Phased Arrays for Atmospheric and Geospace Science

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Superposition Principle

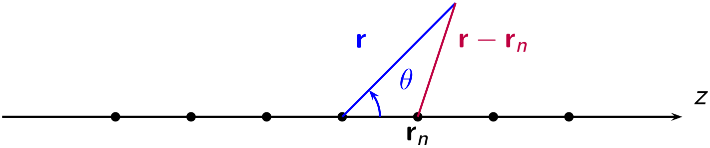
Maxwell's Equations are Linear:

$$\mathbf{J}_1 = \frac{1}{\mu_0}\nabla \times (\mathbf{B}_1) - \epsilon_0 \frac{\partial}{\partial t}(\mathbf{E}_1)$$
$$0 = \nabla \times (\mathbf{E}_1) + \frac{\partial}{\partial t}(\mathbf{B}_1)$$

$$\mathbf{J}_2 = \frac{1}{\mu_0}\nabla \times (\mathbf{B}_2) - \epsilon_0 \frac{\partial}{\partial t}(\mathbf{E}_2)$$
$$0 = \nabla \times (\mathbf{E}_2) + \frac{\partial}{\partial t}(\mathbf{B}_2)$$

$$\mathbf{J}_1 + \mathbf{J}_2 = \frac{1}{\mu_0}\nabla \times (\mathbf{B}_1 + \mathbf{B}_2) - \epsilon_0 \frac{\partial}{\partial t}(\mathbf{E}_1 + \mathbf{E}_2)$$
$$0 = \nabla \times (\mathbf{E}_1 + \mathbf{E}_2) + \frac{\partial}{\partial t}(\mathbf{B}_1 + \mathbf{B}_2)$$

Superposition Applied to Antenna Arrays



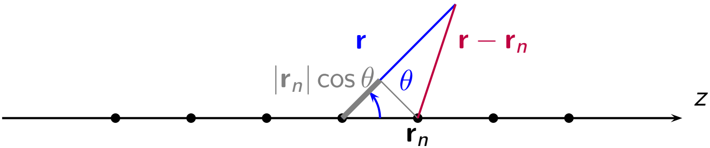
Fields radiated by single element at the origin with applied current I_0:

$$\mathbf{E} = \mathbf{E}_0 I_0 \frac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|}$$

Fields radiated by entire array:

$$\mathbf{E} = \mathbf{E}_0 \sum_{n=0}^{N-1} I_n \frac{e^{-jk|\mathbf{r}-\mathbf{r}_n|}}{|\mathbf{r}-\mathbf{r}_n|}$$

Far Field Approximation (Fraunhofer Zone)



If $\mathbf{r}$ and $\mathbf{r} - \mathbf{r}_n$ are almost parallel lines:

$$\mathbf{r} - \mathbf{r}_n \approx \mathbf{r} - |\mathbf{r}_n| \cos\theta \hat{r}$$

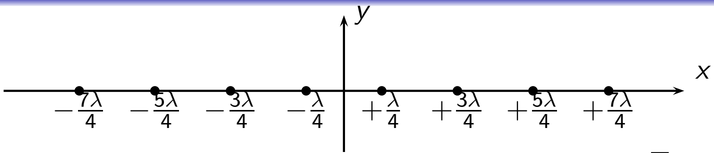
Assume $|\mathbf{r}_n| \ll |\mathbf{r}|$:

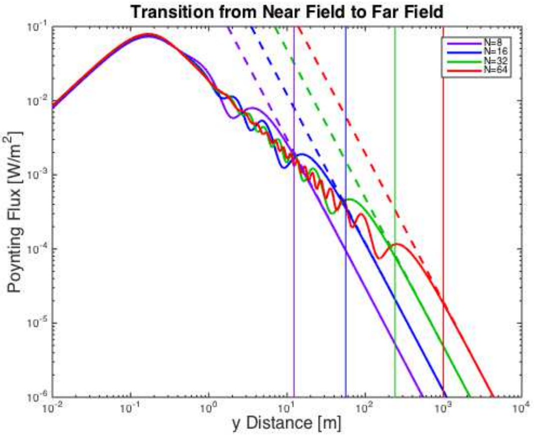
$$|\mathbf{r} - \mathbf{r}_n| \approx |\mathbf{r}| \text{ for demoninator terms}$$

$$-jk\,|\mathbf{r} - \mathbf{r}_n| \approx -jk\,|\mathbf{r}| + jk\,|\mathbf{r}_n| \cos\theta$$

$$\mathbf{E} \approx \underbrace{\mathbf{E}_0 \frac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|}}_{\text{Element Factor}} \underbrace{\sum_{n=0}^{N-1} I_n e^{jk|\mathbf{r}_n|\cos\theta}}_{\text{Array Factor}}$$

Distance to Far Field: Fresnel Numbers





Transition from near to far determined by the **Fresnel Number**:

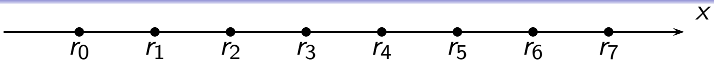
$$\frac{L^2}{r\lambda} \ll 1 \rightarrow \text{Far Field}$$

$$\frac{L^2}{r\lambda} > 1 \rightarrow \text{Near Field}$$

L = Array length

λ = wavelength

1-D Linear Phased Array



$$|\mathbf{r}_n| = nd \qquad I_n = e^{jn\alpha}$$

Array Factor:

$$F = \sum_{n=0}^{N-1} e^{jn\alpha} e^{jknd\cos\theta}$$

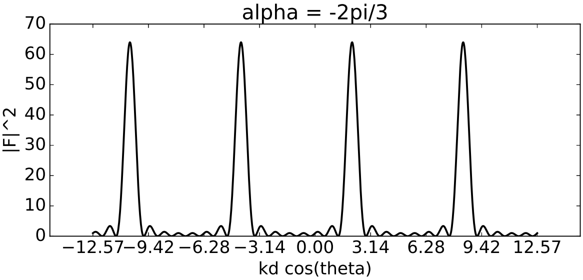
$$= \frac{1 - e^{jN\alpha + jNkd\cos\theta}}{1 - e^{j\alpha + jkd\cos\theta}}$$

$$= e^{j\frac{(N-1)}{2}(kd\cos\theta + \alpha)} \frac{\sin\left[\frac{N}{2}(kd\cos\theta + \alpha)\right]}{\sin\left[\frac{1}{2}(kd\cos\theta + \alpha)\right]}$$

$$|F|^2 = \frac{\sin^2\left[\frac{N}{2}(kd\cos\theta + \alpha)\right]}{\sin^2\left[\frac{1}{2}(kd\cos\theta + \alpha)\right]}$$

1-D Linear Phased Array Cont.

$$|F|^2 = \frac{\sin^2\left[\frac{N}{2}\left(kd\cos\theta + \alpha\right)\right]}{\sin^2\left[\frac{1}{2}\left(kd\cos\theta + \alpha\right)\right]}$$



Peak appears when $kd\cos\theta = -\alpha$

Additional peaks could appear when $kd\cos\theta = -\alpha + 2\pi m$ (Grating Lobes)

Visible Region: $0 < \theta < \pi \rightarrow -kd < kd\cos\theta < kd$

Grating Lobes

- $d < \lambda/2 \rightarrow kd < \pi$: No grating lobes will ever appear
- $\lambda/2 < kd < \lambda \rightarrow \pi < kd < 2\pi$: Grating lobes will only appear at some steering angles
- $d > \lambda \rightarrow kd > 2\pi$: Grating lobes will always appear

Example of linear array with $d = 2\lambda/3$ spacing

Movie

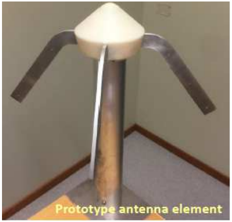
Mutual Coupling

- The true element factor for antennas in an array is different from the same type of antennas in isolation
  - Scattering off of neighboring antennas
  - Inductive coupling involving antenna near-fields

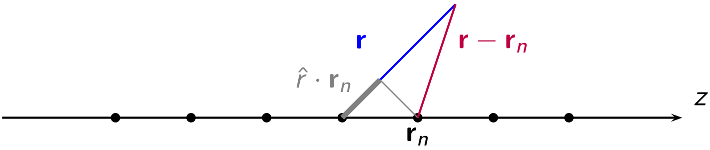
Two possible solutions

- Use a larger antenna separation and live with a limited grating-lobe free steering range.
- Use specially designed antennas to minimize coupling

EISCAT_3D Prototype Drooped Dipole →



Multi-Dimensional Arrays



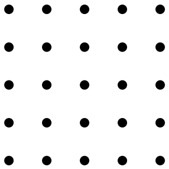
$$-jk\left|\mathbf{r}-\mathbf{r}_n\right| \approx -jk\left|\mathbf{r}\right| + jk\left(\hat{r}\cdot\mathbf{r}_n\right)$$

$$\mathbf{E} \approx \underbrace{\mathbf{E}_0 \frac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|}}_{\text{Element Factor}} \underbrace{\sum_{n=0}^{N-1} I_n e^{jk(\hat{r}\cdot\mathbf{r}_n)}}_{\text{Array Factor}}$$

In spherical coordinates:

$$\hat{r}\cdot\mathbf{r}_n = x_n \cos\phi \sin\theta + y_n \sin\phi \sin\theta + z_n \cos\theta$$

2-D Rectangular Array



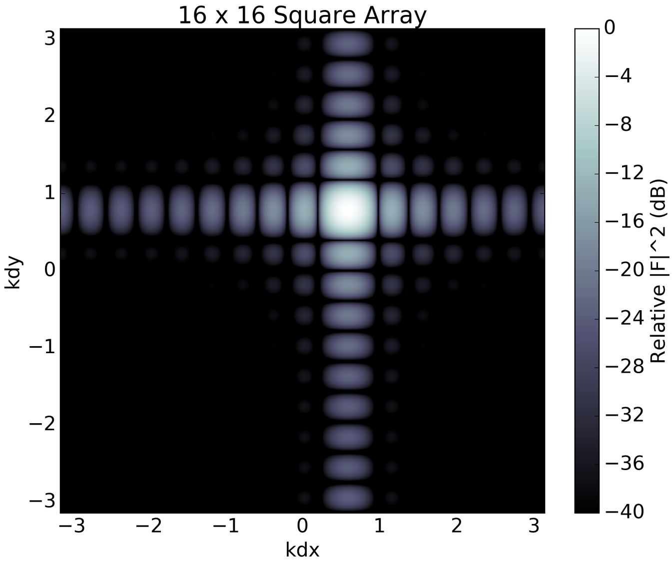
$$\mathbf{r}_{nm} = n d_x \hat{\mathbf{x}} + m d_y \hat{\mathbf{y}} \qquad I_{nm} = e^{j(n\alpha + m\beta)}$$

Array Factor:

$$|F(\theta,\phi)|^2 = \left| \sum_{n=0}^{N_x-1} \sum_{m=0}^{N_y-1} e^{j(nkd_x \cos\phi \sin\theta + n\alpha + mkd_y \sin\phi \sin\theta + m\beta)} \right|^2$$

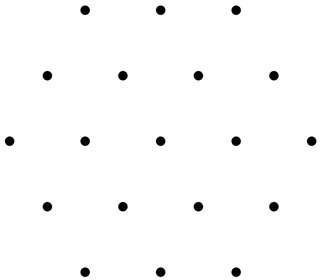
$$= \frac{\sin^2\left[\frac{N_x}{2}\left(kd_x \cos\phi \sin\theta + \alpha\right)\right]}{\sin^2\left[\frac{1}{2}\left(kd_x \cos\phi \sin\theta + \alpha\right)\right]} \frac{\sin^2\left[\frac{N_y}{2}\left(kd_y \sin\phi \sin\theta + \beta\right)\right]}{\sin^2\left[\frac{1}{2}\left(kd_y \sin\phi \sin\theta + \beta\right)\right]}$$

2-D Rectangular Array



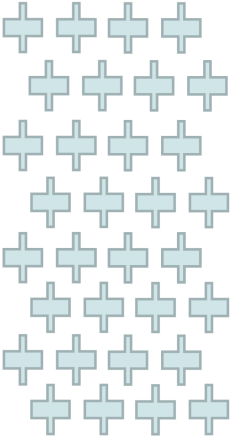
Hexagonal Spacing

Hexagon



Honeycomb Rectangular Array

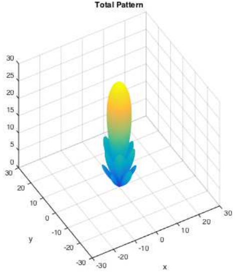
One AMISR panel:



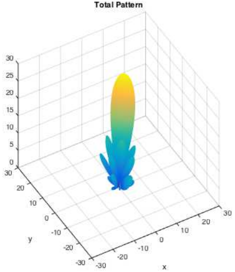
Steering A Hexagonal Array

19-antenna hexagon with $d = 3\lambda/4$.

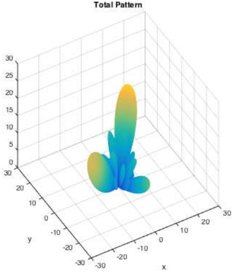
$\theta = 0, \phi = 0$



$\theta = 20^\circ, \phi = 45^\circ$

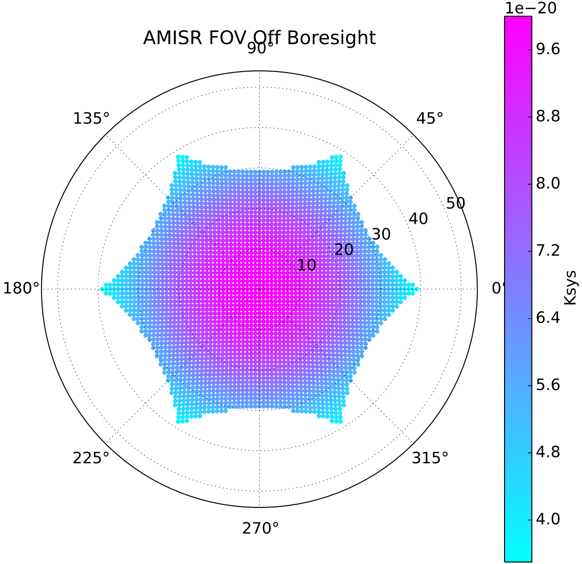


$\theta = 45^\circ, \phi = 45^\circ$



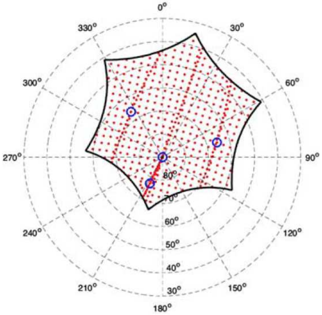
AMISR Antenna Properties

- FOV limited by grating lobe limit $\sim 30^\circ - 40^\circ$
- Antenna gain decreases with steering angle off of boresight
- Antenna works best within $\sim 25^\circ$ off of boresight



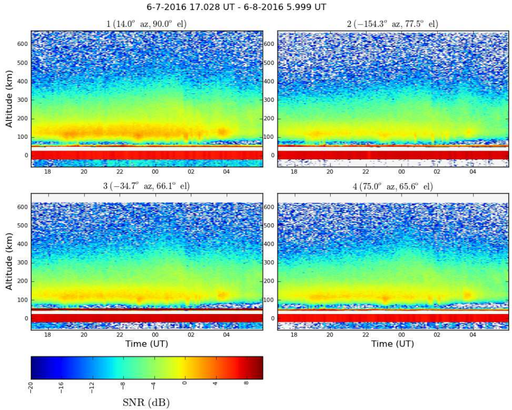
The PFISR Up-B Compromise

IPY Beam Pattern



The Up-B beam is close to the grating lobe limit, and therefore has reduced sensitivity.

Reduced SNR in Up-B (Beam 2)



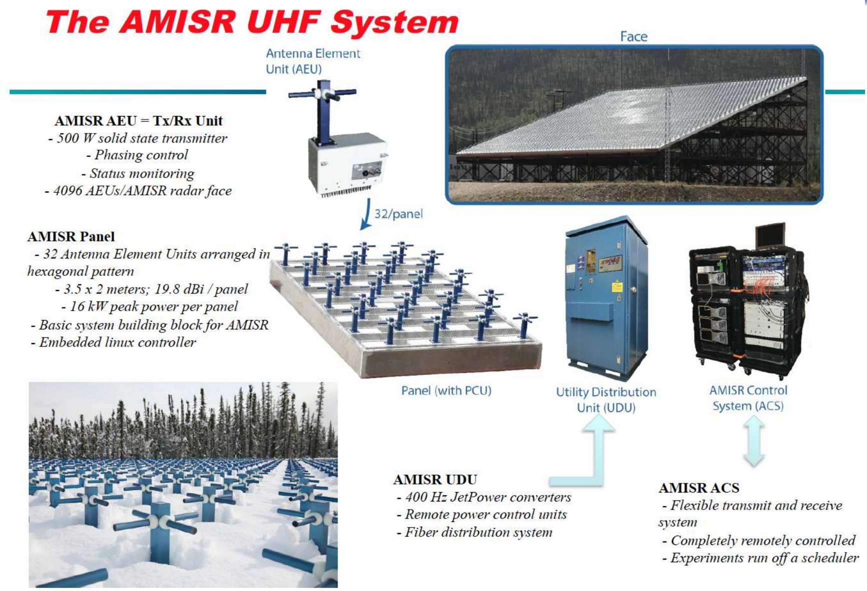
Passive Phased Arrays: Jicamarca

- One transmitter feeds entire array

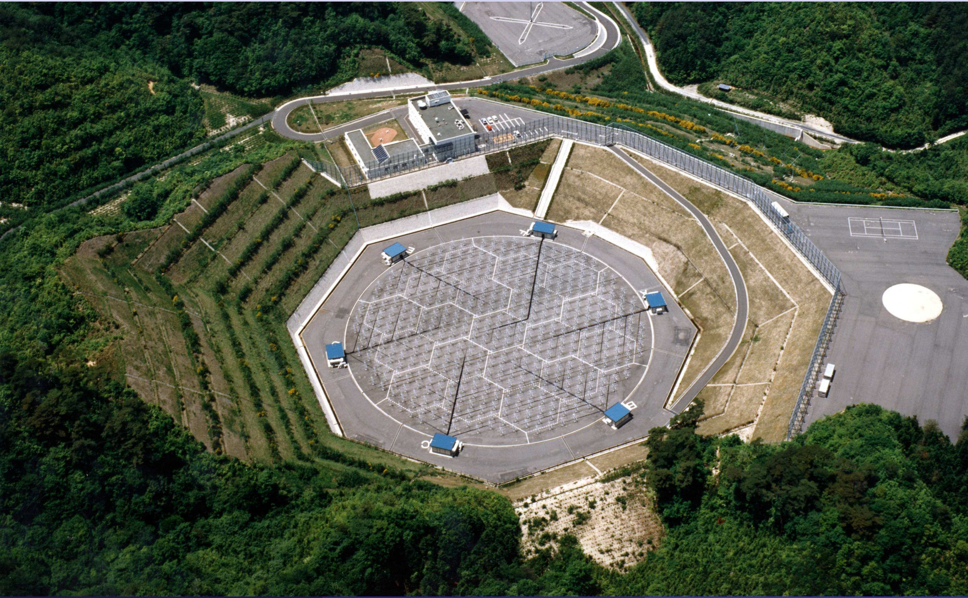
Passive Phased Arrays: Jicamarca

Manual Phasing (Jicamarca)

Active Electronically Steerable Phased Arrays



MU Radar



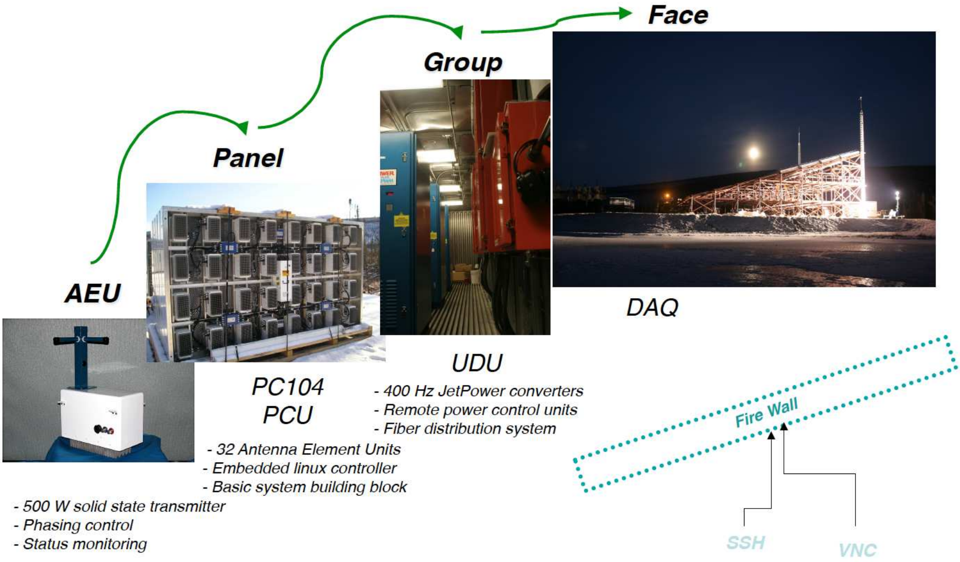
MU Radar

MU Radar Cabling

MU Radar Cabling

MU Radar Power Amplifiers

Advanced Modular Incoherent Scatter Radar

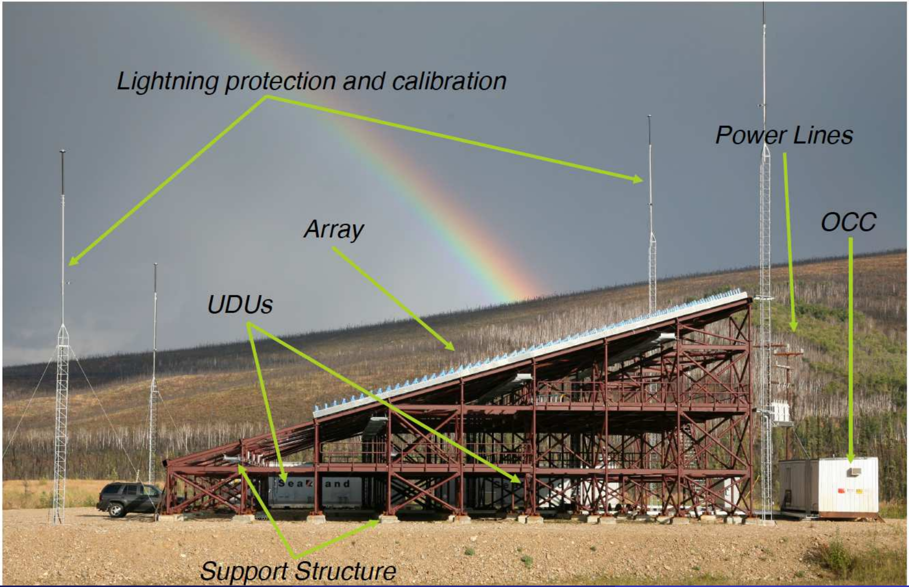


Antenna Element Unit (AEU) Specifications

- Distributed Solid State Power Amplifiers (SSPAs)
- 430-450 MHz instantaneous bandwidth
- 10% Maximum duty cycle
- Minimum PRF interval 500 usec
- Maximum pulsewidth 2 msec
- Passive cooling (no moving parts
- 400 Hz prime power

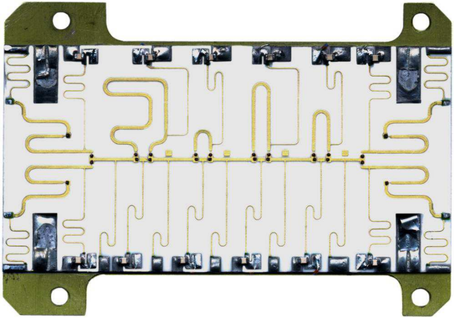
- Crossed dipoles, circular polarization on axis
- Balun built into the antenna support shaft
- Constant impendence over bandwidth and scan angle
- Spacing is hexagonal for efficiency
- Tx/Rx polarizations are opposite and fixed (not measureable)

Poker Flat Incoherent Scatter Radar (PFISR)



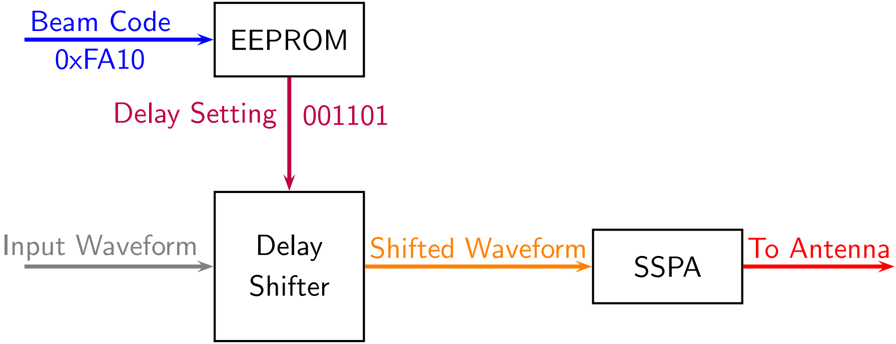
Electronic Steering with Delay Shifters

Example 4-bit delay shifter:

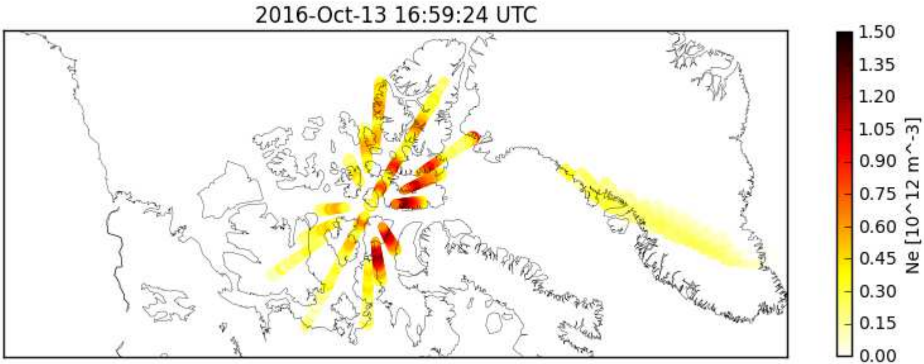


- AMISR uses 6-bit delay shifters
- $2^6 = 64$ steps spaced by $\pi/32 = 5.625°$

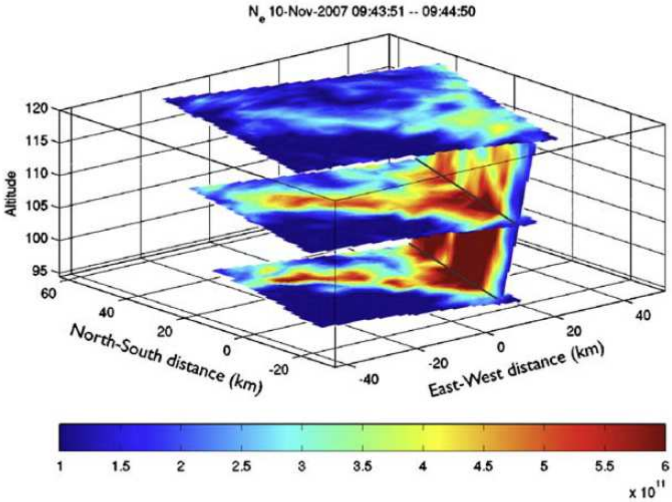
Conceptual Diagram of Steering with AMISR



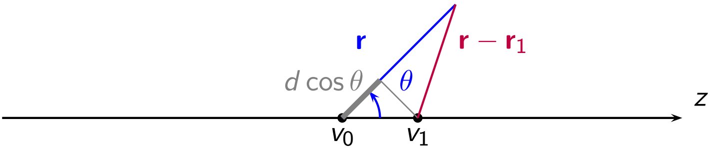
Differences Between AMISR and Scanning Radars



Imaging Auroral Structure [Semeter et al. (2009)]



Two Antenna Inteferometry

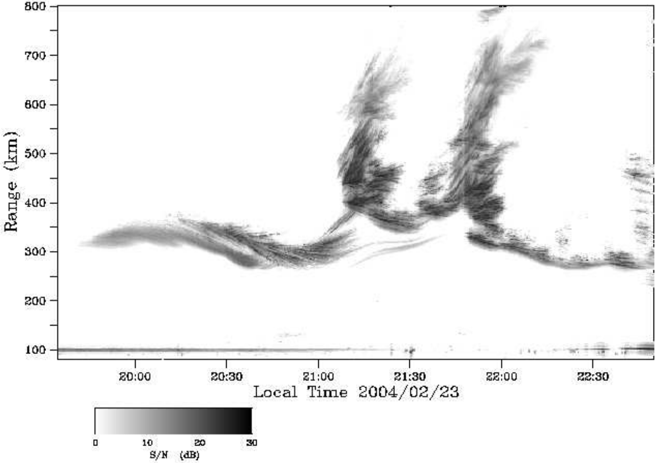


$$\langle v_0 v_1^* \rangle = e^{jkd \cos\theta}$$

- Measure v_0 and v_1 separately and estimate the angle of arrival θ.
- Larger baseline $\rightarrow$ more precise angle estimates.
- Baselines where $kd > \pi$ suffer from 2π ambiguity issues. Related to grating lobe problem.

Interferometric Imaging Configuration at Jicamarca

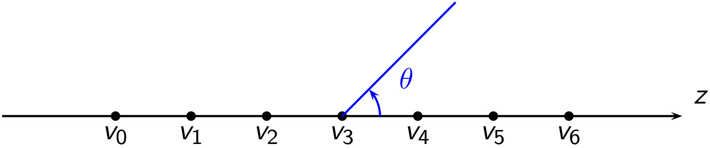
Interferometric Images of Coherent Scatter



Movie

http://landau.geo.cornell.edu/image.html

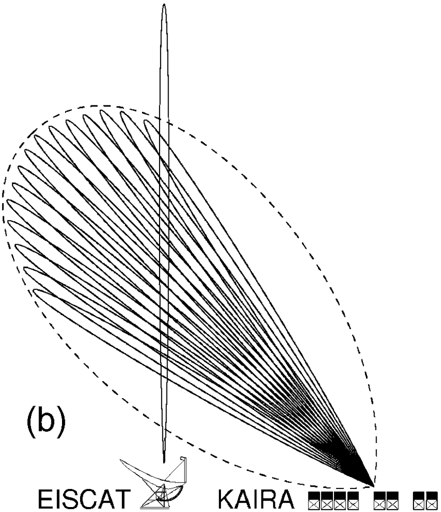
Digital Beam Forming



On reception:

- Digitize the signals v_n on every antenna (expensive!)
- Synthesize any beams you want by forming different linear combinations in software/firmware (computationally intense!)
- Allows you to form custom beam patterns → look at signals of interest while nulling interference.
- Allows you to form any number of different radiation patterns → look in multiple directions at once.

Digital Beam Forming in Multi-static Radar Experiments

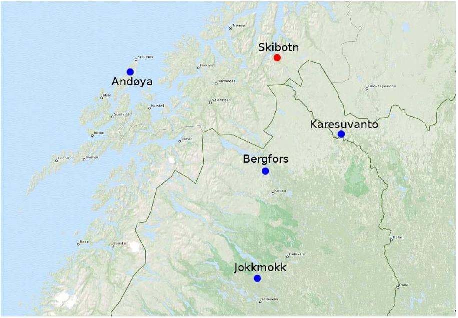


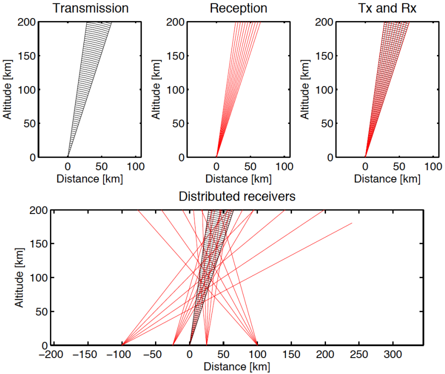
KAIRA = The Kilpisjärvi Atmospheric Imaging Receiver Array

McKay et al. (2015) 10.1109/TGRS.2014.2342252

EISCAT_3D

Major planned facility:





Operational 2021?