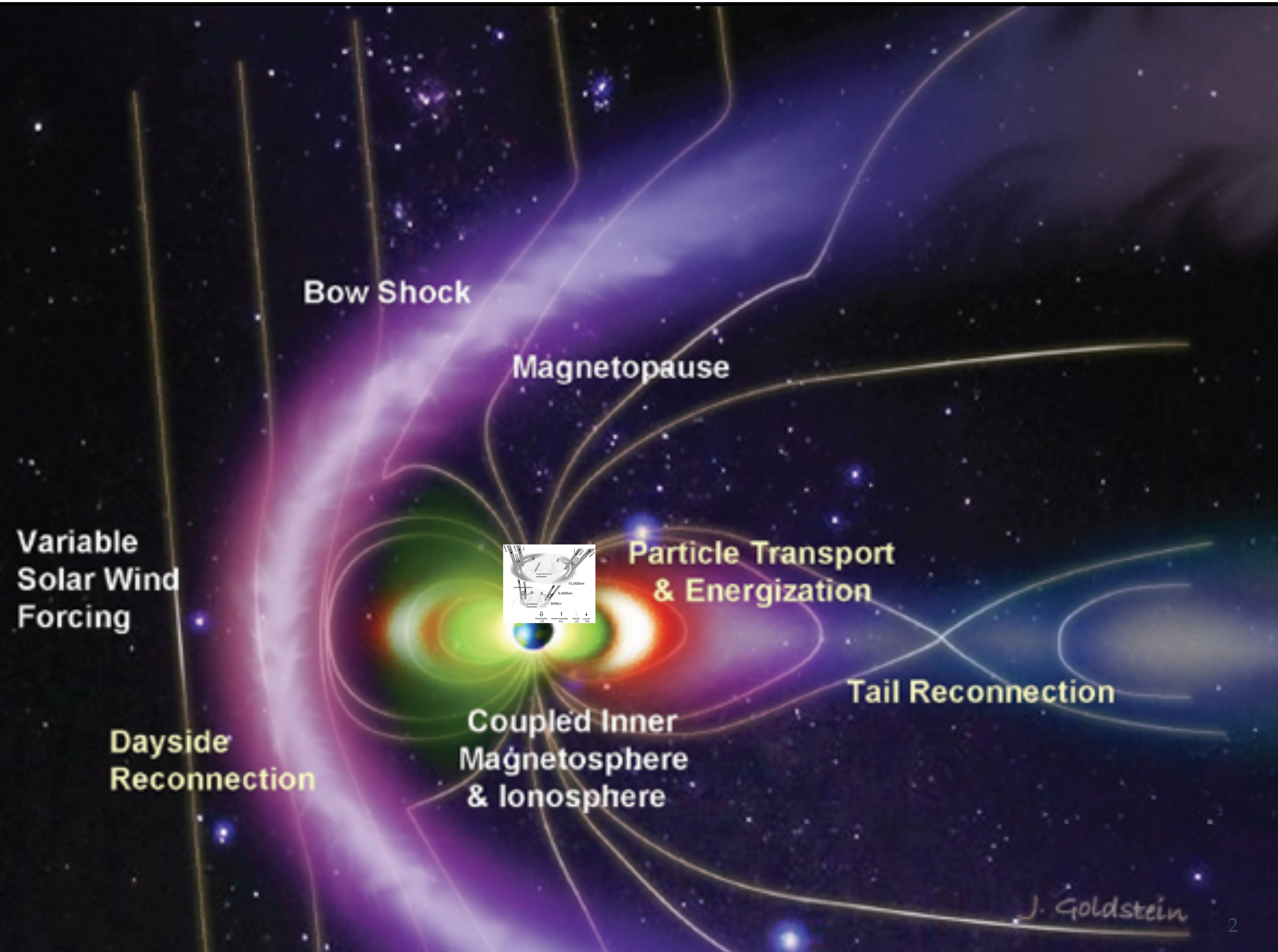
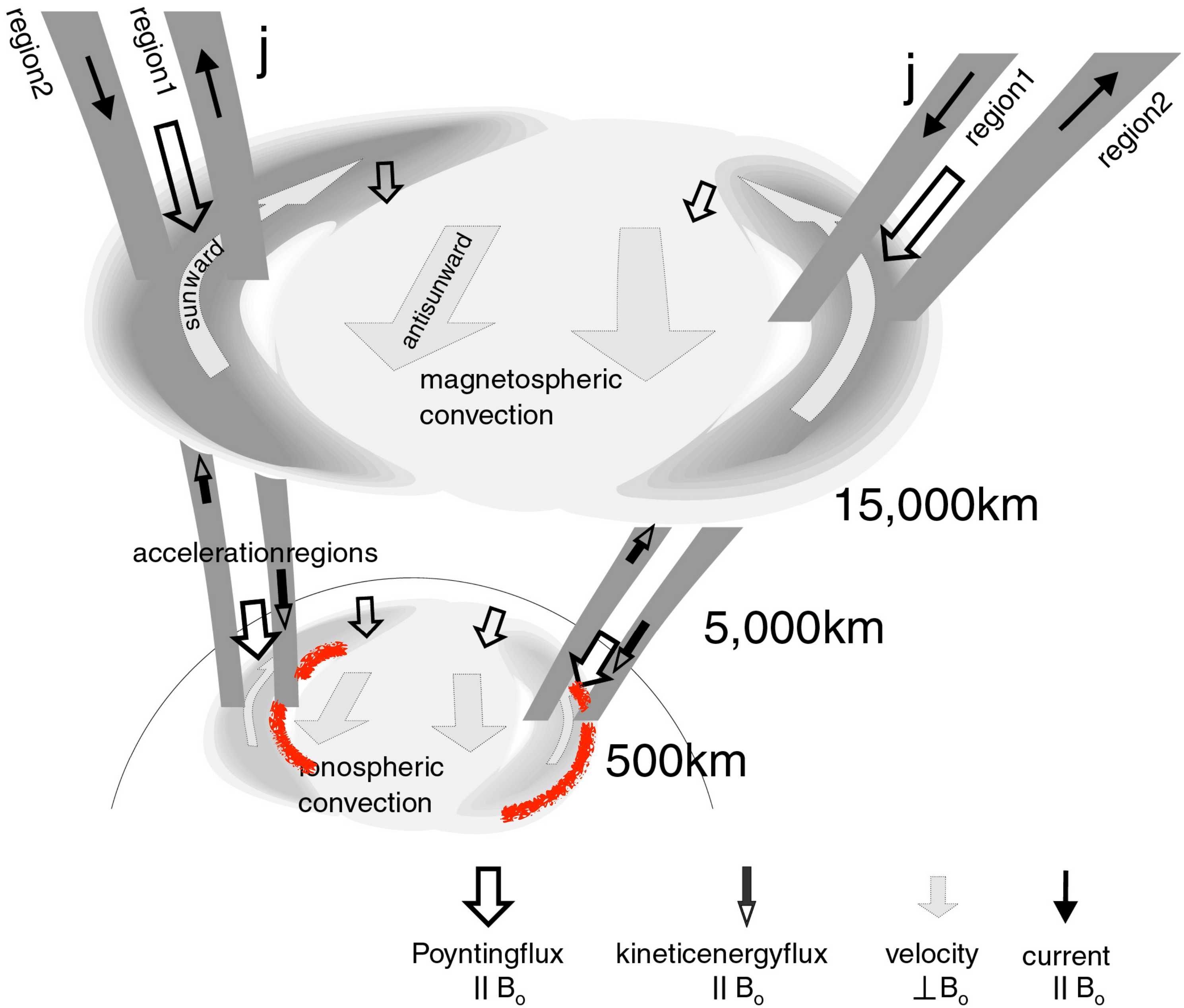
Introduction to ISR Signal Processing

Joshua Semeter, Boston University

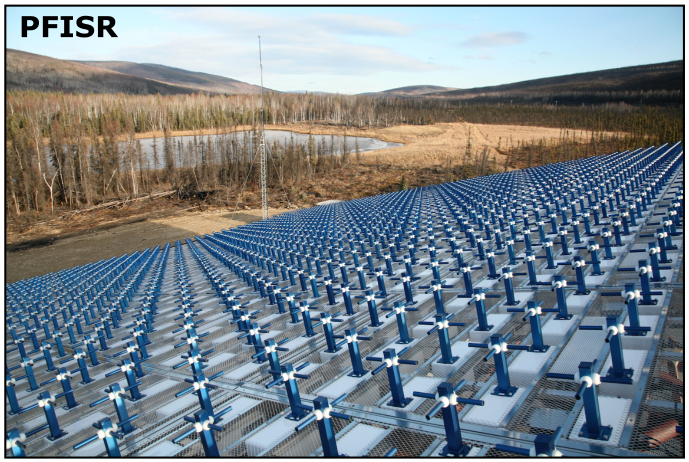

Bow Shock
Magnetopause
Variable
Solar Wind
Forcing
Particle Transport
& Energization
Tail Reconnection
Dayside
Reconnection
Coupled Inner
Magnetosphere
& Ionosphere
J. Goldstein

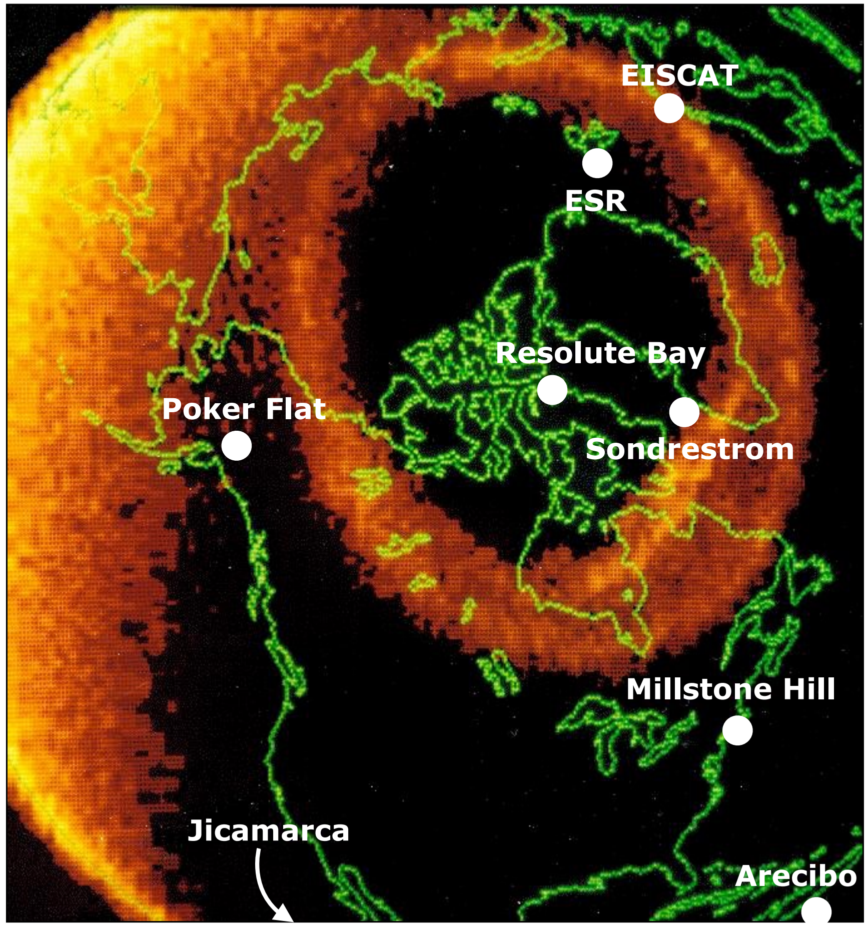
Ionosphere as a projection of the magnetosphere

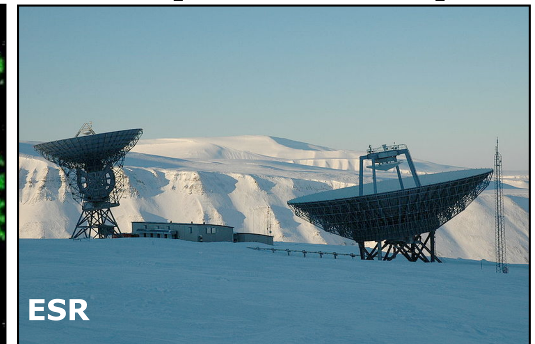


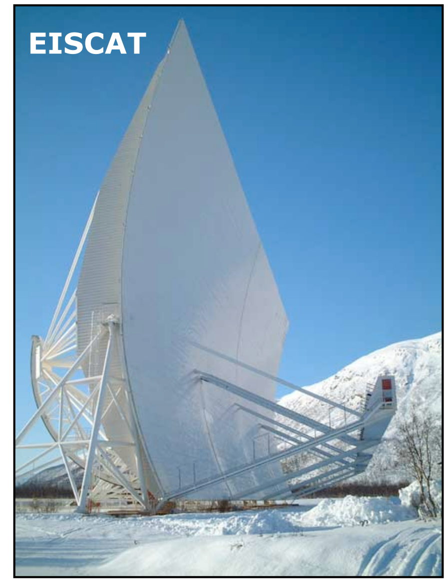
Vaivads, PhD dissertation

Incoherent Scatter Radar (ISR)









Why study ISR?

Requires that you learn about a great many useful and fascinating subjects in substantial depth.

- Plasma physics
- Radar
- Coding (information theory)
- Electronics (Power, RF, DSP)
- Signal Processing
- Inverse theory

Outline

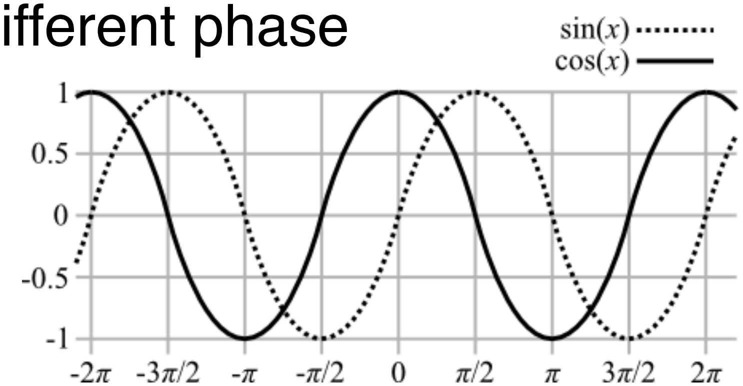
- Mathematical toolbox
- Review of basic radar concepts
- Ionospheric Doppler spectrum
- Range resolution and matched filtering
- I/Q demodulation
- Measuring the autocorrelation function (ACF) and Power Spectral Density (PSD)

Signal Model

sine and cosine are the same function, different phase

$$\cos(\theta) = \sin(\theta + \pi/2)$$

$\pi/2 = 90^\circ$



Euler's identity consolidates these into a single function

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

Make signal oscillate in time: $\theta = \omega t = 2\pi f t$

Add information via amplitude modulation (A.M) or frequency modulation (F.M)

We now have a generic mathematical model of a radio or radar signal.

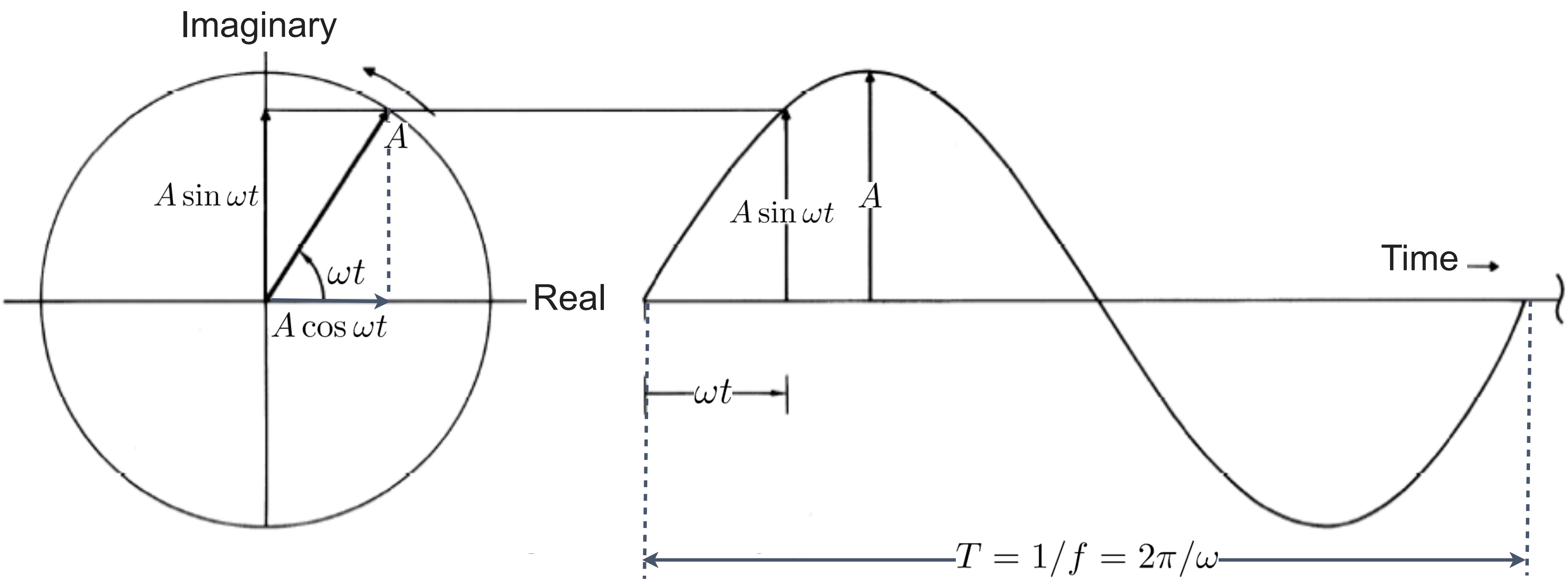
$$s(t) = A(t)e^{j(\omega_o t + \phi(t))}$$

A.M. → $A(t)$; Carrier → $\omega_o t$; F.M. → $\phi(t)$

Or letting $\omega_d = d\phi/dt \rightarrow \phi(t) = \omega_d t$

$$s(t) = A(t)e^{j(\omega_o + \omega_d)t}$$

Complex Exponential Function



ω is the “angular velocity” (radians/s) of the spinning arrow

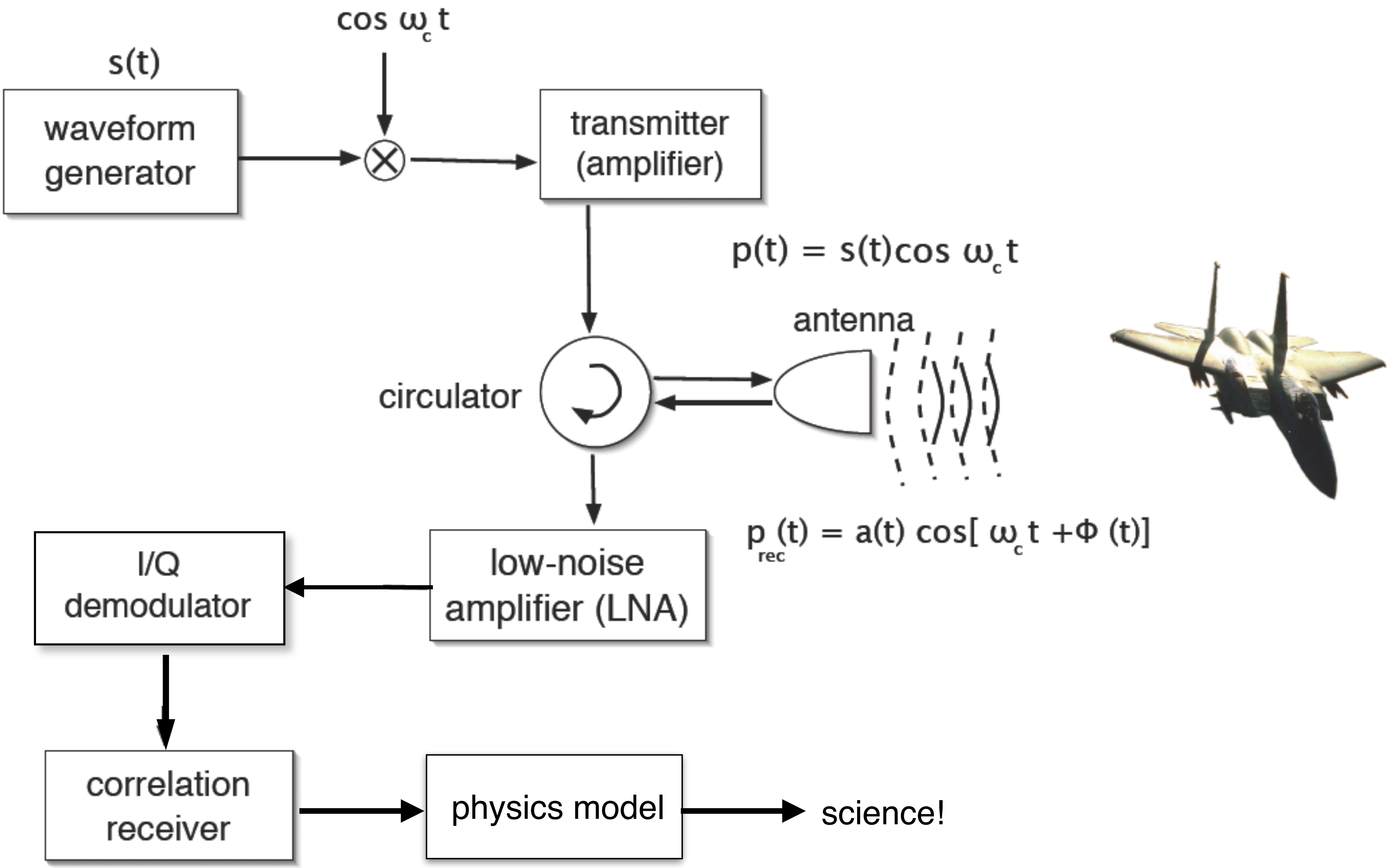
f is the number of complete rotations (2π radians) in one second (1/s or Hz)

We need a signal that tells us **how fast** and in **which direction** the arrow is spinning. This signal is the complex exponential. Invoking the Euler identity,

$$s(t) = Ae^{j\omega t} = A\cos\omega t + jA\sin\omega t = I + jQ$$

I = in-phase component

Q = in-quadrature component



cos ωc t
s(t)
waveform generator
transmitter (amplifier)
p(t) = s(t)cos ωc t
antenna
circulator
prec(t) = a(t) cos[ωc t +Φ (t)]
I/Q demodulator
low-noise amplifier (LNA)
correlation receiver
physics model
science!

Essential mathematical operations

Fourier Transform: Expresses a function as a weighted sum of harmonic functions (i.e., complex exponentials)

$$f(t) = \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega \quad \Longleftrightarrow \quad F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

Convolution: Expresses the action of a linear, time-invariant system on a function.

$$f(t) * g(t) = \int_{-\infty}^{+\infty} f(\tau) g(\tau - t) d\tau \qquad f(t) * g(t) \Longleftrightarrow F(\omega) G(\omega)$$

Correlation: A measure of the degree to which two functions look alike at a given offset.

$$f(t) \circ g(t) = \int_{-\infty}^{+\infty} f^*(\tau) g(t + \tau) d\tau \qquad f(t) \circ g(t) \Longleftrightarrow F^*(\omega) G(\omega)$$

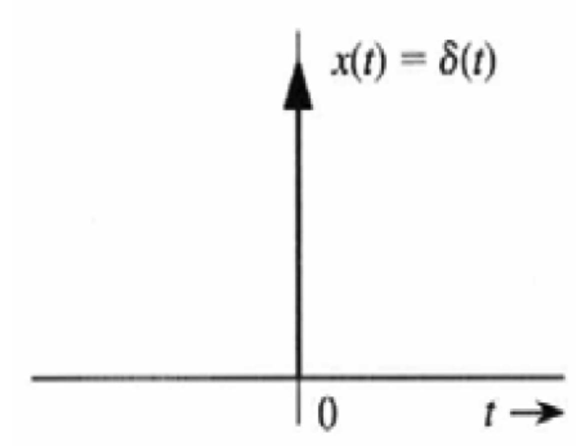
Autocorrelation, Convolution, Power Spectral Density, Wiener-Khinchin Theorem

$$R_{uu} = u(t) \circ u(t) = u(t) * u^*(-t) \qquad R_{uu} \Longleftrightarrow |U(f)|^2$$

Dirac Delta Function

A generalized function, or distribution, with the properties

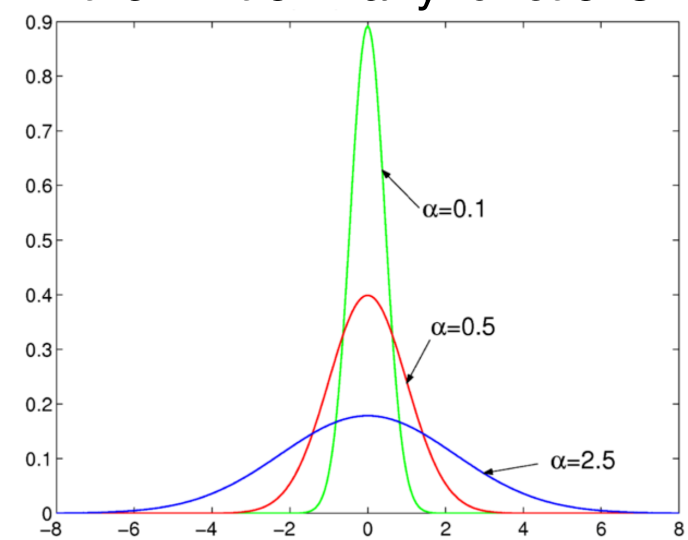
$$\delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases} \qquad \int_{-\infty}^{+\infty} \delta(x)dx = 1$$



From these properties it follows that

$$f(t_0) = \int_{-\infty}^{+\infty} f(t)\underbrace{\delta(t - t_0)}_{\text{nonzero only at } t = t_0}dt \qquad \text{(sampling property)}$$

$\delta(t)$ may be expressed as the limit of many functions



$$\delta(t) = \lim_{a \to 0} \frac{1}{\sqrt{4\pi a}} e^{-t^2/(4a)}$$

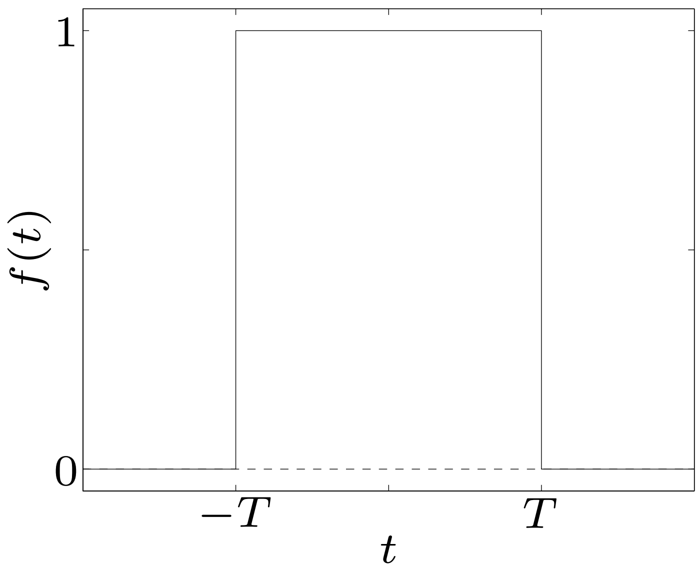
$$F(\omega - \omega_0) = \int_{-\infty}^{+\infty} F(\Omega)\delta(\omega - \omega_0 - \Omega)d\Omega$$

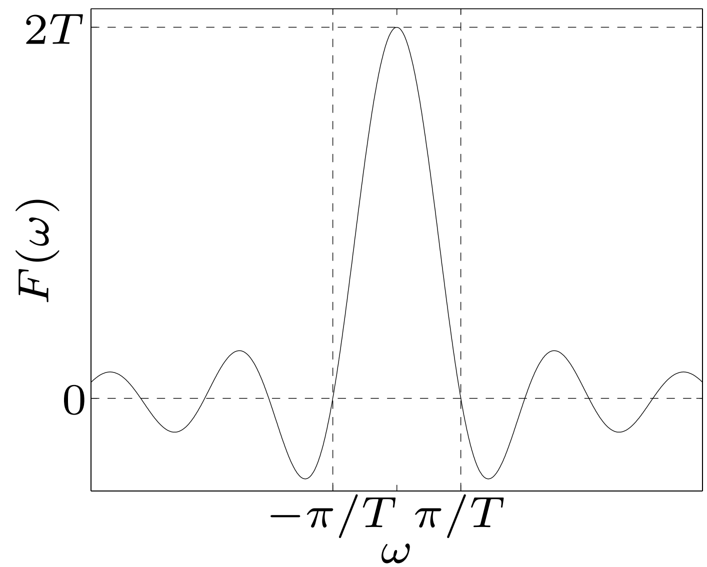
$$= f(\omega) * \delta(\omega - \omega_0) \qquad \text{(shift property)}$$

Fourier transform of two pulses

rectangular pulse: $f(t) = \begin{cases} 1 & -T \le t \le T \\ 0 & |t| > T \end{cases}$

$$F(\omega) = \int_{-T}^{T} e^{-j\omega t}\, dt = \frac{-1}{j\omega}\left(e^{-j\omega T} - e^{j\omega T}\right) = \frac{2\sin\omega T}{\omega}$$





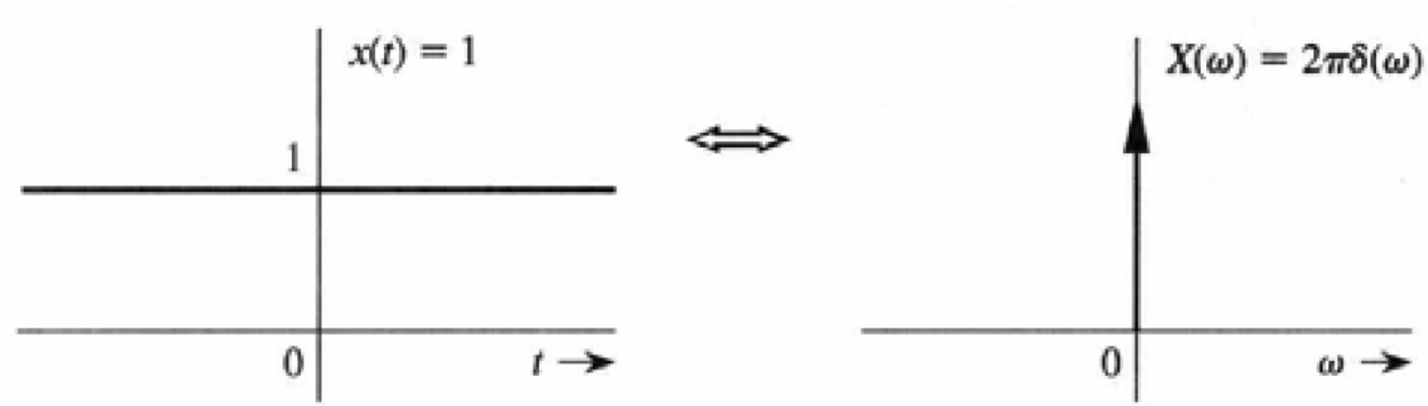
unit impulse: $f(t) = \delta(t)$

$$F(\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t}\, dt = 1$$

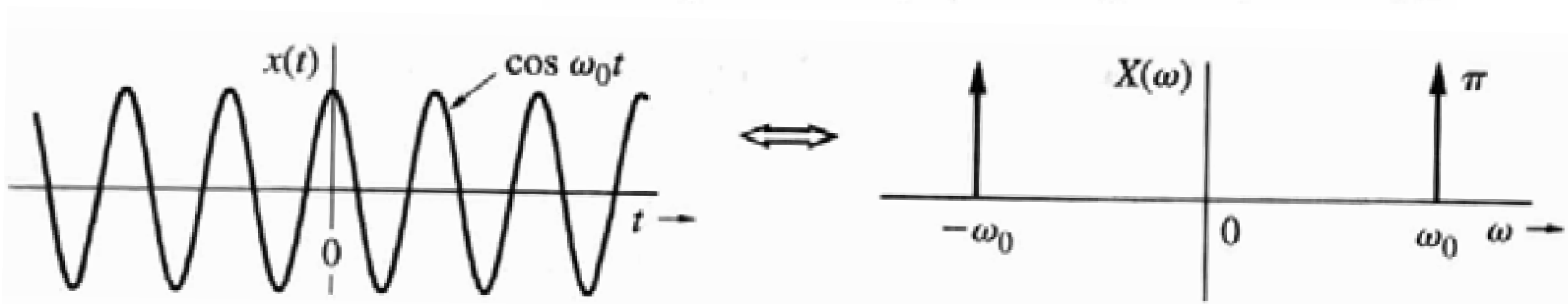
shifted: $f(t) = \delta(t - t_o)$

$F(\omega) =$

Harmonic Functions



$$\cos \omega_0 t \Longleftrightarrow \pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$$



$$\sin \omega_0 t \quad \Longleftrightarrow \quad j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$$

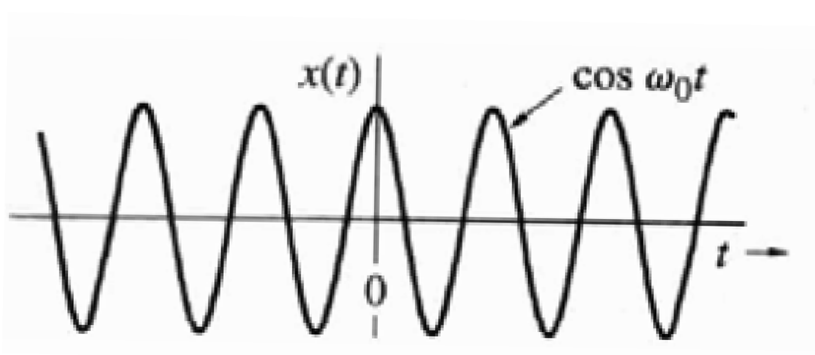
$$e^{j\omega_0 t} \quad \Longleftrightarrow \quad 2\pi\delta(\omega-\omega_0)$$

Convolution

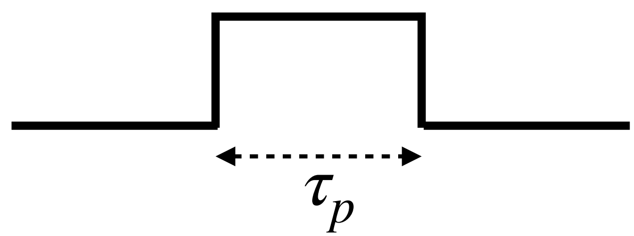
Convolution: Expresses the action of a linear, time-invariant system on a function.

$$f(t) * g(t) = \int_{-\infty}^{+\infty} f(\tau)g(t-\tau)d\tau \iff F(\omega)G(\omega)$$

$$F(\omega) * G(\omega) = \int_{-\infty}^{+\infty} F(\omega)G(\omega-\Omega)d\Omega \iff f(t)g(t)$$



X



?

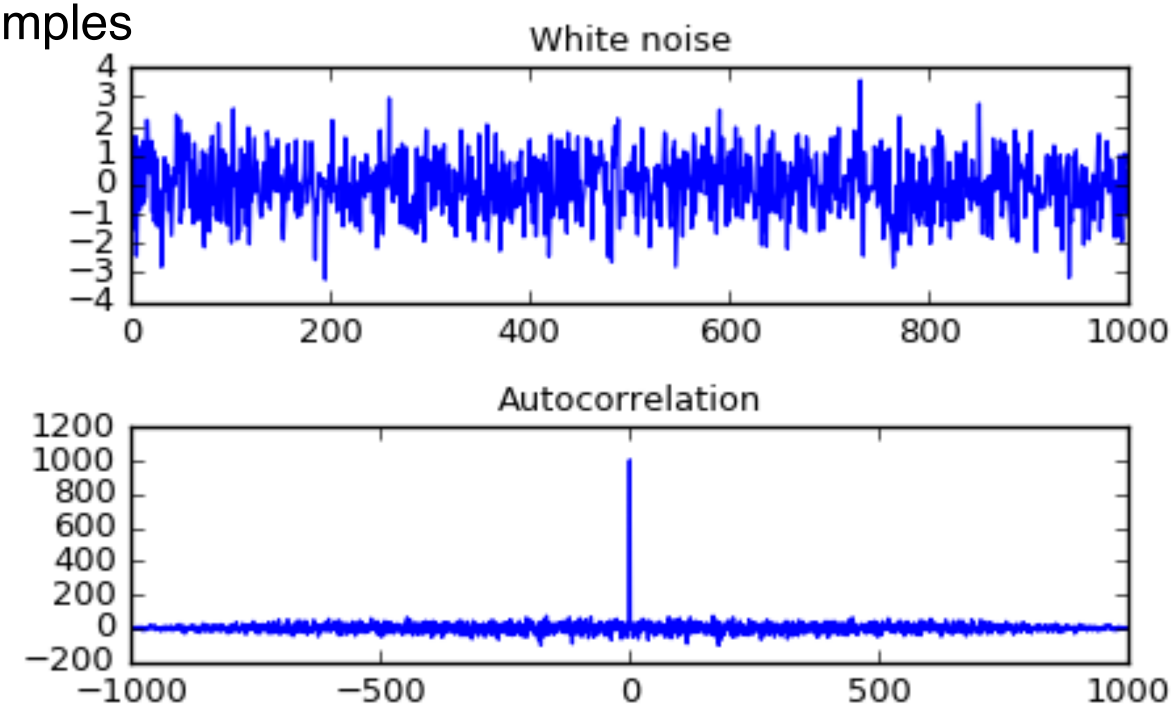
Correlation

<u>**Correlation:**</u> A measure of the degree to which two functions look alike at a given offset. If the two functions are the same, we call this the autocorrelation function, or ACF.

$$R_{ff}(\tau) = \int_{-\infty}^{+\infty} f(t+\tau)\bar{f}(t)dt = f(\tau) * \bar{f}(-\tau)$$

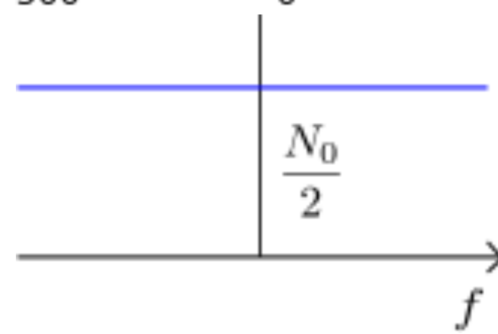
We will be working with discrete samples

$$R_{ff}(k) = \sum_{n=-\infty}^{+\infty} f(n)\bar{(}f)(n-k)$$



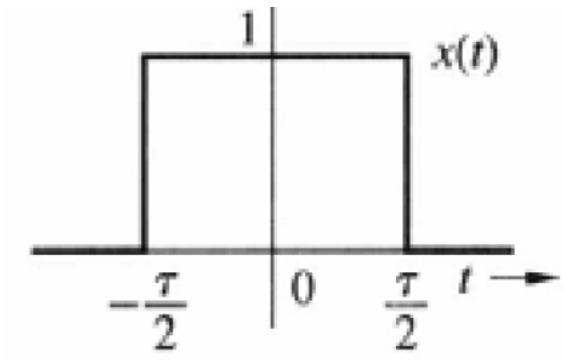
The ‘spectrum’ refers to the power spectrum, which is the Fourier transform of the autocorrelation function

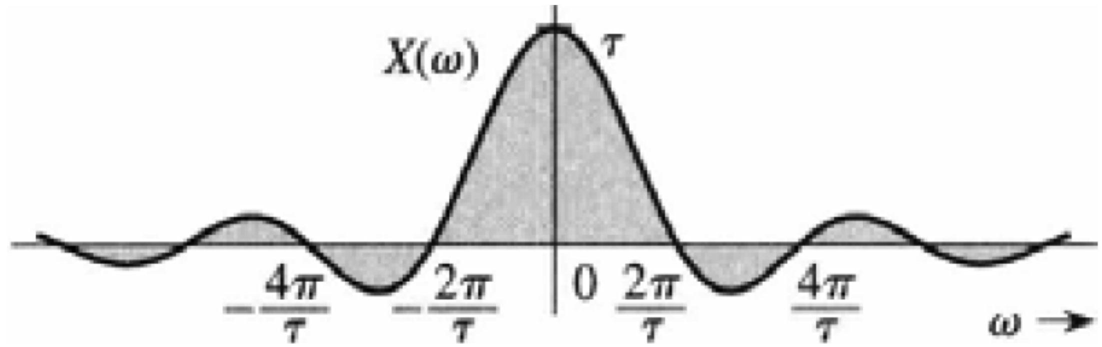
$$R_{ff} \Longleftrightarrow \left| U(\omega) \right|^2$$



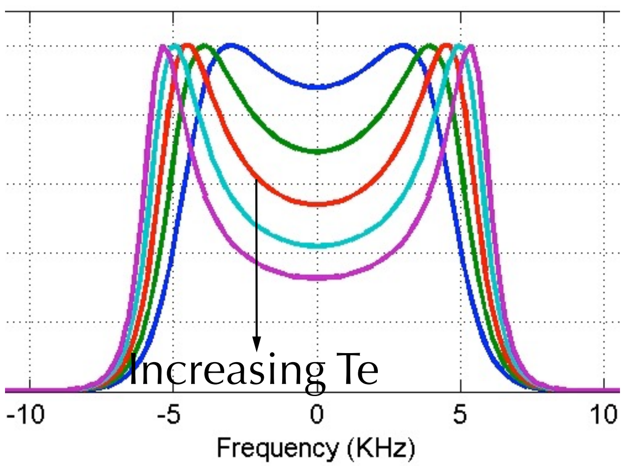
Gate function

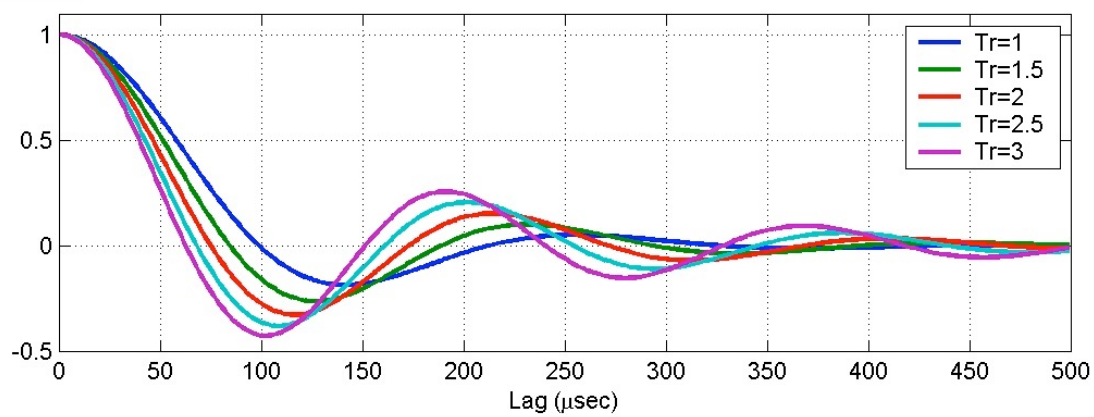
$$\mathrm{rect}(t/\tau) = \begin{cases} 1 & \text{for } -\tau/2 < t < \tau/2 \\ 0 & \text{otherwise} \end{cases} \iff \tau\ \mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$$





ISR spectrum $\iff$ Autocorrelation function (ACF)



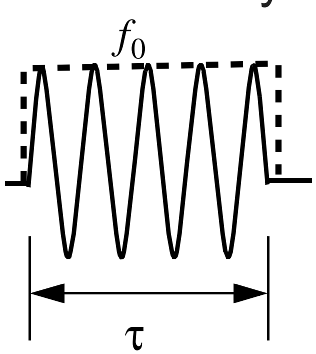


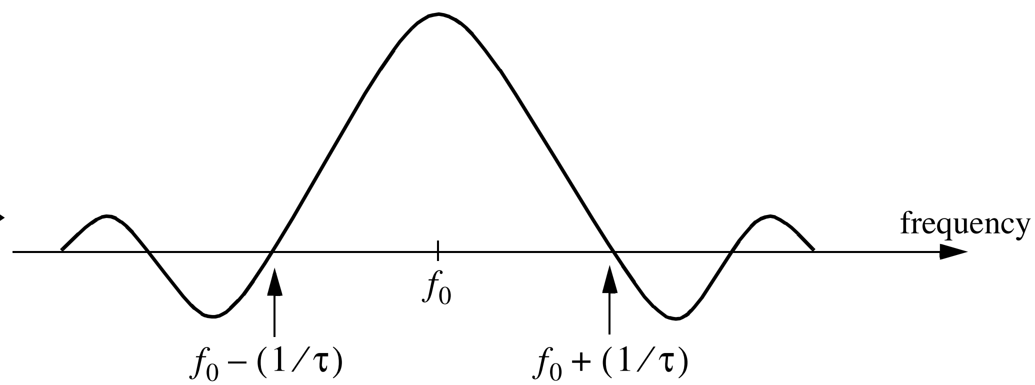
For low Te, the ISR ACF looks like a sinc function. For high Te the ACF becomes more oscillatory and looks more like a cosine (power concentrated at the Doppler frequency corresponding to the ion-acoustic wave speed.

How it all hangs together.

- Duality:
  - Gate function in the time domain represents amplitude modulation
  - Gate function in the frequency domain represents filtering

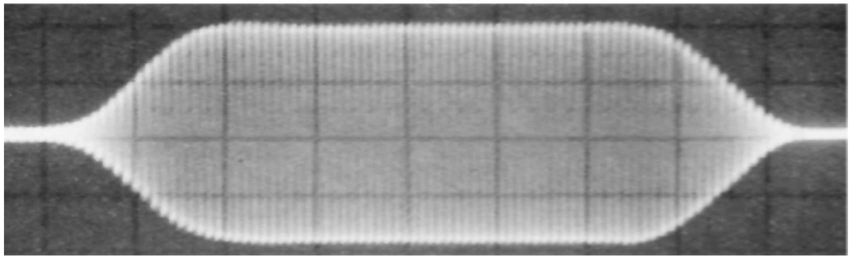
- Limiting cases:
  - Gate function approaches delta function as width goes to 0 with constant area
  - A constant function in time domain is a special case of harmonic function where frequency = 0.
  - A constant function in time domain is a special case of a gate function where width = infinity.





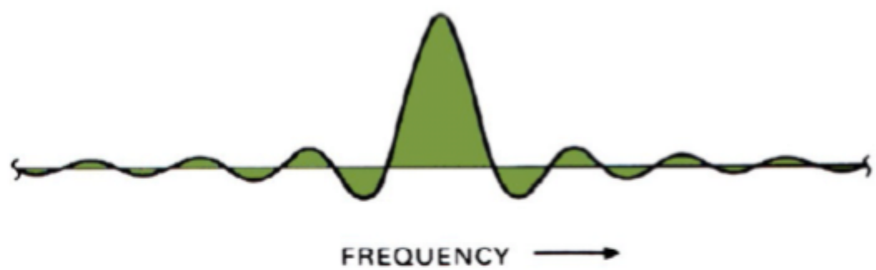
How many cycles are in a typical ISR pulse?
PFISR frequency: 449 MHz
Typical long-pulse length: 480 μs

215,520 cycles!

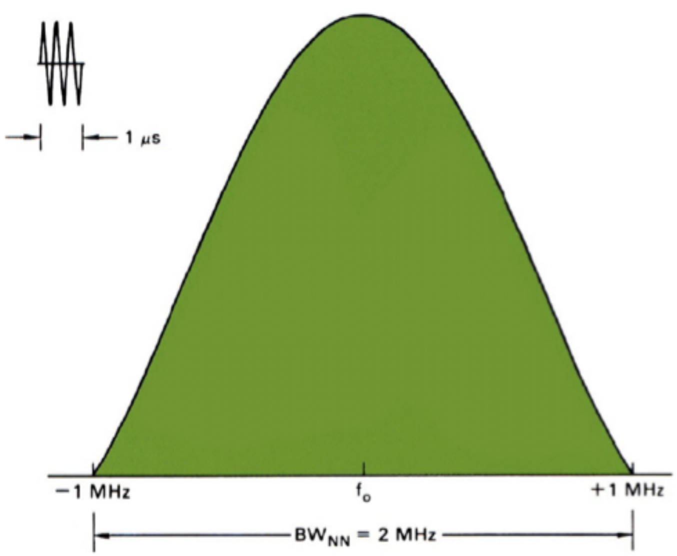


Bandwidth of a pulsed signal

Spectrum of receiver output has sinc shape, with sidelobes half the width of the central lobe and continuously diminishing in amplitude above and below main lobe



A 1 microsecond pulse has a 3 dB bandwidth of 1 MHz



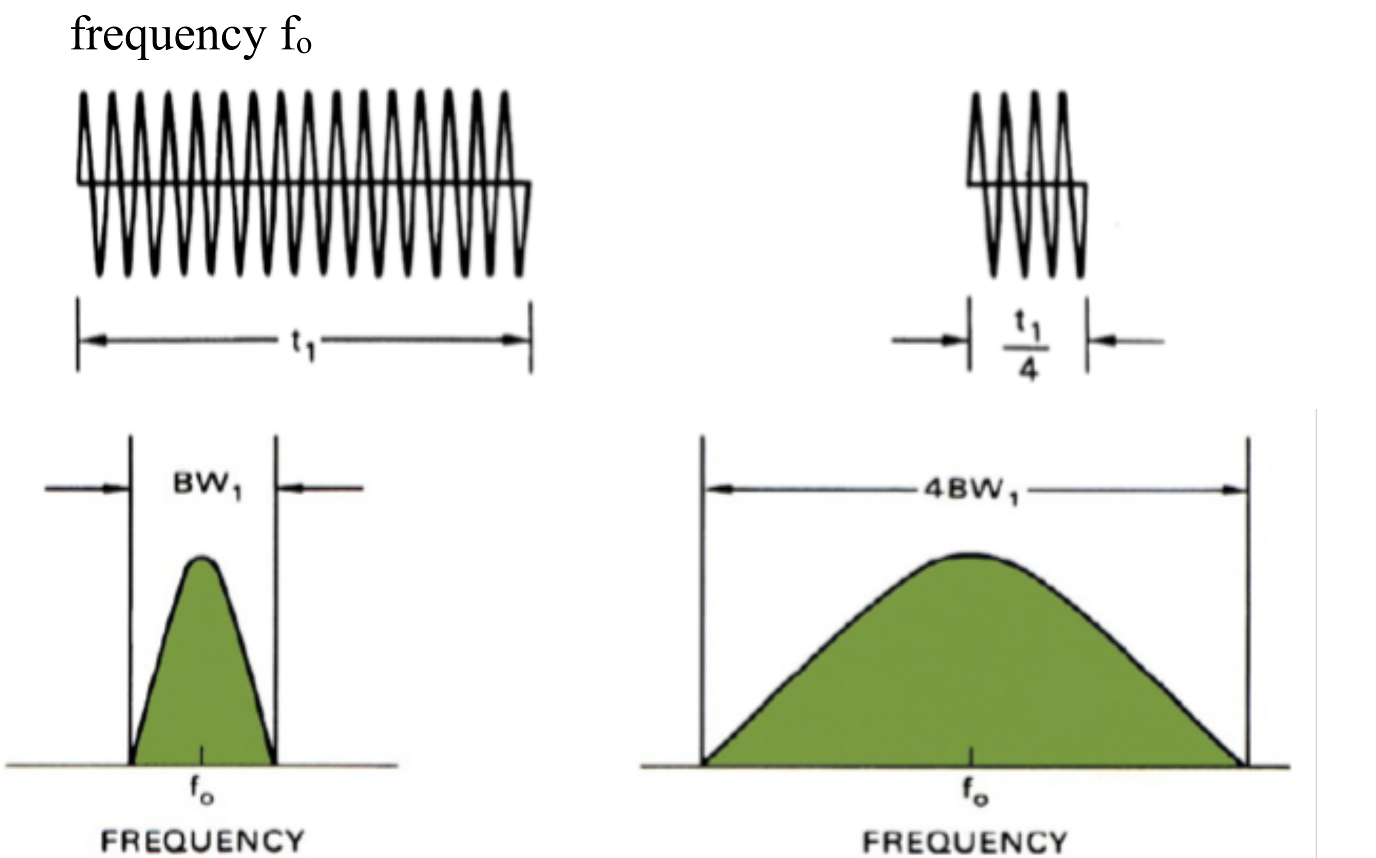
Two possible bandwidth measures:

"null to null" bandwidth $B_{nn} = \frac{2}{\tau}$

"3dB" bandwidth $B_{3dB} = \frac{1}{\tau}$

Unless otherwise specified, assume bandwidth refers to 3 dB bandwidth

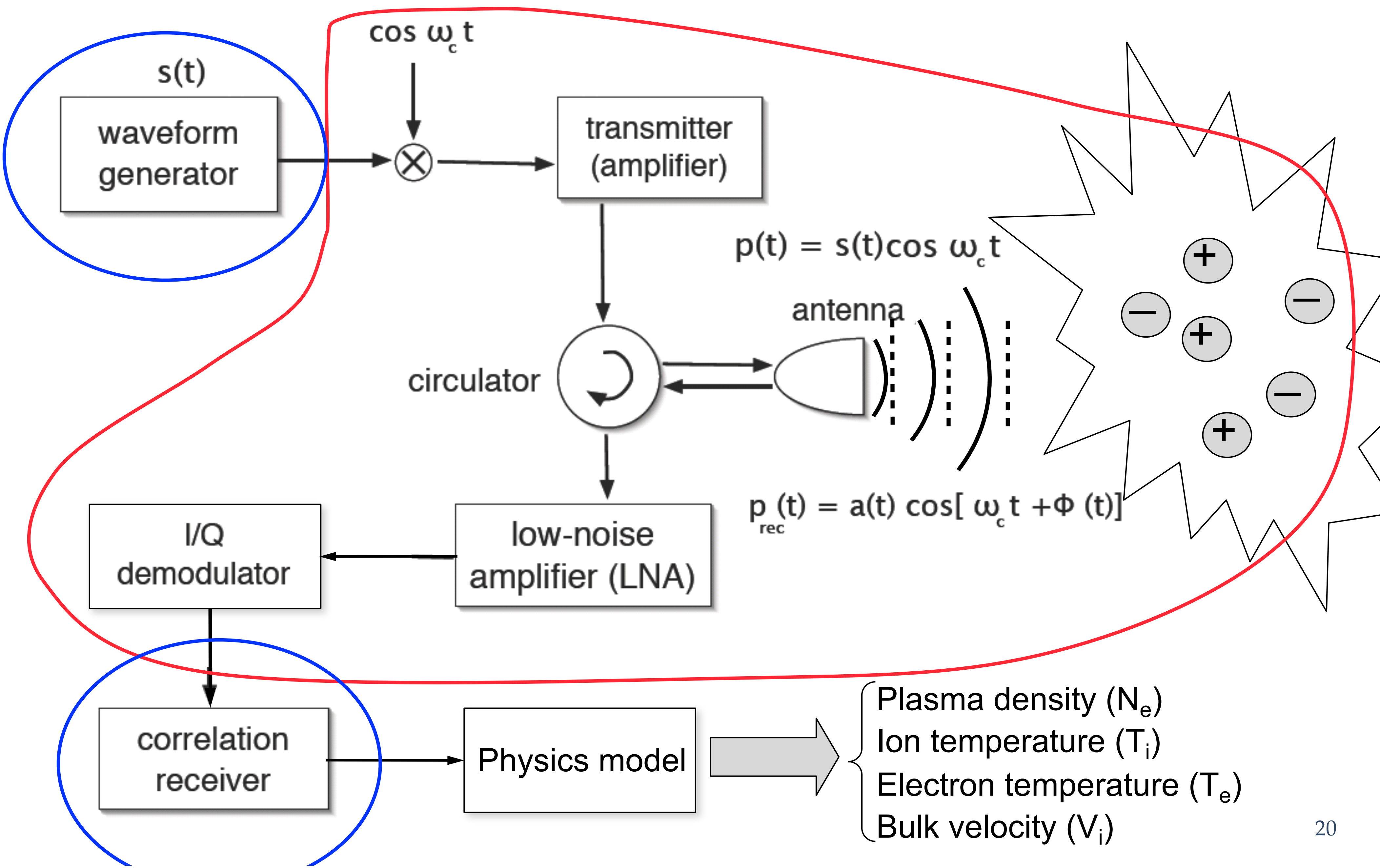
Pulse-Bandwidth Connection



Shorter pulse ⟺ Larger bandwidth

Faster sampling rate ⟺ Larger bandwidth

Components of a Pulsed Doppler Radar



The deciBel (dB)

The relative value of two quantities expressed on a logarithmic scale

$$SNR = 10 \log_{10} \frac{P_1}{P_2} = 20 \log_{10} \frac{V_1}{V_2}$$

(Power $\propto$ Voltage^2)

Factor of:	Scientific Notation	dB
0.1	10^{-1}	-10
0.5	$10^{0.3}$	-3
1	10^{0}	0
2	$10^{0.3}$	3
10	10^{1}	10
100	10^{2}	20
1000	10^{3}	30
1,000,000	10^{6}	60

Other forms used in radar:

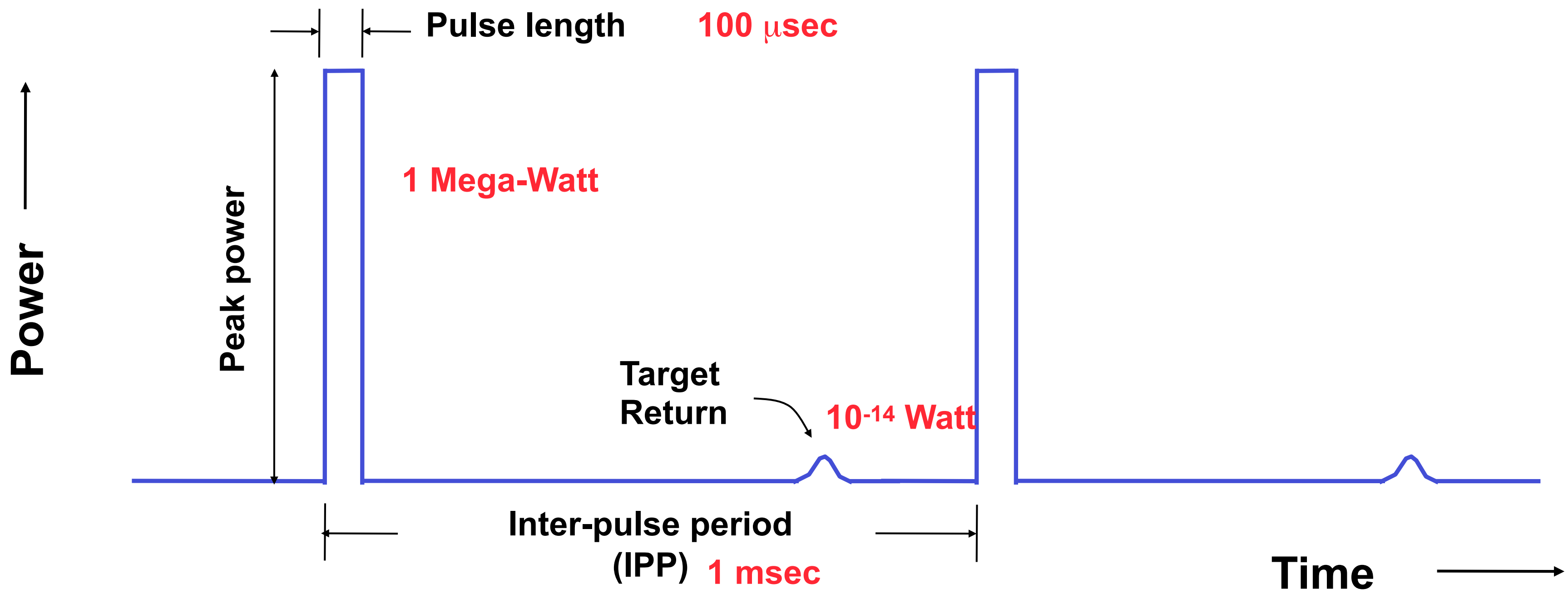
dBW dB relative to 1 Watt

dBm dB relative to 1 mW

dBsm dB relative to 1 m^2 of radar cross section

dBi dB relative to isotropic radiation

Pulsed Radar



Duty cycle = $\dfrac{\text{Pulse length}}{\text{Pulse repetition interval}}$ **10%**

Average power = Peak power * Duty cycle **100 kWatt**

Pulse repetition frequency (PRF) = 1/(IPP) **1 kHz**

Continuous wave (CW) radar: Duty cycle = 100% (always on)

Doppler Frequency Shift

Transmitted signal: $\cos(2\pi f_o t)$

After return from target: $\cos\left[2\pi f_o\left(t + \frac{2R}{c}\right)\right]$

To measure frequency, we need to observe signal for at least one cycle.
So we will need a model of how *R* changes with time. Assume constant velocity:

$$R = R_o + vt$$

Substituting:

$$\cos\left[2\pi\left(f_o + \underbrace{f_o\frac{2v}{c}}_{-f_D}\right)t + \underbrace{\frac{2_o R_o}{c}}_{\text{constant}}\right]$$

$$f_D = -2f_o\left(\frac{v}{c}\right) = -2\left(\frac{v}{\lambda_o}\right) \propto \frac{\text{line-of-sight velocity}}{\text{radar wavelength}}$$

By convention, positive Doppler shift $\Longleftrightarrow$ Target and radar are "closing"

Two key concepts

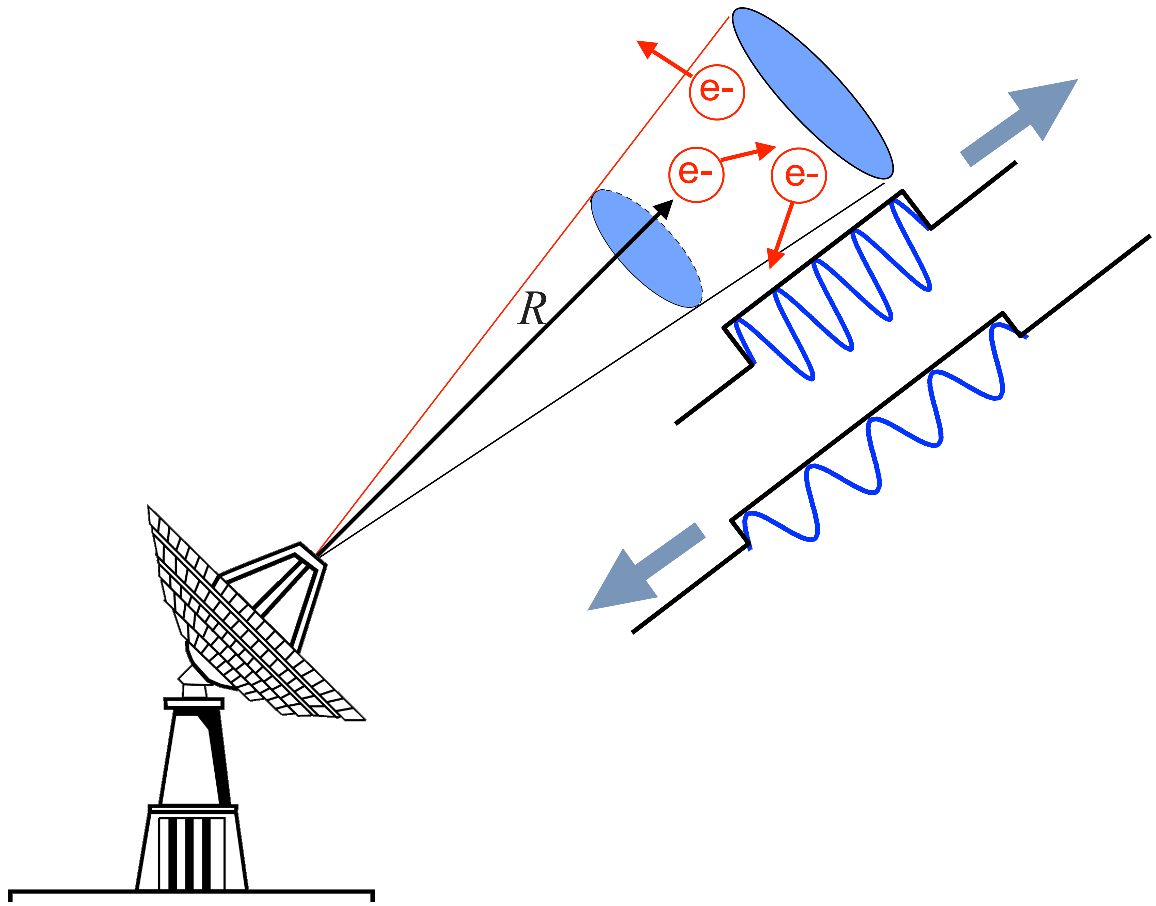
Two key concepts:

Distant ⟺ Time

$R = c\Delta t/2$

Velocity ⟺ Frequency

$v = -f_D \lambda_o/2$



A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

Two key concepts

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$$R = c\Delta t/2$$

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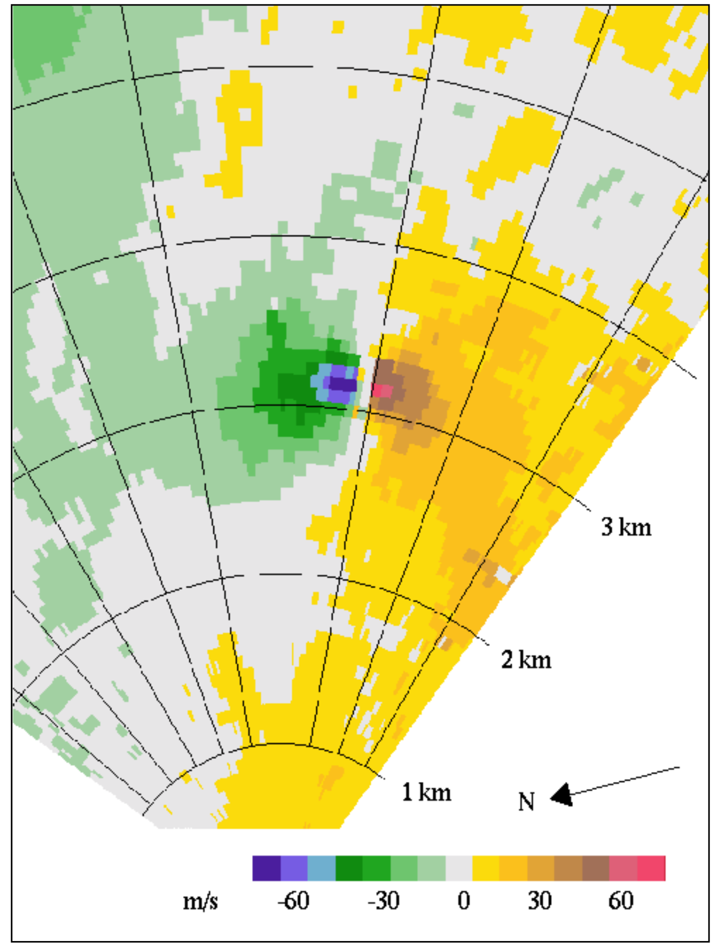
$$v = -f_D\lambda_o/2$$



A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

Concept of a "Doppler Spectrum"

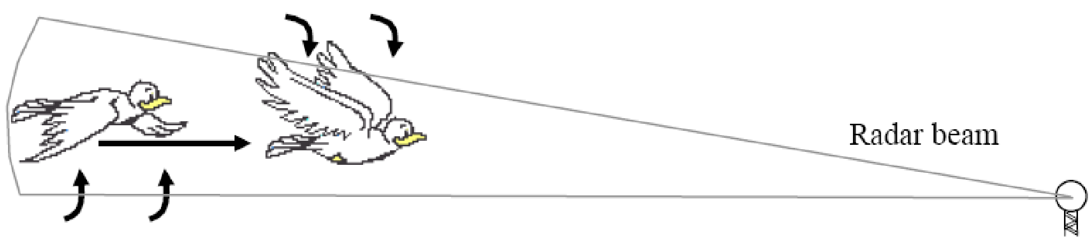
Two key concepts:

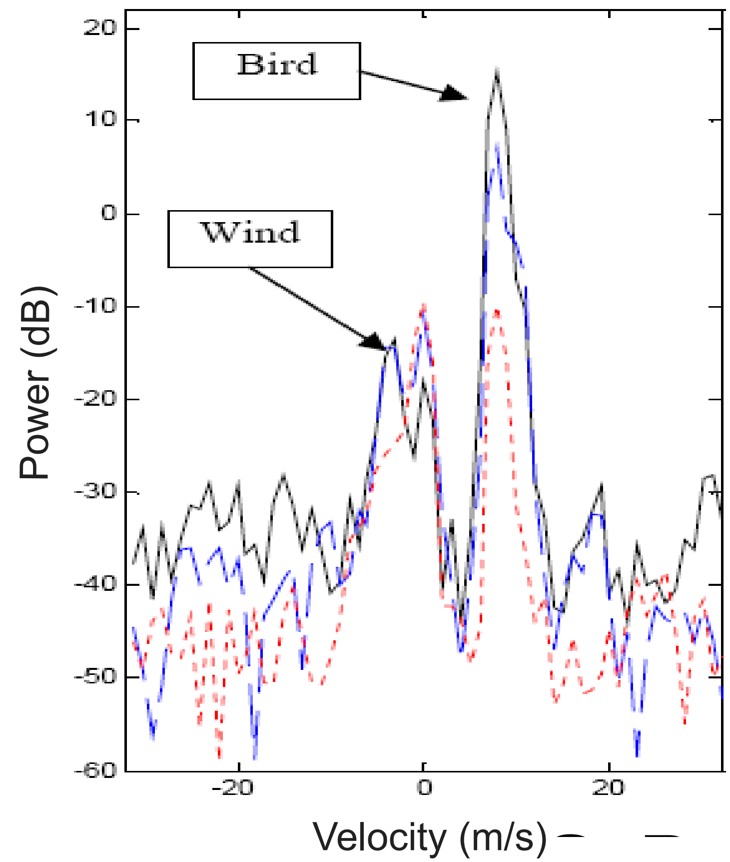
Distant ⟺ Time

$$R = c\Delta t/2$$

Velocity ⟺ Frequency

$$v = -f_D\lambda_o/2$$





If there is a distribution of targets moving at different velocities (e.g., electrons in the ionosphere) then there is no single Doppler shift but, rather, a Doppler spectrum.

What is the Doppler spectrum of the ionosphere at UHF (λ_o of 10 to 30 cm)?

Longitudinal Modes in a Thermal Plasma

Simple dispersion relation

$$f = c/\lambda$$
$$\omega = 2\pi f$$
$$k = 2\pi/\lambda$$

$$\omega = ck$$

k = wave number = “spatial frequency”

Ion-acoustic

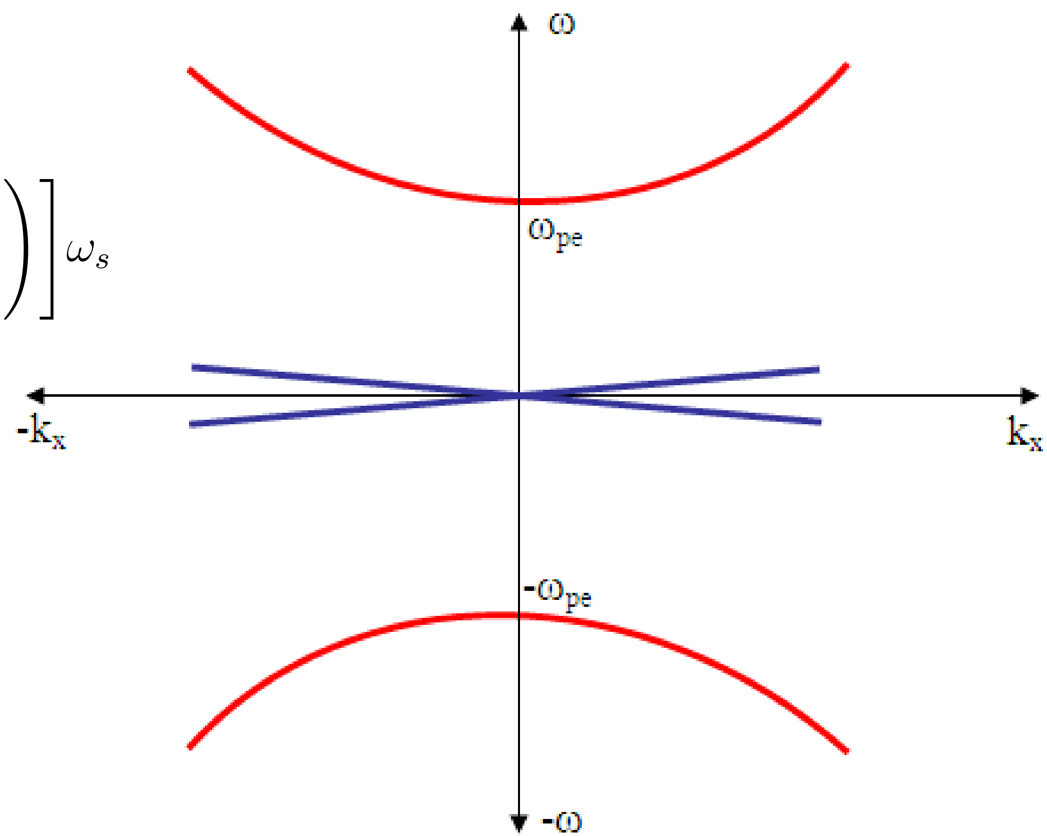
$$\omega_s = C_s k \qquad C_s = \sqrt{k_B(T_e + 3T_i)/m_i}$$

$$\omega_{si} = -\sqrt{\frac{\pi}{8}}\left[\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \left(\frac{T_e}{T_i}\right)^{\frac{3}{2}} \exp\left(-\frac{T_e}{2T_i} - \frac{3}{2}\right)\right]\omega_s$$

Langmuir

$$\omega_L = \sqrt{\omega_{pe}^2 + 3\,k^2\,v_{the}^2} \approx \omega_{pe} + \frac{3}{2} v_{the} \lambda_{De} k^2$$

$$\omega_{Li} \approx -\sqrt{\frac{\pi}{8}}\frac{\omega_{pe}^3}{k^3}\frac{1}{v_{the}^3} exp\left(-\frac{\omega_{pe}^2}{2k^2 v_{the}^2} - \frac{3}{2}\right)\omega_L$$



Incoherent Scatter Radar

Particle-in-cell (PIC):

$$\frac{d\,\mathbf{v}_i}{d\,t} = \frac{q_i}{m_i}(\mathbf{E}(\mathbf{x}_i) + \mathbf{v}_i \times \mathbf{B}(\mathbf{x}_i))$$

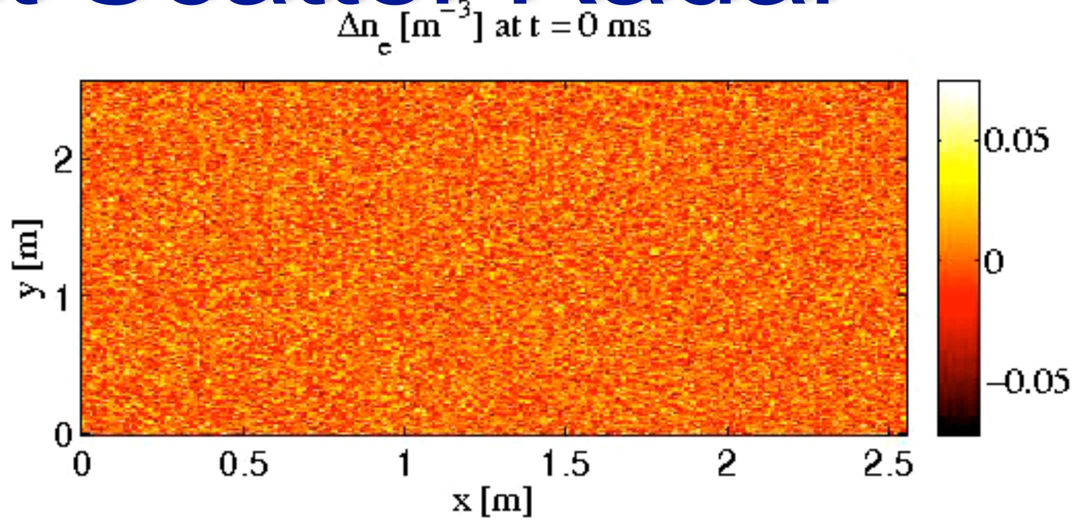
$$\nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t}$$

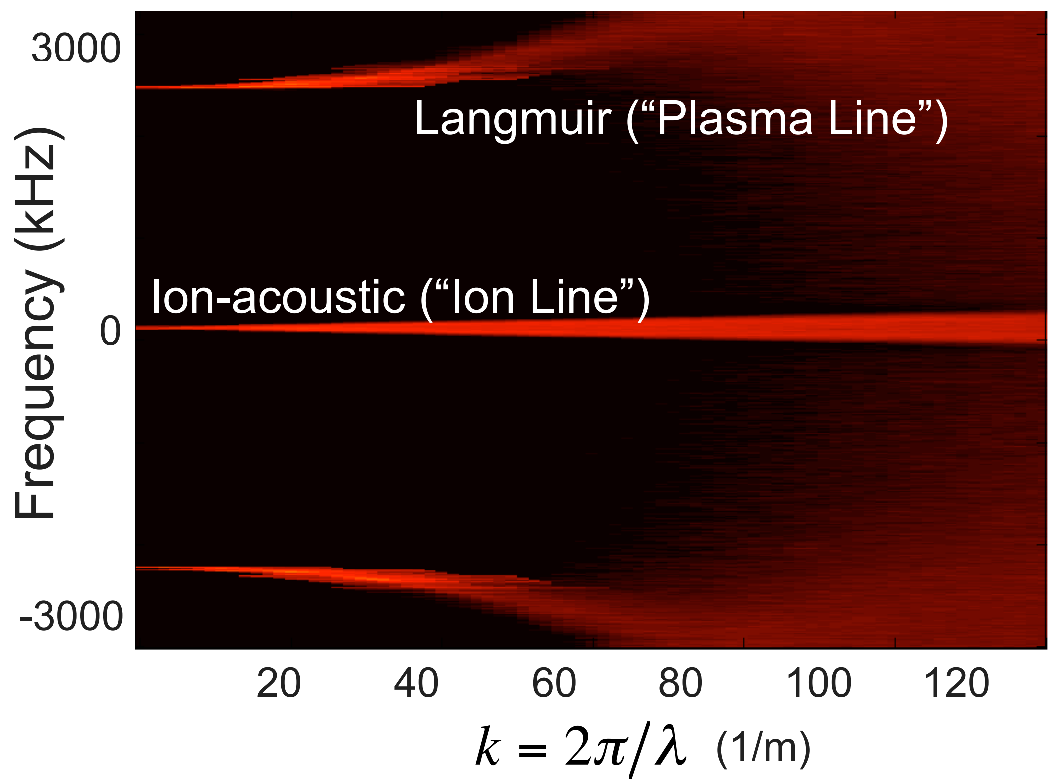
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

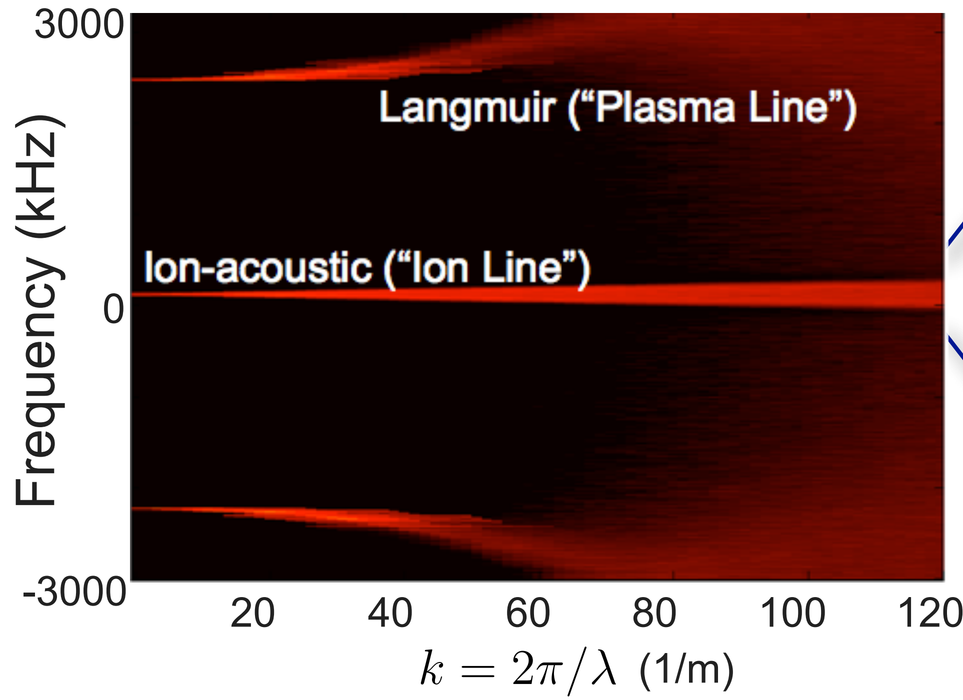
Simple rules yield complex behavior

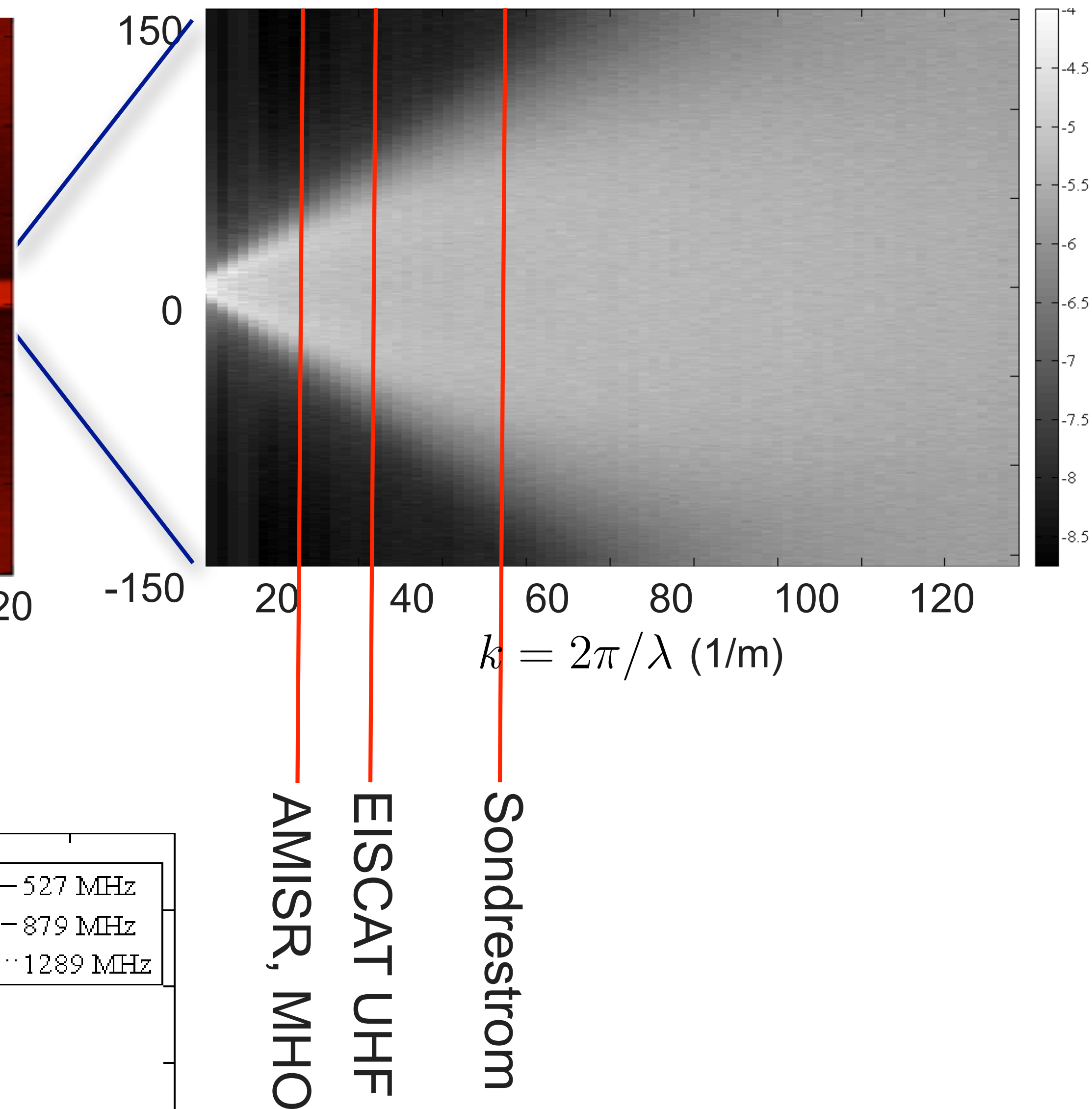


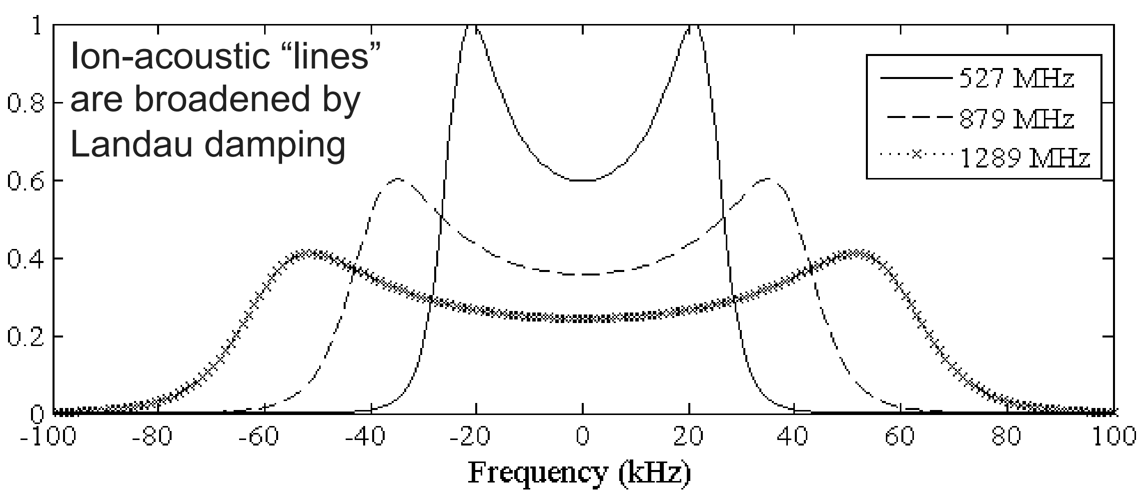


Diaz et al., RS 2008

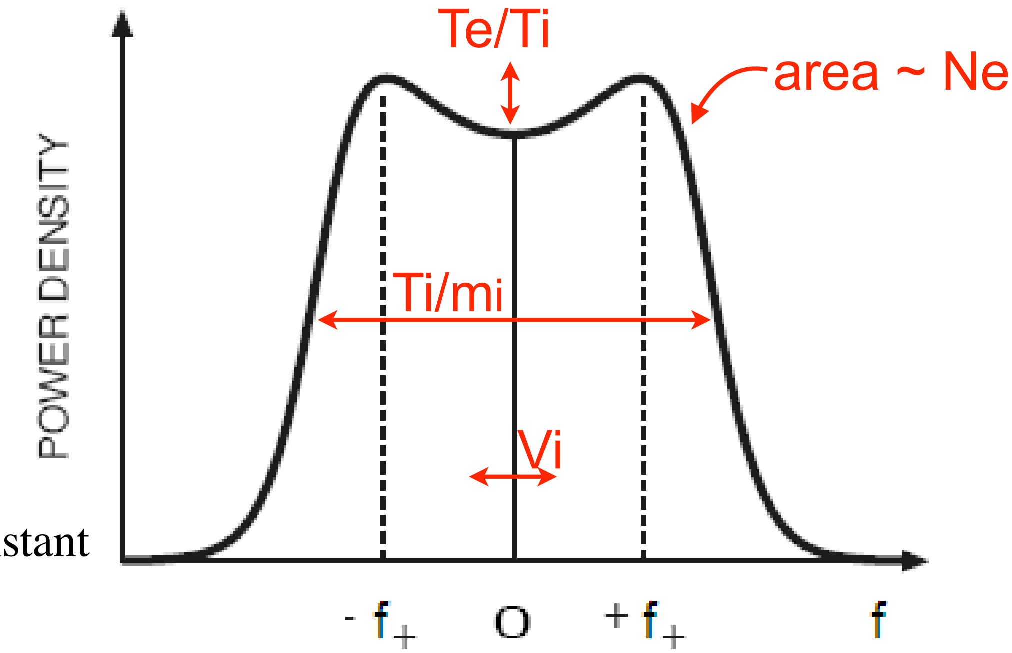
ISR Measures a Cut Through This Surface







ISR in a nutshell



Here's what we measure:

$$SNR = \frac{P_r}{P_n} = \left(\frac{P_t}{4\pi R^2}\right)\left(\frac{\sigma(\omega)}{4\pi R^2}\right)\left(\frac{GA}{KTBN_{sys}}\right)$$

P_r = Received power

P_n = Received noise power

P_t = Transmitted power

ς = Radar cross section

G = Antenna gain

A = Antenna area

k_B = Boltzman's constant

T = Temperature

B = Bandwidth

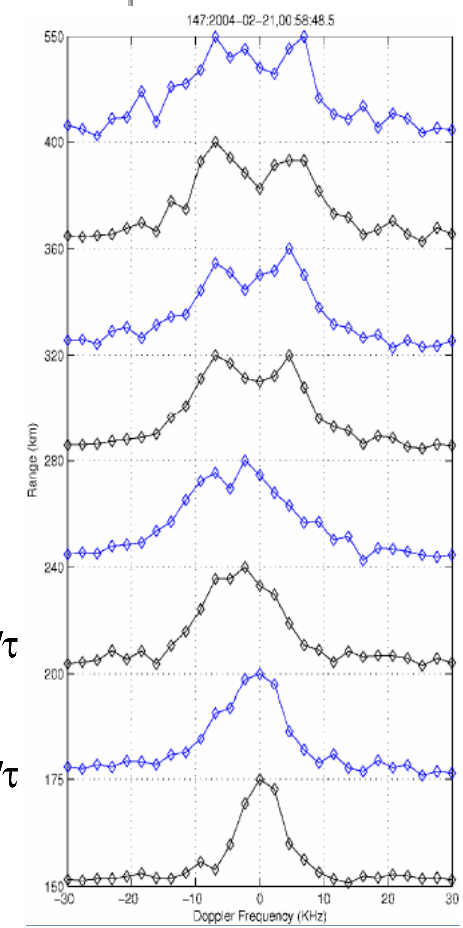
N_{sys} = System noise temperature

Here's the theory:

$$\sigma(\omega) = \frac{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \sum_1 \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right|^2 \overline{\left|N_e^0(\omega)\right|^2} + \left(\frac{\lambda}{4\pi D_e}\right)^4 \left|F_e(\omega)\right|^2 \sum_i \overline{\left|N_i^0(\omega)\right|^2}}{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \left\{\left(\frac{1}{D_e}\right)^2 \times F_e(\omega) + \sum_i \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right\}\right|^2}$$

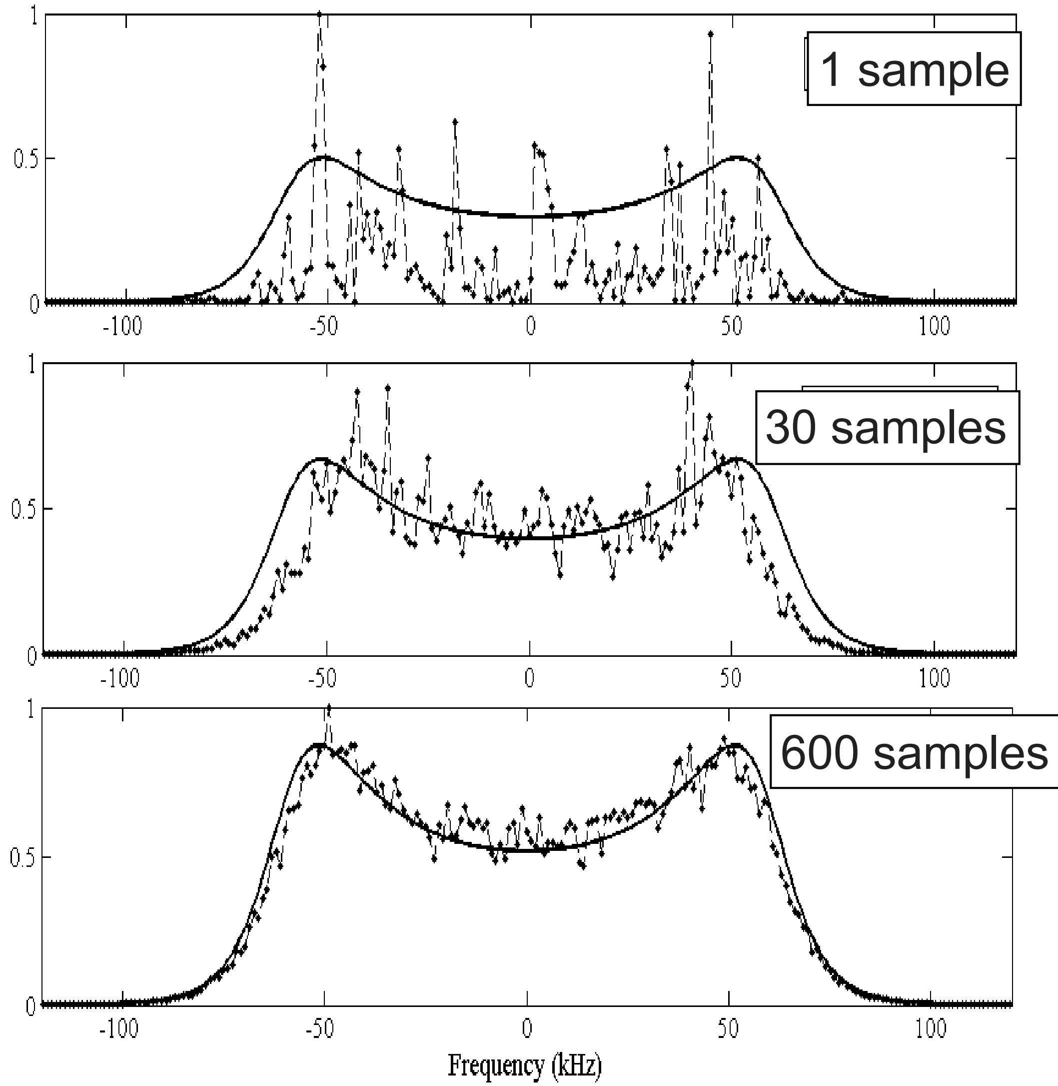
$$F_e(\omega) = 1 - \omega\int_o^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\cos(\omega\tau)\,d\tau$$

$$F_i(\omega) = 1 - \omega\int_o^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\cos(\omega\tau)\,d\tau$$



Incoherent Averaging

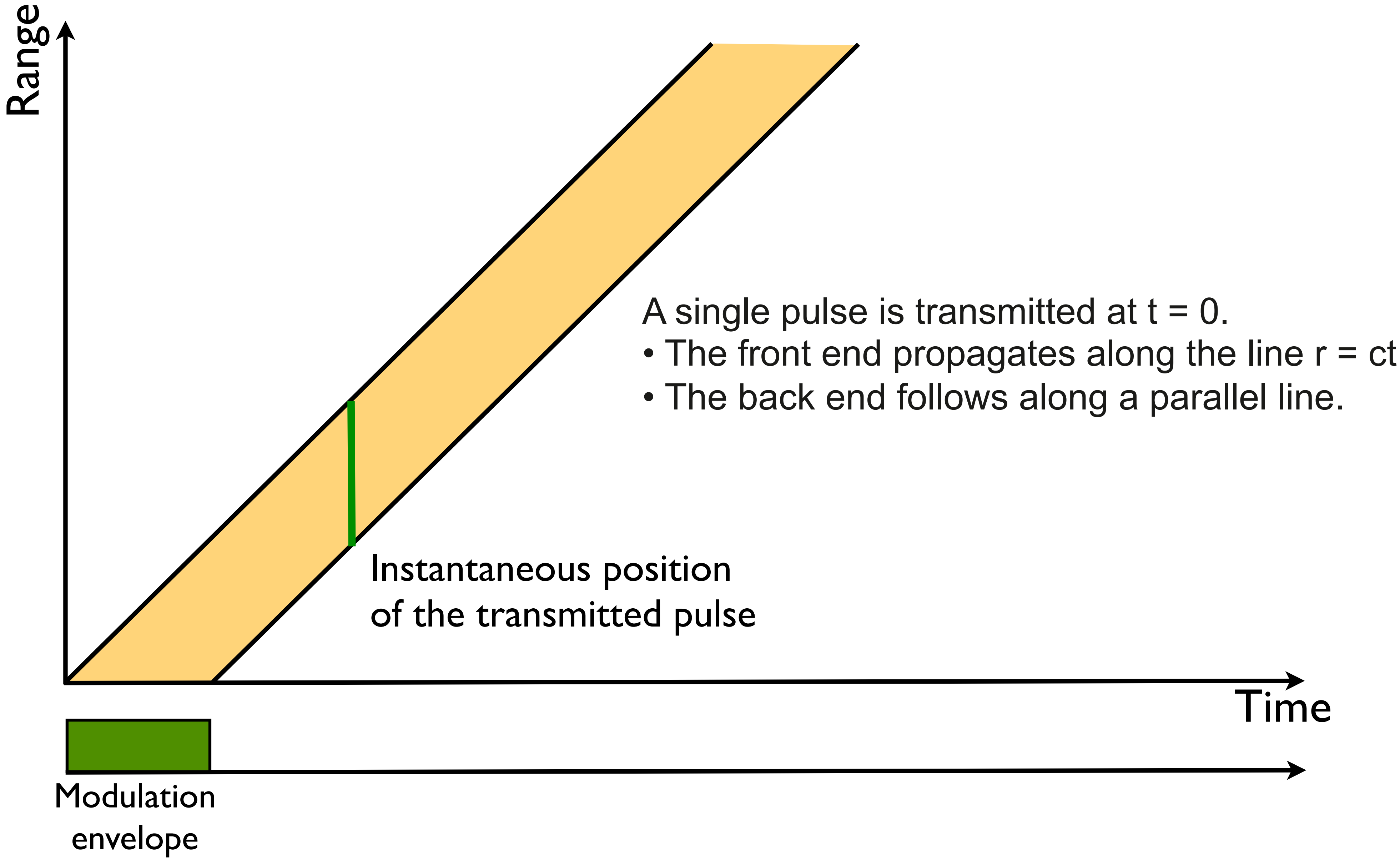
Normalized ISR spectrum for different integration times at 1290 MHz



We are seeking to estimate the power spectrum of a Gaussian random process. This requires that we sample and average many independent "realizations" of the process.

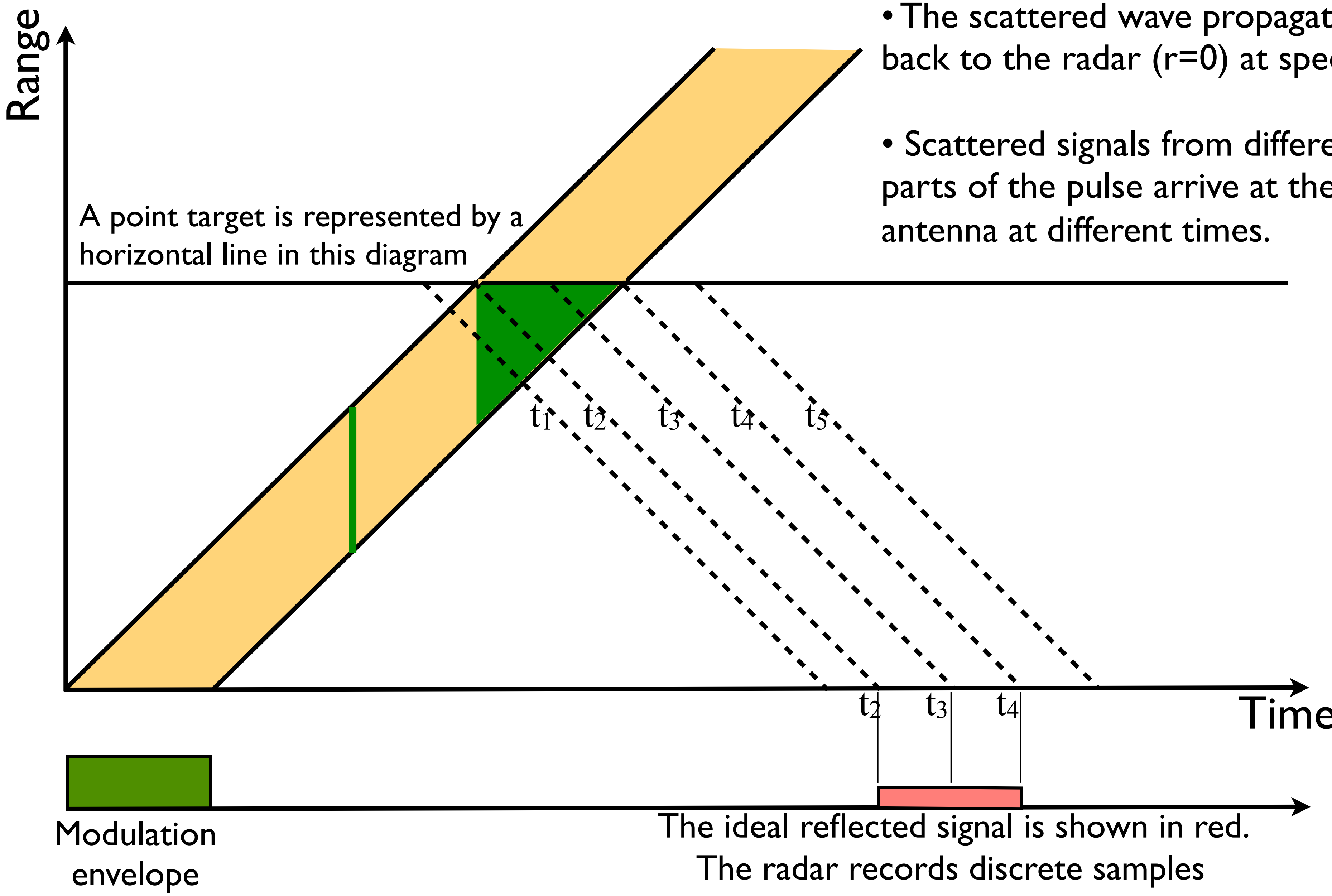
$$\text{Uncertainties} \propto \frac{1}{\sqrt{\text{Number of Samples}}}$$

Range-time analysis



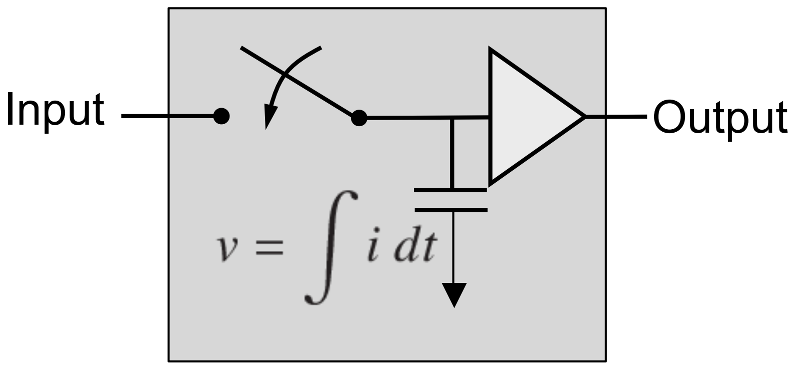
Range-time analysis

- The scattered wave propagates back to the radar (r=0) at speed c.
- Scattered signals from different parts of the pulse arrive at the antenna at different times.

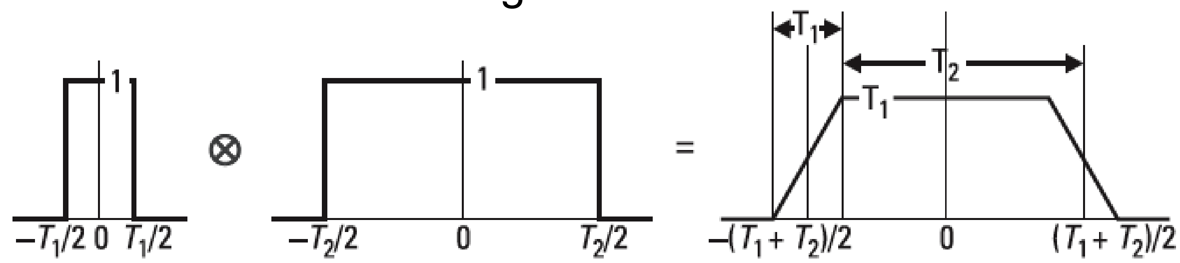


Sampling a signal require time-integration

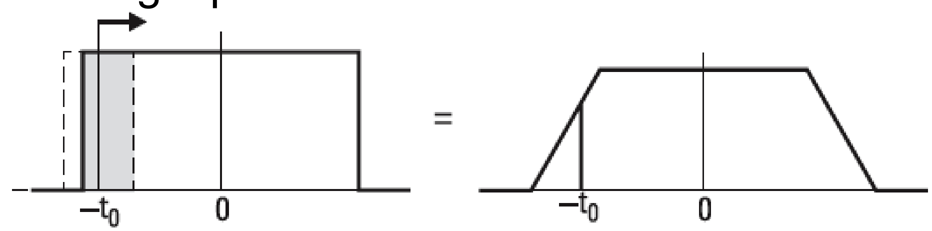
We send a pulse of duration τ. How should we listen for the echo?



Convolution of two rectangle functions

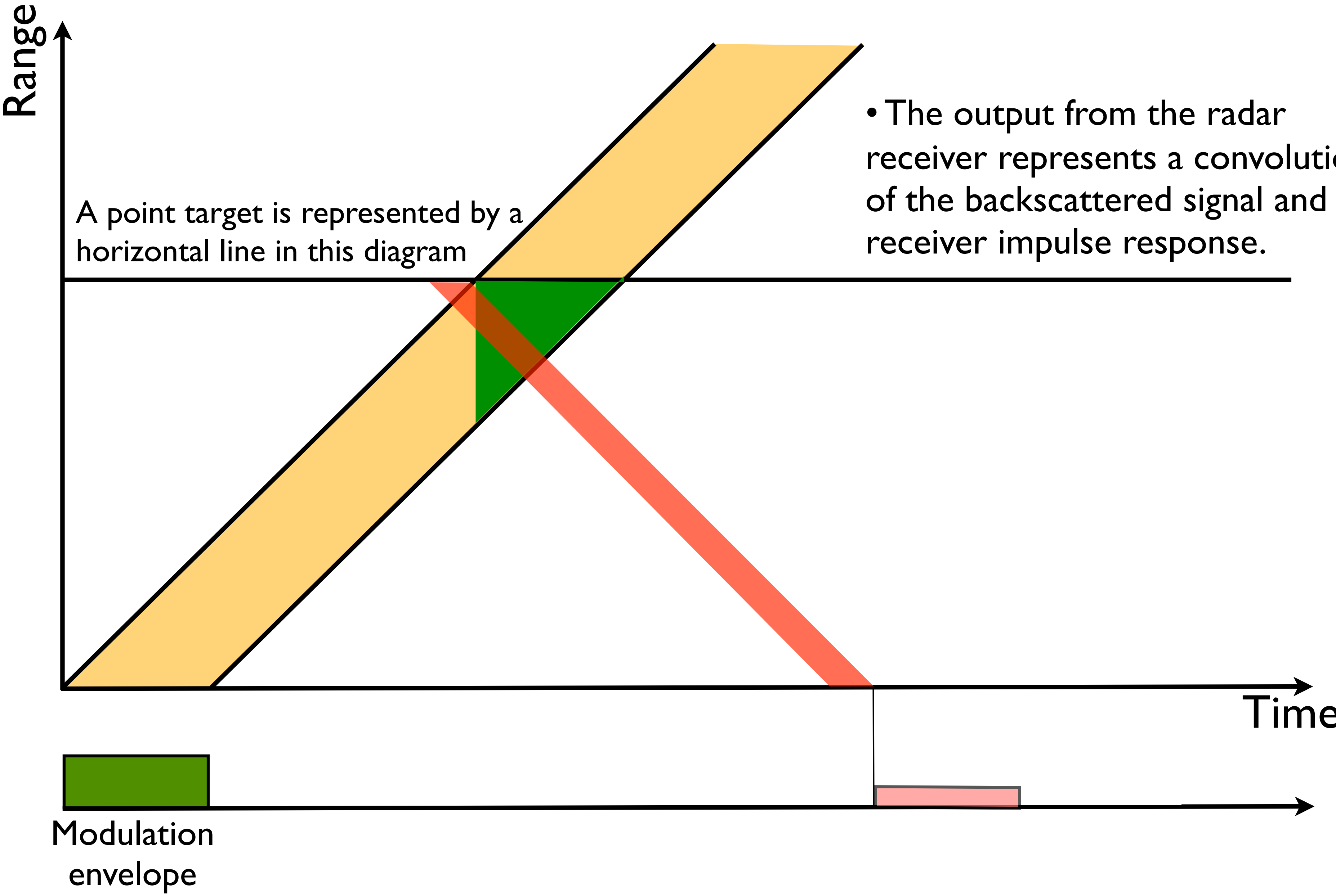


Value at a single point



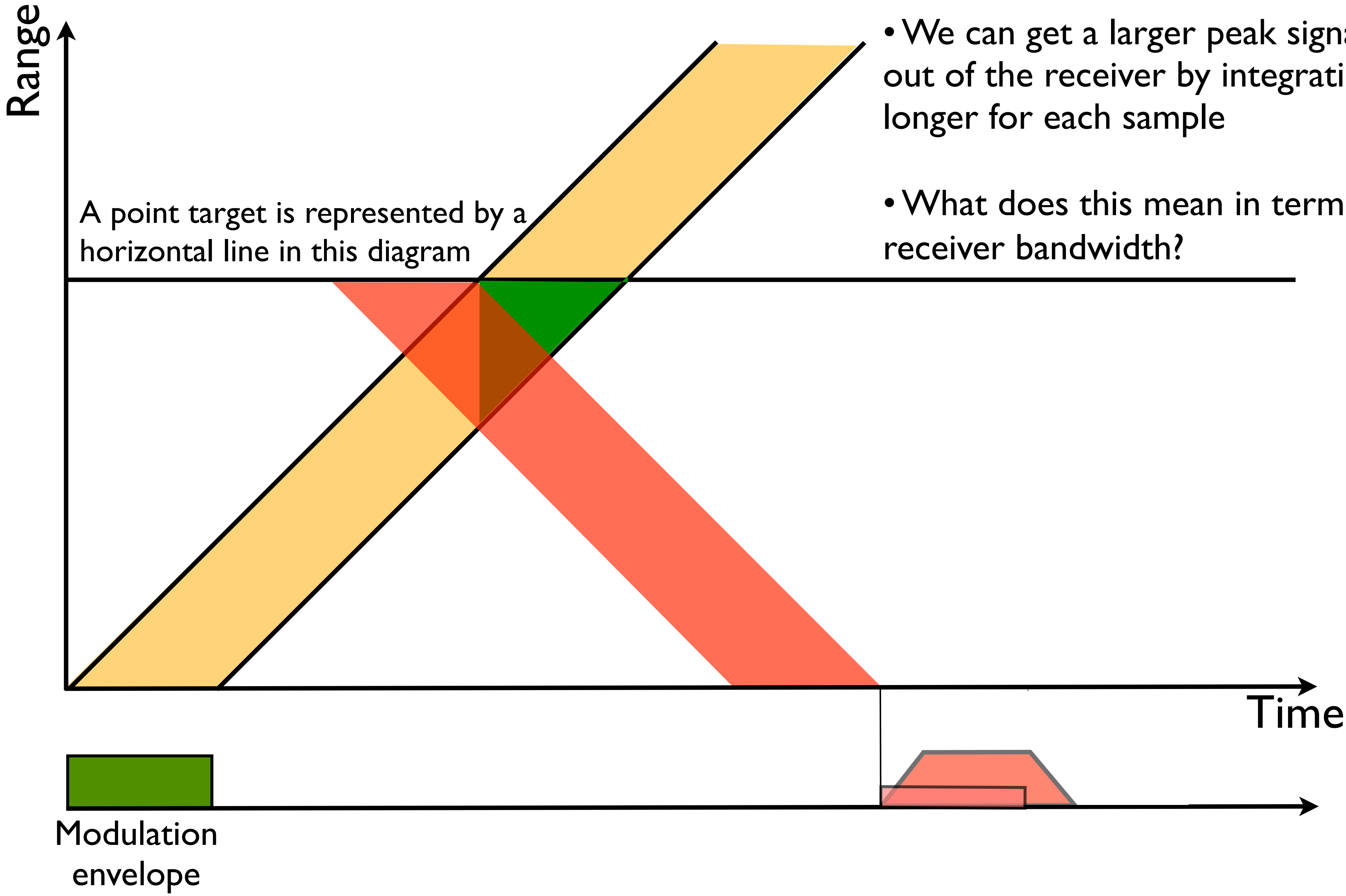
- To determine range to our target, we only need to find the rising edge of the pulse we sent. So make $T_1 << T_2$.
- But that means large receiver bandwidth, lots of noise power, poor SNR.
- Could make $T_1 >> T_2$, then we're integrating noise in time domain.
- So how long should we close the switch?

Sampling the received signal



- The output from the radar receiver represents a convolution of the backscattered signal and the receiver impulse response.

Computing the ACF

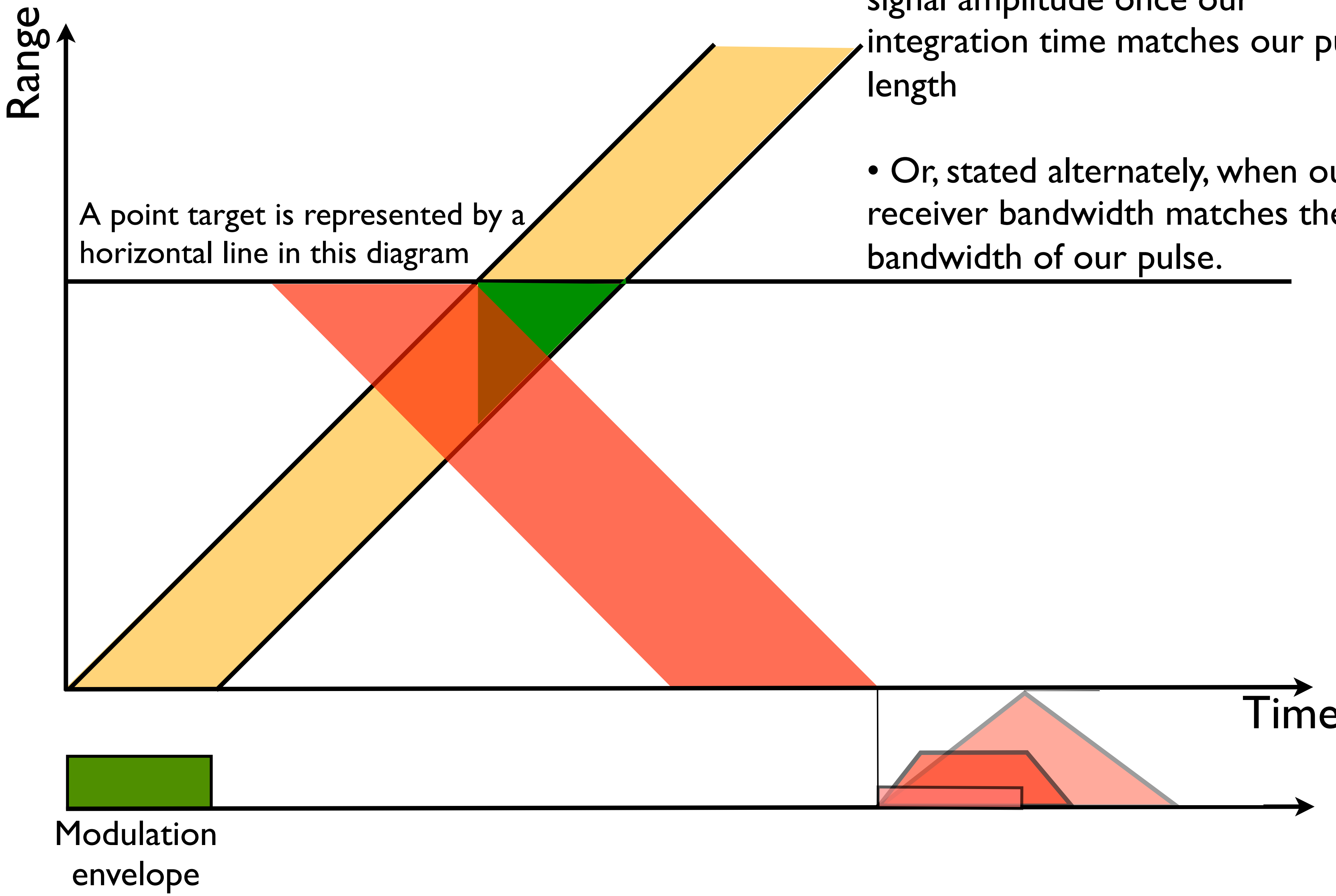


- We can get a larger peak signal out of the receiver by integrating longer for each sample
- What does this mean in terms of receiver bandwidth?

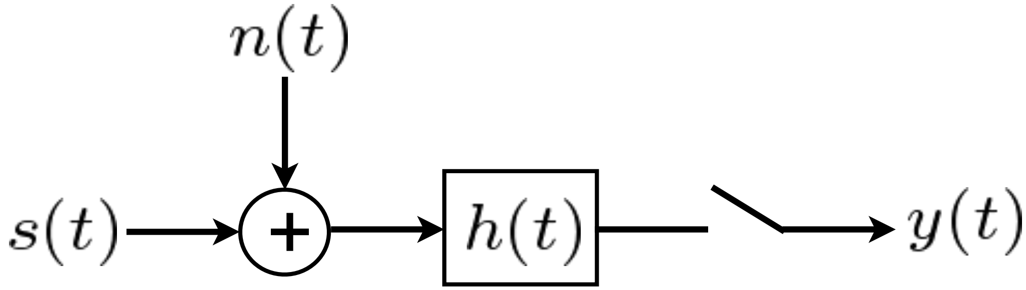
Computing the ACF

- We don't get any more gain in signal amplitude once our integration time matches our pulse length

- Or, stated alternately, when our receiver bandwidth matches the bandwidth of our pulse.



Matched Filter



$$y(t) = \int [s(\tau) + n(\tau)]\, h(t - \tau) d\tau$$

$$= \int H(f) S(f) e^{j2\pi f T} df + \int H(f) N(f) e^{j2\pi f T} df$$

How should we choose *h(t)*⇔*H(f)* such that the output SNR is maximal?

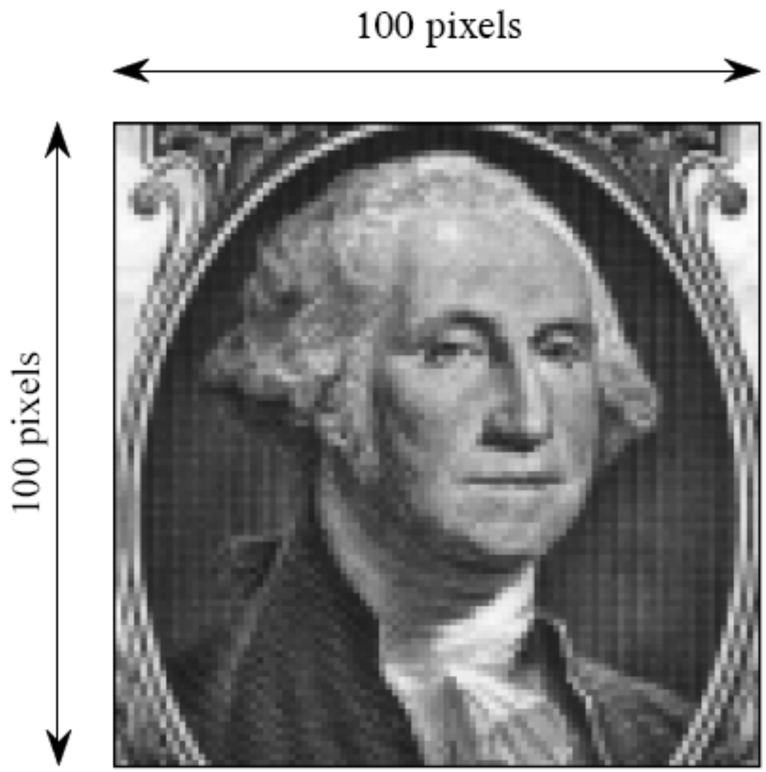
$$SNR = \frac{\left| \int H(f) S(f) e^{j2\pi f T} df \right|^2}{E\left\{ \left| \int H(f) N(f) df \right|^2 \right\}}$$

Assuming white Gaussian noise, it can be shown that max SNR is when

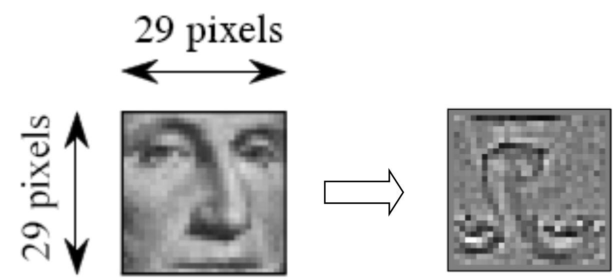
$$\boxed{H(f) = S^*(f) \Longleftrightarrow h(t) = s^*(-t)}$$

Pulse compression and matched filtering

"If you know what you're looking for, it's easier to find."

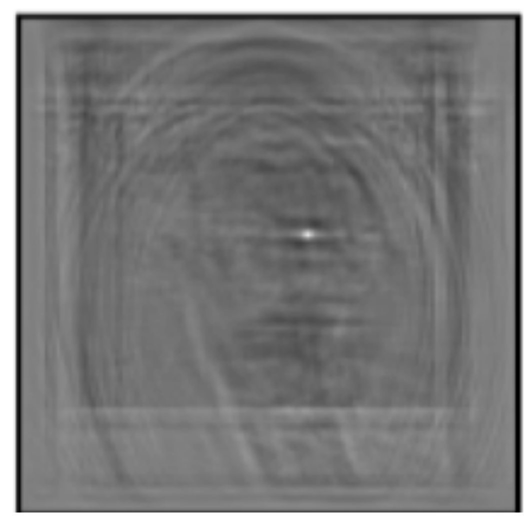


a. Image to be searched



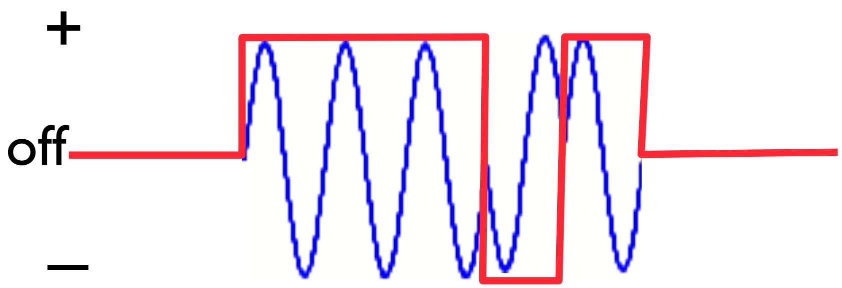
b. Target

c. Kernel



Problem: Find the precise location of the target in the image.
Solution: Correlation

Barker codes



									correlator output
				+	+	+	-	+	
+	+	+	-	+					1
	+	+	+	-	+				-1+1=0
		+	+	+	-	+			1-1+1=1
			+	+	+	-	+		1+1-1-1=0
				+	+	+	-	+	1+1+1+1+1=5

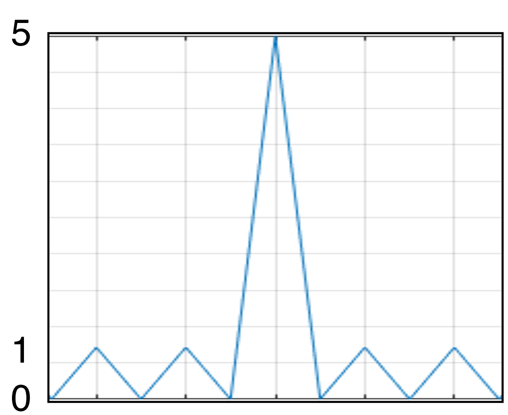
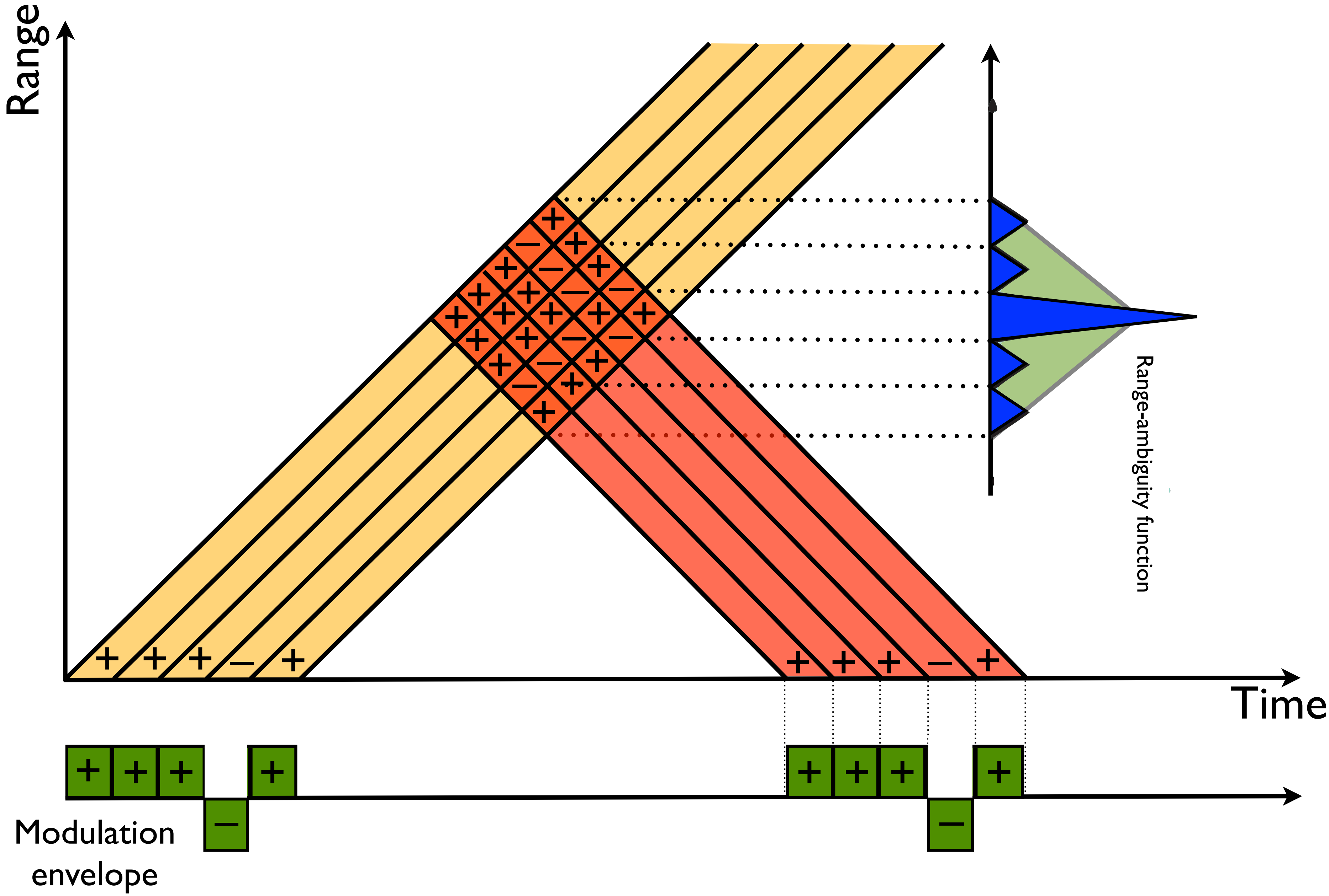


TABLE 6.2 All Known Binary Barker Codes

Code Length	Code
2	11 or 10
3	110
4	1110 or 1101
5	11101
7	1110010
11	11100010010
13	1111100110101

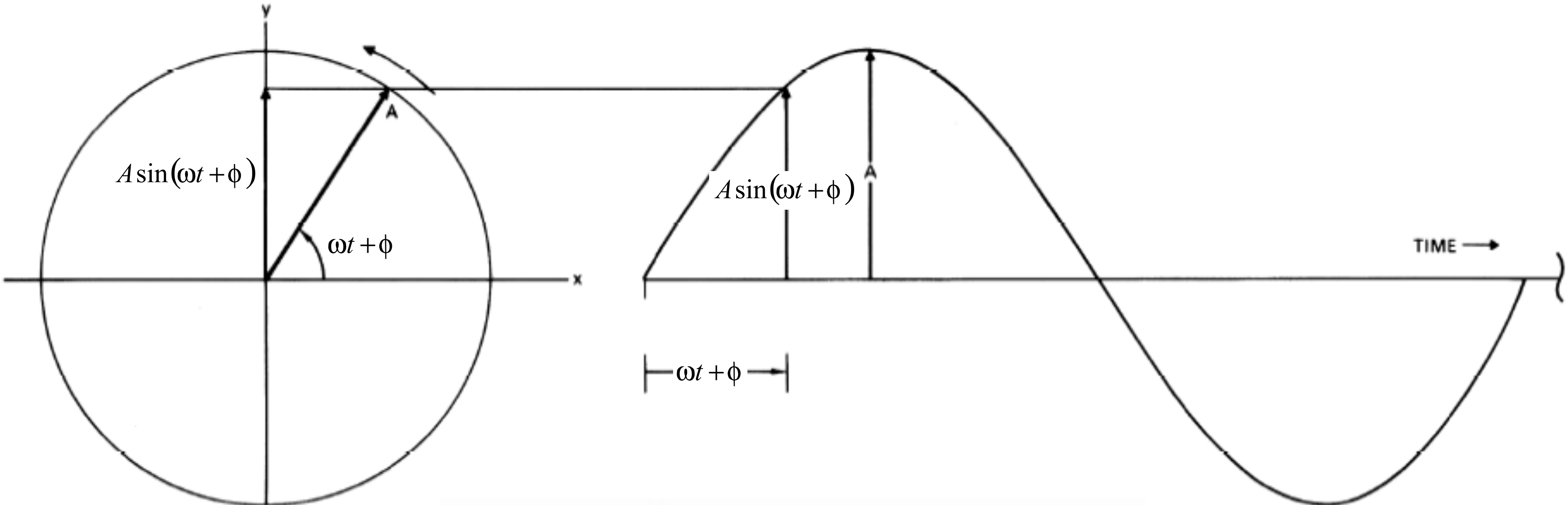
Volume target (e.g., the ionosphere)



Dopplah

Doppler Detection: Intuitive Approach

Phasor diagram is a graphical representation of a sine wave



<u>I & Q components*</u>

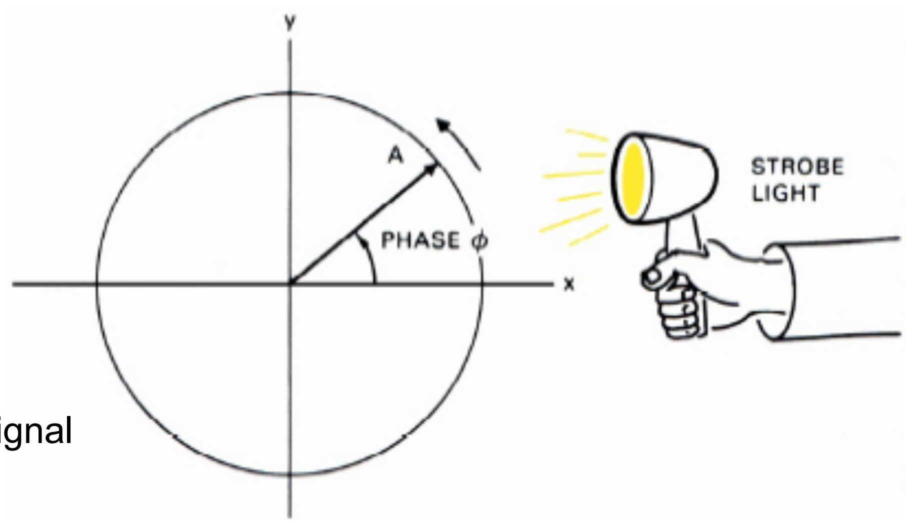
I => in-phase component

Acos(ϕ)

Q => in-quadrature component

Asin(ϕ)

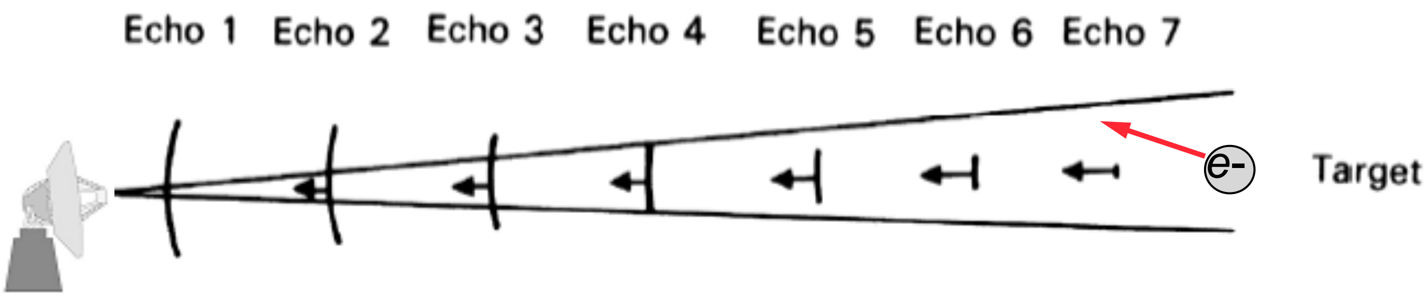
*relative to reference signal

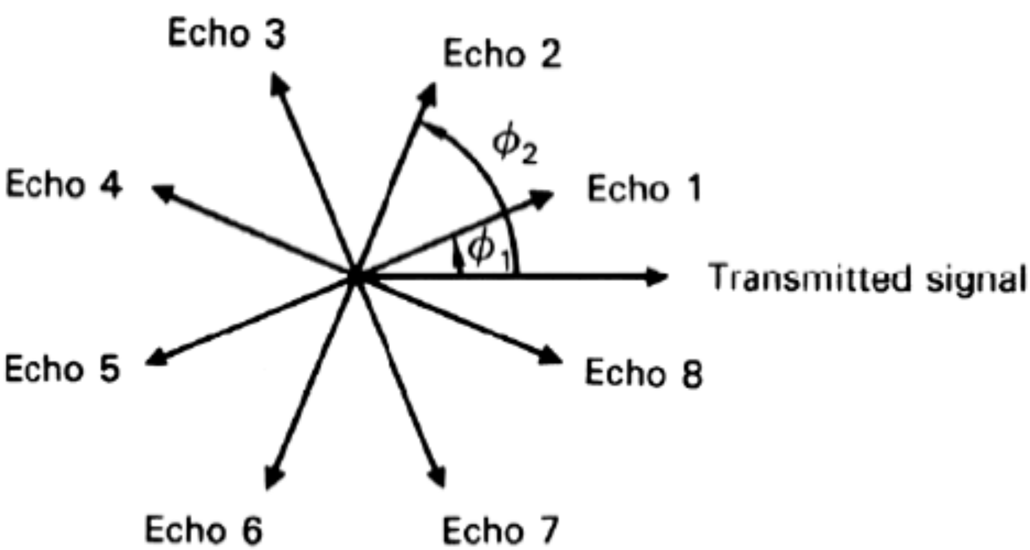


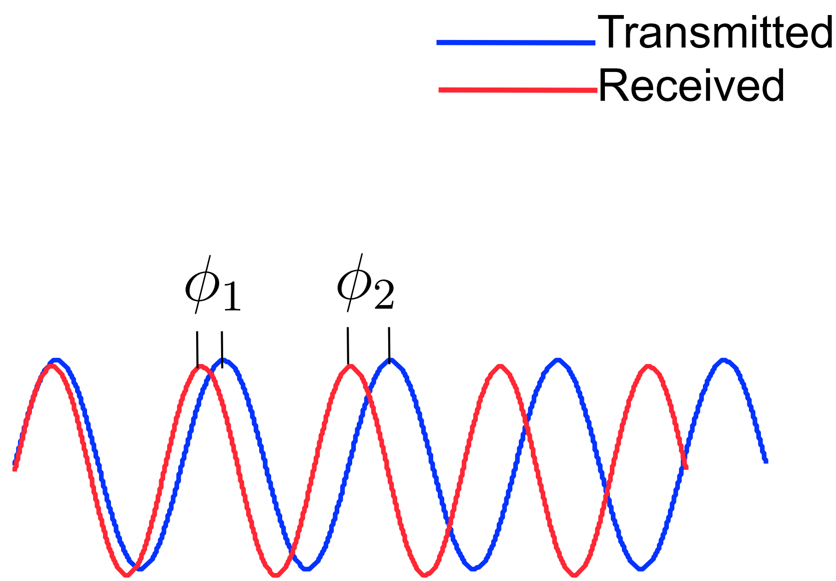
Consider strobe light as cosine reference wave at same frequency but with initial phase = 0

Doppler Detection: Intuitive Approach

Closing on target – positive Doppler shift





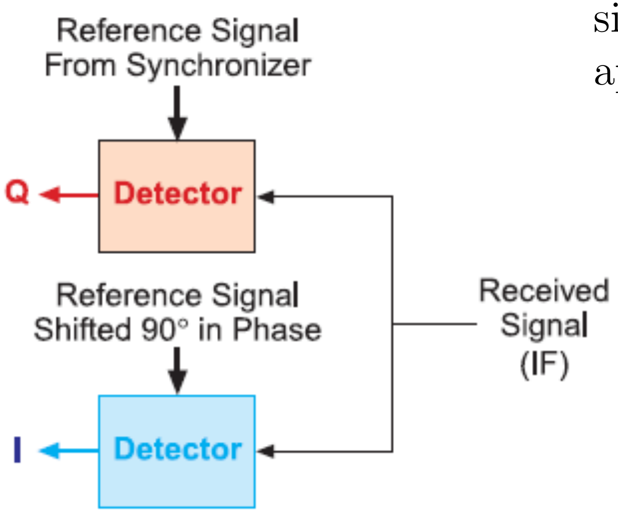


Target's Doppler frequency shows up as a pulse-to-pulse shift in phase.

I and Q Demodulation

We transmit an amplitude-modulated cosine of frequency ω_c. The received signal will have some time varying amplitute $a(t)$ and time-varying phase $\phi(t)$ applied to this,

$$p_{rec}(t) = a(t)\cos(\phi(t) + \omega_c t)$$



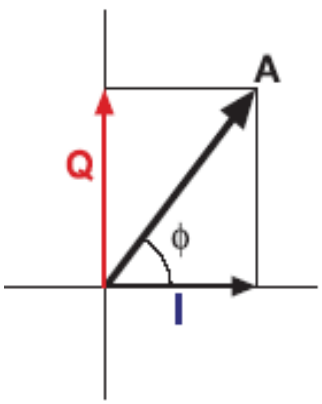
We compute the analytic signal through Euler's identity by "mixing" the signal with cosine and sine

in-phase (I) channel:

$$p_{rec}(t)\cos(\omega_c t) = a(t)\cos(\phi(t) + \omega_c t)\cos(\omega_c t)$$
$$= a(t)\tfrac{1}{2}\left(\underbrace{\cos(\phi(t) + 2\omega_c t)}_{\text{filter out}} + \cos\phi(t)\right)$$

quadrature (Q) channel (90° out of phase):

$$p_{rec}(t)\sin(\omega_c t) = a(t)\cos(\phi(t) + \omega_c t)\sin(\omega_c t)$$
$$= a(t)\tfrac{1}{2}\left(\underbrace{-\sin(\phi(t) + 2\omega_c t)}_{\text{filter out}} + \sin\phi(t)\right)$$



I and Q channels together give the *analytic signal*

$$\boxed{s_{rec}(t) = a(t)e^{i\phi(t)}}$$

The fundamental output of a pulsed Doppler radar is a time series of complex numbers.

I and Q Demodulation in Frequency Domain

Transmitted signal

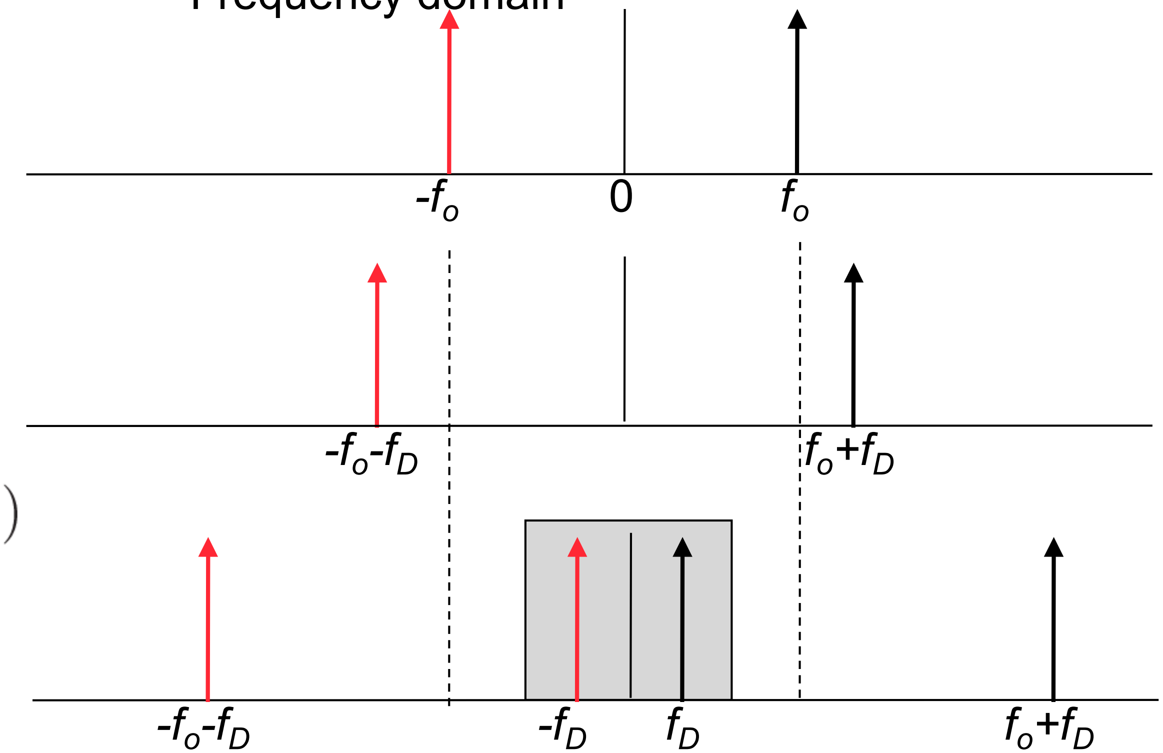
$$\cos(2\pi f_o t)$$

Doppler shifted

$$\cos(2\pi (f_o + f_D) t)$$

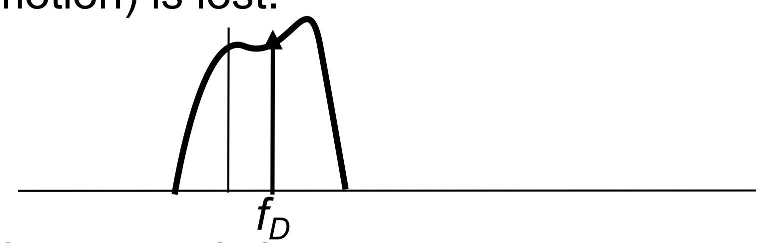
Mixed (multiplied) with carrier $\cos(2\pi f_o t)$

Frequency domain



Cosine is even function, so sign of f_D (and, hence, direction of motion) is lost.
What we need instead is:

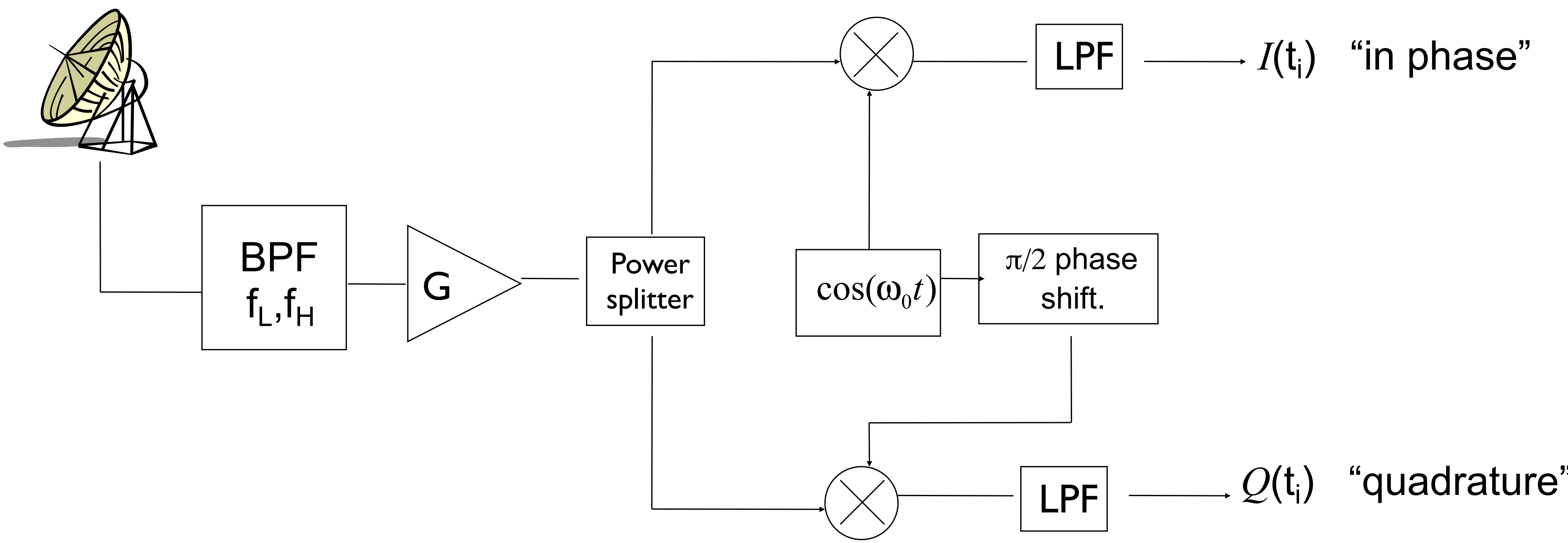
$$e^{j2\pi f_D t} = \cos(2\pi f_D t) + j \sin(2\pi f_D t)$$



The analytic signal $e^{j2\pi f_D t}$ cannot be measured directly, but the cos and sin components via mixing with two oscillators with same frequency but orthogonal phases. The components are called "in phase" (or *I*) and "in quadrature" (or *Q*):

$$Ae^{j2\pi f_D t} = I(t) + jQ(t)$$ FFT $$A\delta(f_D)$$ (for single scatterer)

ISR Receiver: I and Q plus correlation



We have time series of *V(t) =I(t) + jQ(t)*, how do I compute the Doppler spectrum?

Estimate the autocorrelation function (ACF) by computing products of complex voltages ("lag products")

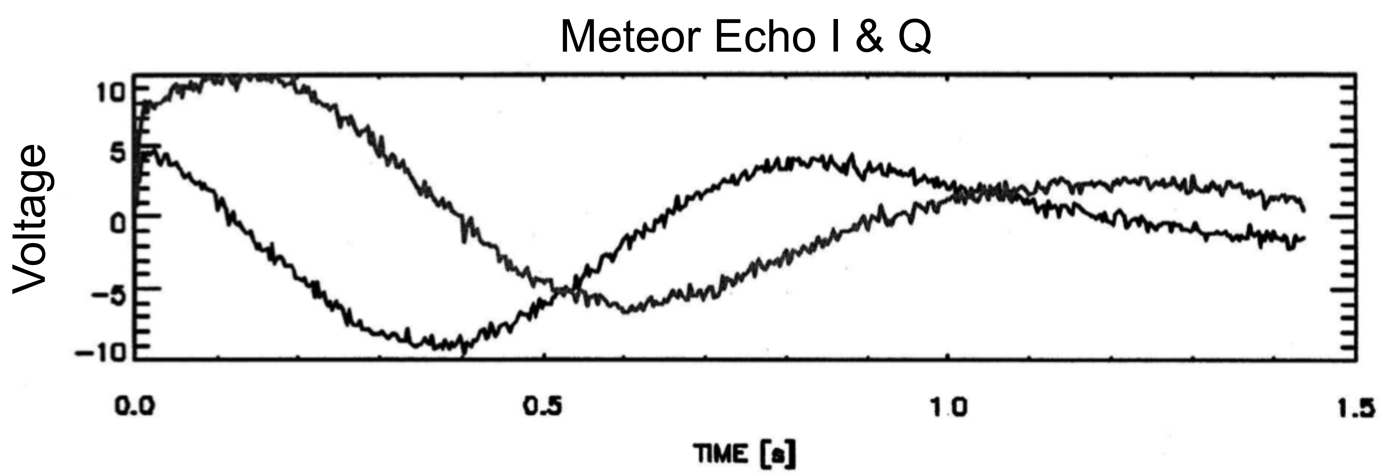
$$R_{vv}(\tau) = \frac{\langle V(t)V^{*}(t+\tau)\rangle}{S}$$

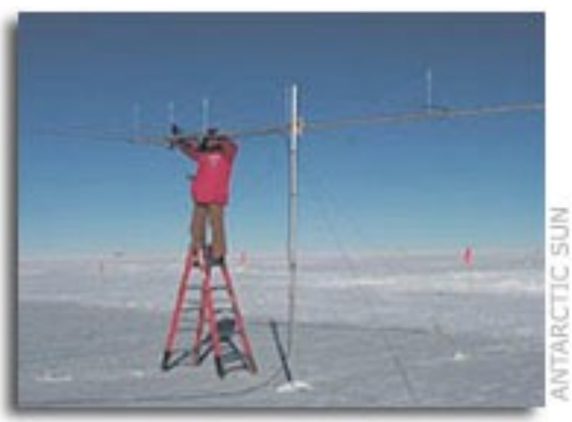
FFT

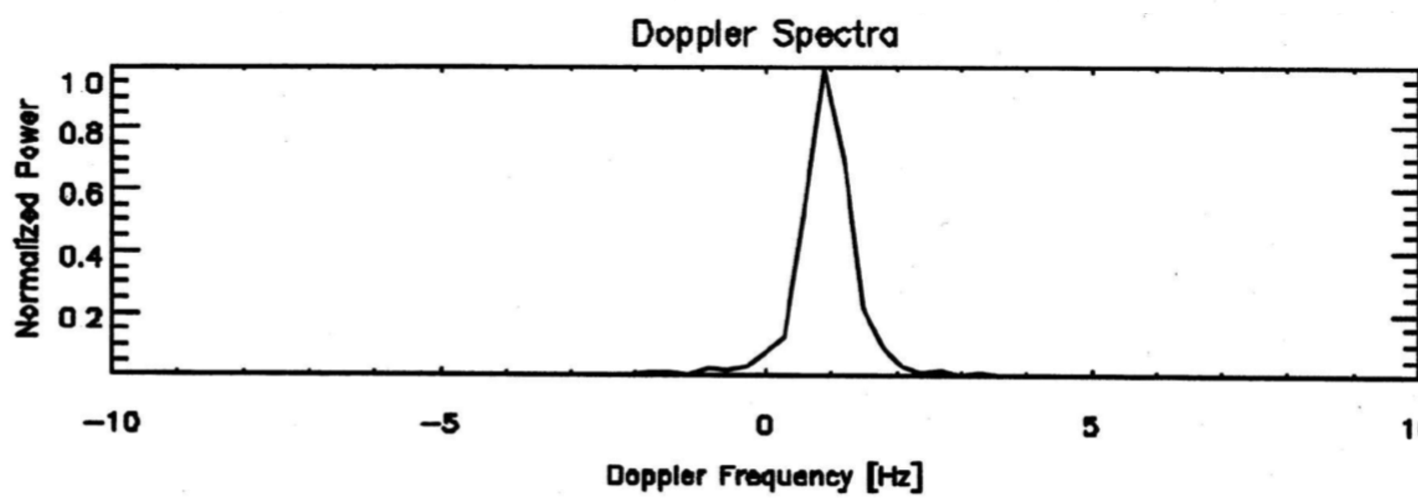
Power spectrum is Fourier Transform of the ACF

Example: Doppler Shift of a Meteor Trail

- Collect N samples of $I(t_k)$ and $Q(t_k)$ from a target
- Compute the complex FFT of $I(t_k)+jQ(t_k)$, and find the maximum in the frequency domain
- Or compute "phase slope" in time domain.







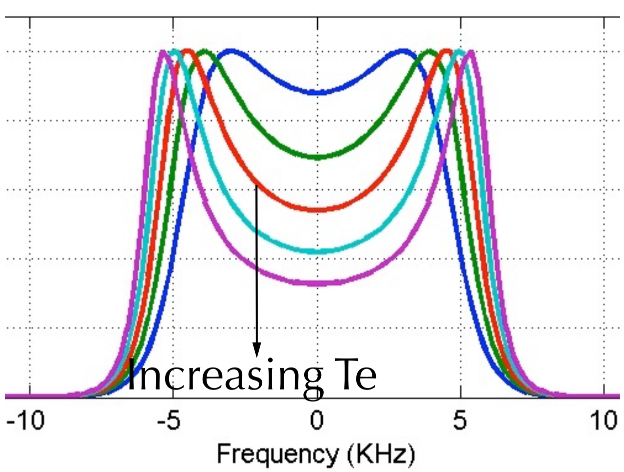
Does this strategy work for ISR?

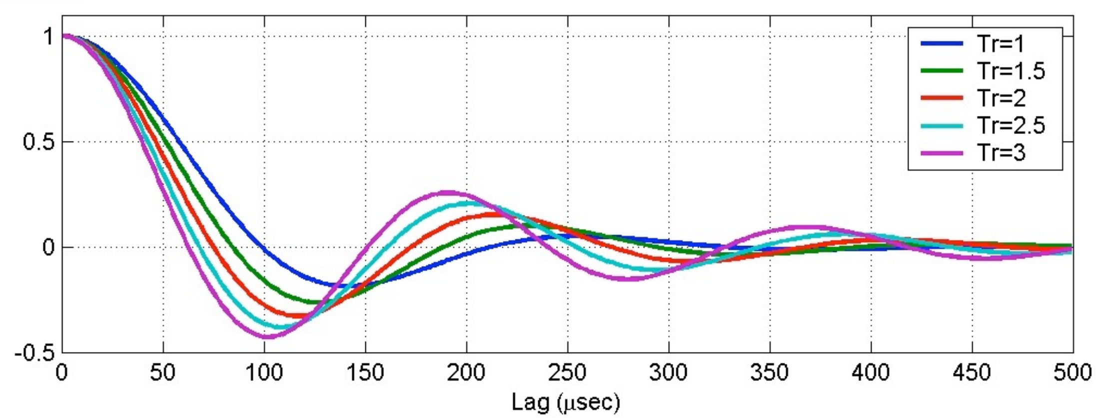
Typical ion-acoustic velocity: 3 km/s
Doppler shift at 450 MHz: 10kHz
Correlation time: 1/10kHz = 0.1 ms
Inter-pulse period (IPP) to reach 500 km: $2R/c$ = 3ms
Required PRF to probe ionosphere (500km range): 300 Hz

Plasma has completely decorrelated by the time we send the next pulse.

Alternately, the Doppler frequency shift imparted by the plasma is much higher than the maximum unambiguous Doppler defined by the pulse-repetition frequency.

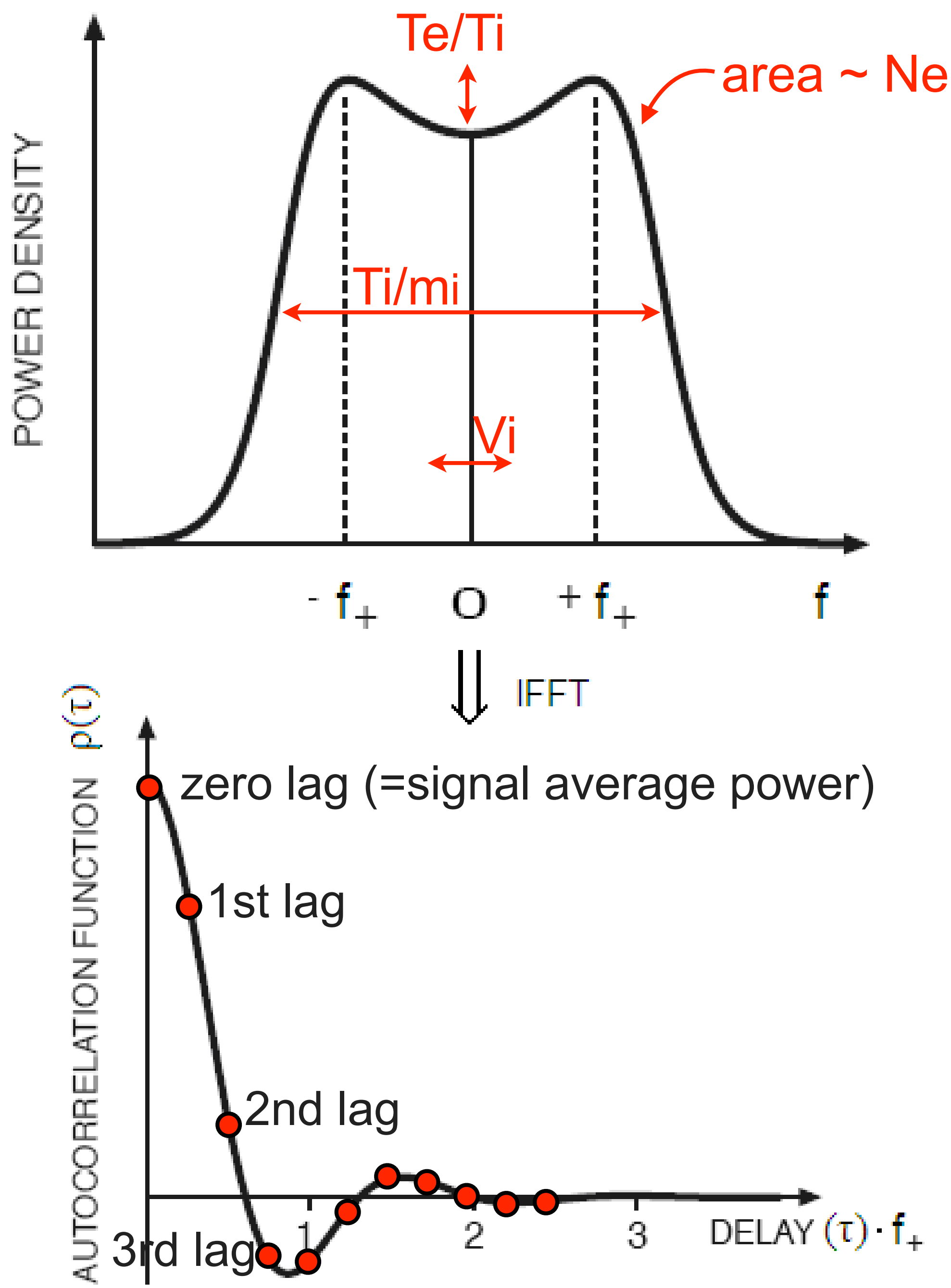
ISR spectrum ⟺ Autocorrelation function (ACF)





Samplin’

Autocorrelation function and power spectrum



Ion temperature (Ti) to ion mass (mi) ratio from the width of the spectra

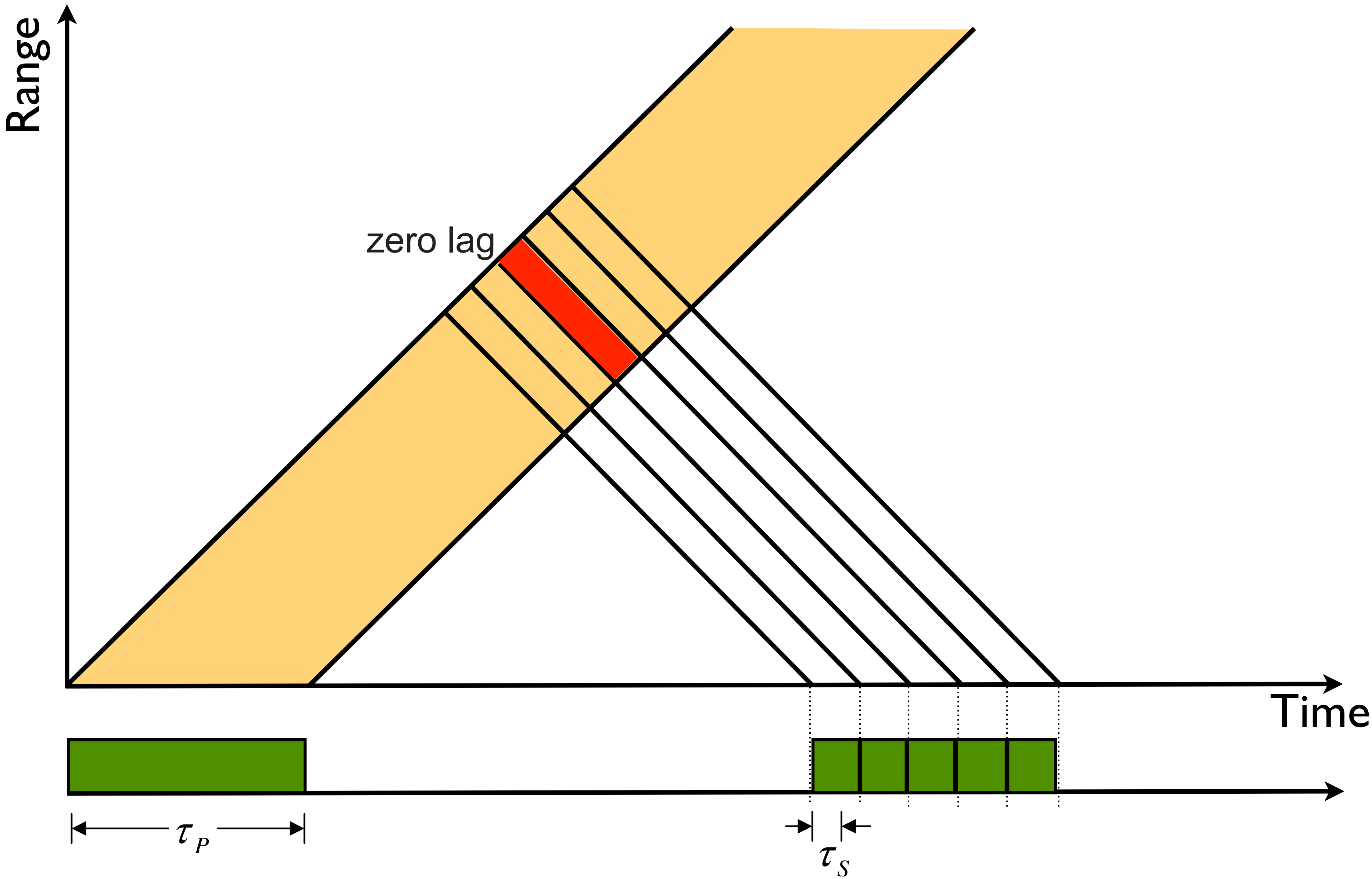
Electron to ion temperature ratio (Te/Ti) from "peak-to-valley" ratio

Electron (= ion) density from total area (corrected for temperatures)

Line-of-sight ion velocity (Vi) from bulk Doppler shift

Our goal is to compute lags

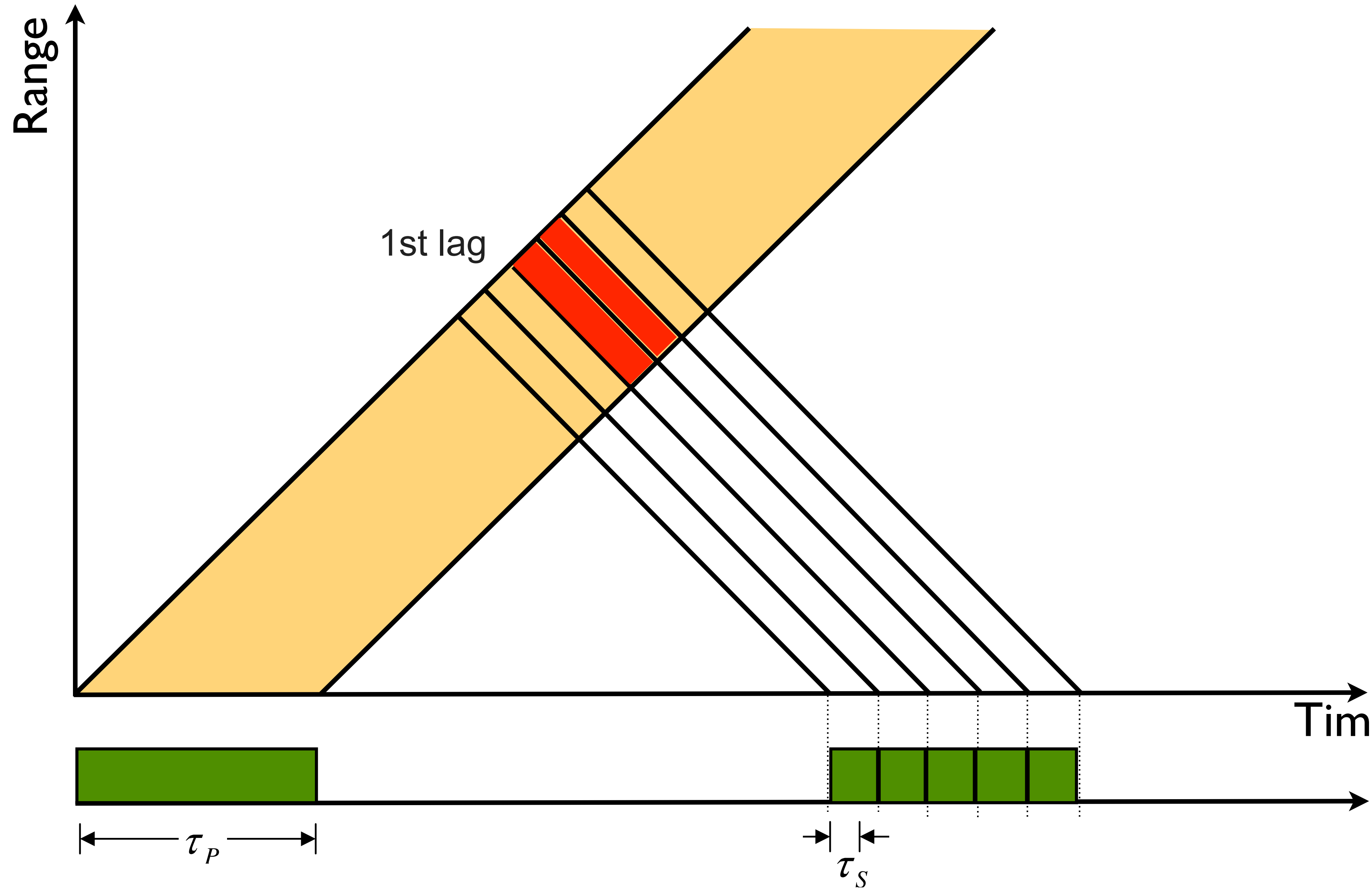
Computing the ACF (and, hence, spectrum)



τ_P = Length of RF pulse

τ_s = Sample Period (typically ~ 1/10 pulse length)

Computing the ACF (and, hence, spectrum)



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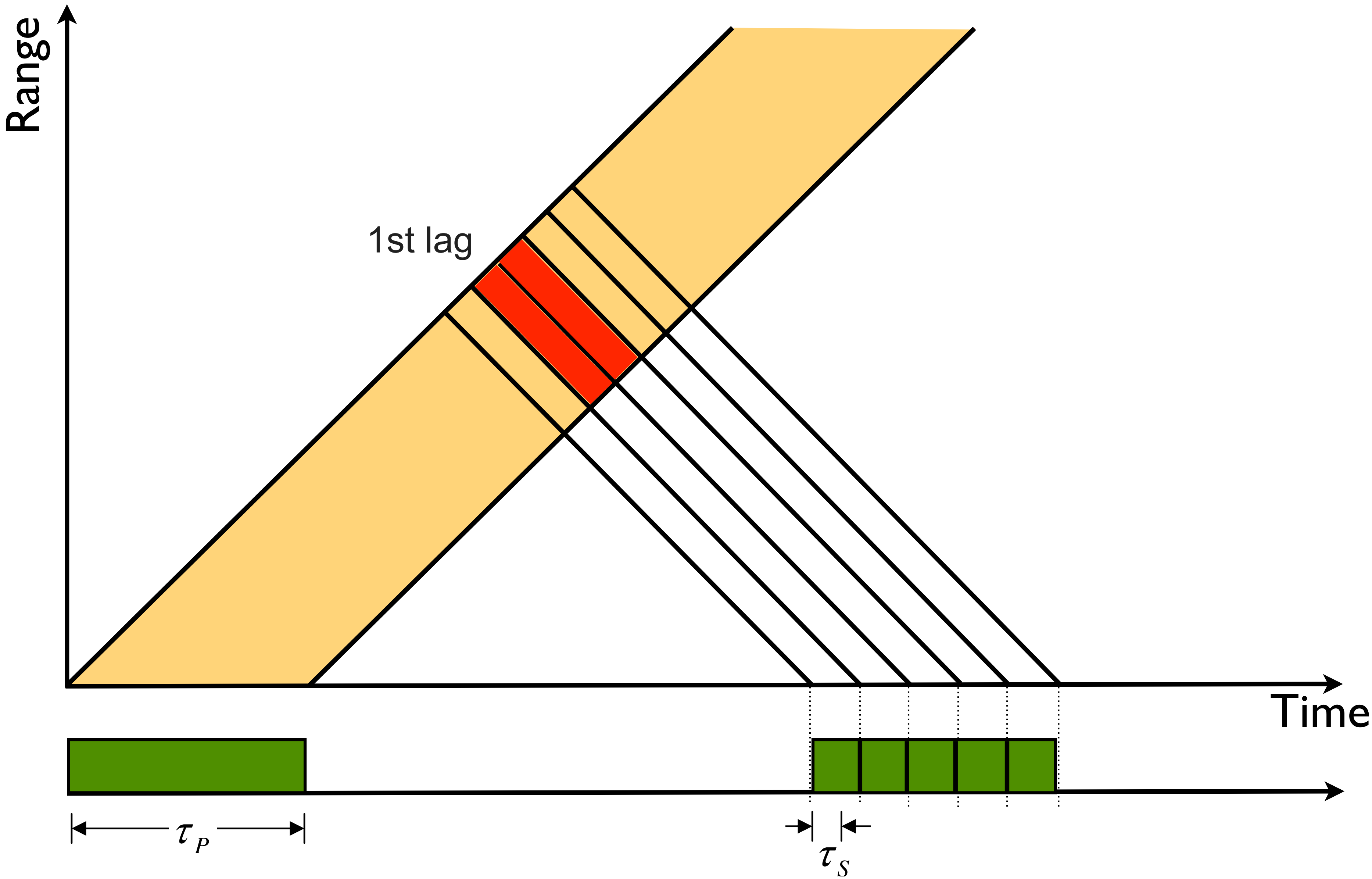

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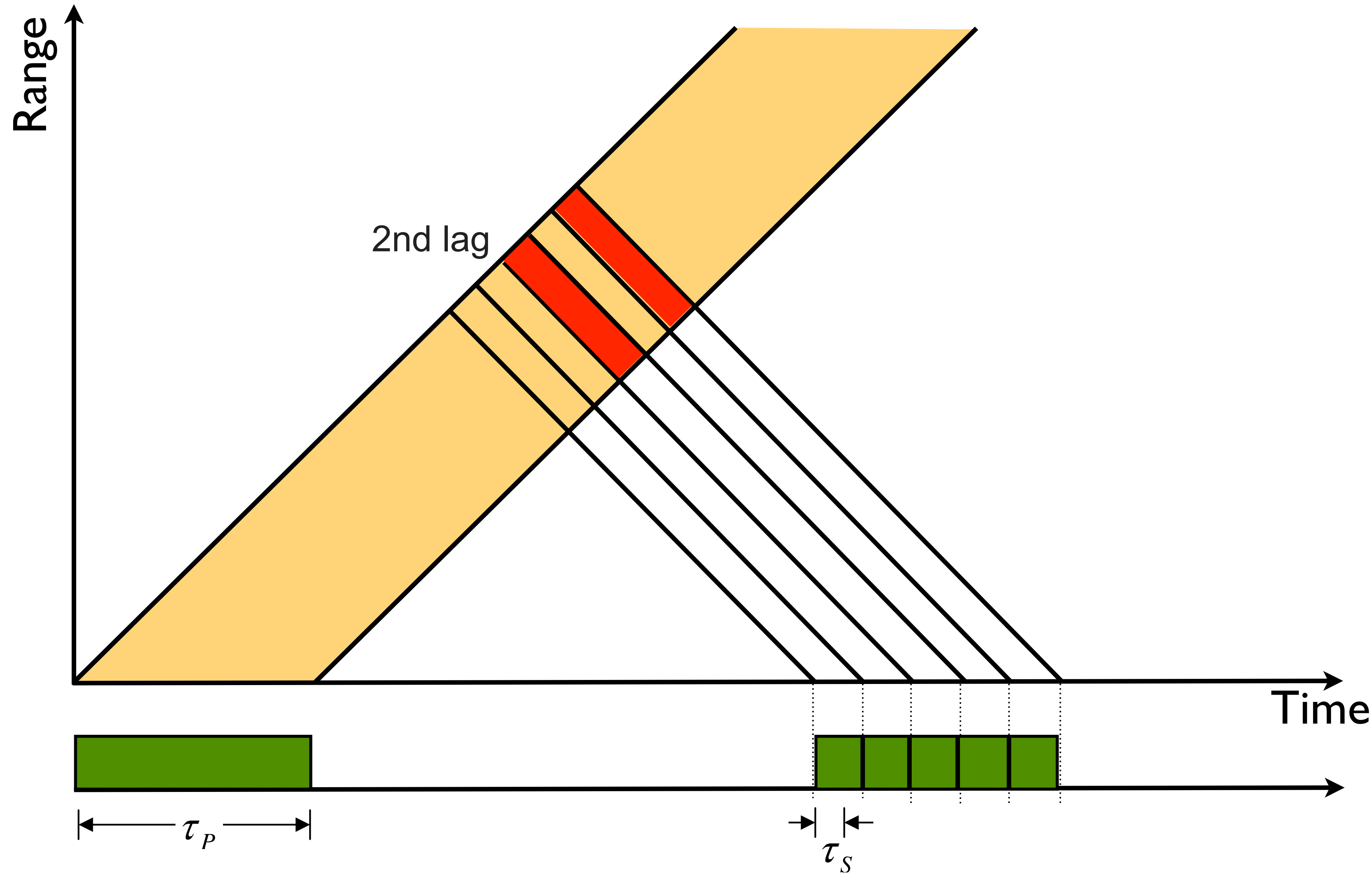

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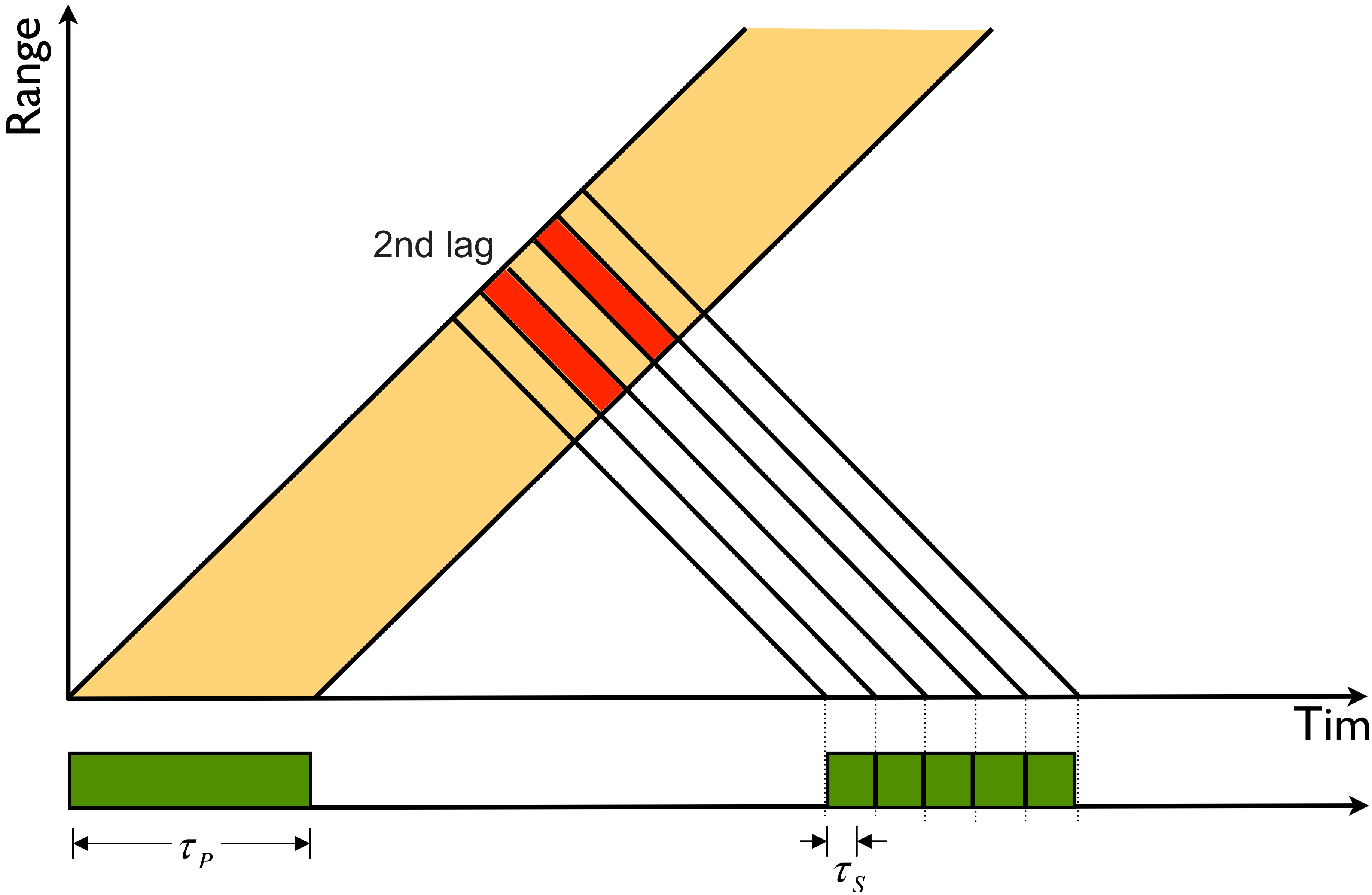

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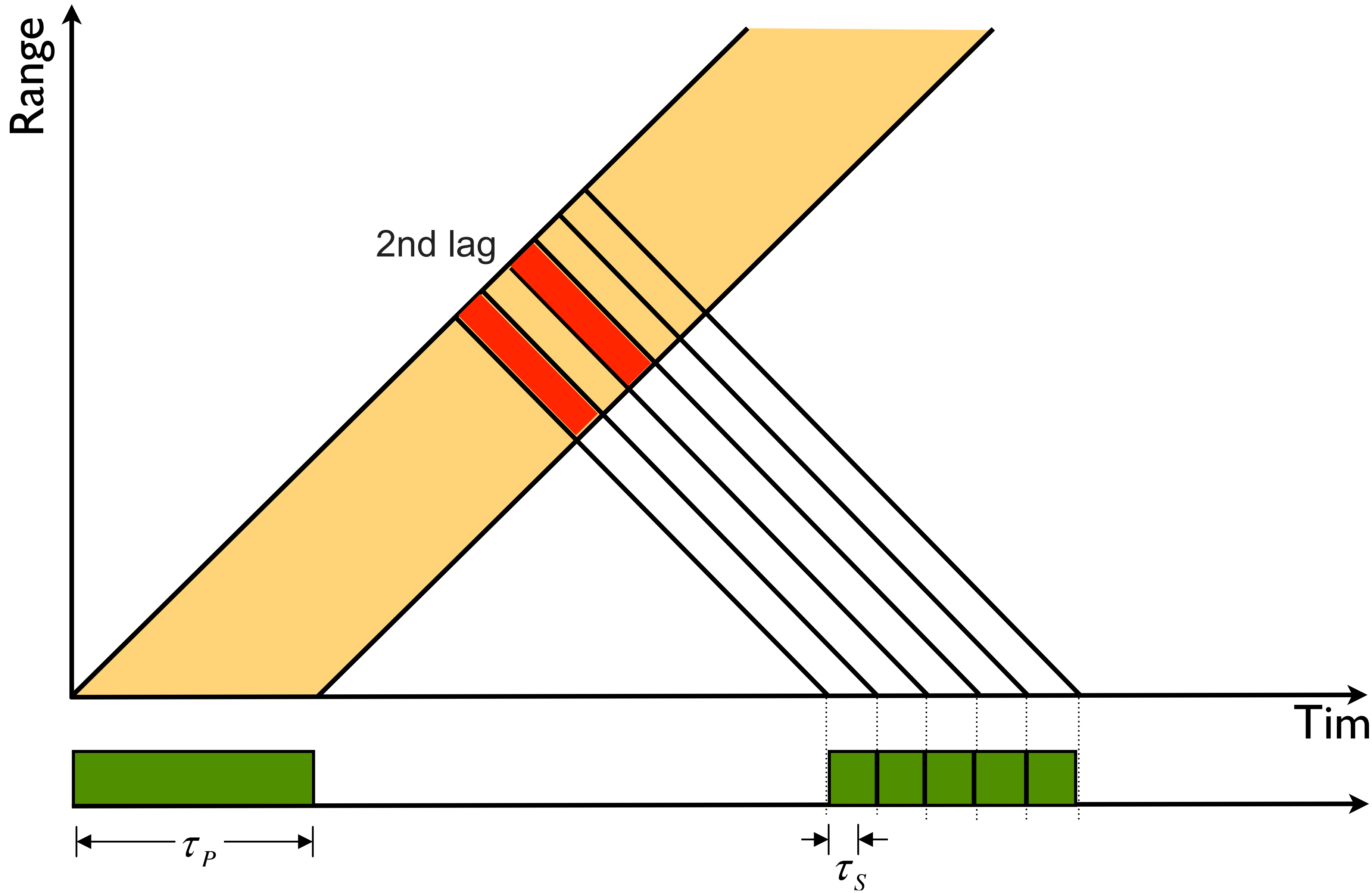

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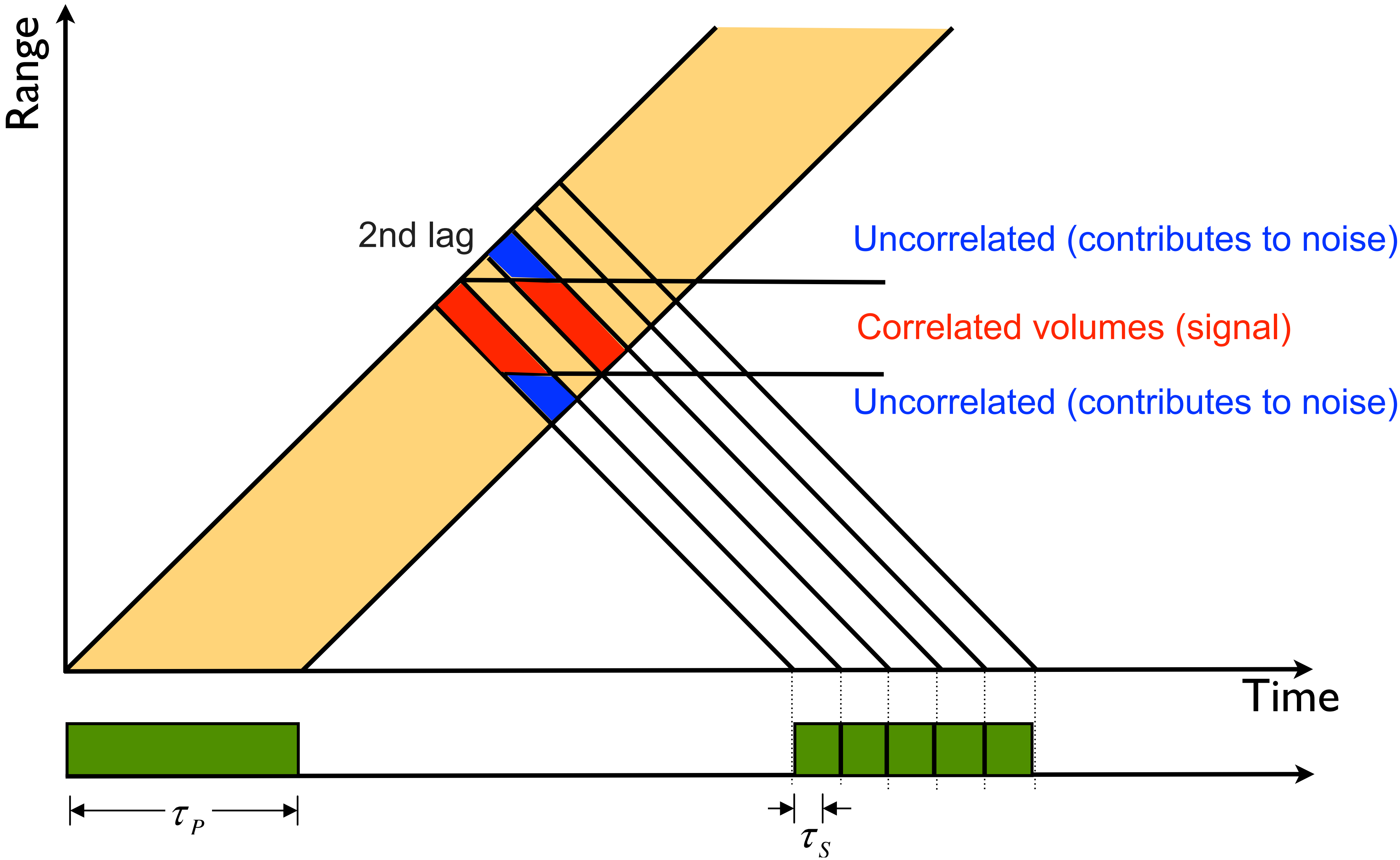
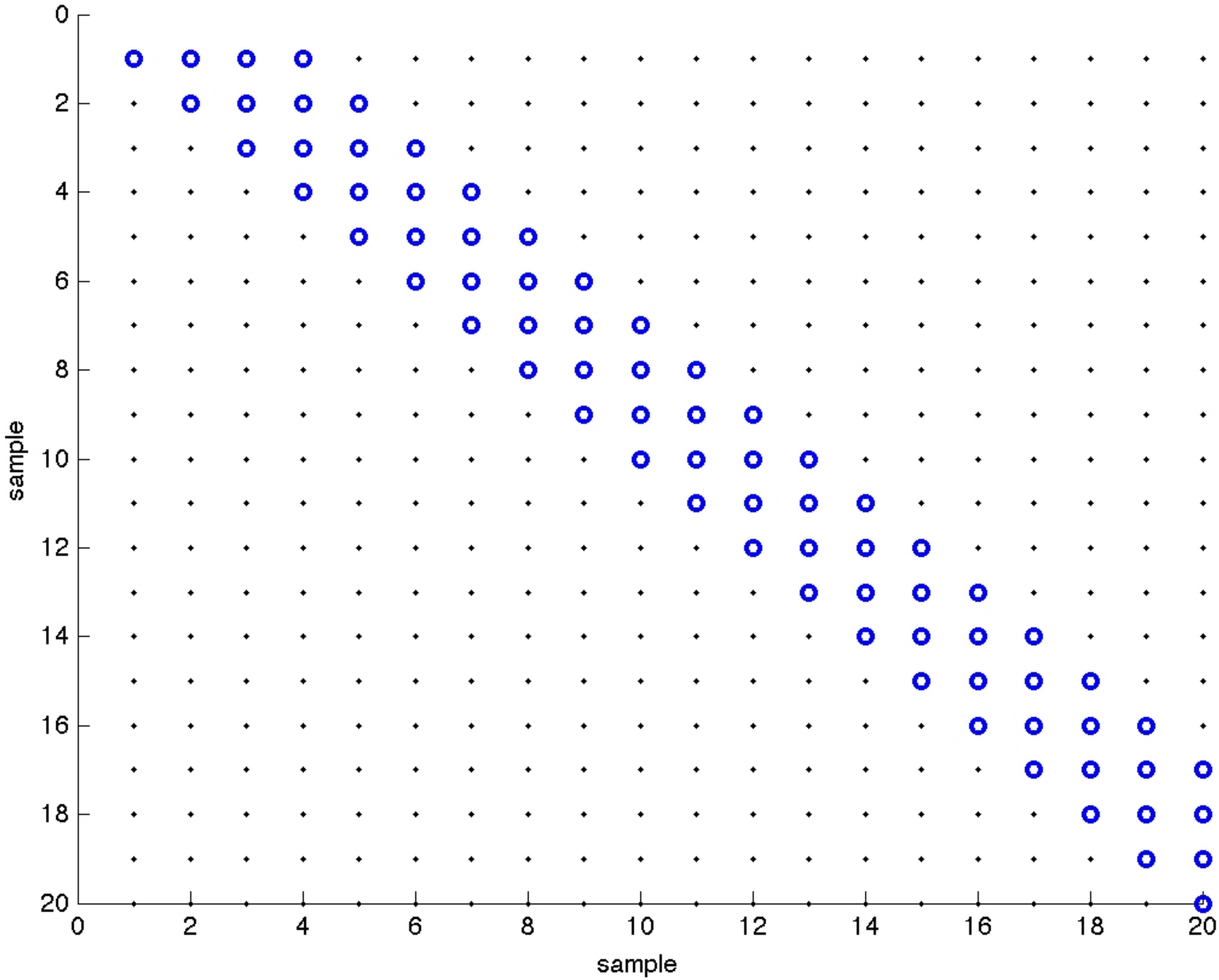


τ_P = Length of RF pulse

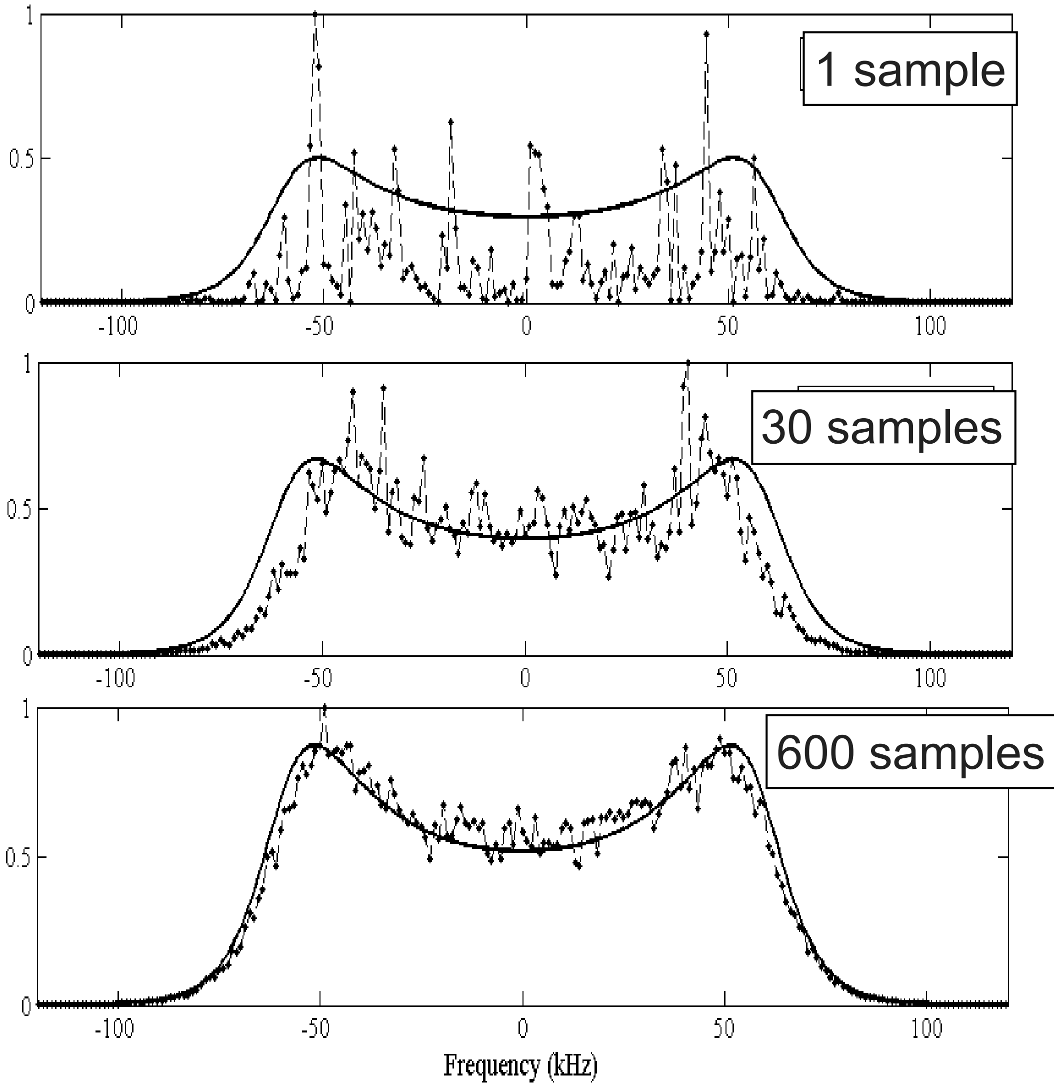
τ_s = Sample Period (typically ~ 1/10 pulse length)

Lag-product matrix



Incoherent Averaging

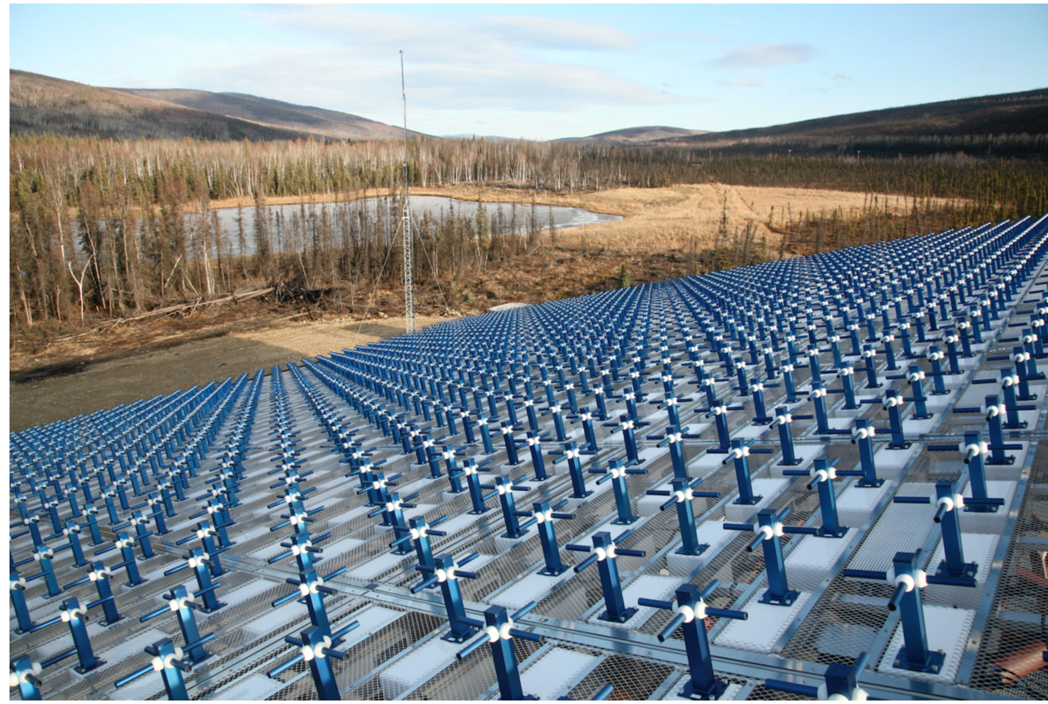
Normalized ISR spectrum for different integration times at 1290 MHz



We are seeking to estimate the power spectrum of a Gaussian random process. This requires that we sample and average many independent "realizations" of the process.

$$\text{Uncertainties} \propto \frac{1}{\sqrt{\text{Number of Samples}}}$$

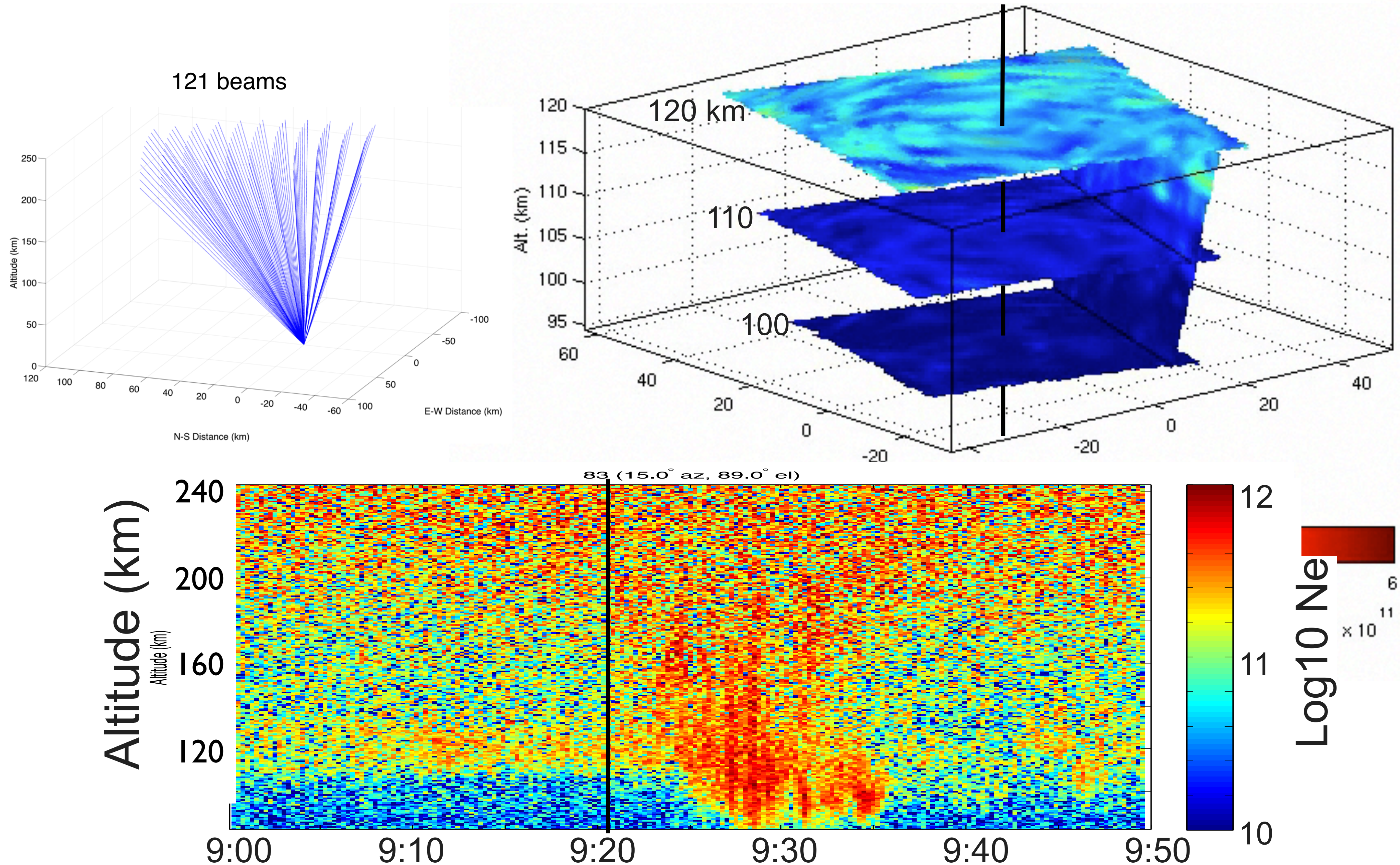
Dish Versus Phased-array



-FOV: Entire sky
-Integration at each position before moving
-Power concentrated at Klystron
-Significant mechanical complexity

-FOV: +/- 15 degrees from boresight
-Integration over all positions simultaneously
-Power distributed
-No moving parts

Three-dimensional ionospheric imaging



Semeter et al., JASTP 2009

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Radar signal processing

- Mahafza, *Radar Systems Analysis and Design Using MATLAB*
- Skolnik, *Introduction to Radar Systems*
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- Levanon, *Radar Principles*
- Blahut, *Theory of Remote Image Formation*
- Curlander, *Synthetic Aperture Radar: Systems and Signal Analysis*

Background (Electromagnetics, Signal Processing:

- Ulaby, *Fundamentals of Engineering Electromagnetics*
- Cheng, *Field and Wave Electromagnetics*
- Oppenheim, Willsky, and Nawab, *Signals and Systems*
- Mitra, *Digital Signal Processing: A Computer-based Approach*

For fun:

http://mathforum.org/mbower/johnandbetty/frame.htm