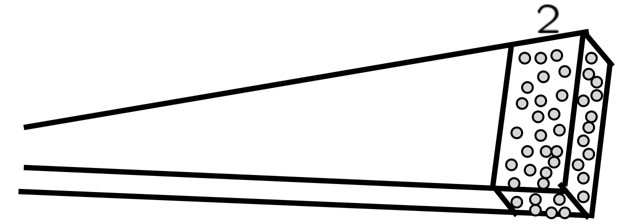
Pulse Compression

Radar Equation Revisited: Energy Matters

Generalized radar equation for one or more scatterers, distributed over a volume:

$$P_r = \int P_t \, \frac{\rho_a^2 A^2}{4\pi\lambda^2 R^4} \, \sigma(\vec{x}) \, dV_s$$

Assume volume is filled with identical, isotropic scatters



$$\int \sigma(\vec{x}) \, dV_s = \frac{c\tau}{2} R^2 \sigma$$

$$P_r = P_t \, \frac{\rho_a^2 A^2}{4\pi\lambda^2 R^4} \, \sigma \, \frac{c\tau}{2} R^2$$

The “soft target” Radar Equation

$$P_r = P_t \, \frac{c\rho_a^2 A^2 \tau}{8\pi\lambda^2 R^2} \, \sigma$$

Notice!

$$\boxed{P_r \propto P_t \, \tau}$$

Units of energy (W x sec = Joules)

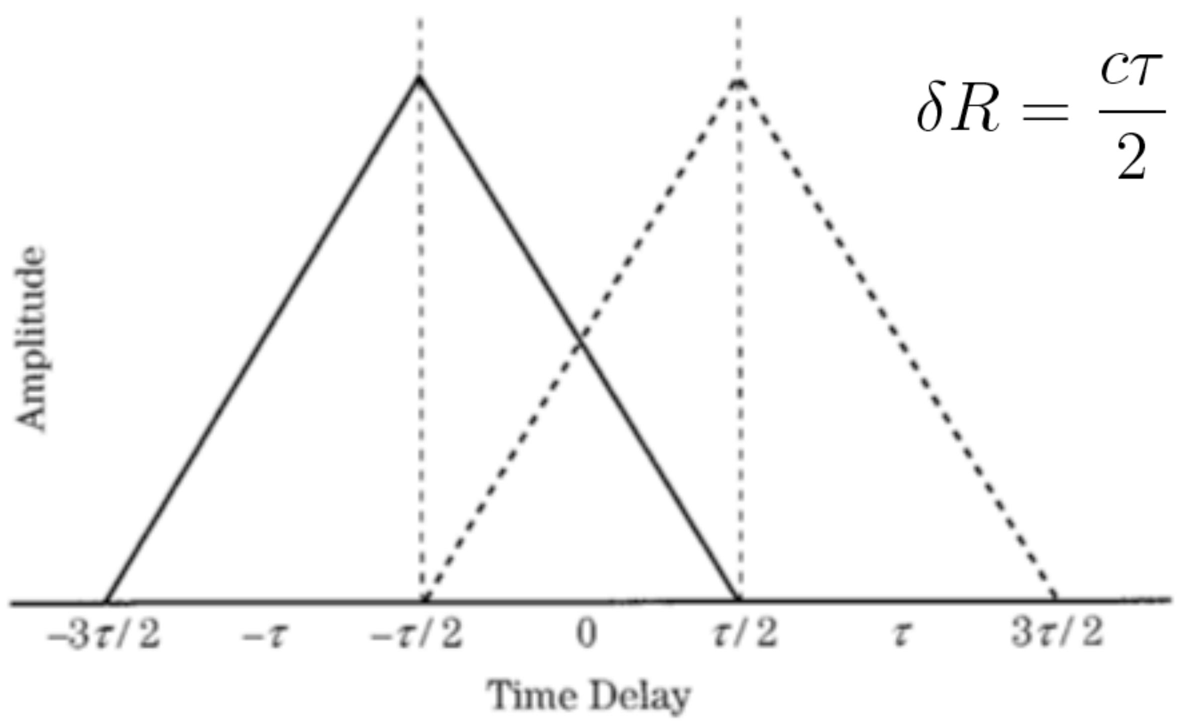
Range Resolution / SNR Coupling

For a given noise level, then, more energy on target means better SNR.

$$P_r \propto P_t \ \tau$$

Longer pulses have more energy.

BUT they have worse range resolution:



$$\delta R = \frac{c\tau}{2}$$

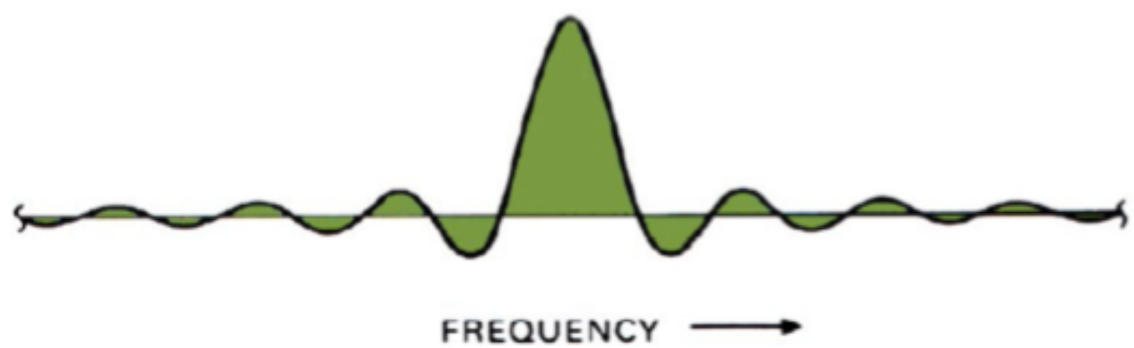
FIGURE 20-4 Individual responses from two point targets separated by the Rayleigh resolution.

Individual matched filter response for Rayleigh separation between peak and first null (-6 dB mainlobe width)

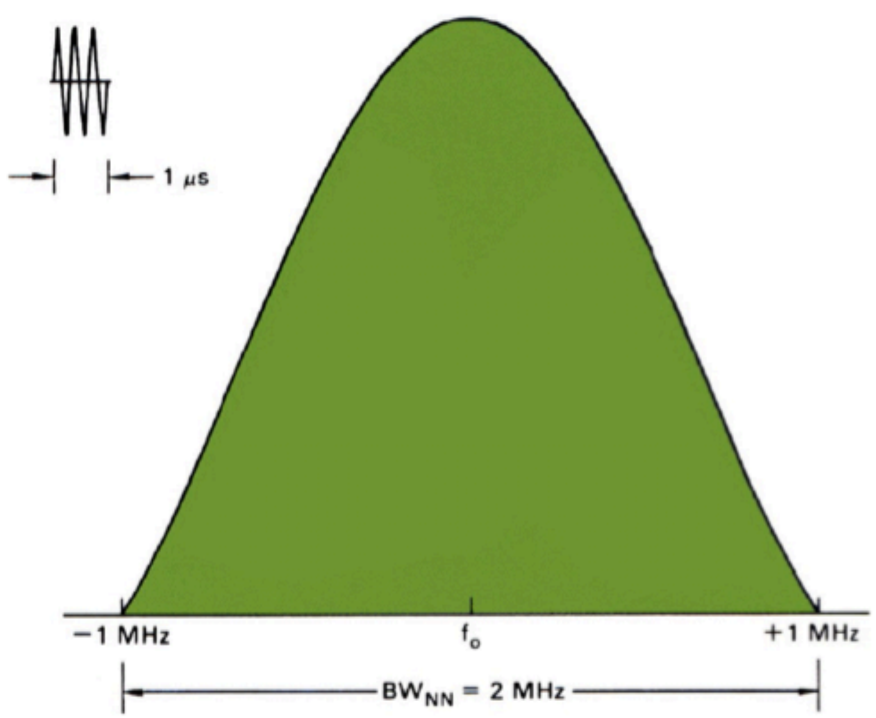
How do we solve this problem?

Bandwidth of a pulsed signal

Spectrum of receiver output has sinc shape, with sidelobes half the width of the central lobe and continuously diminishing in amplitude above and below main lobe



A 1 microsecond pulse has a null-to-null bandwidth of the central lobe = 2 MHz



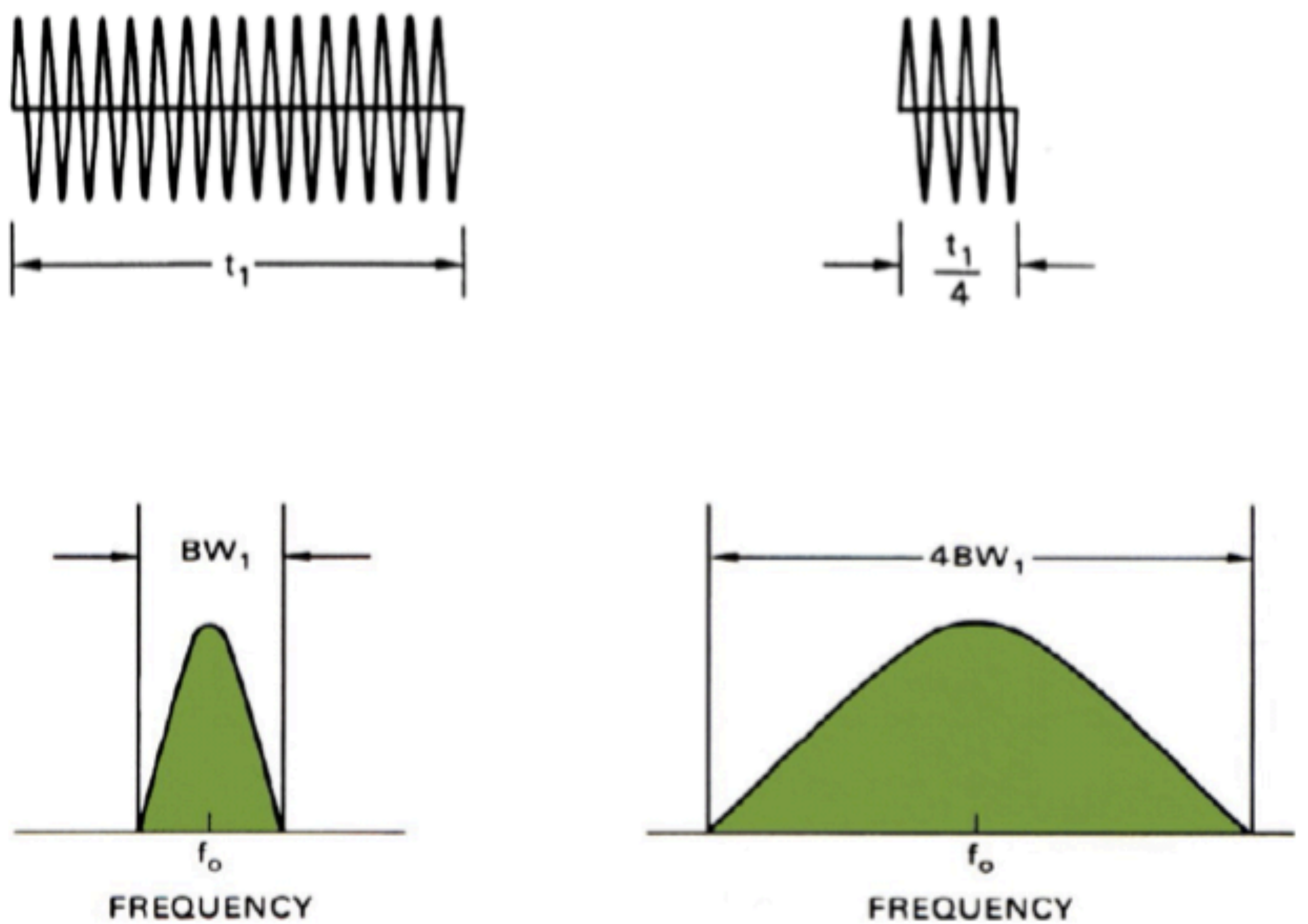
Two possible bandwidth measures:

"null to null" bandwidth $B_{nn} = \frac{2}{\tau}$

"3dB" bandwidth $B_{3dB} = \frac{1}{\tau}$

Unless otherwise specified, assume bandwidth refers to 3 dB bandwidth

Bandwidth is inversely proportional to pulse length



Shorter pulse ⟺ Larger bandwidth

(good range resolution,
worse SNR performance)

Breaking range resolution / energy coupling

Can we increase bandwidth (better range resolution) WITHOUT decreasing pulse length (keep good SNR)? If we can, we could have the best situation.

Ideas?

Breaking range resolution / energy coupling

Can we increase bandwidth (better range resolution) WITHOUT decreasing pulse length (keep good SNR)? If we can, we could have the best situation.

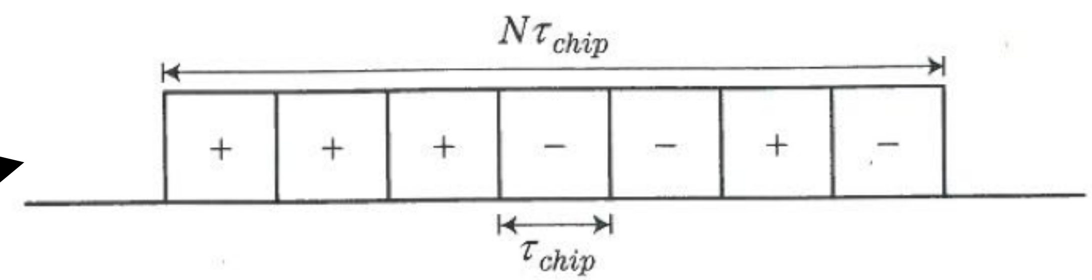
The key:
Use a long total waveform (high energy) but modulate the waveform as we send it (increases bandwidth).

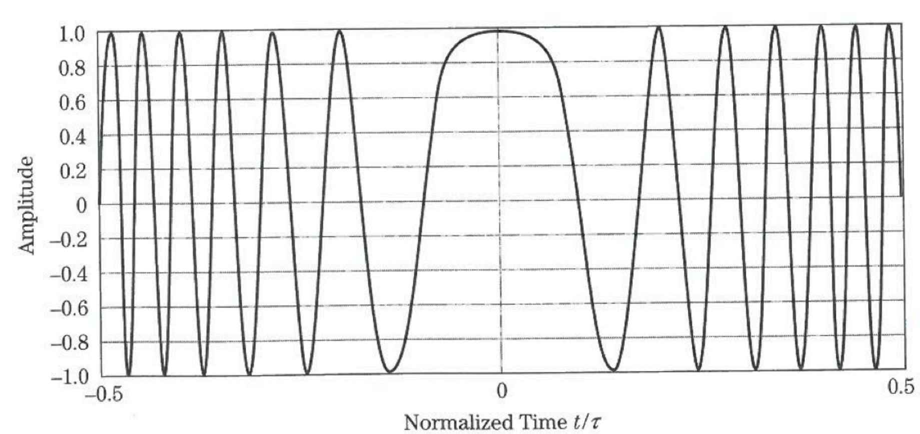
Possibilities:

Amplitude code

Phase code

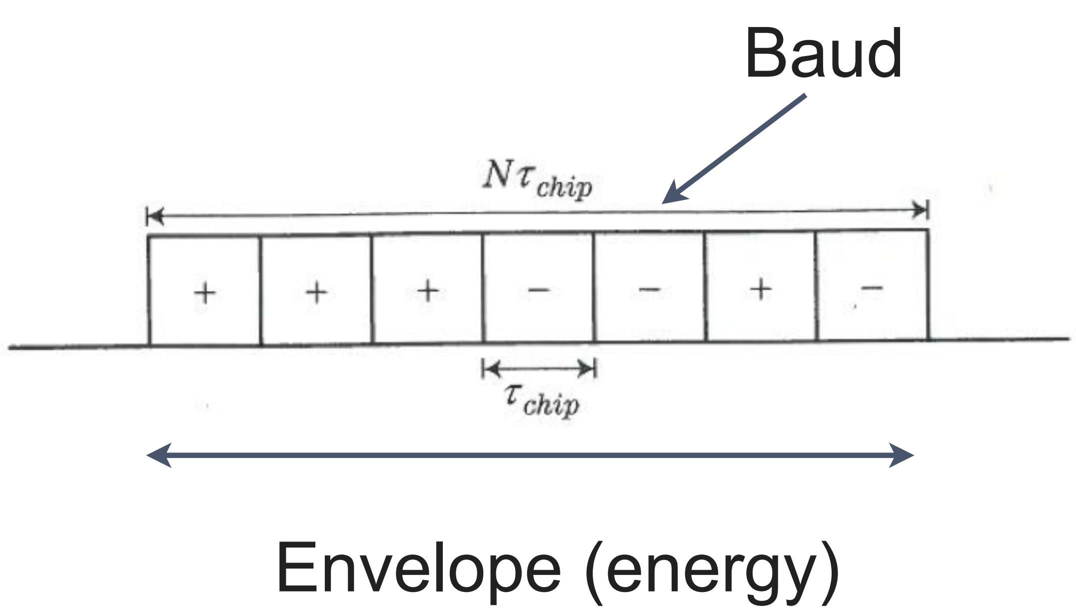
Frequency code



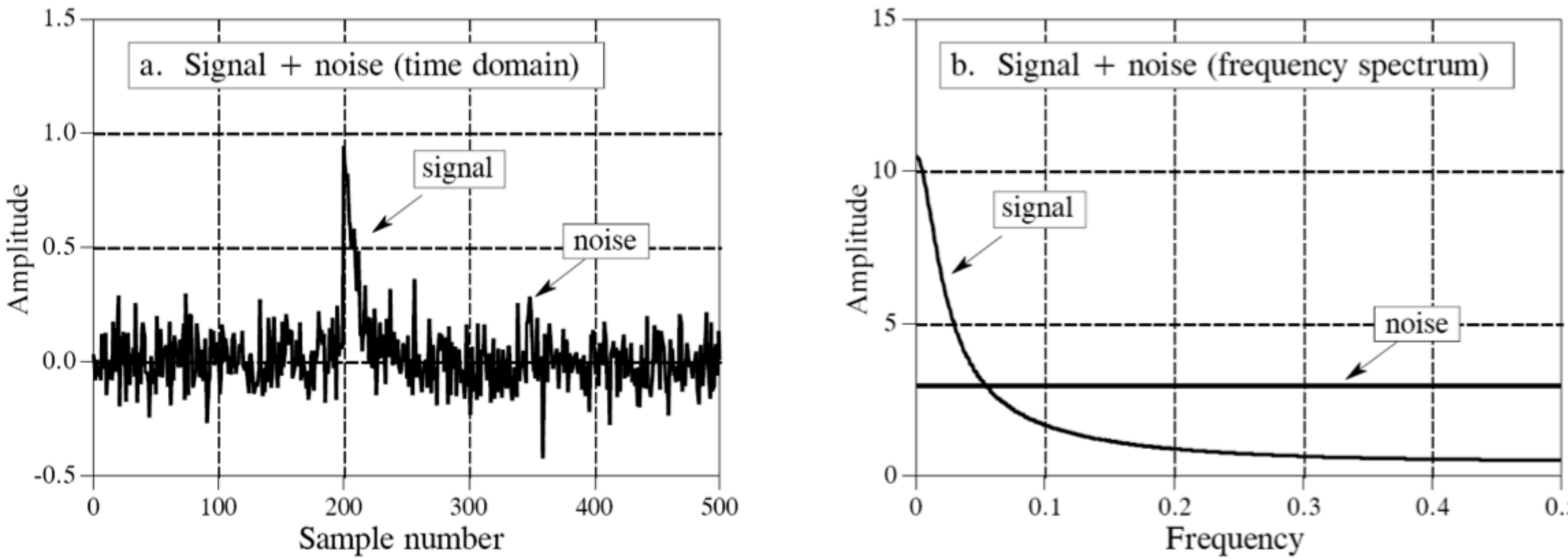


Reassembling the Energy: Matched Filter

We send a modulated pulse of total duration τ. How should we process the echo to find the modulated sequence and pile up all the energy we sent in a range cell the size of an individual 'chip' or phase transition?



Detection of signal in “white Gaussian noise”



Exponential pulse buried in random noise. Sine the signal and noise overlap in both time and frequency domains, the best way to separate them is not obvious.

Most important constraint is to match the bandwidth of signal you are looking for

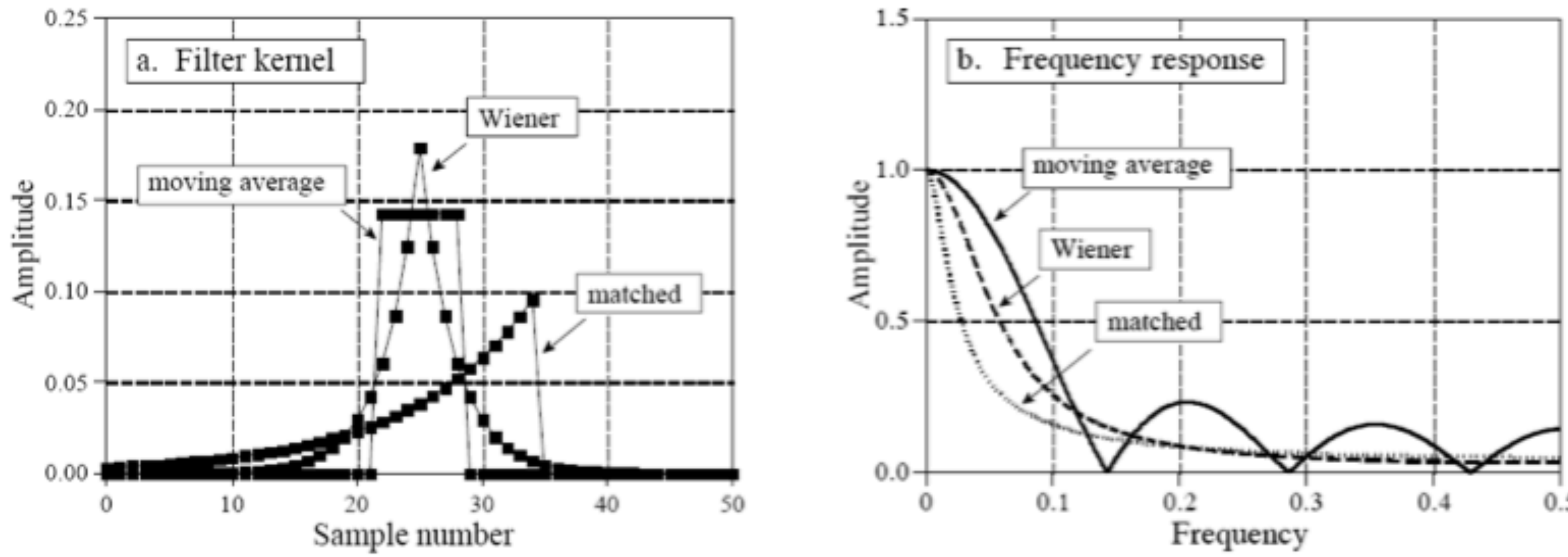
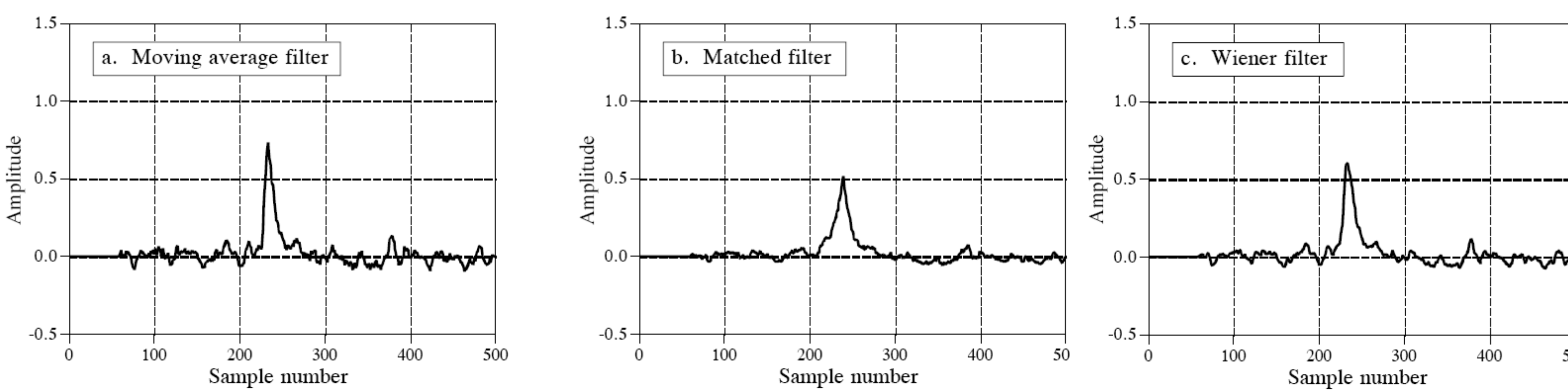
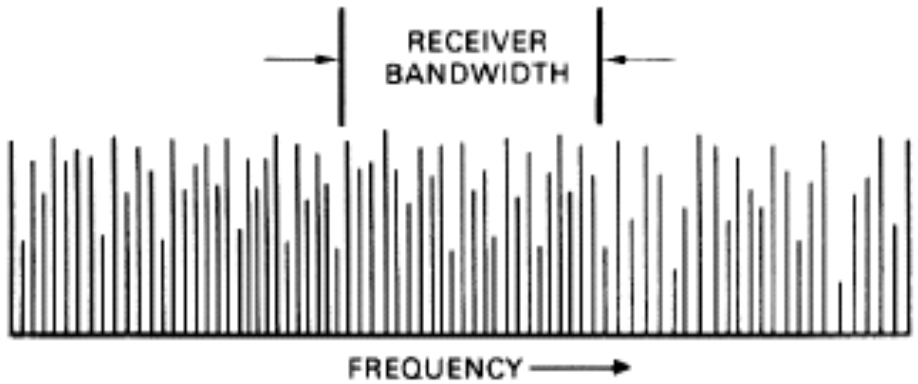


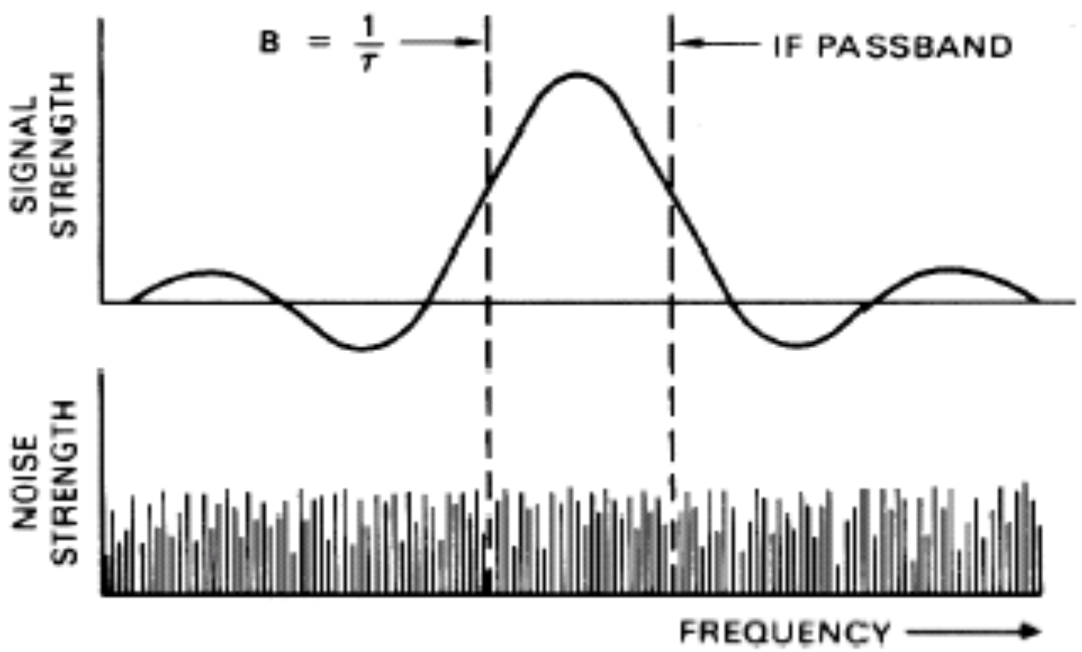
FIGURE 17-8
Example of optimal filters. In (a), three filter kernels are shown, each of which is optimal in some sense. The corresponding frequency responses are shown in (b). The moving average filter is designed to have a rectangular pulse for a filter kernel. In comparison, the filter kernel of the matched filter looks like the signal being detected. The Wiener filter is designed in the frequency domain, based on the relative amounts of signal and noise present at each frequency.



The Matched Filter



6. Noise in receiver output is proportional to bandwidth of receiver.



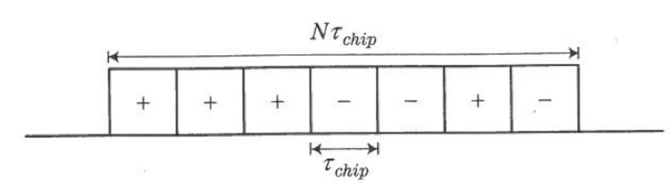
1. Signal-to-noise ratio may be maximized by narrowing the passband of the IF amplifier to the point where only the bulk of the signal energy is passed.

The matched filter is a filter whose impulse response, or transfer function, is determined by a given signal, in a way that will result in the maximum attainable signal-to-noise ratio at the filter output when both the signal and white noise are passed through it.

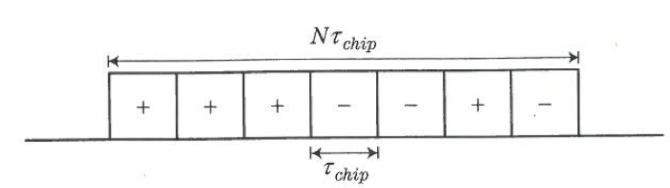
The optimum bandwidth of the filter, B, turns out to be very nearly equal to the bandwidth of the transmitted code sequence.

Its impulse response is the coded sequence you sent!

Note: noise doesn't have the coded sequence you sent (it's random) so it doesn't correlate with the transmitted code - and won't add up coherently!



Target echo



Matched filter

Typically Used Modulation Patterns

Amplitude domain phase code (Barker code): Cross-section with better spatial resolution. Spectral ambiguities though, induced by code properties.

Set of codes, each one a phase pattern (alternating code): measure ACF with good SNR and high spatial resolution. More complex signal processing / bookkeeping.

Barker codes are a nice mathematical set for phase modulation. Decoding (matched filter) done in amplitude domain by multiplying voltages by the known code pattern. Maximum compression for n=13 code.

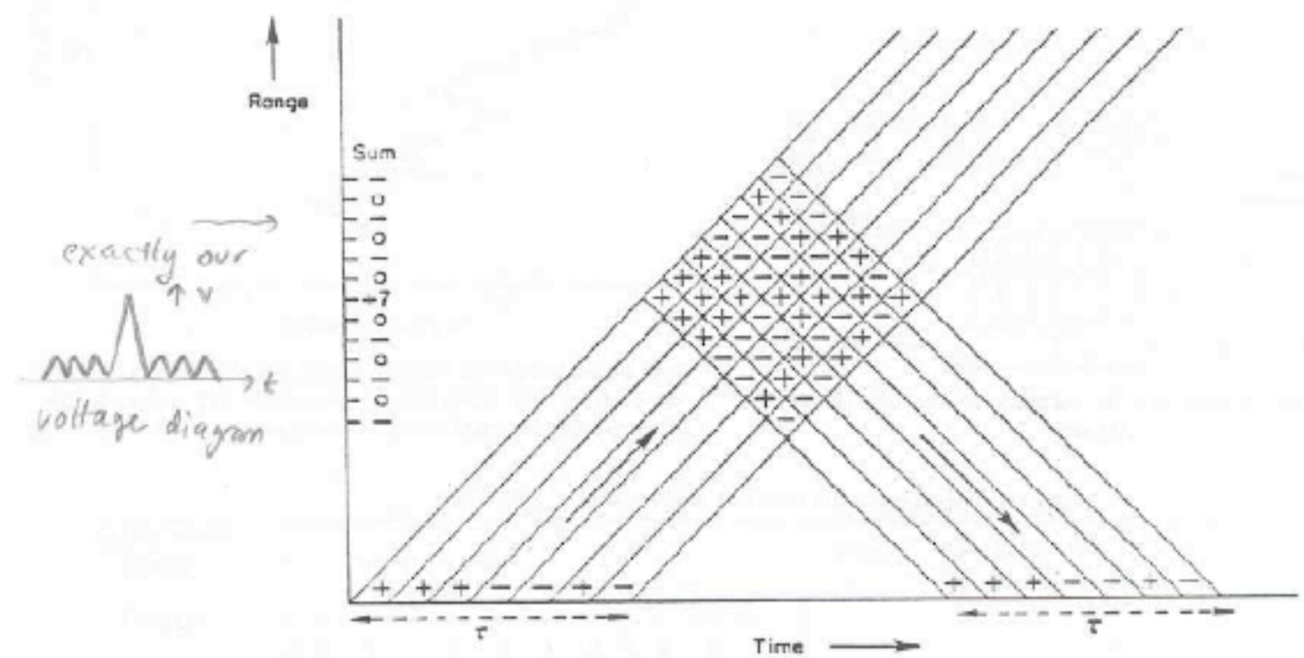


FIG. 9. Range–time diagram for a Barker-coded pulse, which is split into seven elements with + or − phases, as shown. The contributions of the 49 cells are obtained by multiplication, as shown. Only at the central range do these consistently add up to give a sum of +7 contributions. At the other ranges the sums are zero or −1, so nearly all the signal comes from the central range.

Evaluating Codes: Radar Ambiguity Function

Matched filter expression (equivalent to correlating RX signal with a copy of TX pattern)

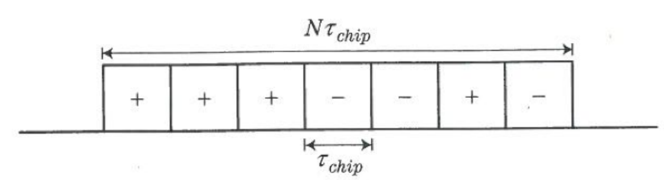
Complex convolution

$$y(t) = \int x_r(\alpha)x^*(\alpha - t)d\alpha$$

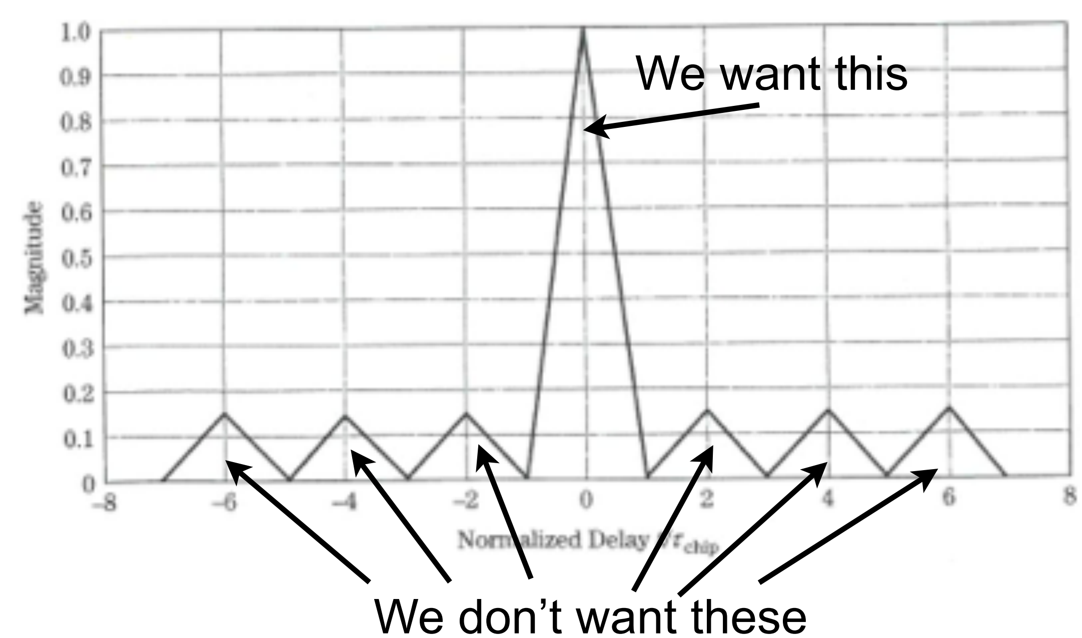
Output RX signal TX pattern

Computing this assuming a perfect returned signal from a point target tells you what the ambiguities are in range when the target is not moving (zero Doppler).

It's the autocorrelation function of the TX pattern!



7 baud Barker code



Evaluating Codes: Radar Ambiguity Function

Matched filter results when target has a Doppler shift

$$A(t, f_d) = \left| \int x(\alpha) \exp\left(j 2\pi f_d \alpha\right) x^*\left(\alpha - t\right) d\alpha \right|$$

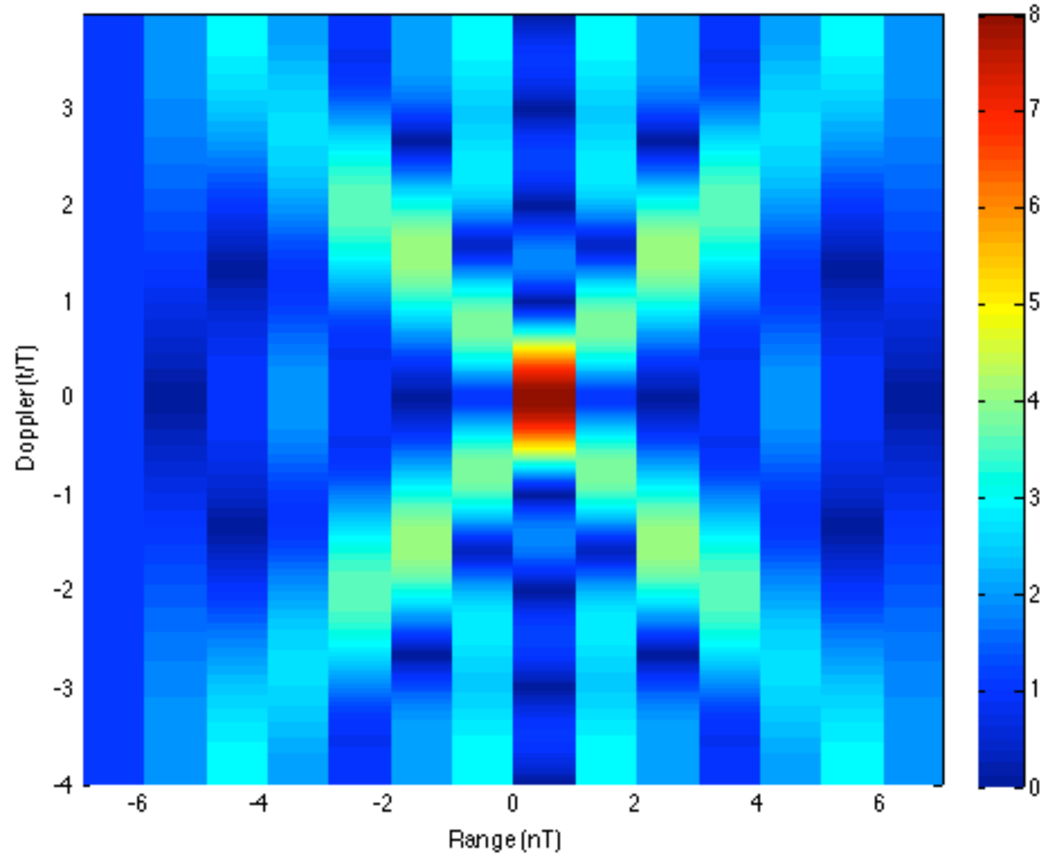
Output

RX signal (now with Doppler)

TX pattern

Evaluate in a 2D sense over range AND Doppler = Radar Ambiguity Function.

Shows the code's response to range and Doppler in places other than where you want to be looking.



Ideal radar ambiguity function: Delta function in range and Doppler at (0, 0).

This is not one of those!

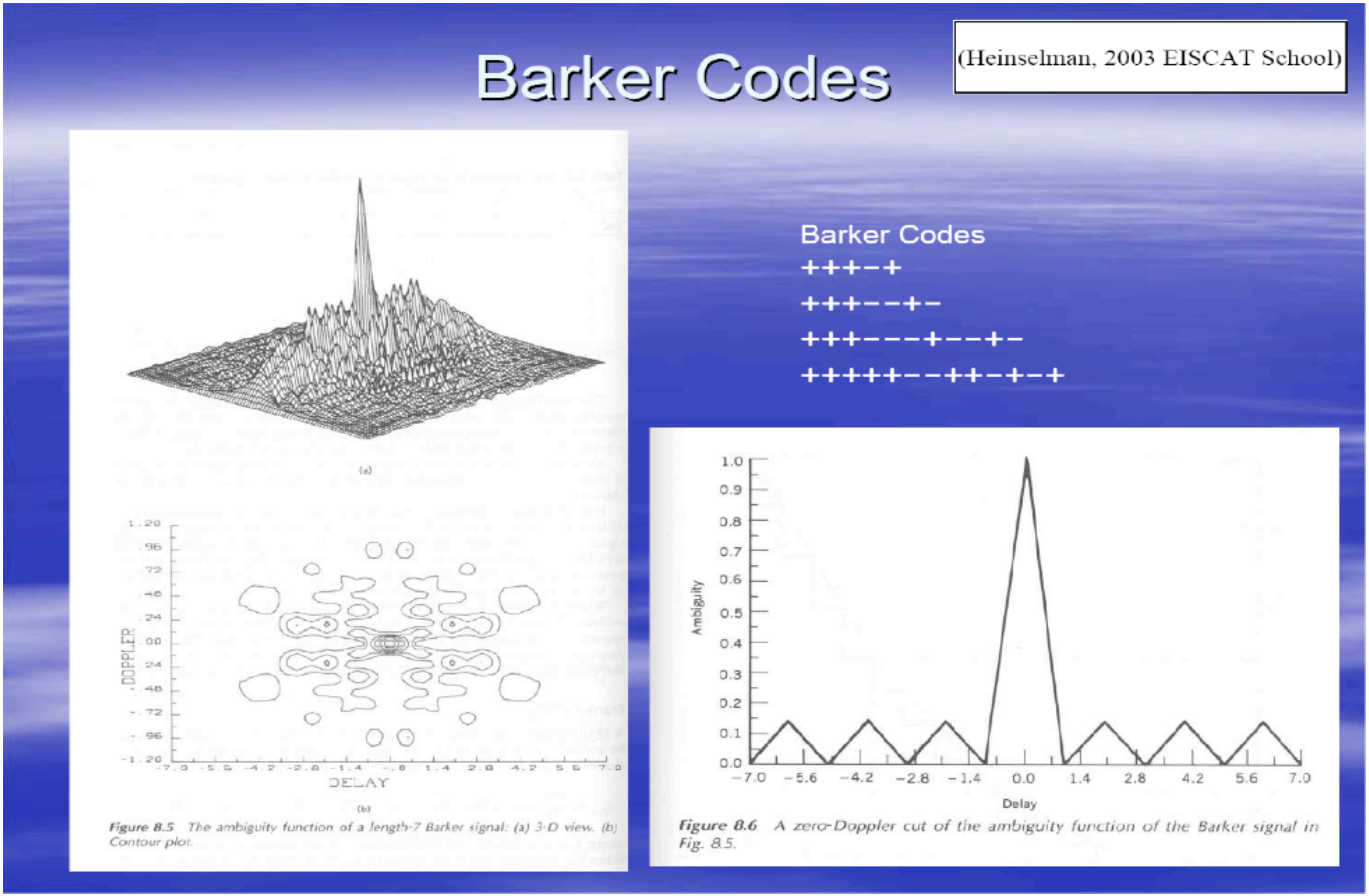
Image processing analogy: Blurring kernel

Gaussian blurring kernel

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

(Wikipedia / CC)

Barker Codes



Alternating Codes: Avoiding Code Induced Ambiguities

Barker code was one modulation pattern repeated identically for each pulse (and decoded in the voltage domain).

What if we used a set of codes, each with a different sidelobe pattern?

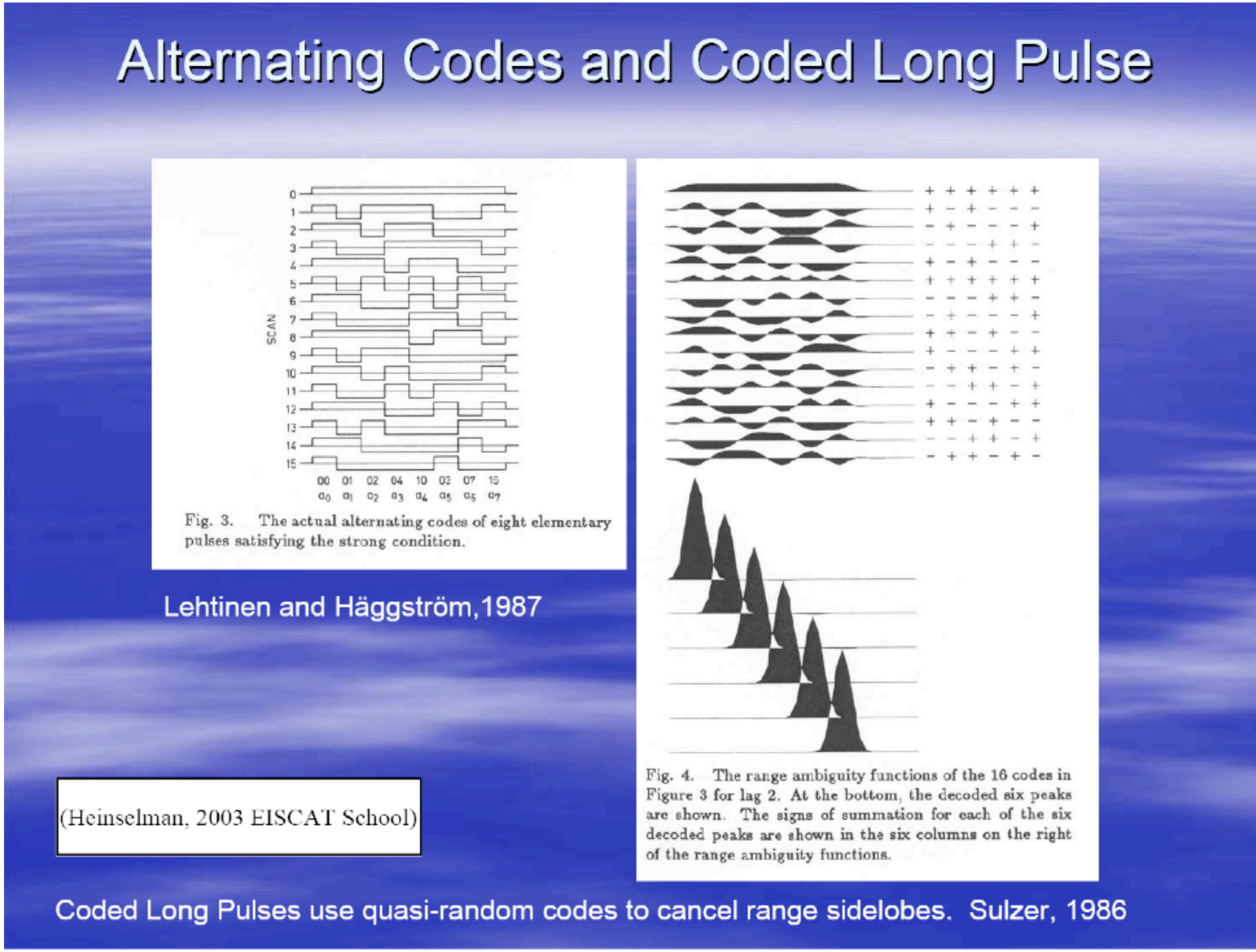
What if, further, we arranged these codes mathematically so that when we decoded them, formed their ACFs, and added the ACFs all together, we could cancel all the range sidelobes except for the central peak?

Codes which do this: alternating codes (Lehtinen and Haggstrom, 1987).

The penalty is a lot of bookkeeping: we need to match filter each individual code, save its statistical ACFs separately, and then add together all the ACF estimates at the very end. Sidelobe cancellation occurs in the second moment (power) domain – some ranges in ambiguity function give positive contribution for individual codes, some give negative contributions.

Alternating Codes

(coded long pulses use a similar idea, but cancellation happens in a statistical rather than deterministic sense.)



Poker Flat Alternating Codes

16 baud "strong" alternating code
32 codes in total set
Total time for code set transmission: about 0.25 seconds (so ionosphere parameters must remain stationary over this time interval)

480 usec total modulation length = 64 km total range
Final range resolution = 480 / 16 = 30 usec = 4.5 km
16 points measured on ACF / power spectrum

(NB: almost the identical parameter set has been used at Millstone Hill since the mid 1990s.)