Introduction to ISR Signal Processing

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Why study ISR?

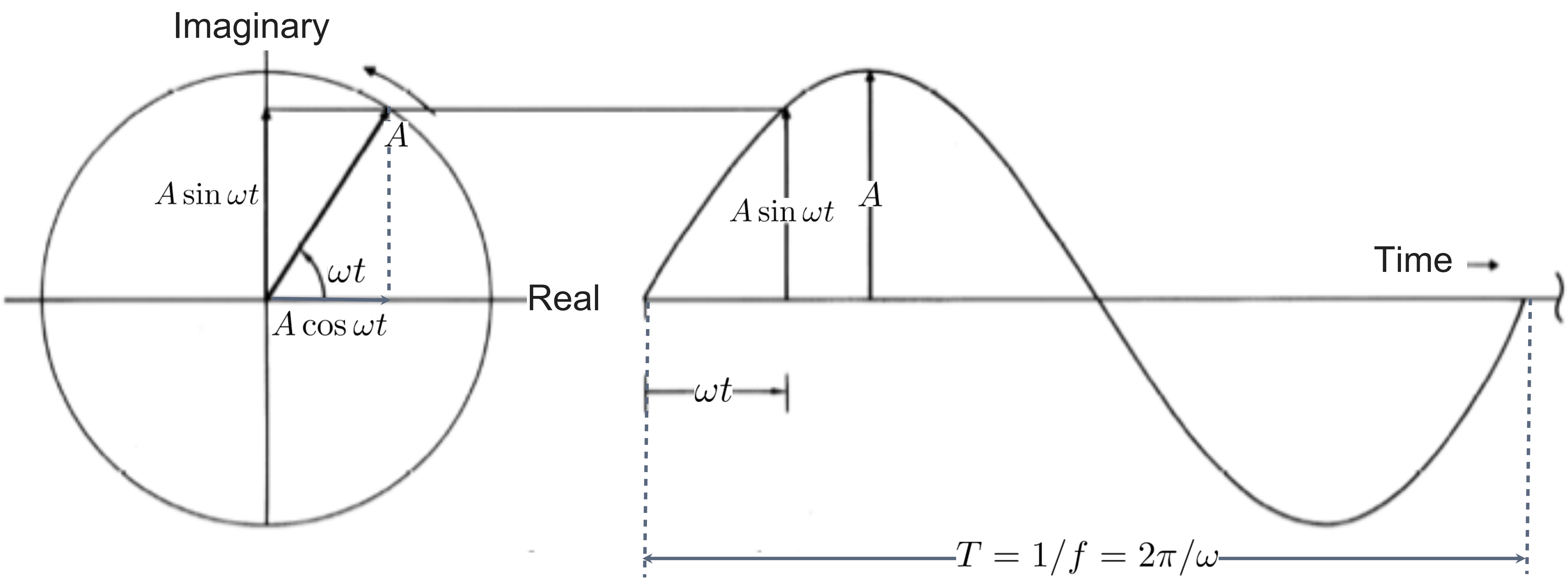
Requires that you learn about a great many useful and fascinating subjects in substantial depth.

- Plasma physics/Space physics
- Radar
- Coding (information theory)
- Electronics (Power, RF, DSP)
- Signal Processing
- Inverse theory

Topics for this talk

- Mathematical toolbox
- Review of basic radar concepts
- Ionospheric Doppler spectrum
- Range resolution and matched filtering
- I/Q demodulation
- Autocorrelation function (ACF) and Power Spectral Density (PSD)

Euler identity and the complex plane



ω is the "angular velocity" (radians/s) of the spinning arrow

f is the number of complete rotations (2π radians) in one second (1/s or Hz)

We need a signal that tells us **how fast** and in **which direction** the arrow is spinning. This signal is the complex exponential. Invoking the Euler identity,

$$s(t) = Ae^{j\omega t} = A\cos\omega t + jA\sin\omega t = I + jQ$$

I = in-phase component

Q = in-quadrature component

Exponentials are eigenfunctions of linear, time-invariant systems!

Essential mathematical operations

Fourier Transform: Expresses a function as a weighted sum of harmonic functions (i.e., complex exponentials)

$$f(t) = \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega \quad \Longleftrightarrow \quad F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

Convolution: Expresses the action of a linear, time-invariant system on a function.

$$f(t) * g(t) = \int_{-\infty}^{+\infty} f(\tau) g(\tau - t) d\tau \qquad f(t) * g(t) \Longleftrightarrow F(f)G(f)$$

Correlation: A measure of the degree to which two functions look alike at a given offset.

$$f(t) \circ g(t) = \int_{-\infty}^{+\infty} f^*(\tau) g(t + \tau) d\tau \qquad f(t) \circ g(t) \Longleftrightarrow F^*(f)G(f)$$

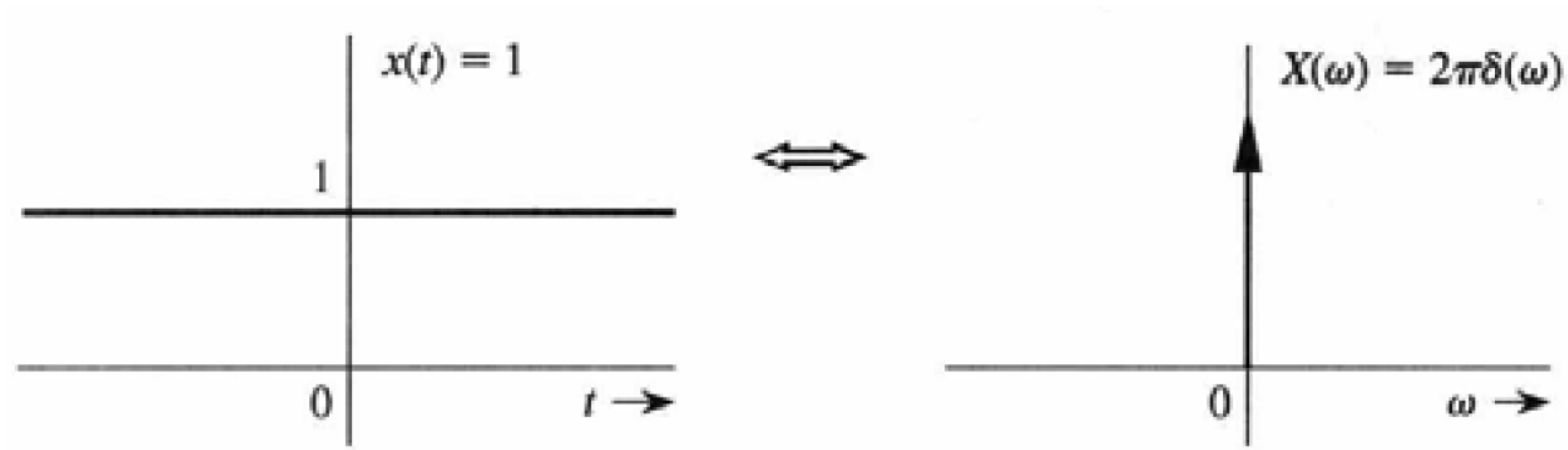
Autocorrelation, Convolution, Power Spectral Density, Wiener-Khinchin Theorem

$$R_{uu} = u(t) \circ u(t) = u(t) * u^*(-t) \qquad R_{uu} \Longleftrightarrow |U(f)|^2$$

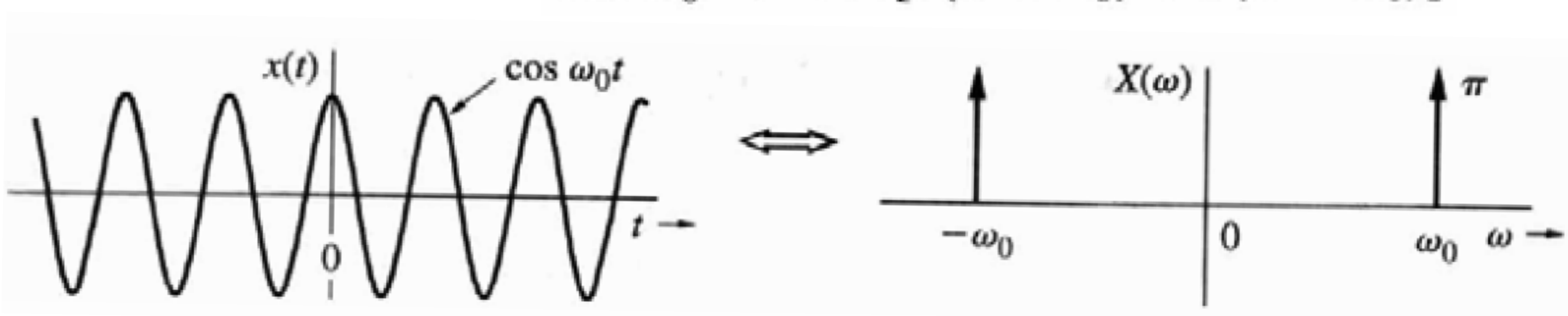
Fourier Transform Properties

Operation	Time Function	Fourier Transform
Linearity	$af_1(t) + bf_2(t)$	$aF_1(\omega) + bF_2(\omega)$
Time shift	$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
Time scaling	$f(at)$	$\frac{1}{\lvert a\rvert} F\left(\frac{\omega}{a}\right)$
Time transformation	$f(at - t_0)$	$\frac{1}{\lvert a\rvert} F\left(\frac{\omega}{a}\right)e^{-j\omega t_0/a}$
Duality	$F(t)$	$2\pi f(-\omega)$
Frequency shift	$f(t)e^{j\omega_0 t}$	$F(\omega-\omega_0)$
Convolution	$f_1(t)*f_2(t)$	$F_1(\omega)F_2(\omega)$
	$f_1(t)f_2(t)$	$\frac{1}{2\pi}F_1(\omega)*F_2(\omega)$
Differentiation	$\frac{d^n[f(t)]}{dt^n}$	$(j\omega)^n F(\omega)$
	$(-jt)^n f(t)$	$\frac{d^n[F(\omega)]}{d\omega^n}$
Integration	$\int_{-\infty}^{t} f(\tau)d\tau$	$\frac{1}{j\omega}F(\omega) + \pi F(0)\delta(\omega)$

Harmonic Functions



$$\cos \omega_0 t \Longleftrightarrow \pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$$

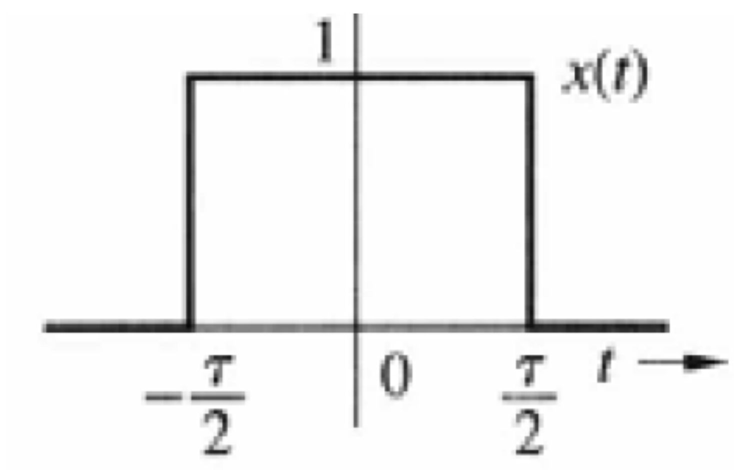


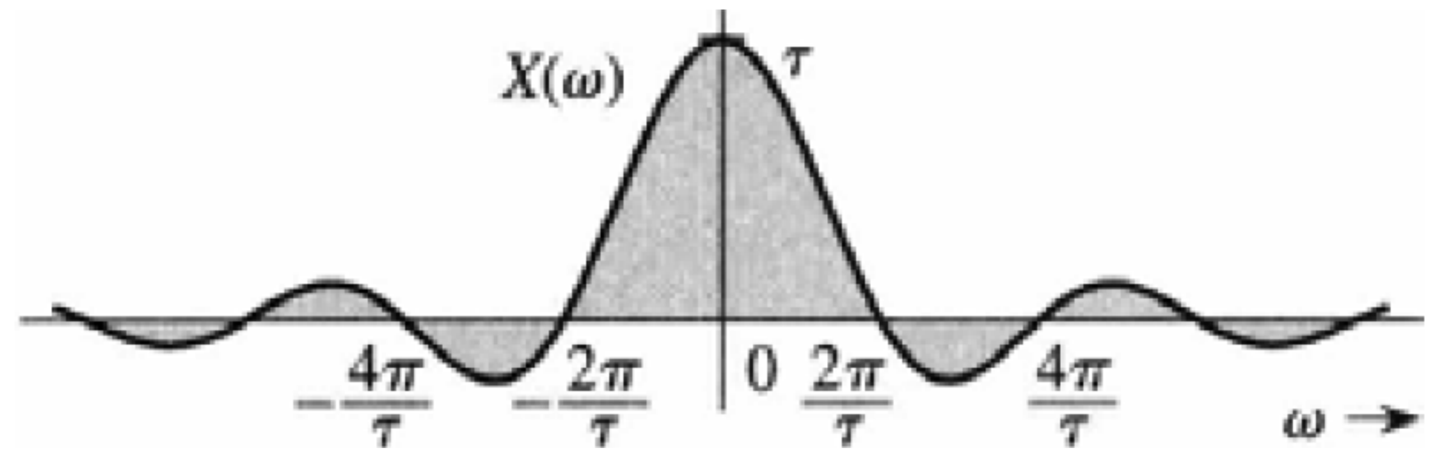
$$\sin \omega_0 t \qquad \Longleftrightarrow \qquad j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$$

$$e^{j\omega_0 t} \qquad \Longleftrightarrow \qquad 2\pi\delta(\omega - \omega_0)$$

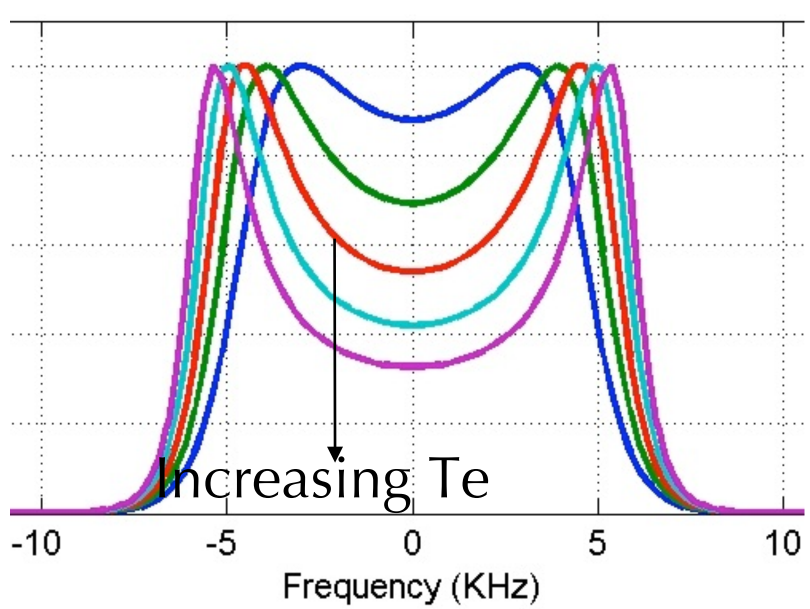
Gate function

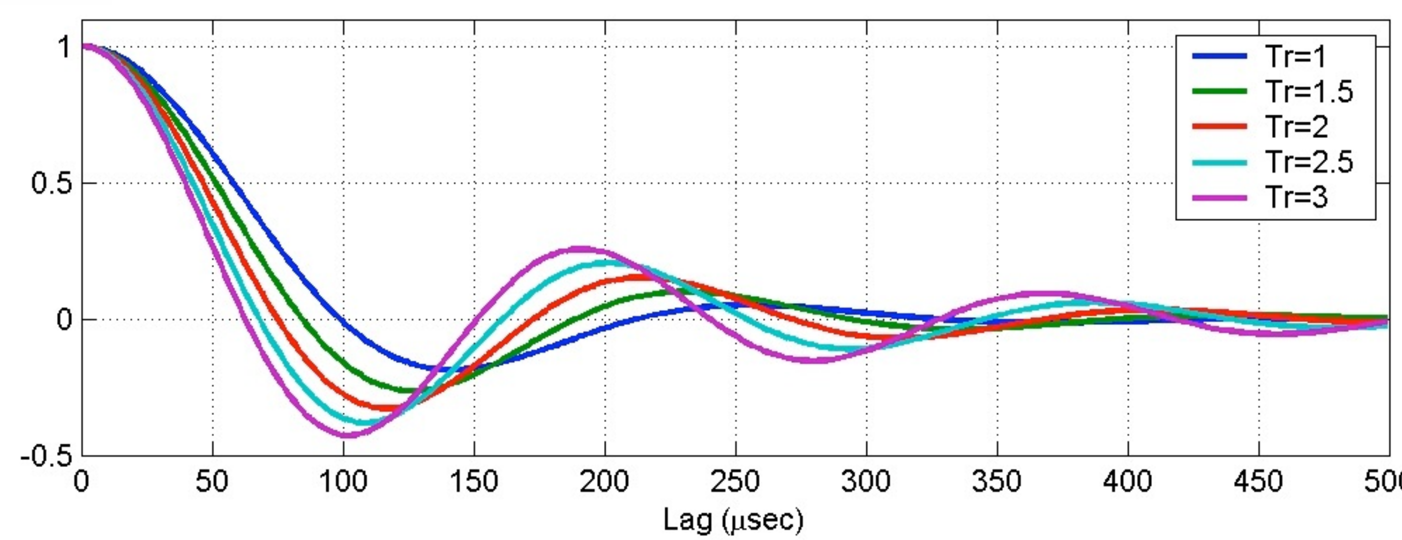
$$\mathrm{rect}(t/\tau) = \begin{cases} 1 & \text{for } -\tau/2 < t < \tau/2 \\ 0 & \text{otherwise} \end{cases} \iff \tau \,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$$





ISR spectrum $\iff$ Autocorrelation function (ACF)



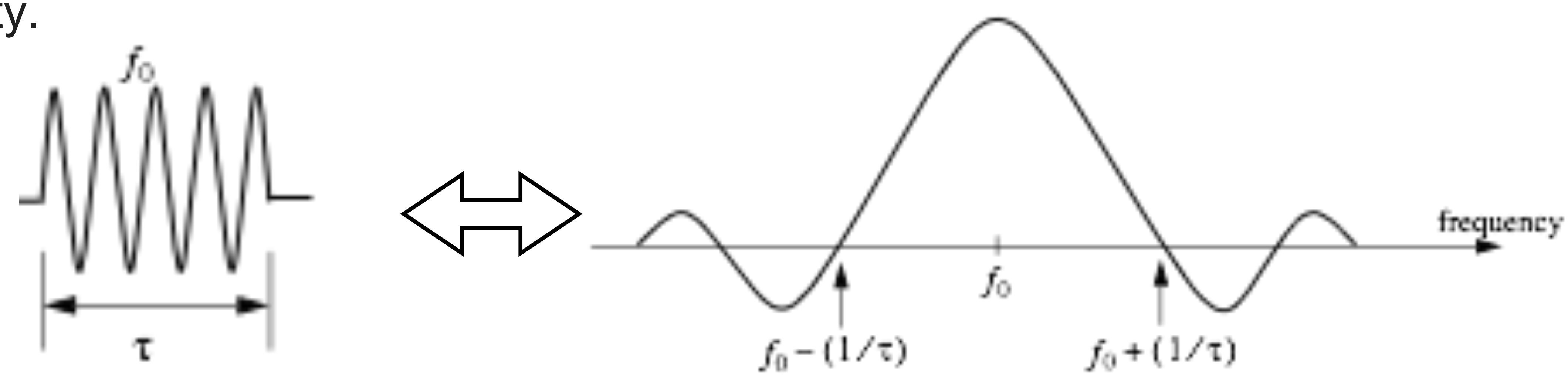


Not surprisingly, the ISR ACF looks like a sinc function, but with longer oscillations when the Te/Ti ratio gets large…

How it all hangs together.

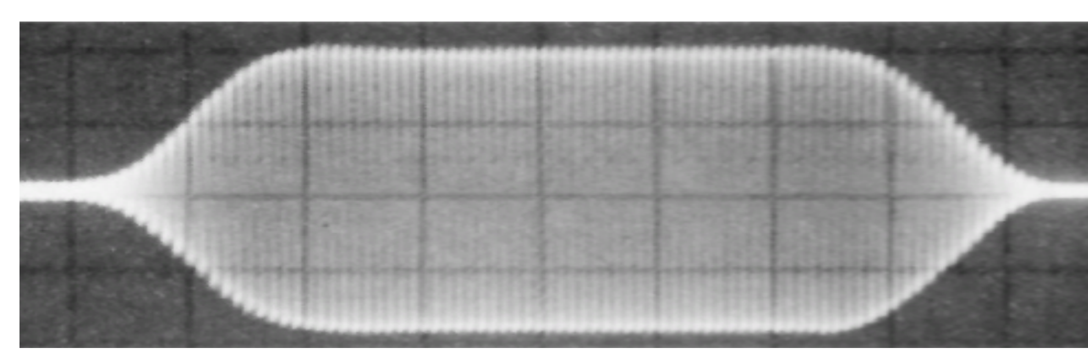
- Duality:
  - Gate function in the time domain represents amplitude modulation
  - Gate function in the frequency domain represents filtering

- Limiting cases:
  - Gate function approaches delta function as width goes to 0 with constant area
  - A constant function in time domain is a special case of harmonic function where frequency = 0.
  - A constant function in time domain is a special case of a gate function where width = infinity.



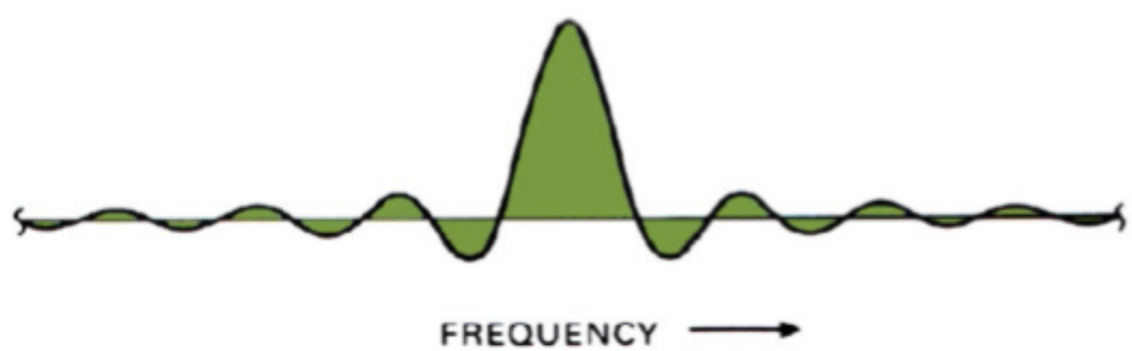
How many cycles are in a typical ISR pulse?
PFISR frequency: 449 MHz
Typical long-pulse length: 480 μs

⇨ 215,520 cycles!

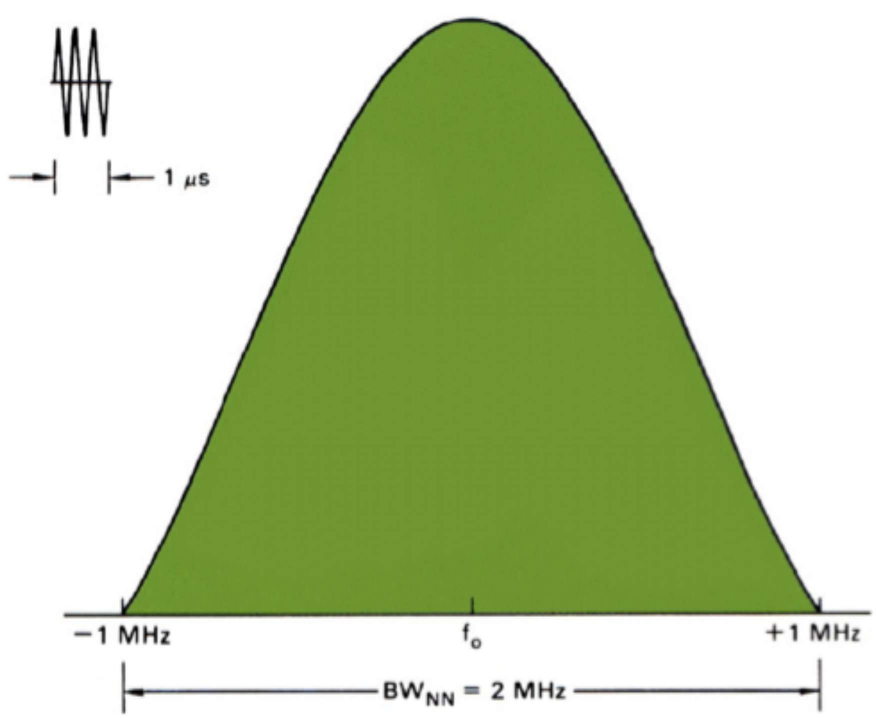


Bandwidth of a pulsed signal

Spectrum of receiver output has sinc shape, with sidelobes half the width of the central lobe and continuously diminishing in amplitude above and below main lobe



A 1 microsecond pulse has a null-to-null bandwidth of the central lobe = 2 MHz



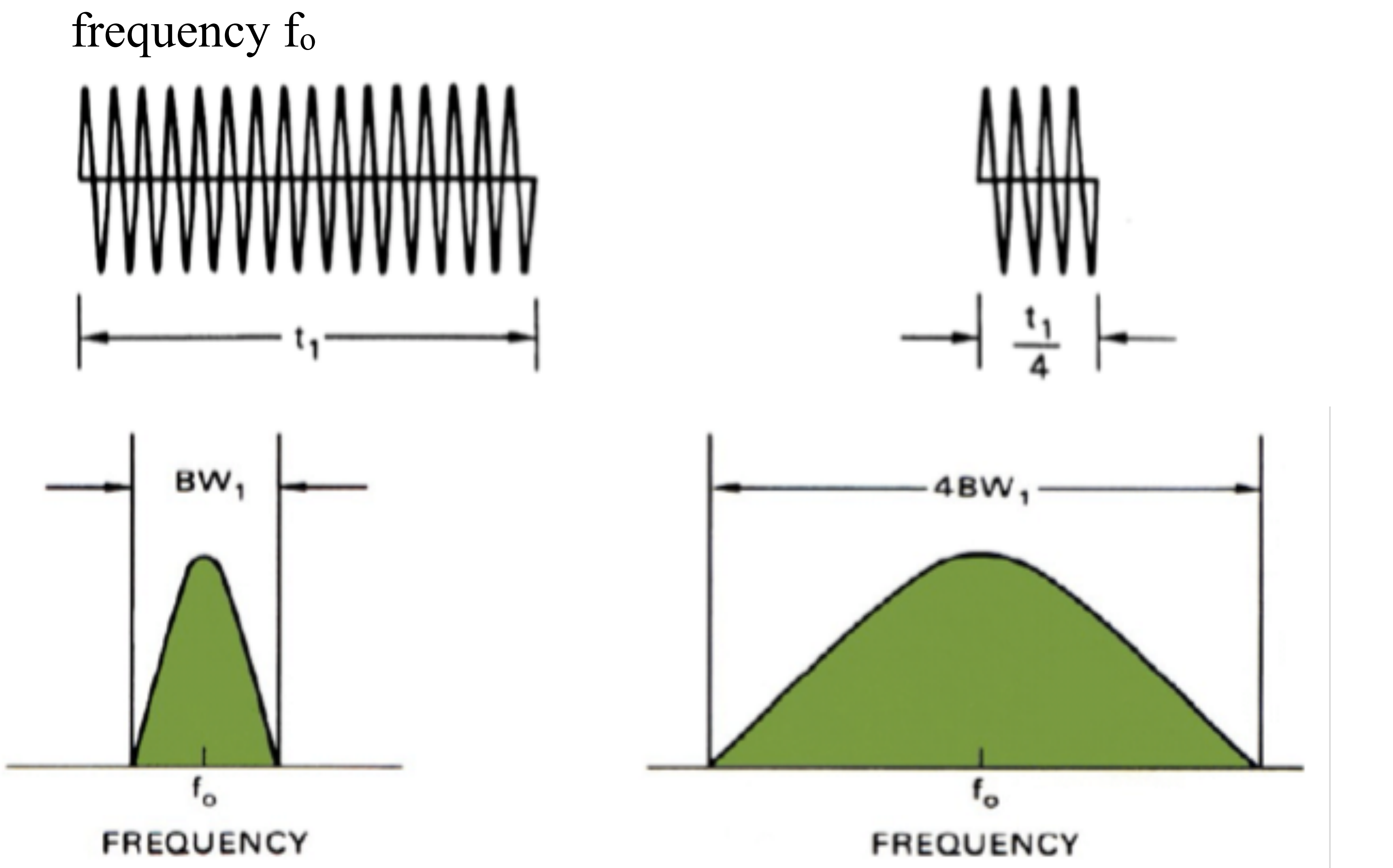
Two possible bandwidth measures:

"null to null" bandwidth $B_{nn} = \frac{2}{\tau}$

"3dB" bandwidth $B_{3dB} = \frac{1}{\tau}$

Unless otherwise specified, assume bandwidth refers to 3 dB bandwidth

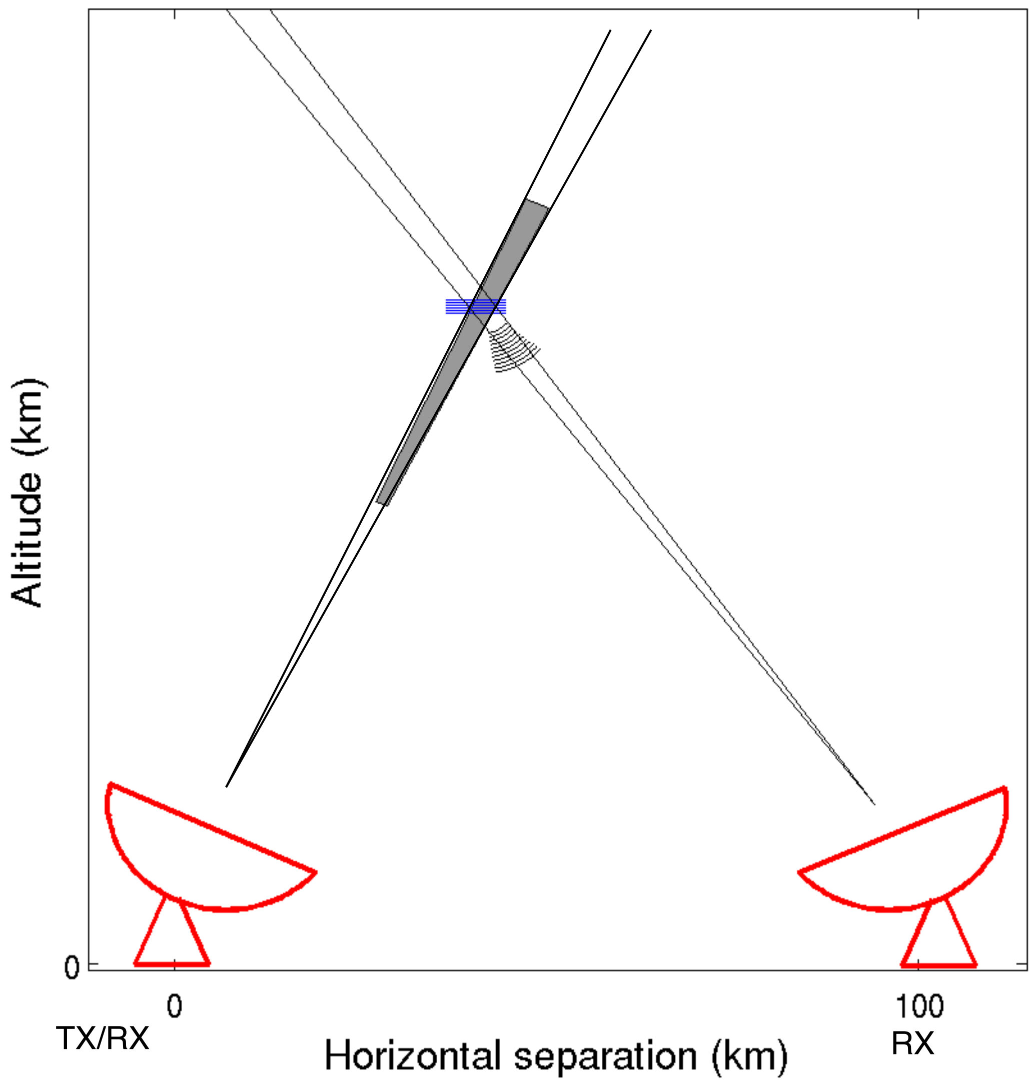
Pulse-Bandwidth Connection

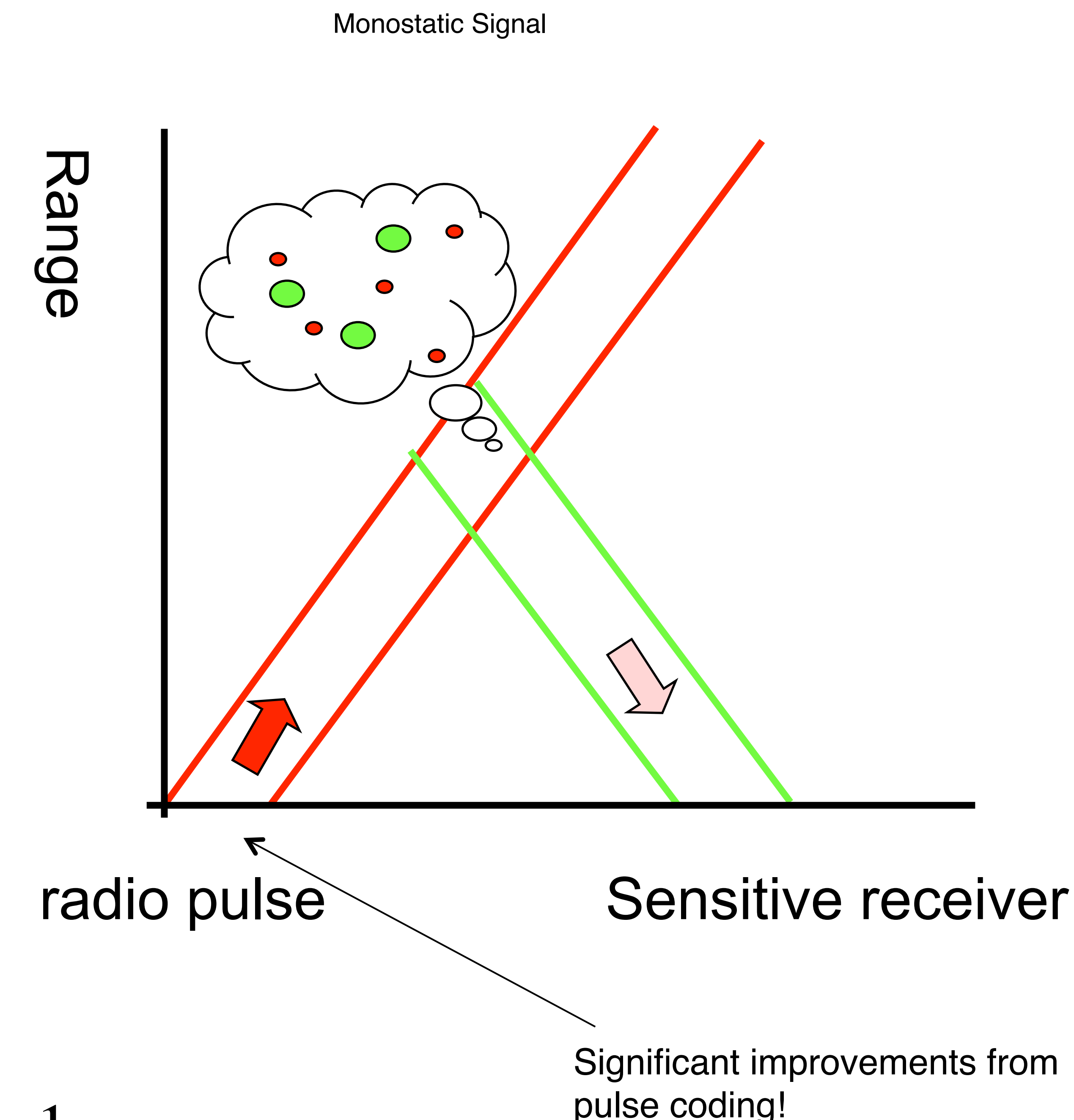


Shorter pulse ⟺ Larger bandwidth

Faster sampling rate ⟺ Larger bandwidth

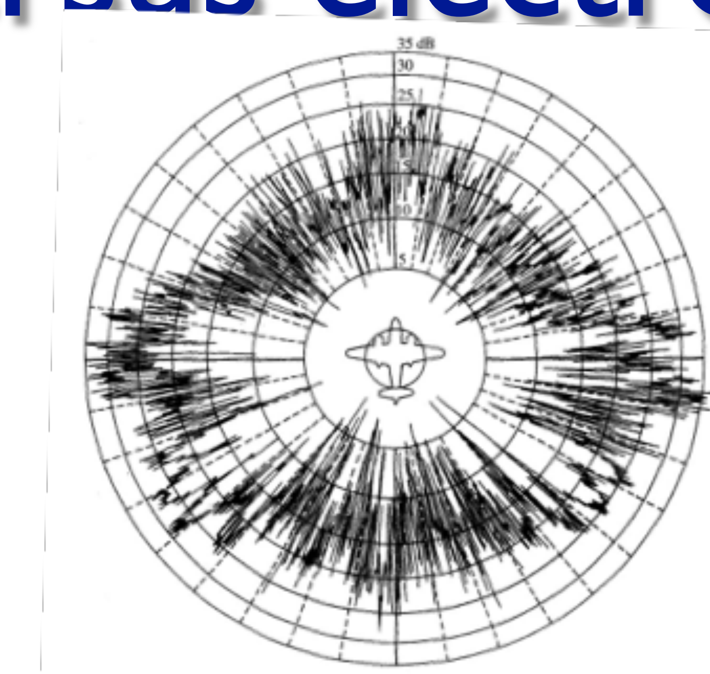
Radar Basics

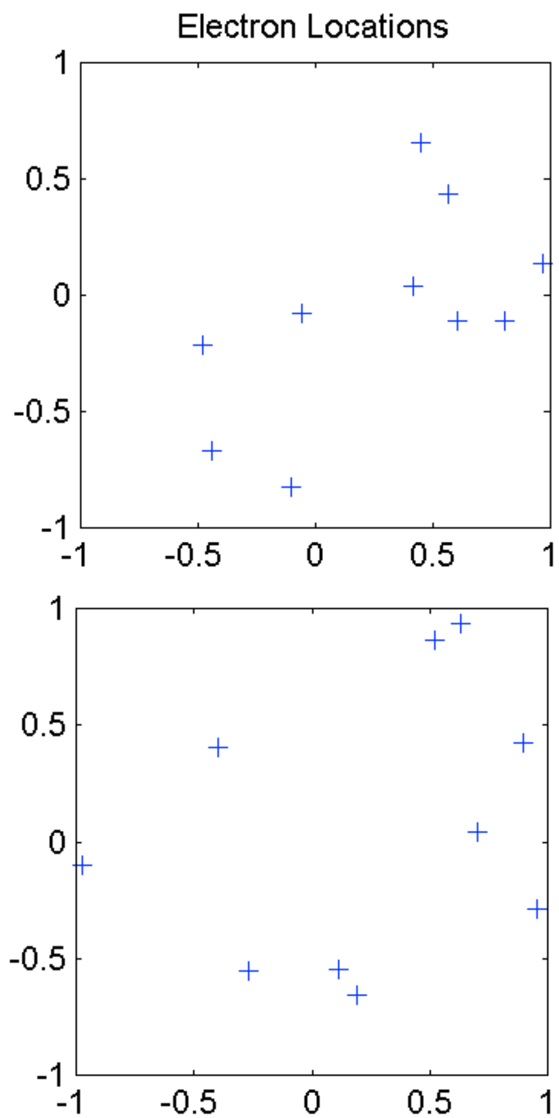


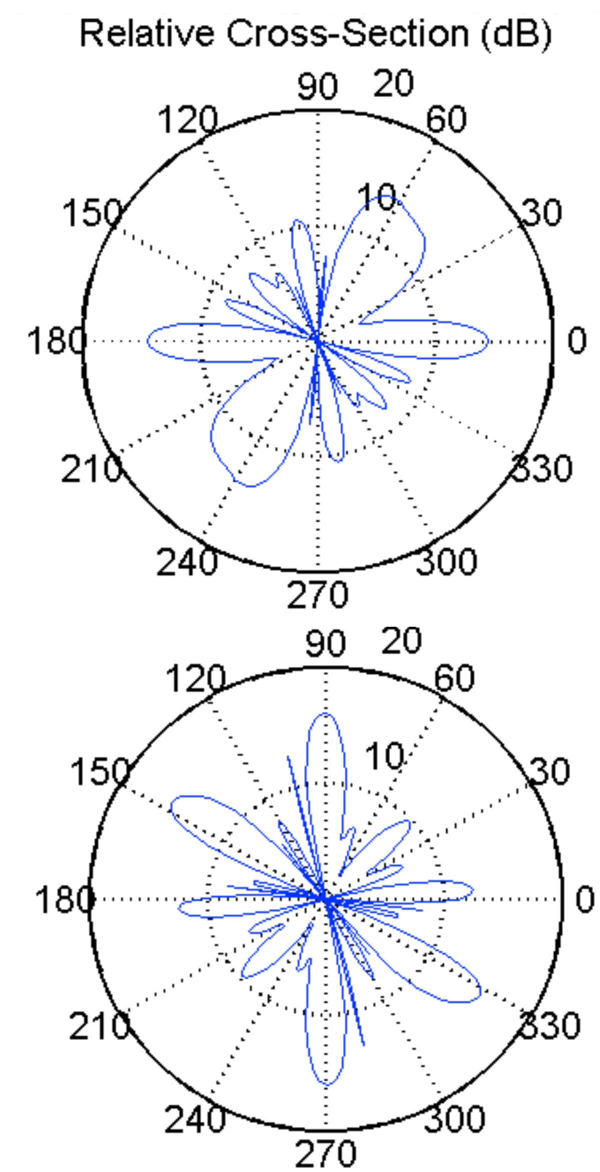


$$\mathrm{P}_r = P_t \frac{G_t(\theta,\phi)}{4\pi R^2}\sigma(\omega,\vec{k},\chi)\frac{1}{4\pi R^2}A_{eff}(\theta,\phi)$$

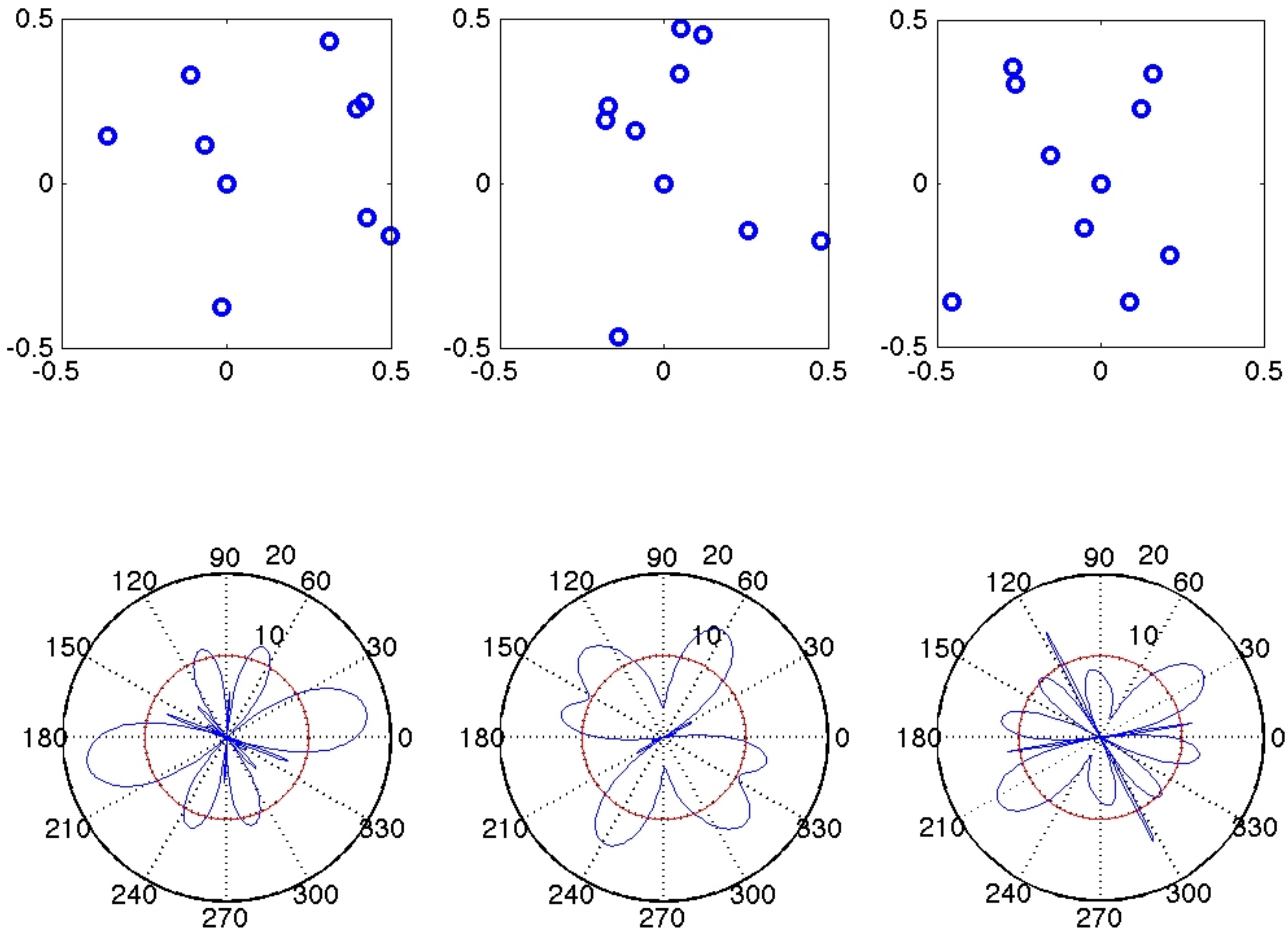
RCS: hard target versus electrons



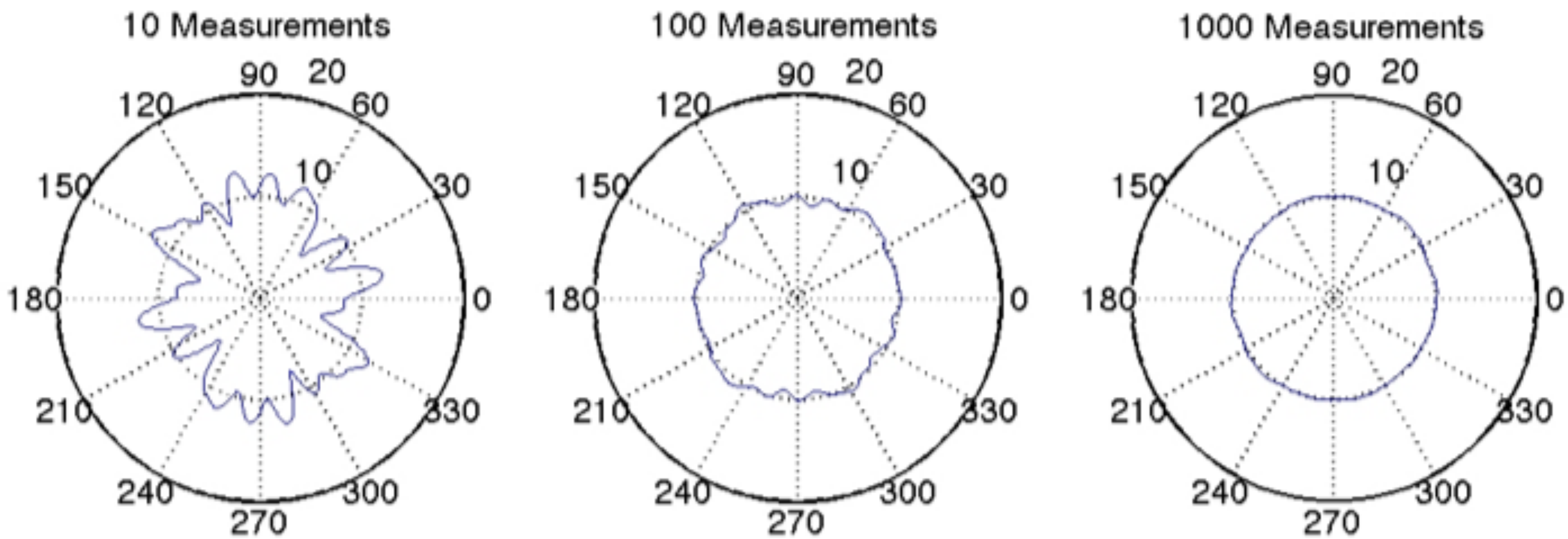




‘Incoherent’ electron positions



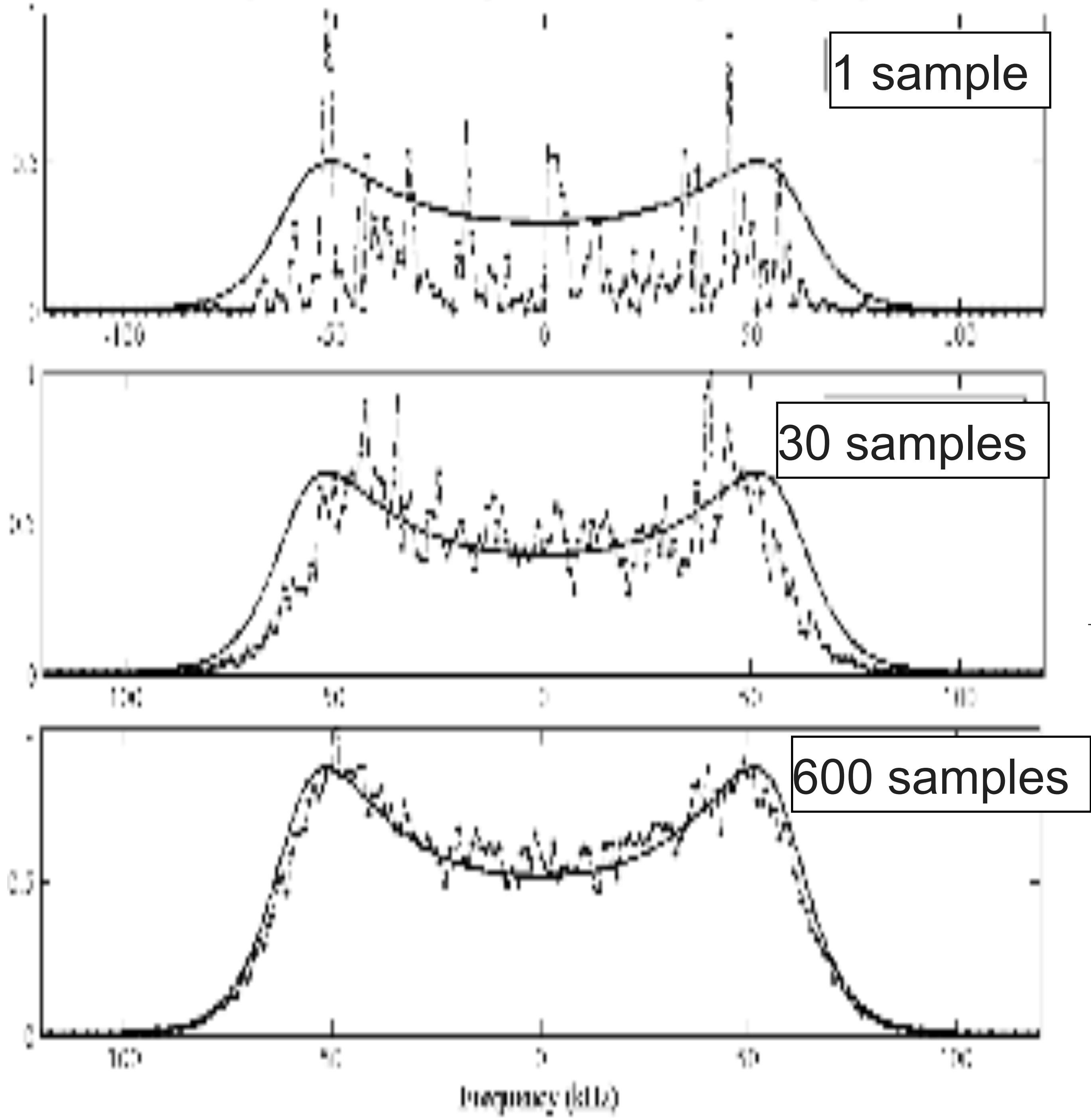
Incoherent integration



$$\text{Uncertainties} \propto \frac{1}{\sqrt{\text{Number of Samples}}}$$

Incoherent Averaging

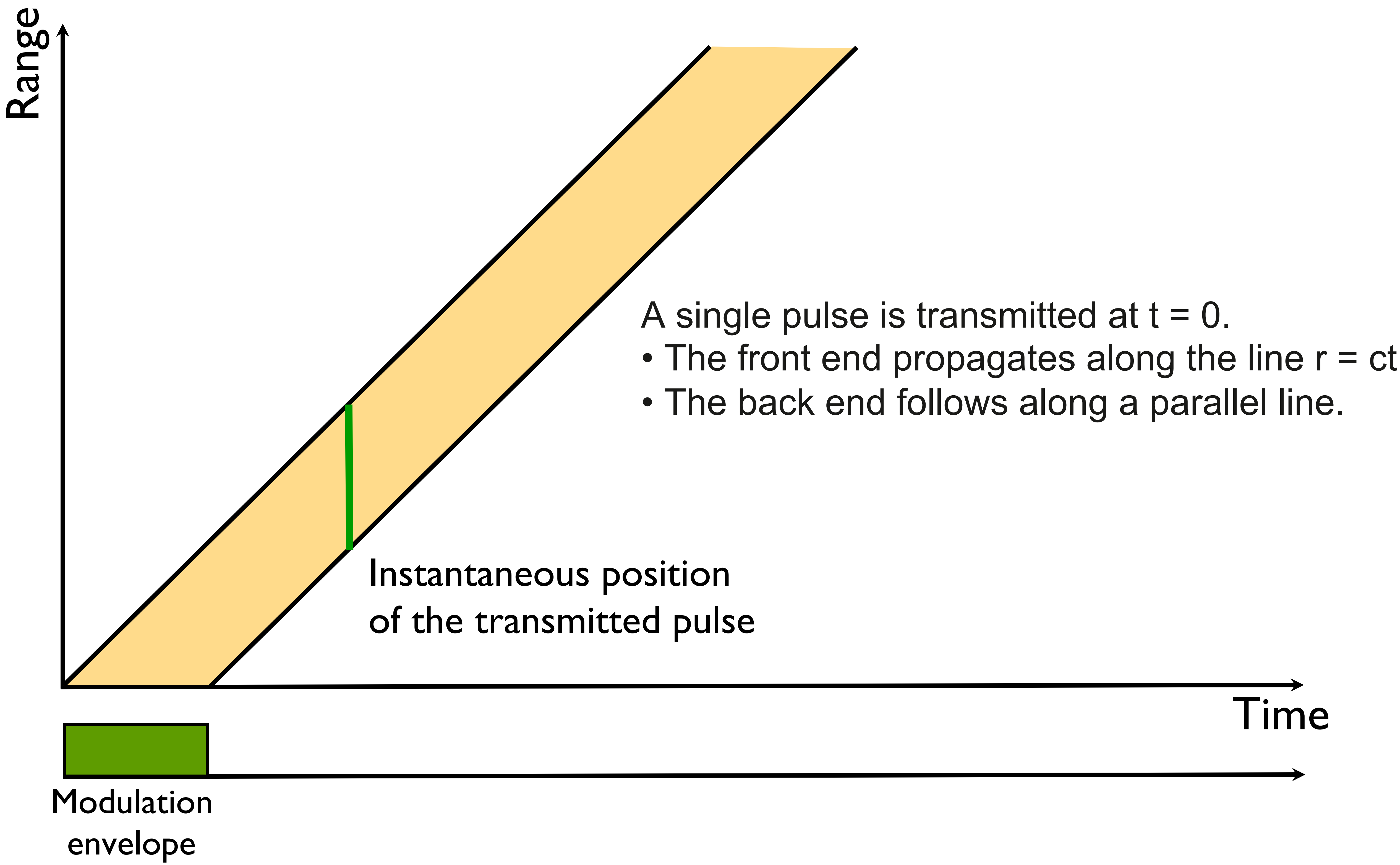
Normalized ISR spectrum for different integration times at 1290 MHz



We are seeking to estimate the power spectrum of a Gaussian random process. This requires that we sample and average many independent "realizations" of the process.

$$\text{Uncertainties} \propto \frac{1}{\sqrt{\text{Number of Samples}}}$$

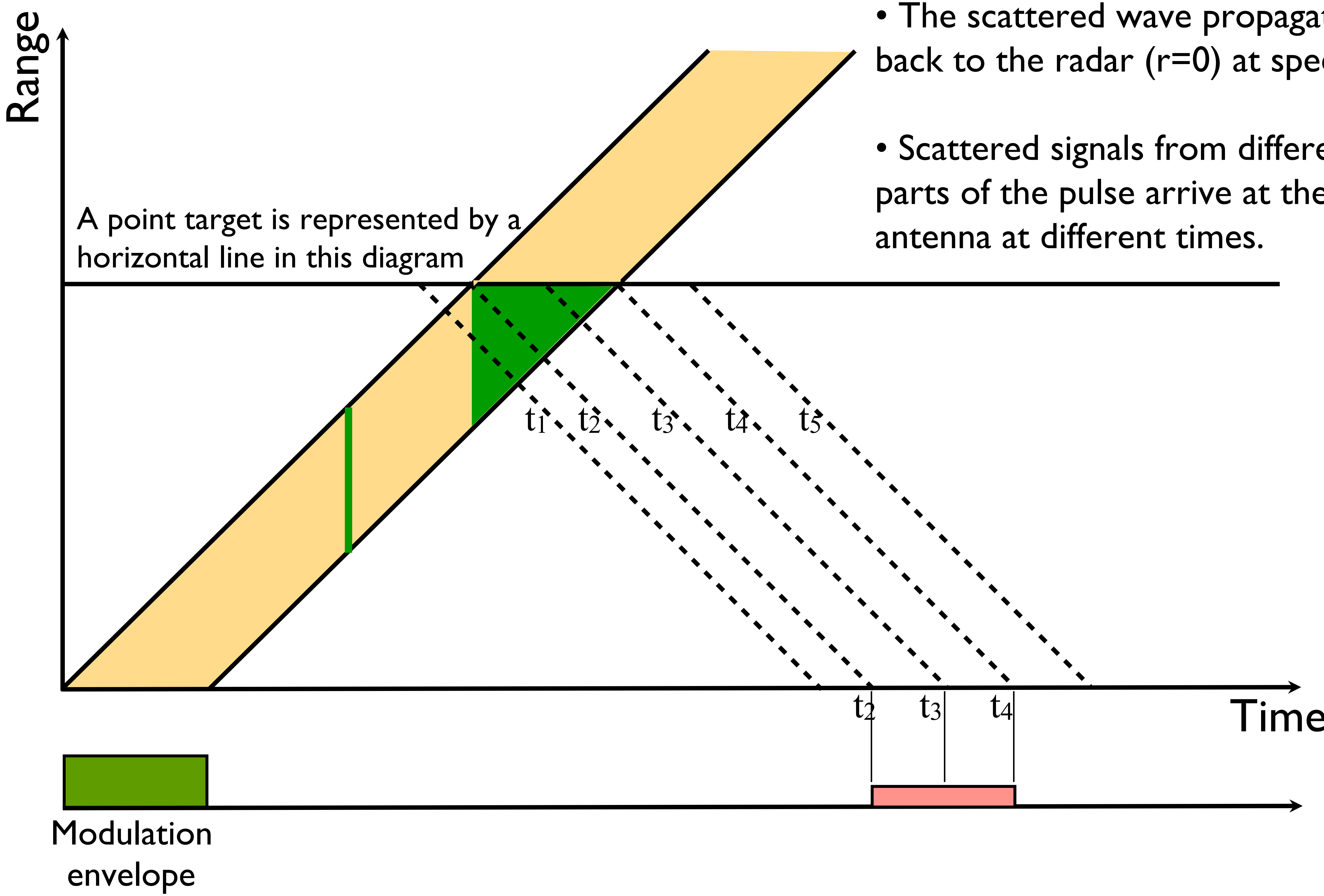
Range-time analysis



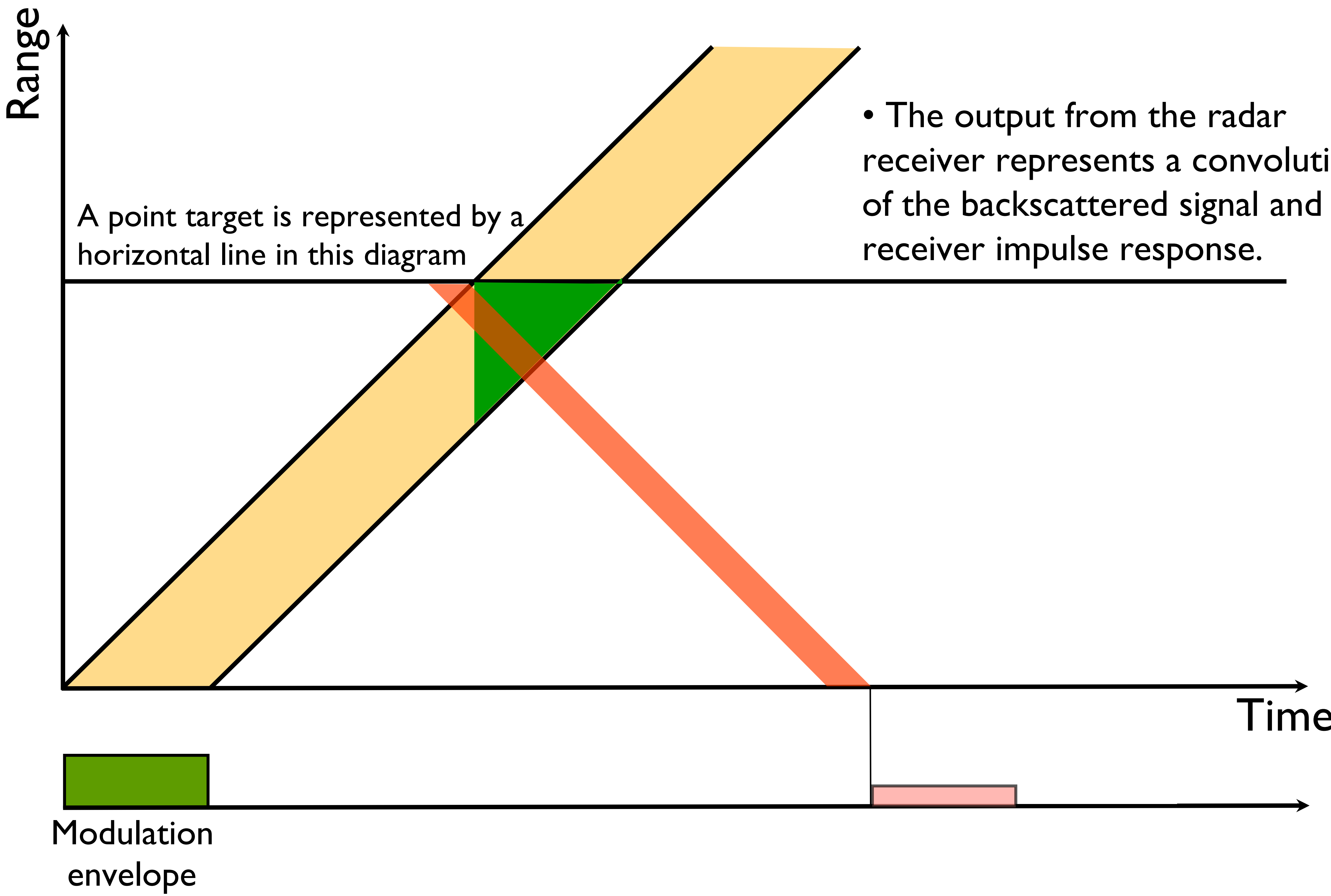
Range-time analysis

• The scattered wave propagates back to the radar (r=0) at speed c.

• Scattered signals from different parts of the pulse arrive at the antenna at different times.

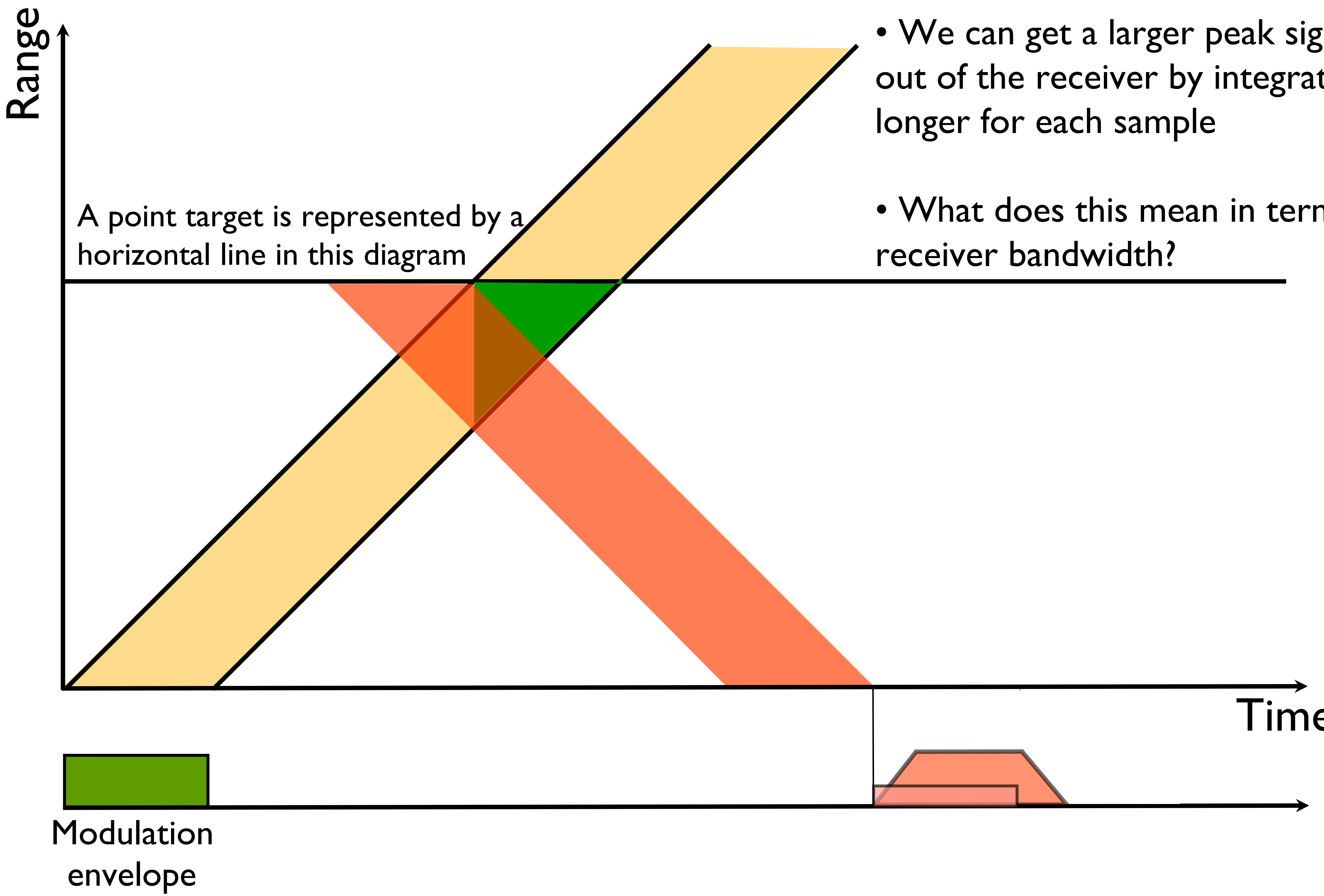


Sampling the received signal



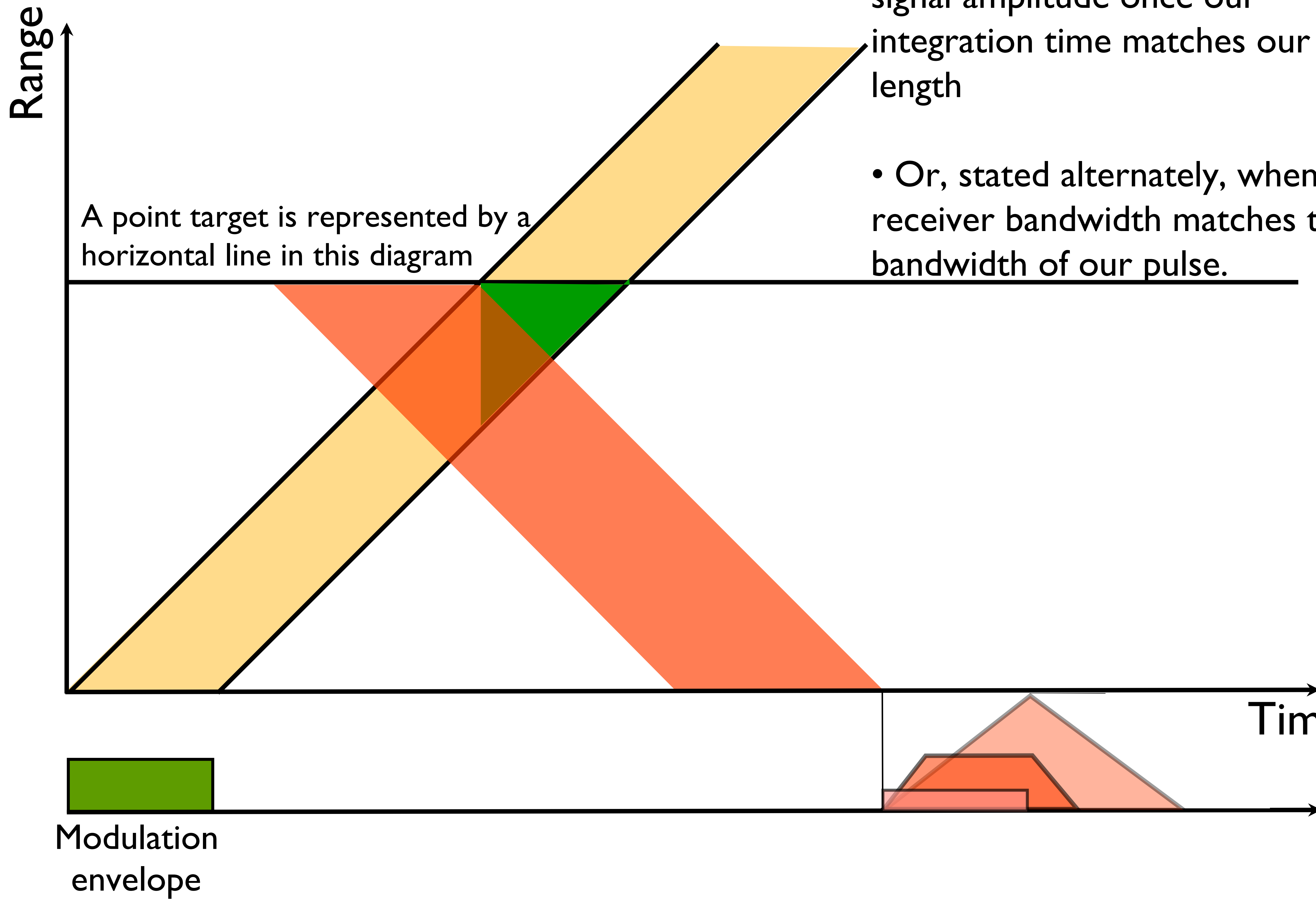
- The output from the radar receiver represents a convolution of the backscattered signal and the receiver impulse response.

Computing the ACF



- We can get a larger peak signal out of the receiver by integrating longer for each sample
- What does this mean in terms of receiver bandwidth?

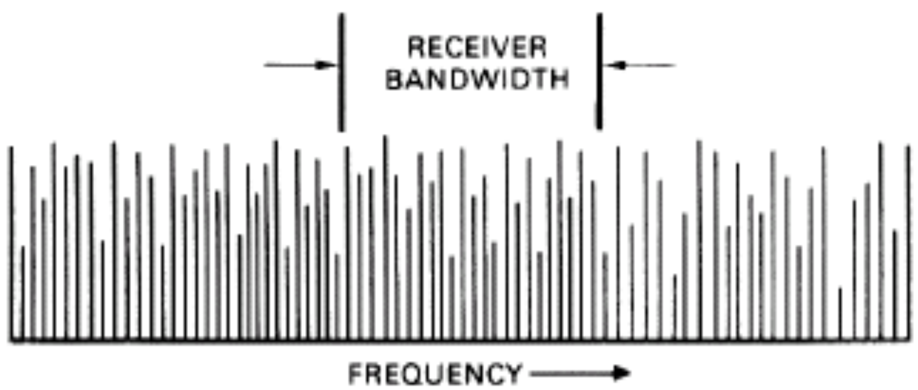
Computing the ACF



• We don't get any more gain in signal amplitude once our integration time matches our pulse length

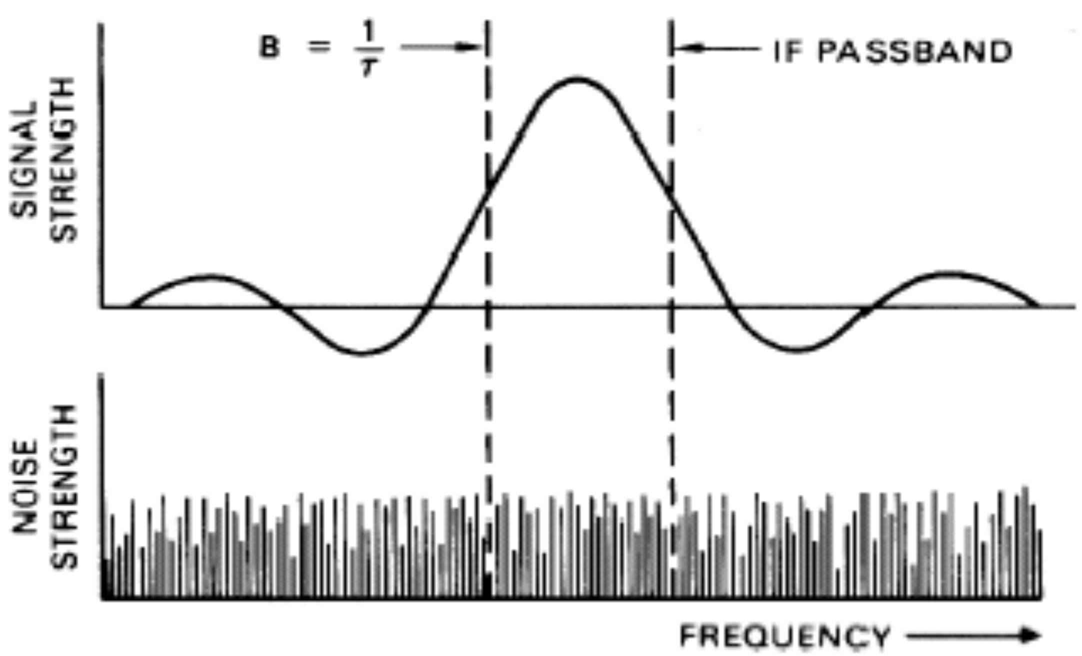
• Or, stated alternately, when our receiver bandwidth matches the bandwidth of our pulse.

The bandwidth-noise connection



6. Noise in receiver output is proportional to bandwidth of receiver.

The matched filter is a filter whose impulse response, or transfer function, is determined by a given signal, in a way that will result in the maximum attainable signal-to-noise ratio at the filter output when both the signal and white noise are passed through it.

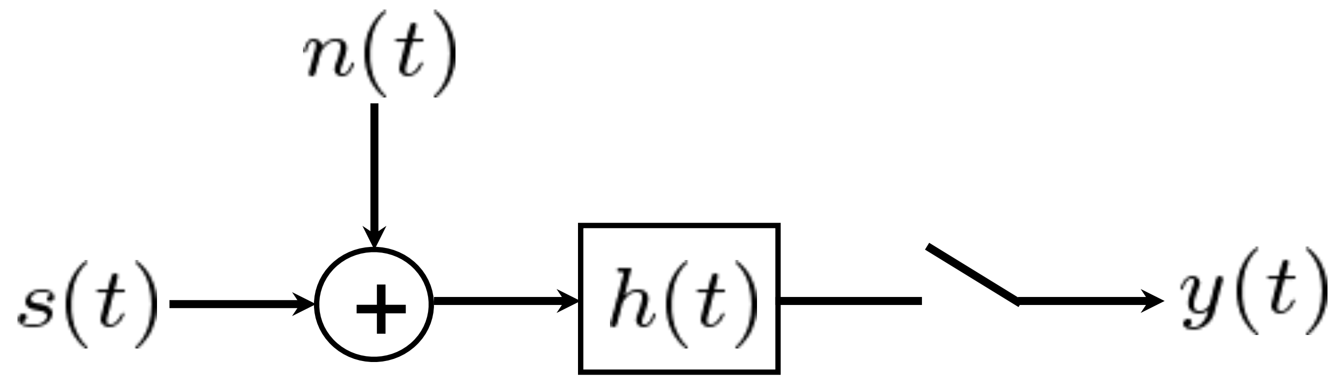


1. Signal-to-noise ratio may be maximized by narrowing the passband of the IF amplifier to the point where only the bulk of the signal energy is passed.

The optimum bandwidth of the filter, B, turns out to be very nearly equal to the inverse of the transmitted pulse width.

To improve range resolution, we can reduce τ (pulse width), but that means increasing the bandwidth of transmitted signal = More noise…

Matched Filter



$$y(t) = \int [s(\tau) + n(\tau)]\, h(t - \tau) d\tau$$
$$= \int H(f) S(f) e^{j2\pi f T} df + \int H(f) N(f) e^{j2\pi f T} df$$

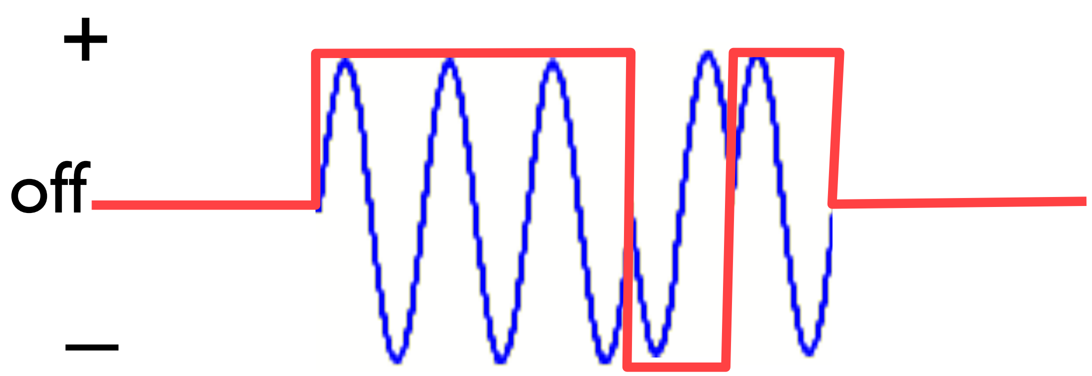
How should we choose *h(t)*⇔*H(f)* such that the output SNR is maximal?

$$SNR = \frac{\left| \int H(f) S(f) e^{j2\pi f T} df \right|^2}{E\left\{ \left| \int H(f) N(f) df \right|^2 \right\}}$$

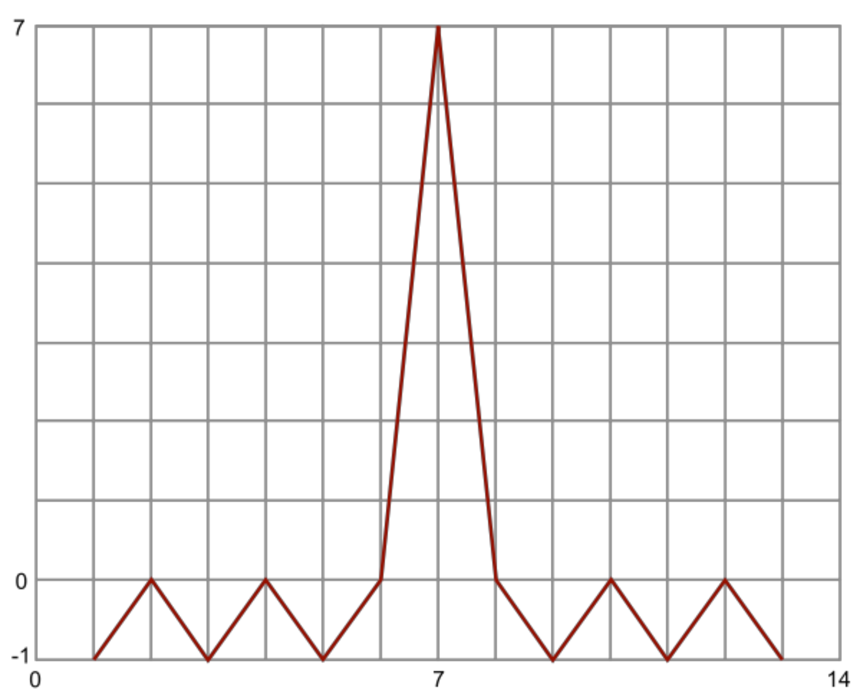
Assuming white Gaussian noise, it can be shown that max SNR is when

$$\boxed{H(f) = S^*(f) \Longleftrightarrow h(t) = s^*(-t)}$$

Barker codes



									correlator output
				+	+	+	-	+	correlator output
+	+	+	-	+					1
	+	+	+	-	+				-1+1=0
		+	+	+	-	+			1-1+1=1
			+	+	+	-	+		1+1-1-1=0
				+	+	+	-	+	1+1+1+1+1=5

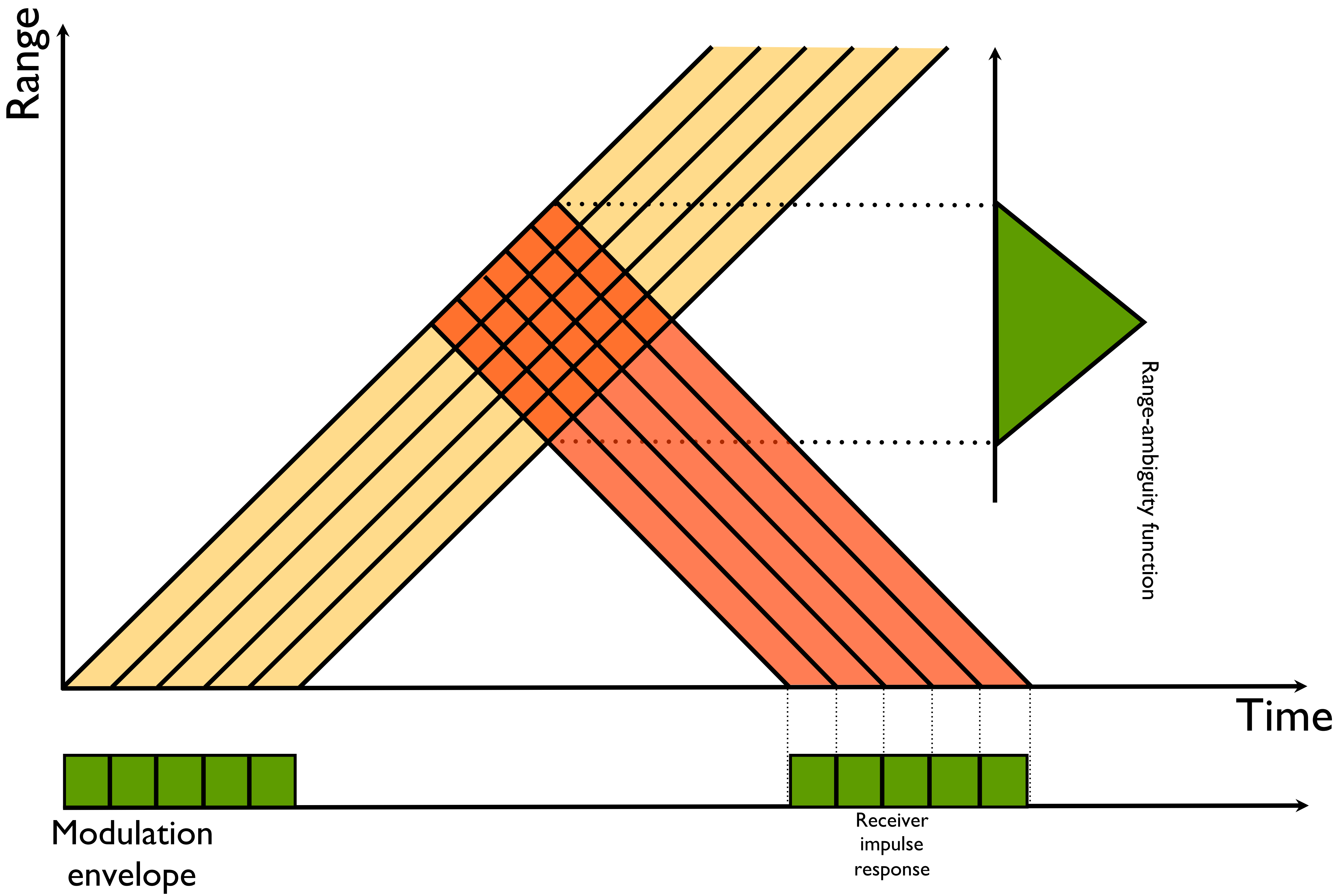


Autocorrelation function of Barker-7 code

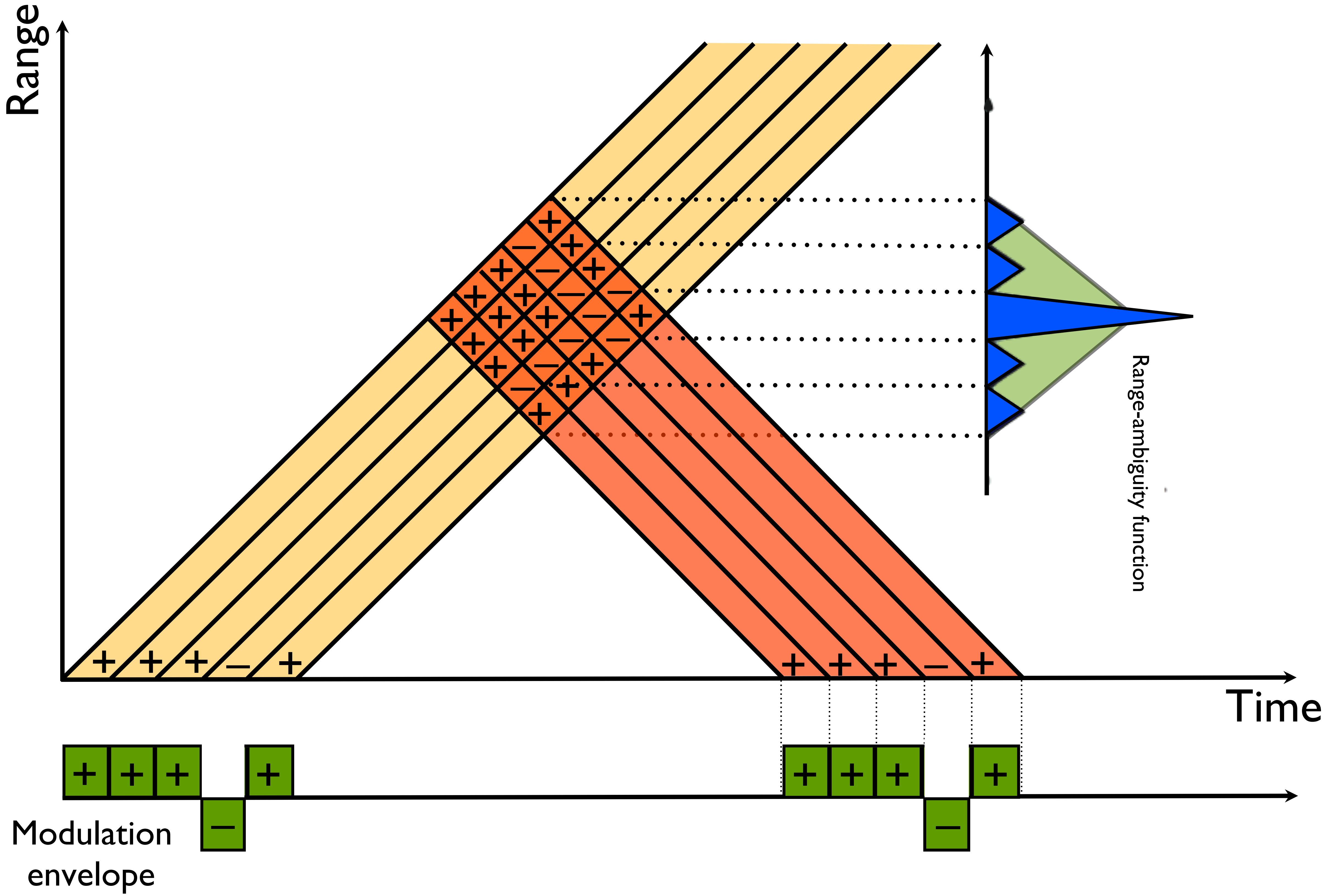
TABLE 6.2 All Known Binary Barker Codes

Code Length	Code
2	11 or 10
3	110
4	1110 or 1101
5	11101
7	1110010
11	11100010010
13	1111100110101

Volume target (e.g., the ionosphere)



Volume target (e.g., the ionosphere)



Ambiguity Functions

- A standard way to compare different pulse coding strategies
- Based on the principle of a 'matched filter'
  - Output of the matched filter maximizes the attainable SNR when both signal and white noise are applied to the input
  - Impulse response of the matched filter is the complex conjugate of the time-reversed version of the signal

$$h(t) = s^*(t_M - t)$$

$$H(f) = S^*(f)\exp(-j2\pi f t_M)$$

where

$h(t)$ is the impulse response of the matched filter

$s(t)$ is the signal to be detected

t_M is the measurement time

t, f are time and frequency

Ambiguity Functions

- The ambiguity function is defined as the absolute value of the envelope of the output of a matched filter when the input to the filter is a Doppler shifted version of the original signal

$$|X(\tau, f)| = \left| \int_{-\infty}^{\infty} u(t) u^*(t-\tau) \exp(j2\pi f t) dt \right|$$

$u(t)$ is the complex envelope of the signal

τ is the additional delay

f is the frequency shift (Doppler)

Ambiguity Functions

For $u(t)$ with unit energy

$$|X(\tau, f)| \leq |X(0,0)| = 1$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|X(\tau, f)|^2 d\tau df = 1$$

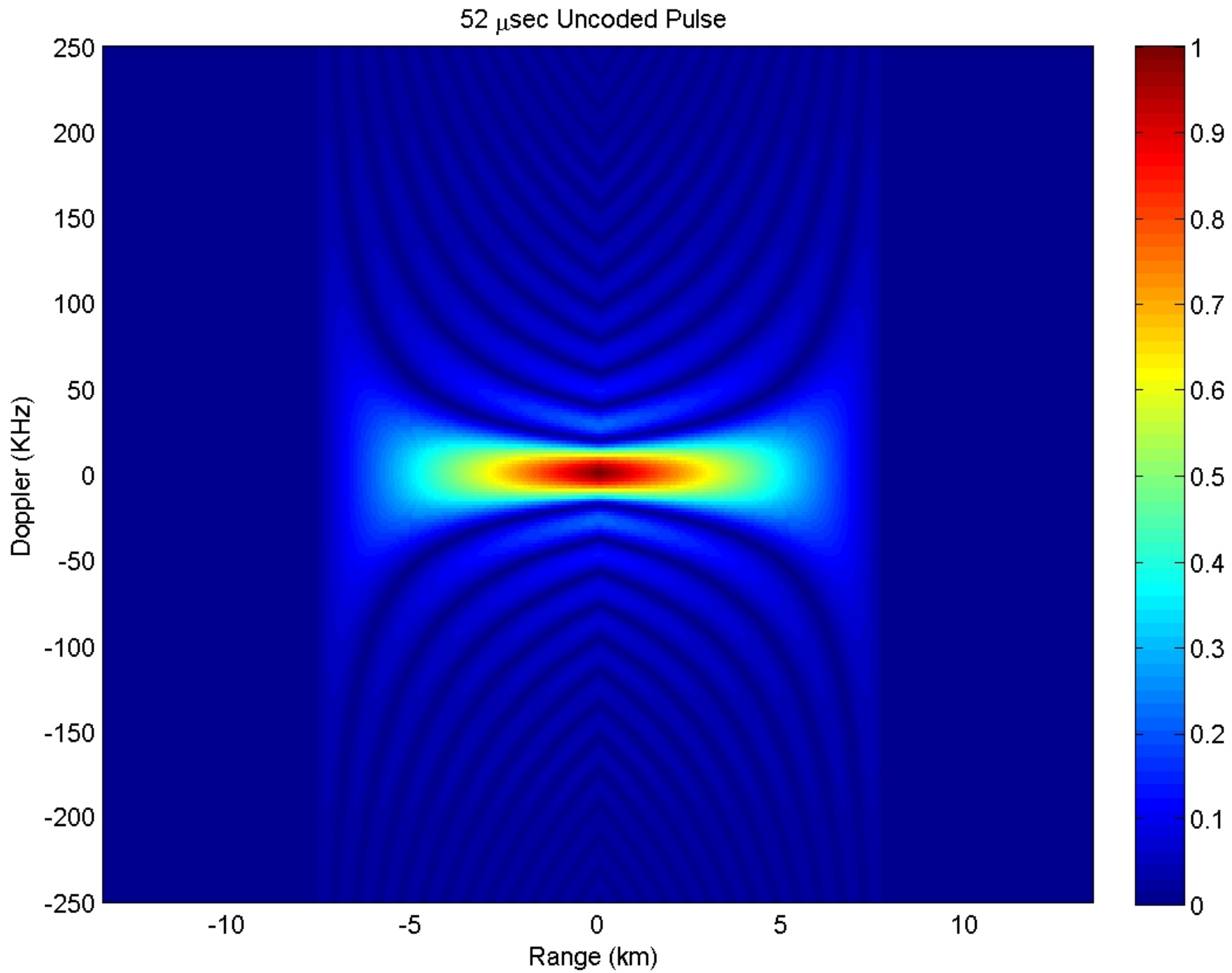
and for all signals

$$|X(-\tau, -f)| = |X(\tau, f)|$$

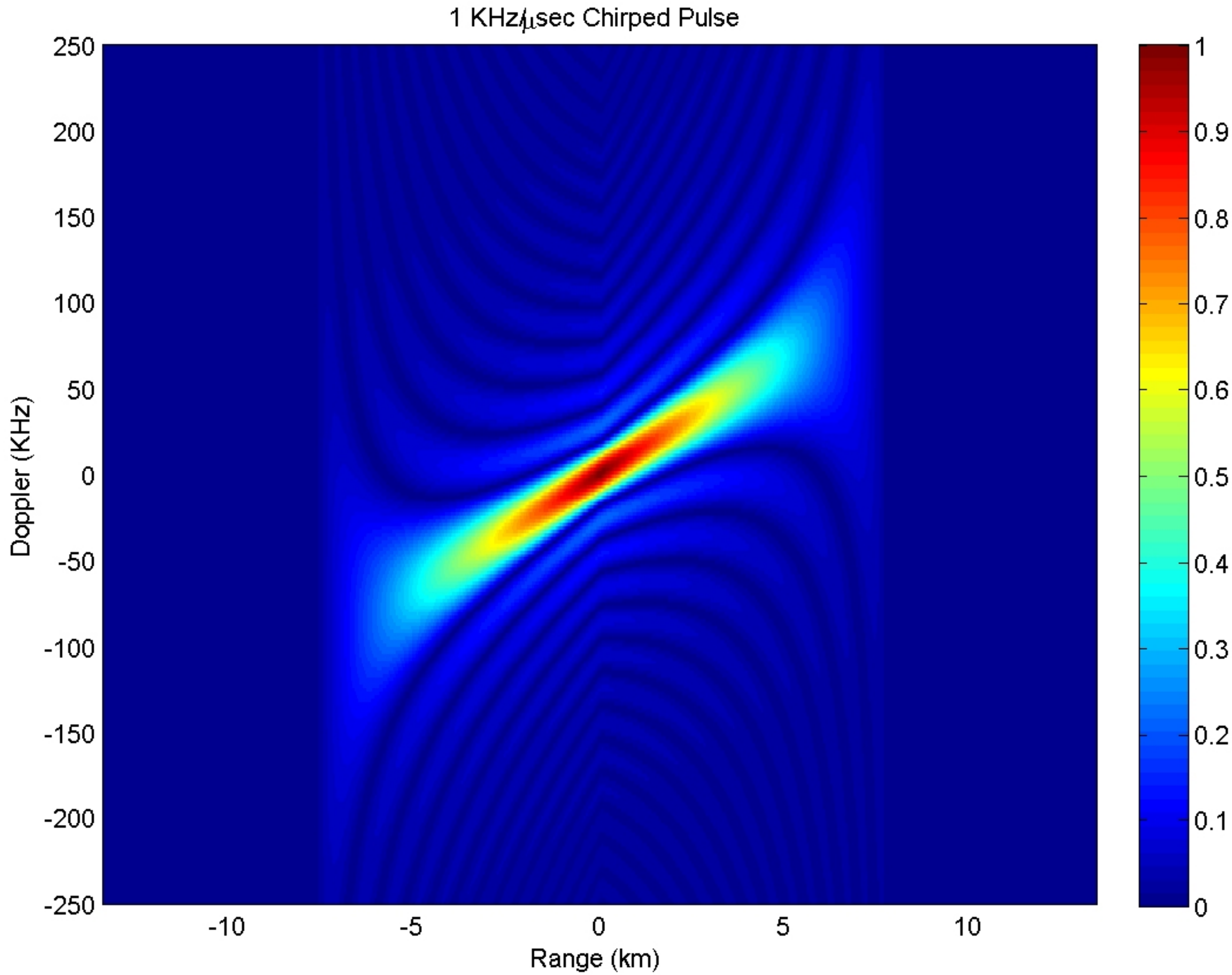
$$\text{if } u(t) \leftrightarrow |X(\tau, f)|$$

$$\text{then } u(t)\exp(j\pi kt^2) \leftrightarrow |X(\tau, f + k\tau)|$$

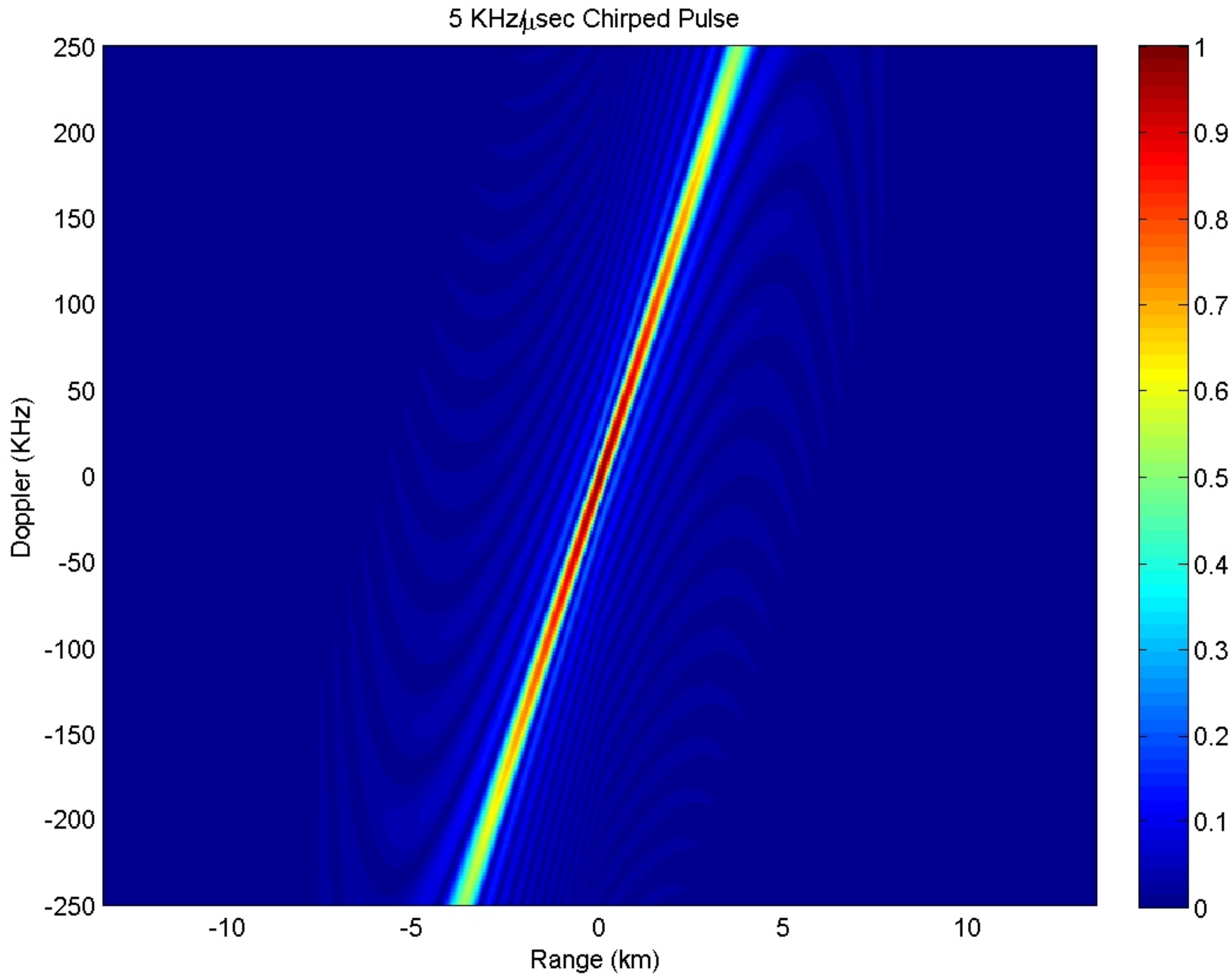
Ambiguity Function



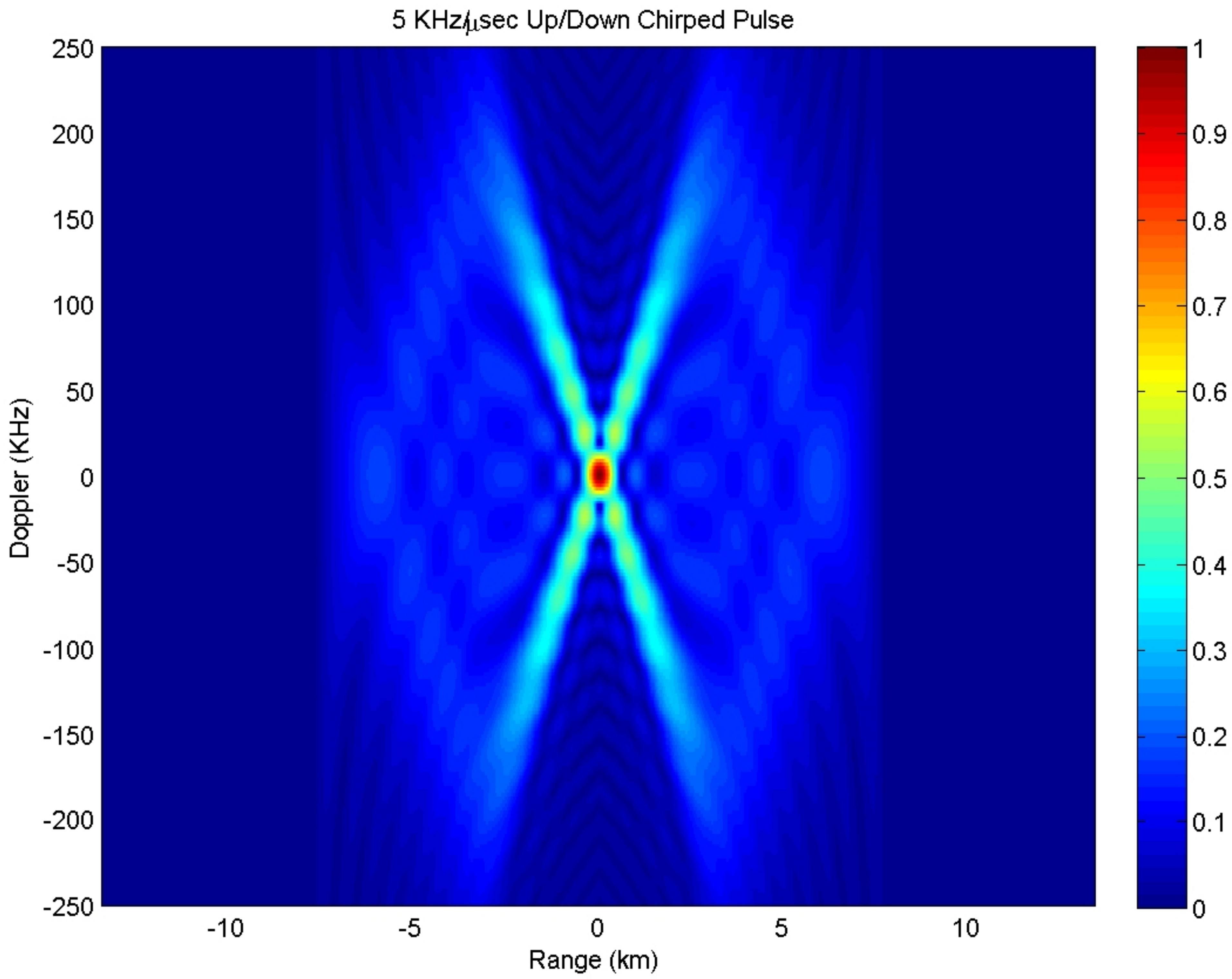
Ambiguity Function



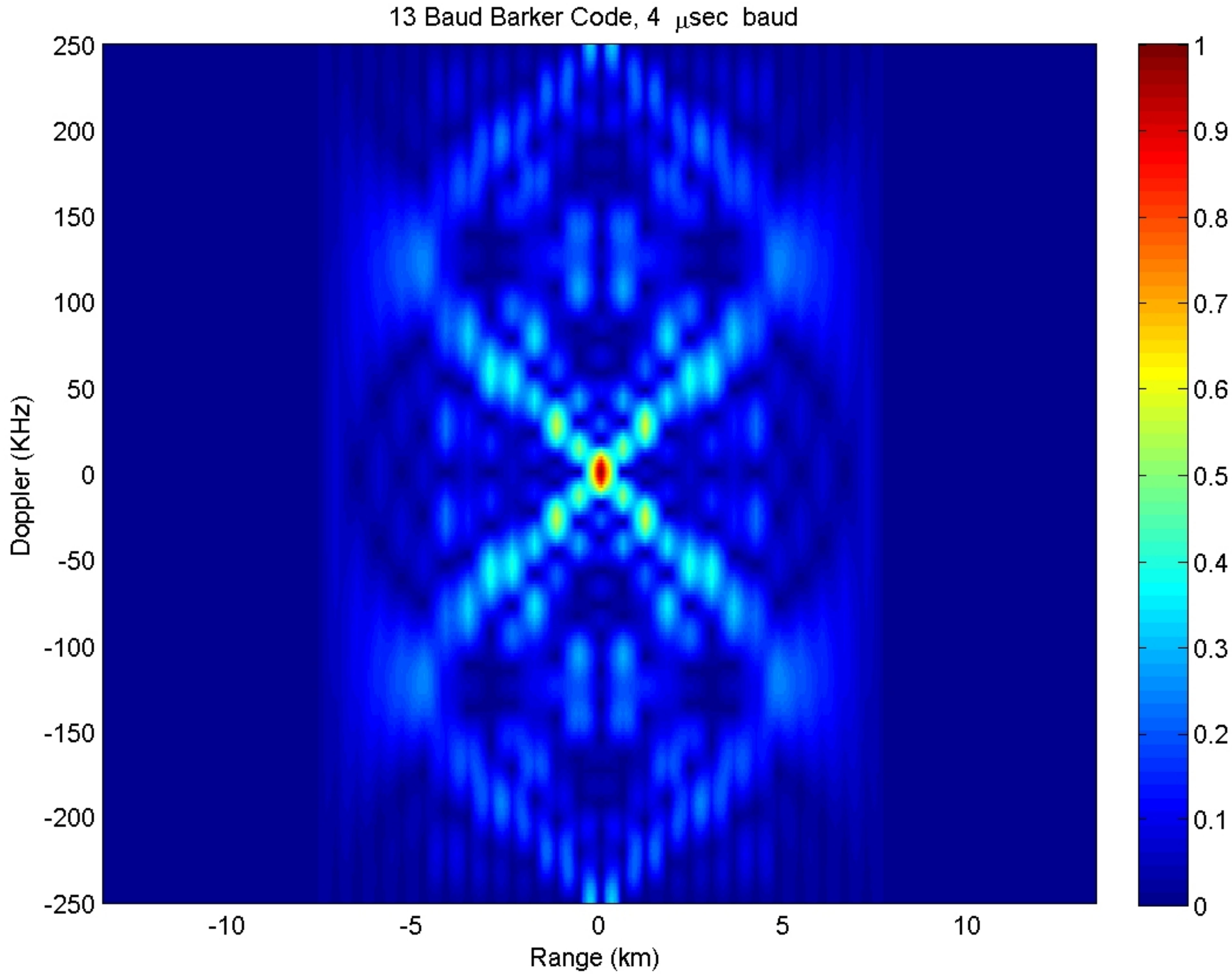
Ambiguity Function



Ambiguity Function



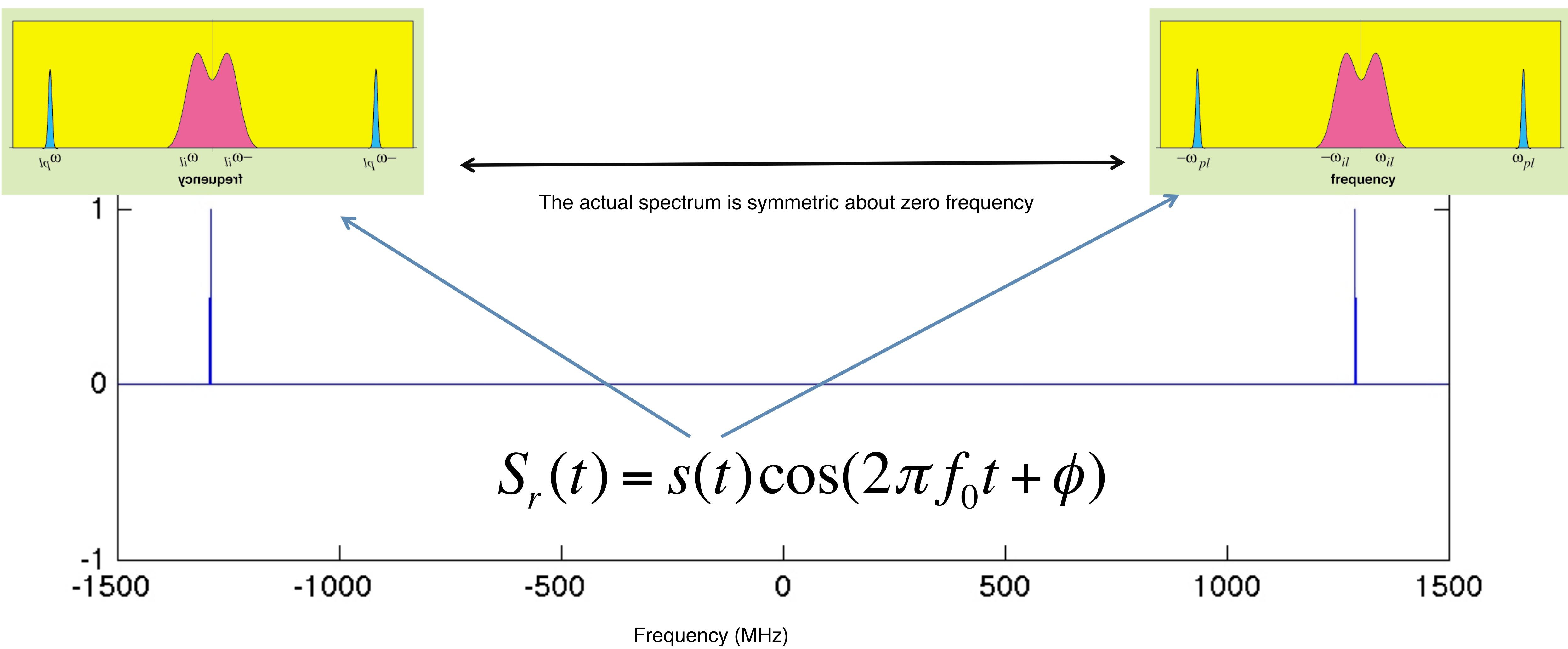
Ambiguity Function



Received signal?

Expected value (statistically speaking) of the Power Spectral Density of the received signal!

The individual sides are **not** symmetric about their center frequencies!

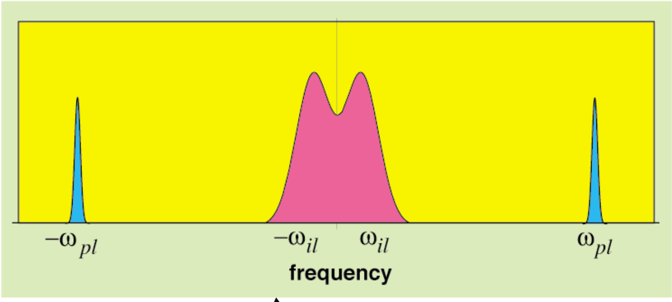


$$S_r(t) = s(t)\cos(2\pi f_0 t + \phi)$$

Received signal?

So, can we just sample the received signal and pick this out in software? I can mix and filter, after all! Well, we need to sample pretty fast to do it this way (the Nyquist frequency would be GHz at Sondrestrom).
Instead, we normally mix it down to an intermediate frequency and sample that. Then the final mixing to baseband (real and imaginary) is done digitally.

$$S_r(t) = s(t)\cos(2\pi f_0 t + \phi)$$



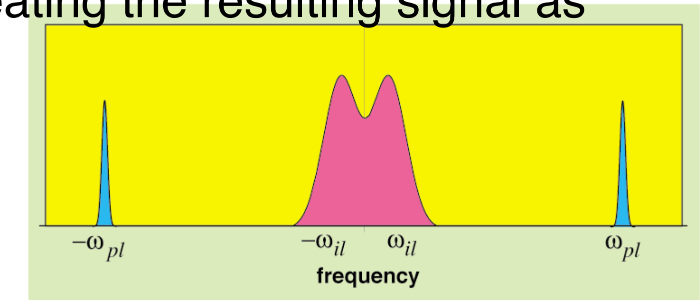
$$S_r(t)\cos(2\pi f_1 t) = s(t)\frac{1}{2}\left[\cos(2\pi(f_0 - f_1)t + \phi) + \cos(2\pi(f_0 + f_1)t + \phi)\right]$$

The signal represented here must be complex because it will not, in general, be symmetric.

Intermediate frequency

Received signal?

Given that the signal we are interested in must be complex, how do we get such a measurement? What does that mean?
We look at shifting the signal all the way down to center it on the interesting bit. This means mixing with the carrier frequency.
This has to be done in a way that maintains both the cosine (in-phase, real) and sine (quadrature, imaginary) components.
We also have to keep from shifting the positive-frequency part of s(t) on top of the negative frequency part of s(t).
This, it turns out, can be handled by multiplying the signal by both cosine and sine and treating the resulting signal as real and imaginary parts.



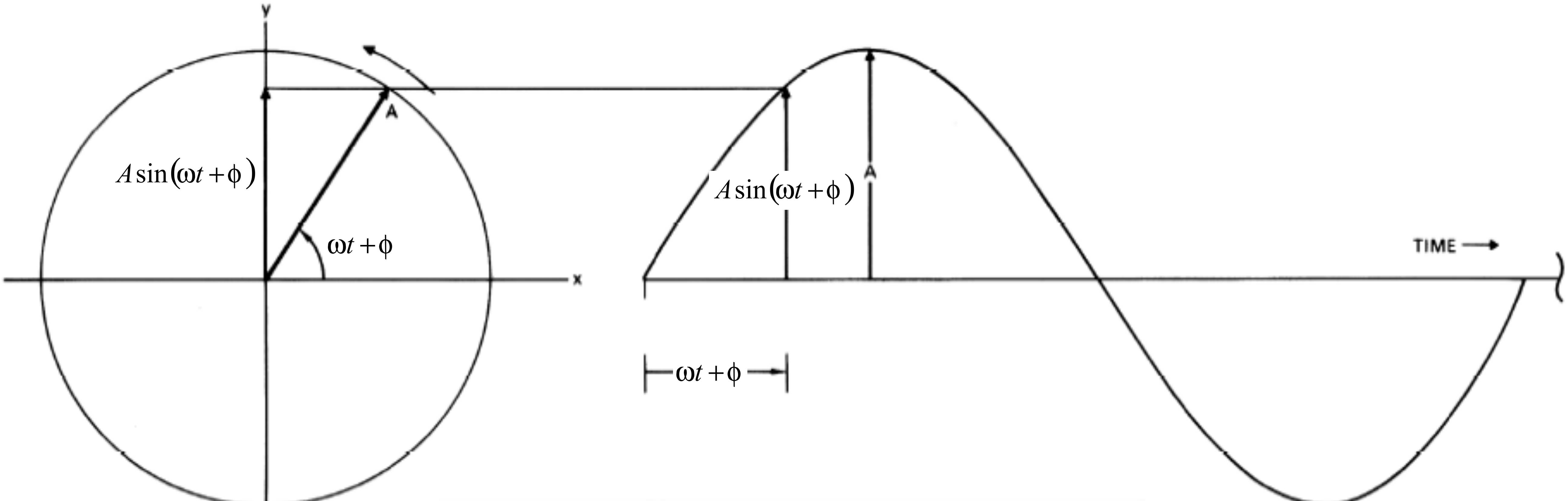
$$S_r(t) = s(t)\cos(2\pi f_0 t + \phi)$$

Real $$I = S_r(t)\cos(2\pi f_1 t) = s(t)\frac{1}{2}\left[\cos(\phi) + \cos(2\pi(f_0 + f_1)t + \phi)\right]$$

Imag $$Q = S_r(t)\sin(2\pi f_1 t) = s(t)\frac{1}{2}\left[-\sin(\phi) + \sin(2\pi(f_0 + f_1)t + \phi)\right]$$

Doppler Detection: Intuitive Approach

Phasor diagram is a graphical representation of a sine wave



<u>I & Q components*</u>

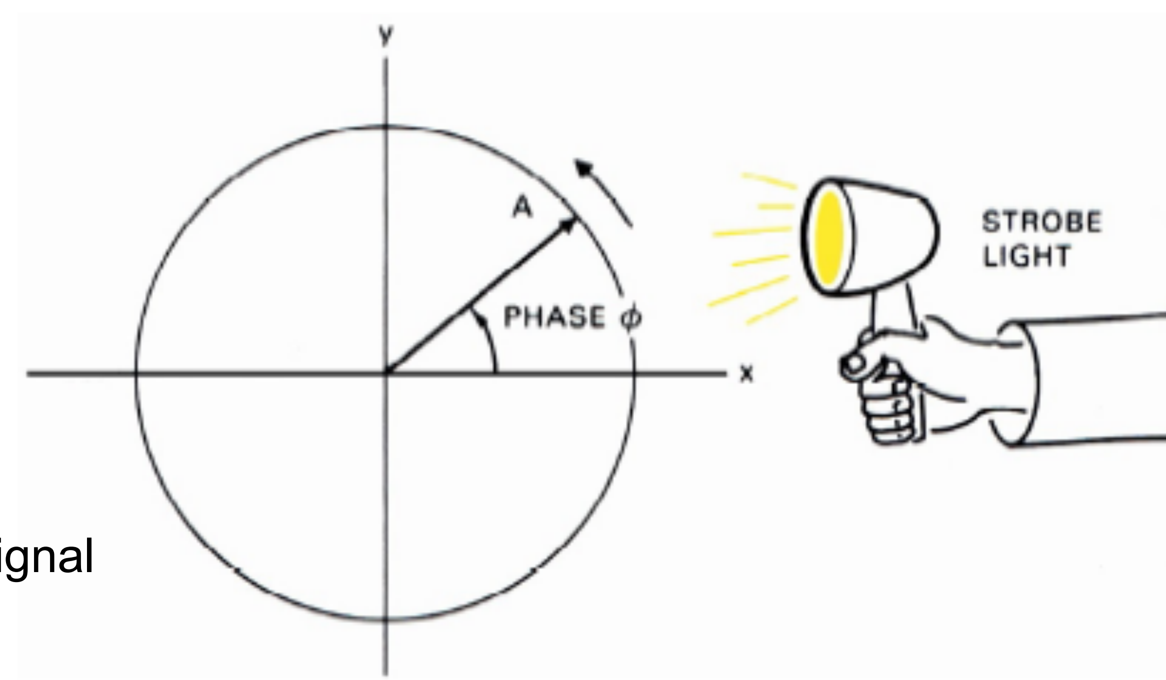
I => in-phase component

Acos(ϕ)

Q => in-quadrature component

Asin(ϕ)

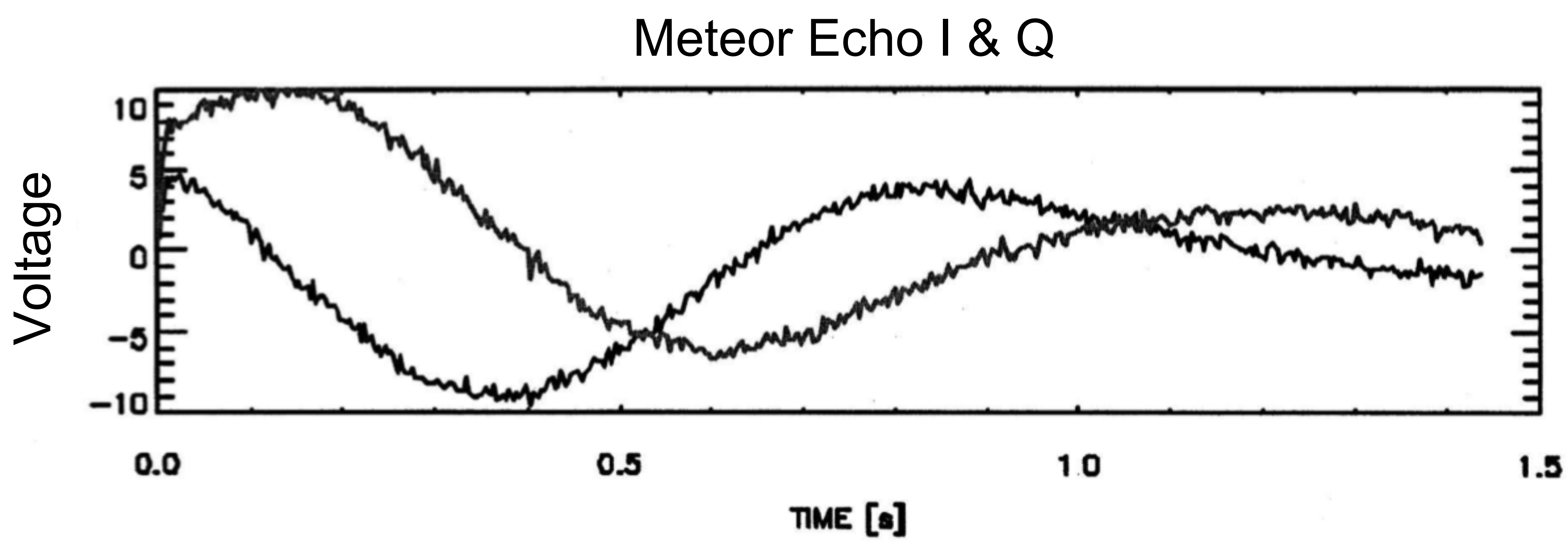
*relative to reference signal

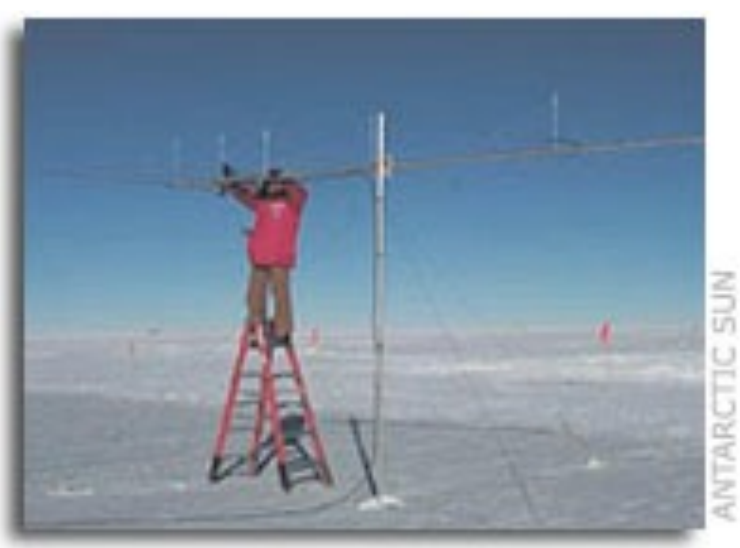


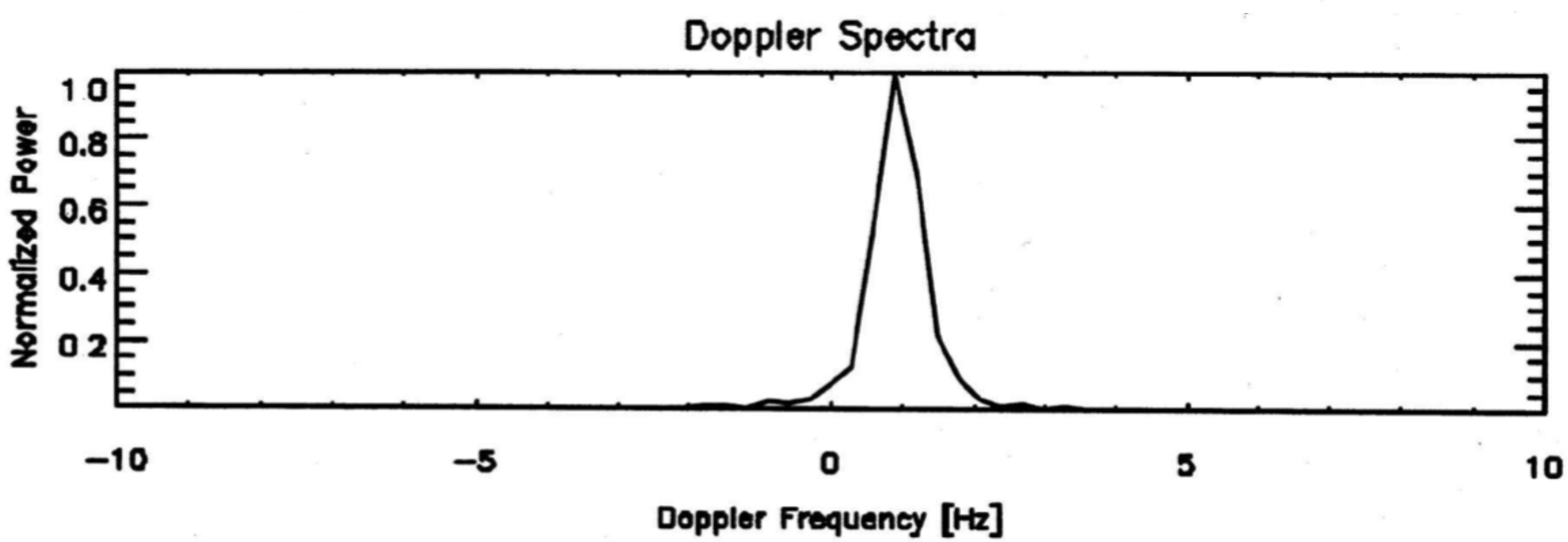
Consider strobe light as cosine reference wave at same frequency but with initial phase = 0

Example: Doppler Shift of a Meteor Trail

- Collect N samples of $I(t_k)$ and $Q(t_k)$ from a target
- Compute the complex FFT of $I(t_k)+jQ(t_k)$, and find the maximum in the frequency domain
- Or compute “phase slope” in time domain.





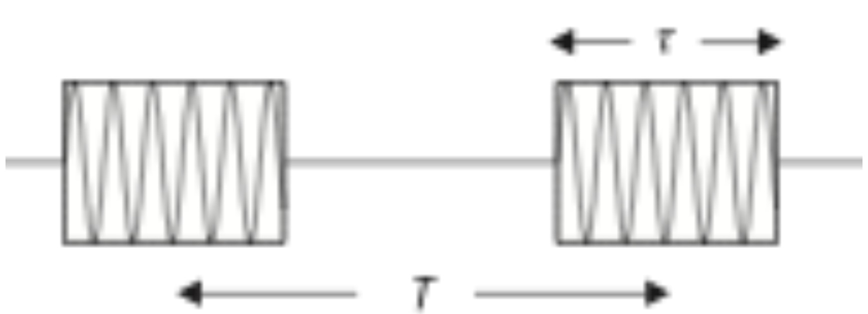


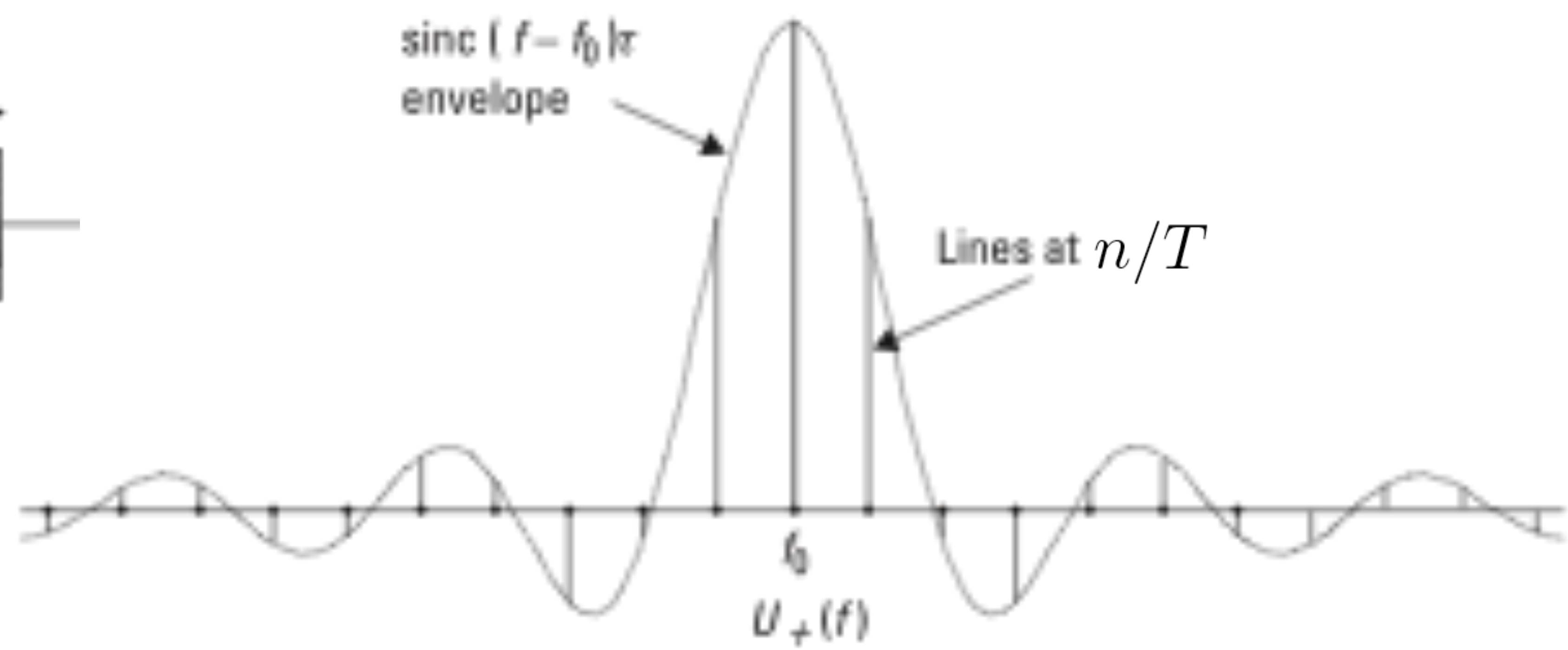
Does this strategy work for ISR?

Typical ion-acoustic velocity: 3 km/s
Doppler shift at 450 MHz: 10kHz
Correlation time: 1/10kHz = 0.1 ms
Required PRF to probe ionosphere (500km range): 300 Hz

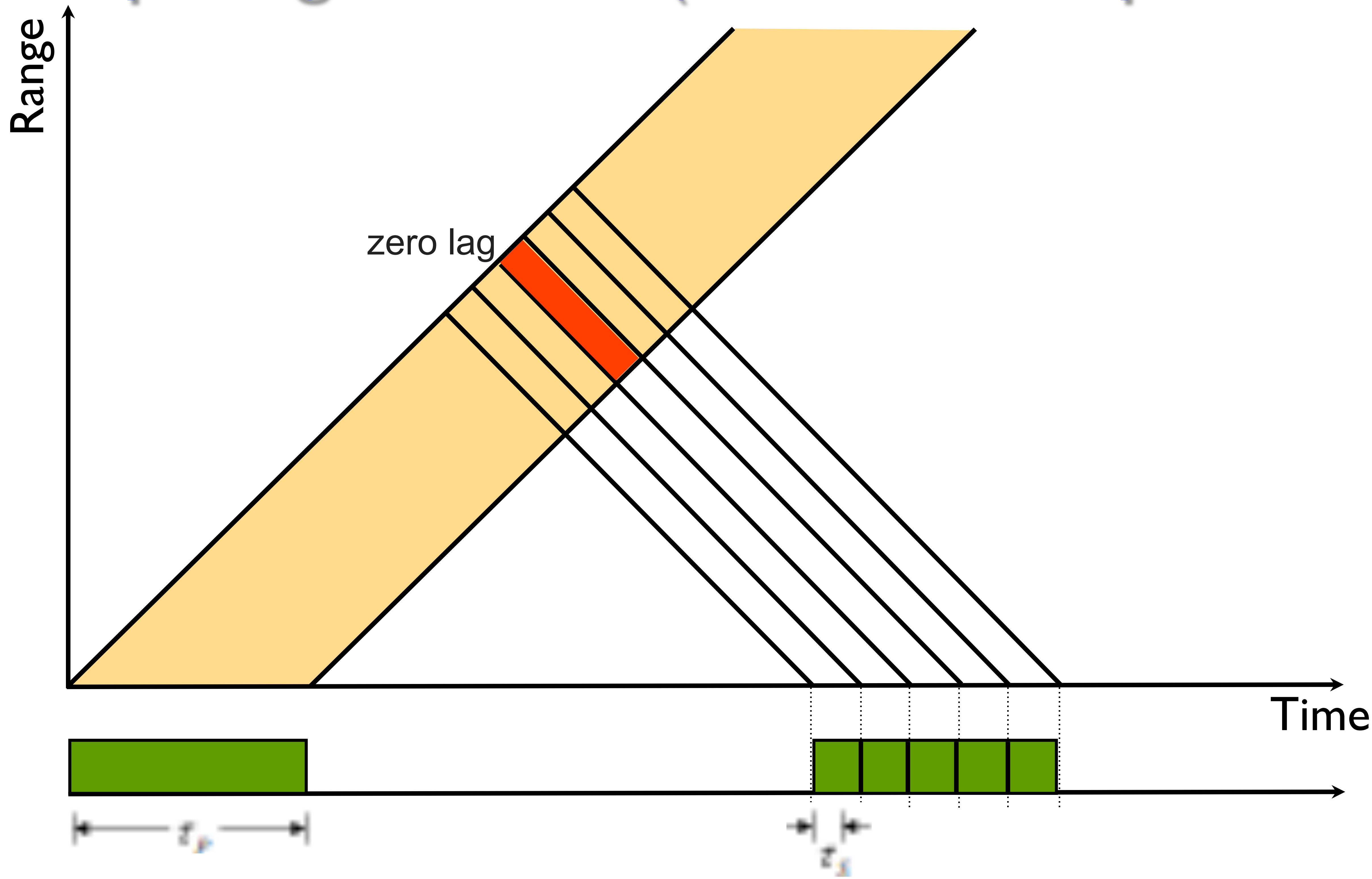
Plasma has completely decorrelated by the time we send the next pulse.

Alternately, the Doppler shift is well beyond the max unambiguous Doppler defined by the Inter-Pulse Period *T*.





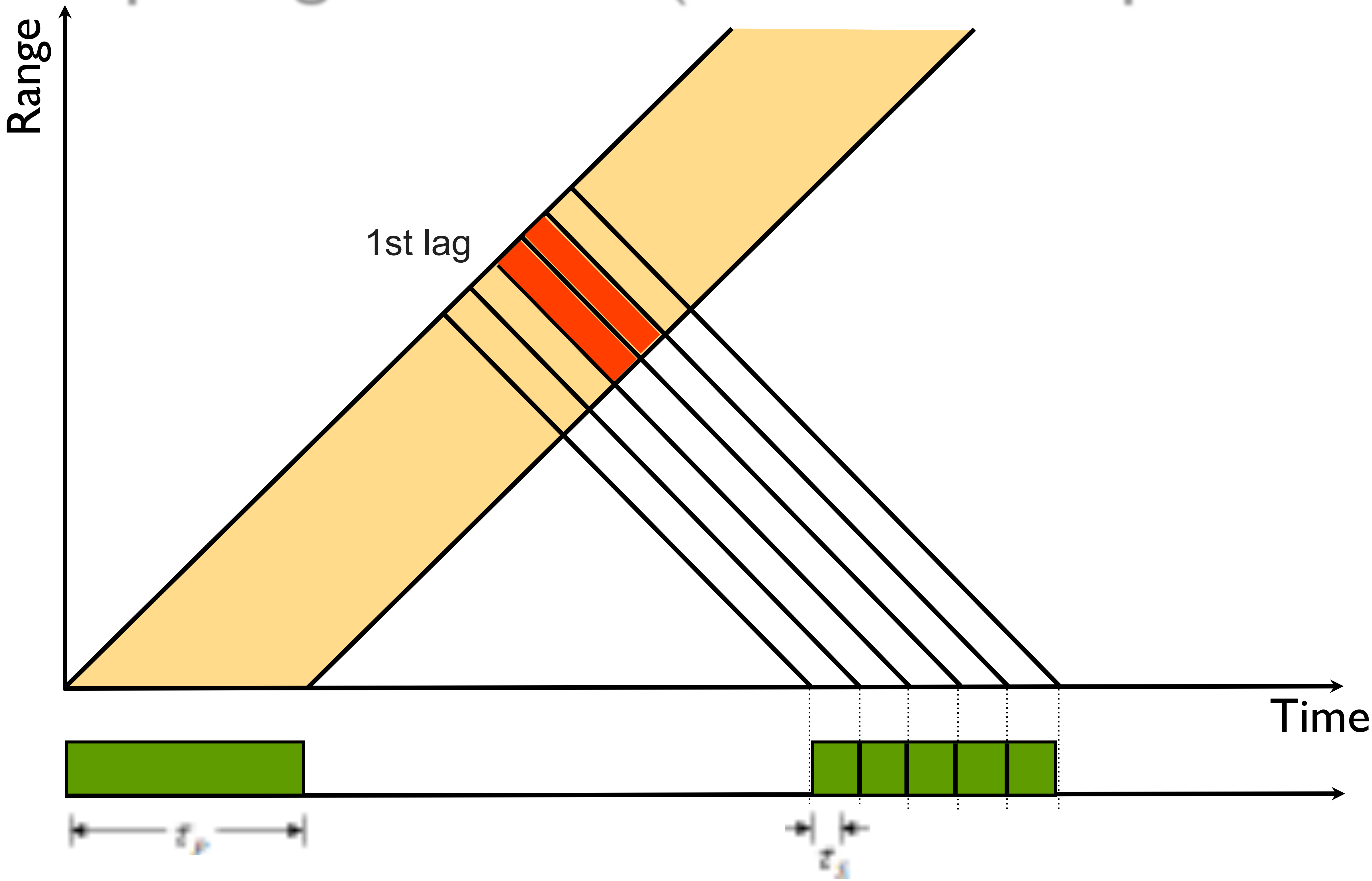
Computing the ACF (and, hence, spectrum)



τ_p = Length of RF pulse

τ_s = Sample Period (typically ~ 1/10 pulse length)

Computing the ACF (and, hence, spectrum)



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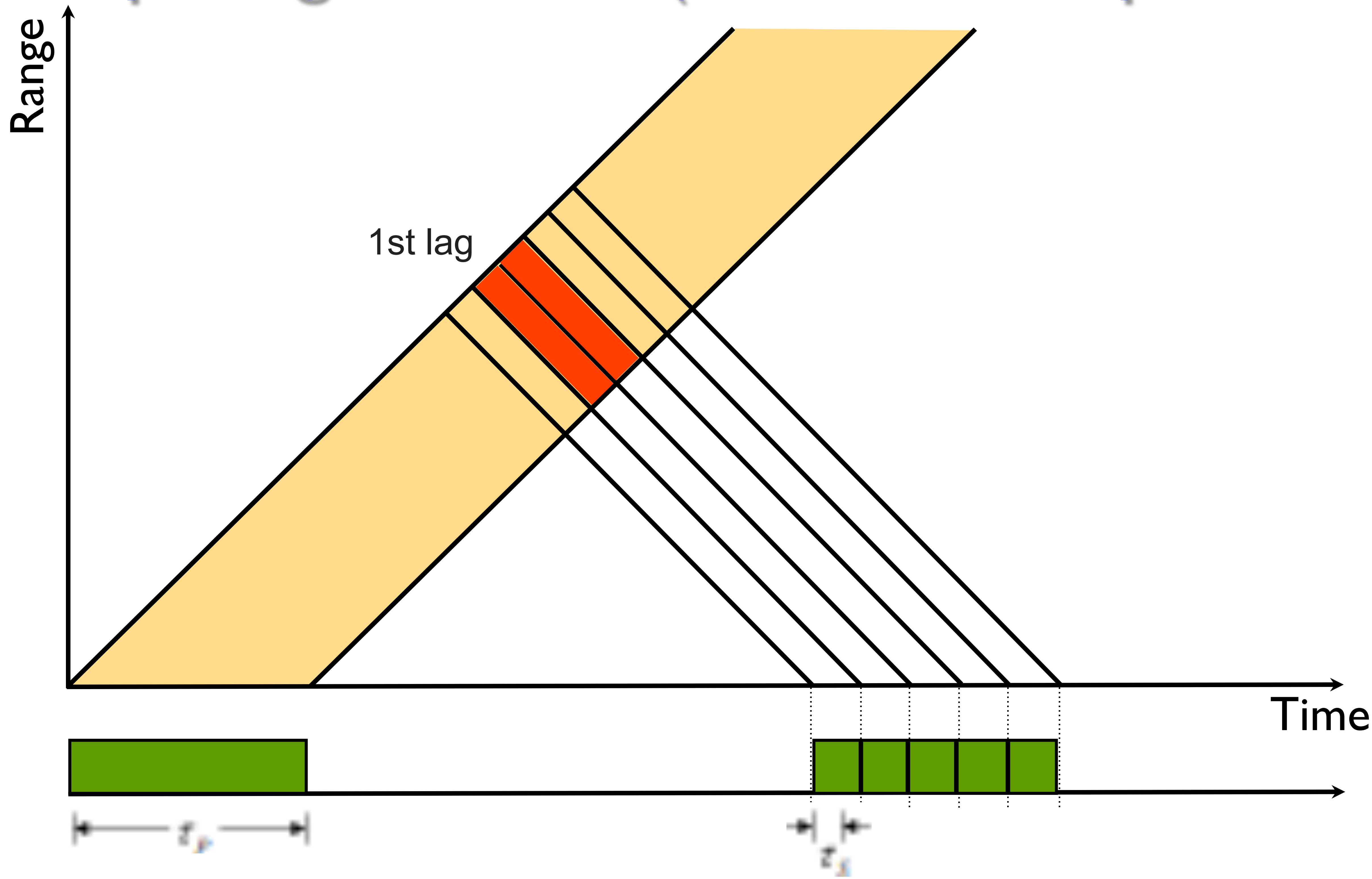

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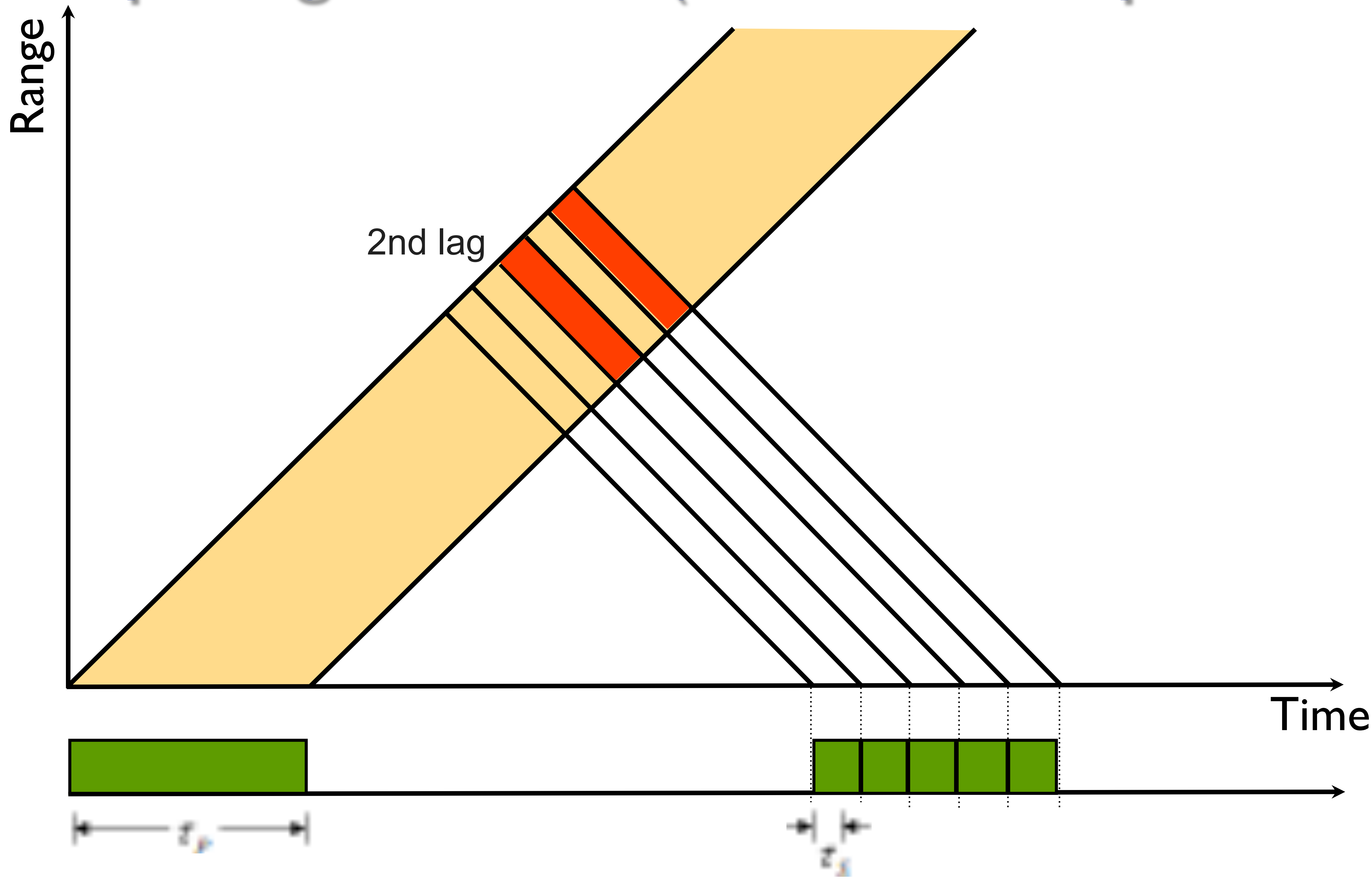

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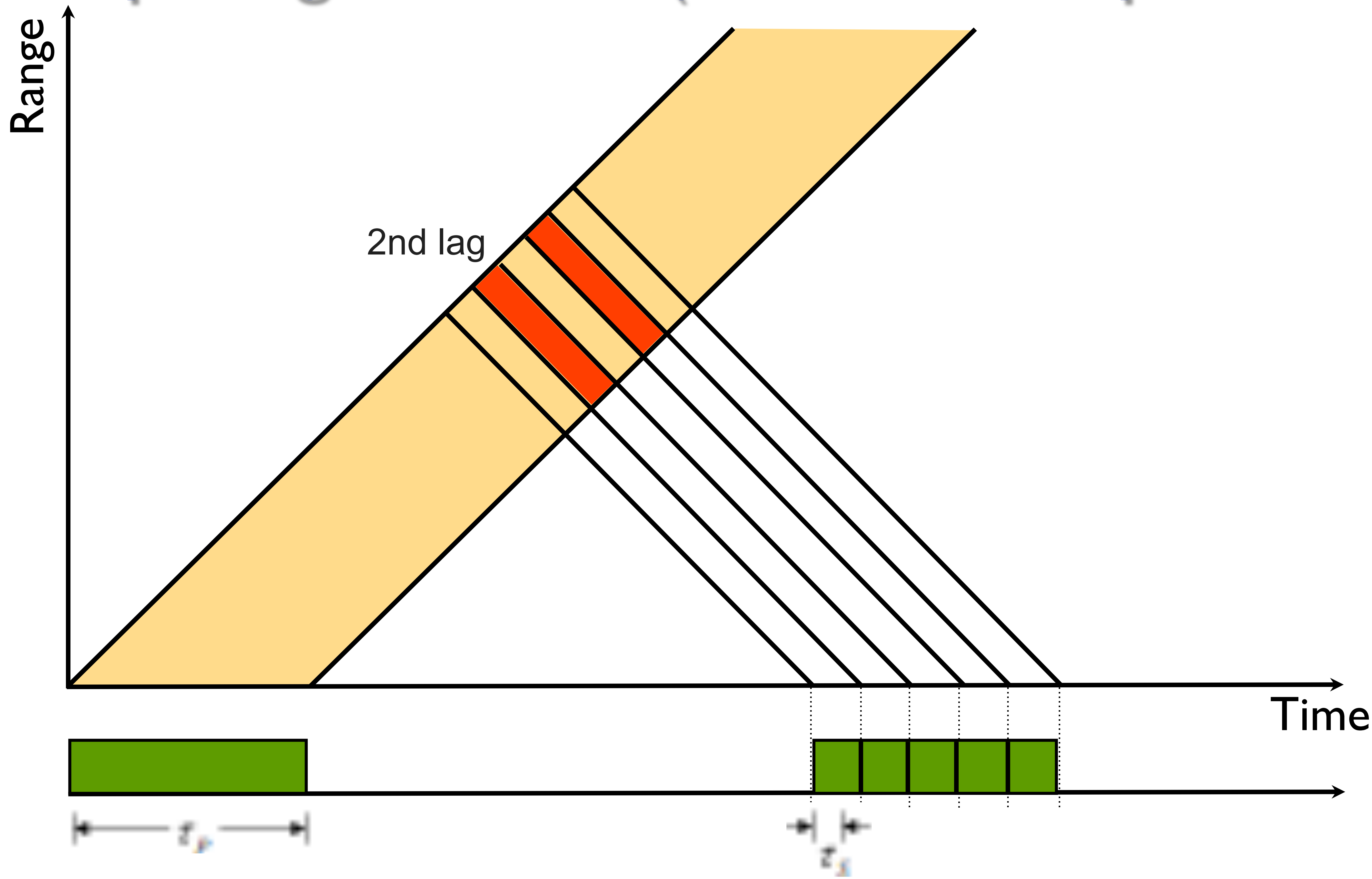
Computing the ACF (and, hence, spectrum)



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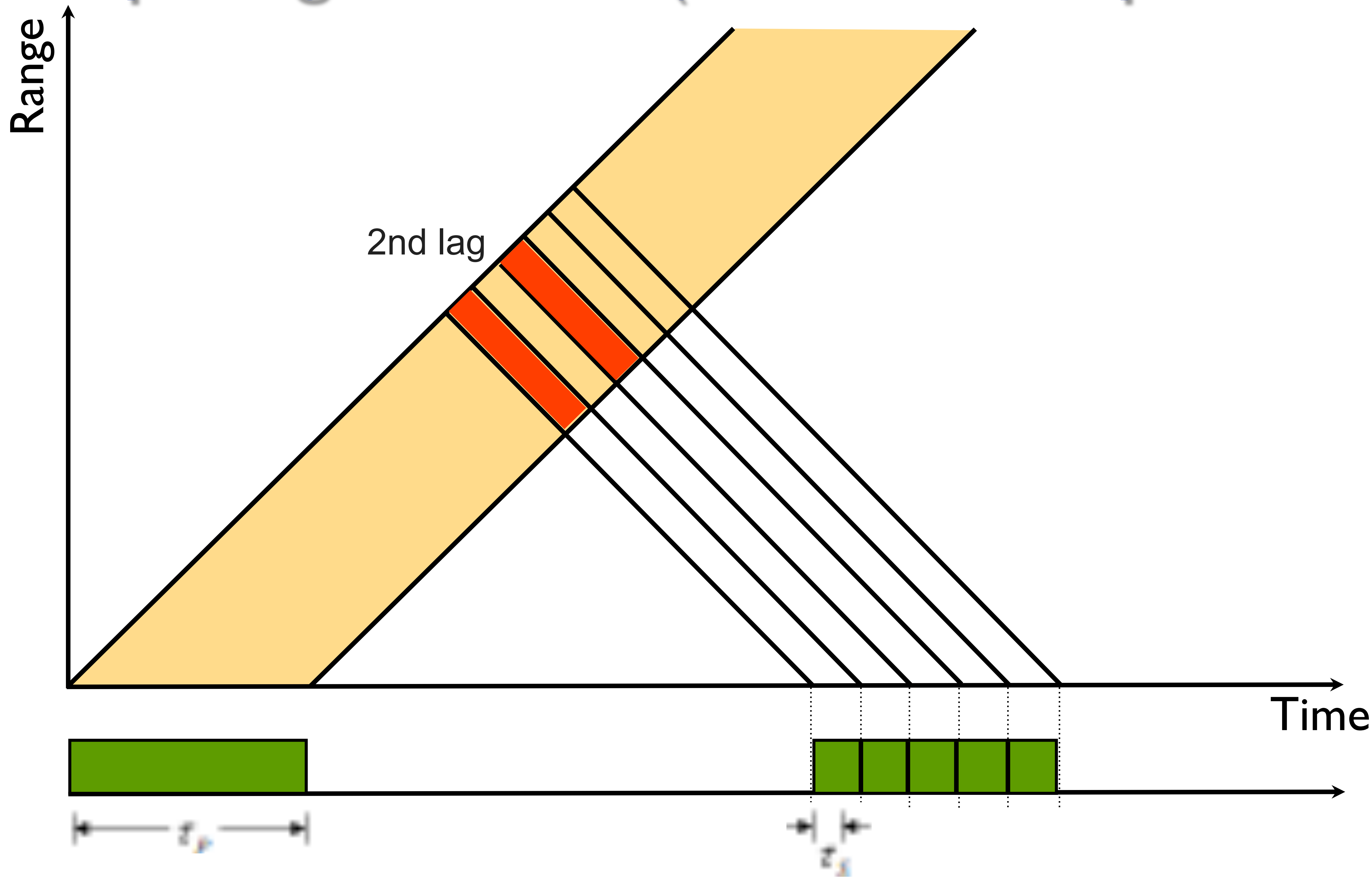
Computing the ACF (and, hence, spectrum)



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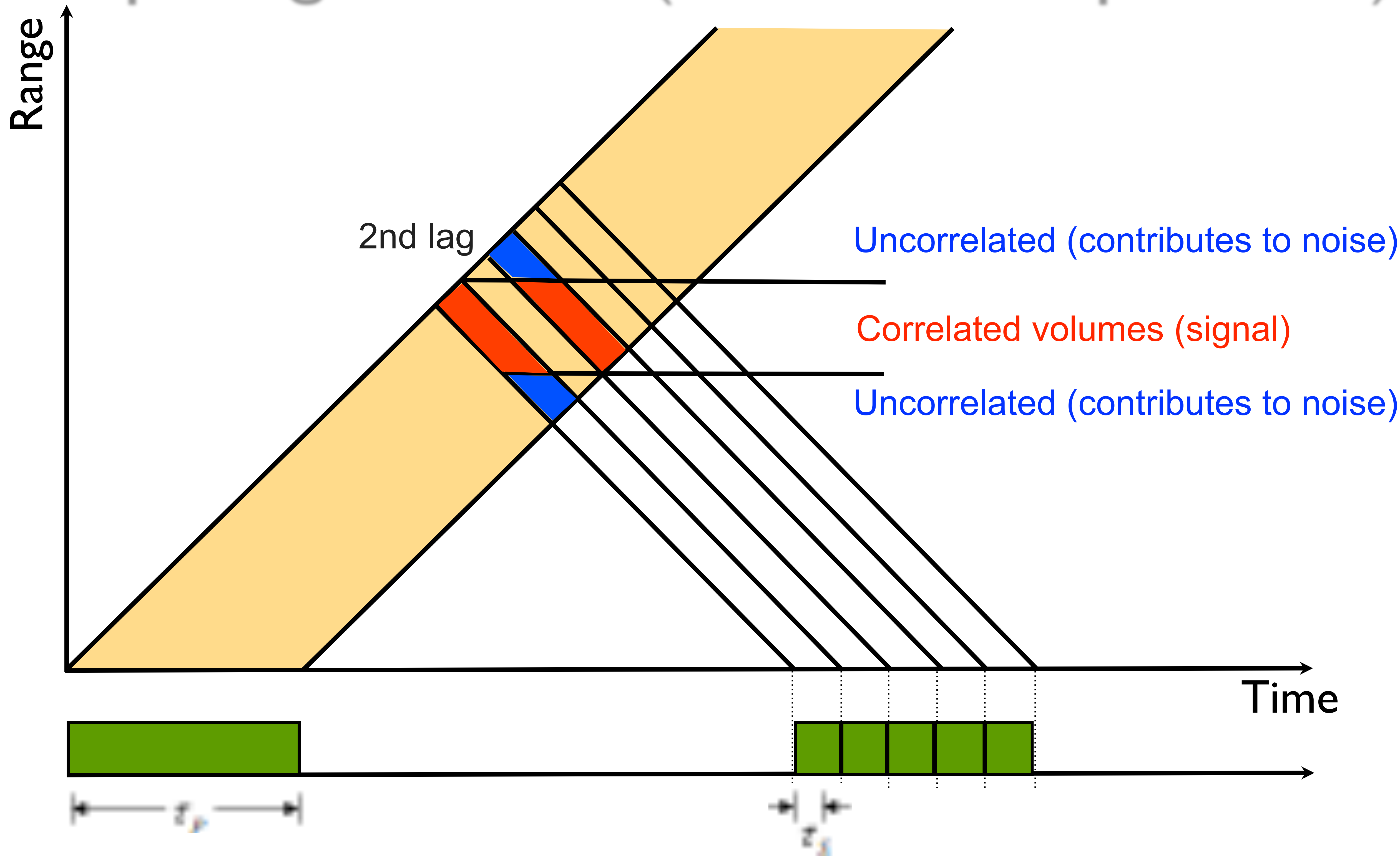
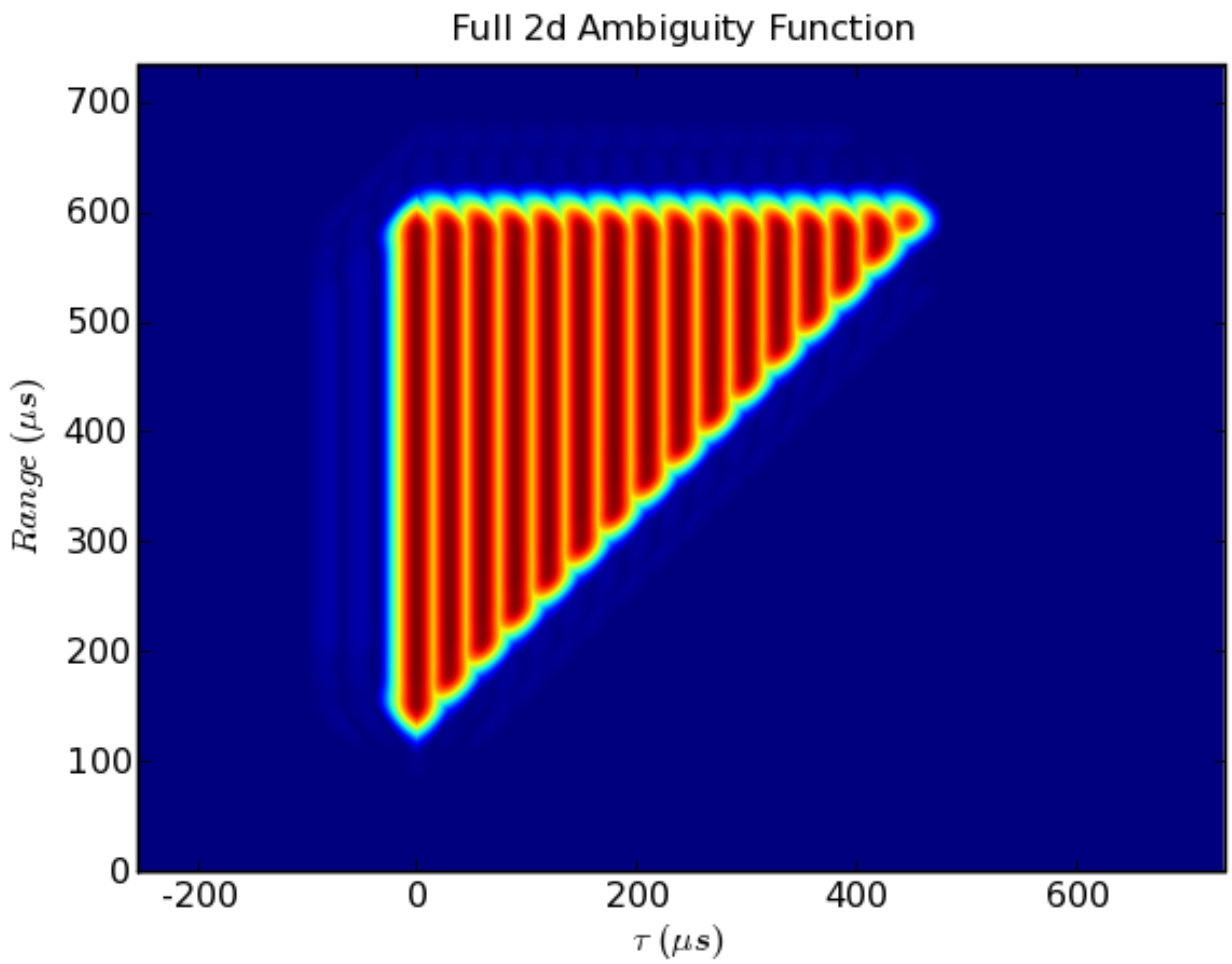
Computing the ACF (and, hence, spectrum)



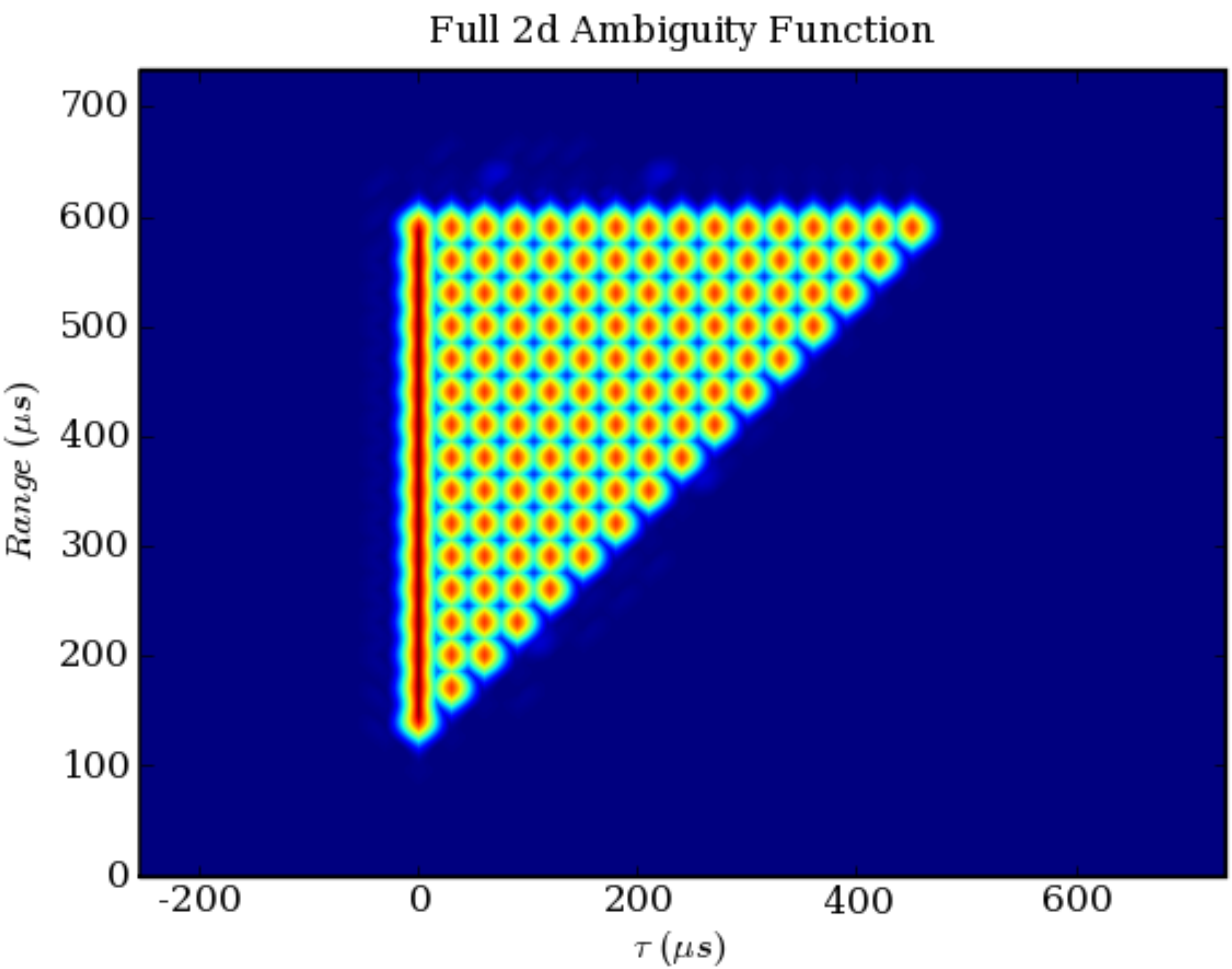
τ_p = Length of RF pulse

τ_s = Sample Period (typically ~ 1/10 pulse length)

Ambiguity Function (smearing in range and lag)



Alternating Code
(smearing in range and lag)



Longitudinal Modes in a Thermal Plasma

Ion-acoustic

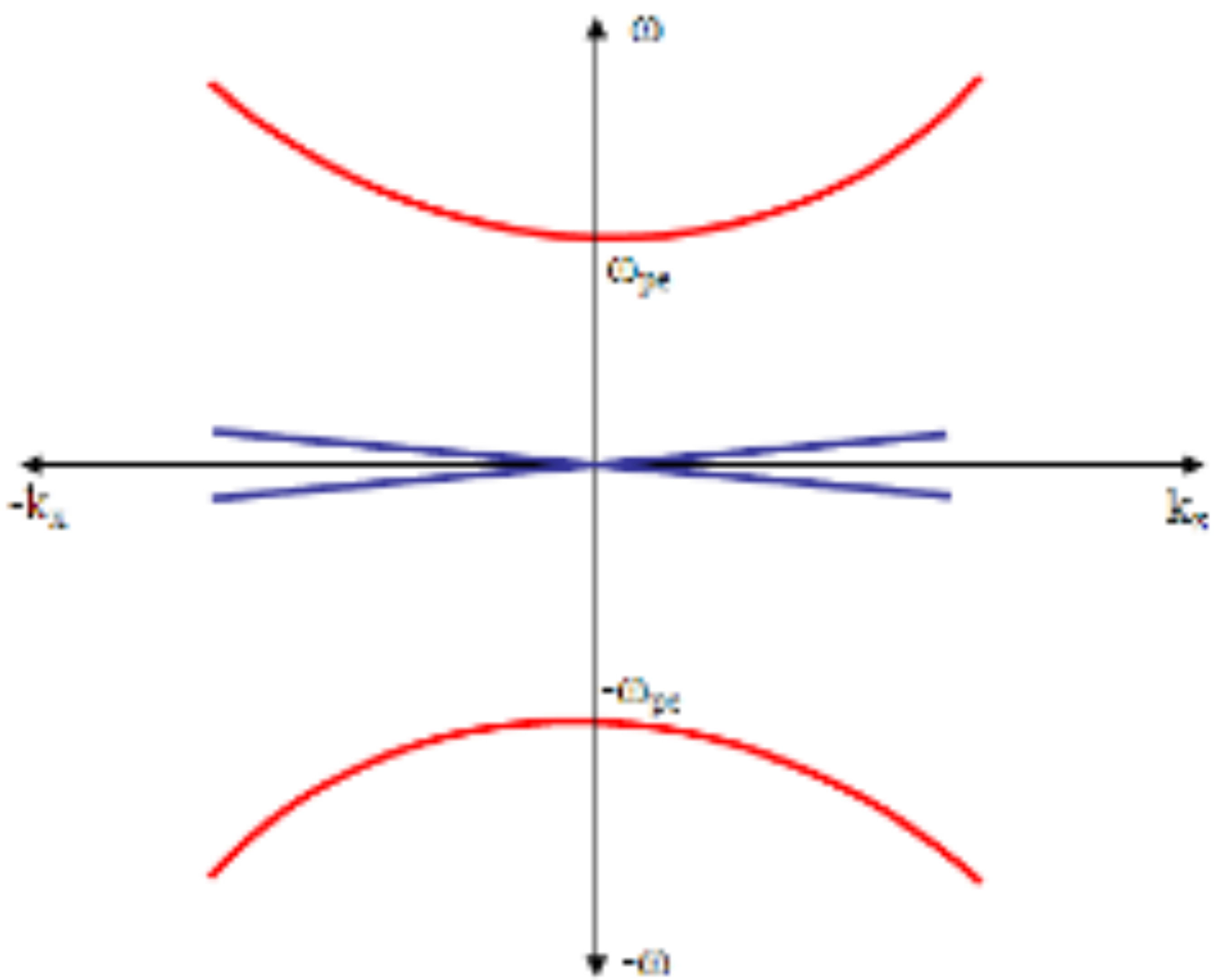
$$\omega_s = C_s k \qquad C_s = \sqrt{k_B(T_e + 3T_i)/m_i}$$

$$\omega_{si} = -\sqrt{\frac{\pi}{8}}\left[\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \left(\frac{T_e}{T_i}\right)^{\frac{3}{2}} \exp\left(-\frac{T_e}{2T_i} - \frac{3}{2}\right)\right]\omega_s$$

Langmuir

$$\omega_L = \sqrt{\omega_{pe}^2 + 3\,k^2\,v_{the}^2} \approx \omega_{pe} + \frac{3}{2} v_{the} \lambda_{De} k^2$$

$$\omega_{Li} \approx -\sqrt{\frac{\pi}{8}} \frac{\omega_{pe}^3}{k^3} \frac{1}{v_{the}^3} exp\left(-\frac{\omega_{pe}^2}{2k^2 v_{the}^2} - \frac{3}{2}\right)\omega_L$$



Simulated ISR Doppler Spectrum

Particle-in-cell (PIC):

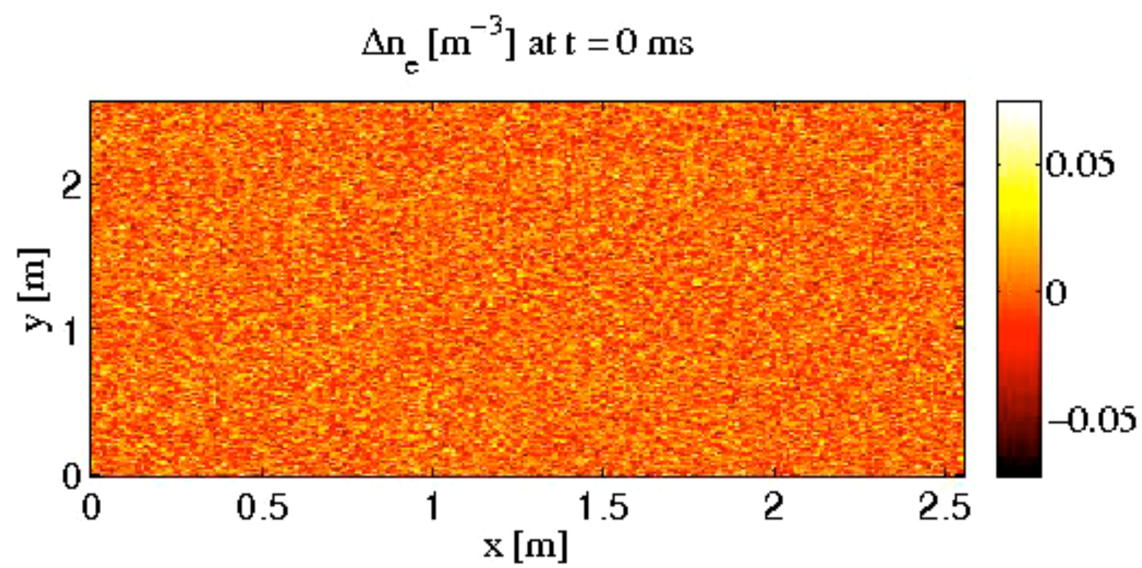
$$\frac{d\,\mathbf{v}_i}{d\,t} = \frac{q_i}{m_i}(\mathbf{E}(\mathbf{x}_i) + \mathbf{v}_i \times \mathbf{B}(\mathbf{x}_i))$$

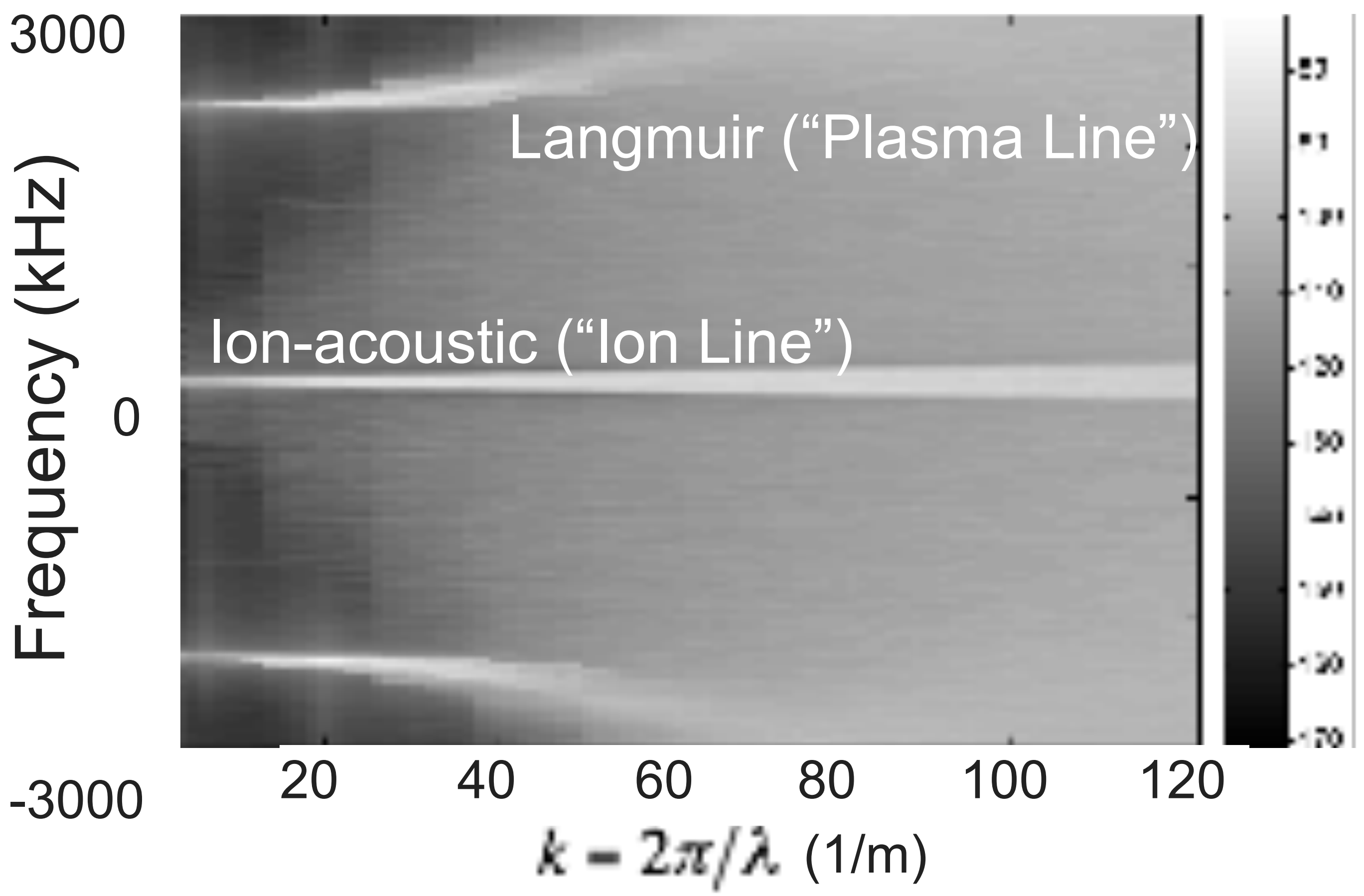
$$\nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

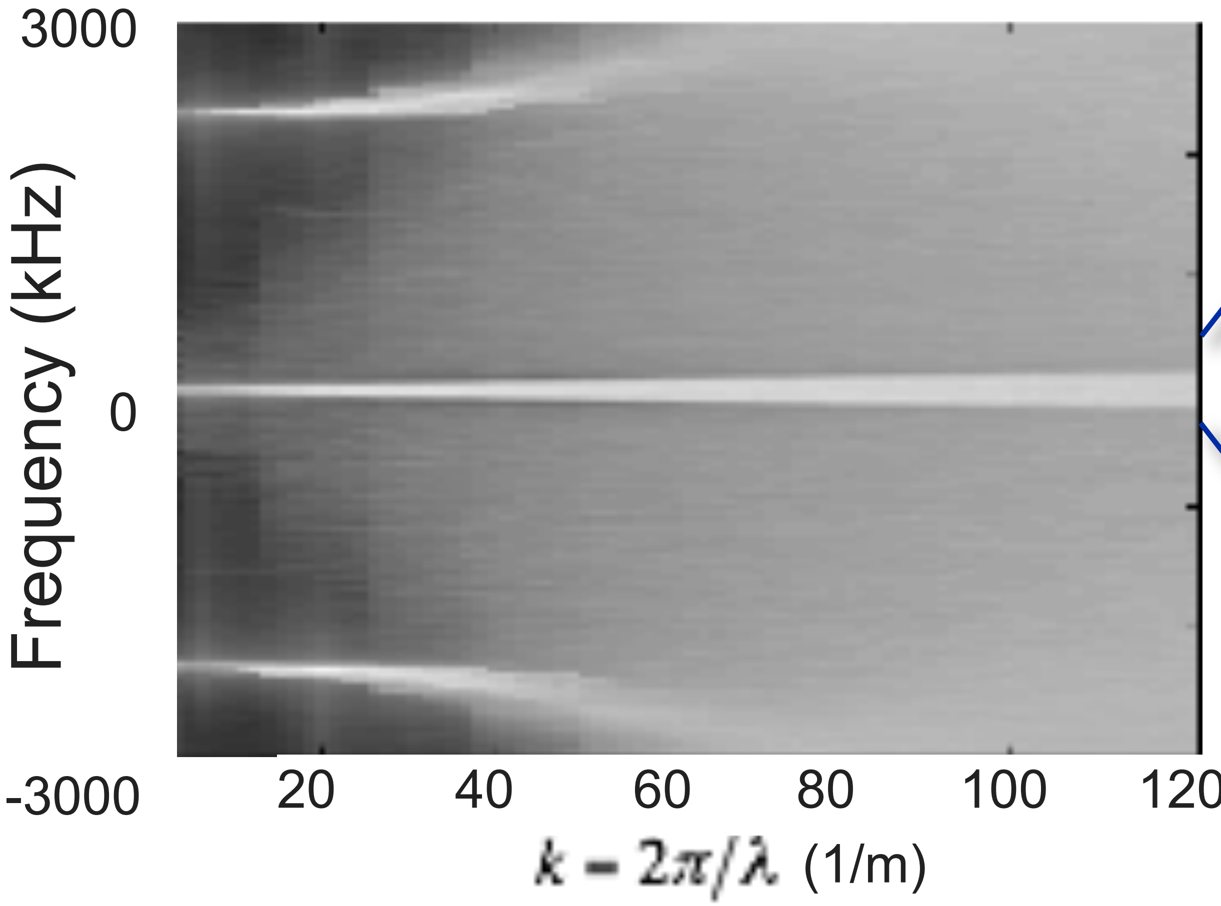
$$\nabla \cdot \mathbf{B} = 0$$

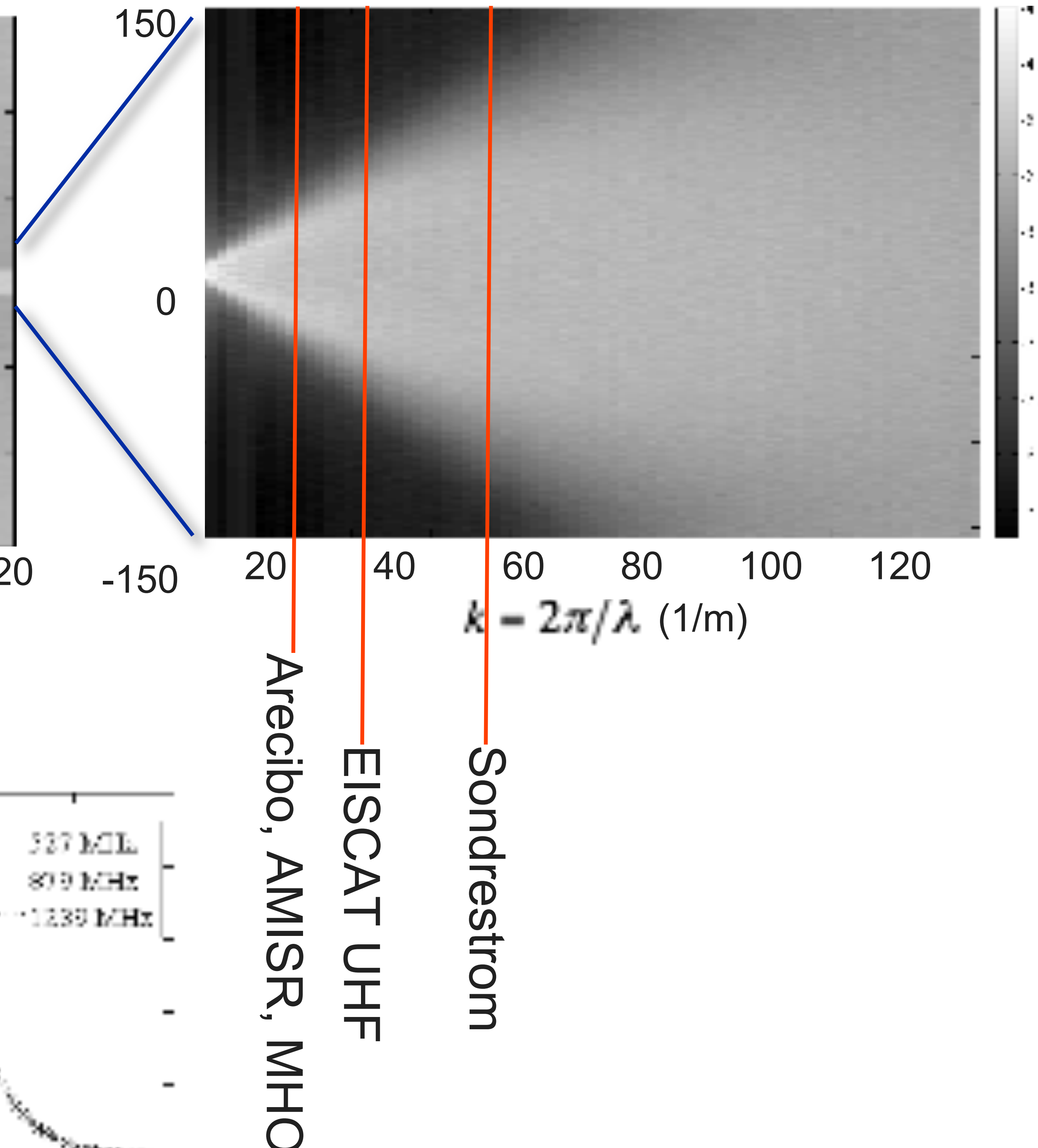


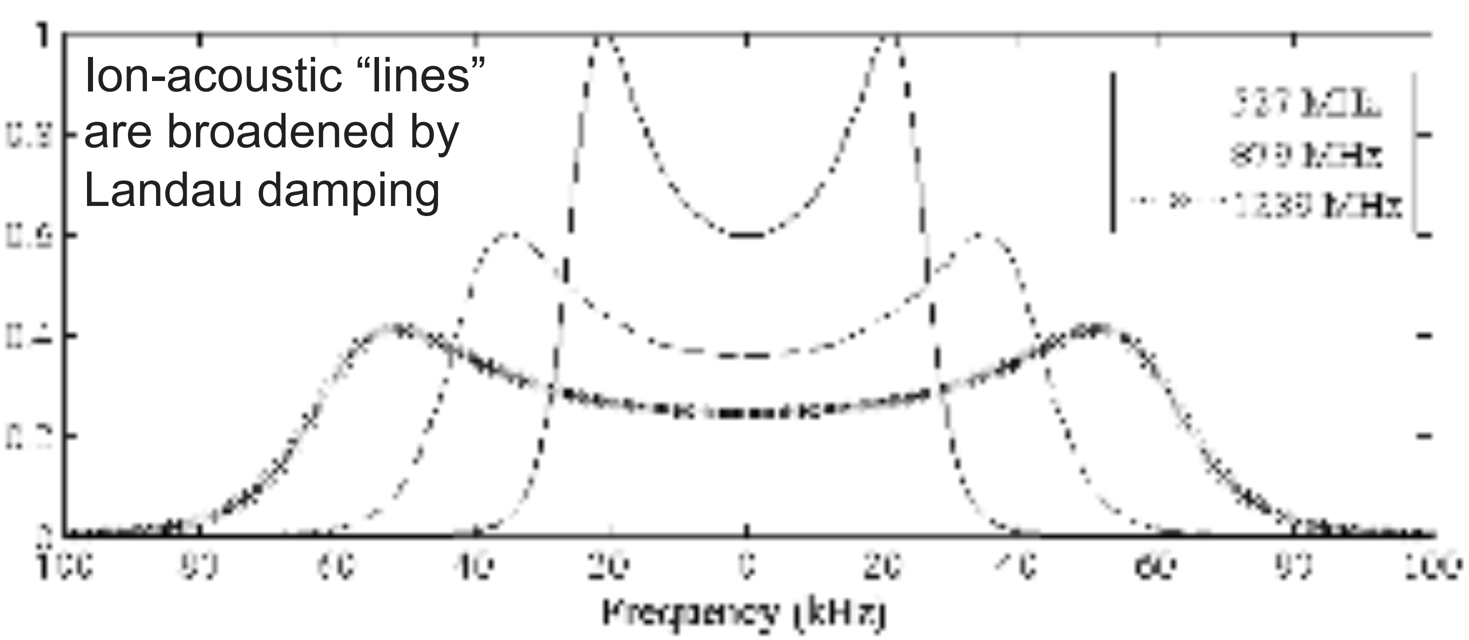


Simple rules yield complex behavior

ISR Measures a Cut Through This Surface







Bibliography

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Radar signal processing

- Mahafza, *Radar Systems Analysis and Design Using MATLAB*
- Skolnik, *Introduction to Radar Systems*
- Peebles, *Radar Principles*
- Levanon, *Radar Principles*
- Blahut, *Theory of Remote Image Formation*
- Curlander, *Synthetic Aperture Radar: Systems and Signal Analysis*

Background (Electromagnetics, Signal Processing:

- Ulaby, *Fundamentals of Engineering Electromagnetics*
- Cheng, *Field and Wave Electromagnetics*
- Oppenheim, Willsky, and Nawab, *Signals and Systems*
- Mitra, *Digital Signal Processing: A Computer-based Approach*

For fun:

http://mathforum.org/mbower/johnandbetty/frame.htm

END