Radar School - Group 5

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Objective

Explore the time and space gradients present in the polar ionosphere. This is a basic study to get an idea of the gross order of magnitude estimate of the change in time and space in temperatures, electron density and ion velocity.

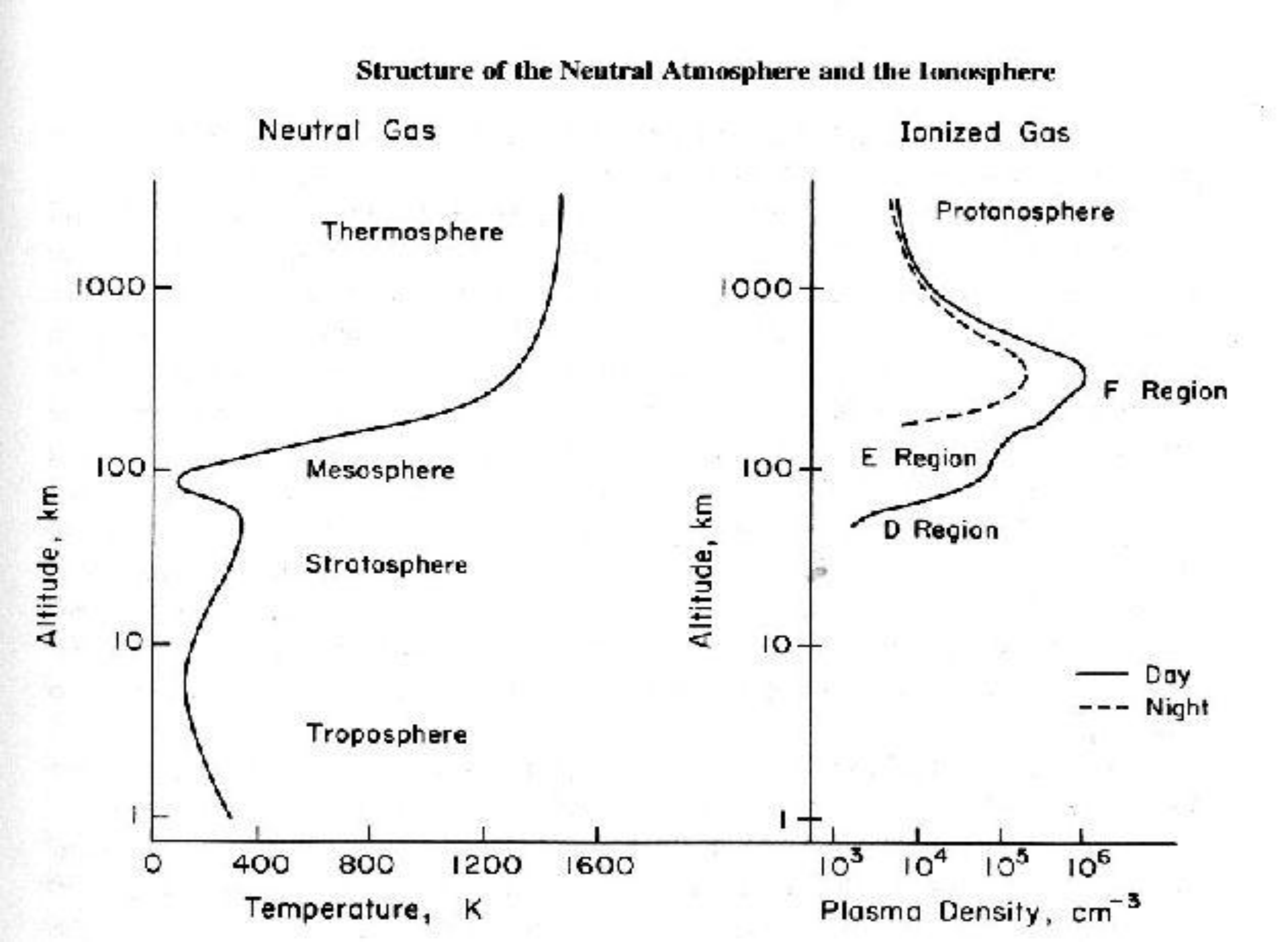
Outline

- Background
- Question
- Experiment

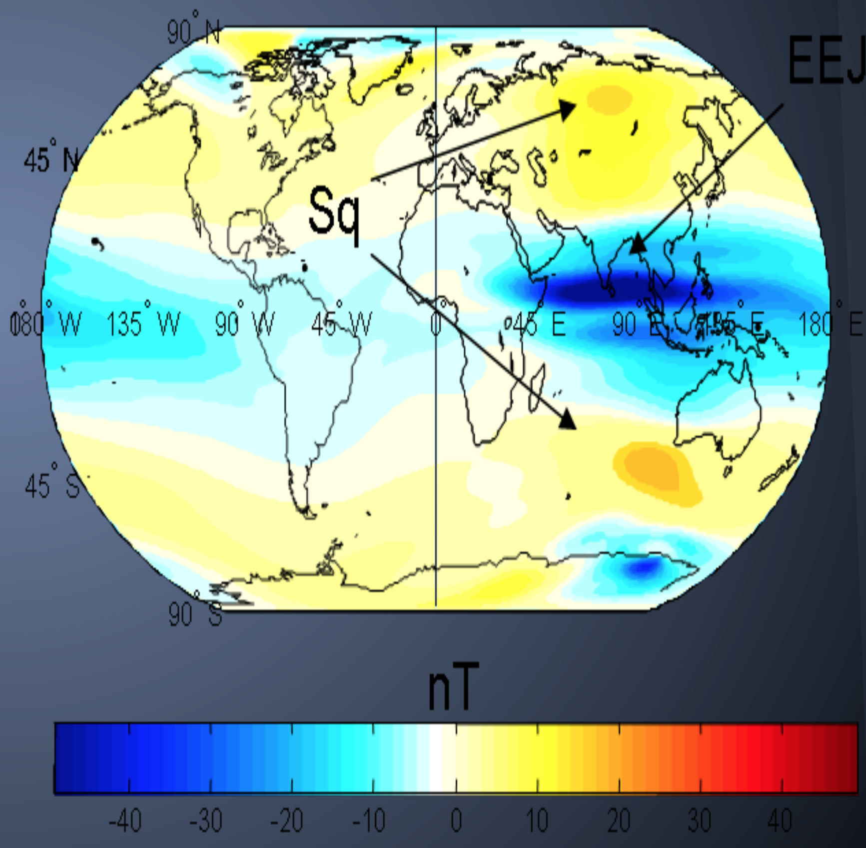
Background

- **Neutral atmosphere:** is an atmosphere consisting of neutral gas.
- Ionosphere: is defined as the layer of the Earth's atmosphere that is ionized by solar and cosmic radiation.
- Auroral and Equatorial Electroject
- Incoherent Scatter Radar
- IPP, PRF, Ne, Te, Ti

Neutral atmosphere and Ionosphere



Auroral and Equatorial Electrojet



Incoherent Scatter Radar

It is a technique to study Earth’s ionosphere.

Related:

- PFISR
- AMISR
- RISR

IPP, PRF, Te, Ti, ACFs

- IPP, is the elapsed time from the beginning of one pulse to the next.
- PRF, is the number of pulses per unit.
- Ne, Electron density, is the probability of an electron being present in a specific location.
- Te, Ti, Electron and Ion temperature.
- ACFs - autocorrelation functions

Kp-index

It is the global geomagnetic storm index and is based on 3h measurements.

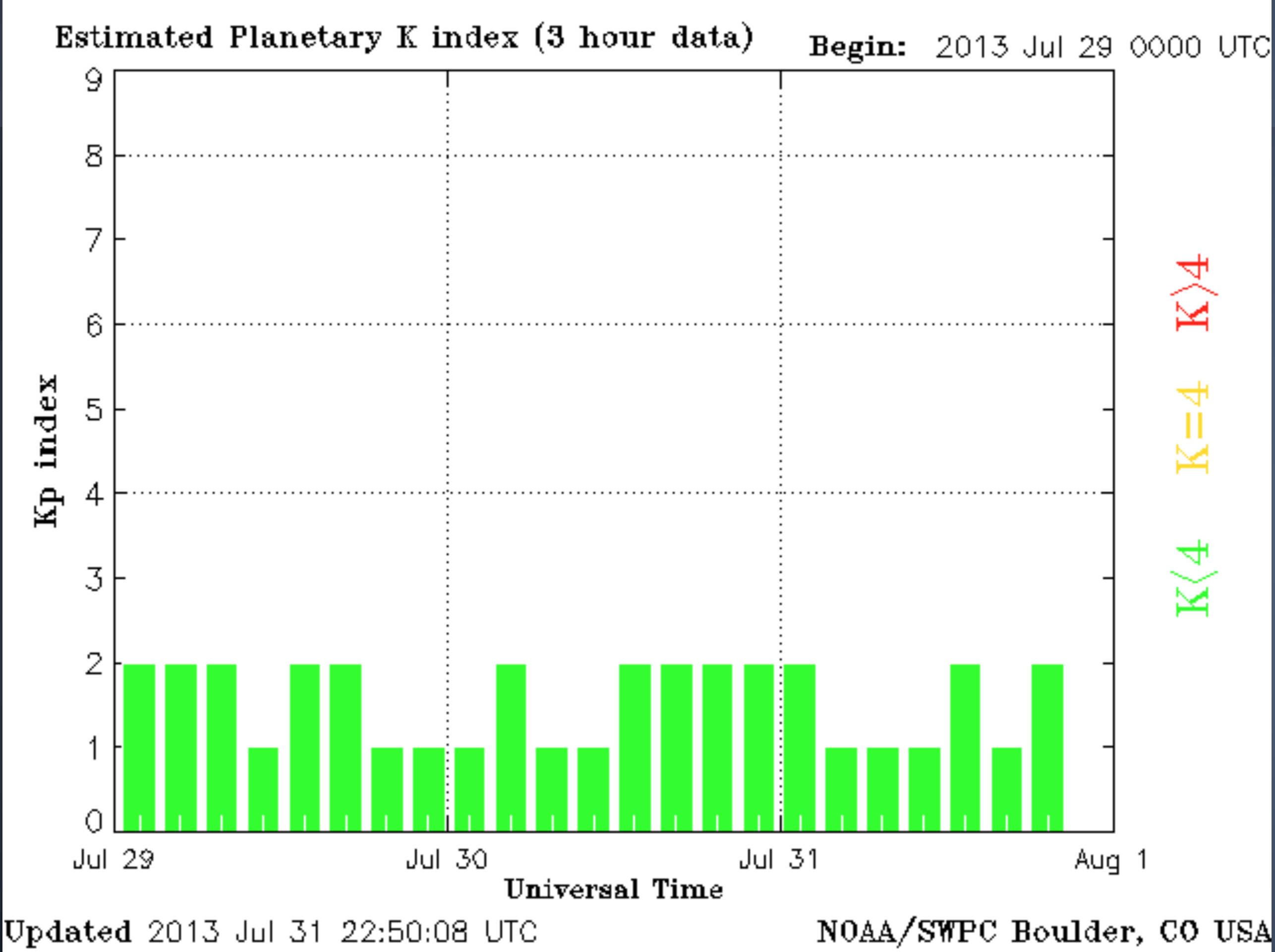
Based on the Kp index, you know if Geomagnetic activity is calm or strong.

kp index could be:

- <=4, calm
- =5, minor storm
- >=6, major storm
- 9, extreme storm.

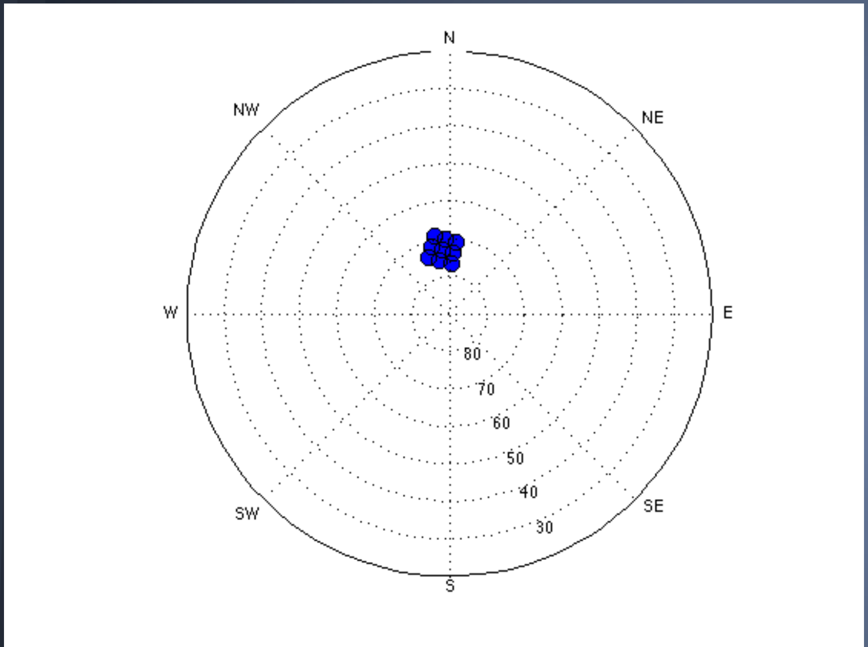
Reference: http://www.spaceweatherlive.com/en/help/the-kp-index

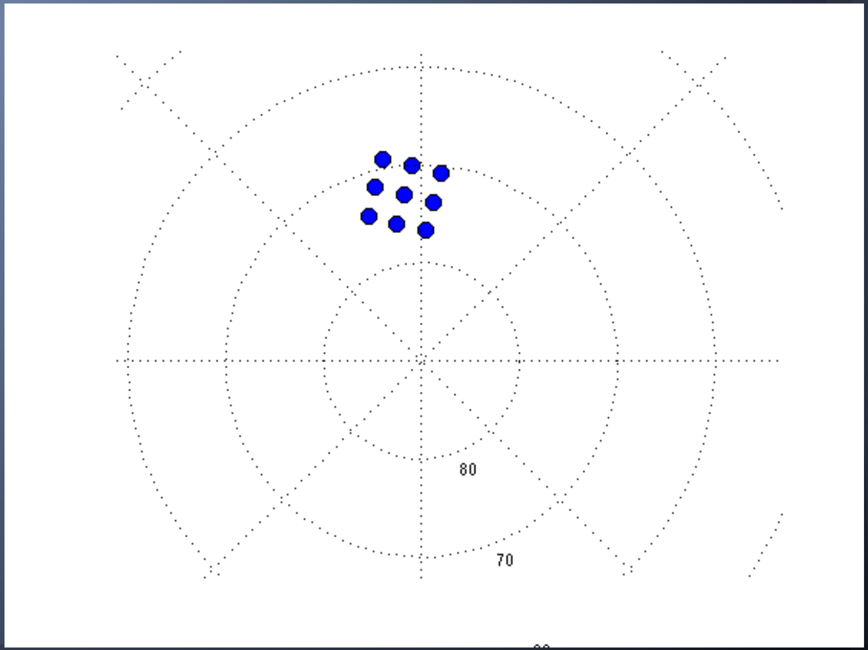
Kp-index=1 (period 31-01)



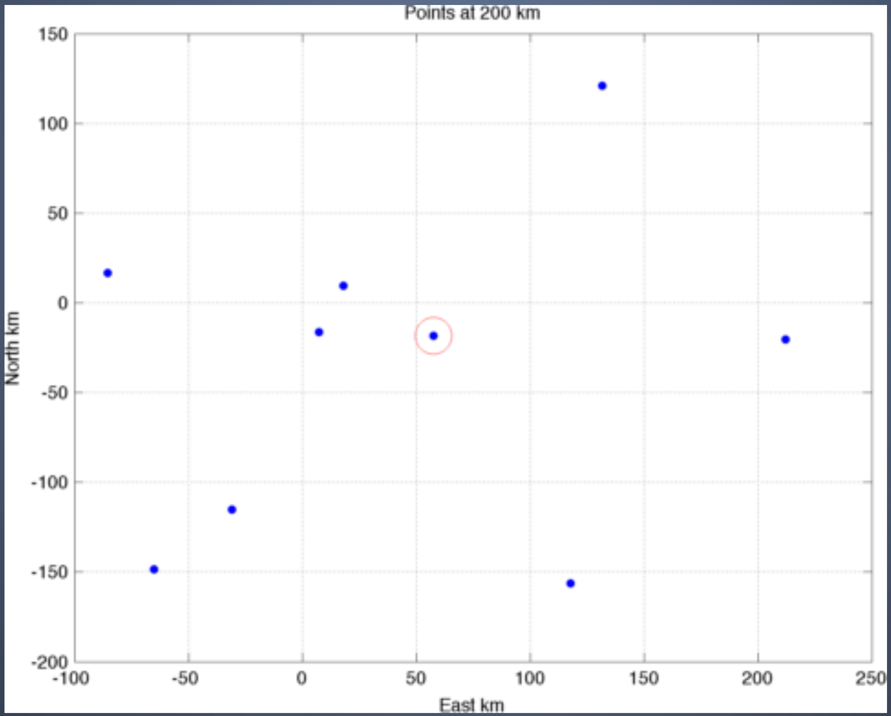
Reference: http://www.swpc.noaa.gov/rt_plots/kp_3d.html

Beams





Beams (Plain Content)



Correlation

cor·re·la·tion [kawr-*uh*-**ley**-shuh n, kor-] ? Show IPA

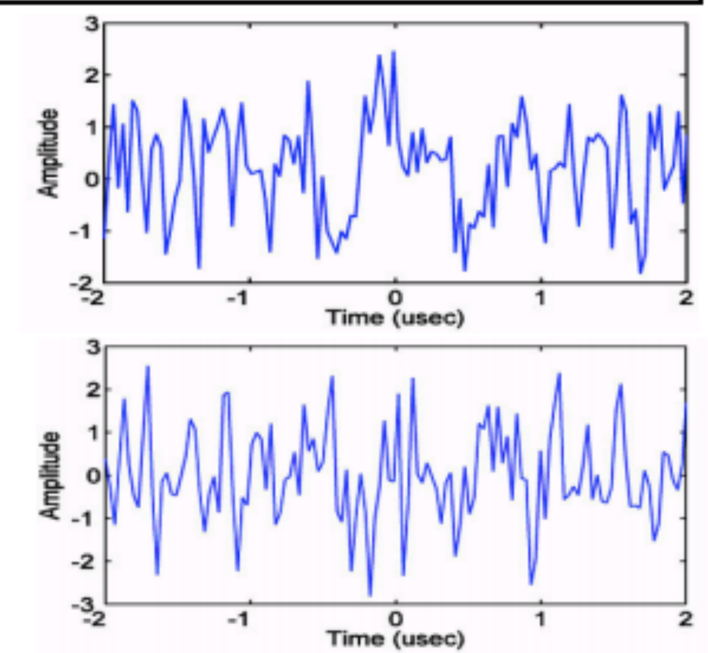
noun

1. mutual relation of two or more things, parts, etc.: *Studies find a positive correlation between severity of illness and nutritional status of the patients.* ***Synonyms:*** similarity, correspondence, matching; parallelism, equivalence; interdependence, interrelationship, interconnection.
2. the act of correlating or state of being correlated.
3. *Statistics.* the degree to which two or more attributes or measurements on the same group of elements show a tendency to vary together.
4. *Physiology* . the interdependence or reciprocal relations of organs or functions.
5. *Geology* . the demonstrable equivalence, in age or lithology, of two or more stratigraphic units, as formations or members of such.

Question?

If I give you a time series of two signals could you define **some measure** for me of "how correlated the two signals are".

This is an ill posed question. What is well correlated? Correlated in what? Time? Space?



It turns out that **a** good measure of how correlated two functions are is the convolution of those two functions. The convolution tells us how much two signals overlap. The more they overlap, the more they are alike the higher the convolution is. We can check the overlap of the two functions as we slide them with respect to each other as a function of time. This gives us:

- The convolution as a function of time,
- How much these functions overlap as a function of time
- One good measure of how well these two functions are correlated as a function of time

The convolution is defined as:

$$(f * g)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau$$

http://upload.wikimedia.org/wikipedia/commons/6/6a/Convolution_of_box_signal_with_itself2.gif

Notice that when these two functions are on top of eachother the value of the convolution is maximum. This is the shift/lag/time at which they are most alike, therefore this is when the convolution is the largest.

Convolution: $(f * g)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau$

!Caution: There are other productive ways to think of the convolution function such as a "smoothing operation", or a "filter". Don't forget to explore these thoughts.

You can see that the convolution can be a measure of how correlated two functions are as a function of how much they have been slid with respect to each other.

Thus we say that the more common area two functions have the more that the two functions are correlated at a given amount of slide with respect to each other (slide = lag).

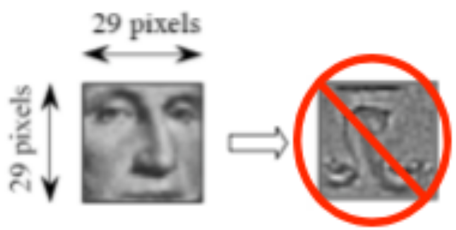
The convolution is precisely what we define as correlation! The only difference is that the correlation operation has been generalized for complex functions.

ACF = Autocorrelation Function

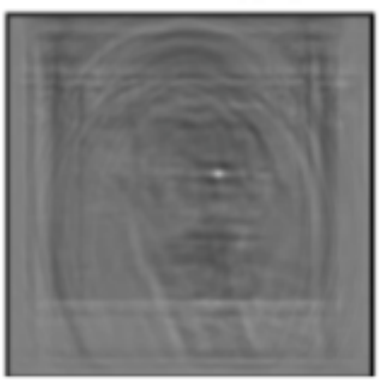
$$\text{ACF(f(t))} = \int_{-\infty}^{\infty} f(t)\overline{f}(t-\tau)\, \mathrm{d}t$$

Note! The ACF is the convolution of a function with itself if the function is real valued.

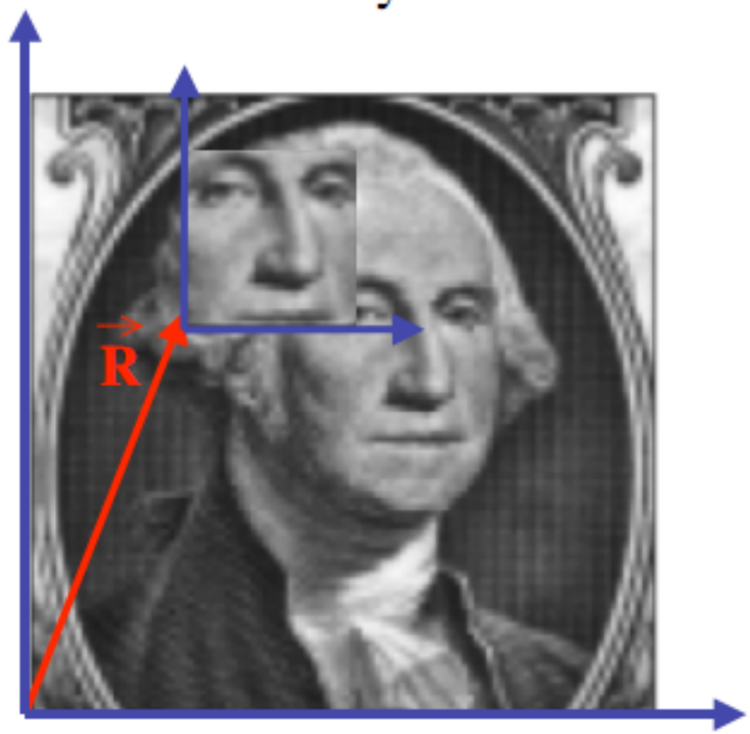
a. Image to be searched



b. Target c. Kernel



Lets think of it this way:



If we find the $\vec{R}$ where the overlap is maximized then we will find the $\vec{R}$ where the two images are the "most correlated". Proof of this is in this example.

This is only instructional for real valued functions. Just as a 4-D matrix cannot be shown as an object but rather by indicies you cannot use this sort of example for an ACF of complex valued functions.

We now have a good mental image of one good measure of correlation between two real valued functions. How should we picture this measure of correlation?

The autocorrelation is the sum/integral/overlap of:

$f(t)f^*(t)$ what is a complex number times its complex conjugate?

$(x+iy)(x-iy) = |x|^2 - ixy + ixy + |y|^2 = |x|^2 + |y|^2 = R^2$ Square magnitude!

For a complex function we define it to be well correlated at a lag τ if the square magnitude of the function has a large overlap.

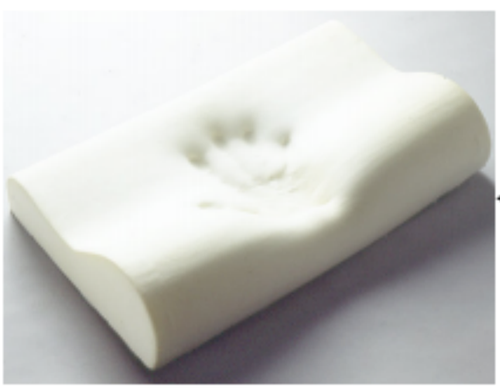
If you want to understand the ACF go home and meditate on this statement.

So how is this useful in our field?

Just like vectors the ACF is not only an interesting mathematical object that we can have some intuition for but it has a very useful place in our world.

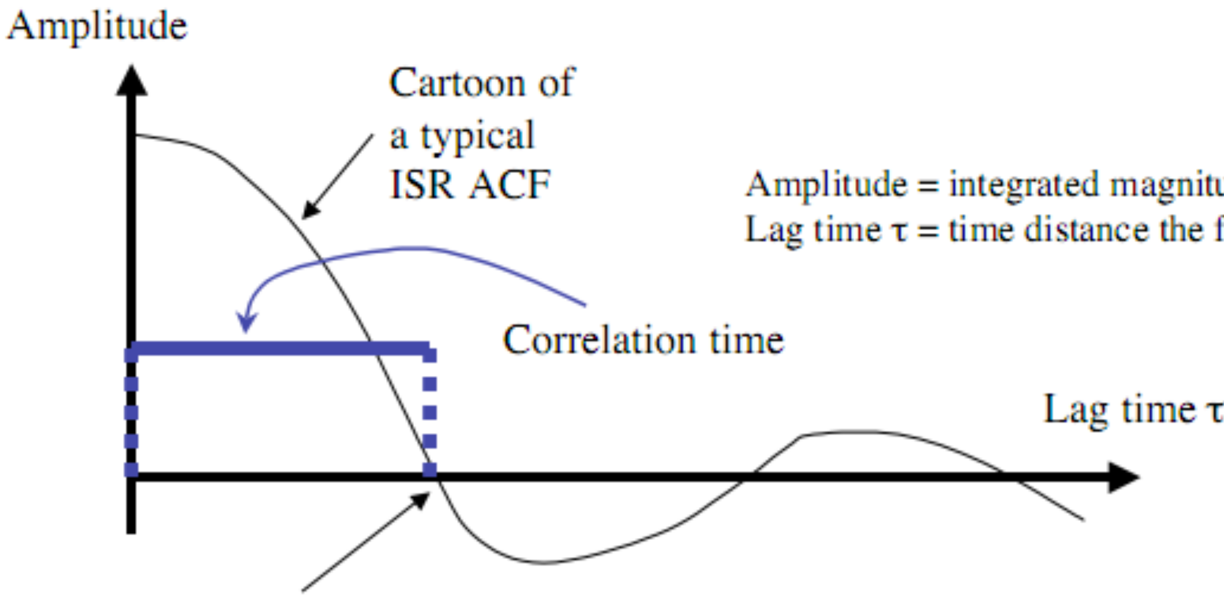
If we compute the ACF of a radar signal it tells us information about the physics of the plasma.

So what does it tell us? It tells us how correlated the plasma is in time, or how long the plasma remembers and retains its properties.



Memory foam pillow has a much larger ACF for much longer lags τ.

ACF of a Return from an ISR



Amplitude = integrated magnitude, VV*t (units volts^2 t)
Lag time τ = time distance the function has been shifted

First crossing time **(!big deal!).** This time is defined as the **correlation time**.
It encloses much of the physics that we are looking for.

The correlation time of the plasma in the ionosphere gives us a measure of how long the plasma ‘remembers’ what happened to it in the past. Just like the memory foam pillow would have a correlation time that indicates how long it takes for your hand print to disappear so the plasma has a time that tells us how long a previous state affects a future state.

So Why Does the Plasma Decorrelate and What Changes the Correlation Time?

The only way to do this is turn the mathematical crank and do the math. I will simply state the result:

The plasma decorrelates because there is a distribution in the velocities of the electrons.

Two instructive examples:

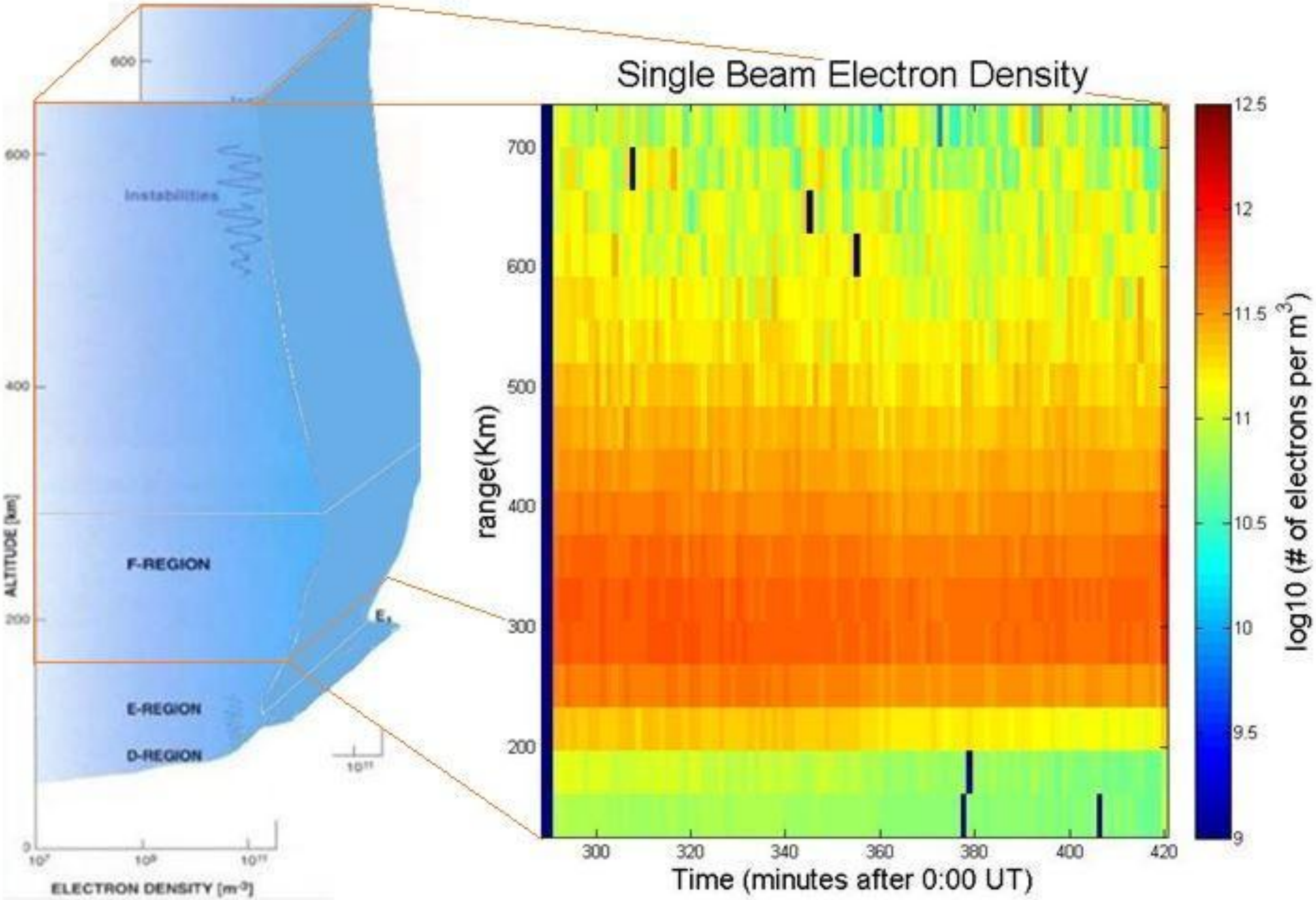
- If the distribution of velocities was flat or any velocity is equally probable; even those indefinitely high (just as many particles going the speed of light as particles going 10 $^{m}/_{s}$).
    - The decorrelation time=0
- There is no distribution of velocities. All electrons have exactly the same velocity magnitude.
    - The plasma will never decorrelate. Decorrelation time = infinity.

What Does This Tell us About the Physics

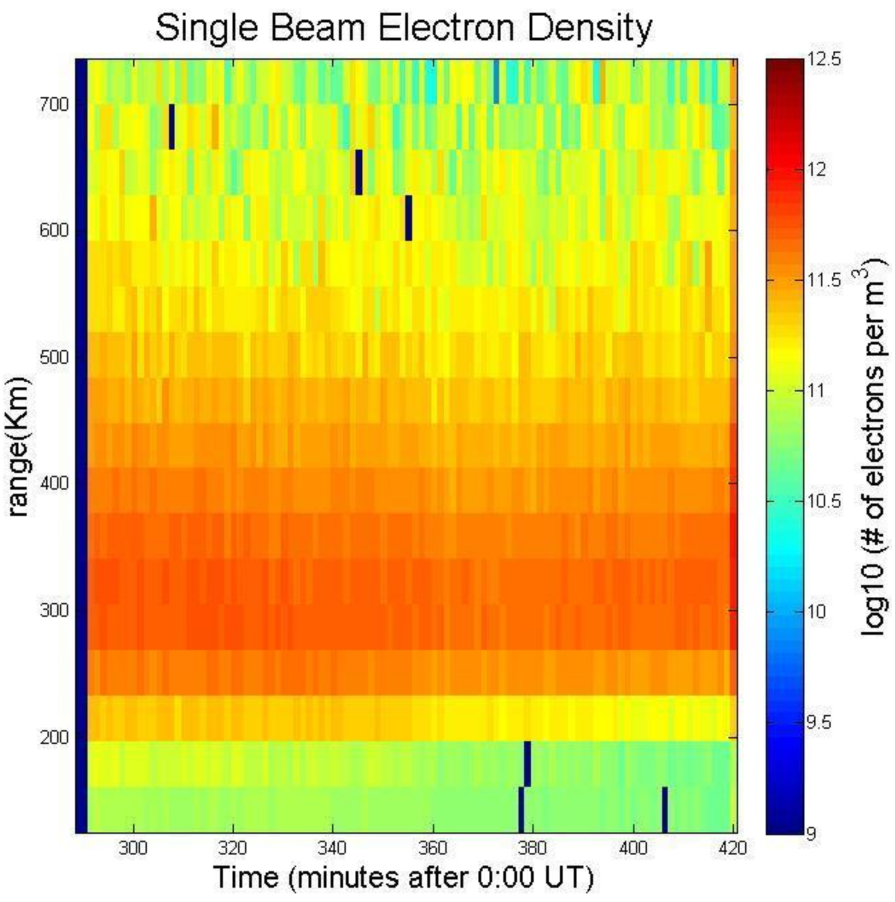
-Therefore the decorrelation time is a measure of the **distribution** of velocities.

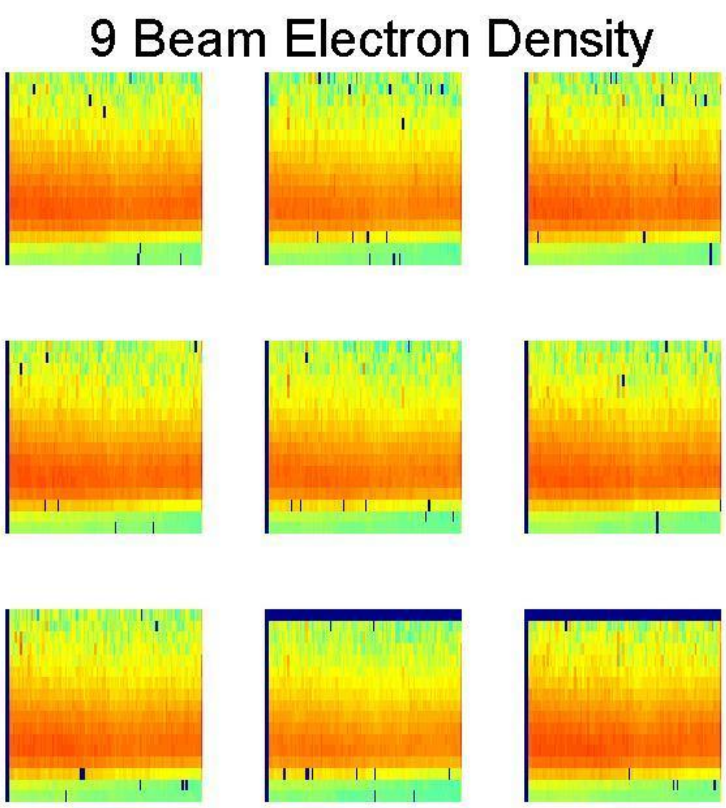
-Which is a measure of the temperature of the electrons.

Electron Density

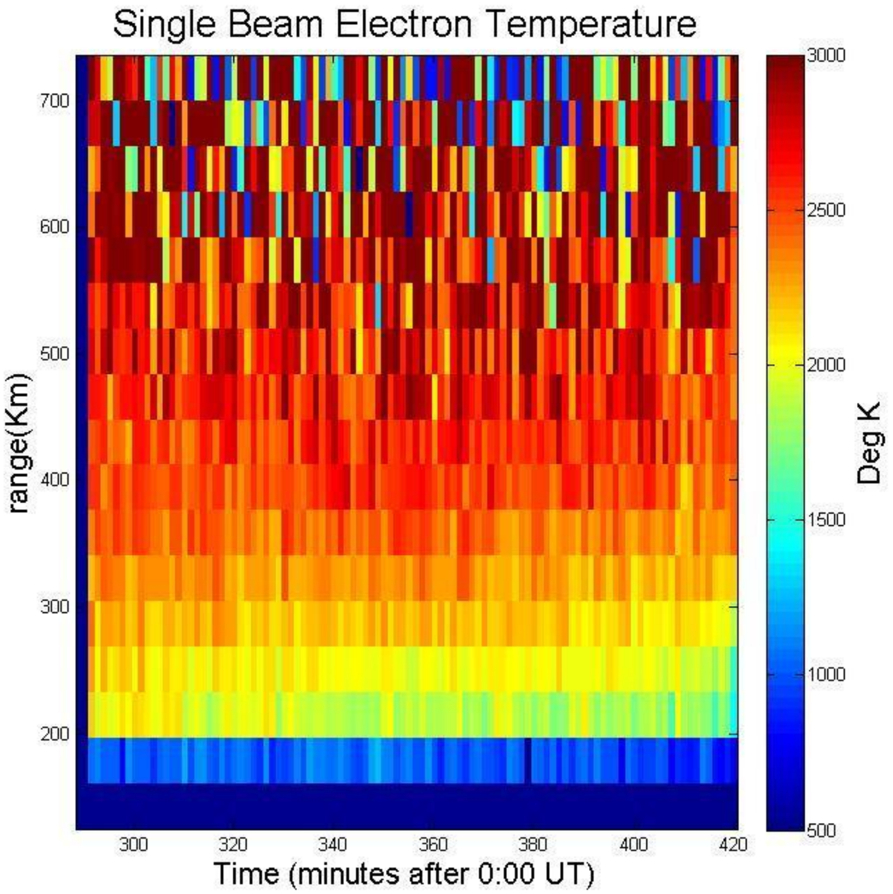


Electron Density

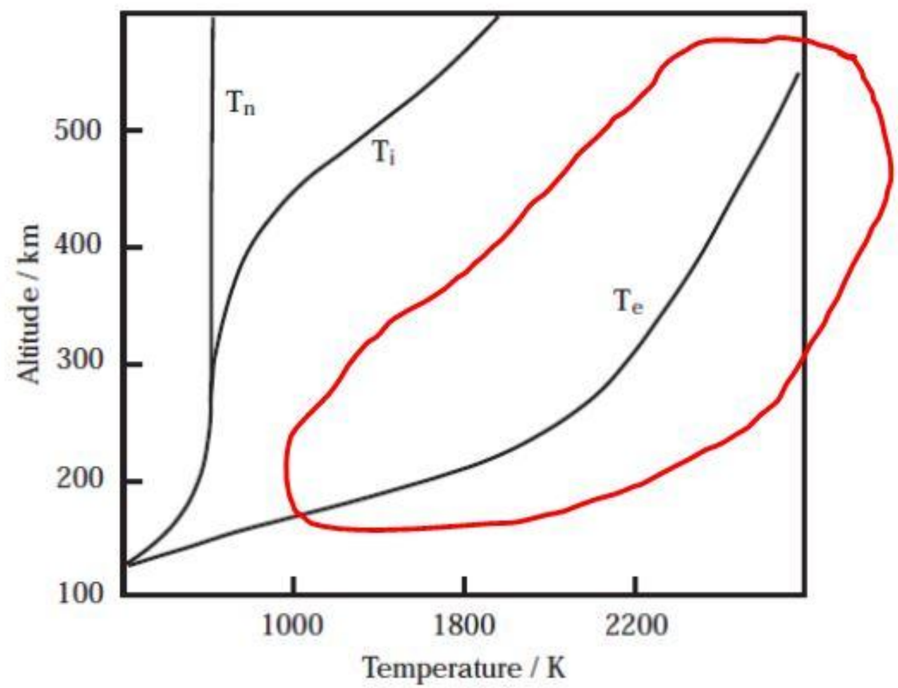




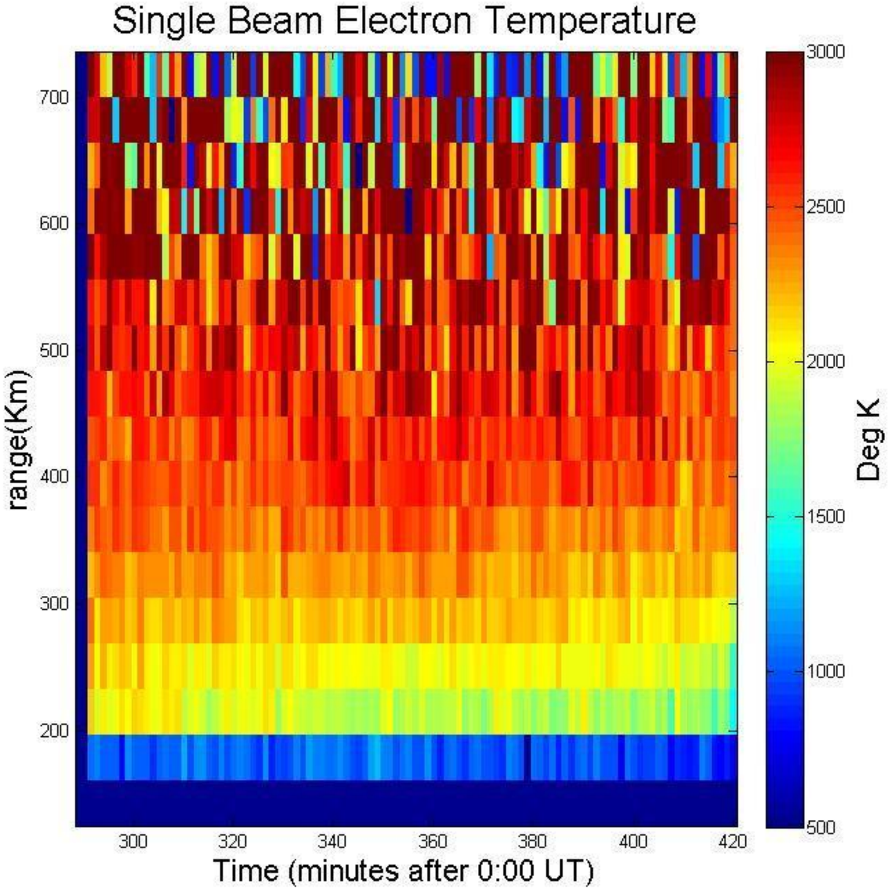
Electron Temperature

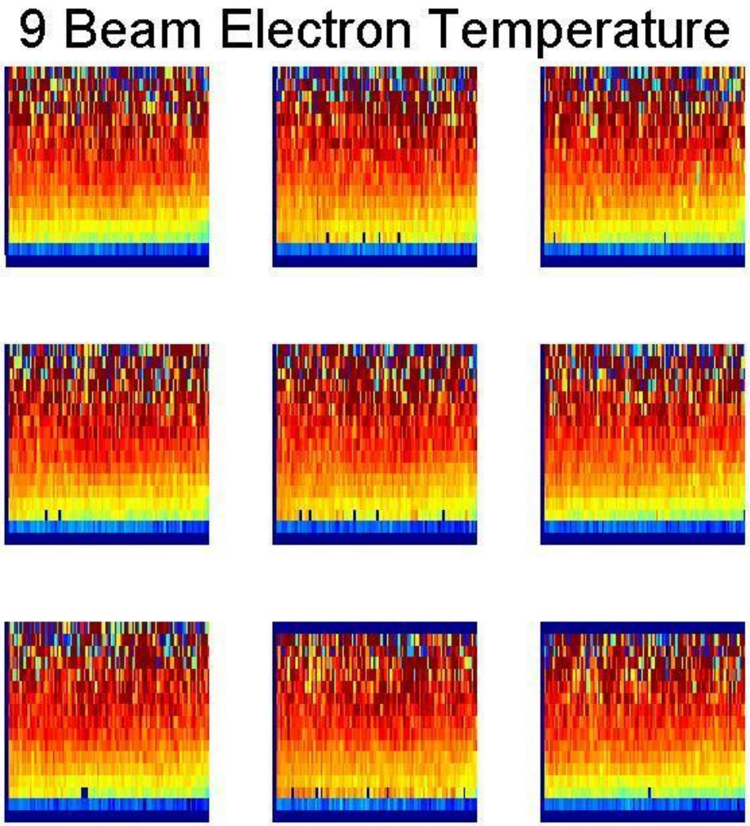


Ionosphere Temperatures

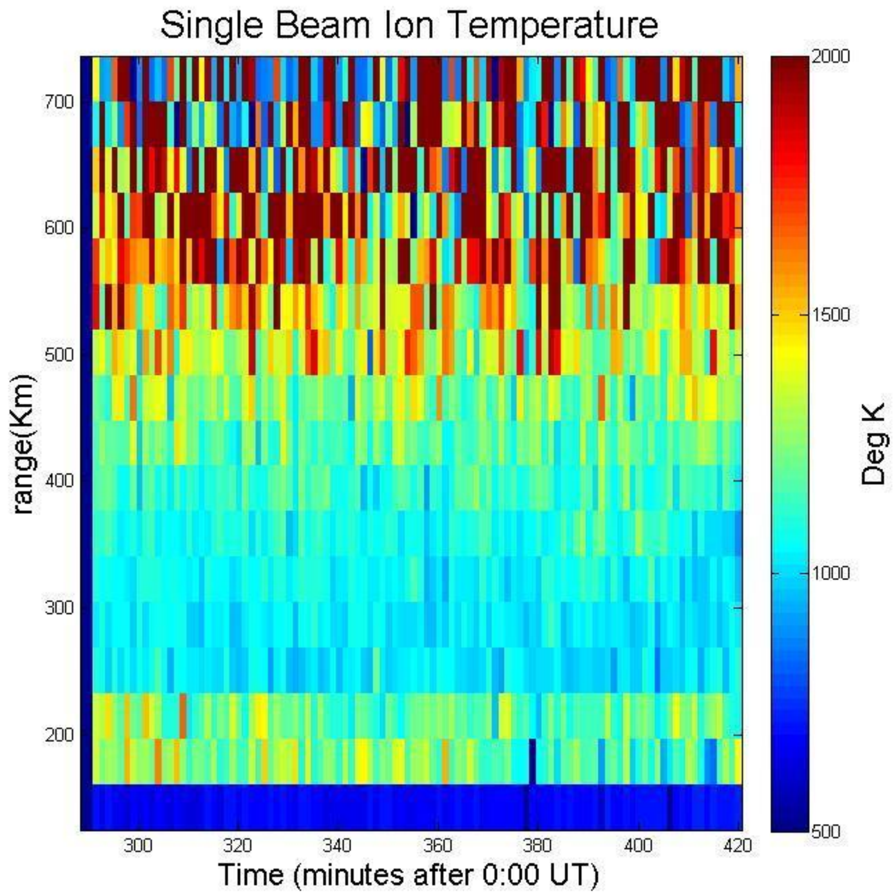


Electron Temperature

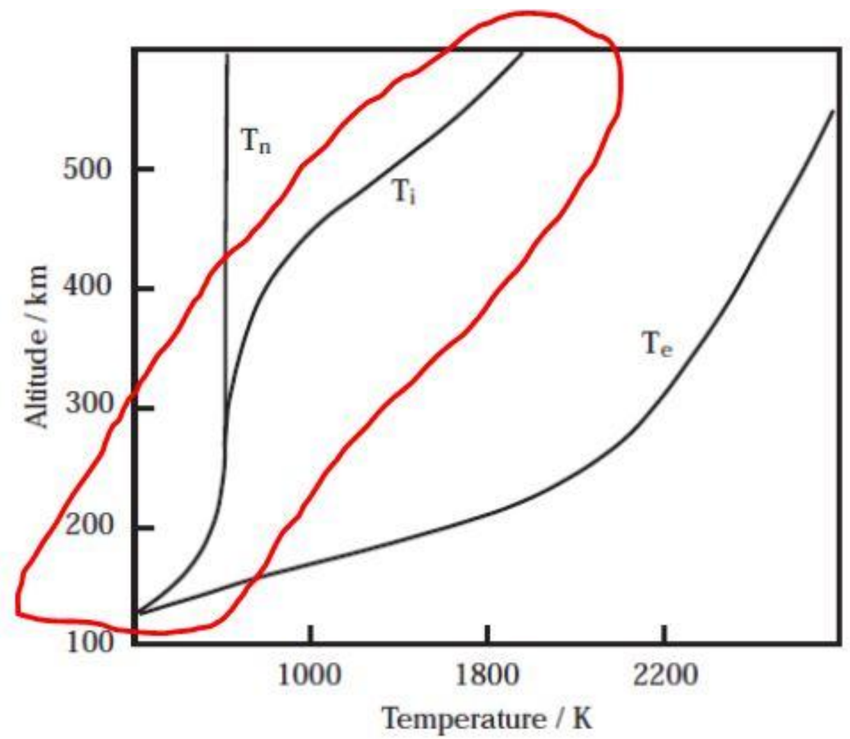




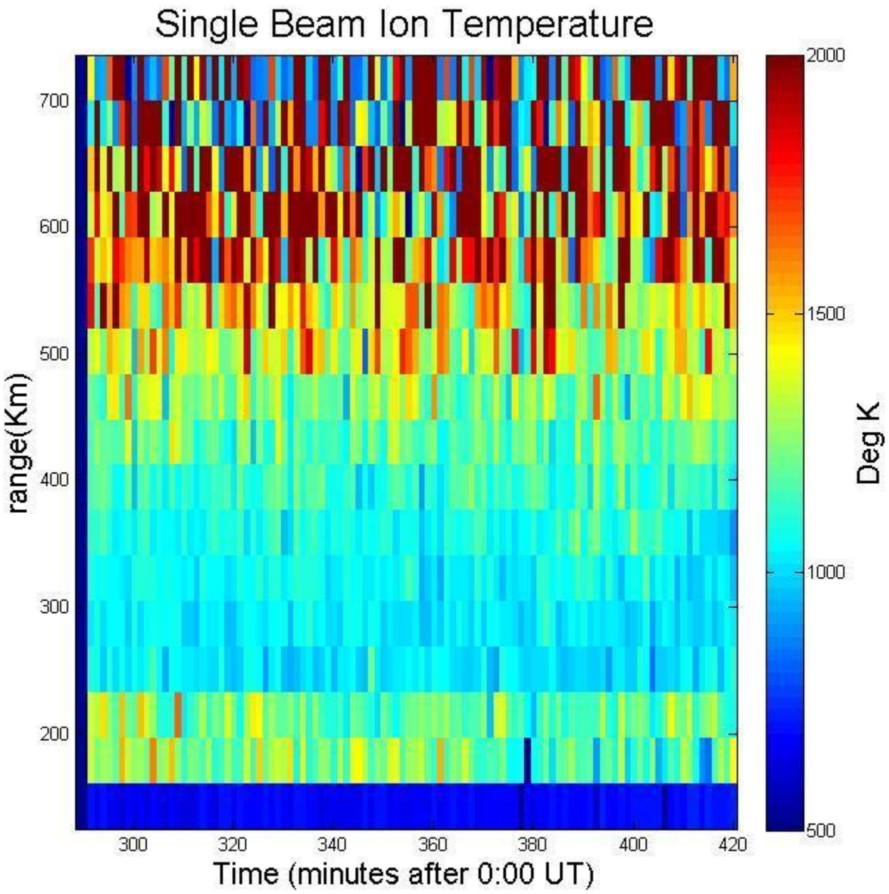
Ion Temperature

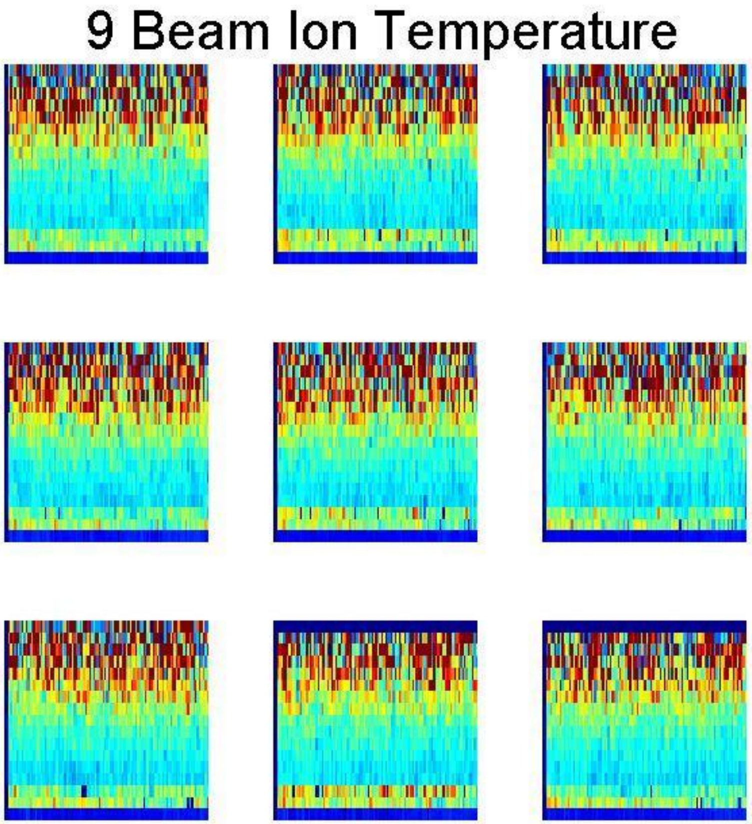


Ionosphere Temperatures



Ion Temperature





SuperDARN

-Super Dual Auroral Radar Network

-Network of over 30 low-power (16 300 W transmitters) HF radars

-20 Total elements (16 element main array, 4 element interferometer array)

-Measures coherent ionospheric structures

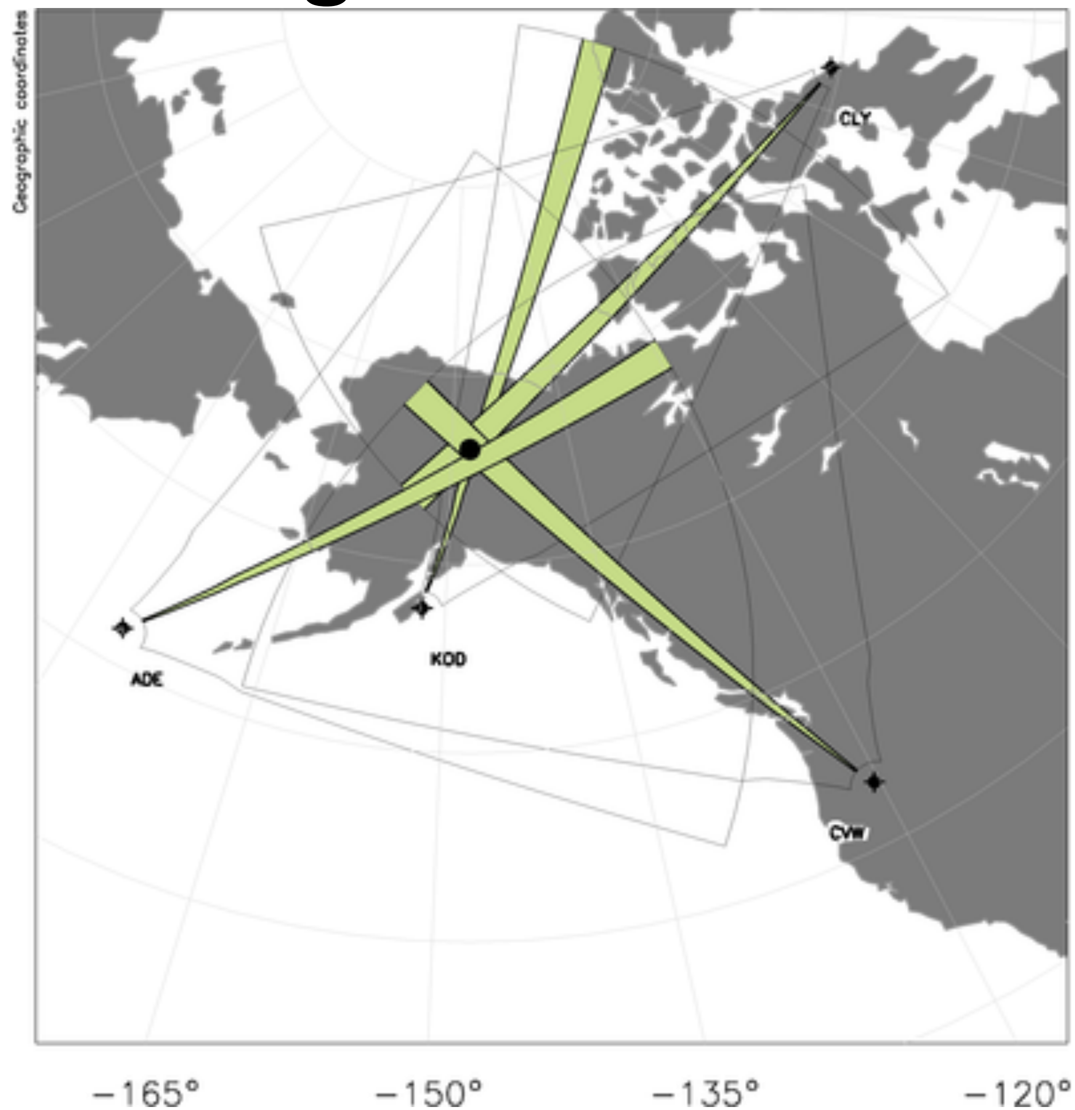
SuperDARN coverage for PFISR

-Adak, AK
(UAF) ~2.2 km

-Kodiak, AK
(UAF) ~1 km

-Christmas Valley, OR
(Darmouth) ~3 km

-Clyde River, CA
(USask) ~3 km

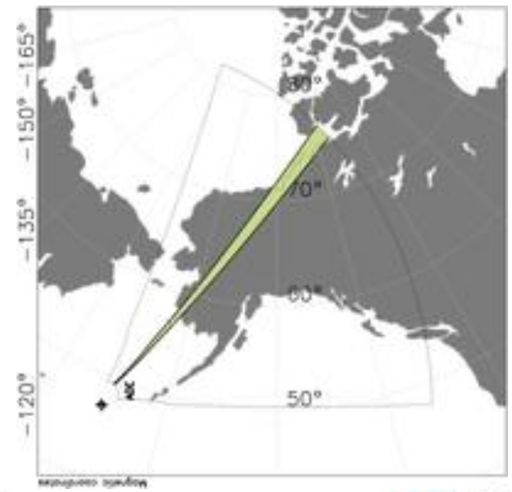


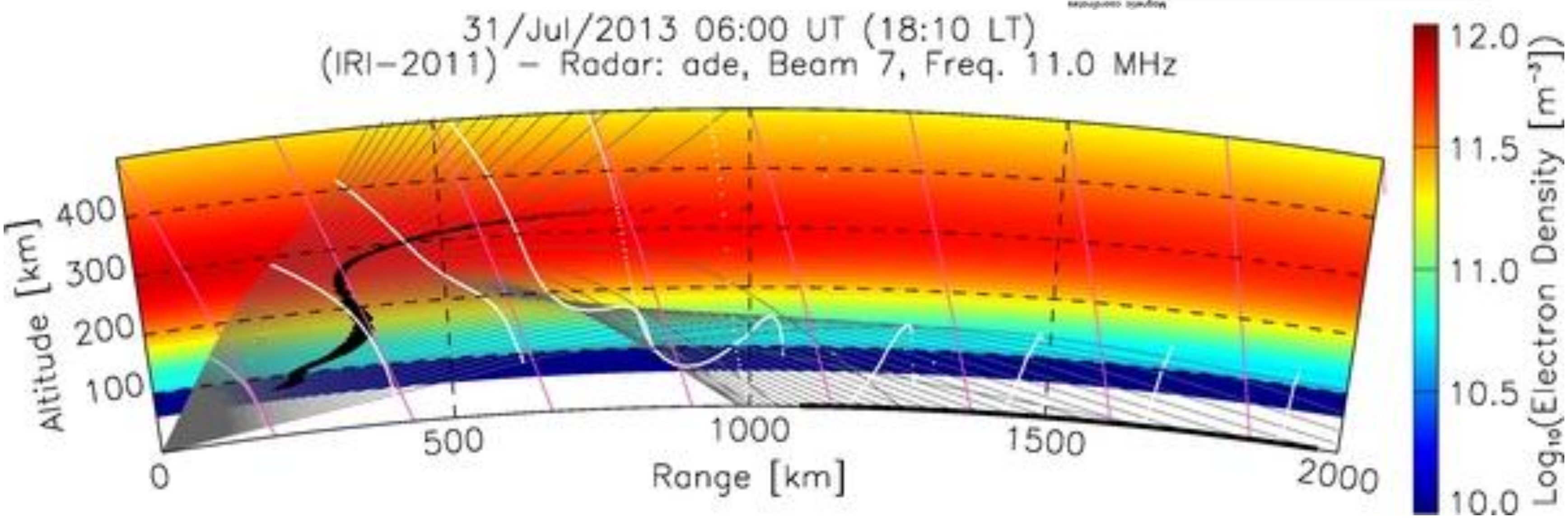
Visualization (Ray Tracing)

-Rays refract in the ionosphere!

$$f_{critical} \cong 9\sqrt{N}$$

$$f_{muf} = \frac{f_{critical}}{\sin\theta}$$





Measurement Decisions

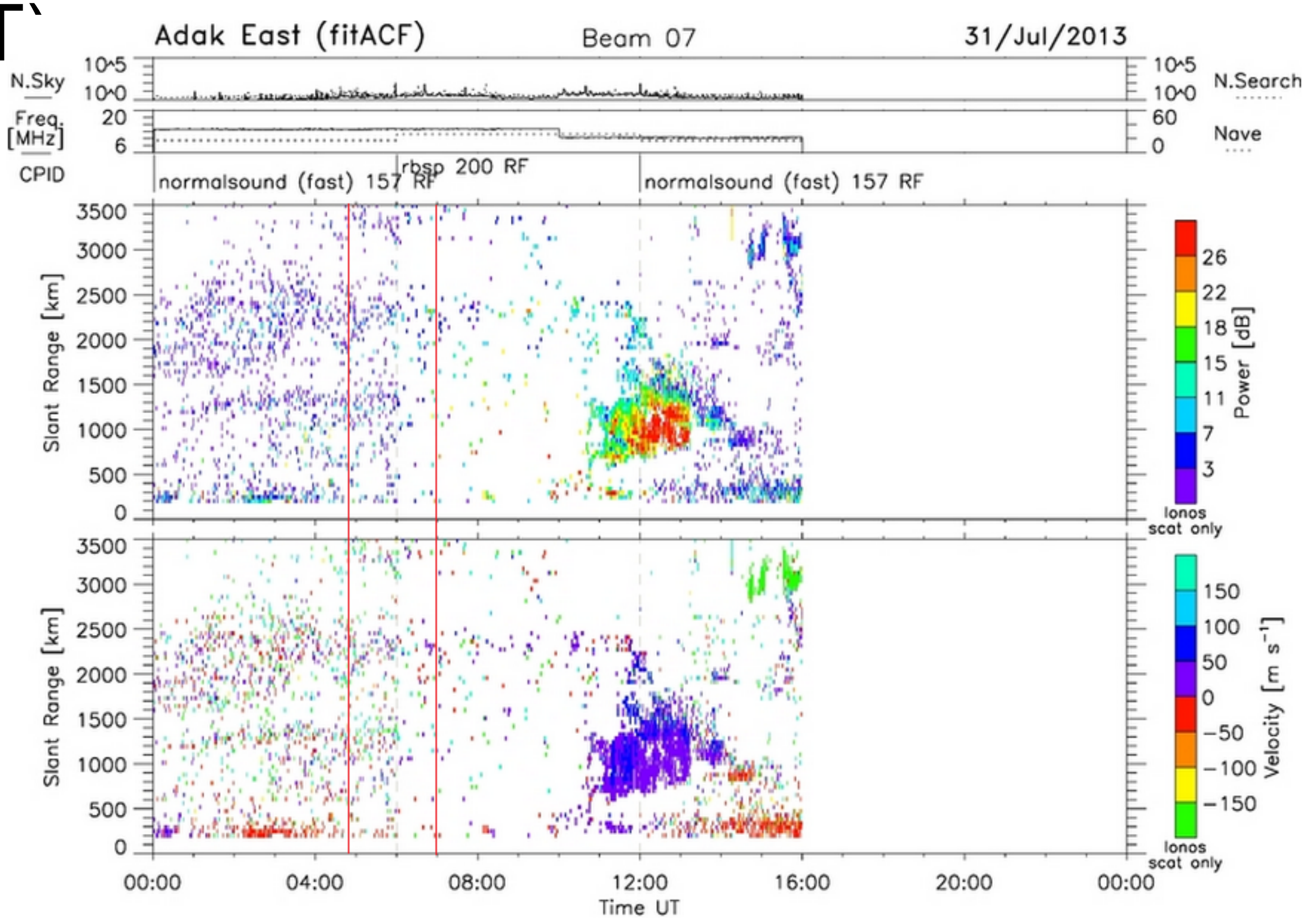
-Will use Adak East radar (Kodiak has no data for the 31st... typical SuperDARN state)

-Look for possible ionospheric structures seen by both PFISR and SuperDARN

-Compare velocities to PFISR measurements and look for correlations!

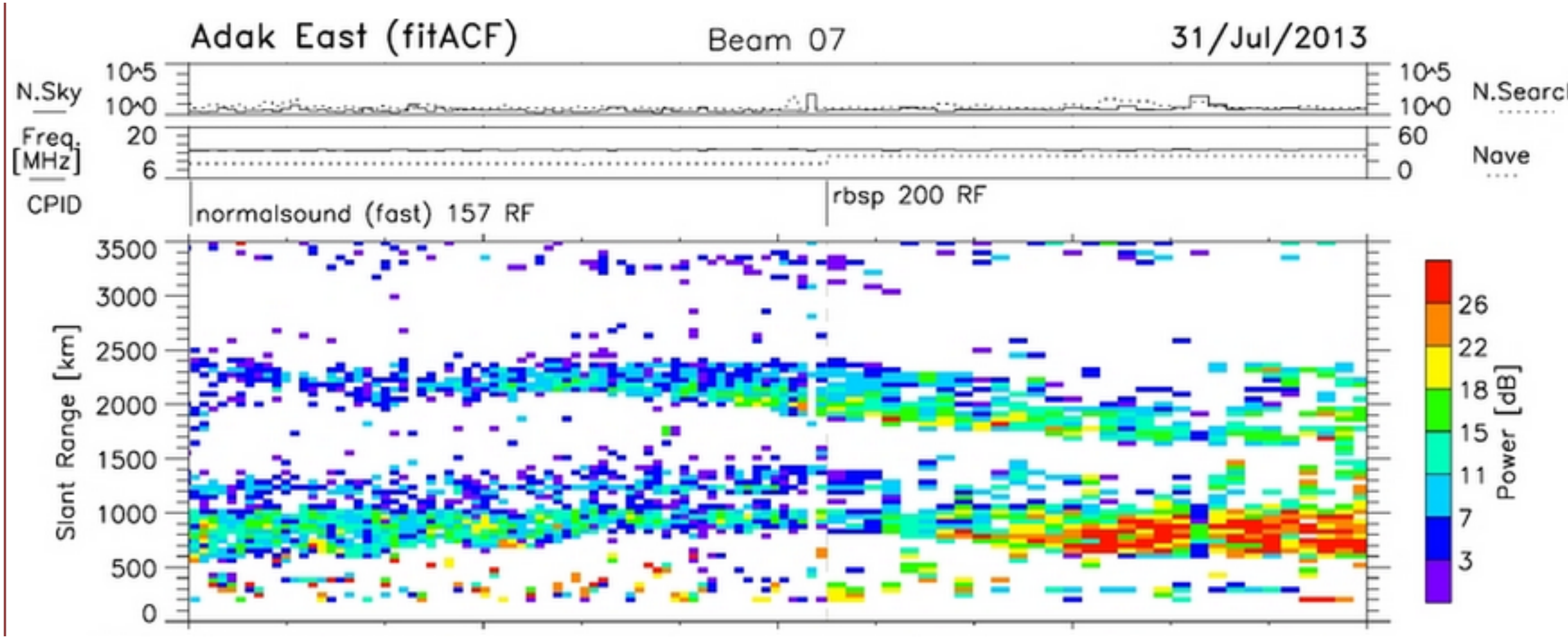
THERE'S NOTHING

No good ionospheric scatter (4:50 UT - 7:00 UT`



Ground Scatter

Anything interesting in SuperDARN data...?



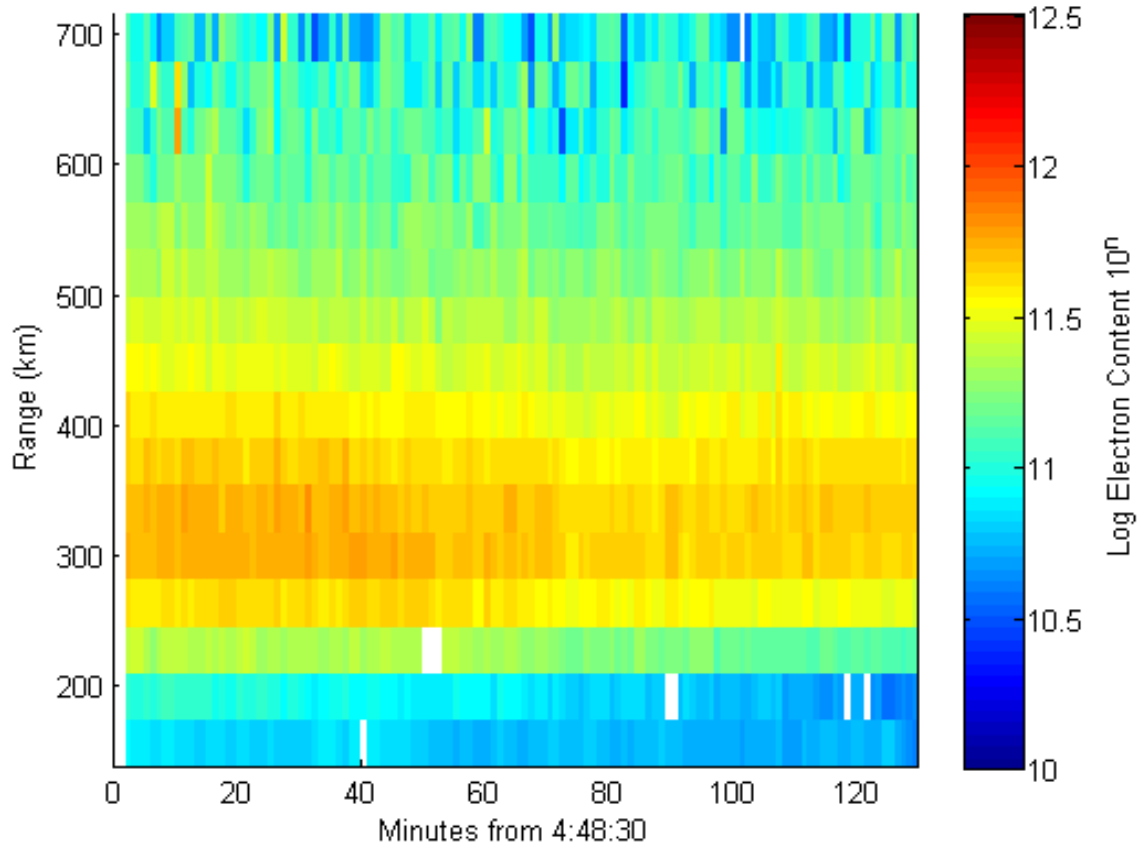
(Data is 4:50 UT to 7:00 UT)

PFISR Data

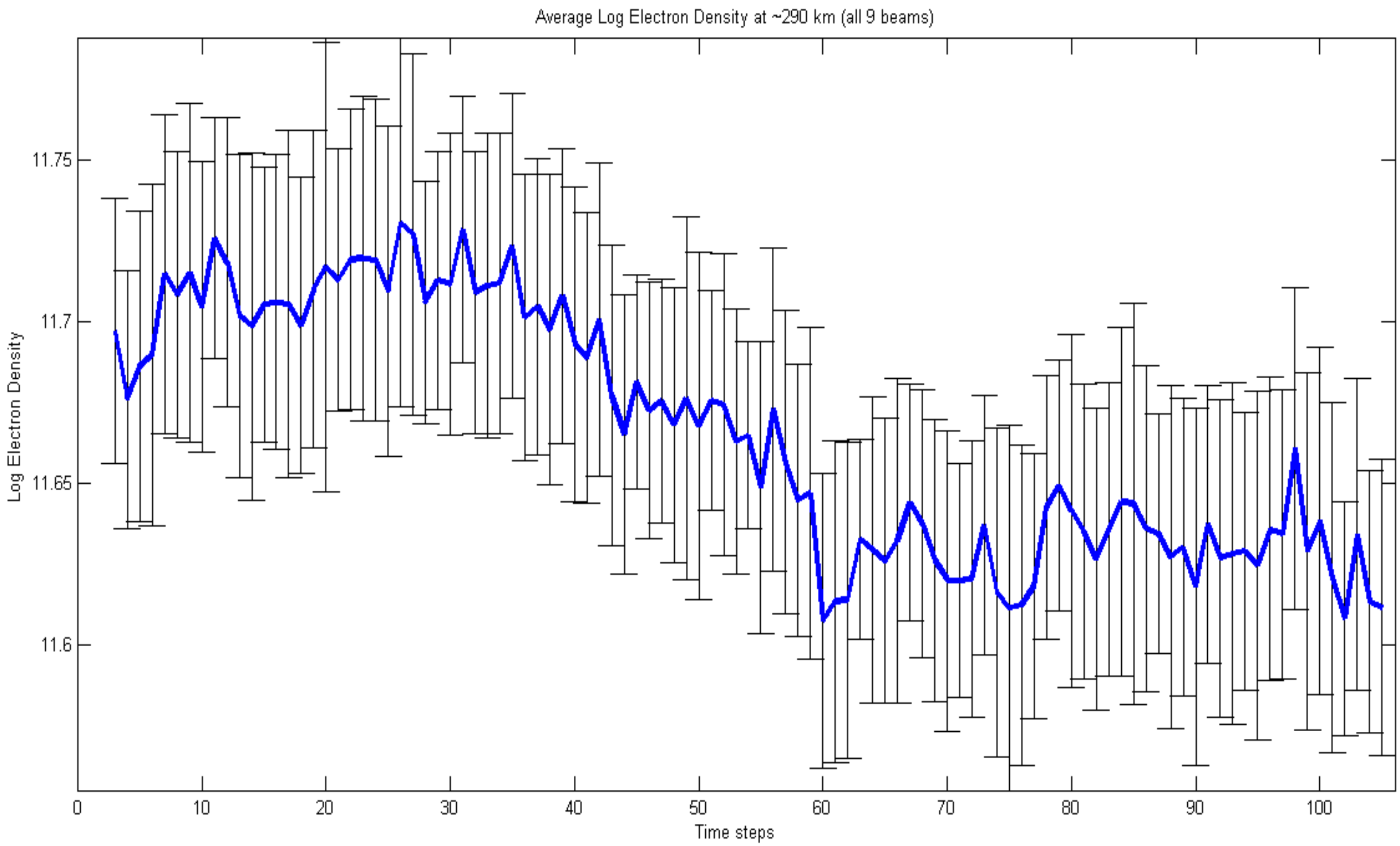
Possible decreasing trend in electron density!

-Log scale total electron content

-Data is from 4:48 to 7:00 UT



Trend seen in all 9 beams! (PFISR)



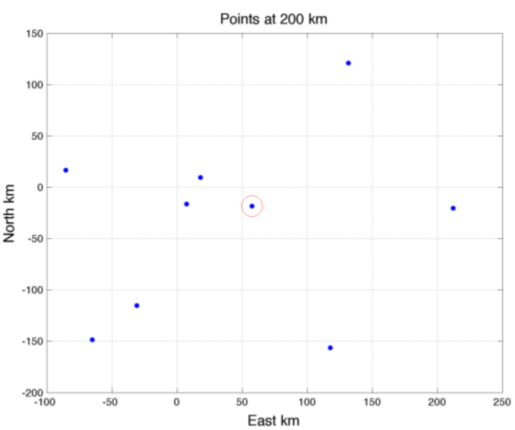
Data from 4:48 UT to 7:00 UT

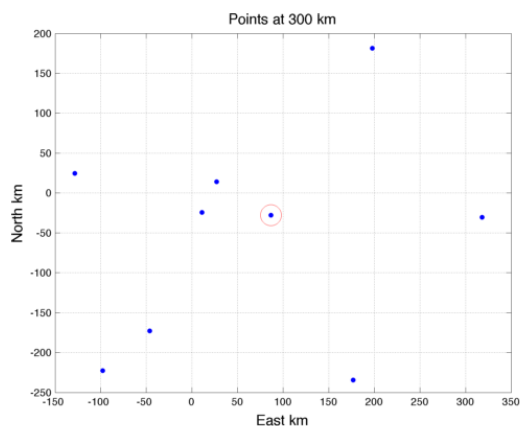
Gradient Measurement

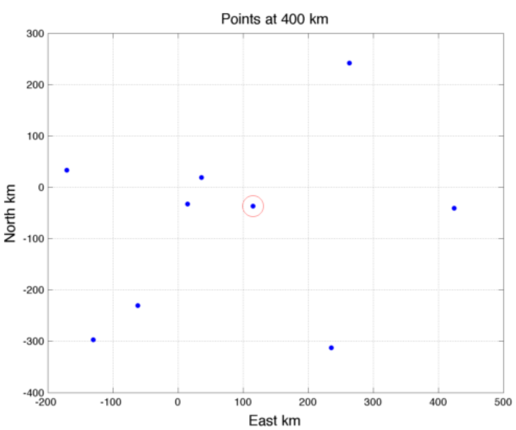
- Want to understand horizontal gradients
- First question how big are the gradients compared to error?
- What trade offs are there between measuring both error and the gradient?

Compare Error vs Gradient

- Chose three altitudes to look at
  - 200km, 300km, 400km
- Nearest Neighbors interpolation in altitude
  - Take the nearest range gate as error and parameter measurement

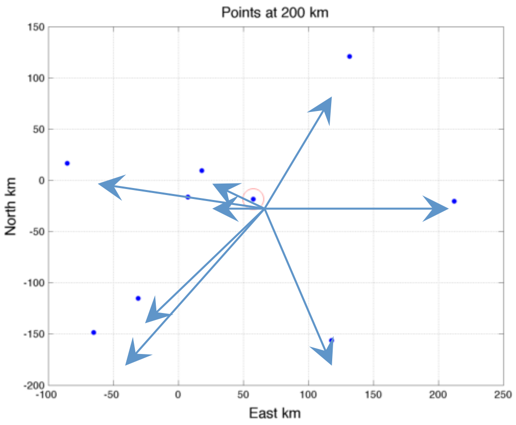


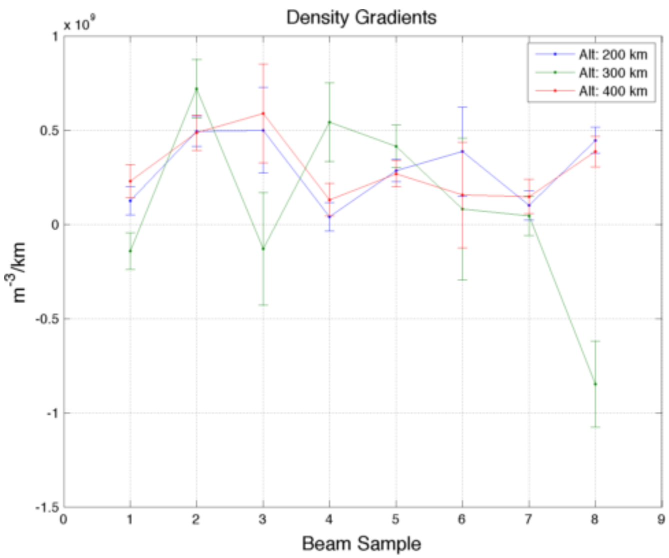




Compare Error vs Gradient

- Difference from each point
- With the errors assume independent normal random variables
  - Should assume in some way they are correlated





Inverse Method

- Assume measurement can be represented as a plane

$$ax + by + c = Ne$$

- Can make this in term of an inverse problem

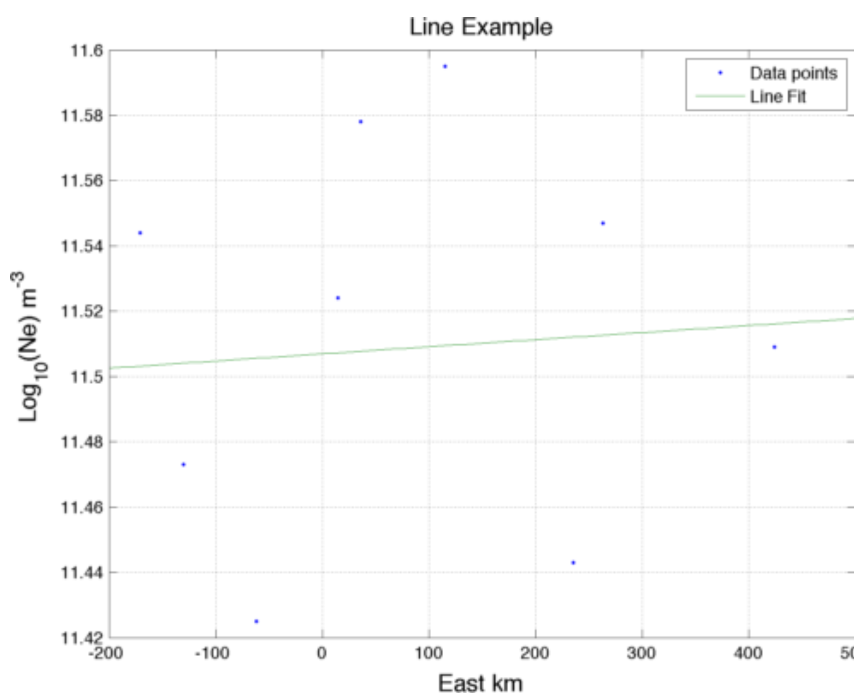
$$\mathbf{m} = \mathbf{A}\mathbf{x} + \xi$$

$$\hat{\mathbf{x}} = \left(\mathbf{A}^T \Sigma^{-1} \mathbf{A}\right)^{-1} \mathbf{A}^T \Sigma^{-1} \mathbf{m}$$

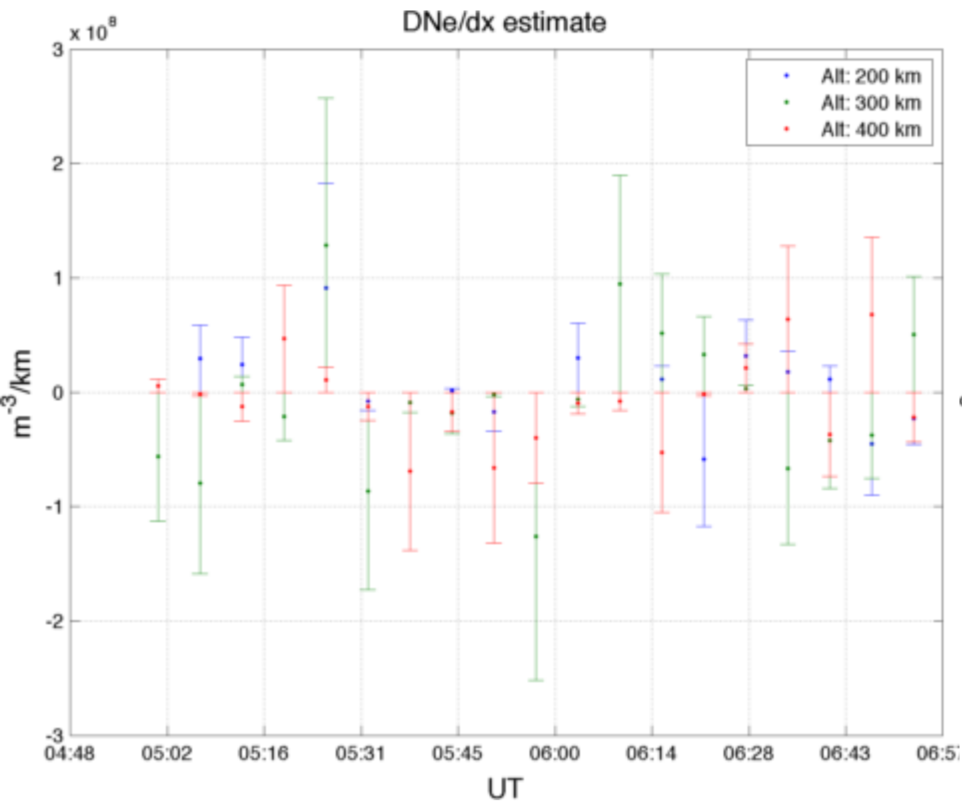
$$\Sigma_{posterior} = \left(\mathbf{A}^T \Sigma^{-1} \mathbf{A}\right)^{-1}$$

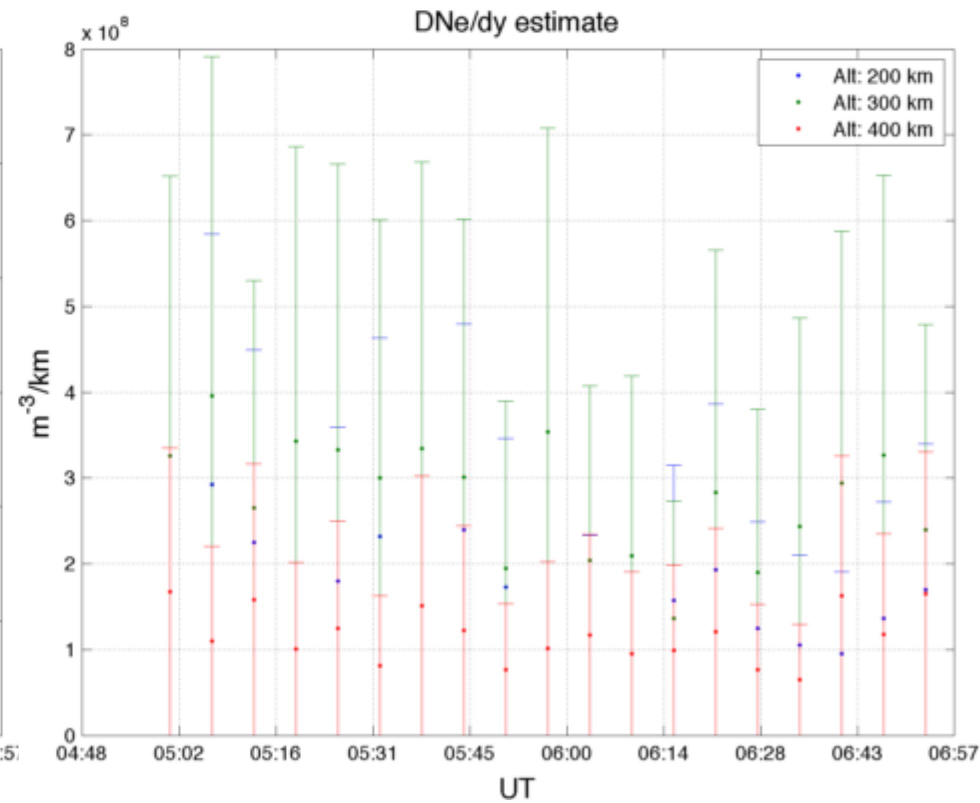
Inverse Method

- Similar to a line fit
- Gets an error for the slope
  - Slope = 1.62e7 m^{-3}/km
  - Error = 6.29e7 m^{-3}/km

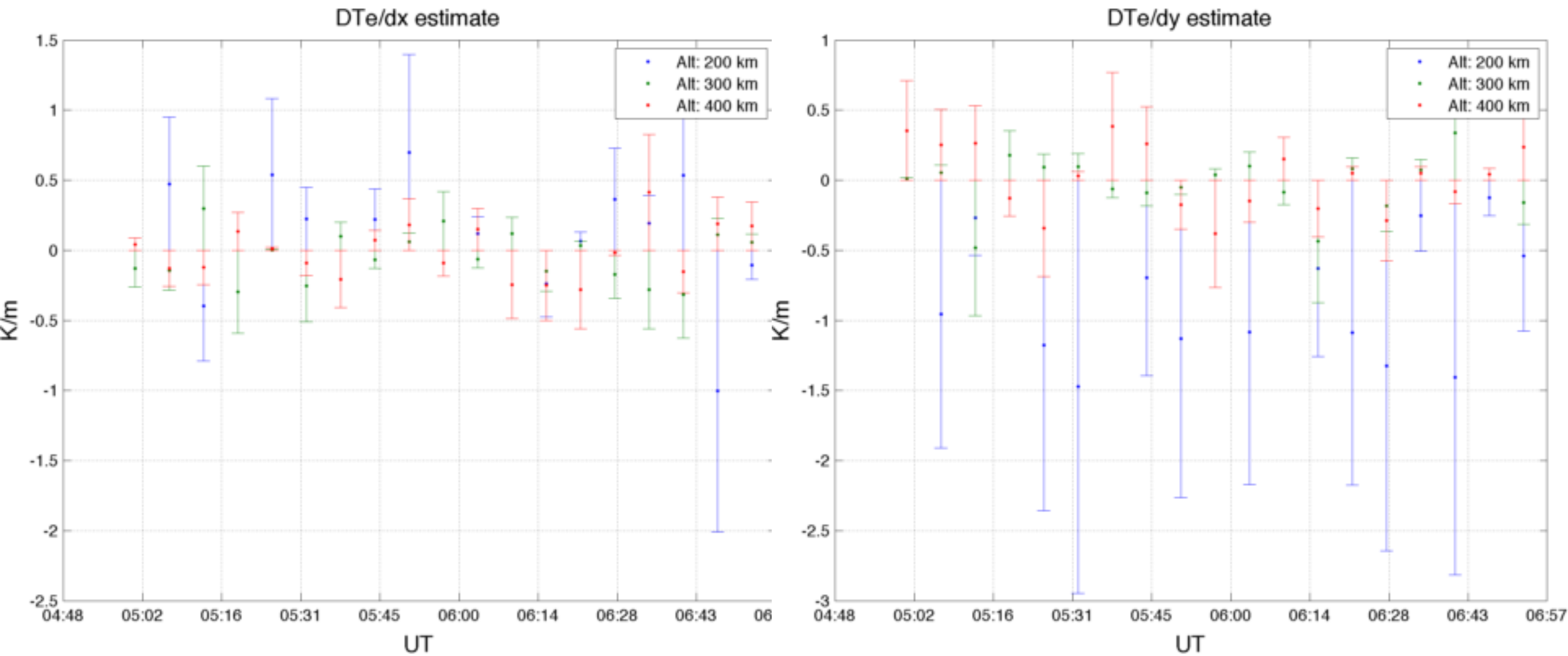


Density Gradient

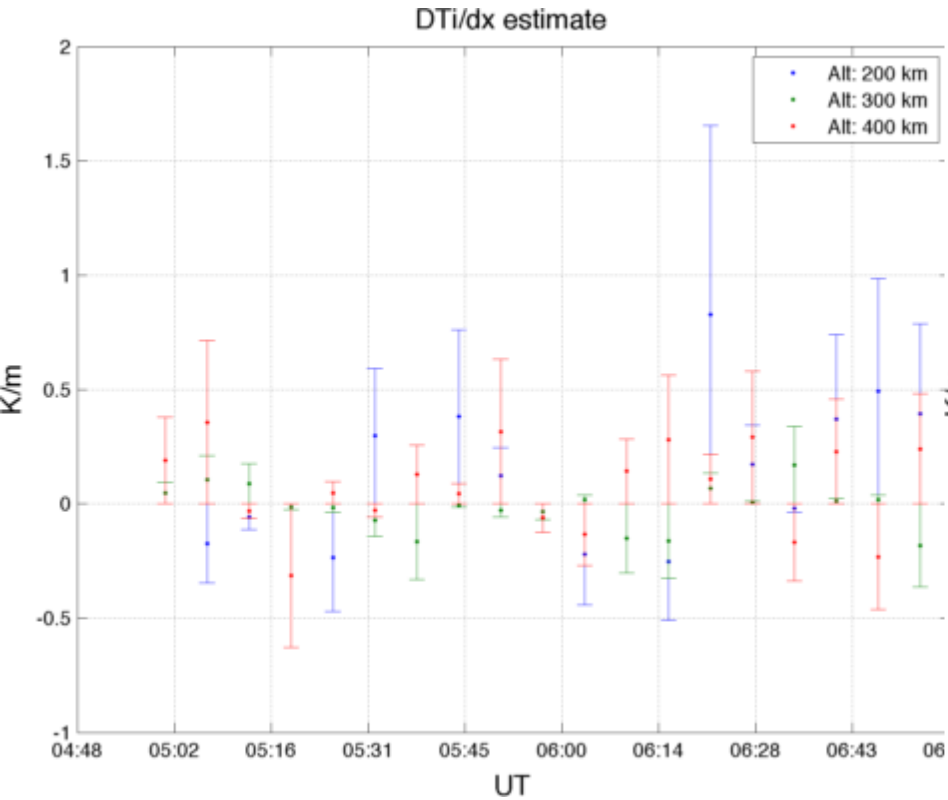




Te Gradient


DTe/dx estimate
DTe/dy estimate
Alt: 200 km
Alt: 300 km
Alt: 400 km
K/m
UT
04:48
05:02
05:16
05:31
05:45
06:00
06:14
06:28
06:43
06:57

Ti Gradient


DTi/dx estimate
Alt: 200 km
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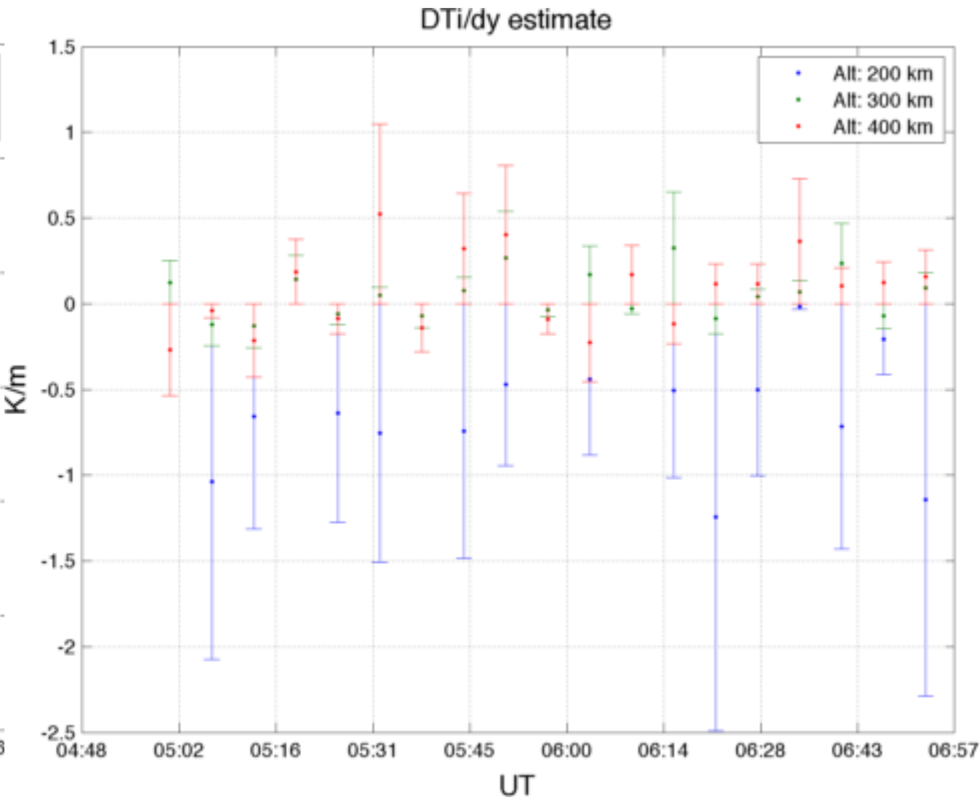
DTi/dy estimate
Alt: 200 km
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05:45
06:00
06:14
06:28
06:43
06:57

Results

- We see southward Temperature gradients
- We also see Northward Density increase