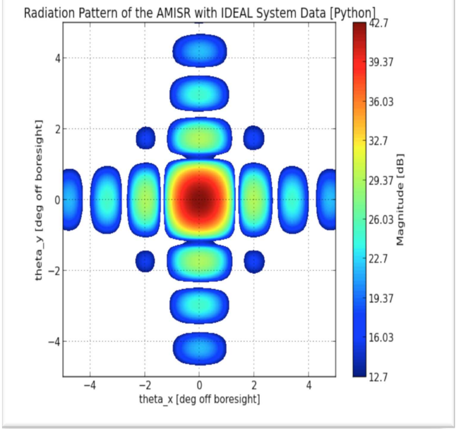


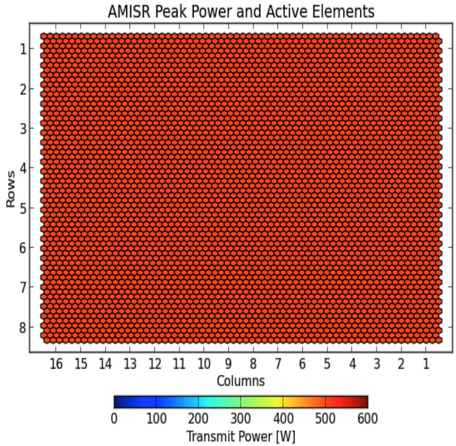
- AMISR uses 6-bit delay shifters
- $2^6 = 64$ steps spaced by $\pi/32 = 5.625^\circ$

Conceptual Diagram of Steering with AMISR

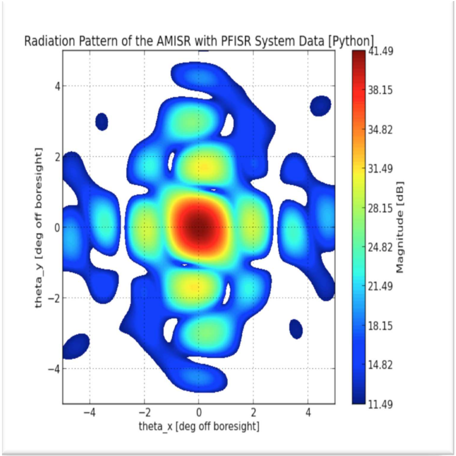


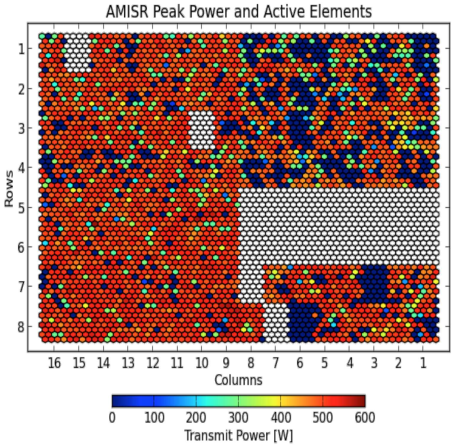
Ideal AMISR Radiation Pattern



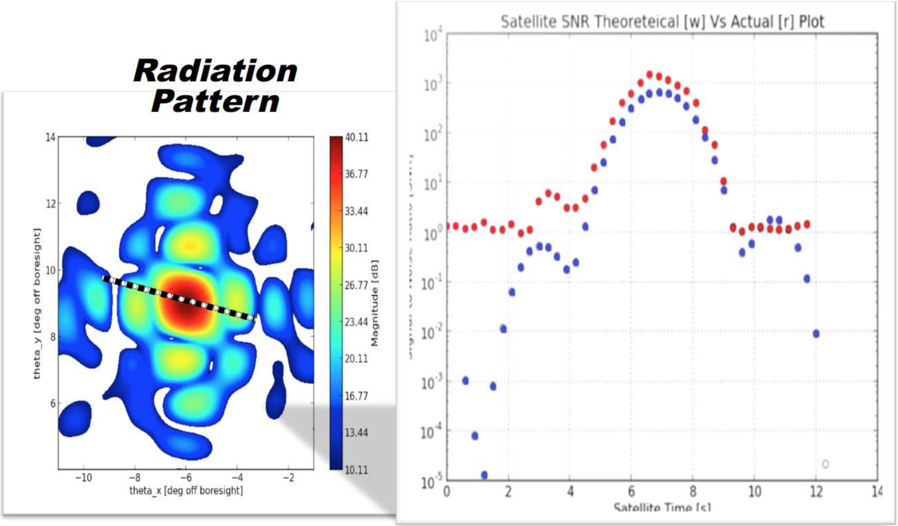


AMISR Graceful Degradation

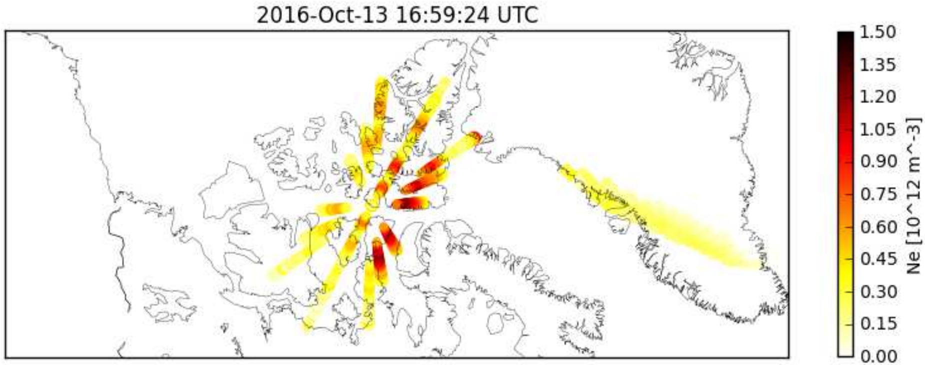




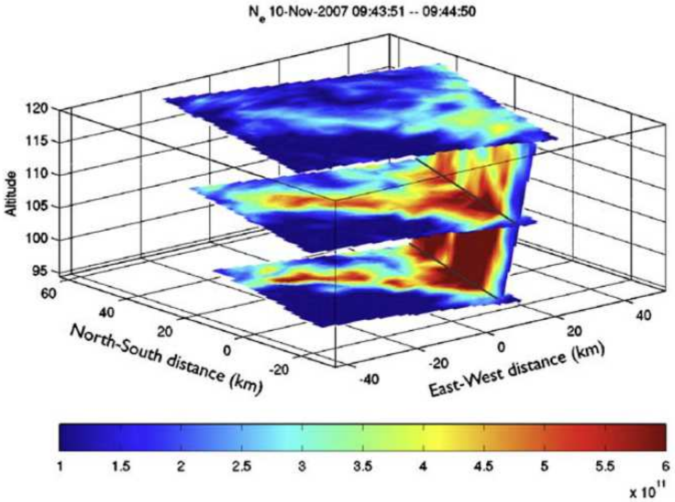
AMISR Beamwidth During Satellite Pass



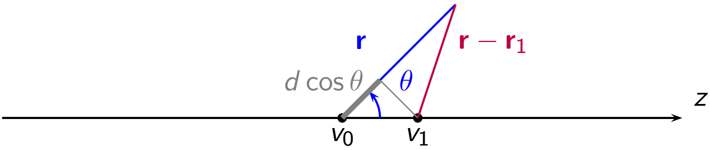
Differences Between AMISR and Scanning Radars



Imaging Auroral Structure [Semeter et al. (2009)]



Two Antenna Inteferometry

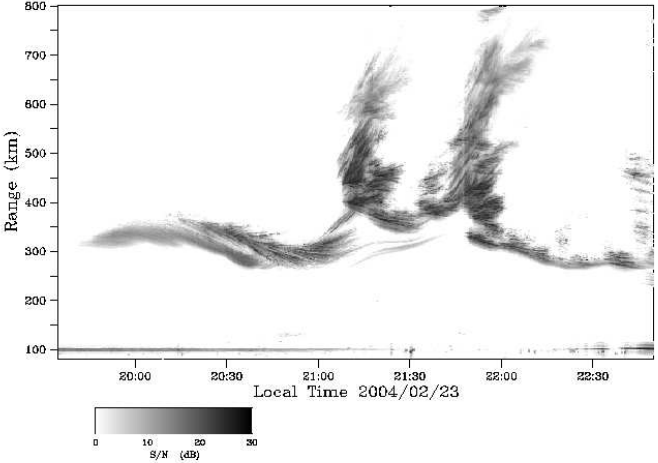


$$\langle v_0 v_1^* \rangle = e^{jkd \cos\theta}$$

- Measure v_0 and v_1 separately and estimate the angle of arrival θ.
- Larger baseline $\rightarrow$ more precise angle estimates.
- Baselines where $kd > \pi$ suffer from 2π ambiguity issues. Related to grating lobe problem.

Interferometric Imaging Configuration at Jicamarca

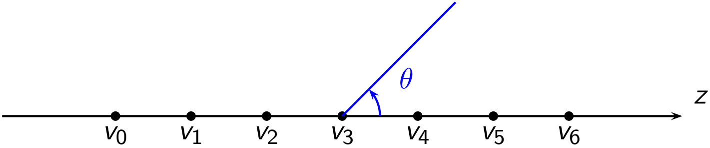
Interferometric Images of Coherent Scatter



Movie

http://landau.geo.cornell.edu/image.html

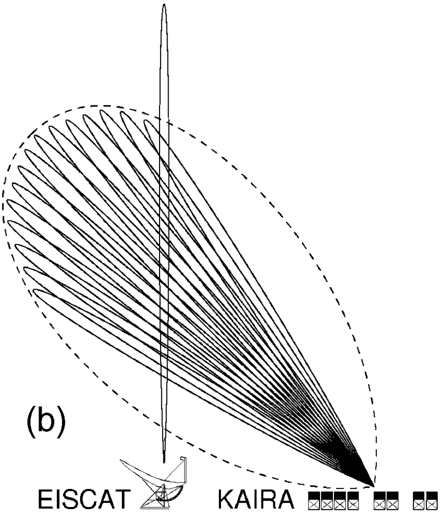
Digital Beam Forming



On reception:

- Digitize the signals v_n on every antenna (expensive!)
- Synthesize any beams you want by forming different linear combinations in software/firmware (computationally intense!)
- Allows you to form custom beam patterns → look at signals of interest while nulling interference.
- Allows you to form any number of different radiation patterns → look in multiple directions at once.

Digital Beam Forming in Multi-static Radar Experiments

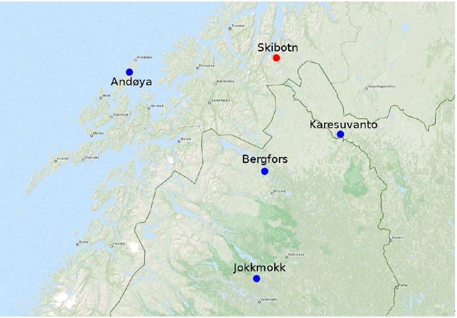


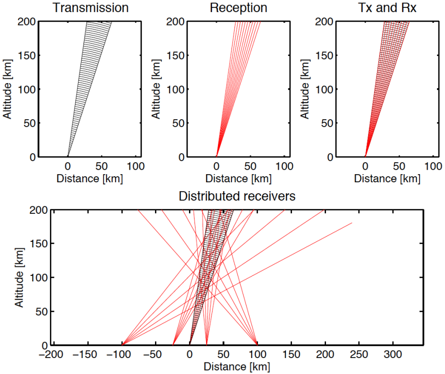
KAIRA = The Kilpisjärvi Atmospheric Imaging Receiver Array

McKay et al. (2015)
10.1109/TGRS.2014.2342252

EISCAT_3D

Major planned facility:





Operational 2021?