ISR Experiments, Data Reduction, and Analysis

Michael J. Nicolls

ISR Summer School, July 2014

Outline

Overspread Targets

(a.k.a, frequency and range aliased targets)

- For a target with a bandwidth B, you must sample at a rate F_s exceeding B (e.g., for IS at 450 MHz, $B \sim 40$ kHz).
- For a target which could be as far away as R_{max}, radar pulses must be at least $2R_{max}/c$ apart.

Thus, there is a competition between distance and bandwidth.

Overspread Targets

(a.k.a, frequency and range aliased targets)

- For a target with a bandwidth B, you must sample at a rate F_s exceeding B (e.g., for IS at 450 MHz, $B \sim 40$ kHz).
- For a target which could be as far away as R_{max}, radar pulses must be at least $2R_{max}/c$ apart.

Thus, there is a competition between distance and bandwidth.

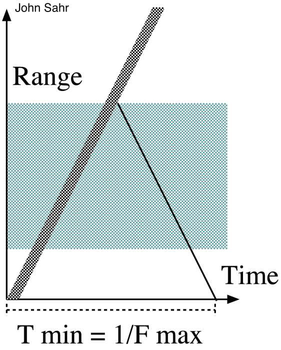
Is an ISR target (probed at 450 MHz, $B \sim 40$ kHz)
at a range $R \sim 750$ km overspread?

Overspread Targets

(a.k.a, frequency and range aliased targets)

- For a target with a bandwidth B, you must sample at a rate F_s exceeding B (e.g., for IS at 450 MHz, $B \sim 40$ kHz).
- For a target which could be as far away as R_{max}, radar pulses must be at least $2R_{max}/c$ apart.

Thus, there is a competition between distance and ba



Is an ISR target (probed at 450 MHz, $B \sim 40$ kHz) at a range $R \sim 750$ km overspread?

- $B < F_s < \frac{c}{2R_{max}}$
- or: $B\frac{2R_{max}}{c} < 1$
- At 450 MHz, $B \sim 40$ kHz, $R \sim 750$ km (5 ms) $\rightarrow$ highly overspread
- Do we get the range right or the spectrum right??

Overspread Targets

Any ideas how to resolve this?

Overspread Targets

Any ideas how to resolve this?

- Use the fact that the random scattering process from non-overlapping range bins is uncorrelated.
- Construct autocorrelation function estimate, $R(\tau) = \mathcal{F}[P(f)]$

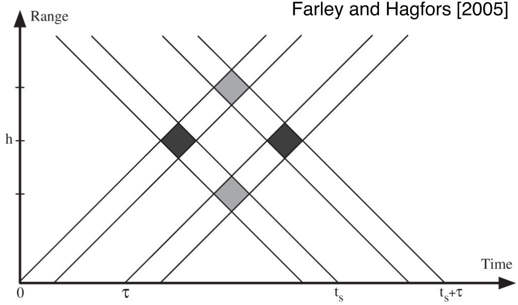
Overspread Targets

Any ideas how to resolve this?

- Use the fact that the random scattering process from non-overlapping range bins is uncorrelated.
- Construct autocorrelation function estimate, $R(\tau) = \mathcal{F}[P(f)]$

Simplest scheme to measure correlation at a given lag - double pulse:

Farley and Hagfors [2005]



$v_1 = v_h(t_s) + v_{h-\delta}(t_s)$, $v_2 = v_h(t_s + \tau) + v_{h+\delta}(t_s + \tau)$, $\langle v_1 v_2^* \rangle = ?$

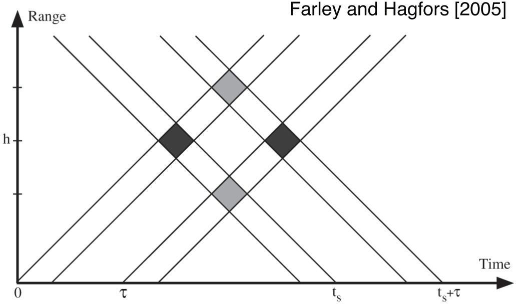
Overspread Targets

Any ideas how to resolve this?

- Use the fact that the random scattering process from non-overlapping range bins is uncorrelated.
- Construct autocorrelation function estimate, $R(\tau) = \mathcal{F}[P(f)]$

Simplest scheme to measure correlation at a given lag - double pulse:

Farley and Hagfors [2005]

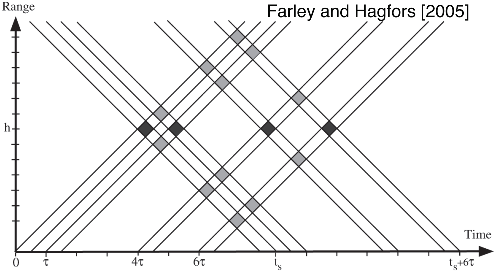


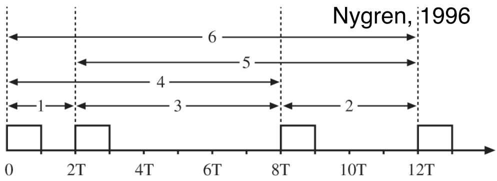
$v_1 = v_h(t_s) + v_{h-\delta}(t_s)$, $v_2 = v_h(t_s + \tau) + v_{h+\delta}(t_s + \tau)$, $\langle v_1 v_2^* \rangle = ?$

$$v_1 v_2^* = v_h(t_s)v_h^*(t_s + \tau) + v_h(t_s)v_{h+\delta}^*(t_s + \tau) + v_{h-\delta}(t_s)v_h^*(t_s + \tau) + v_{h-\delta}(t_s)v_{h+\delta}^*(t_s + \tau)$$

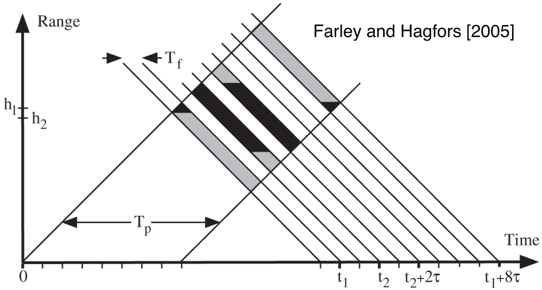
$$\langle v_1 v_2^* \rangle = \langle v_h(t_s)v_h^*(t_s + \tau)\rangle$$

Generalization - Multipulses





Generalization - Long pulses



Farley and Hagfors [2005]

$v_1 = v(t)$, $v_2 = v(t+\tau) \rightarrow v_1 = x_1 + ix_2$, $v_2 = x_3 + ix_4$ where the scattering process is represented by the 4-dimensional joint Gaussian probability distribution, $p(x_1, x_2, x_3, x_4)$.

Measurement Statistics

$v_1 = v(t)$, $v_2 = v(t+\tau) \rightarrow v_1 = x_1 + ix_2$, $v_2 = x_3 + ix_4$ where the scattering process is represented by the 4-dimensional joint Gaussian probability distribution, $p(x_1, x_2, x_3, x_4)$.
Defining ρ is the normalized acf (complex) and $S = 2\sigma^2$ is the signal power.

$$\langle v_1 v_2^* \rangle = S\rho(\tau) = \langle (x_1 + ix_2)(x_3 - ix_4) \rangle = c_{13} + c_{24} + i(c_{23} - c_{14})$$

Measurement Statistics

$v_1 = v(t)$, $v_2 = v(t+\tau) \rightarrow v_1 = x_1 + ix_2$, $v_2 = x_3 + ix_4$ where the scattering process is represented by the 4-dimensional joint Gaussian probability distribution, $p(x_1, x_2, x_3, x_4)$.
Defining ρ is the normalized acf (complex) and $S = 2\sigma^2$ is the signal power.

$$\langle v_1 v_2^* \rangle = S\rho(\tau) = \langle (x_1 + ix_2)(x_3 - ix_4) \rangle = c_{13} + c_{24} + i(c_{23} - c_{14})$$

Covariance matrix of p is then:

$$C = \sigma^2 \begin{bmatrix} 1 & 0 & \rho_r & -\rho_I \\ 0 & 1 & \rho_I & \rho_r \\ \rho_r & \rho_I & 1 & 0 \\ -\rho_I & \rho_r & 0 & 1 \end{bmatrix}$$

Measurement Statistics - Signal Power

Obvious estimator of $S = \langle |v(t)|^2 \rangle$ with K samples is

$$\hat{S} = \frac{1}{K} \sum_{i}^{K} v_i v_i^*$$

Measurement Statistics - Signal Power

Obvious estimator of $S = \langle |v(t)|^2 \rangle$ with K samples is

$$\hat{S} = \frac{1}{K} \sum_{i}^{K} v_i v_i^*$$

This is an unbiased estimator of S:

$$\langle \hat{S} \rangle = \left\langle \frac{1}{K} \sum_{i}^{K} v_i v_i^* \right\rangle = \frac{1}{K} \sum_{i}^{K} \langle |v_i|^2 \rangle = \langle |v_i|^2 \rangle = S$$

Measurement Statistics - Signal Power

Obvious estimator of $S = \langle |v(t)|^2 \rangle$ with K samples is

$$\hat{S} = \frac{1}{K} \sum_{i}^{K} v_i v_i^*$$

This is an unbiased estimator of S:

$$\langle \hat{S} \rangle = \left\langle \frac{1}{K} \sum_{i}^{K} v_i v_i^* \right\rangle = \frac{1}{K} \sum_{i}^{K} \langle |v_i|^2 \rangle = \langle |v_i|^2 \rangle = S$$

With variance:

$$\sigma_{\hat{S}}^2 = \langle (\hat{S} - S)^2 \rangle = \langle \hat{S}^2 \rangle - S^2$$

$$\langle \hat{S}^2 \rangle = ?$$

Measurement Statistics - Signal Power

fourth-moment theorem: $\langle v_1 v_2 v_3 v_4 \rangle = \langle v_1 v_2 \rangle \langle v_3 v_4 \rangle + \langle v_1 v_3 \rangle \langle v_2 v_4 \rangle + \langle v_1 v_4 \rangle \langle v_2 v_3 \rangle$.

$$\langle \hat{S}^2 \rangle = \frac{1}{K^2} \left\langle \sum_{i=1}^{K} v_i v_i^* \sum_{j=1}^{K} v_j v_j^* \right\rangle$$

Measurement Statistics - Signal Power

fourth-moment theorem: $\langle v_1 v_2 v_3 v_4 \rangle = \langle v_1 v_2 \rangle \langle v_3 v_4 \rangle + \langle v_1 v_3 \rangle \langle v_2 v_4 \rangle + \langle v_1 v_4 \rangle \langle v_2 v_3 \rangle$.

$$\langle \hat{S}^2 \rangle = \frac{1}{K^2} \left\langle \sum_{i=1}^{K} v_i v_i^* \sum_{j=1}^{K} v_j v_j^* \right\rangle$$

There are K terms where $i = j \rightarrow \langle v_i v_i^* v_i v_i^* \rangle = 2S^2$

Measurement Statistics - Signal Power

fourth-moment theorem: $\langle v_1 v_2 v_3 v_4 \rangle = \langle v_1 v_2 \rangle \langle v_3 v_4 \rangle + \langle v_1 v_3 \rangle \langle v_2 v_4 \rangle + \langle v_1 v_4 \rangle \langle v_2 v_3 \rangle$.

$$\langle \hat{S}^2 \rangle = \frac{1}{K^2} \left\langle \sum_{i=1}^{K} v_i v_i^* \sum_{j=1}^{K} v_j v_j^* \right\rangle$$

There are K terms where $i = j \rightarrow \langle v_i v_i^* v_i v_i^* \rangle = 2S^2$
and $K^2 - K$ terms where $i \neq j \rightarrow \langle v_i v_i^* v_j v_j^* \rangle_{i \neq j} = S^2$

Measurement Statistics - Signal Power

fourth-moment theorem: $\langle v_1 v_2 v_3 v_4 \rangle = \langle v_1 v_2 \rangle \langle v_3 v_4 \rangle + \langle v_1 v_3 \rangle \langle v_2 v_4 \rangle + \langle v_1 v_4 \rangle \langle v_2 v_3 \rangle$.

$$\langle \hat{S}^2 \rangle = \frac{1}{K^2} \left\langle \sum_{i=1}^{K} v_i v_i^* \sum_{j=1}^{K} v_j v_j^* \right\rangle$$

There are K terms where $i = j \rightarrow \langle v_i v_i^* v_i v_i^* \rangle = 2S^2$
and $K^2 - K$ terms where $i \neq j \rightarrow \langle v_i v_i^* v_j v_j^* \rangle_{i \neq j} = S^2$

$$\begin{aligned} \langle \hat{S}^2 \rangle &= \frac{1}{K^2} \left[K 2 S^2 + (K^2 - K) S^2 \right] \\ &= S^2 (1 + 1/K) \\ \sigma_{\hat{S}}^2 &= S^2 / K \end{aligned}$$

Measurement Statistics - Signal Power

fourth-moment theorem: $\langle v_1 v_2 v_3 v_4 \rangle = \langle v_1 v_2 \rangle \langle v_3 v_4 \rangle + \langle v_1 v_3 \rangle \langle v_2 v_4 \rangle + \langle v_1 v_4 \rangle \langle v_2 v_3 \rangle$.

$$\langle \hat{S}^2 \rangle = \frac{1}{K^2} \left\langle \sum_{i=1}^{K} v_i v_i^* \sum_{j=1}^{K} v_j v_j^* \right\rangle$$

There are K terms where $i = j \rightarrow \langle v_i v_i^* v_i v_i^* \rangle = 2S^2$
and $K^2 - K$ terms where $i \neq j \rightarrow \langle v_i v_i^* v_j v_j^* \rangle_{i \neq j} = S^2$

$$\begin{aligned} \langle \hat{S}^2 \rangle &= \frac{1}{K^2} \left[K2S^2 + (K^2 - K)S^2 \right] \\ &= S^2(1 + 1/K) \\ \sigma_{\hat{S}}^2 &= S^2/K \end{aligned}$$

How many samples to achieve 5% precision? 1% precision?

Measurement Statistics - Signal Power

fourth-moment theorem: $\langle v_1 v_2 v_3 v_4 \rangle = \langle v_1 v_2 \rangle \langle v_3 v_4 \rangle + \langle v_1 v_3 \rangle \langle v_2 v_4 \rangle + \langle v_1 v_4 \rangle \langle v_2 v_3 \rangle$.

$$\langle \hat{S}^2 \rangle = \frac{1}{K^2} \left\langle \sum_{i=1}^{K} v_i v_i^* \sum_{j=1}^{K} v_j v_j^* \right\rangle$$

There are K terms where $i = j \rightarrow \langle v_i v_i^* v_i v_i^* \rangle = 2S^2$
and $K^2 - K$ terms where $i \neq j \rightarrow \langle v_i v_i^* v_j v_j^* \rangle_{i \neq j} = S^2$

$$\begin{aligned} \langle \hat{S}^2 \rangle &= \frac{1}{K^2} \left[K2S^2 + (K^2 - K)S^2 \right] \\ &= S^2(1 + 1/K) \\ \sigma^2_{\hat{S}} &= S^2/K \end{aligned}$$

How many samples to achieve 5% precision? 1% precision?

$$5\% : K = 1/(0.05)^2 = 400; 1\% : K = 1/(0.01)^2 = 10^4.$$

Measurement Statistics - Additive Noise

Power estimator is now our estimator of the total signal minus our estimate of the noise power,

$$\hat{S} = \hat{P}_{SN} - \hat{N}$$

Measurement Statistics - Additive Noise

Power estimator is now our estimator of the total signal minus our estimate of the noise power,

$$\hat{S} = \hat{P}_{SN} - \hat{N}$$

Variance:

$$\begin{aligned}
\sigma^2_{\hat{S}} &= \sigma^2_{\hat{P}_{SN}} + \sigma^2_N \\
&= (S+N)^2/K + N^2/K_N \\
&\quad \text{Assume } K_N \gg K \\
&\approx (S+N)^2/K
\end{aligned}$$

Measurement Statistics - Additive Noise

Power estimator is now our estimator of the total signal minus our estimate of the noise power,

$$\hat{S} = \hat{P}_{SN} - \hat{N}$$

Variance:

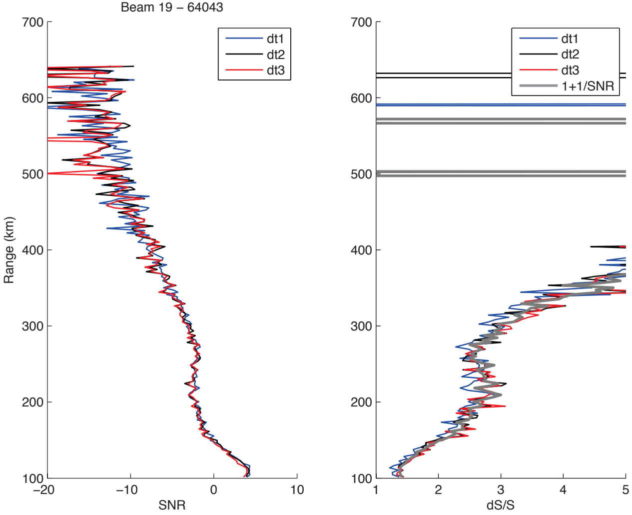
$$\begin{aligned} \sigma^2_{\hat{S}} &= \sigma^2_{\hat{P}_{SN}} + \sigma^2_N \\ &= (S+N)^2/K + N^2/K_N \\ & \quad \text{Assume } K_N \gg K \\ &\approx (S+N)^2/K \end{aligned}$$

$$\frac{\sigma_{\hat{S}}}{S} \approx \frac{1}{\sqrt{K}}\left(1 + \frac{N}{S}\right)$$

Implications of this formula?

PFISR Data - Additive Noise Example

41 beam experiment, tri-frequency 240 us pulses, ∼2500 pulses per beam in 5 minutes



Ambiguity Function

(Note possibly different than most standard radar treatments, following *Nygren, 1996*)

Ambiguity Function

(Note possibly different than most standard radar treatments, following *Nygren, 1996*)

After modulation with a pulse transmission envelope $\mathrm{env}(t)$,

$$v(t) = \int_{\mathbf{r}} \mathrm{env}(t - \frac{2R}{c})\delta v(t, \mathbf{r})$$

where $\langle \delta v(t, \mathbf{r}) \delta v^*(t', \mathbf{r}') \rangle = RP_e \sigma_e(t - t', \mathbf{r})\delta(\mathbf{r} - \mathbf{r}') d\mathbf{r} d\mathbf{r}'$

Ambiguity Function

(Note possibly different than most standard radar treatments, following *Nygren, 1996*)

After modulation with a pulse transmission envelope $\mathrm{env}(t)$,

$$v(t) = \int_{\mathbf{r}} \mathrm{env}(t - \frac{2R}{c})\delta v(t, \mathbf{r})$$

where $\langle \delta v(t, \mathbf{r}) \delta v^*(t', \mathbf{r}') \rangle = R P_e \sigma_e(t - t', \mathbf{r}) \delta(\mathbf{r} - \mathbf{r}') d\mathbf{r} d\mathbf{r}'$

For any realizable measurement, must pass through a receiver filter with impulse response $h(t)$:

$$v_h(t) = v(t) \star h(t) = \int_{-\infty}^{\infty} h(t - \tau) v(\tau) d\tau = \int_{-\infty}^{\infty} \left[\int_{\mathbf{r}} W_t^A(\tau, \mathbf{r}) \delta v(\tau, \mathbf{r}) \right] d\tau$$

where $W_t^A(\tau, \mathbf{r}) = h(t - \tau)\mathrm{env}(\tau - \frac{2R}{c})$ is the amplitude ambiguity function.

Ambiguity Function

(Note possibly different than most standard radar treatments, following *Nygren, 1996*)

After modulation with a pulse transmission envelope $\mathrm{env}(t)$,

$$v(t) = \int_{\mathbf{r}} \mathrm{env}(t - \frac{2R}{c}) \delta v(t, \mathbf{r})$$

where $\langle \delta v(t, \mathbf{r}) \delta v^*(t', \mathbf{r}') \rangle = R P_e \sigma_e(t - t', \mathbf{r}) \delta(\mathbf{r} - \mathbf{r}') d\mathbf{r} d\mathbf{r}'$

For any realizable measurement, must pass through a receiver filter with impulse response $h(t)$:

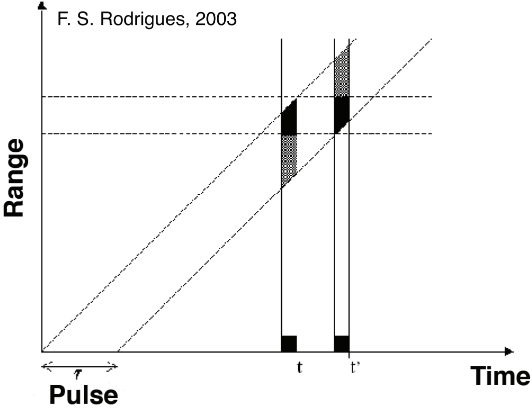
$$v_h(t) = v(t) \star h(t) = \int_{-\infty}^{\infty} h(t - \tau) v(\tau) d\tau = \int_{-\infty}^{\infty} \left[\int_{\mathbf{r}} W_t^A(\tau, \mathbf{r}) \delta v(\tau, \mathbf{r}) \right] d\tau$$

where $W_t^A(\tau, \mathbf{r}) = h(t - \tau) \mathrm{env}(\tau - \frac{2R}{c})$ is the amplitude ambiguity function.

The received signal is a weighted sum of elementary signals from all volume elements times, and the weight in this sum is given by the amplitude ambiguity function. (Nygren, 1996).

Ambiguity Function

Long-pulse of length τ, sampled at t and t' with a box-car impulse response.



Ambiguity Function

What we really care about is the ambiguity for a lagged product (estimate of the autocorrelation function at a given lag).

$$\langle v_h(t) v_h^*(t') \rangle = R \int_{\mathbf{r}} P_e(\mathbf{r}) \left[\int_{-\infty}^{\infty} W_{t,t'}(\nu, \mathbf{r}) \sigma_e(\nu, \mathbf{r}) d\nu \right] d\mathbf{r}$$

where $\nu = t - t'$ and

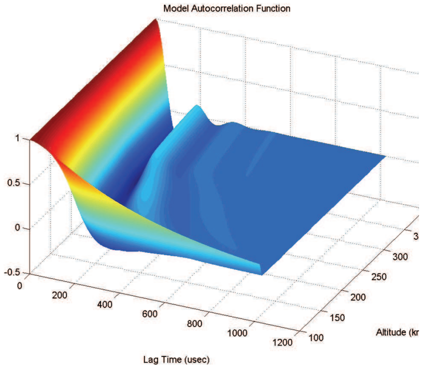
$$W_{t,t'}(\nu, \mathbf{r}) = \int_{-\infty}^{\infty} W_t^A(\tau, \mathbf{r}) W_{t'}^{A*}(\tau - \nu, \mathbf{r}) d\tau$$

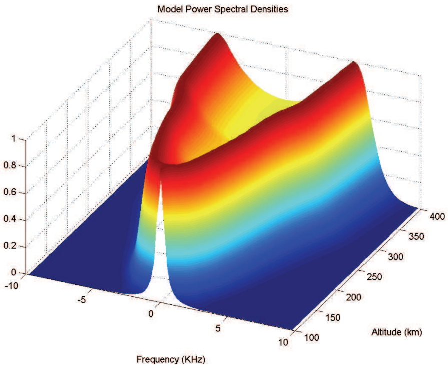
(cross-correlation of two amplitude ambiguity functions, in time direction)

The estimated lagged product is a weighted average of the plasma acf in both space and time. These weights are given by $W_{t,t'}$.

Experiments

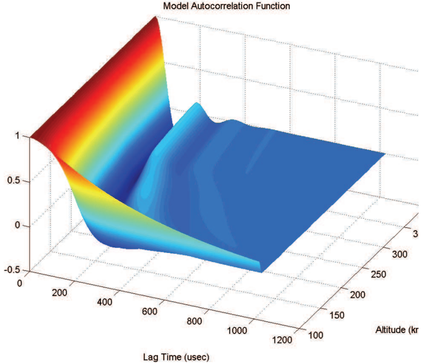
To design an effective an experiment, we need to know our target. Why?

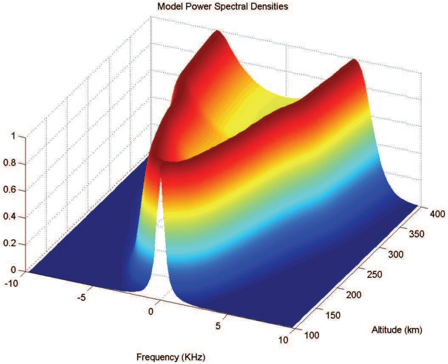




Experiments

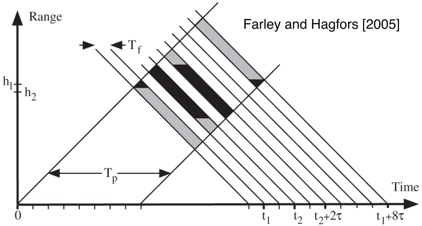
To design an effective an experiment, we need to know our target. Why?





Much of what I present next will be specific to ISRs within a specific range of frequencies (∼VHF-UHF).

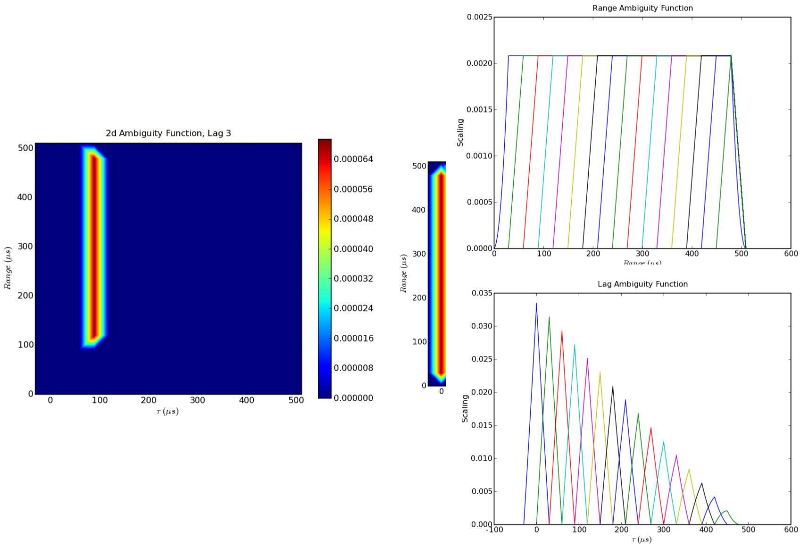
Standard *F*-region Experiment - Long Pulse



- At high altitudes, use a single long pulse with mismatched filter (oversampled) to measure all lags of the ACF at once
- Sacrifice range resolution
- E.g., 300-500 μs pulse (F region) or even 1-2 ms (topside)

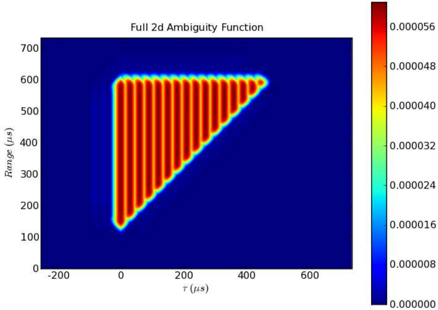
Long Pulse Ambiguity Function

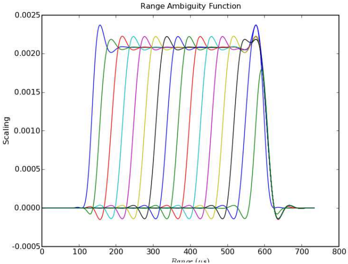
Ambiguity function with a boxcar filter. 480 μs long pulse, 30 μs sampling.

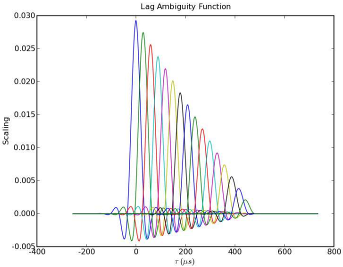


Long Pulse Ambiguity Function

- Ambiguity function including filter effects.
- 480 μs long pulse, 30 μs sampling.
- With filter effects.







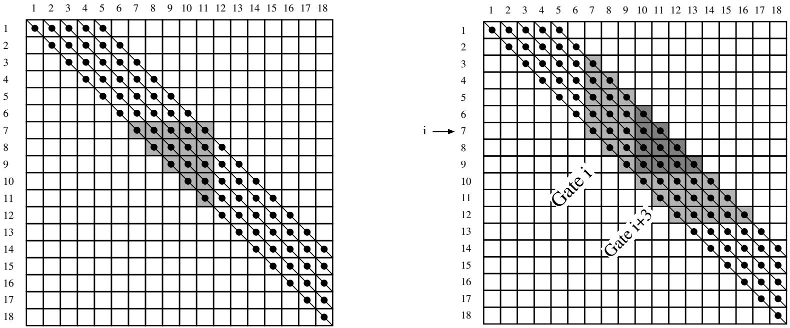
Long Pulse Gating

The different lags of the long pulse have very different range ambiguity functions. Is this a problem?

Long Pulse Gating

The different lags of the long pulse have very different range ambiguity functions. Is this a problem?

“Simple solution” - Gating using elements of the so-called lag-profile matrix.

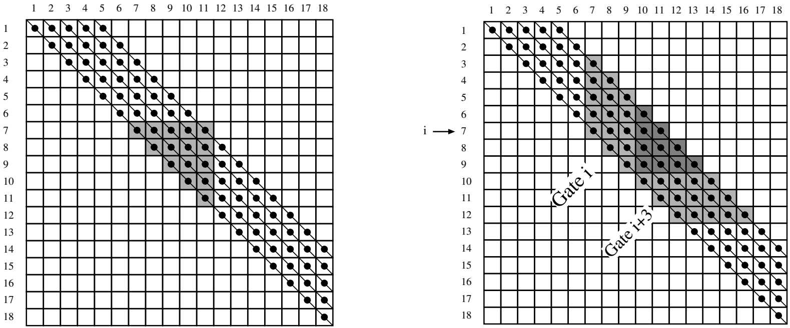


Nygren, 1996

Long Pulse Gating

The different lags of the long pulse have very different range ambiguity functions. Is this a problem?

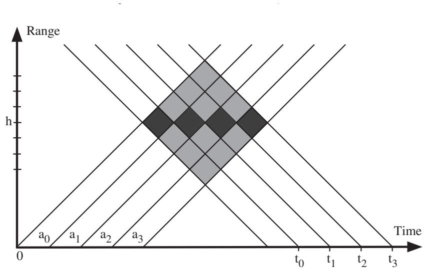
"Simple solution" - Gating using elements of the so-called lag-profile matrix.



Nygren, 1996

A better method - treat as an inverse problem: deconvolution or full profile methodologies. These are active areas of research.

Standard *E*-region Experiment - Coded Pulse

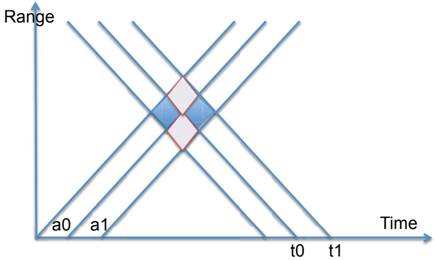


Farley and Hagfors [2005]

E.g., consider lag estimate using $v(t_0)$ and $v^*(t_1)$ - choose a_n such that clutter terms cancel.

- At lower altitudes, we require better range resolution.
- For this, we utilize binary coded pulse ACF measurements (do not compress pulse or eliminate clutter like BC - eliminate correlation of clutter)
- Random (CLP) or alternating (cyclic codes)
- E.g., for AMISR standard experiment is 480 μs, 16-baud (4.5 km), randomized strong code (32 pulses) with an uncoded 30 μs pulse for zero-lag normalization.

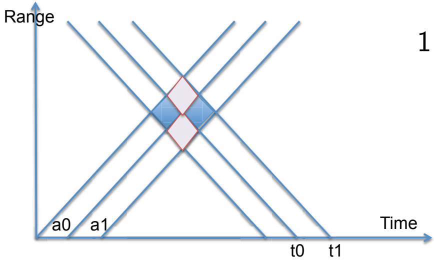
Exercise - 2-baud Alternating Code



Lag estimate using $v(t_0)$ and $v^*(t_1)$ - choose a_0 and a_1 such that clutter terms cancel.

hint: a_0 and a_1 binary [+1,-1]

Exercise - 2-baud Alternating Code

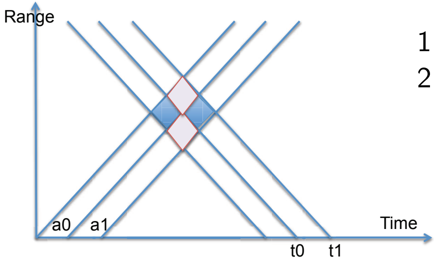


1. How many pulses do you need?

Lag estimate using $v(t_0)$ and $v^*(t_1)$ - choose a_0 and a_1 such that clutter terms cancel.

hint: a_0 and a_1 binary [+1,-1]

Exercise - 2-baud Alternating Code



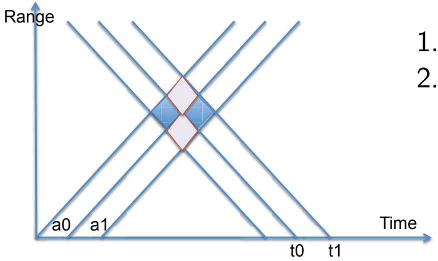
1. How many pulses do you need?
2. Fill out the following table:

	a_0	a_1
*Pulse*1	?	?
*Pulse*2	?	?

Lag estimate using $v(t_0)$ and $v^*(t_1)$ - choose a_0 and a_1 such that clutter terms cancel.

hint: a_0 and a_1 binary [+1,-1]

Exercise - 2-baud Alternating Code



1. How many pulses do you need?
2. Fill out the following table:

	a_0	a_1
$Pulse1$	?	?
$Pulse2$	?	?

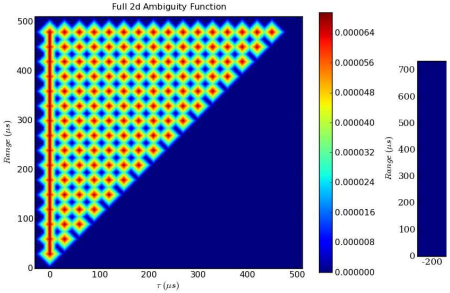
Lag estimate using $v(t_0)$ and $v^*(t_1)$ - choose a_0 and a_1 such that clutter terms cancel.

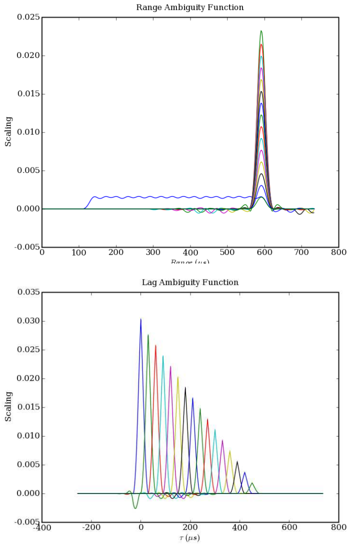
$$\langle a_0 v_0 a_1 v_1^* \rangle = \ldots$$

hint: a_0 and a_1 binary [+1,-1]

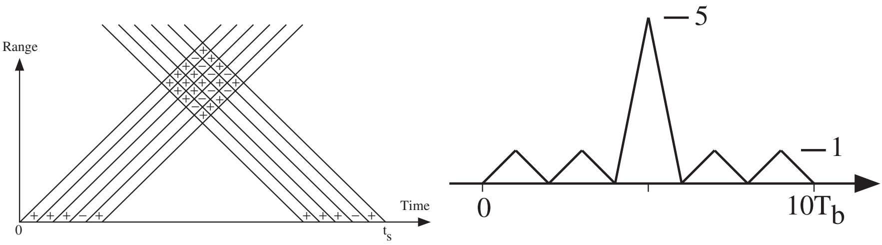
Standard *E*-region Experiment - Ambiguity Function

Ambiguity function including filter effects. 480 μs (16-baud, 30 μs baud, 32 pulse).





Standard E/F-region Power Measurement



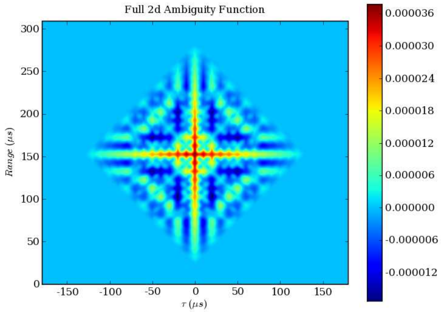
Farley and Hagfors [2005]

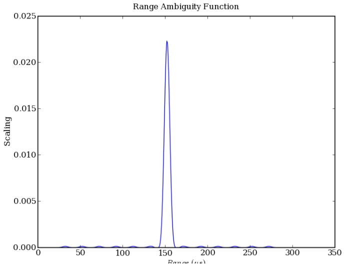
- Pulse compression code allow for high sensitivity, high range resolution power measurements.
- Plasma must remain correlated over pulse length (limits range of use for most systems).
- Typical code is 13-baud Barker code, 130 μs.

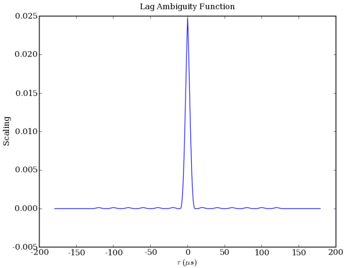
E/F-region Power Measurement - Ambiguity Function

Ambiguity function including filter effects.

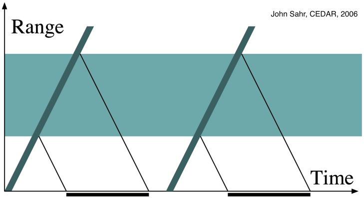
130 μs (13-baud, 10 μs baud, 5 μs sampling).







Standard *D*-region Experiments



- Long correlation times (narrow spectral widths) in the *D* region require pulse-to-pulse techniques
- E.g., PFISR employs coded double-pulse techniques that give range resolutions up to 600 m and spectral resolutions up to 1 Hz.

Mode	Pulse	Baud	δR	τ	IPP	δf	Nyquist	δt
0	130 μs	10 μs	1.5 km	5 μs (0.75 km)	2 ms	2 Hz	250 Hz	1 s
1	260 μs	10 μs	1.5 km	5 μs (0.75 km)	4 ms	1 Hz	125 Hz	2.5 s
2	130 μs	10 μs	1.5 km	5 μs (0.75 km)	2 ms	2 Hz	250 Hz	1.8 s
3	280 μs	10 μs	1.5 km	5 μs (0.75 km)	3 ms	1.3 Hz	167 Hz	2.7 s
4	112 μs	4 μs	0.6 km	2 μs (0.3 km)	3 ms	1.3 Hz	167 Hz	2.7 s

General

A typical experiment consists of:

- Data samples
- Noise samples
- Cal pulse samples

General

A typical experiment consists of:

- Data samples
- Noise samples
- Cal pulse samples

Given experiment is complicated by:

- Interleaving of pulses (possibly on different frequencies)
- Clutter considerations, Noise & Cal sample placement
- Maximization of duty cycle
- Beam pointing, Distribution of pulses, Integration time considerations
- All this can be very complicated

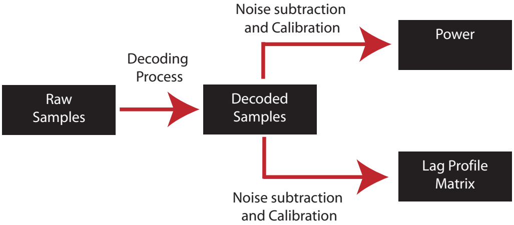
General

A typical experiment consists of:

- Data samples
- Noise samples
- Cal pulse samples

Given experiment is complicated by:

- Interleaving of pulses (possibly on different frequencies)
- Clutter considerations, Noise & Cal sample placement
- Maximization of duty cycle
- Beam pointing, Distribution of pulses, Integration time considerations
- All this can be very complicated



Power Estimation

Received power can be written as

$$P_r = \frac{P_t \tau_p}{r^2} K_{sys} \frac{N_e}{(1 + k^2\lambda_D^2)(1 + k^2\lambda_D^2 + T_r)} \text{ Watts}$$

where
P_r - received power (Watts)
P_t - transmit power (Watts)
τ_p - pulse length (seconds)
r - range (meters)
N_e - electron density (m^{-3})
k - Bragg scattering wavenumber (rad/m)
λ_D - Debye length (m)
T_r - electron to ion temperature ratio
K_{sys} - system constant (m^5/s)

Power Estimation

Received signal power needs to be calibrated to absolute units of Watts. To do this, we in general (a) take noise samples and (b) inject a calibration pulse (at each AEU for AMISR), which is then summed in the same way as the signal. The absolute calibration power in Watts is:

$$P_{cal} = k_B T_{cal} B \quad \text{Watts}$$

where
k_B - Boltzmann constant (J/kg K)
T_{cal} - temperature of calibration source (K)
B - receiver bandwidth (Hz)

Power Estimation

Received signal power needs to be calibrated to absolute units of Watts. To do this, we in general (a) take noise samples and (b) inject a calibration pulse (at each AEU for AMISR), which is then summed in the same way as the signal. The absolute calibration power in Watts is:

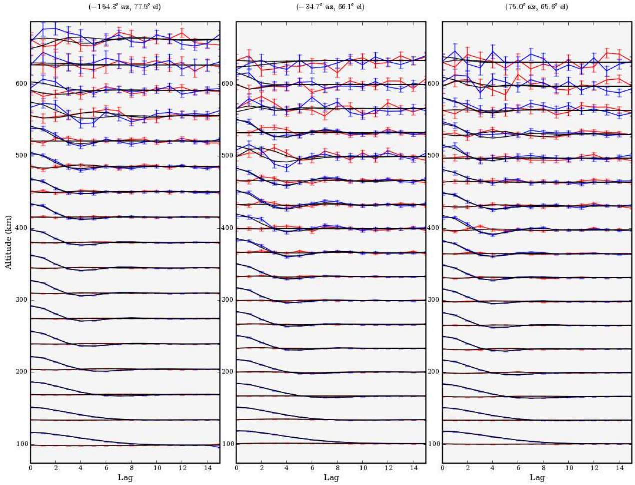
$$P_{cal} = k_B T_{cal} B \quad \mathrm{Watts}$$

where
k_B - Boltzmann constant (J/kg K)
T_{cal} - temperature of calibration source (K)
B - receiver bandwidth (Hz)

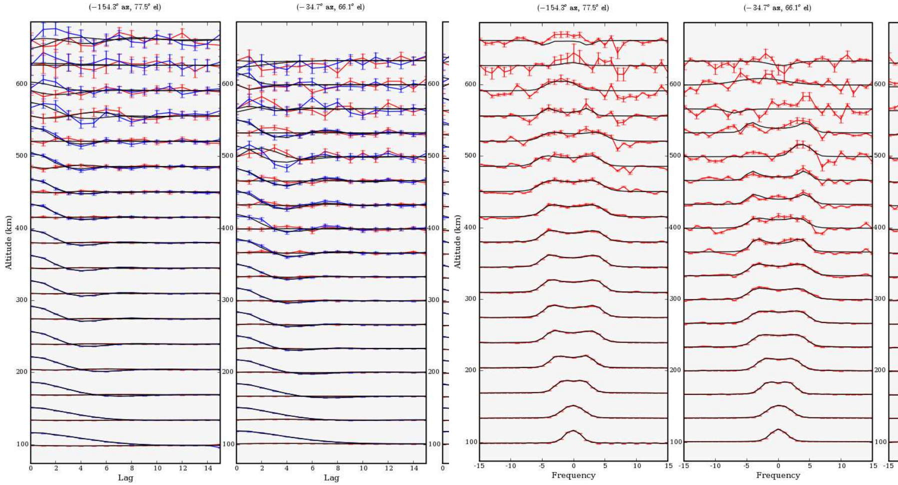
The measurement of the calibration power (after noise subtraction) can then be used as a yardstick to convert the received power to Watts. This is done as,

$$P_r = P_{cal} * (Signal - Noise)/(Cal - Noise) \quad \mathrm{Watts}$$

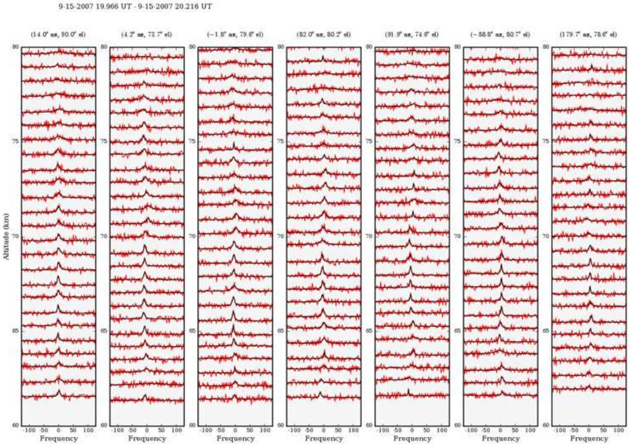
ACF / Spectra Estimation - E/F region



ACF / Spectra Estimation - E/F region



ACF / Spectra Estimation - *D* region



Electron Density

Recall,

$$P_r = \frac{P_t \tau_p}{r^2} K_{sys} \frac{N_e}{(1 + k^2\lambda_D^2)(1 + k^2\lambda_D^2 + T_r)} \text{ Watts}$$

Electron Density

Recall,

$$P_r = \frac{P_t \tau_p}{r^2} K_{sys} \frac{N_e}{(1 + k^2\lambda_D^2)(1 + k^2\lambda_D^2 + T_r)} \text{ Watts}$$

Calibrated received power can easily be inverted to determine N_e (if one makes assumptions about T_r), but what about K_{sys}?

Electron Density

Recall,

$$P_r = \frac{P_t \tau_p}{r^2} K_{sys} \frac{N_e}{(1 + k^2\lambda_D^2)(1 + k^2\lambda_D^2 + T_r)} \text{ Watts}$$

Calibrated received power can easily be inverted to determine N_e (if one makes assumptions about T_r), but what about K_{sys}?

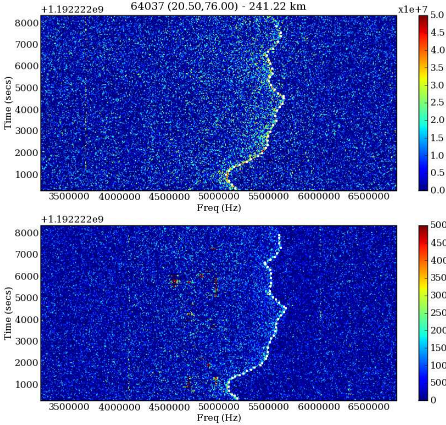
Within K_{sys} is embedded information on the gain, which might vary with look-angle [e.g., AMISR] or change with time [hopefully slowly].

Electron Density

$$f_r^2 \approx f_p^2 + \frac{3k^2}{4\pi^2}\frac{k_B T_e}{m_e} + f_c^2 \sin^2 \alpha$$

where

f_r - plasma line frequency (Hz)
f_p - plasma frequency (Hz)
T_e - electron temperature (K)
m_e - electron mass (kg)
f_c - electron cyclotron frequency (Hz)
α - magnetic aspect angle

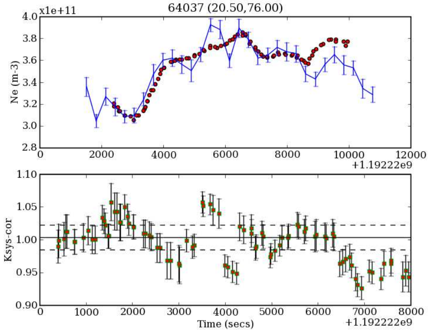


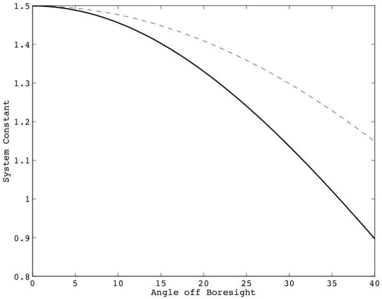
Electron Density

$$K_{sys} = A\cos^{B}(\theta_{BS}) \quad \mathrm{m}^5/\mathrm{s}$$

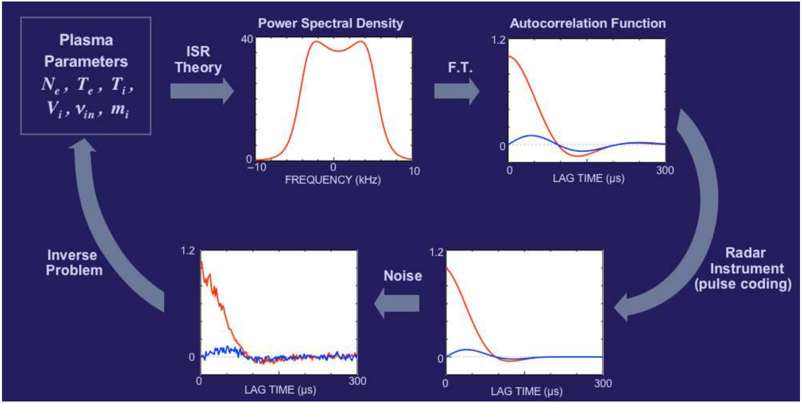
θ_{BS} - angle off boresight

A,B - constants





Fitting Spectra



Fitting Spectra

General Complicating Factors:

- Range smearing
- Lag smearing
- Pulse coding effects / "Self"-clutter
- Clutter (geophysical and not - e.g., mountains, irregularities, turbulence, non-Maxwellian)
- Signal strength / statistics
- Time stationarity

Fitting Spectra

General Complicating Factors:

- Range smearing
- Lag smearing
- Pulse coding effects / "Self"-clutter
- Clutter (geophysical and not - e.g., mountains, irregularities, turbulence, non-Maxwellian)
- Signal strength / statistics
- Time stationarity

Specific Based on Altitude:

- *F*-region/Topside - Light ion composition
- Bottomside - Molecular ion composition
- *E*-region - Collision frequency, Temperature
- *D*-region - Complete ambiguity

Fitting Spectra

General Complicating Factors:

- Range smearing
- Lag smearing
- Pulse coding effects / "Self"-clutter
- Clutter (geophysical and not - e.g., mountains, irregularities, turbulence, non-Maxwellian)
- Signal strength / statistics
- Time stationarity

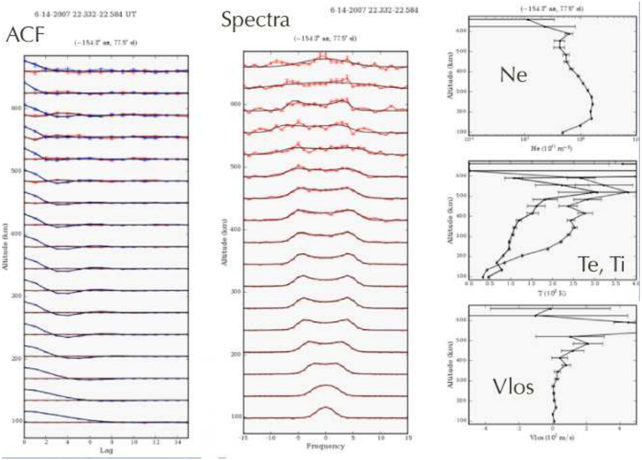
Specific Based on Altitude:

- F-region/Topside - Light ion composition
- Bottomside - Molecular ion composition
- E-region - Collision frequency, Temperature
- D-region - Complete ambiguity

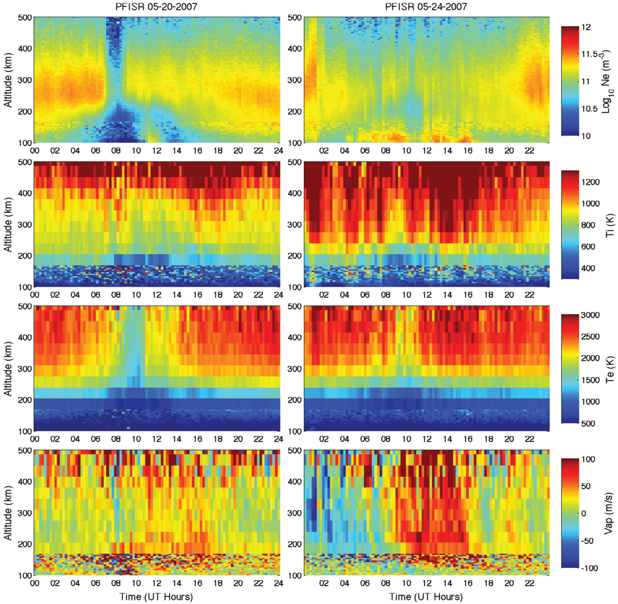
Approach:

- F-region - T_e, T_i, v_{los}, N_e
- Bottomside - Assume a composition profile
- E-region - $<\sim 105km$, assume $T_e = T_i$
- D-region - Fit a Lorentzian (width, Doppler, N_e)

Fitting Spectra - Example



Fitting Spectra - Example



Ions: Magnetized or Unmagnetized?

Depends on ratio of gyrofrequency (qB/m_i) to collision frequency (ν_{in})

- Both winds and electric fields matter for the ions. Simple steady-state ion-momentum eqn:

$$0 = e(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - m_i \nu_{in} (\mathbf{v}_i - \mathbf{u})$$

$$C = \begin{bmatrix} (1+\kappa_i^2)^{-1} & -\kappa_i (1+\kappa_i^2)^{-1} & 0 \\ \kappa_i (1+\kappa_i^2)^{-1} & (1+\kappa_i^2)^{-1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $\kappa_i = eB/m_i\nu_{in} = \Omega_i/\nu_{in}$.

Ions: Magnetized or Unmagnetized?

Depends on ratio of gyrofrequency (qB/m_i) to collision frequency (ν_{in})

- Both winds and electric fields matter for the ions. Simple steady-state ion-momentum eqn:

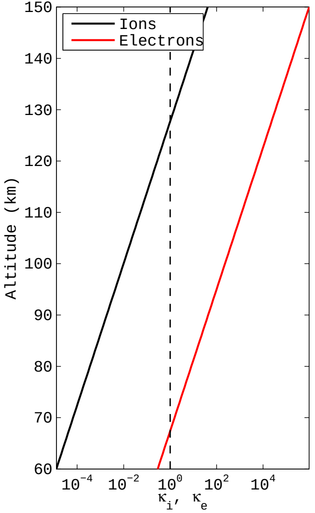
$$0 = e(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - m_i \nu_{in}(\mathbf{v}_i - \mathbf{u})$$

$$C = \begin{bmatrix} (1+\kappa_i^2)^{-1} & -\kappa_i(1+\kappa_i^2)^{-1} & 0 \\ \kappa_i(1+\kappa_i^2)^{-1} & (1+\kappa_i^2)^{-1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

  where $\kappa_i = eB/m_i\nu_{in} = \Omega_i/\nu_{in}$. The vector velocity can then be solved for $\mathbf{v}_i = b_i C\mathbf{E} + C\mathbf{u}$ where $b_i = e/m_i\nu_{in} = \kappa_i/B$

- Whereas electrons are collisionless $\mathbf{v_e} = \mathbf{E} \times \mathbf{B}/B^2$
- Currents flow even in the absence of winds:

$$\mathbf{J} = n_e e(\mathbf{v}_i - \mathbf{v}_e) = \sigma \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$$



Vector Velocities - Preliminaries

LOS Velocity measurement can be represented as:

$$v_{los}^i = k_x^i v_x + k_y^i v_y + k_z^i v_z$$

Vector Velocities - Preliminaries

LOS Velocity measurement can be represented as:

$$v_{los}^i = k_x^i v_x + k_y^i v_y + k_z^i v_z$$

where the radar **k** vector in geographic coordinates is:

$$\mathbf{k} = \begin{bmatrix} k_e \\ k_n \\ k_z \end{bmatrix} = \begin{bmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} R^{-1}$$

Vector Velocities - Preliminaries

LOS Velocity measurement can be represented as:

$$v_{los}^i = k_x^i v_x + k_y^i v_y + k_z^i v_z$$

where the radar **k** vector in geographic coordinates is:

$$\mathbf{k} = \begin{bmatrix} k_e \\ k_n \\ k_z \end{bmatrix} = \begin{bmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} R^{-1}$$

If we can neglect Earth curvature ("high enough" elevation angles),

$$\mathbf{k} = \begin{bmatrix} k_e \\ k_n \\ k_z \end{bmatrix} = \begin{bmatrix} \cos\theta\sin\phi \\ \cos\theta\cos\phi \\ \sin\theta \end{bmatrix}$$

where θ, ϕ are elevation and azimuth angles, respectively.

Vector Velocities - Preliminaries

For a local geomagnetic coordinate system we can use the rotation matrix,

$$R_{geo \rightarrow gmag} = \begin{bmatrix} \cos\delta & -\sin\delta & 0 \\ \sin I \sin\delta & \cos\delta \sin I & \cos I \\ -\cos I \sin\delta & -\cos I \cos\delta & \sin I \end{bmatrix}$$

where δ ($\sim 22^\circ$ for PFISR) and I ($\sim 77.5^\circ$ for PFISR) are the declination and dip angles, respectively.

Vector Velocities - Preliminaries

For a local geomagnetic coordinate system we can use the rotation matrix,

$$R_{geo \rightarrow gmag} = \begin{bmatrix} \cos\delta & -\sin\delta & 0 \\ \sin I \sin\delta & \cos\delta \sin I & \cos I \\ -\cos I \sin\delta & -\cos I \cos\delta & \sin I \end{bmatrix}$$

where δ ($\sim 22^\circ$ for PFISR) and I ($\sim 77.5^\circ$ for PFISR) are the declination and dip angles, respectively. Then,

$$\mathbf{k} = \begin{bmatrix} k_{pe} \\ k_{pn} \\ k_{ap} \end{bmatrix} = \begin{bmatrix} k_e \cos\delta - k_n \sin\delta \\ k_z \cos I + \sin I (k_n \cos\delta + k_e \sin\delta) \\ k_z \sin I - \cos I (k_n \cos\delta + k_e \sin\delta) \end{bmatrix}.$$

Vector Velocities - Two Point

Two LOS velocity measurements can be written as,

$$\begin{bmatrix} v_{los}^1 \\ v_{los}^2 \end{bmatrix} = \begin{bmatrix} k_{pe}^1 & k_{pn}^1 & k_{ap}^1 \\ k_{pe}^2 & k_{pn}^2 & k_{ap}^2 \end{bmatrix} \begin{bmatrix} v_{pe} \\ v_{pn} \\ v_{ap} \end{bmatrix}$$

Vector Velocities - Two Point

Two LOS velocity measurements can be written as,

$$\begin{bmatrix} v_{los}^1 \\ v_{los}^2 \end{bmatrix} = \begin{bmatrix} k_{pe}^1 & k_{pn}^1 & k_{ap}^1 \\ k_{pe}^2 & k_{pn}^2 & k_{ap}^2 \end{bmatrix} \begin{bmatrix} v_{pe} \\ v_{pn} \\ v_{ap} \end{bmatrix}$$

Can be solved for v_{pn} and v_{pe} assuming $v_{ap} \approx 0$,

$$v_{pn} = \frac{v_{los}^1 - \frac{k_{pe}^1}{k_{pe}^2} v_{los}^2 - v_{ap}\left(k_{ap}^1 - k_{ap}^2 \frac{k_{pe}^1}{k_{pe}^2}\right)}{k_{pn}^1 \left(1 - \frac{k_{pn}^2}{k_{pn}^1} \frac{k_{pe}^1}{k_{pe}^2}\right)} \approx \frac{v_{los}^1 - \frac{k_{pe}^1}{k_{pe}^2} v_{los}^2}{k_{pn}^1 \left(1 - \frac{k_{pn}^2}{k_{pn}^1} \frac{k_{pe}^1}{k_{pe}^2}\right)}$$

Vector Velocities - Two Point

Two LOS velocity measurements can be written as,

$$\begin{bmatrix} v_{los}^1 \\ v_{los}^2 \end{bmatrix} = \begin{bmatrix} k_{pe}^1 & k_{pn}^1 & k_{ap}^1 \\ k_{pe}^2 & k_{pn}^2 & k_{ap}^2 \end{bmatrix} \begin{bmatrix} v_{pe} \\ v_{pn} \\ v_{ap} \end{bmatrix}$$

Can be solved for v_{pn} and v_{pe} assuming $v_{ap} \approx 0$,

$$v_{pn} = \frac{v_{los}^1 - \frac{k_{pe}^1}{k_{pe}^2} v_{los}^2 - v_{ap}\left(k_{ap}^1 - k_{ap}^2 \frac{k_{pe}^1}{k_{pe}^2}\right)}{k_{pn}^1\left(1 - \frac{k_{pn}^2}{k_{pn}^1}\frac{k_{pe}^1}{k_{pe}^2}\right)} \approx \frac{v_{los}^1 - \frac{k_{pe}^1}{k_{pe}^2} v_{los}^2}{k_{pn}^1\left(1 - \frac{k_{pn}^2}{k_{pn}^1}\frac{k_{pe}^1}{k_{pe}^2}\right)}$$

Implies that you need look directions with different **k** vectors.

Vector Velocities - Generalization

Multiple measurements can be written as,

$$\begin{bmatrix} v_{los}^1 \\ v_{los}^2 \\ \vdots \\ v_{los}^n \end{bmatrix} = \begin{bmatrix} k_{pe}^1 & k_{pn}^1 & k_{ap}^1 \\ k_{pe}^2 & k_{pn}^2 & k_{ap}^2 \\ \vdots & \vdots & \vdots \\ k_{pe}^n & k_{pn}^n & k_{ap}^n \end{bmatrix} \begin{bmatrix} v_{pe} \\ v_{pn} \\ v_{ap} \end{bmatrix} + \begin{bmatrix} e_{los}^1 \\ e_{los}^2 \\ \vdots \\ e_{los}^n \end{bmatrix}$$

or

$$\mathbf{v}_{los} = A\mathbf{v}_i + \mathbf{e}_{los}$$

Vector Velocities - Generalization

Multiple measurements can be written as,

$$\begin{bmatrix} v_{los}^1 \\ v_{los}^2 \\ \vdots \\ v_{los}^n \end{bmatrix} = \begin{bmatrix} k_{pe}^1 & k_{pn}^1 & k_{ap}^1 \\ k_{pe}^2 & k_{pn}^2 & k_{ap}^2 \\ \vdots & \vdots & \vdots \\ k_{pe}^n & k_{pn}^n & k_{ap}^n \end{bmatrix} \begin{bmatrix} v_{pe} \\ v_{pn} \\ v_{ap} \end{bmatrix} + \begin{bmatrix} e_{los}^1 \\ e_{los}^2 \\ \vdots \\ e_{los}^n \end{bmatrix}$$

or

$$\mathbf{v}_{los} = A\mathbf{v}_i + \mathbf{e}_{los}$$

Treat $\mathbf{v}_i$ as a Gaussian random variable (Bayesian), use linear theory to derive a least-squares estimator.

Vector Velocities - Generalization

Multiple measurements can be written as,

$$\begin{bmatrix} v_{los}^1 \\ v_{los}^2 \\ \vdots \\ v_{los}^n \end{bmatrix} = \begin{bmatrix} k_{pe}^1 & k_{pn}^1 & k_{ap}^1 \\ k_{pe}^2 & k_{pn}^2 & k_{ap}^2 \\ \vdots & \vdots & \vdots \\ k_{pe}^n & k_{pn}^n & k_{ap}^n \end{bmatrix} \begin{bmatrix} v_{pe} \\ v_{pn} \\ v_{ap} \end{bmatrix} + \begin{bmatrix} e_{los}^1 \\ e_{los}^2 \\ \vdots \\ e_{los}^n \end{bmatrix}$$

or

$$\mathbf{v}_{los} = A\mathbf{v}_i + \mathbf{e}_{los}$$

Treat $\mathbf{v}_i$ as a Gaussian random variable (Bayesian), use linear theory to derive a least-squares estimator. $\mathbf{v}_i$ zero mean, Σ_v (*a priori*). Measurements zero mean, covariance Σ_e.

Vector Velocities - Generalization

Multiple measurements can be written as,

$$\begin{bmatrix} v_{los}^1 \\ v_{los}^2 \\ \vdots \\ v_{los}^n \end{bmatrix} = \begin{bmatrix} k_{pe}^1 & k_{pn}^1 & k_{ap}^1 \\ k_{pe}^2 & k_{pn}^2 & k_{ap}^2 \\ \vdots & \vdots & \vdots \\ k_{pe}^n & k_{pn}^n & k_{ap}^n \end{bmatrix} \begin{bmatrix} v_{pe} \\ v_{pn} \\ v_{ap} \end{bmatrix} + \begin{bmatrix} e_{los}^1 \\ e_{los}^2 \\ \vdots \\ e_{los}^n \end{bmatrix}$$

or

$$\mathbf{v}_{los} = A\mathbf{v}_i + \mathbf{e}_{los}$$

Treat $\mathbf{v}_i$ as a Gaussian random variable (Bayesian), use linear theory to derive a least-squares estimator. $\mathbf{v}_i$ zero mean, Σ_v (*a priori*). Measurements zero mean, covariance Σ_e. Solution,

$$\hat{\mathbf{v}}_i = \Sigma_v A^T (A\Sigma_v A^T + \Sigma_e)^{-1} \mathbf{v}_{los}$$

Error covariance,

$$\Sigma_{\hat{v}} = \Sigma_v - \Sigma_v A^T (A\Sigma_v A^T + \Sigma_e)^{-1} A\Sigma_v = (A^T \Sigma_e^{-1} A + \Sigma_v^{-1})^{-1}$$

Electric Fields

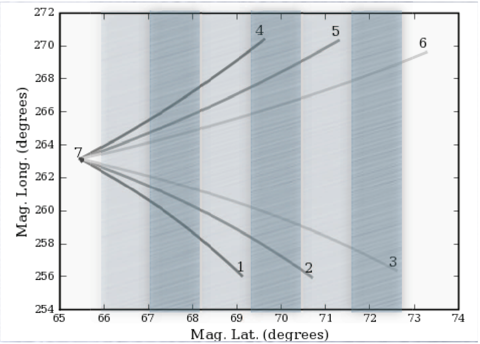
- While above approach can be used to resolve vectors as a function of altitude (or anything else), we often want to resolve vectors as a function of invariant latitude.

Electric Fields

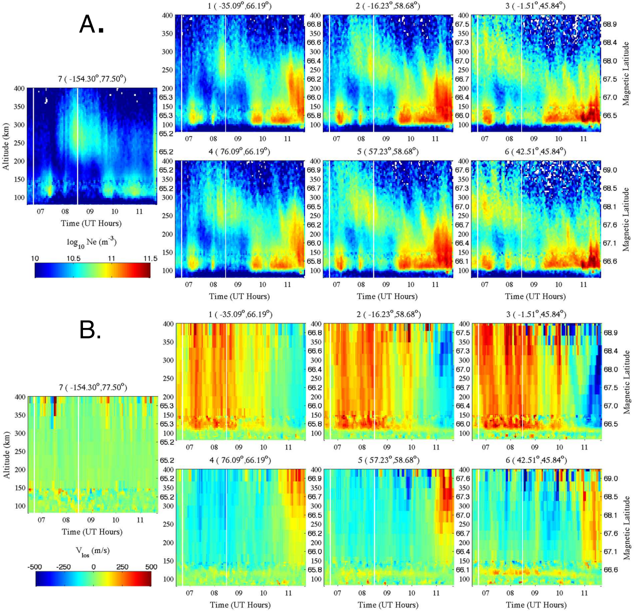
- While above approach can be used to resolve vectors as a function of altitude (or anything else), we often want to resolve vectors as a function of invariant latitude.
- In the F region (above $\sim 150 - 175$ km), plasma is $\mathbf{E} \times \mathbf{B}$ drifting.

Electric Fields

- While above approach can be used to resolve vectors as a function of altitude (or anything else), we often want to resolve vectors as a function of invariant latitude.
- In the F region (above $\sim 150 - 175$ km), plasma is $\mathbf{E} \times \mathbf{B}$ drifting.



Electric Fields - Example

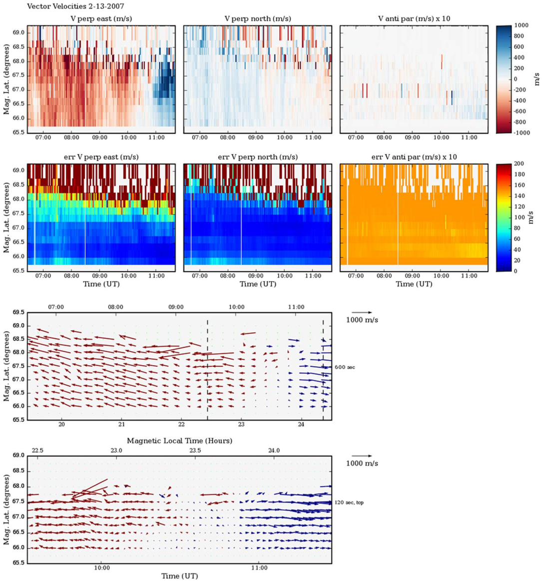


Electron Density

LOS Velocities

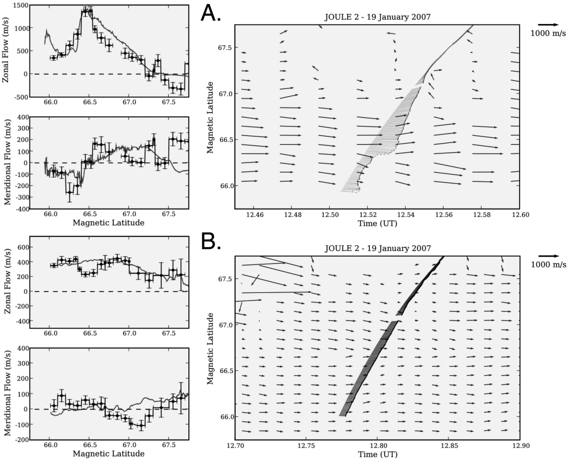
Electric Fields - Example

Resolved Vectors



Electric Fields - Example

Comparison to rocket-measured E-fields.



E-Region Winds

At lower altitudes, the ions become collisional and transition from $\mathbf{E} \times \mathbf{B}$ drifting at high altitudes to drifting with the neutral winds at low altitudes.

E-Region Winds

At lower altitudes, the ions become collisional and transition from $\mathbf{E} \times \mathbf{B}$ drifting at high altitudes to drifting with the neutral winds at low altitudes.

The steady state ion momentum equations relate the vector velocities (as a function of altitude) to electric fields and neutral winds

$$0 = e(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - m_i \nu_{in} (\mathbf{v}_i - \mathbf{u})$$

E-Region Winds

At lower altitudes, the ions become collisional and transition from $\mathbf{E} \times \mathbf{B}$ drifting at high altitudes to drifting with the neutral winds at low altitudes.
The steady state ion momentum equations relate the vector velocities (as a function of altitude) to electric fields and neutral winds

$$0 = e(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - m_i \nu_{in}(\mathbf{v}_i - \mathbf{u})$$

Defining the matrix C as,

$$C = \begin{bmatrix} (1+\kappa_i^2)^{-1} & -\kappa_i(1+\kappa_i^2)^{-1} & 0 \\ \kappa_i(1+\kappa_i^2)^{-1} & (1+\kappa_i^2)^{-1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $\kappa_i = eB/m_i\nu_{in} = \Omega_i/\nu_{in}$.

E-Region Winds

At lower altitudes, the ions become collisional and transition from $\mathbf{E} \times \mathbf{B}$ drifting at high altitudes to drifting with the neutral winds at low altitudes.
The steady state ion momentum equations relate the vector velocities (as a function of altitude) to electric fields and neutral winds

$$0 = e(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - m_i \nu_{in}(\mathbf{v}_i - \mathbf{u})$$

Defining the matrix C as,

$$C = \begin{bmatrix} (1+\kappa_i^2)^{-1} & -\kappa_i(1+\kappa_i^2)^{-1} & 0 \\ \kappa_i(1+\kappa_i^2)^{-1} & (1+\kappa_i^2)^{-1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $\kappa_i = eB/m_i\nu_{in} = \Omega_i/\nu_{in}$. The vector velocity can then be solved for

$$\mathbf{v}_i = b_i C\mathbf{E} + C\mathbf{u}$$

where $b_i = e/m_i\nu_{in} = \kappa_i/B$

E-Region Winds

$$\mathbf{v}_i = b_i C \mathbf{E} + C \mathbf{u}$$

E-Region Winds

$$\mathbf{v}_i = b_i C\mathbf{E} + C\mathbf{u}$$

Defining a new matrix as

$$D = [b_i C \;\; C]$$

E-Region Winds

$$\mathbf{v}_i = b_i C\mathbf{E} + C\mathbf{u}$$

Defining a new matrix as

$$D = [b_i C \;\; C]$$

we can write the forward model

$$\mathbf{v}_{los} = (A \cdot D)\mathbf{x} + \mathbf{e}_{los}.$$

E-Region Winds

$$\mathbf{v}_i = b_i C \mathbf{E} + C \mathbf{u}$$

Defining a new matrix as

$$D = [b_i C \;\; C]$$

we can write the forward model

$$\mathbf{v}_{los} = (A \cdot D)\mathbf{x} + \mathbf{e}_{los}.$$

An obvious problem is the ambiguity in terms of **E** and **u**. Solution is to invert all measurements from all altitudes at once, allowing winds to vary with altitude but the electric field to map along field lines.

E-Region Winds

$$\mathbf{v}_i = b_i C\mathbf{E} + C\mathbf{u}$$

Defining a new matrix as

$$D = [b_i C \;\; C]$$

we can write the forward model

$$\mathbf{v}_{los} = (A \cdot D)\mathbf{x} + \mathbf{e}_{los}.$$

An obvious problem is the ambiguity in terms of **E** and **u**. Solution is to invert all measurements from all altitudes at once, allowing winds to vary with altitude but the electric field to map along field lines. Forward model becomes,

$$\mathbf{x} = [E_{pe} \;\; E_{pn} \;\; E_{||} \;\; u^1_{pe} \;\; u^1_{pn} \;\; u^1_{||} \;\; u^2_{pe} \;\; u^2_{pn} \;\; u^2_{||} \; \ldots \; u^n_{pe} \;\; u^n_{pn} \;\; u^n_{||}]^T$$

E-Region Winds

$$\mathbf{v}_i = b_i C\mathbf{E} + C\mathbf{u}$$

Defining a new matrix as

$$D = [b_i C \;\; C]$$

we can write the forward model

$$\mathbf{v}_{los} = (A \cdot D)\mathbf{x} + \mathbf{e}_{los}.$$

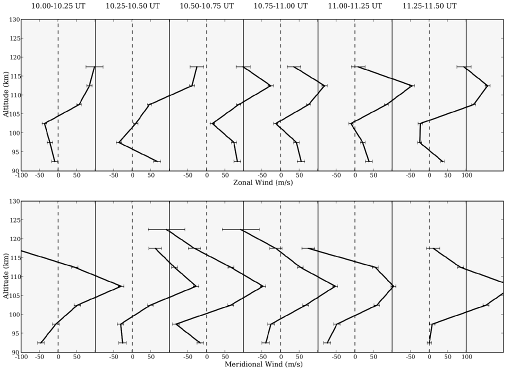
An obvious problem is the ambiguity in terms of **E** and **u**. Solution is to invert all measurements from all altitudes at once, allowing winds to vary with altitude but the electric field to map along field lines. Forward model becomes,

$$\mathbf{x} = [E_{pe} \;\; E_{pn} \;\; E_{||} \;\; u_{pe}^1 \;\; u_{pn}^1 \;\; u_{||}^1 \;\; u_{pe}^2 \;\; u_{pn}^2 \;\; u_{||}^2 \; \ldots \; u_{pe}^n \;\; u_{pn}^n \;\; u_{||}^n]^T$$

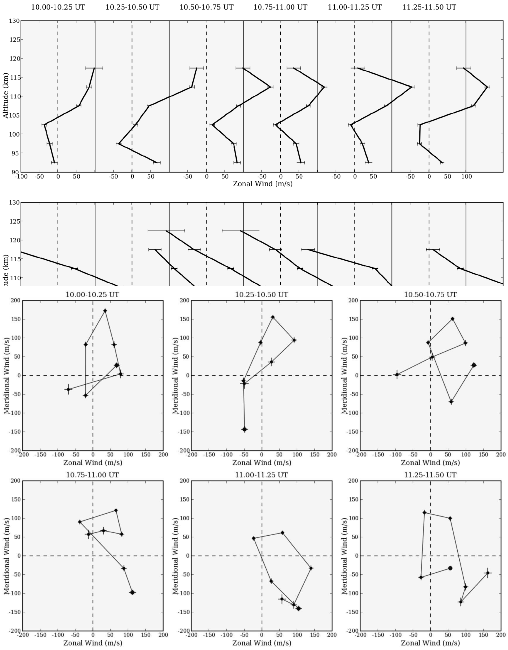
This allows for direct constraint of both the vertical wind and the parallel electric field, both of which we expect to be small.

$$\Sigma_v^{gmag} = J_{geo\rightarrow gmag} \Sigma_v^{geo} J_{geo\rightarrow gmag}^T$$

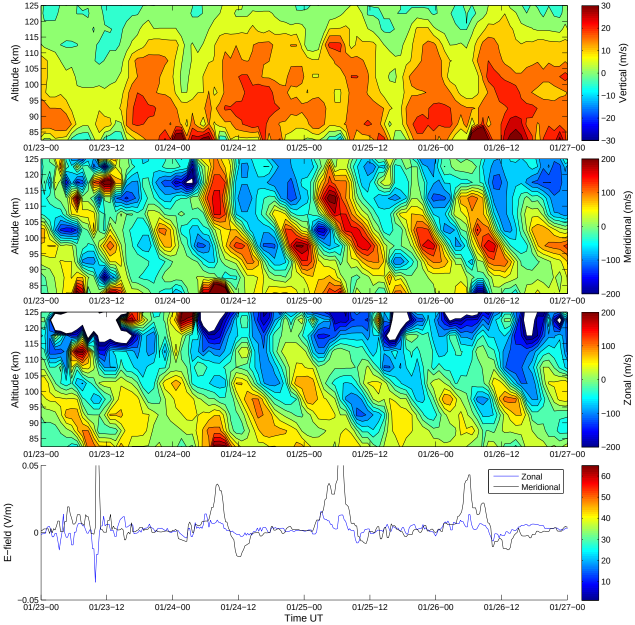
E-Region Winds - Example



E-Region Winds - Example



E-Region Winds - Example



Collision Frequency

Two approaches (that I know of) for assessing collision frequency:

Collision Frequency

Two approaches (that I know of) for assessing collision frequency:

1. Direct fits at lower altitudes (spectral width $\sim\propto T_n/\nu_{in}$)

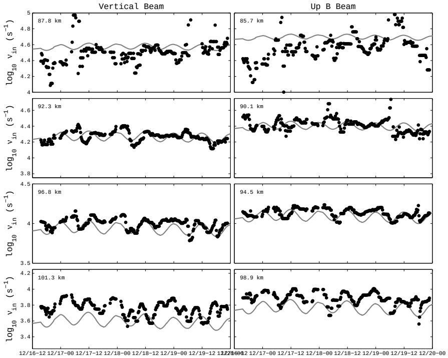
Collision Frequency

Two approaches (that I know of) for assessing collision frequency:

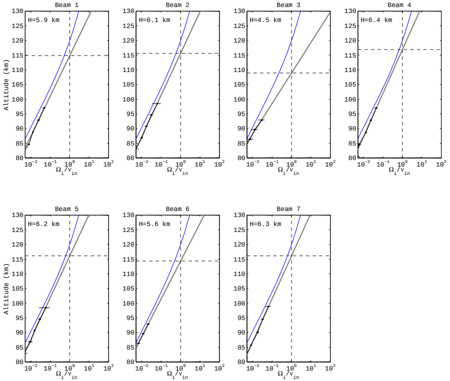
1. Direct fits at lower altitudes (spectral width $\sim\propto T_n/\nu_{in}$)
2. Examination of variation of LOS velocity with altitude

Collision Frequency - Method 1

Semi-diurnal variation over several days.



Altitude profile and extrapolation.



Collision Frequency - Method 2 - Example

The rotation of the LOS velocity with altitude is a good indicator of collision frequency effects.

Collision Frequency - Method 2 - Example

The rotation of the LOS velocity with altitude is a good indicator of collision frequency effects.
E.g., take the vertical beam,

$$v_z = v_{\perp n} \cos I + v_{||} \sin I$$

Collision Frequency - Method 2 - Example

The rotation of the LOS velocity with altitude is a good indicator of collision frequency effects.
E.g., take the vertical beam,

$$v_z = v_{\perp n} \cos I + v_{||} \sin I$$

Perp-north and parallel components given by,

$$v_{\perp n} = \kappa_i (1 + \kappa_i^2)^{-1} (b_i E_{\perp e} + u_{\perp e}) + (1 + \kappa_i^2)^{-1} (b_i E_{\perp n} + u_{\perp n})$$

$$v_{||} = u_{||} + b_i E_{||}$$

Collision Frequency - Method 2 - Example

The rotation of the LOS velocity with altitude is a good indicator of collision frequency effects.
E.g., take the vertical beam,

$$v_z = v_{\perp n} \cos I + v_{||} \sin I$$

Perp-north and parallel components given by,

$$v_{\perp n} = \kappa_i (1 + \kappa_i^2)^{-1} (b_i E_{\perp e} + u_{\perp e}) + (1 + \kappa_i^2)^{-1} (b_i E_{\perp n} + u_{\perp n})$$

$$v_{||} = u_{||} + b_i E_{||}$$

Define a new variable,

$$v_z' = v_z - v_{||} \sin I$$

Collision Frequency - Method 2 - Example

The rotation of the LOS velocity with altitude is a good indicator of collision frequency effects.
E.g., take the vertical beam,

$$v_z = v_{\perp n} \cos I + v_{||} \sin I$$

Perp-north and parallel components given by,

$$v_{\perp n} = \kappa_i (1+\kappa_i^2)^{-1} (b_i E_{\perp e} + u_{\perp e}) + (1+\kappa_i^2)^{-1} (b_i E_{\perp n} + u_{\perp n})$$

$$v_{||} = u_{||} + b_i E_{||}$$

Define a new variable,

$$v_z' = v_z - v_{||} \sin I$$

Under strong convection (electric field) conditions, neglect winds

$$v_z' \sim b_i (1+\kappa_i^2)^{-1} [\kappa_i E_{\perp e} + E_{\perp n}] \cos I$$

Collision Frequency - Method 2 - Example

$$v_z' \sim b_i(1+\kappa_i^2)^{-1}\left[\kappa_i E_{\perp e} + E_{\perp n}\right]\cos I$$

Collision Frequency - Method 2 - Example

$$v'_z \sim b_i(1+\kappa_i^2)^{-1}\left[\kappa_i E_{\perp e} + E_{\perp n}\right]\cos I$$

If $\kappa_i(z) = \kappa_0 e^{(z-z_0)/H}$, vertical ion velocity will maximize at

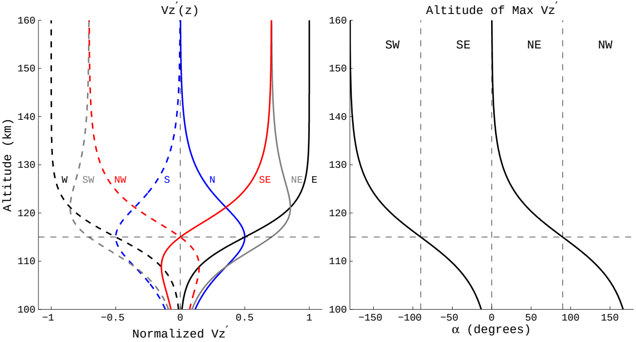
$$z_{\max\ v'_z} = z_0 + H\ln\kappa_0^{-1} + H\ln\left[\frac{\cos\alpha \pm 1}{\sin\alpha}\right]$$

Collision Frequency - Method 2 - Example

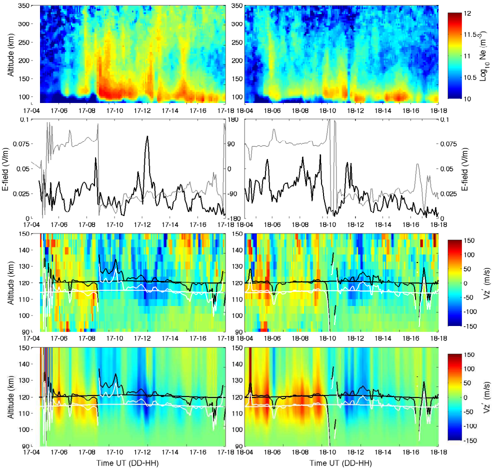
$$v'_z \sim b_i(1+\kappa_i^2)^{-1}\left[\kappa_i E_{\perp e} + E_{\perp n}\right]\cos I$$

If $\kappa_i(z) = \kappa_0 e^{(z-z_0)/H}$, vertical ion velocity will maximize at

$$z_{\max\ v'_z} = z_0 + H\ln\kappa_0^{-1} + H\ln\left[\frac{\cos\alpha \pm 1}{\sin\alpha}\right]$$

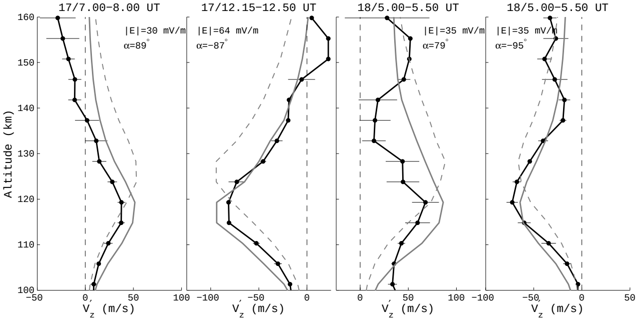


Collision Frequency - Method 2



Collision Frequency - Method 2

Profiles of v_z' during high convection conditions.
Dashed - with MSIS; Solid - scaled by a factor of 2.



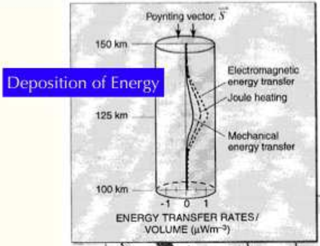
Conductivities / Currents / Joule Heating Rates

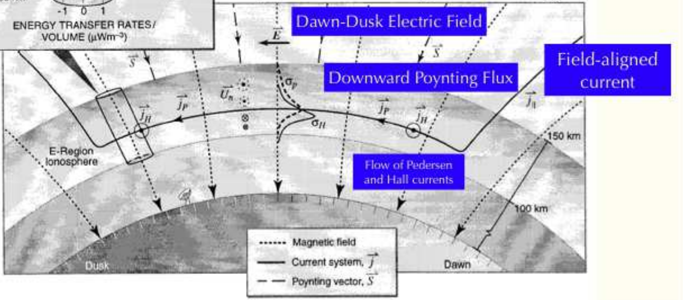
- Supplied electromagnetic energy is converted to heat (Joule heating) or momentum (Lorentz forcing)

$$q_j = \mathbf{j} \cdot \mathbf{E}' = \mathbf{j} \cdot (\mathbf{E} + \mathbf{U} \times \mathbf{B}) = \sigma_P |\mathbf{E} + \mathbf{U} \times \mathbf{B}|^2$$

$$q_m = \mathbf{U} \cdot (\mathbf{j} \times \mathbf{B})$$

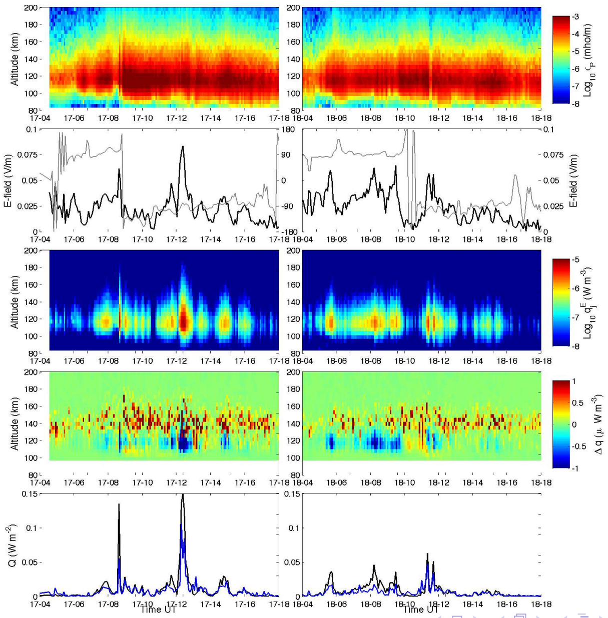
- q_m negative - neutral gas is a sink (load)
- q_m positive - neutral gas is a source (generator)





Thayer [2000]

Conductivities / Currents / Joule Heating Rates



Active Areas of Reserch

1. Full profile / deconvolution techniques for IS fitting
2. Taking advantage of space and time information; Optimal inference of parameters
3. Optimization and standardization of approaches
4. Additional parameters: molecular ion composition, height-resolved plasma lines, topside parameters, etc.
5. Additional parameters ++: *D*-region momentum fluxes, higher altitude winds, etc.
6. etc.