

The short introduction to Incoherent Scatter (IS) Theory

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# Outline

- Some basic radar concepts
- The ionosphere
- Incoherent Scatter Radars and theory

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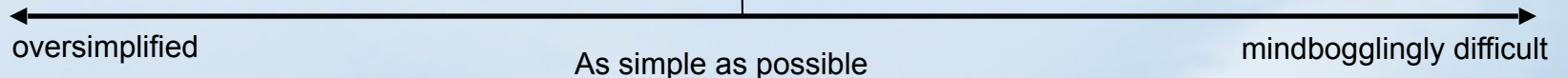
Albert Einstein

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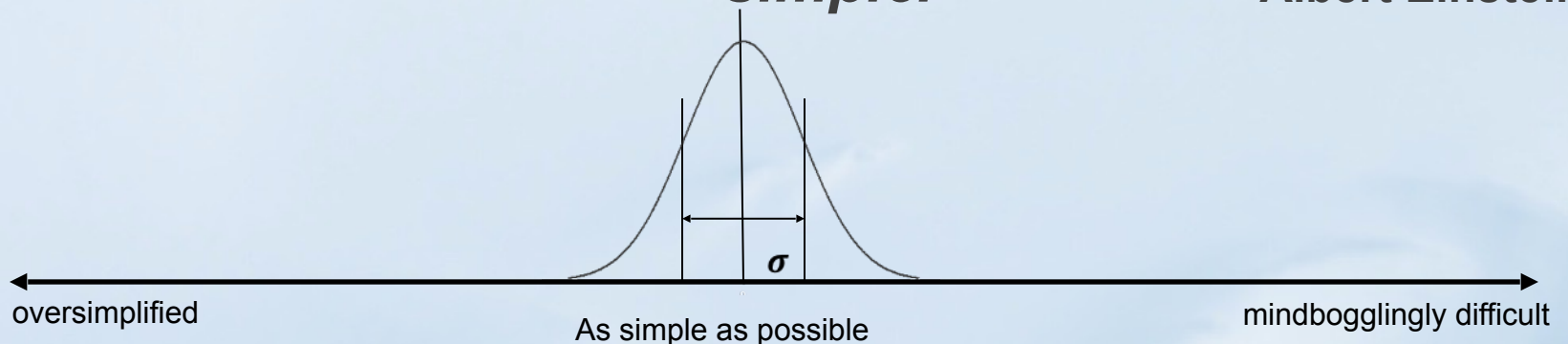
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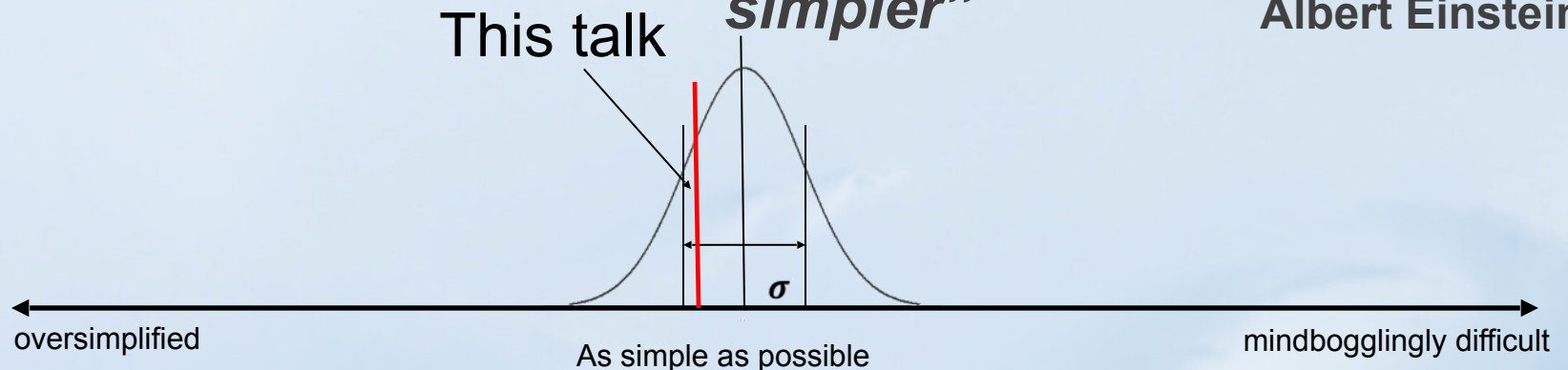
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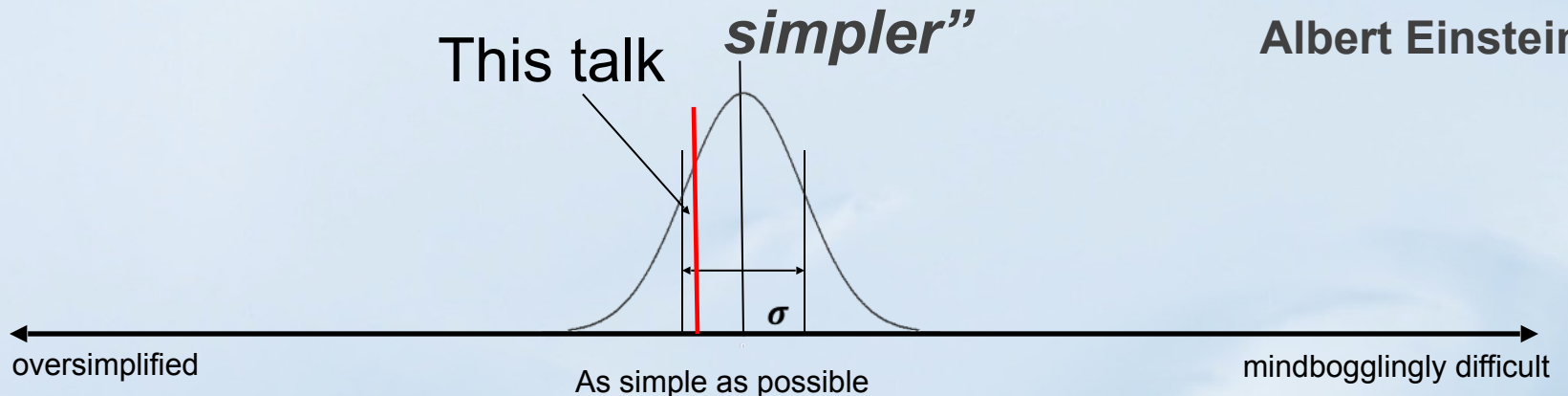


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- Inco

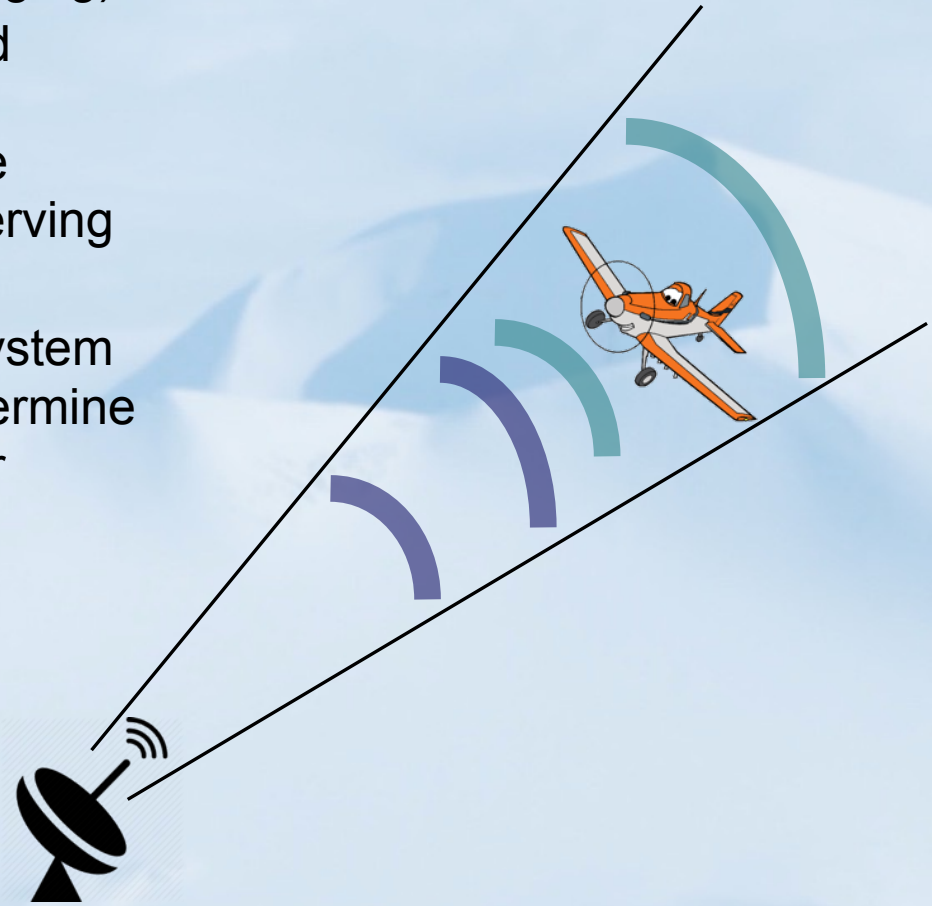
***"For every complex problem there is an answer that is clear, simple, and wrong"***  
H. L. Mencken

*... be made as simple as possible, but no simpler"*  
Albert Einstein



# Radar

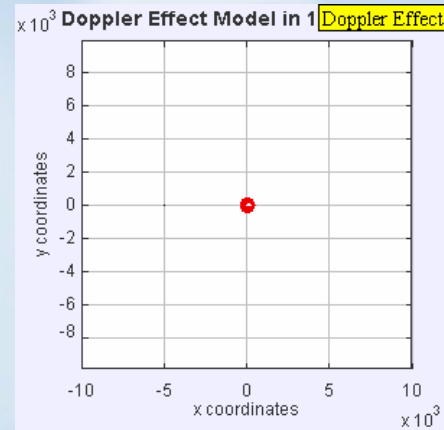
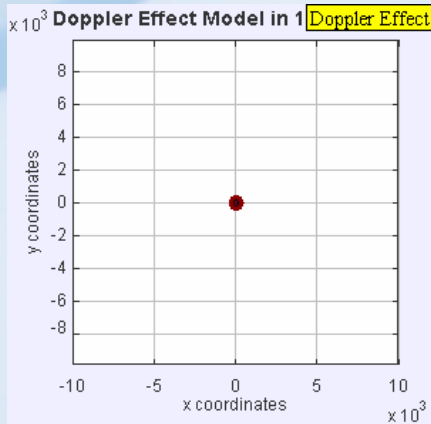
- RADAR (RAdio Detection And Ranging)
  - is a technique for detecting and studying remote targets by transmitting a radio wave in the direction of the target and observing the reflection of the wave.
  - **Radar** is an object detection system which uses radio waves to determine the range, altitude, direction, or speed of objects. (wikipedia)



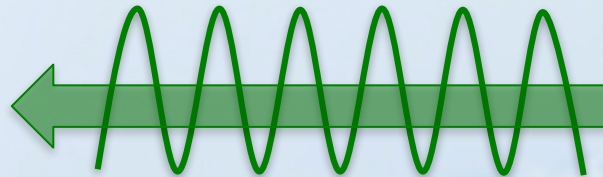
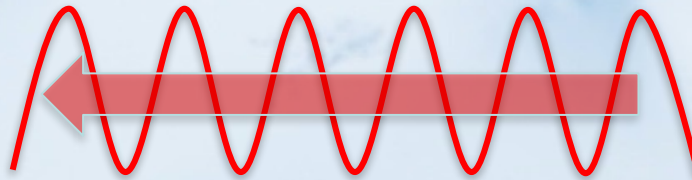
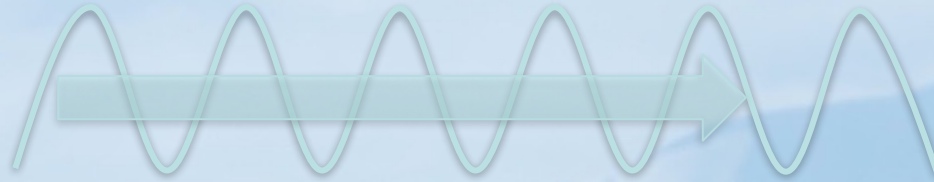


# Doppler Radar

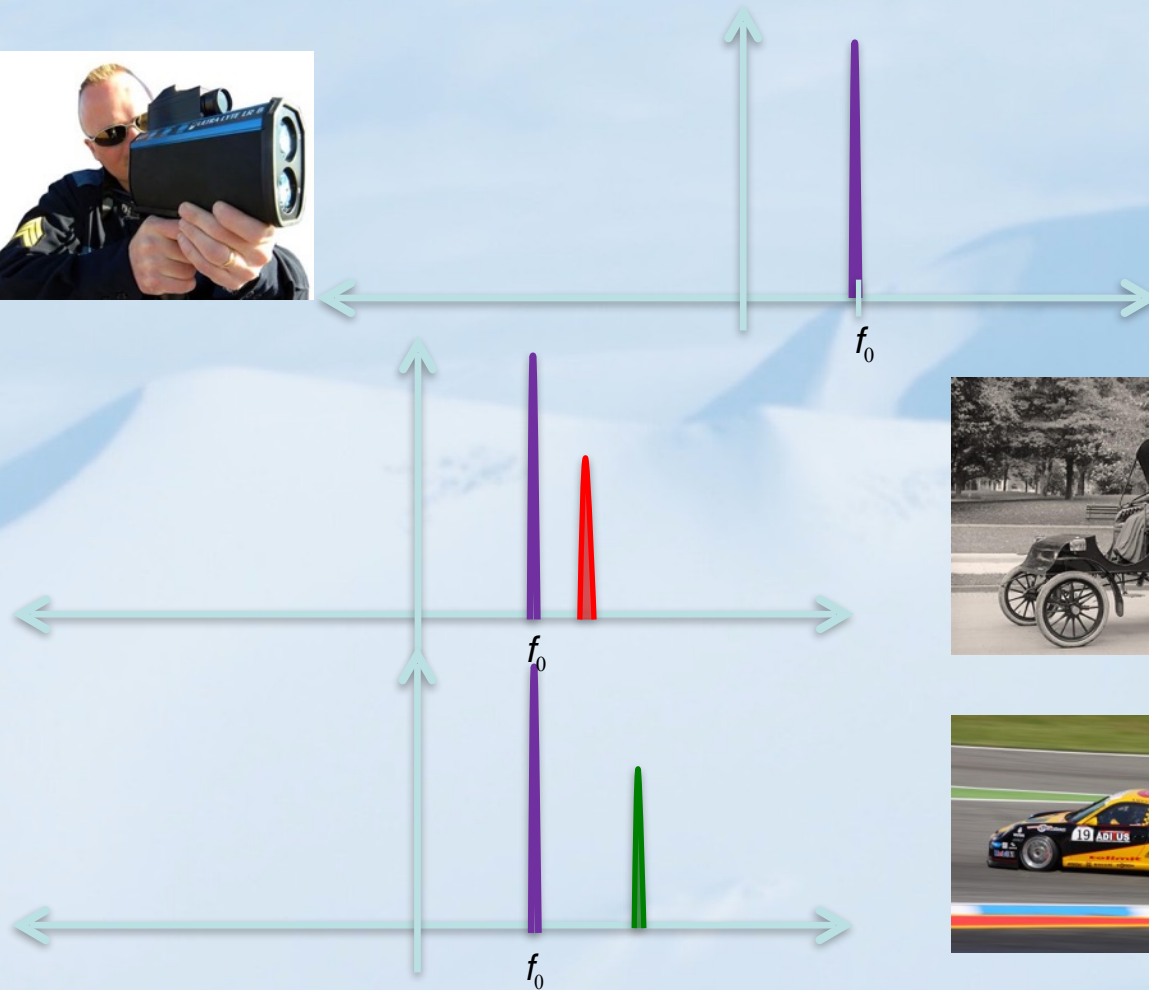
- In order to detect the velocity of the target we are taking advantage of a concept called Doppler Shifts
- We are rather familiar with it when it comes to sound waves



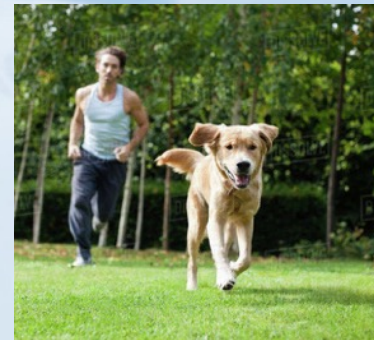
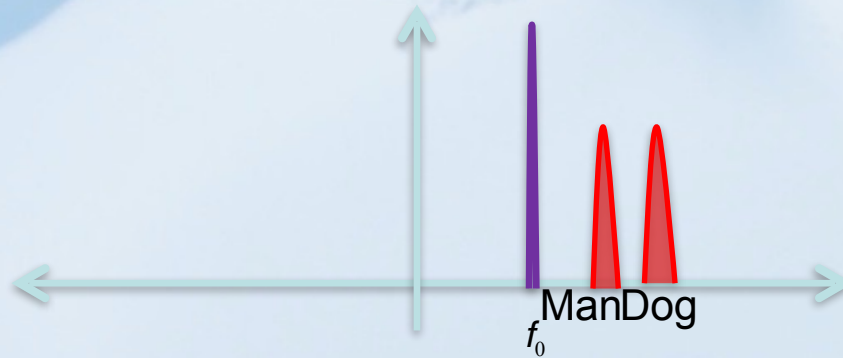
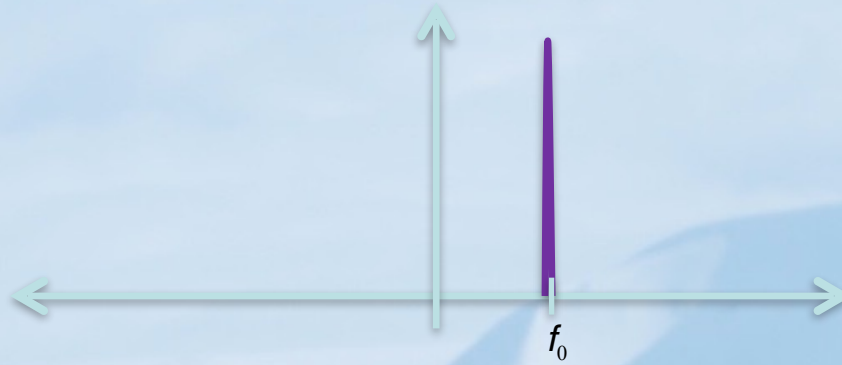
# Doppler Radar – time domain

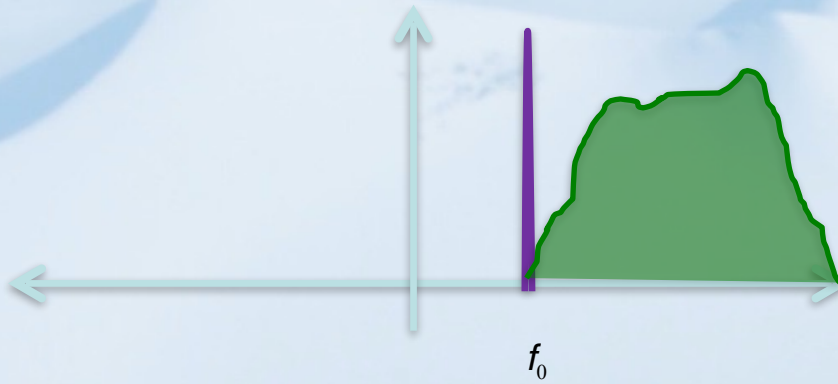
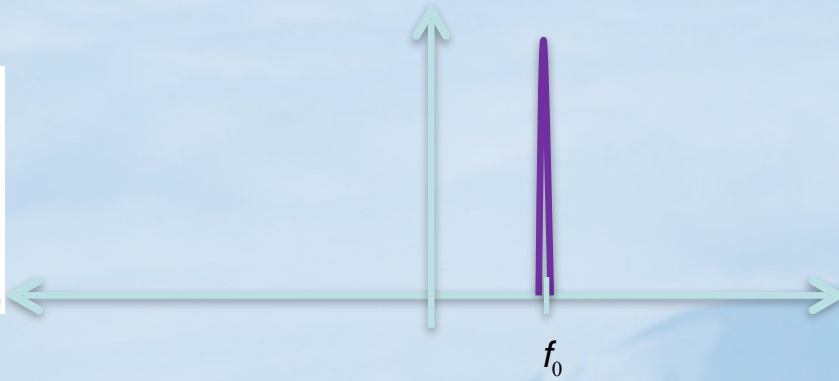


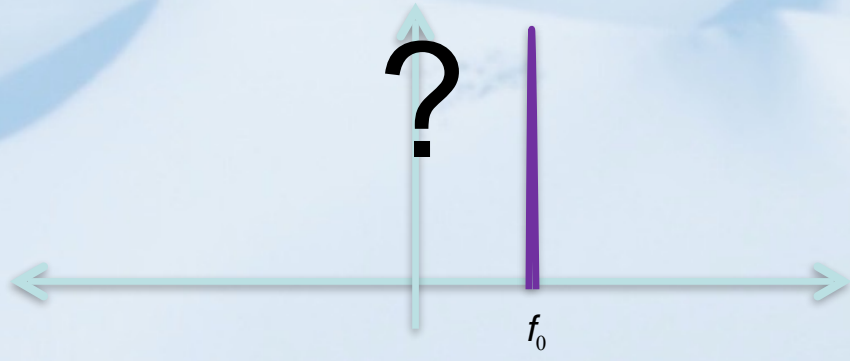
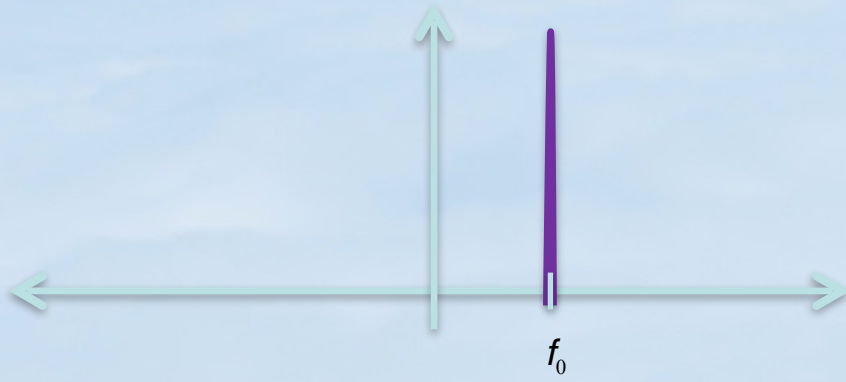
# Doppler Radar – frequency domain

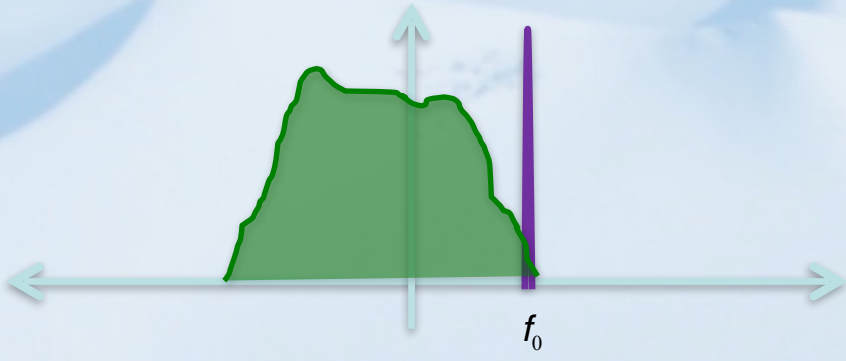
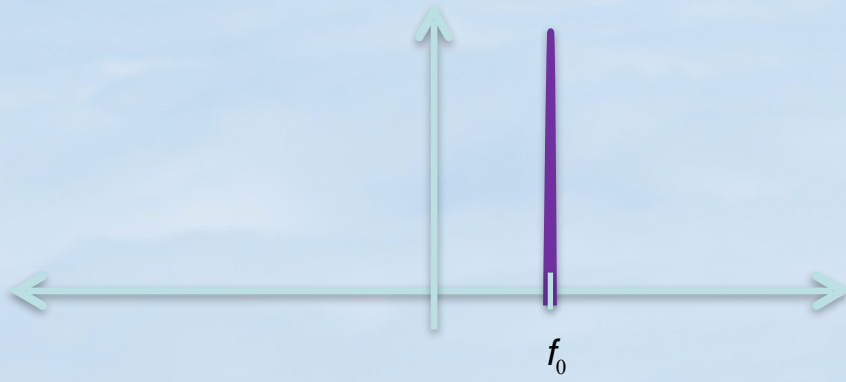


# Doppler Radar – frequency domain









**How can we use this method here?**





**...and at slightly higher latitudes here?**



Photo: Anja Strømme

The answer is....

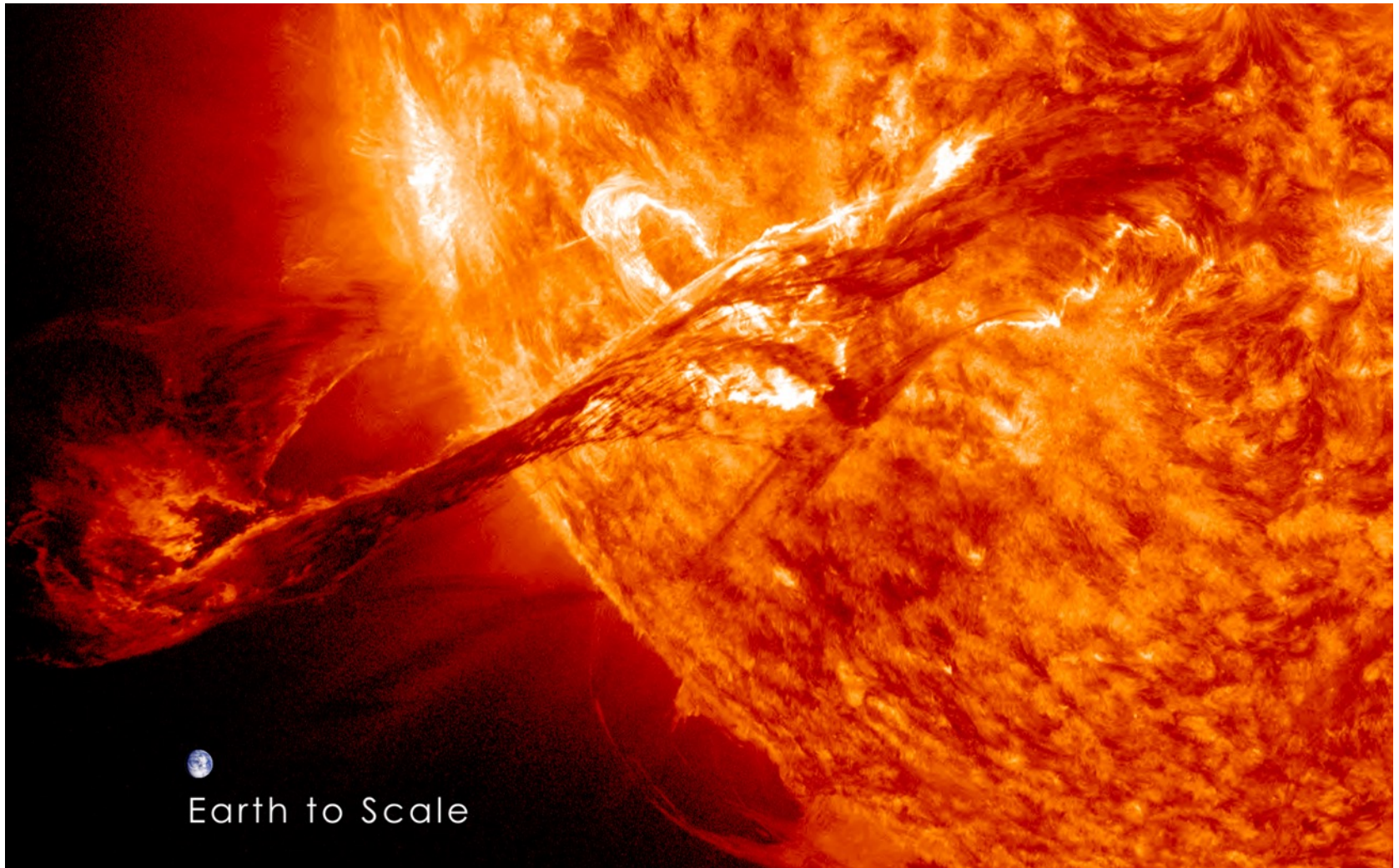




The answer is....

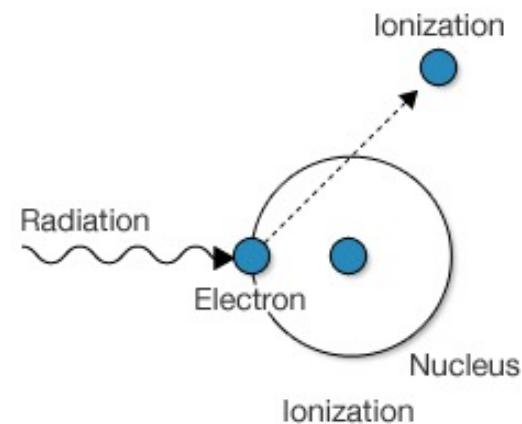
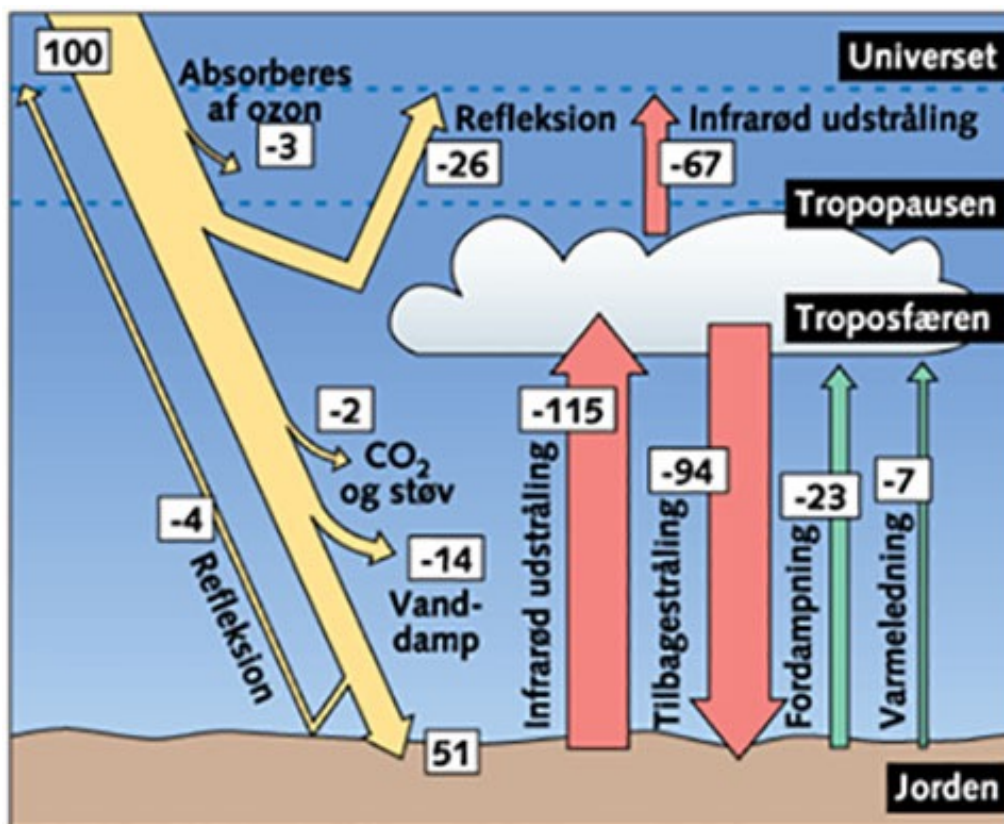
Electrons!

**Like most things on Earth – it all starts on the Sun...**

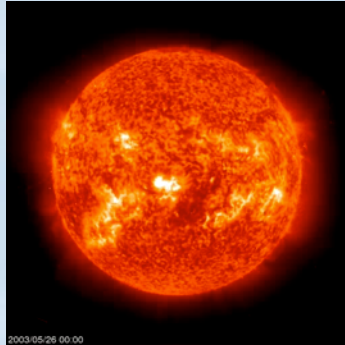


# UV radiation

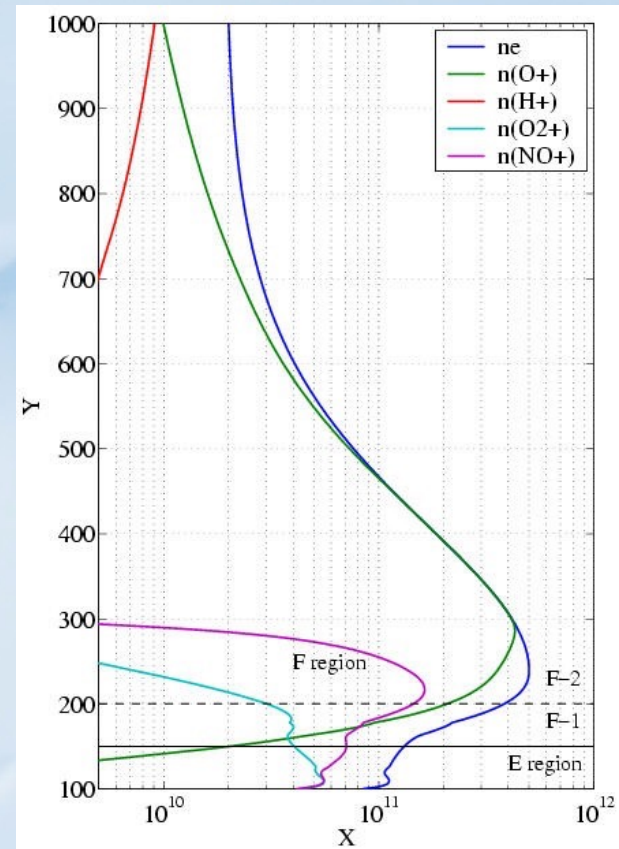
UV and EUV radiation is driving Earth's energy balance, and are responsible for an ionosphere with charged particles in the upper part of our atmosphere.



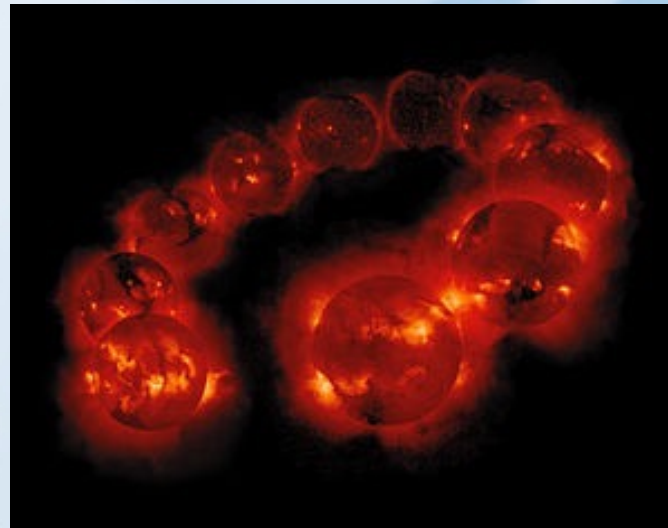
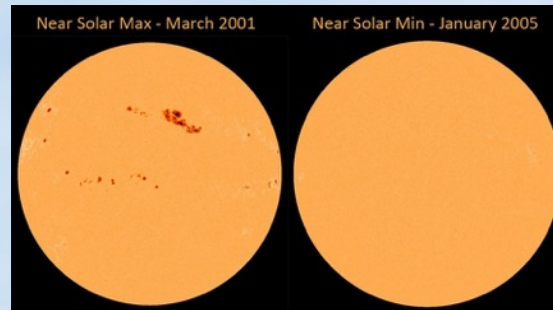
# Electron Density profile



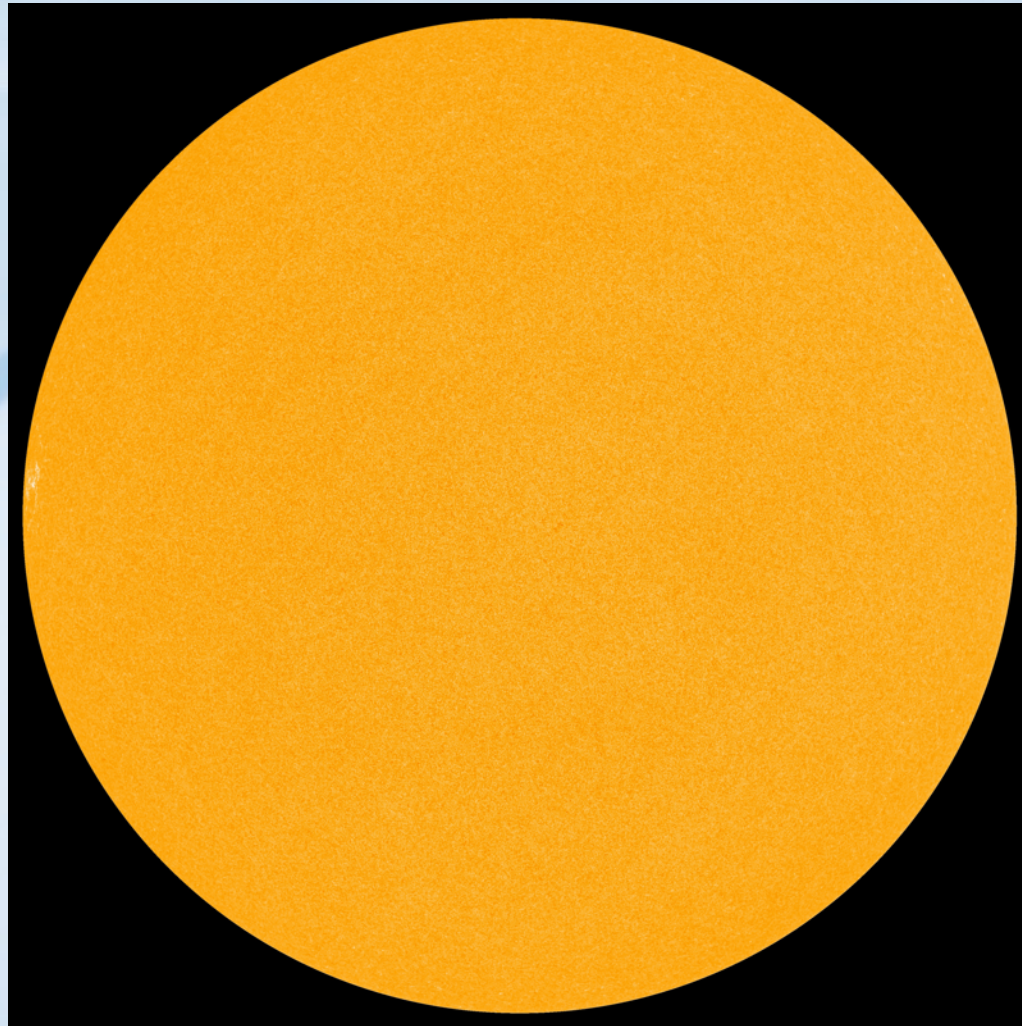
A complex balance between the ionization and recombination and diffusion results in an altitude dependent electron (and ion) density profile.



# The Sun is an active star!

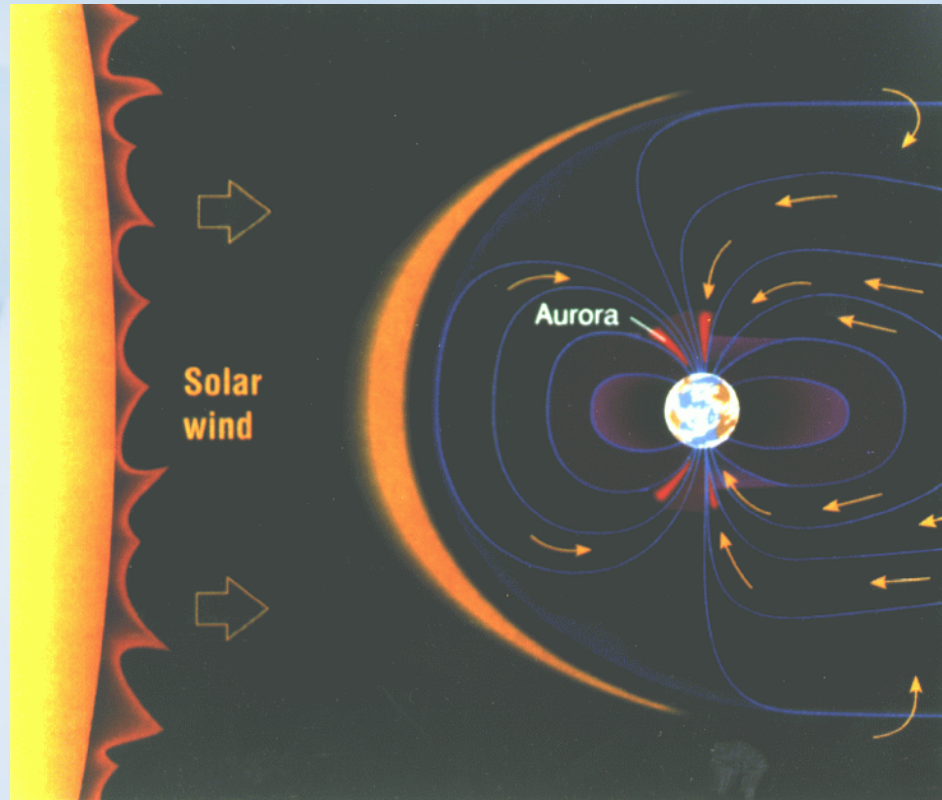


**The Sun is an active star... Most days...!**

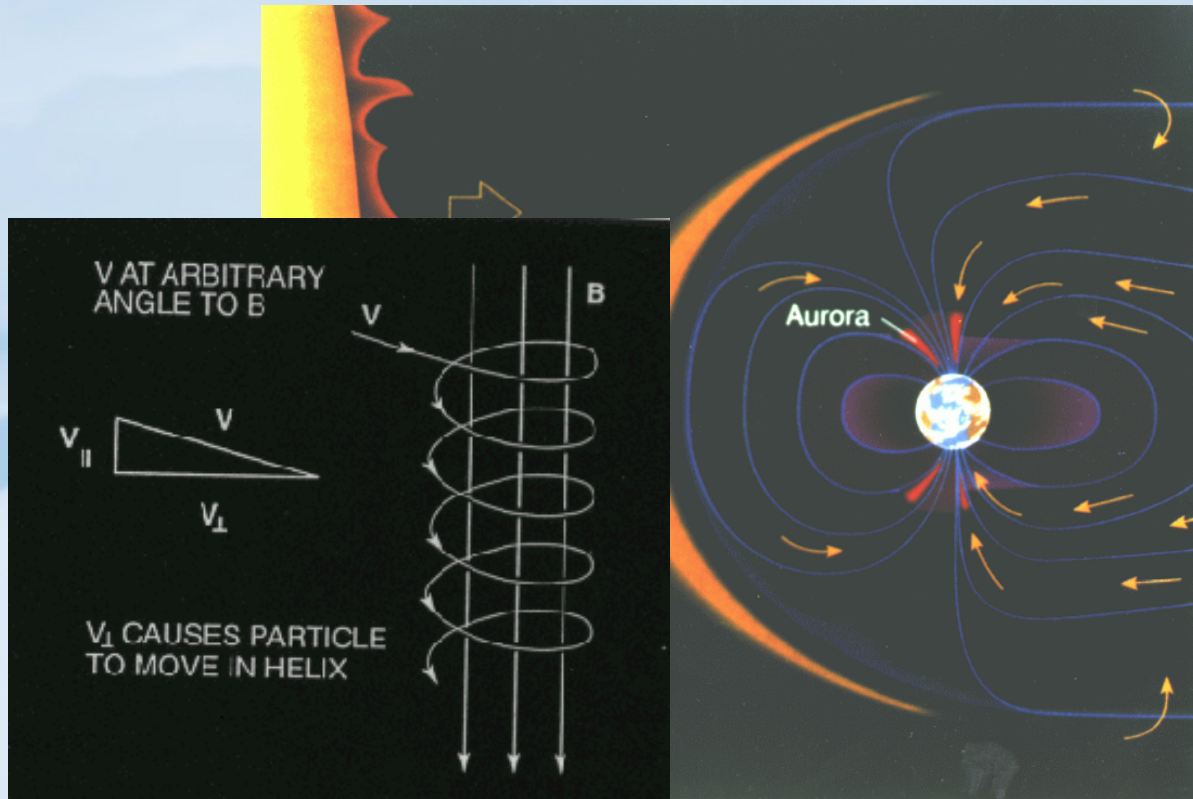




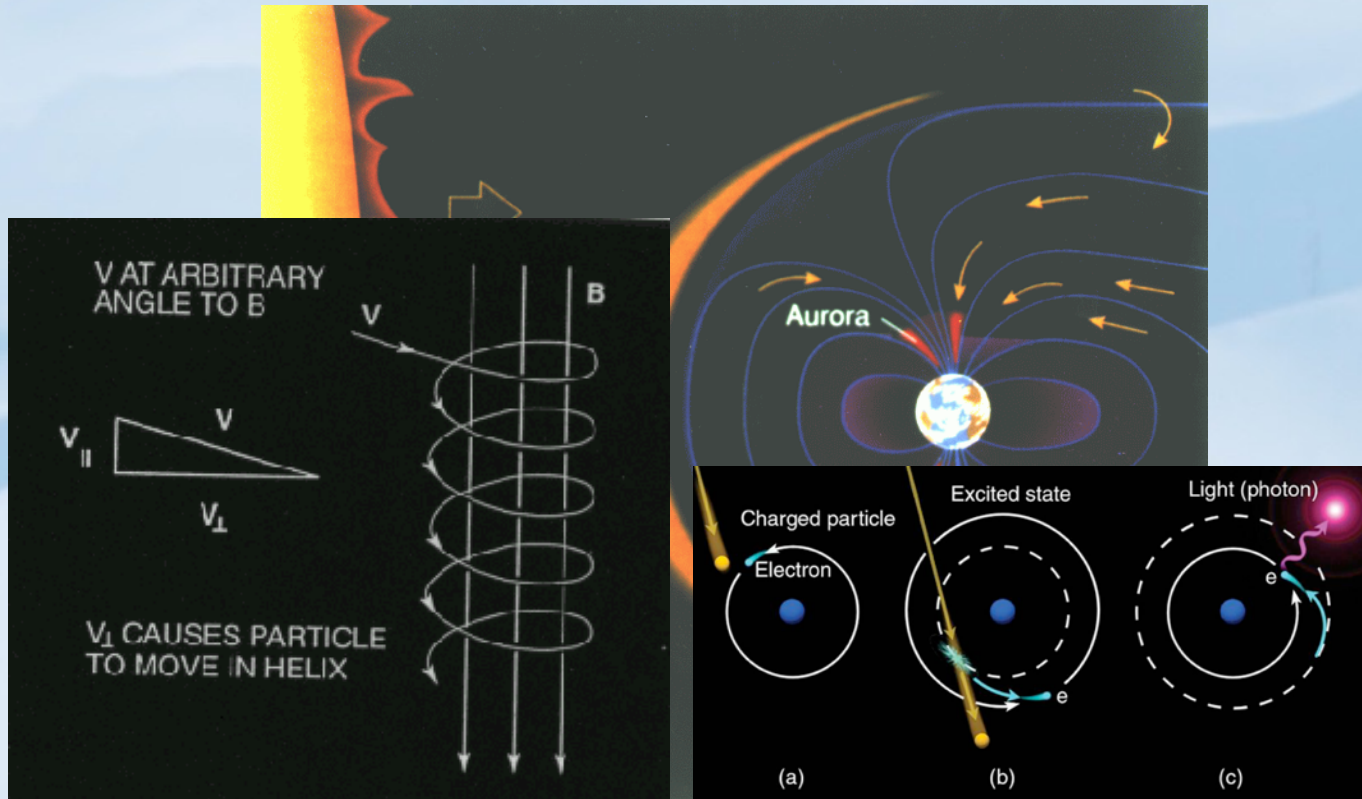
**At high latitudes electron (and proton) with solar wind origin creates additional ionization, seen as aurora borealis/australis displays**



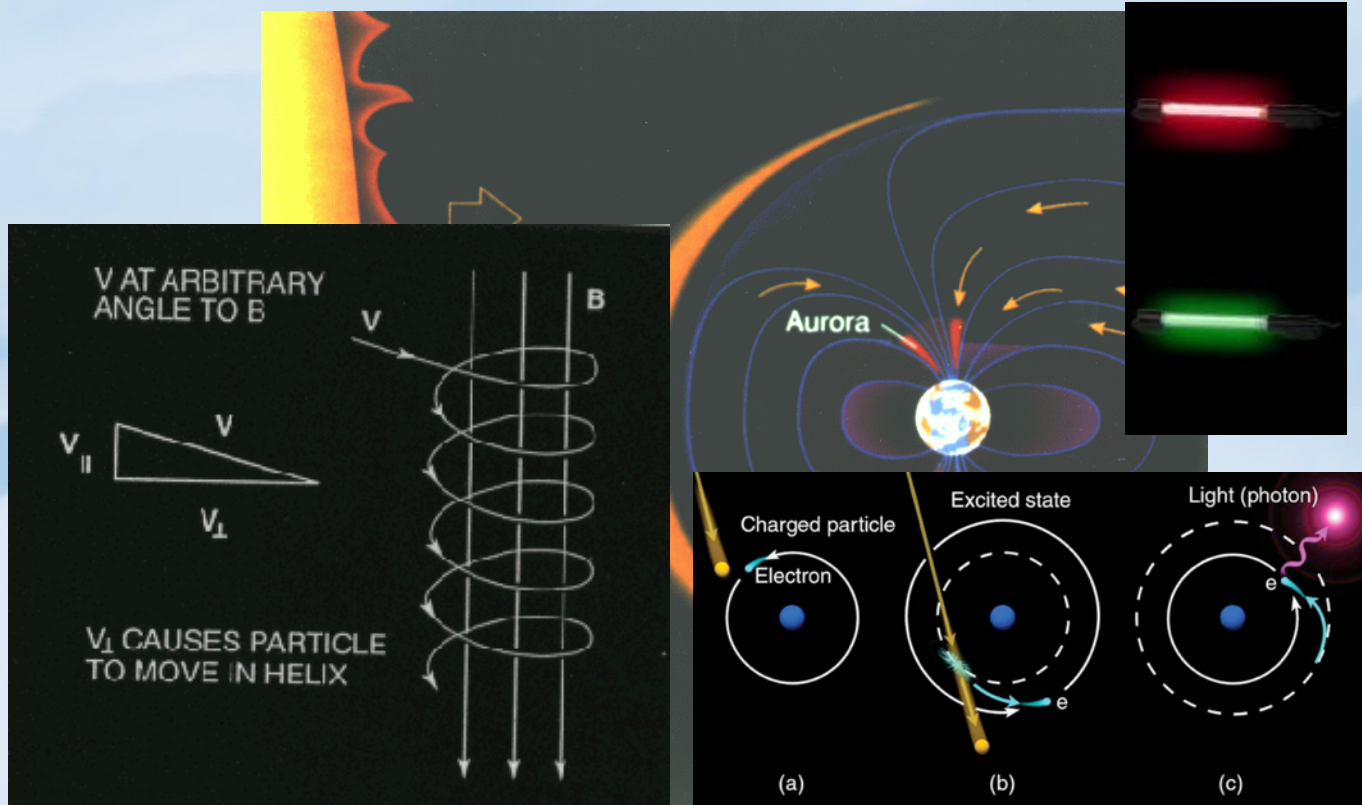
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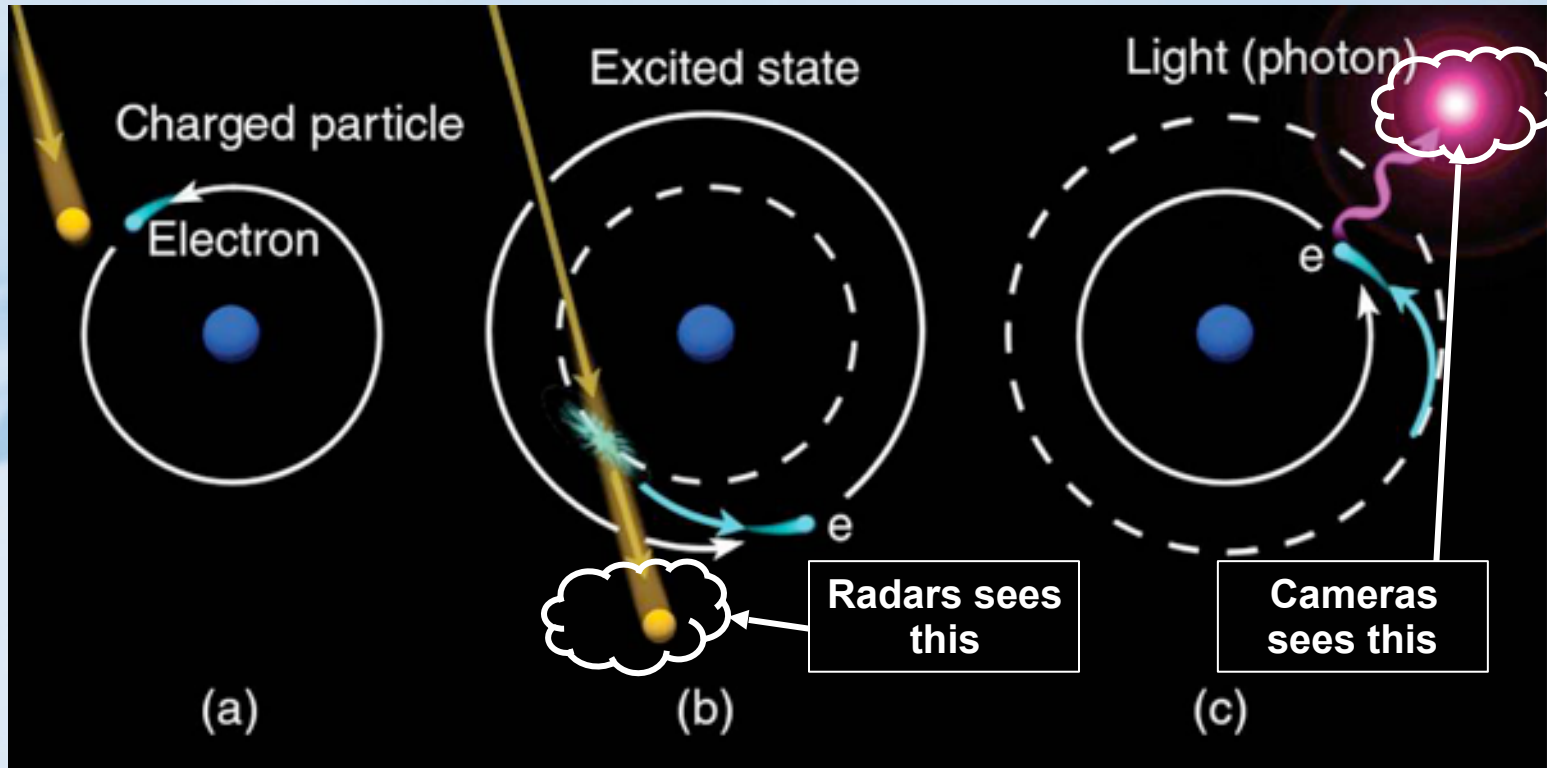
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# Optical versus Radar Aurora

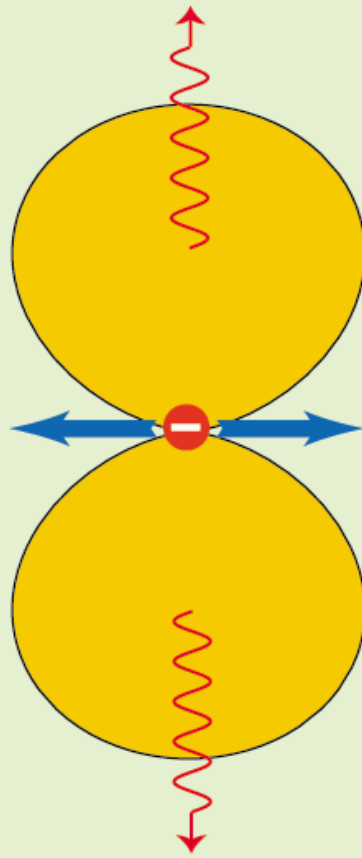


# Scattering from charged particles



- When a charged particle is subjected to an electromagnetic wave, it starts oscillating with the wave
- When a charged particle oscillates, it emits an electromagnetic wave
- So by radiating an electron with a radar wave, the electron becomes a tiny antenna itself and radiates power in all directions

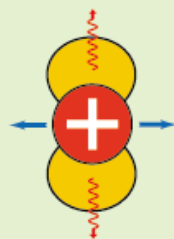
**electron**



$\mathbf{k}$

$\mathbf{E} \sin(\omega t - \mathbf{k} \cdot \mathbf{x})$

**ion**



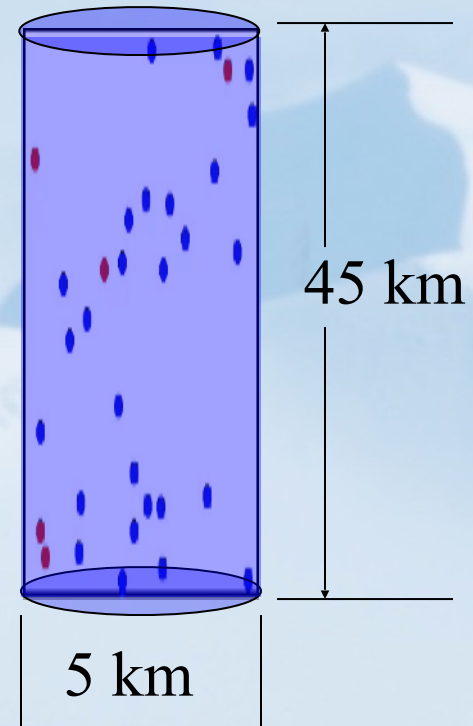
$$\frac{\sigma_{ion}}{\sigma_{ele}} = \left( \frac{m_{ele}}{m_{ion}} \right)^2$$



# Total cross section estimate:

Consider an antenna with a 1-degree beam measuring the ionospheric plasma at 300 km range and using a 300 microsecond pulse.

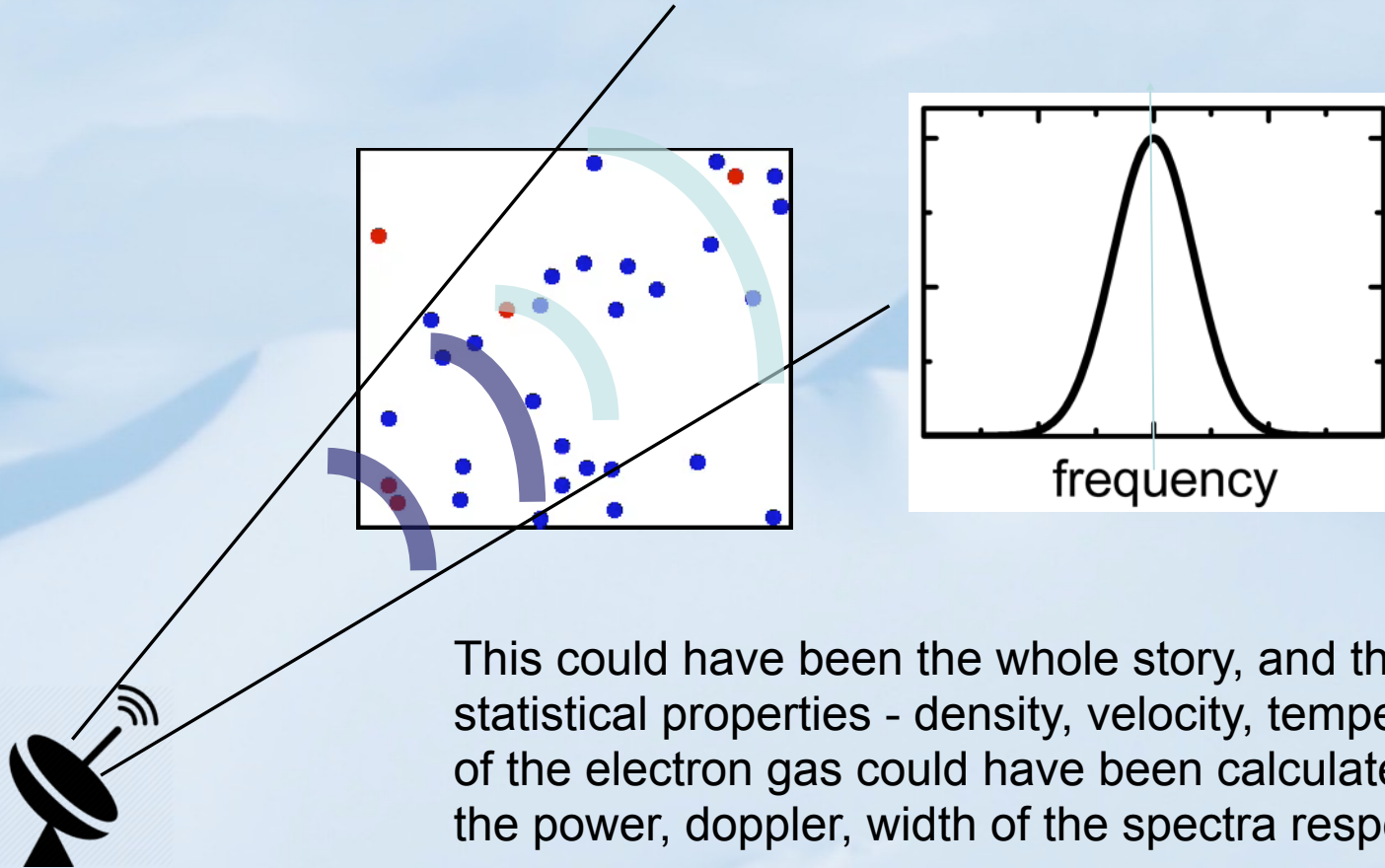
If the electron density is  $10^{12} \text{ m}^{-3}$ , the total number of electrons scattering into a given measurement is  $\sim 8.8 \times 10^{23}$ . This yields a total cross-section of  $88 \text{ mm}^2$  – we need a big radar!



*“The possibility that incoherent scattering from electrons in the ionosphere, vibrating independently, might be observed by radar techniques has apparently been considered by many workers although seldom seriously because of the enormous sensitivity required...”*

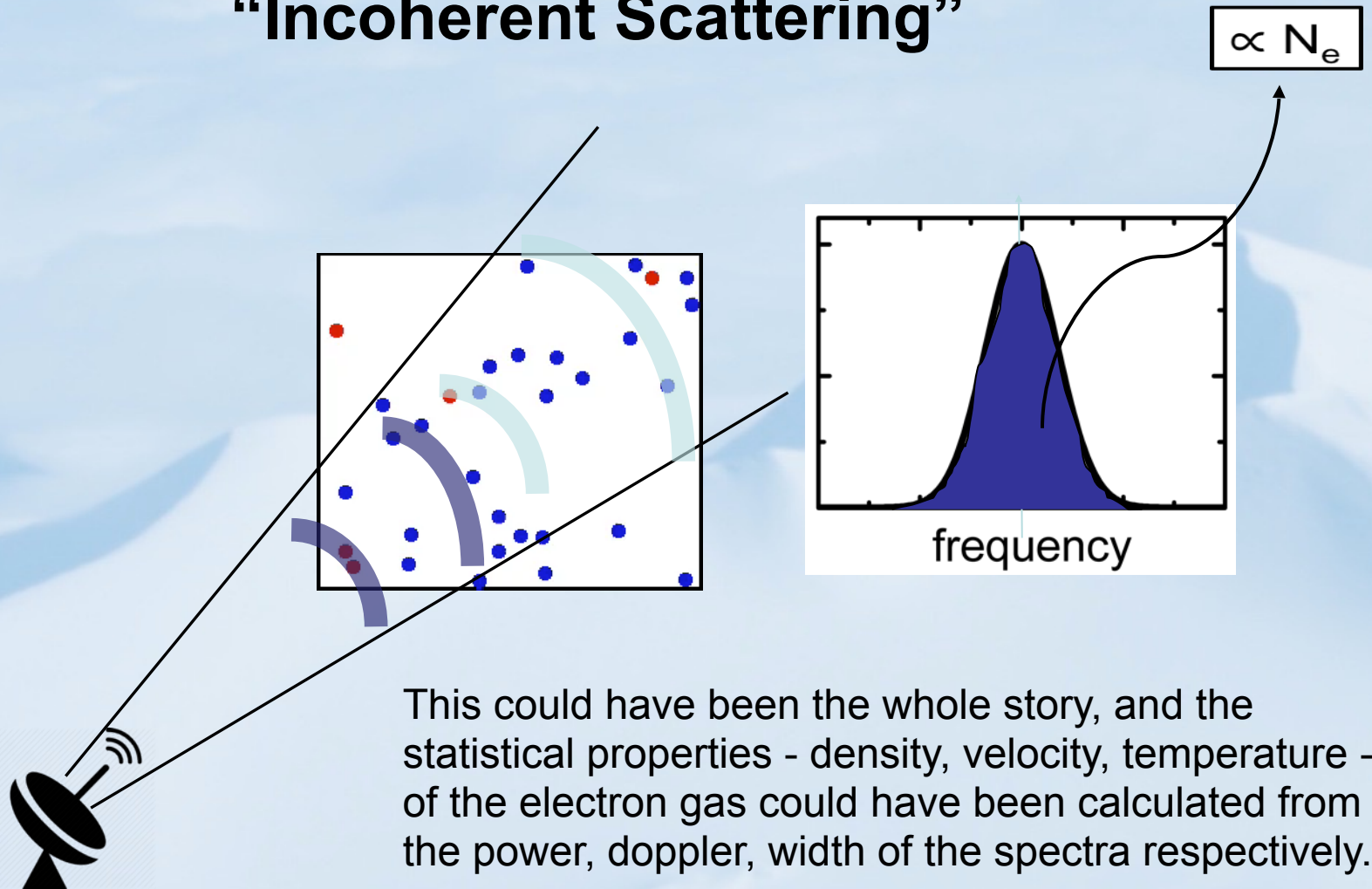
**K.L. Bowles [Cornell PhD 1955]**, Observations of vertical incidence scatter from the ionosphere at 41 Mc/sec. *Physical Review Letters* 1958:

# Thermal fluctuating electrons “Incoherent Scattering”

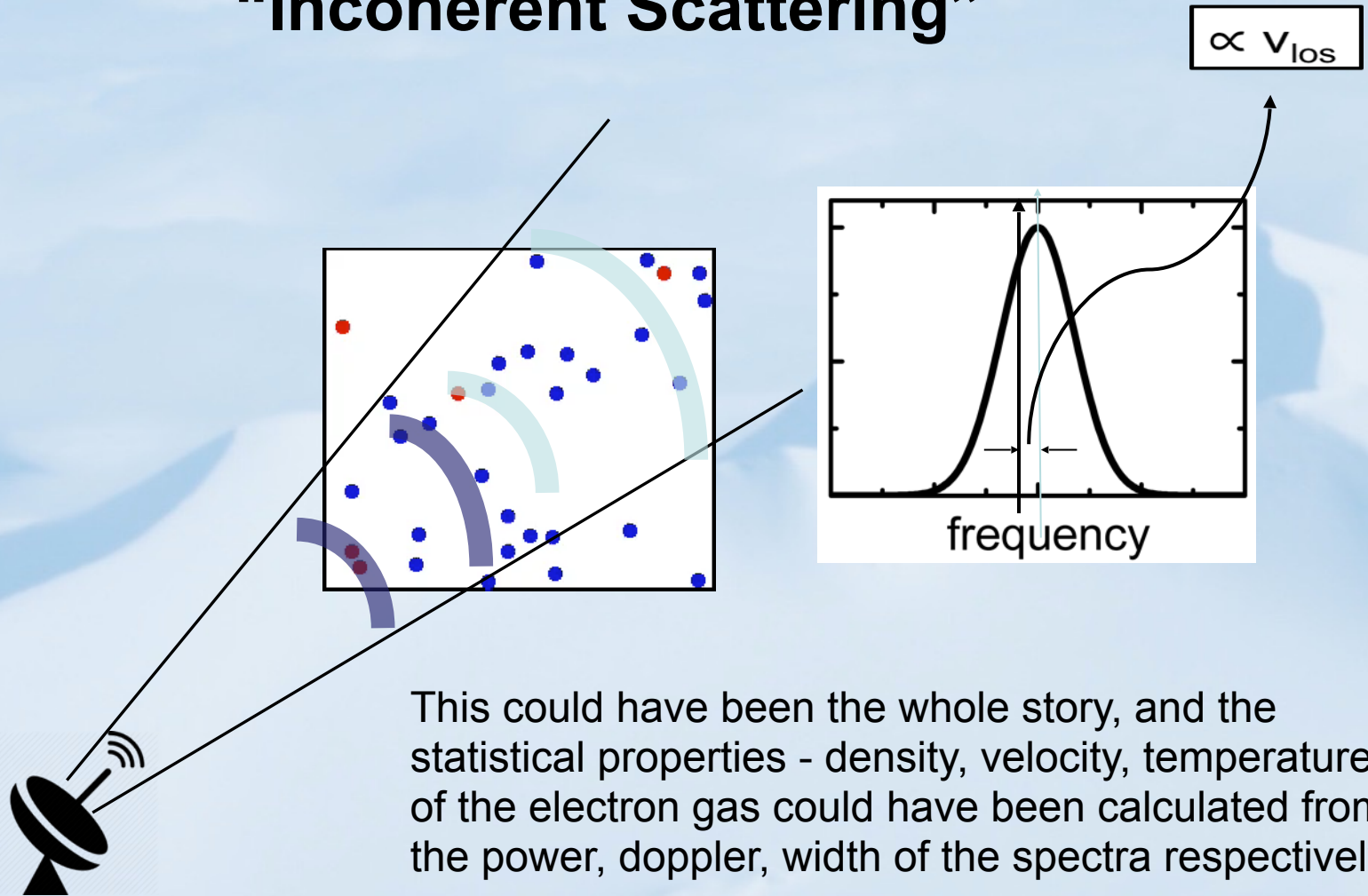


This could have been the whole story, and the statistical properties - density, velocity, temperature - of the electron gas could have been calculated from the power, doppler, width of the spectra respectively...

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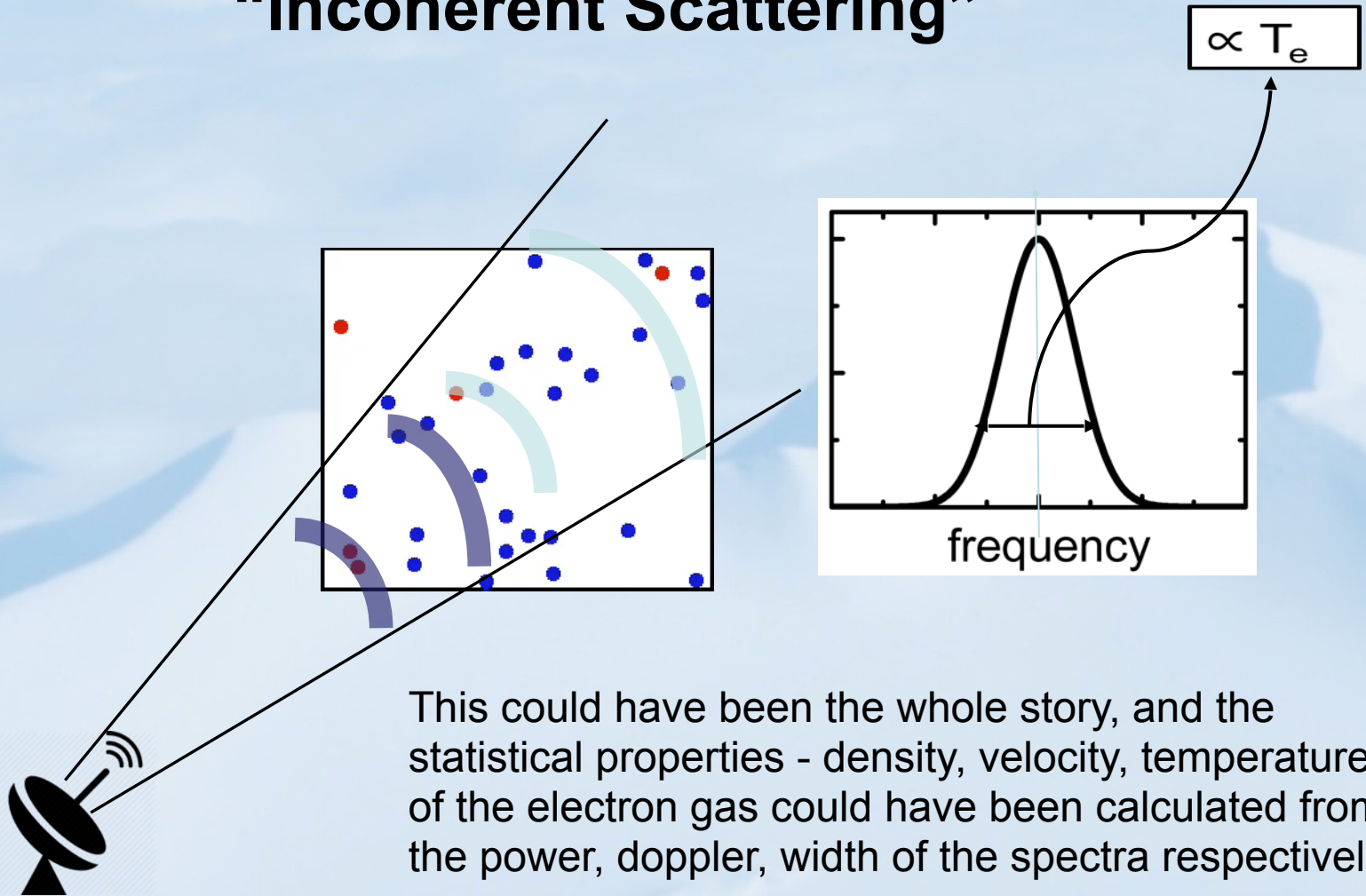


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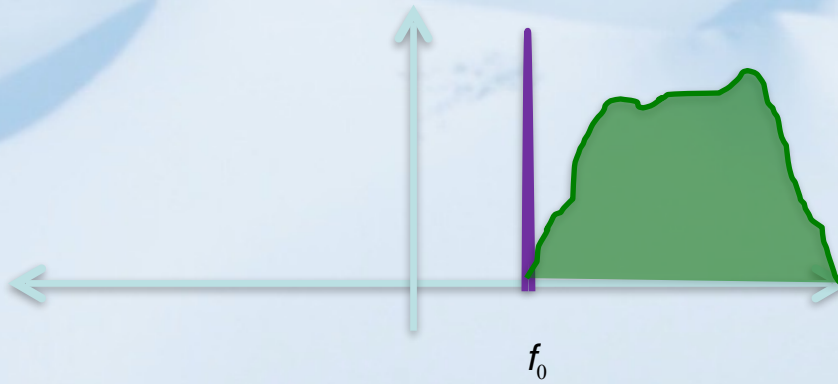
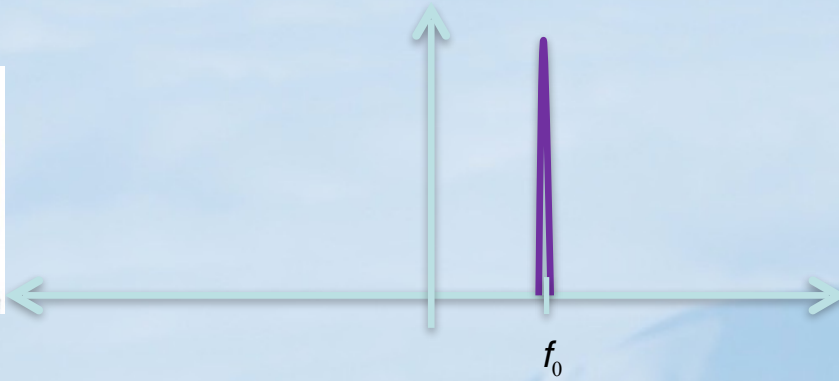


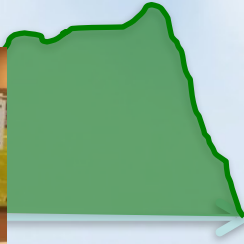
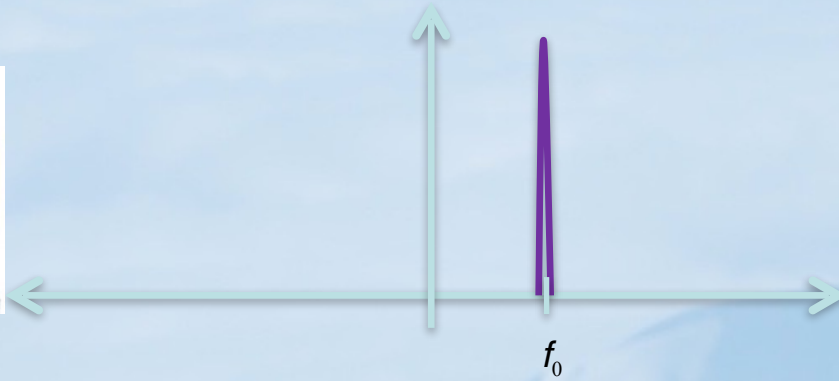
In order to detect this you need Arecibo



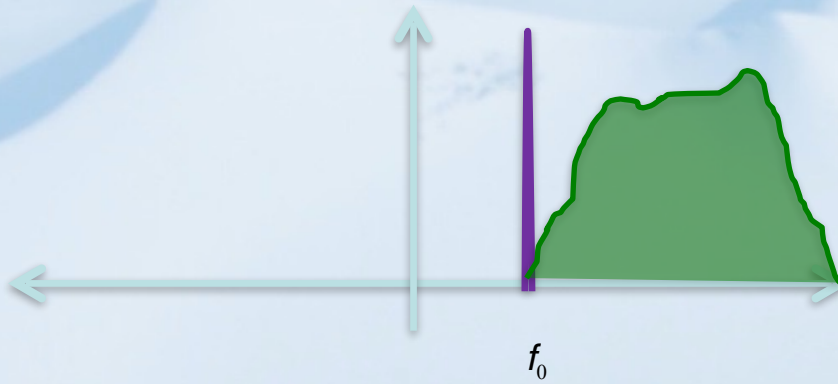
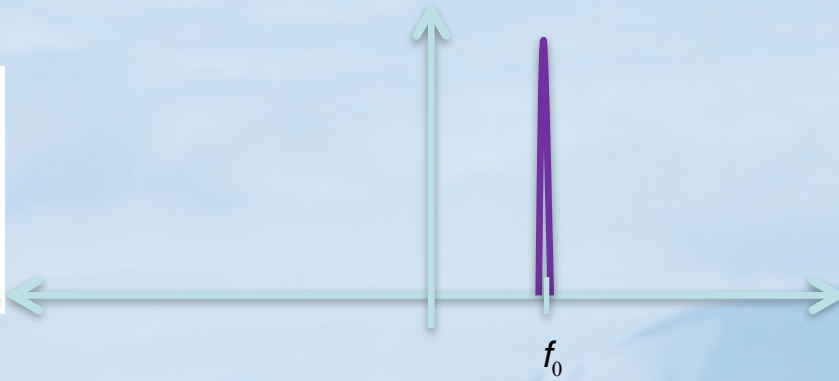
What if there are collective behaviours  
in the ionospheric plasma....

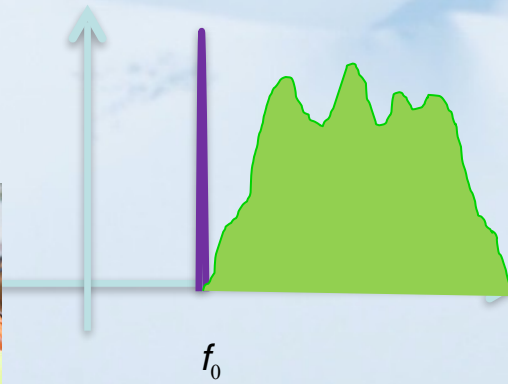
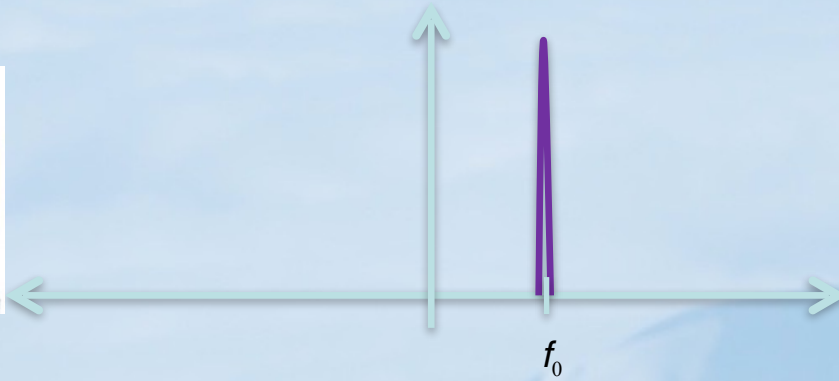




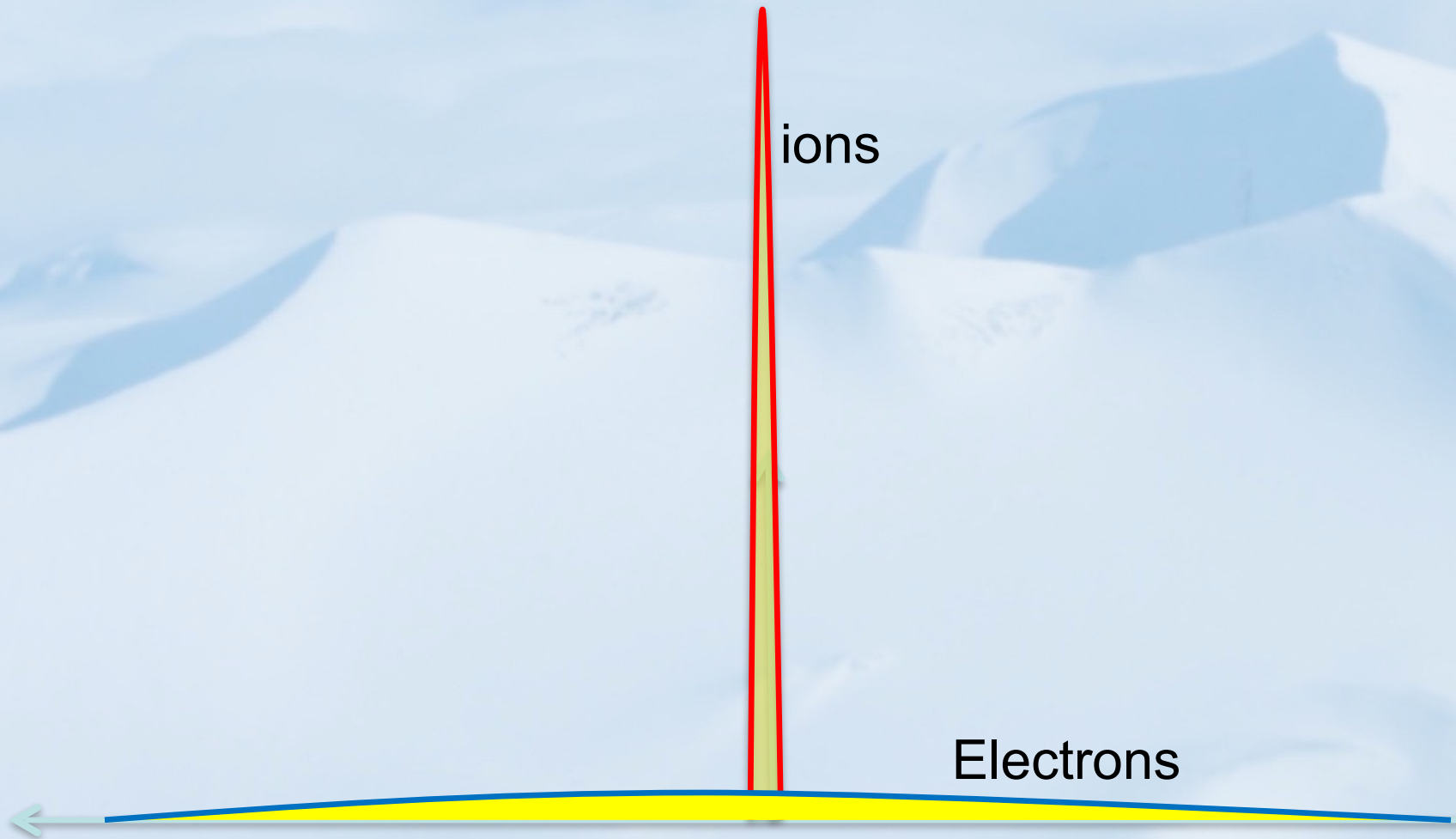








The ionospheric ions acts as sloooow pacers for the electron gas



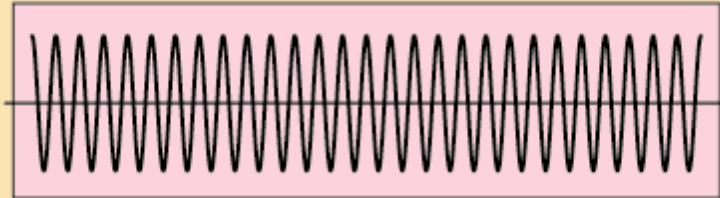
# Incoherent scattering - the short story



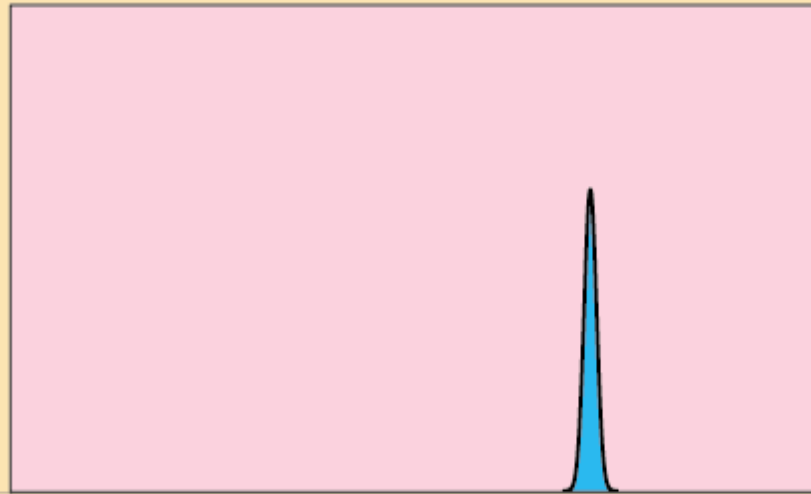
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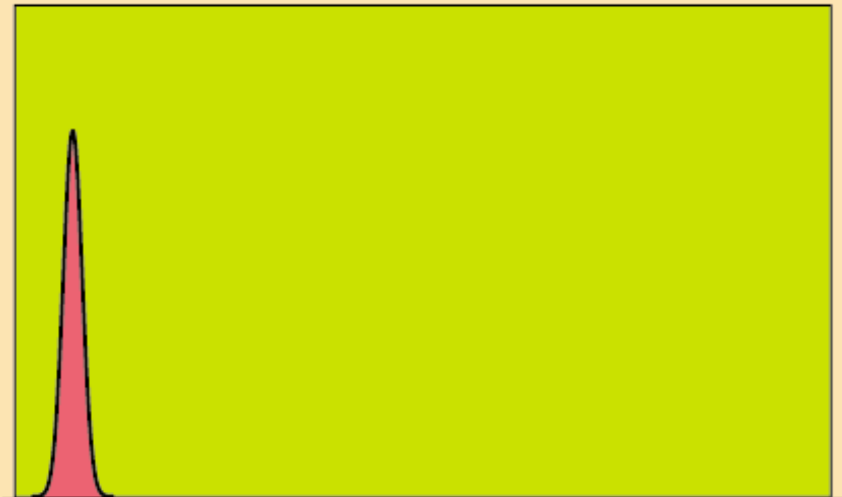
time



frequency



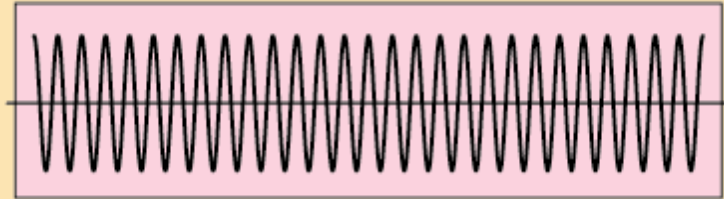
time



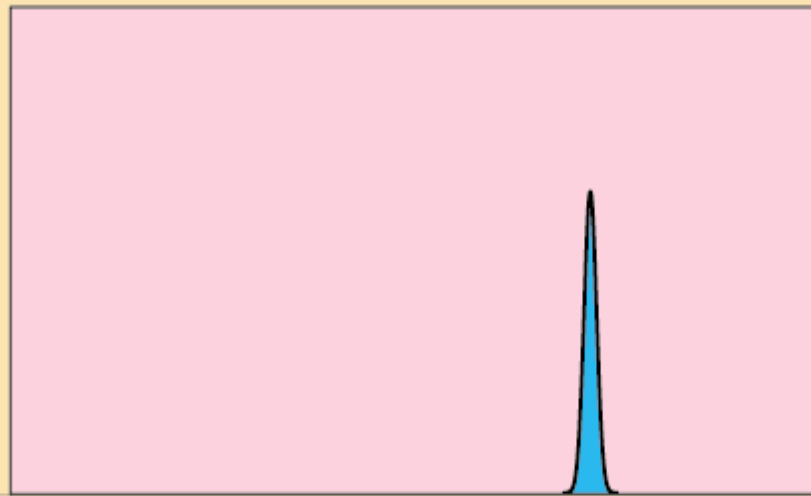
frequency



Langmuir waves



time



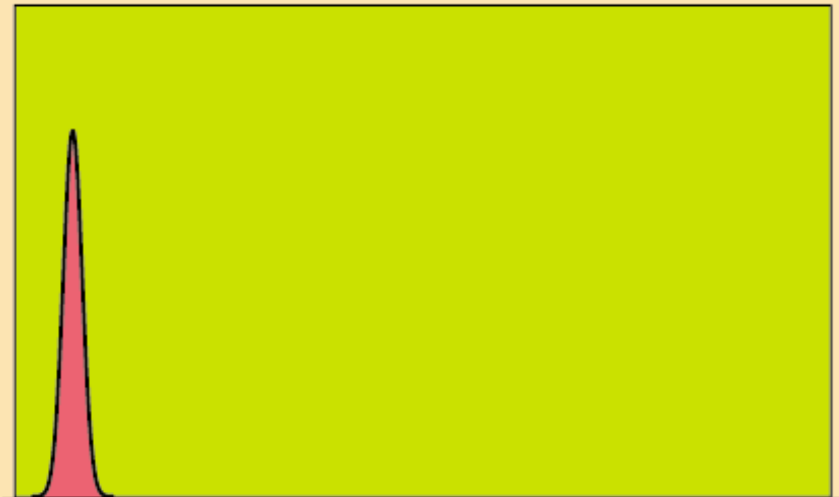
frequency



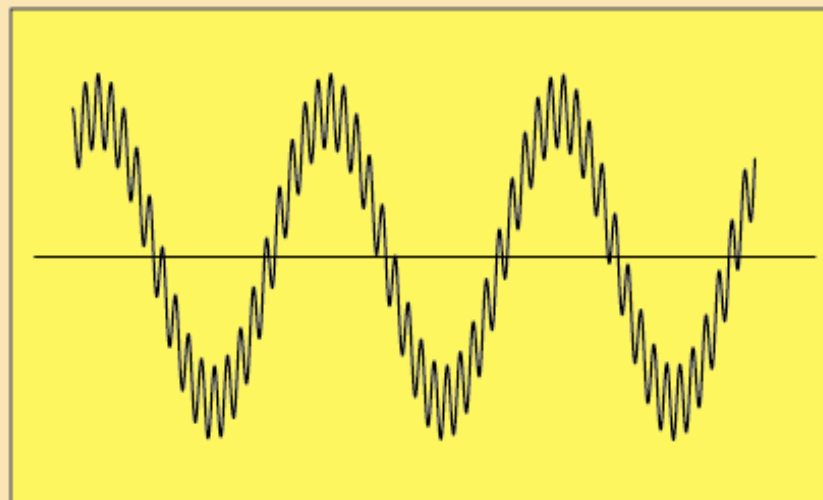
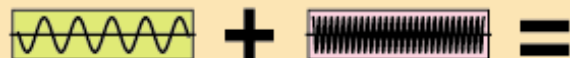
Ion acoustic waves



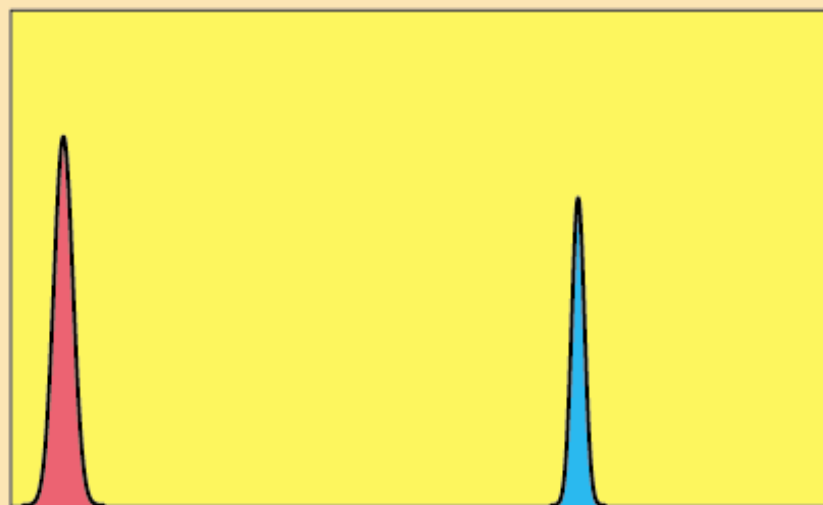
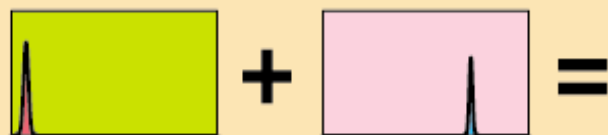
time



frequency

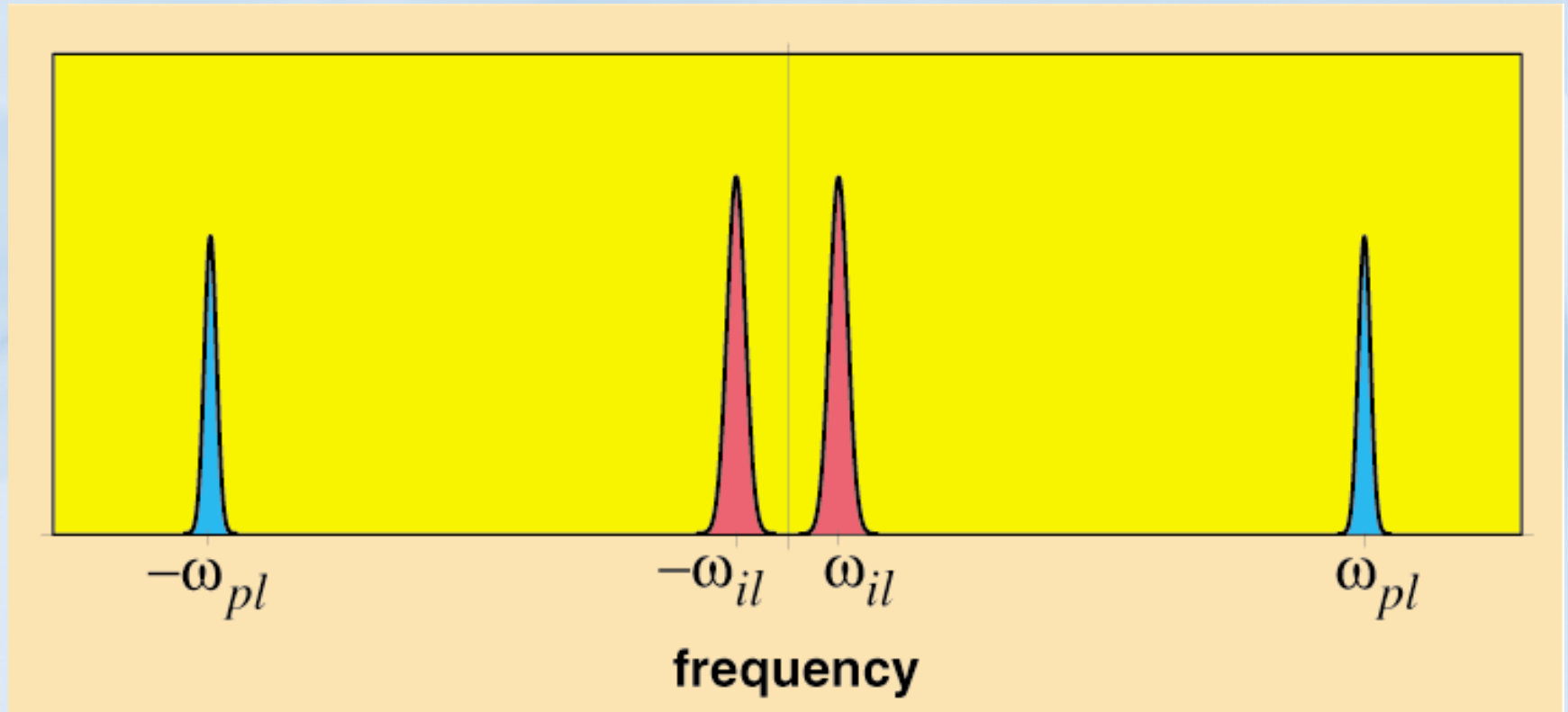


time

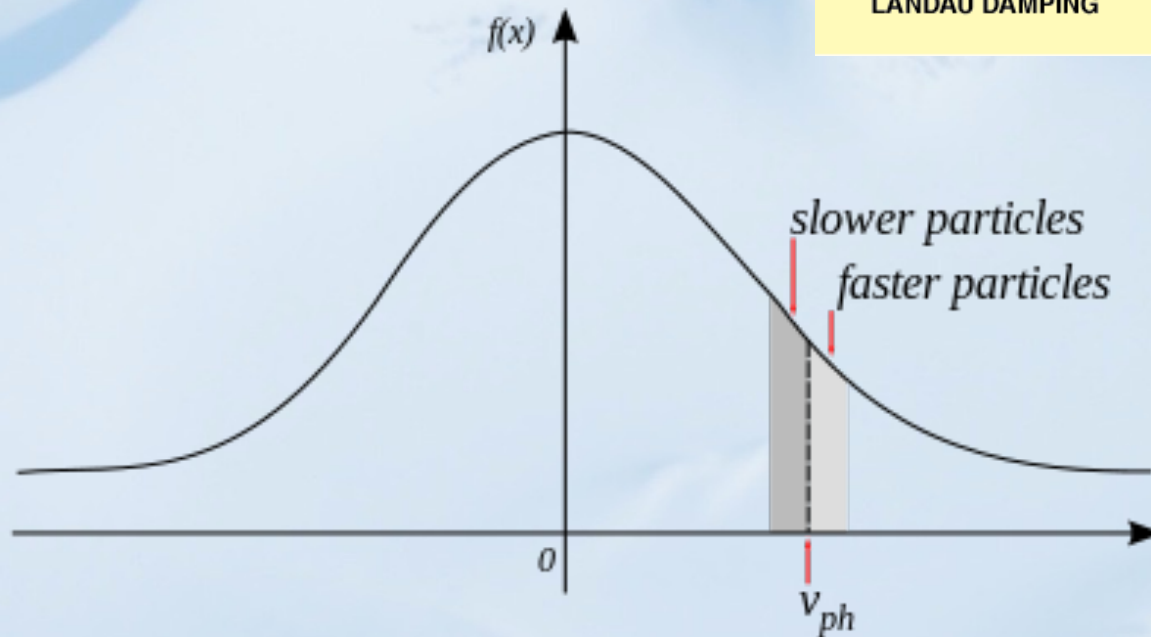
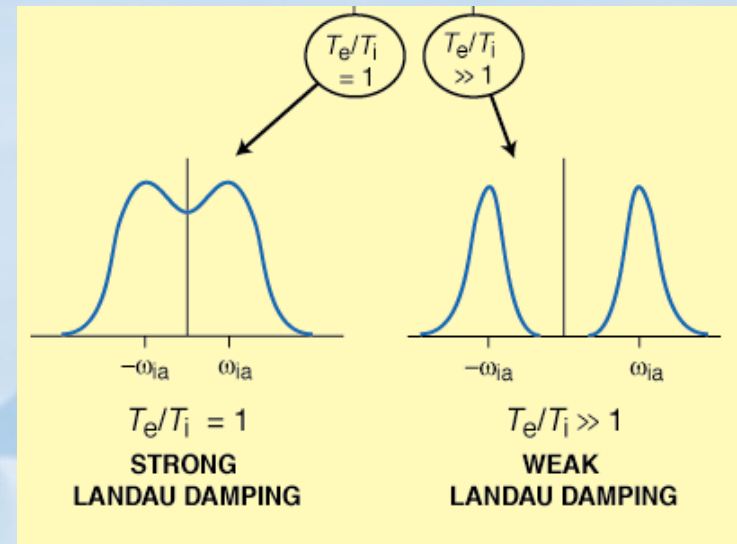
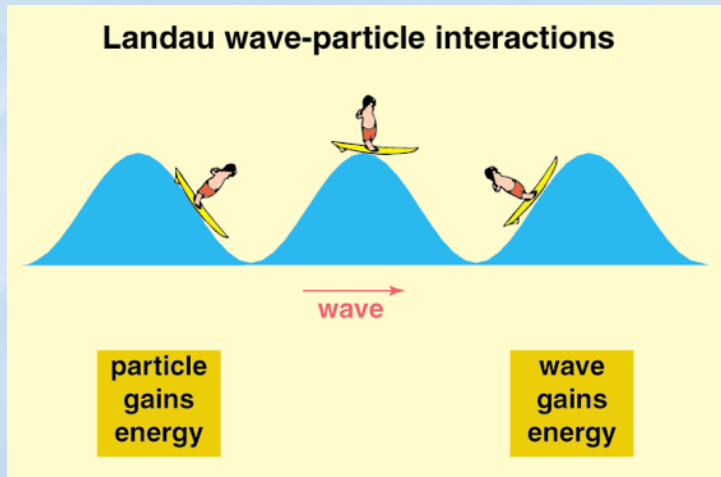


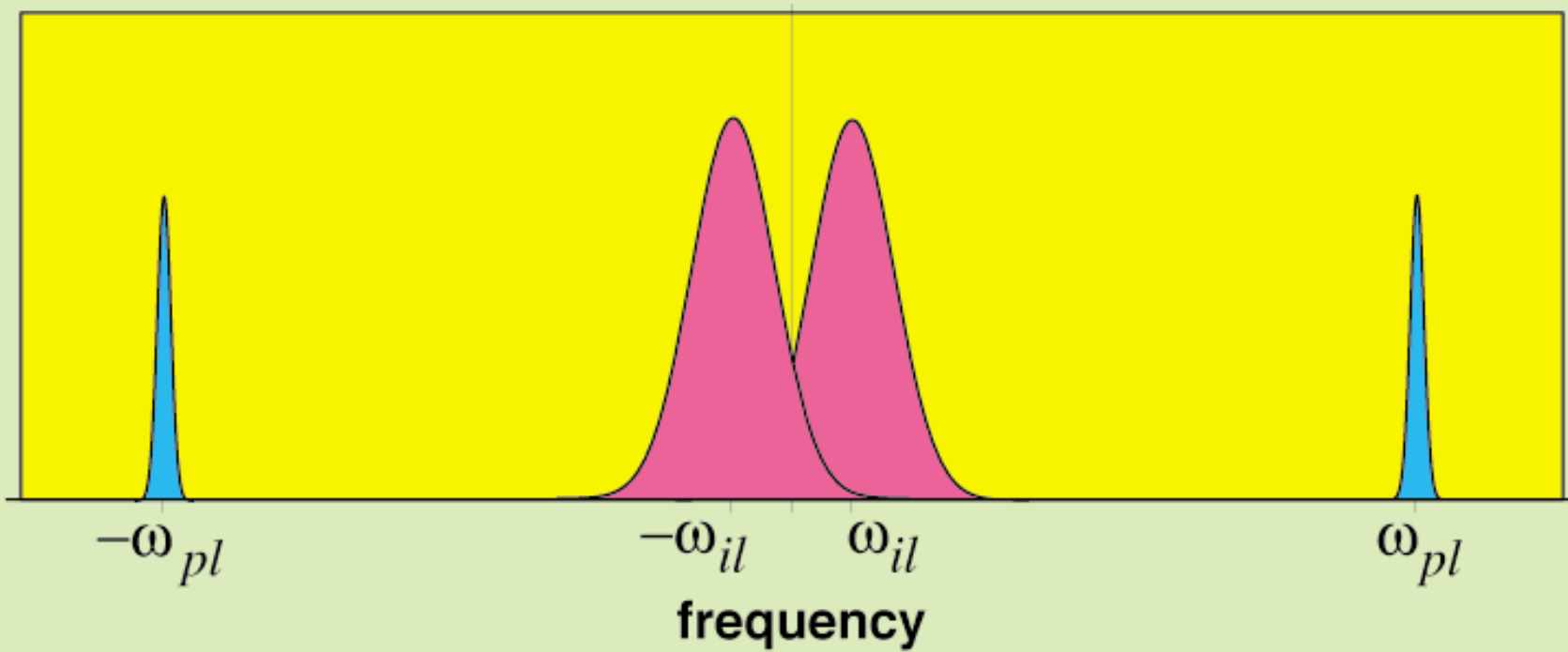
frequency

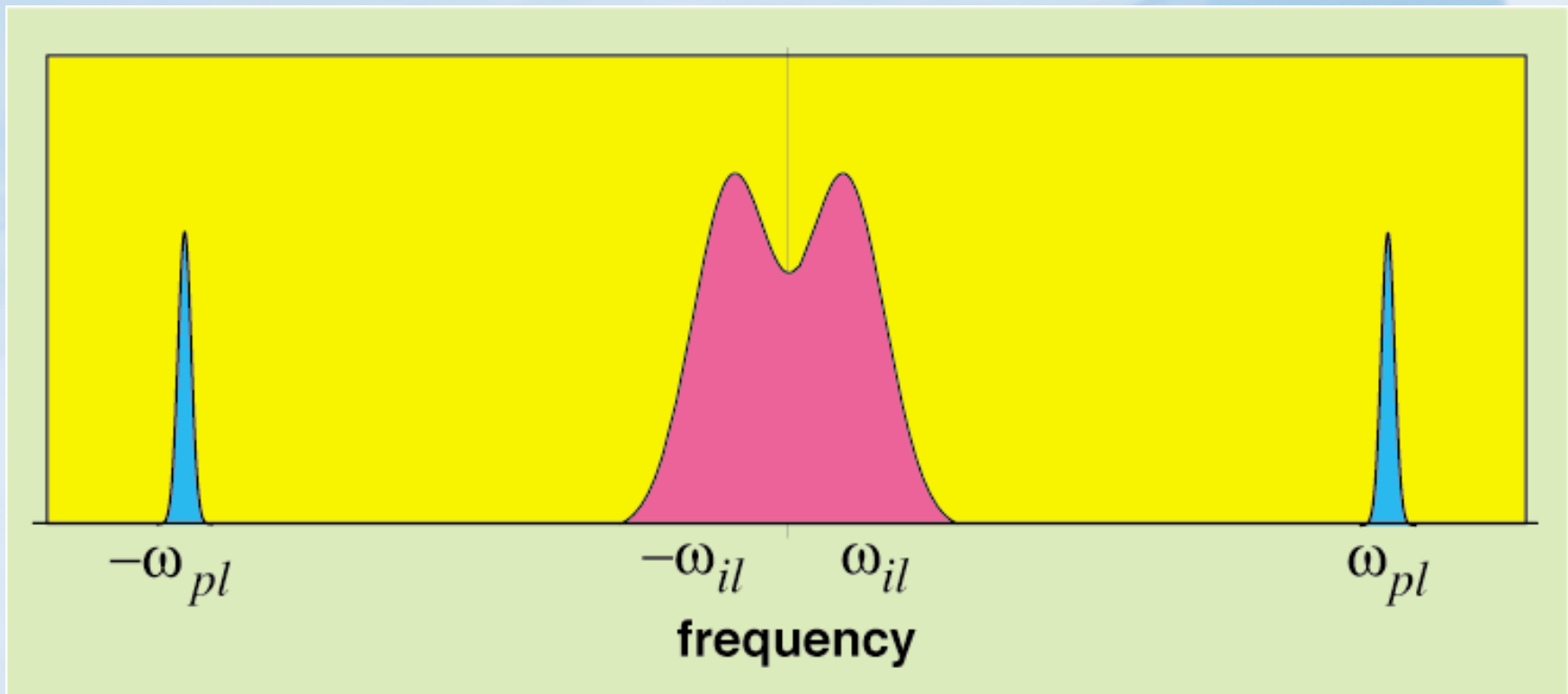
# Plasma Wave Approach (cont'd)



# Landau damping







## Incoherent Scattering Spectrum

$$S_e(\mathbf{k}, \omega) = N_e \left| 1 - \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_e(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v}) + N_i \left| \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_i(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$

Plasma line

Ion line

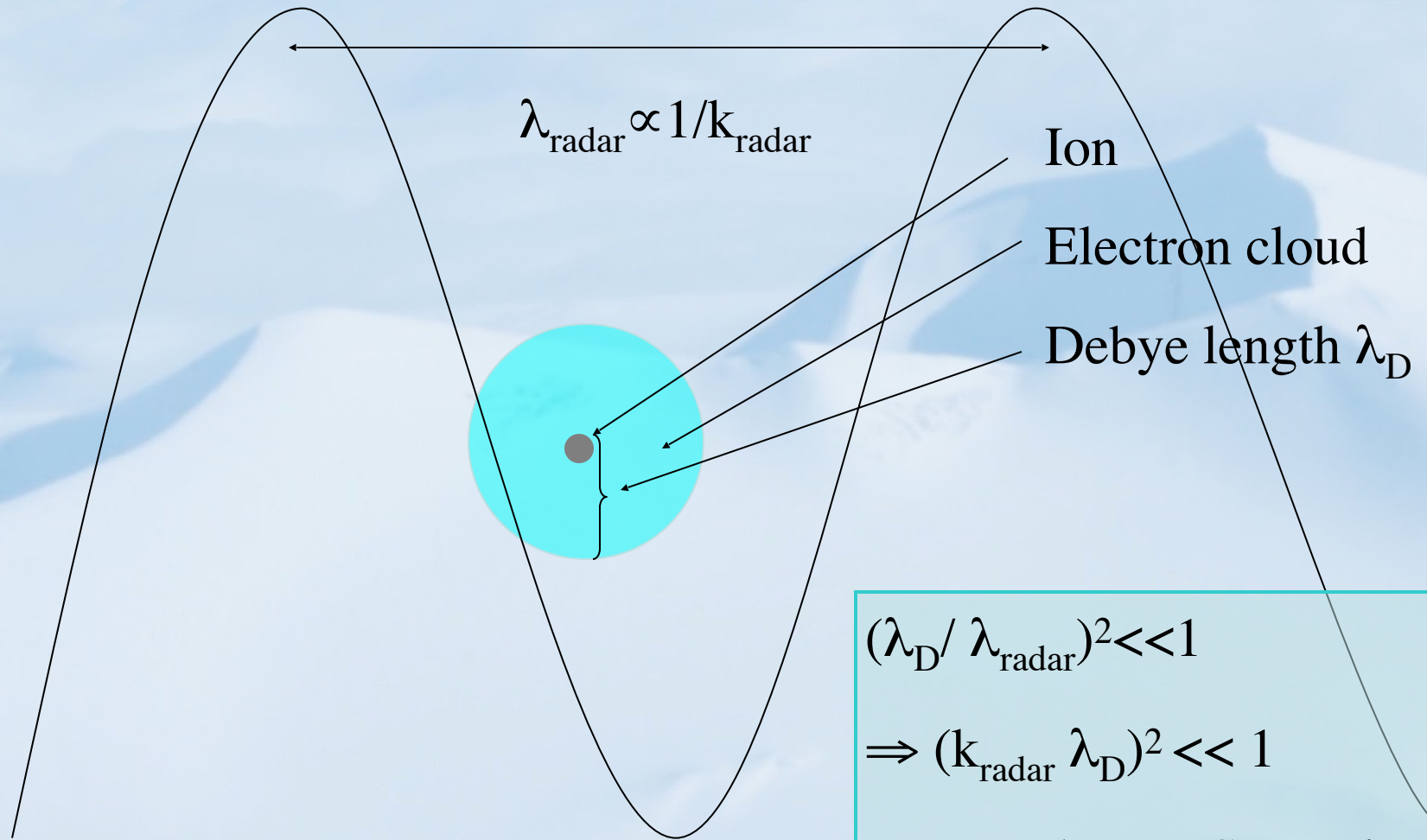
electric susceptibility  $\chi_{e,i}(\mathbf{k}, \omega)$

dielectric constant function  $\epsilon(\mathbf{k}, \omega)$

velocity distribution function  $f_{e,i}(\mathbf{v})$



# Debye length dependence



$$(\lambda_D / \lambda_{\text{radar}})^2 \ll 1$$

$$\Rightarrow (k_{\text{radar}} \lambda_D)^2 \ll 1$$

$\Rightarrow$  “Incoherent Scattering”

**Plasma Line**  $S_{PL}(\mathbf{k}, \omega)$

**Ion Line**  $S_{IL}(\mathbf{k}, \omega)$

$$S_e(\mathbf{k}, \omega) = N_e \left| 1 - \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_e(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v}) + N_i \left| \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_i(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$

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$$\epsilon(\mathbf{k}, \omega) = 0$$

$$\omega_{pl}(k) \approx \omega_{pe} (1 + 3\lambda_D^2 k^2)$$

$$\omega_{ia}(k) \approx k \sqrt{\frac{T_e + 3T_i}{m_i}}$$

**Plasma Line**  $S_{PL}(\mathbf{k}, \omega)$

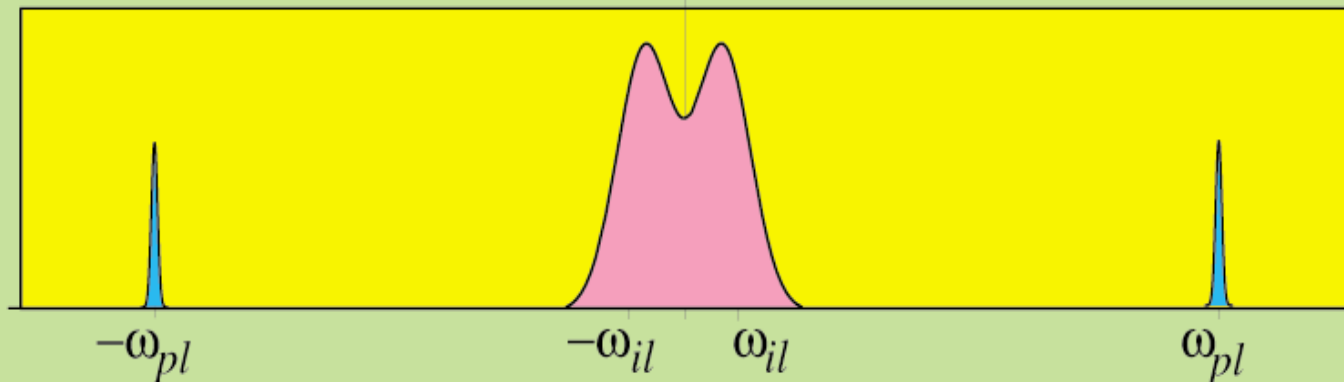
**Ion Line**  $S_{IL}(\mathbf{k}, \omega)$

$$S_e(\mathbf{k}, \omega) = N_e \left| 1 - \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_e(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v}) + N_i \left| \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_i(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$

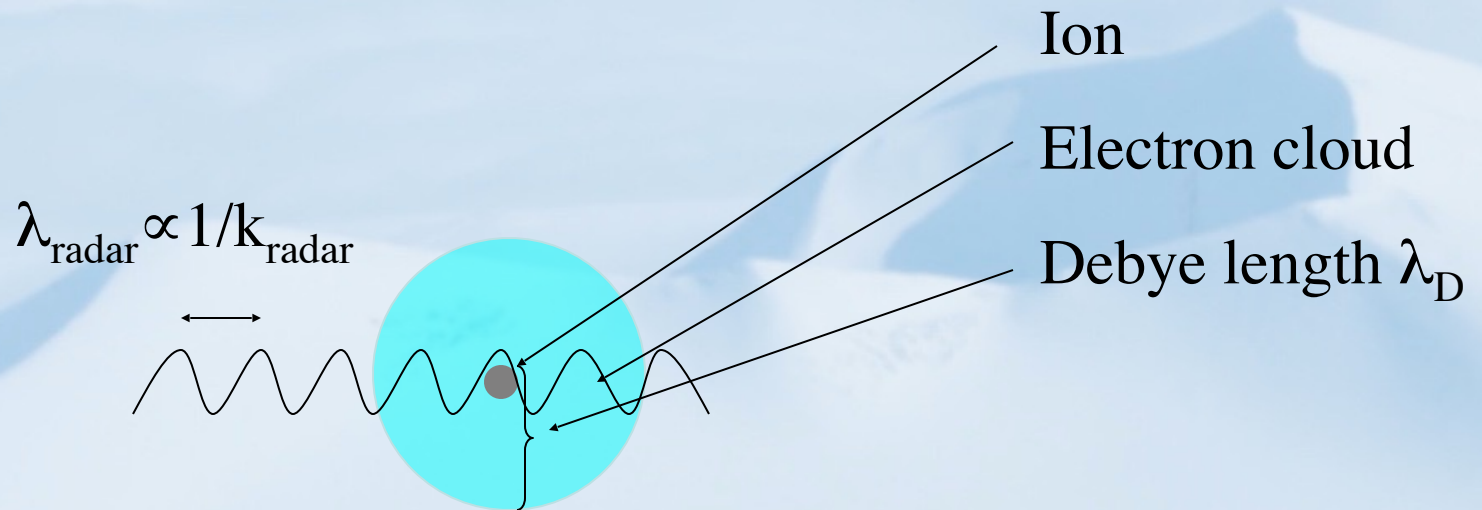
$$\epsilon(\mathbf{k}, \omega) = 0$$

$$\omega_{pl}(k) \approx \omega_{pe} (1 + 3\lambda_D^2 k^2)$$

$$\omega_{ia}(k) \approx k \sqrt{\frac{T_e + 3T_i}{m_i}}$$



# Debye length dependence



$$(\lambda_D / \lambda_{\text{radar}})^2 > 1$$

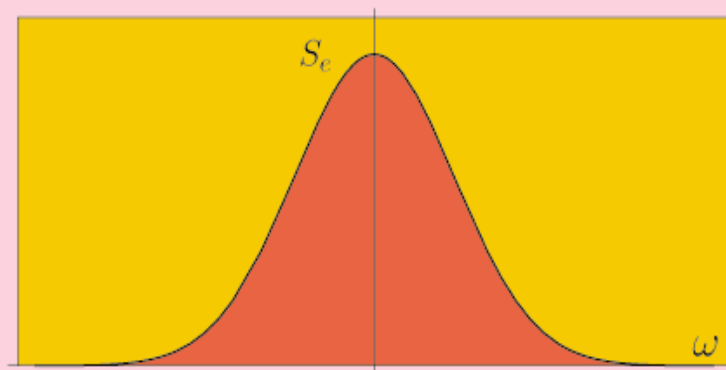
$$\Rightarrow (k_{\text{radar}} \lambda_D)^2 > 1$$

$\Rightarrow$  No collective interactions

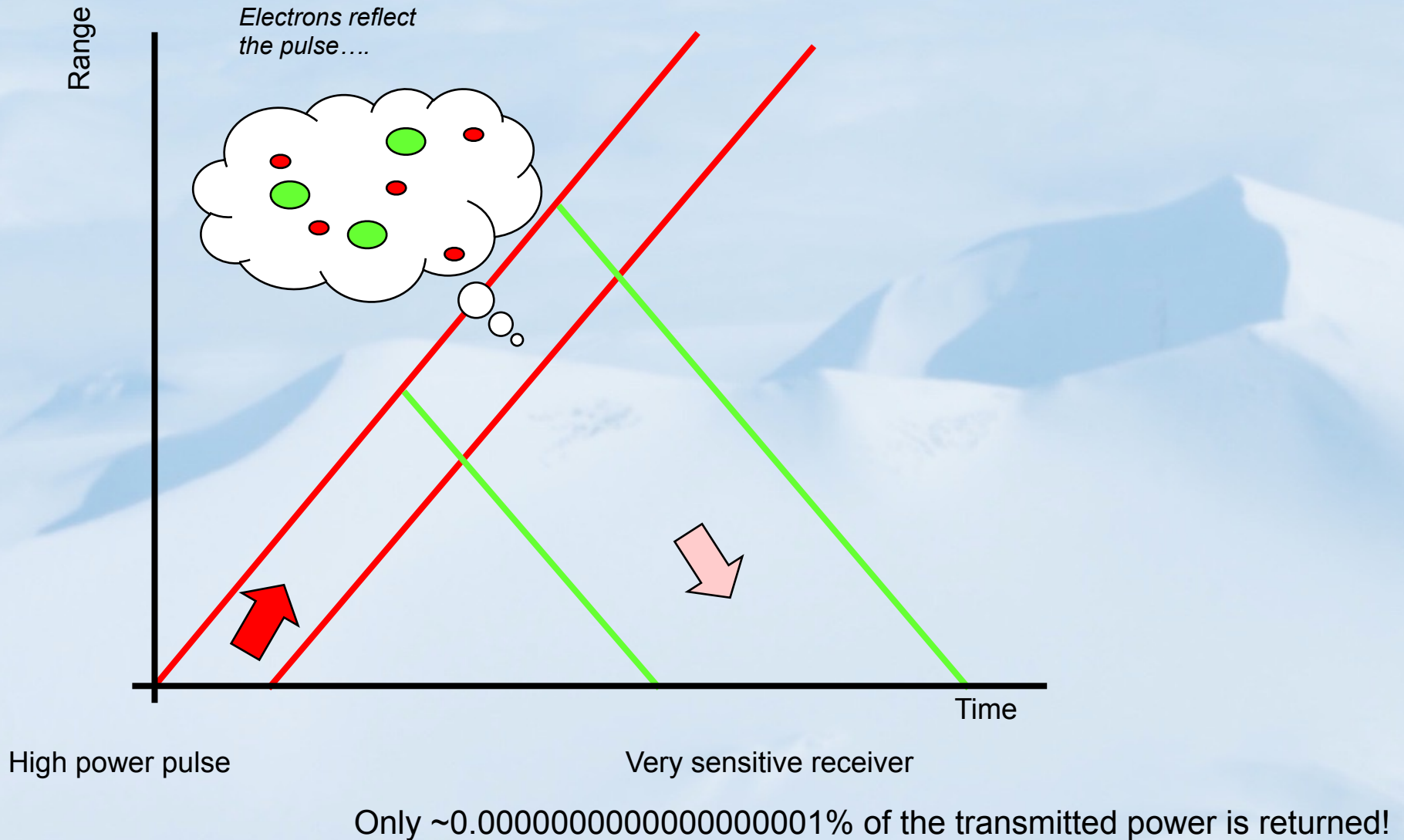
**no collective interactions**

$$S_e(\mathbf{k}, \omega) = N_e \left| 1 - \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_e(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v}) + N_i \left| \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_i(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$

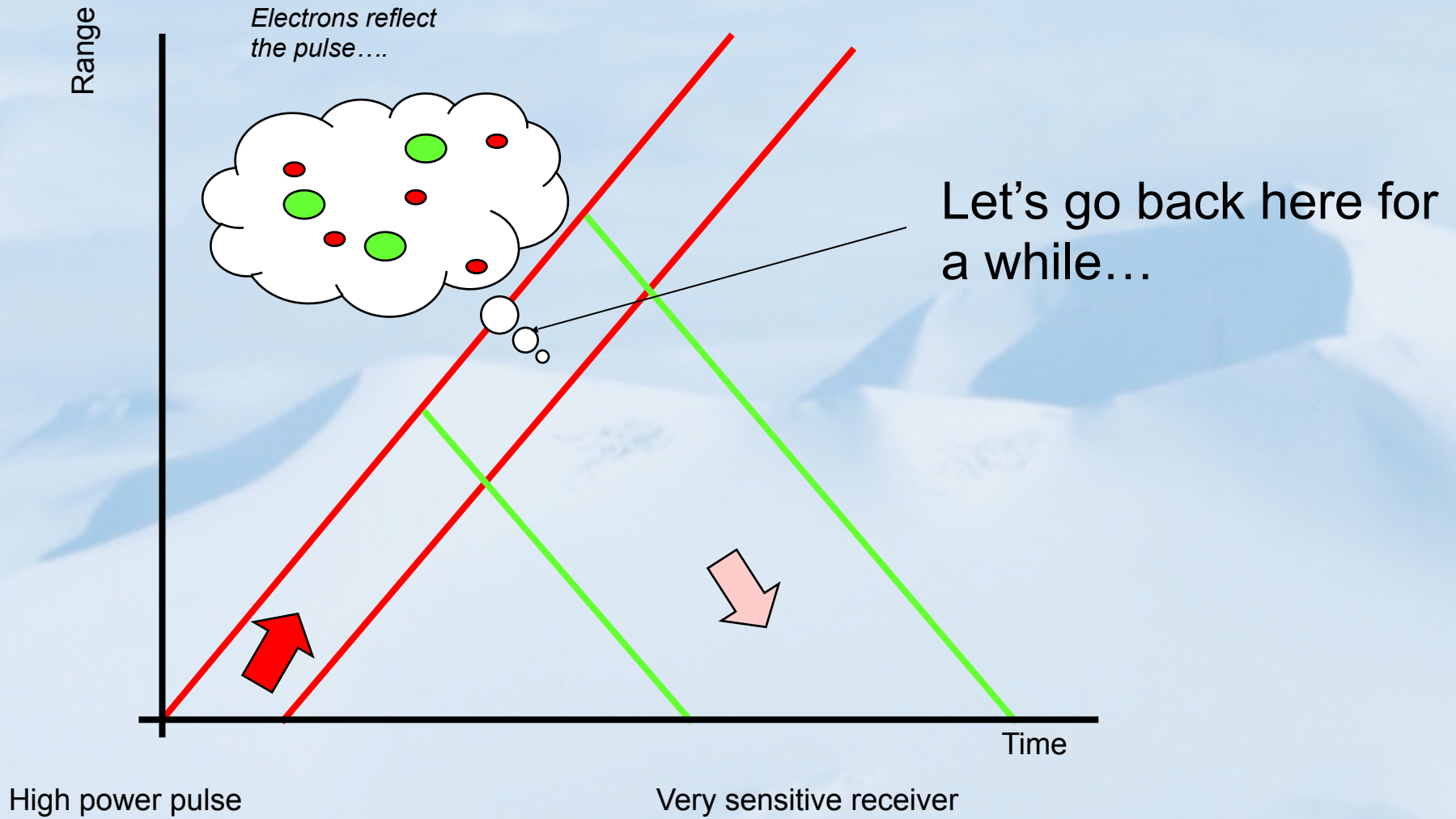
$$S_e(\mathbf{k}, \omega) = N_e \int d\mathbf{v} f_e(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$



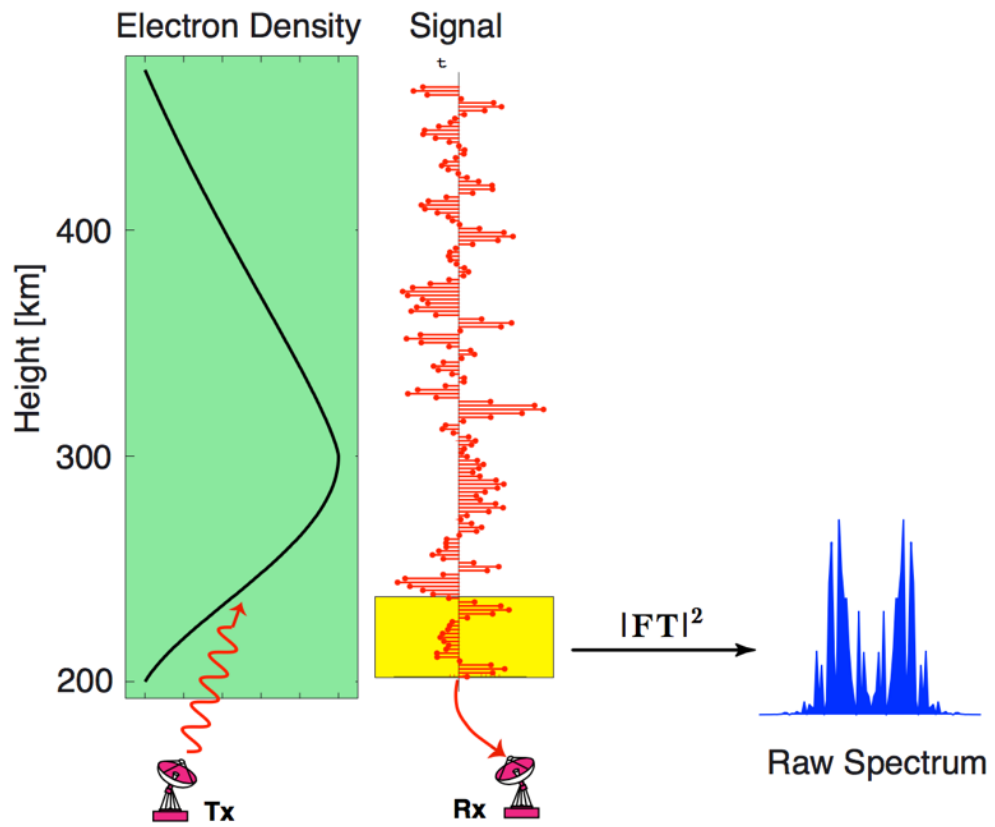
# How ISRs work...

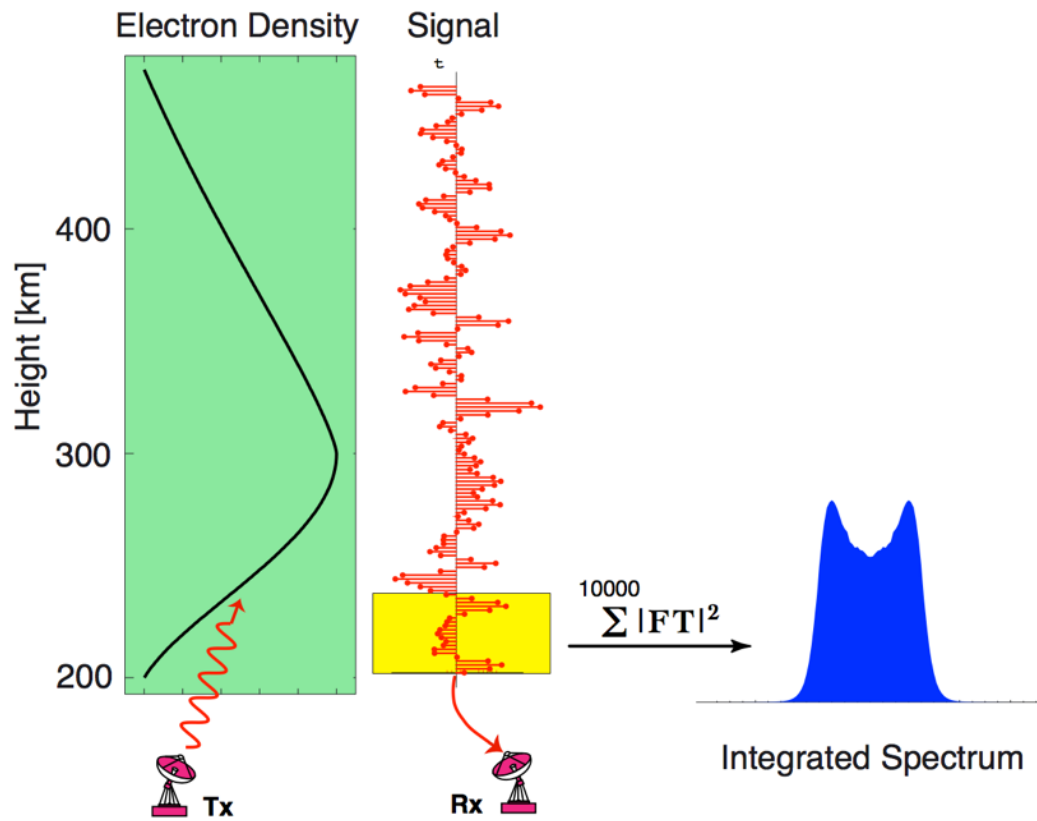


# How ISRs work...



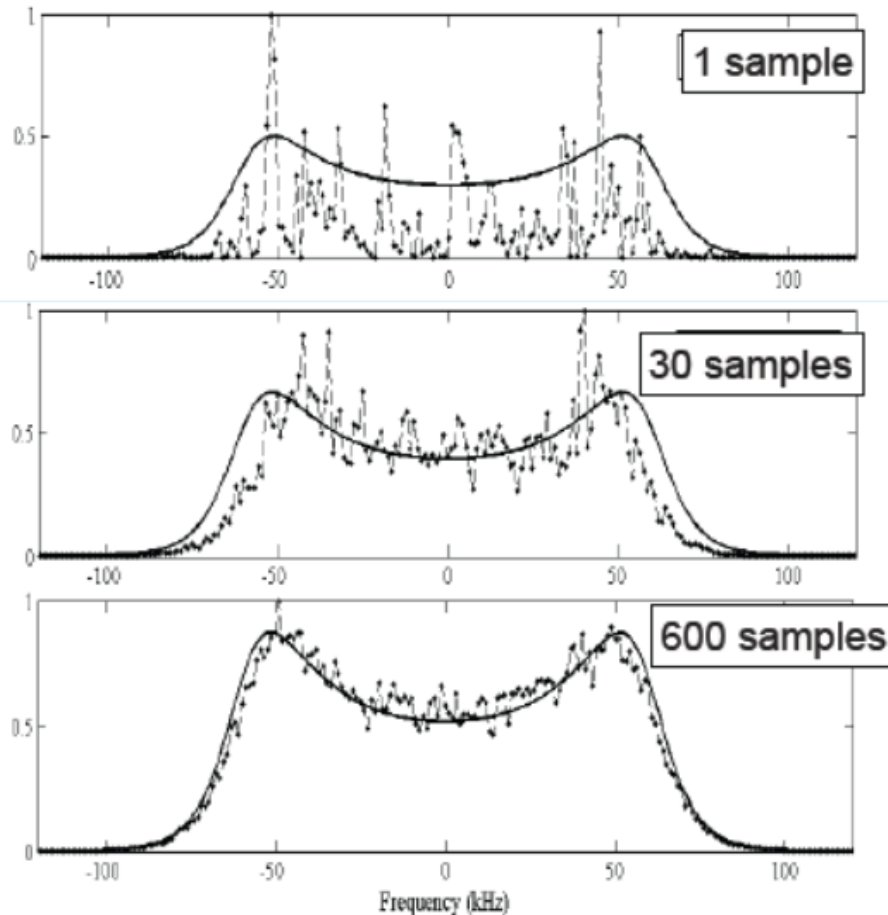






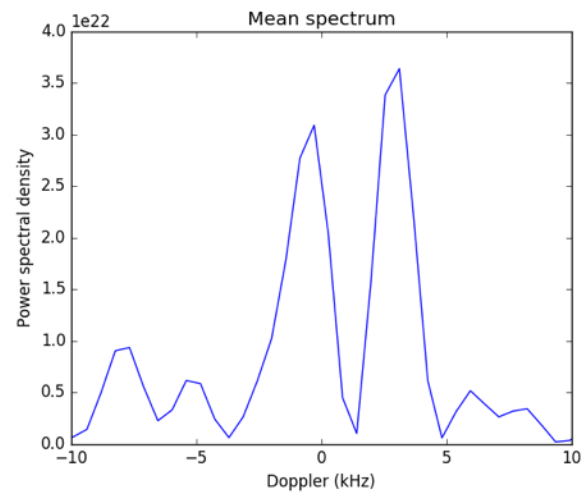
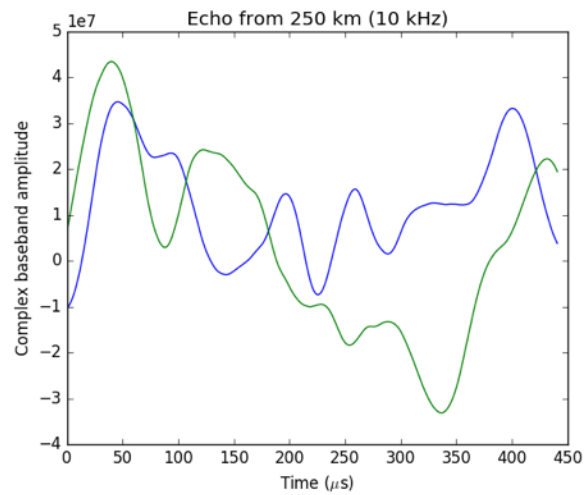
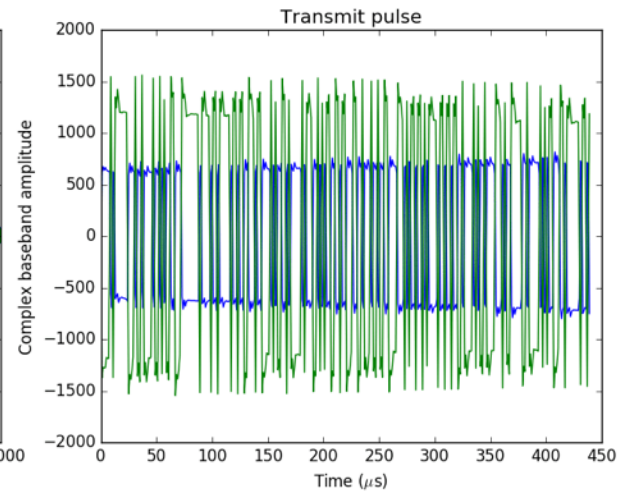
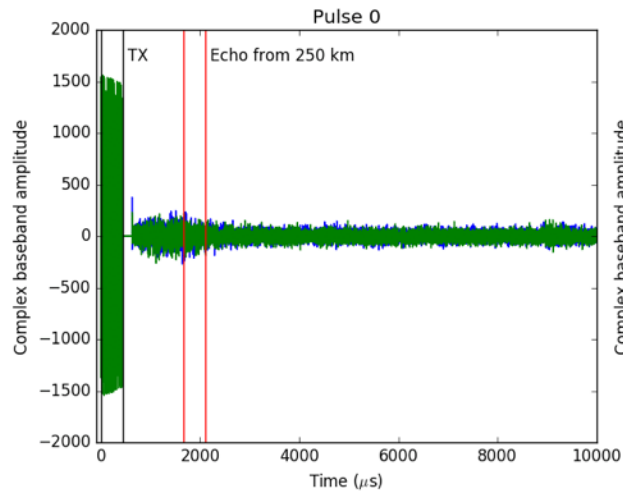
# Incoherent Averaging

Normalized ISR spectrum for different integration times at 1290 MHz

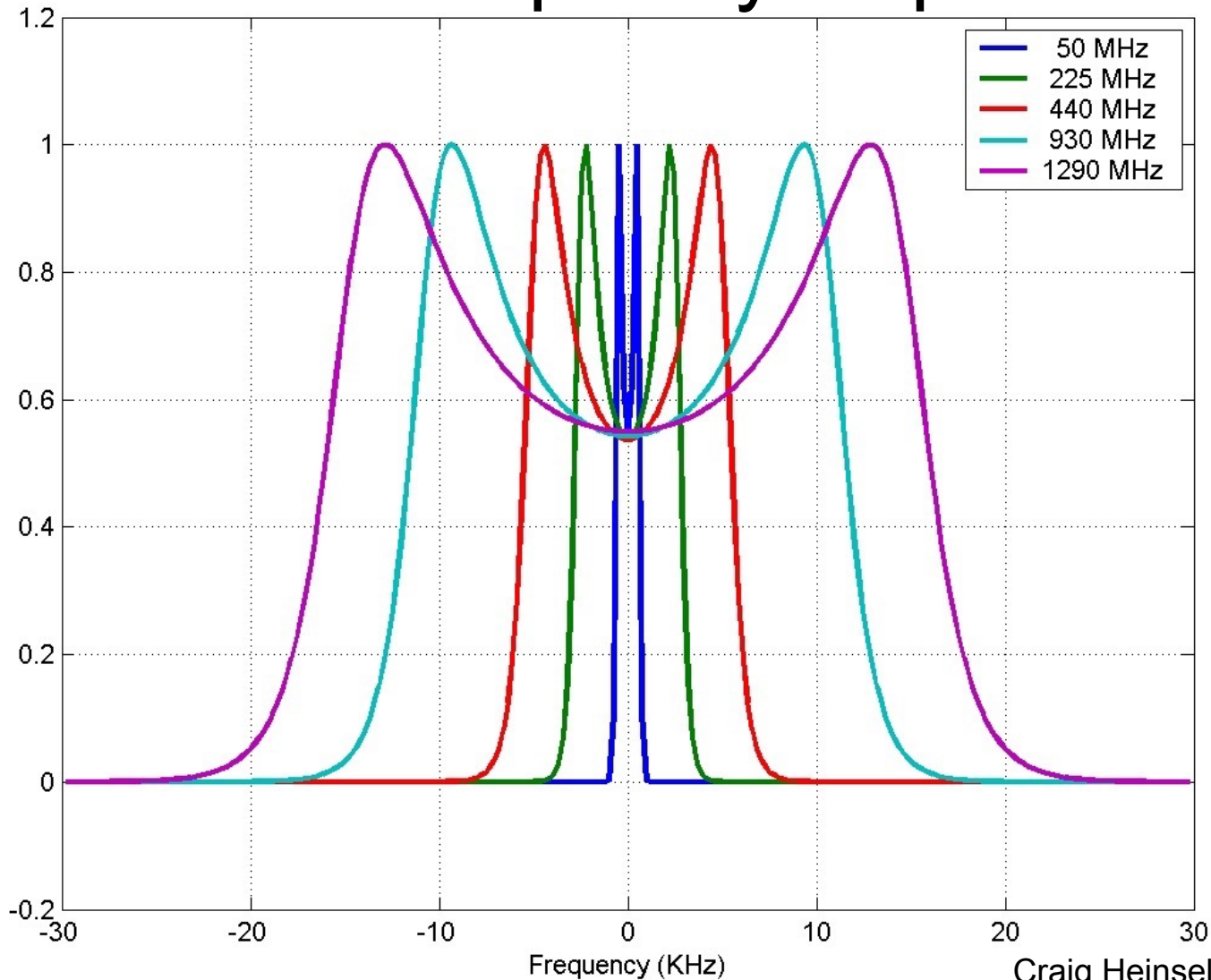


We are seeking to estimate the power spectrum of a Gaussian random process. This requires that we sample and average many independent “realizations” of the process.

$$\text{Uncertainties} \propto \frac{1}{\sqrt{\text{Number of Samples}}}$$

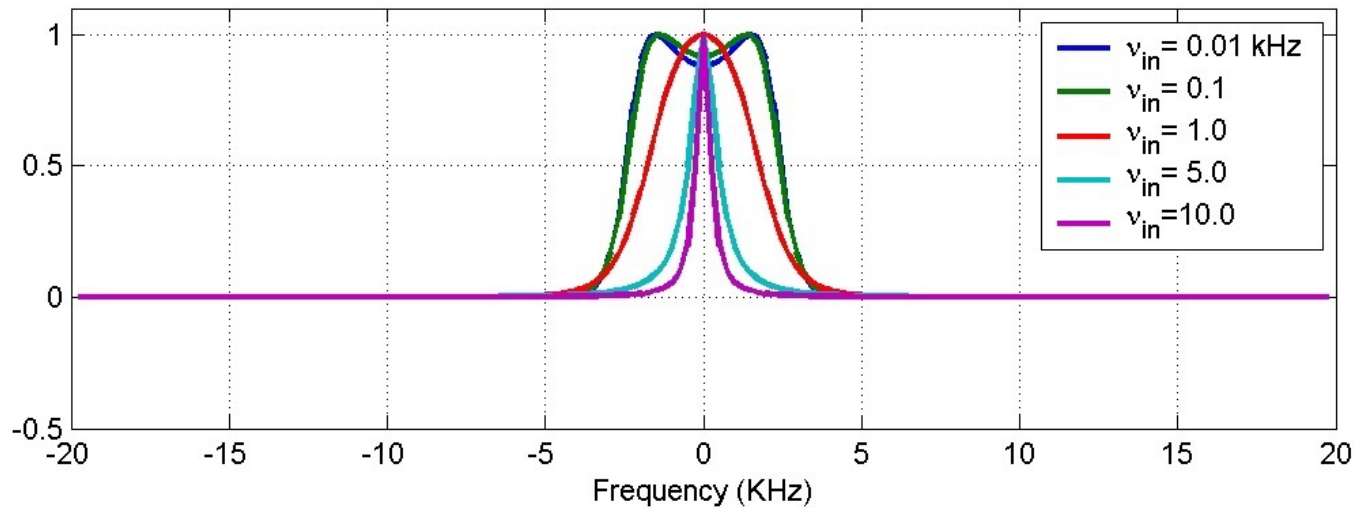


# Radar Frequency Dependencies

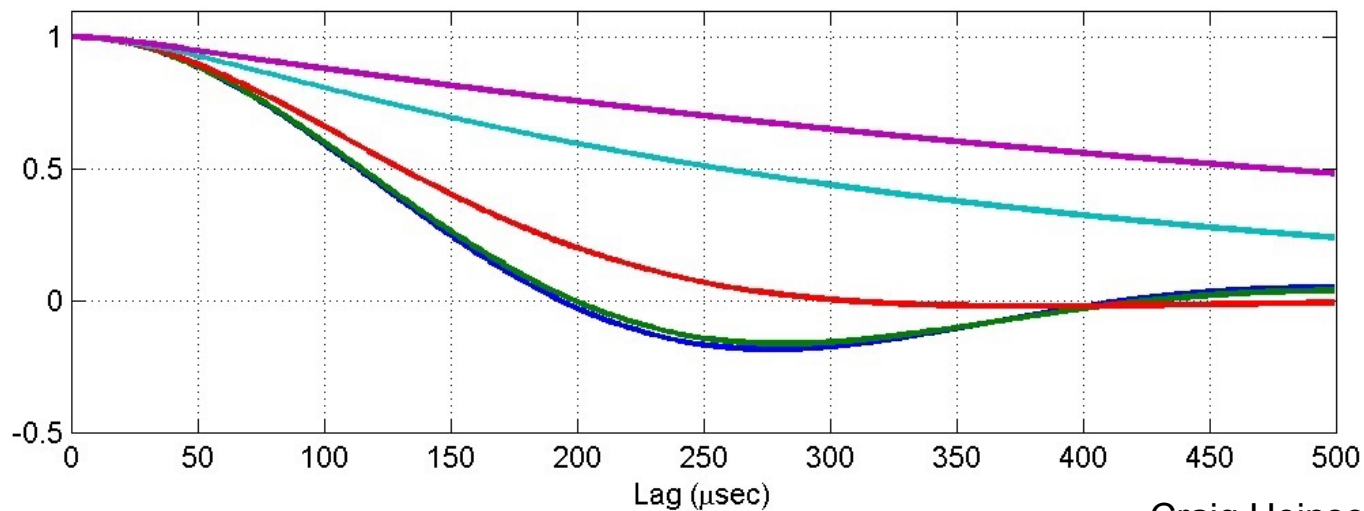


Parameters  
Ne:  $10^{12} \text{ m}^{-3}$   
Ti: 1000 K  
Te: 2000 K  
Comp: 100% O<sup>+</sup>  
 $v_{in}$ :  $10^{-6} \text{ KHz}$

# Ion-Neutral Collision Frequency

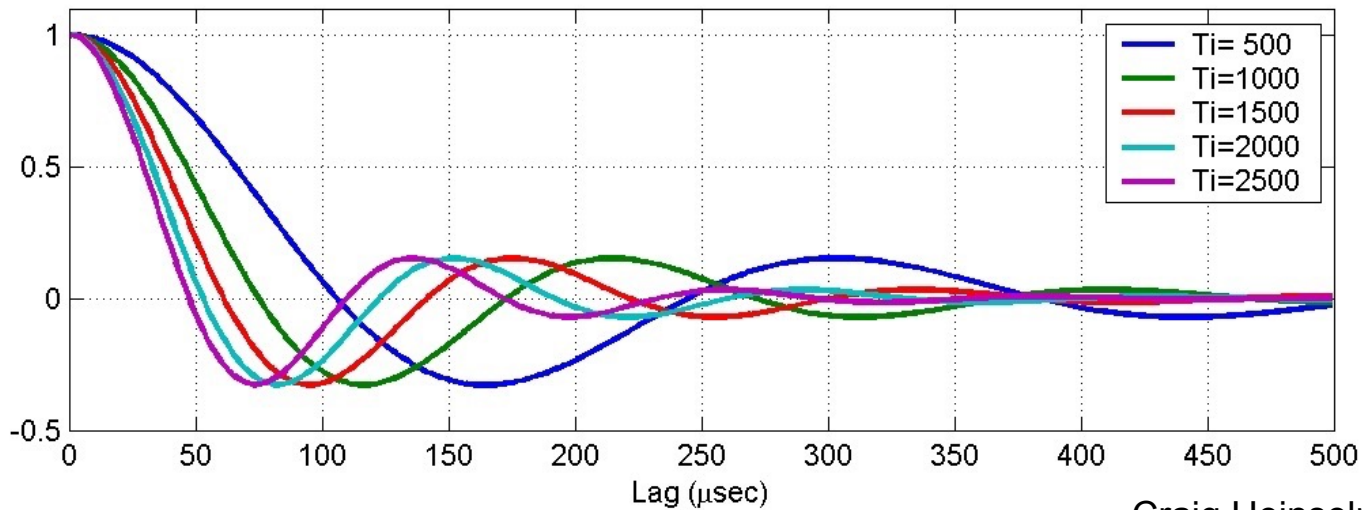
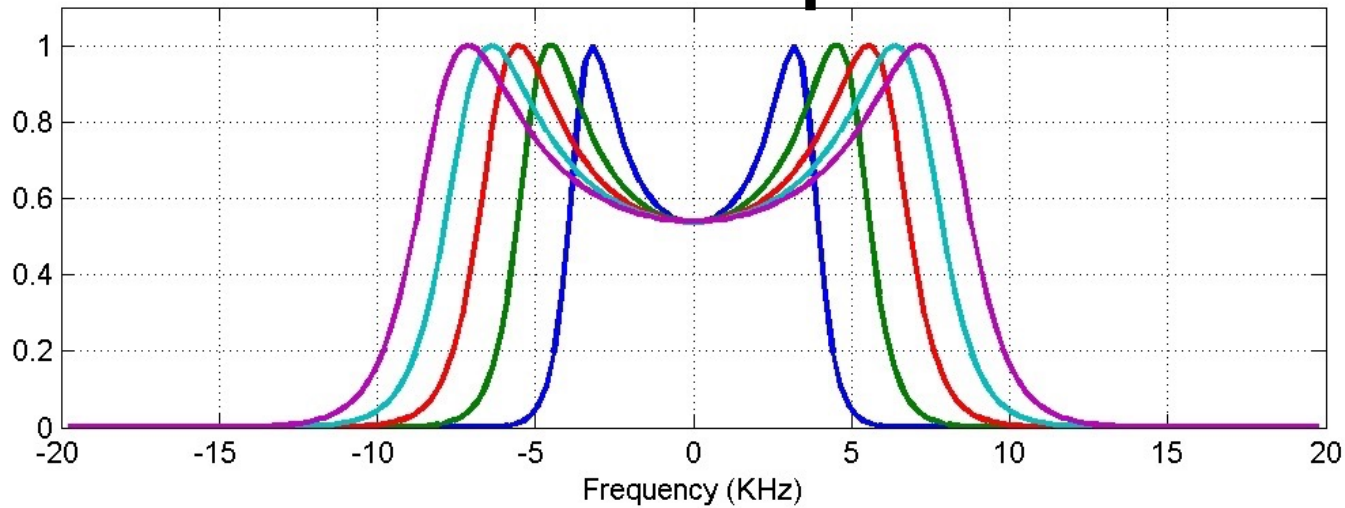


Parameters  
Freq: 449 MHz  
Ne:  $10^{12} \text{ m}^{-3}$   
Ti: 500 K  
Te: 500 K  
Comp: 100% NO<sup>+</sup>

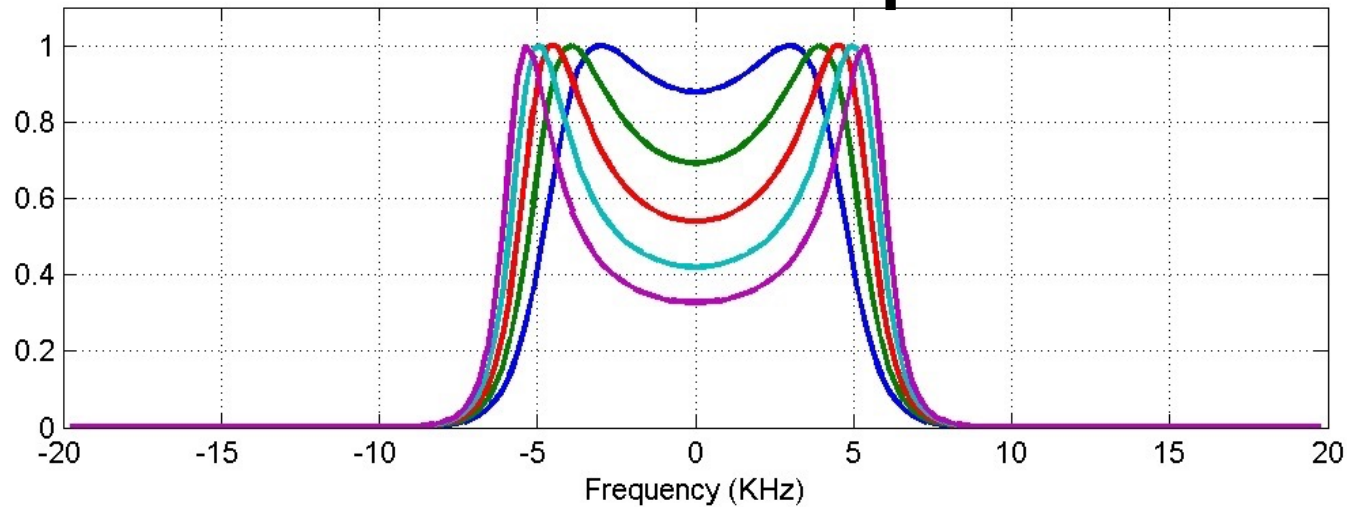


# Ion Temperature

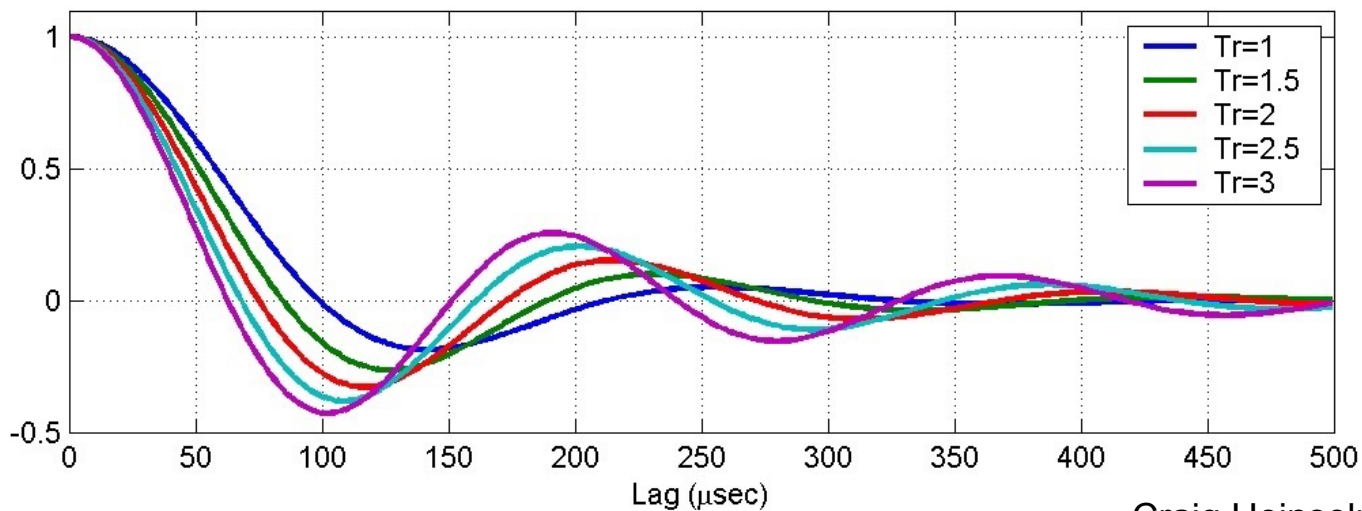
Parameters  
Freq: 449 MHz  
Ne:  $10^{12} \text{ m}^{-3}$   
Te:  $2 * T_i$   
Comp: 100% O<sup>+</sup>  
 $v_{in}$ :  $10^{-6} \text{ KHz}$



# Electron/Ion Temperature Ratio

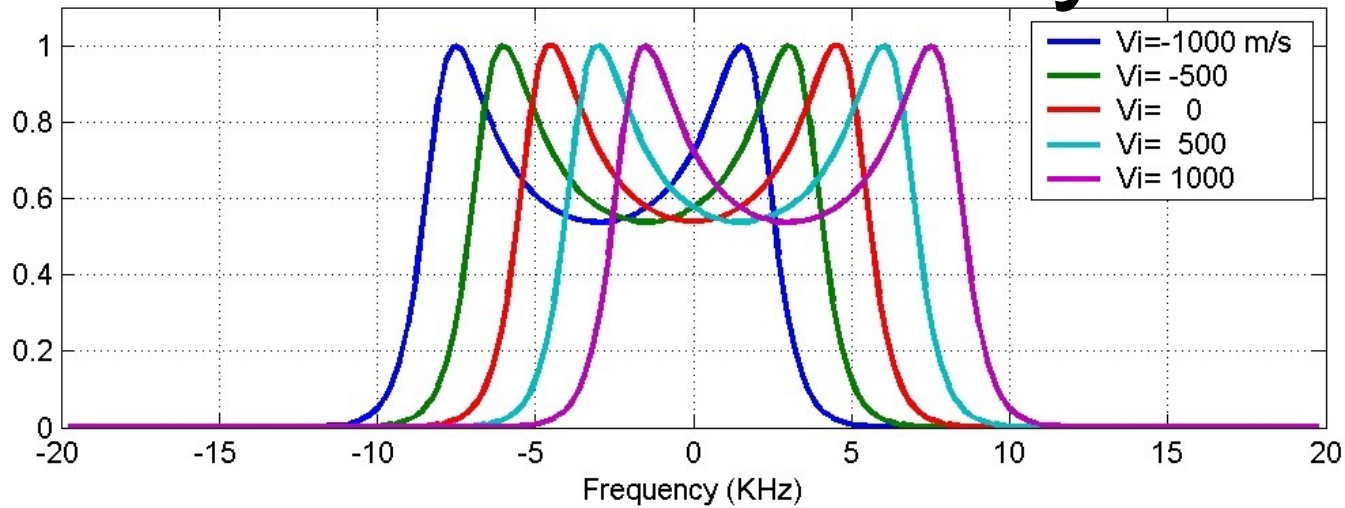


Parameters  
Freq: 449 MHz  
Ne:  $10^{12} \text{ m}^{-3}$   
Ti: 1000 K  
Comp: 100% O<sup>+</sup>  
 $v_{in}$ :  $10^{-6} \text{ KHz}$

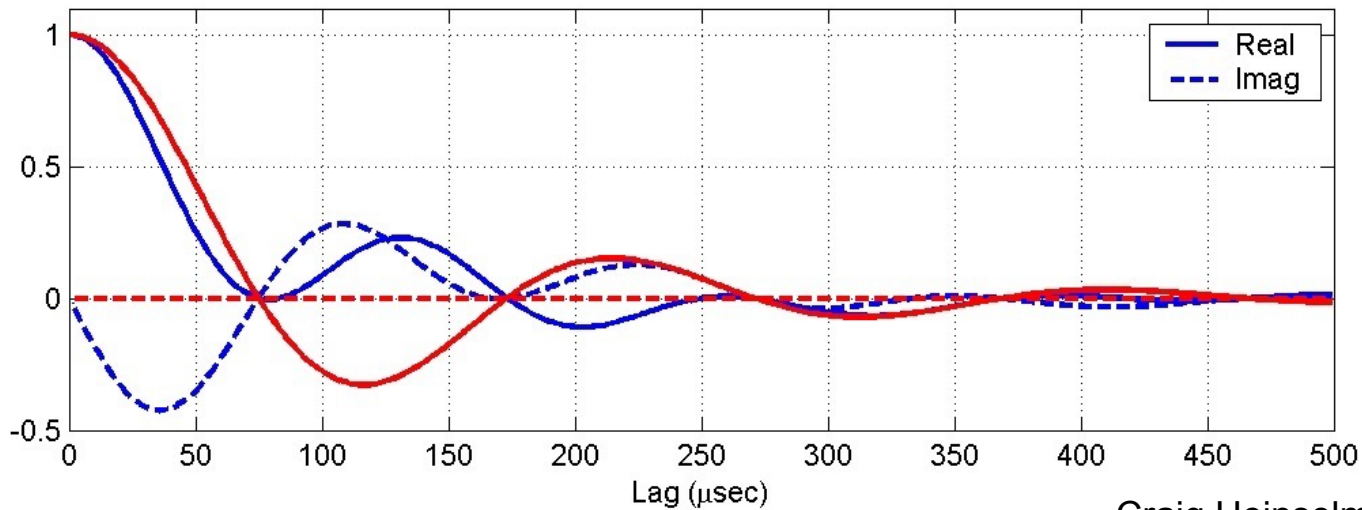




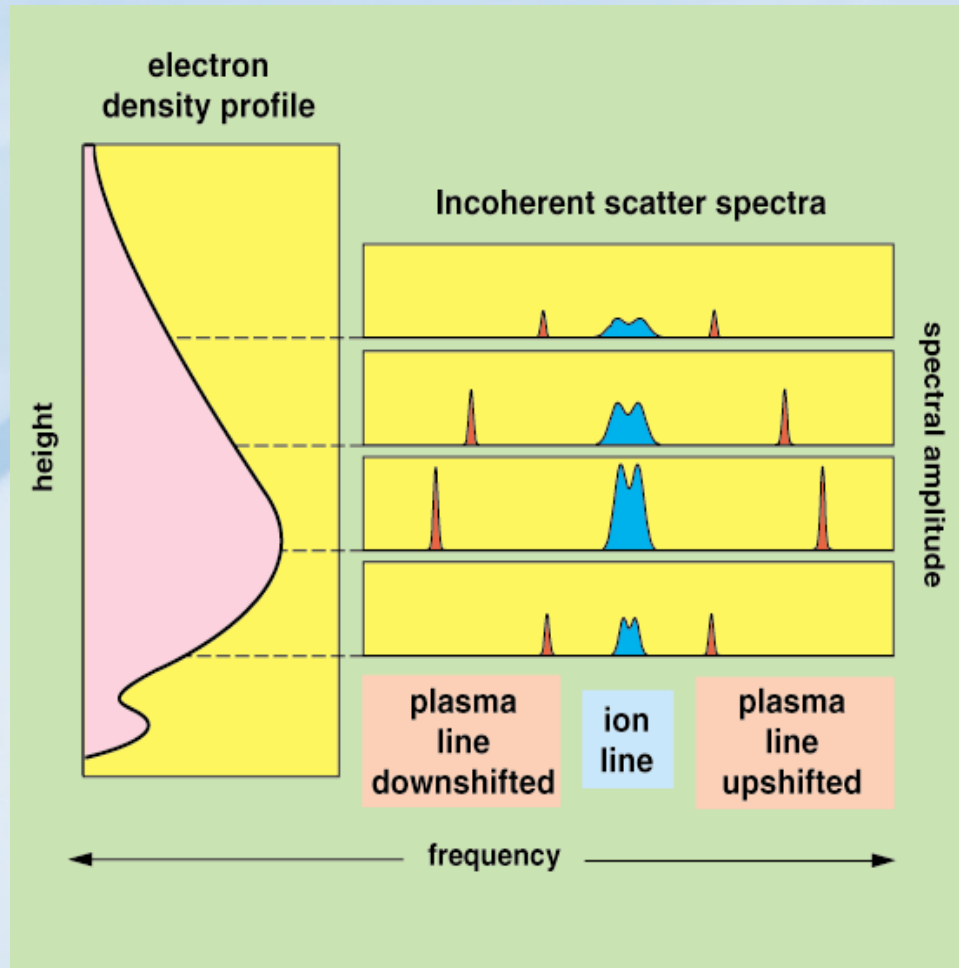
# Ion Velocity



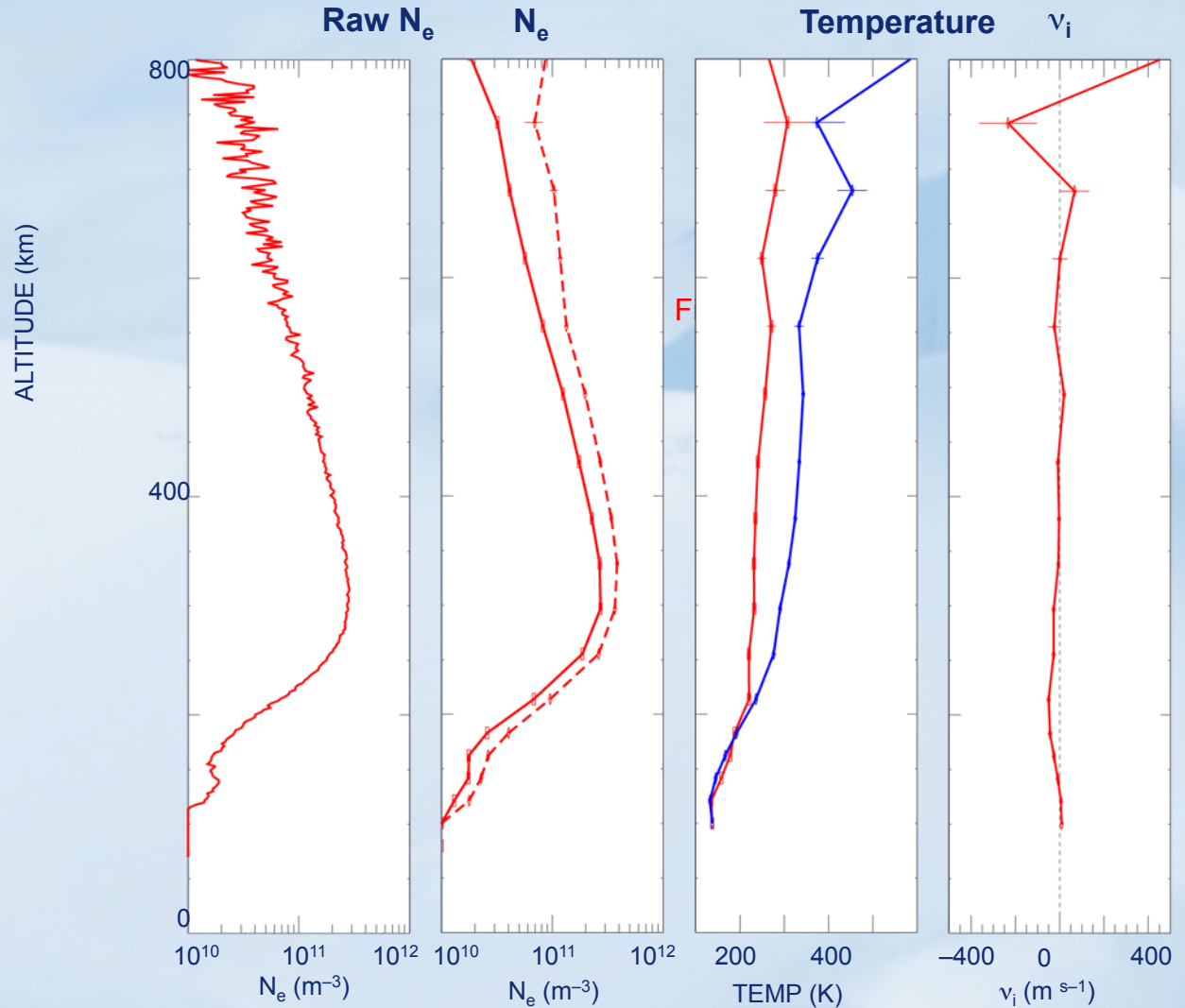
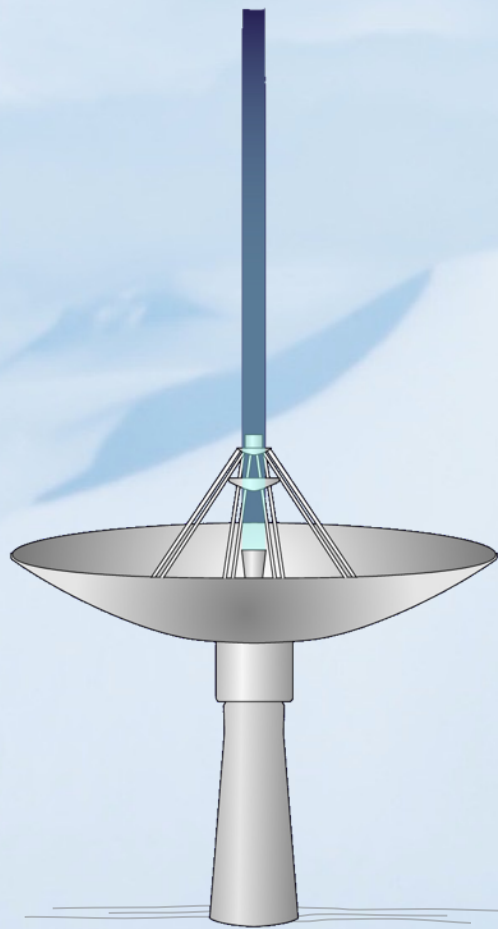
**Parameters**  
Freq: 449 MHz  
Ne:  $10^{12} \text{ m}^{-3}$   
Ti: 1000 K  
Te: 2000 K  
Comp: 100% O<sup>+</sup>  
 $v_{in}$ :  $10^{-6}$  KHz



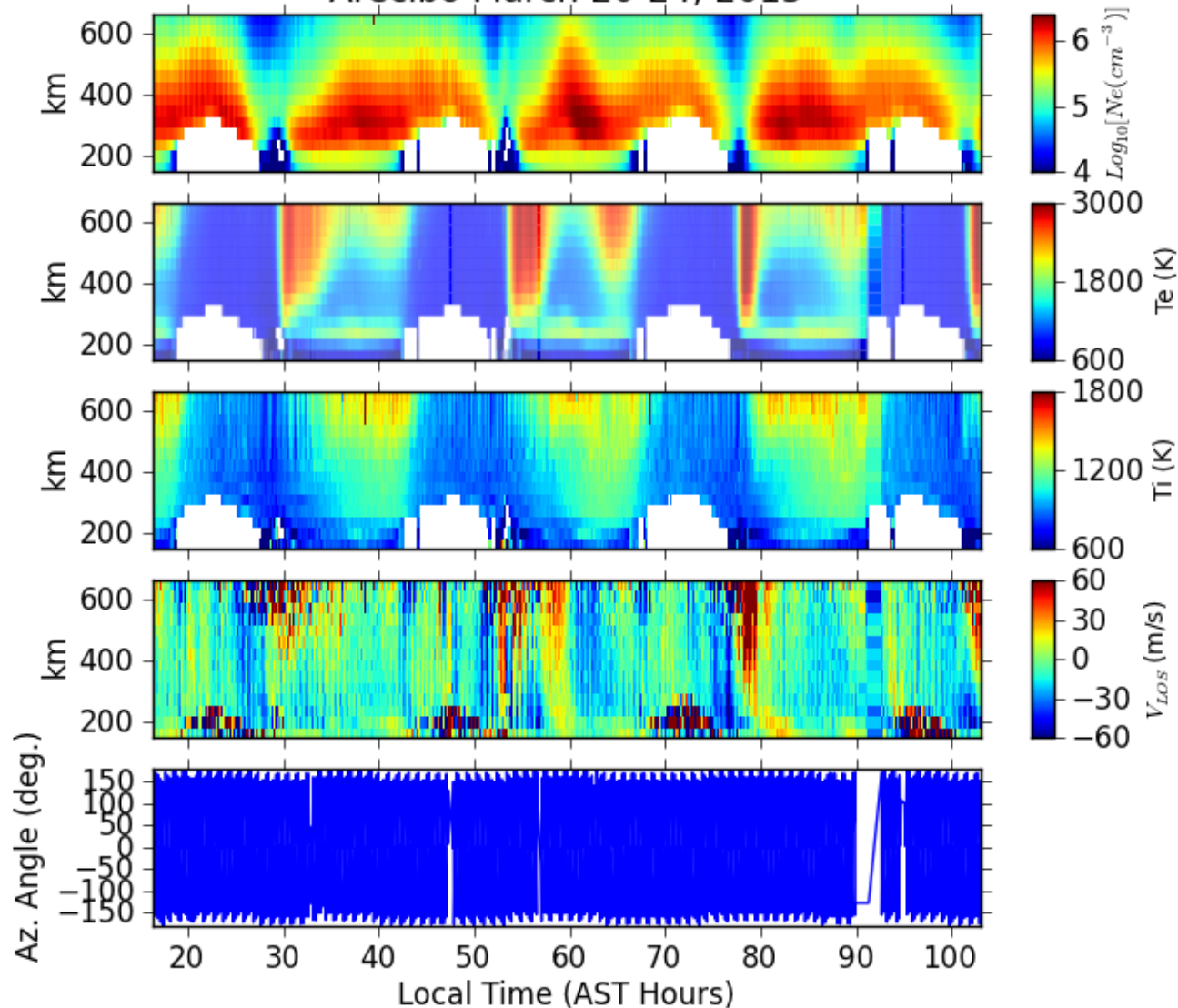
# Spectral space as a function of altitude



# Plasma Parameter Profile



# Arecibo March 20-24, 2015



And this is the level data we will work on in the MADRIGAL session...

# Questions you should ask

(...although some might cause facepalming and potential religious wars among the lecturers)

- What is the difference between coherent and incoherent scatter?
- What is the significance of under- versus over-spread targets?
- What is a hard target versus a soft target?
- What is the difference between a beam filling versus a point target?