First Order Compressive Axial Buckling Analysis on Ascent Vehicles

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1 The rocket as a beam

1.1 Free body diagram

We will model the rocket as an isotropic beam, that can only deform along its axial direction. All non-axial loads are ignored in the analysis, but should be included in safety factors and design considerations. The beam loads are as in Fig. 1.

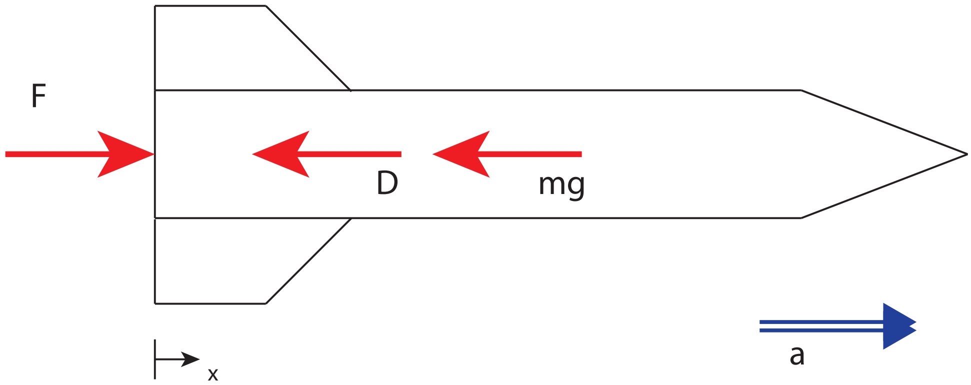

Figure 1: Rocket Free body diagram

The rocket sees a thrust $F = F(t)$, drag $D = F(t)$ and a gravitational force mg where $m = m(t)$ is the rockets mass at the time of analysis. These result in an acceleration $a = a(t)$ which is positive during the burn and negative after. We can write:

$$ma = F - D - mg \tag{1}$$

$$a = \frac{F - D}{m} - g \tag{2}$$

1.2 Take a cut

Now we can cut the beam and analyse the tension in the beam at the section that the cut was taken, Fig. 2.

Therefore we can write:

$$m'a = -T(x) - m'g - D' \tag{3}$$

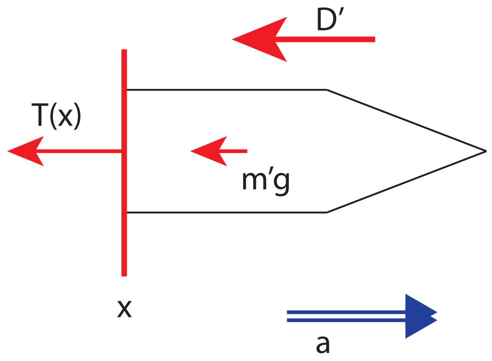

Figure 2: Take a cut on the beam and analyse the tension

$$\therefore \boxed{T(x) = -m'(x)(a+g) - D'(x)} \tag{4}$$

where the prime indicates that we only consider the fraction that was above the cut line, ie. D' refers to the fraction of drag force that acts on the rocket above the point x. A negative value indicates the material is under compression.

From an aerodynamics point of view this is a really weird idea, but I am imagining this from a structural perspective where we study the distribution of pressures and shear forces on the rocket, and integrate it appropriately and therefore determine how much of the drag force acts on the rocket above x. For now, we assume we know this function $D' = D'(x)$, which to first order can be assumed to either entirely acting at $x = x_{tip}$, ie. $D'(x) = D \quad \forall x$ or linearly and evenly distributed along the length of the rocket, ie, $D'(x) = \frac{x_{tip}-x}{x_{tip}} D$.

The mass distribution can be estimated from the CAD.

1.3 Determine the buckling load

From a NASA paper (*Buckling of Thin Walled Truncated Cones, NASA SP -8019, 1986*, we have an estimate on the buckling load of thin walled nose cones:

$$P_{crit} = \gamma \frac{2\pi E t^2 \cos^2 \alpha}{\sqrt{3(1-\mu^2}} \tag{5}$$

where γ correction factor between experimental and theoretical due to imperfections, E is the Youngs Modulus, t is the wall thickness, α is the semi-vertex angle of the cone, μ is the Poisson's ratio of the material.

Based on the one load test we did (we tested until we heard some delimination, not buckling failure, the appropriate value for γ is 0.14, while the NASA paper (which was for isotropic materials) suggested 0.33. It is probably fair for us to assume 0.14 to account for the anisotropy of the fiberglass, and the inconsistency of our manual lay-up methods, and a safety factor on the load should be applied, perhaps 1.5.

Using the above sections we can predict the compressive load on the nose cone, and using the last equation we can then estimate the required thickness of fiberglass, and round the value up to find the number of layers of fiberglass needed.