ISR Experiments, Data Reduction, and Analysis

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AMISR Summer School, July 2013

Outline

Overspread Targets

(a.k.a, frequency and range aliased targets)

- For a target with a bandwidth B, you must sample at a rate F_s exceeding B (e.g., for IS at 450 MHz, $B \sim 40$ kHz).
- For a target which could be as far away as R_{max}, radar pulses must be at least $2R_{max}/c$ apart.

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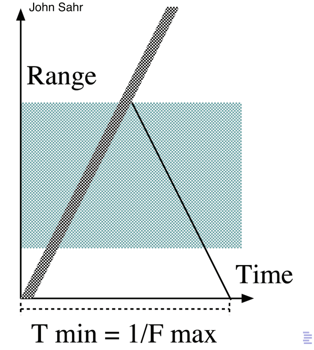
- $B < F_s < \frac{c}{2R_{max}}$
- or: $B\frac{2R_{max}}{c} < 1$
- At 450 MHz, $B \sim 40$ kHz, $R \sim 750$ km (5 ms) $\rightarrow$ highly overspread
- Do we get the range right or the spectrum right??



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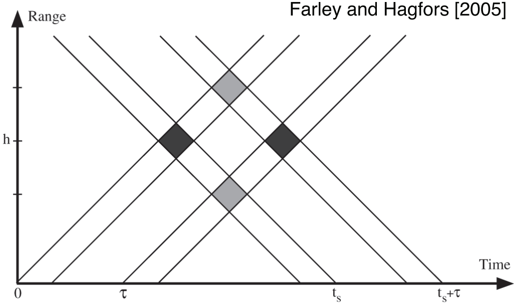
Farley and Hagfors [2005]



$v_1 = v_h(t_s) + v_{h-\delta}(t_s), \; v_2 = v_h(t_s+\tau) + v_{h+\delta}(t_s+\tau), \; \langle v_1 v_2^* \rangle = ?$

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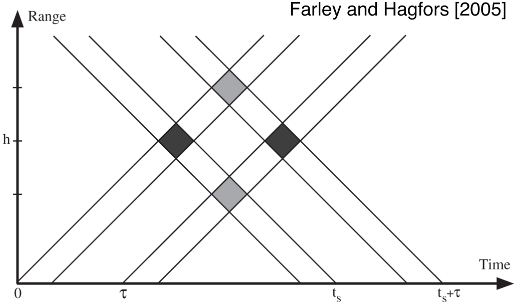

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$$v_1 v_2^* = v_h(t_s)v_h^*(t_s+\tau) + v_h(t_s)v_{h+\delta}^*(t_s+\tau) + v_{h-\delta}(t_s)v_h^*(t_s+\tau) + v_{h-\delta}(t_s)v_{h+\delta}^*(t_s+\tau)$$

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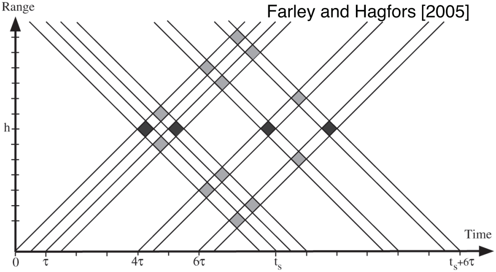
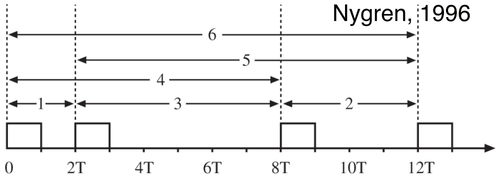
Generalization - Multipulses





Measurement Statistics

$v_1 = v(t)$, $v_2 = v(t+\tau) \rightarrow v_1 = x_1 + ix_2$, $v_2 = x_3 + ix_4$ where the scattering process is represented by the 4-dimensional joint Gaussian probability distribution, $p(x_1, x_2, x_3, x_4)$.

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Defining ρ is the normalized acf (complex) and $S = 2\sigma^2$ is the signal power.

$$\langle v_1 v_2^* \rangle = S\rho(\tau) = \langle (x_1 + ix_2)(x_3 - ix_4) \rangle = c_{13} + c_{24} + i(c_{23} - c_{14})$$

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Covariance matrix of p is then:

$$C = \sigma^2 \begin{bmatrix} 1 & 0 & \rho_r & -\rho_I \\ 0 & 1 & \rho_I & \rho_r \\ \rho_r & \rho_I & 1 & 0 \\ -\rho_I & \rho_r & 0 & 1 \end{bmatrix}$$

Measurement Statistics - Signal Power

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With variance:

$$\sigma_{\hat{S}}^2 = \langle (\hat{S} - S)^2 \rangle = \langle \hat{S}^2 \rangle - S^2$$

$$\langle \hat{S}^2 \rangle = ?$$

Measurement Statistics - Signal Power

fourth-moment theorem: $\langle v_1 v_2 v_3 v_4 \rangle = \langle v_1 v_2 \rangle \langle v_3 v_4 \rangle + \langle v_1 v_3 \rangle \langle v_2 v_4 \rangle + \langle v_1 v_4 \rangle \langle v_2 v_3 \rangle$.

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\langle \hat{S}^2 \rangle &= \frac{1}{K^2} \left[K 2S^2 + (K^2 - K) S^2 \right] \\
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$$5\% : K = 1/(0.05)^2 = 400; 1\% : K = 1/(0.01)^2 = 10^4.$$

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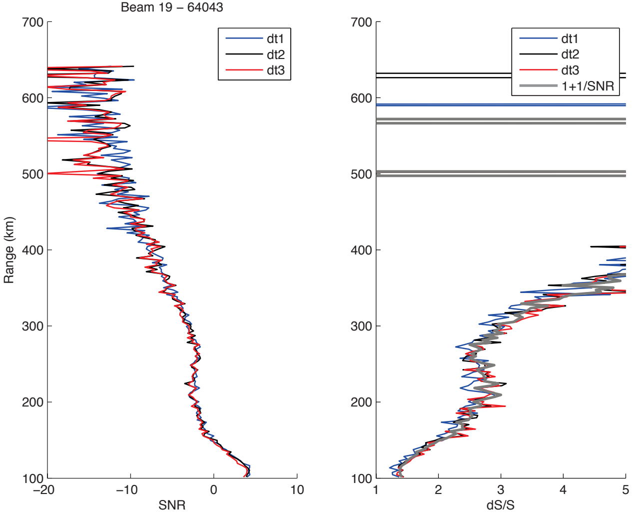
$$\frac{\sigma_{\hat{S}}}{S} \approx \frac{1}{\sqrt{K}}\left(1 + \frac{N}{S}\right)$$

Implications of this formula?

PFISR Data - Additive Noise Example

41 beam experiment, tri-frequency 240 us pulses, ∼2500 pulses per beam in 5 minutes



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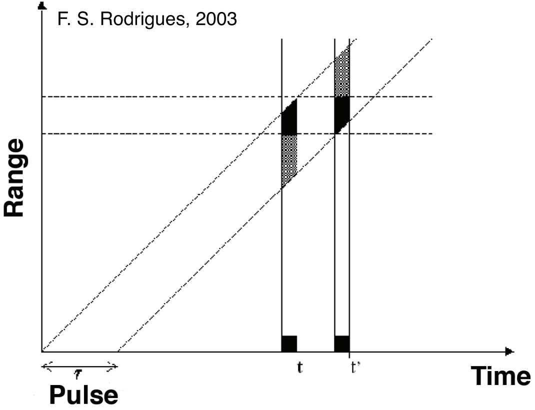
The received signal is a weighted sum of elementary signals from all volume elements times, and the weight in this sum is given by the amplitude ambiguity function. (Nygren, 1996).

Ambiguity Function

Long-pulse of length τ, sampled at t and t' with a box-car impulse response.



Ambiguity Function

What we really care about is the ambiguity for a lagged product (estimate of the autocorrelation function at a given lag).

$$\langle v_h(t) v_h^*(t') \rangle = R \int_{\mathbf{r}} P_e(\mathbf{r}) \left[\int_{-\infty}^{\infty} W_{t,t'}(\nu, \mathbf{r}) \sigma_e(\nu, \mathbf{r}) d\nu \right] d\mathbf{r}$$

where $\nu = t - t'$ and

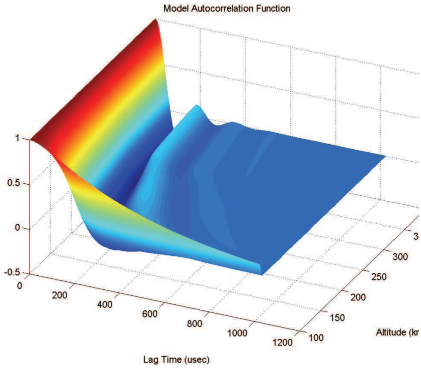
$$W_{t,t'}(\nu, \mathbf{r}) = \int_{-\infty}^{\infty} W_t^A(\tau, \mathbf{r}) W_{t'}^{A*}(\tau - \nu, \mathbf{r}) d\tau$$

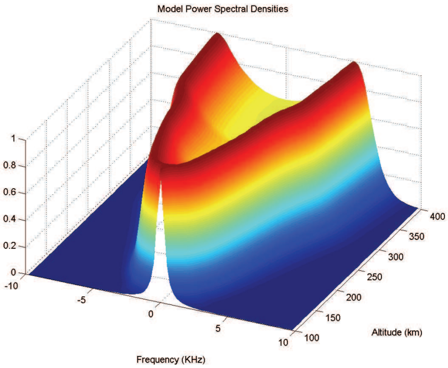
(cross-correlation of two amplitude ambiguity functions, in time direction)

The estimated lagged product is a weighted average of the plasma acf in both space and time. These weights are given by $W_{t,t'}$.

Experiments

To design an effective an experiment, we need to know our target. Why?





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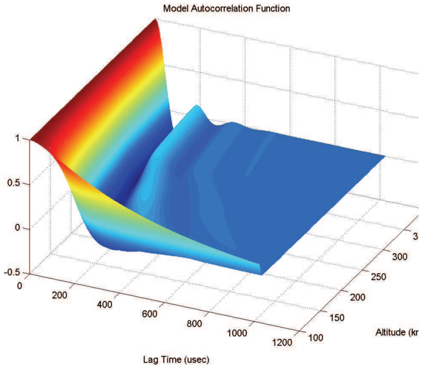
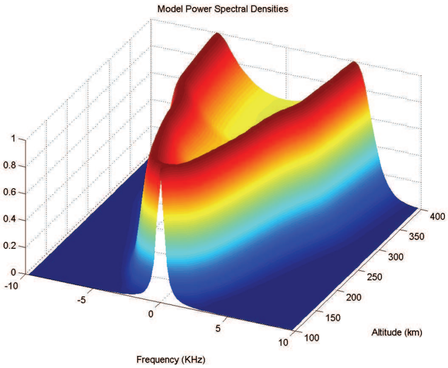
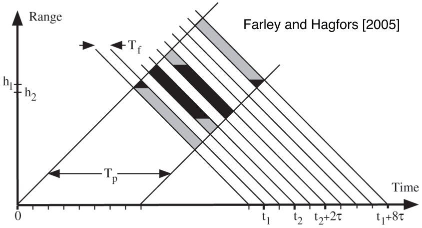




Much of what I present next will be specific to ISRs within a specific range of frequencies (∼VHF-UHF).

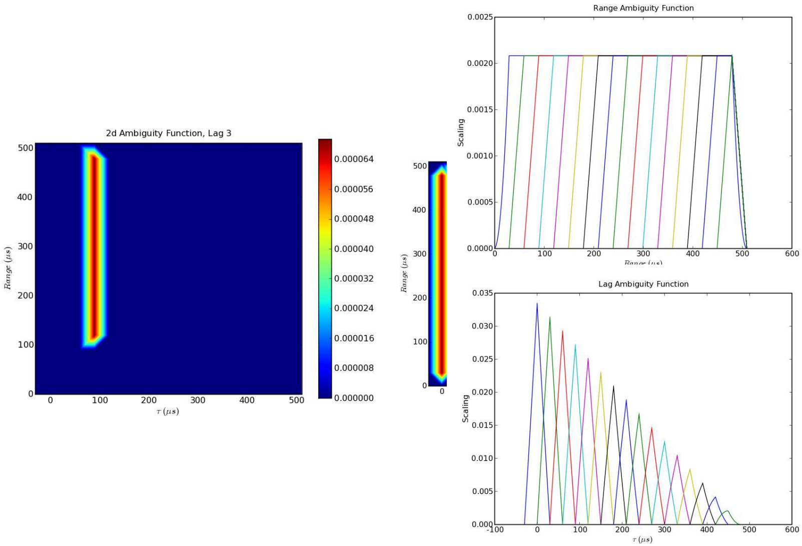
Standard *F*-region Experiment - Long Pulse



- At high altitudes, use a single long pulse with mismatched filter (oversampled) to measure all lags of the ACF at once
- Sacrifice range resolution
- E.g., 300-500 μs pulse (F region) or even 1-2 ms (topside)

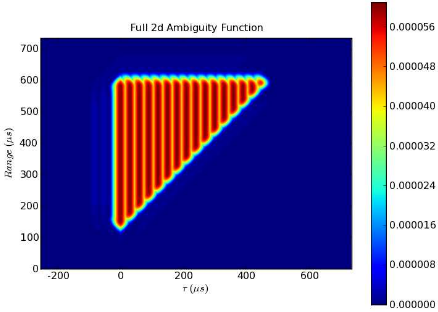
Long Pulse Ambiguity Function

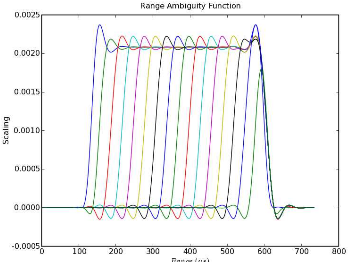
Ambiguity function with a boxcar filter. 480 μs long pulse, 30 μs sampling.

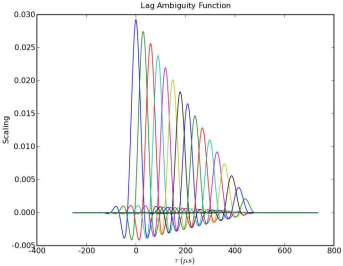


Long Pulse Ambiguity Function

- Ambiguity function including filter effects.
- 480 μs long pulse, 30 μs sampling.
- With filter effects.







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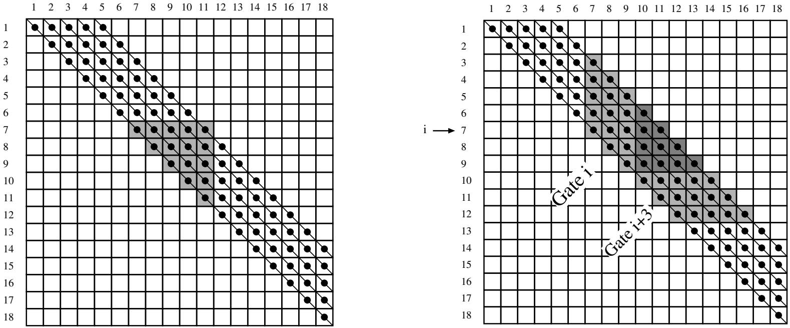

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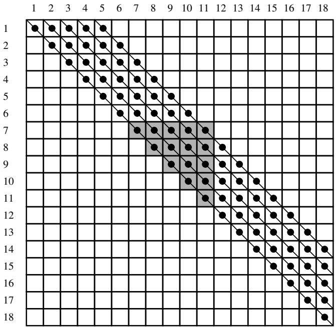
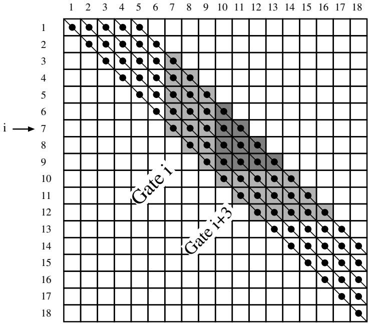
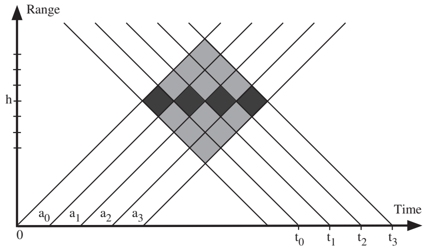




Nygren, 1996

A better method - treat as an inverse problem: deconvolution or full profile methodologies. These are active areas of research.

Standard *E*-region Experiment - Coded Pulse

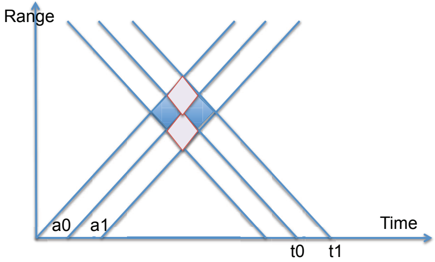


Farley and Hagfors [2005]

E.g., consider lag estimate using $v(t_0)$ and $v^*(t_1)$ - choose a_n such that clutter terms cancel.

- At lower altitudes, we require better range resolution.
- For this, we utilize binary coded pulse ACF measurements (do not compress pulse or eliminate clutter like BC - eliminate correlation of clutter)
- Random (CLP) or alternating (cyclic codes)
- E.g., for AMISR standard experiment is 480 μs, 16-baud (4.5 km), randomized strong code (32 pulses) with an uncoded 30 μs pulse for zero-lag normalization.

Exercise - 2-baud Alternating Code



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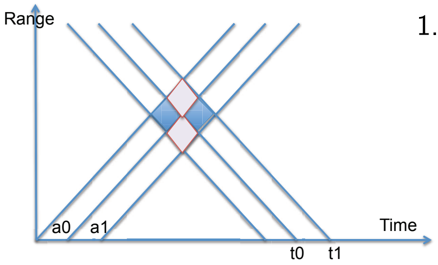

1. How many pulses do you need?

Lag estimate using $v(t_0)$ and $v^*(t_1)$ - choose a_0 and a_1 such that clutter terms cancel.

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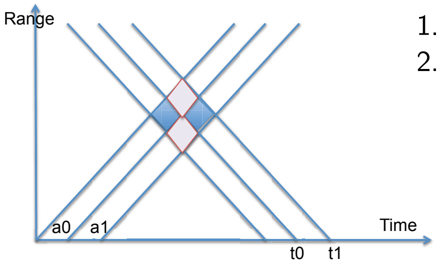

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1. How many pulses do you need?
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	a_0	a_1
*Pulse*1	?	?
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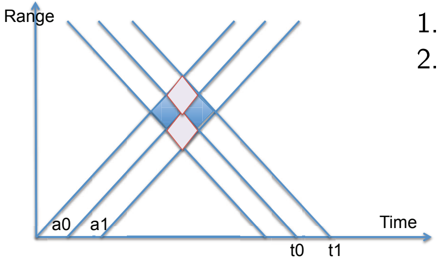

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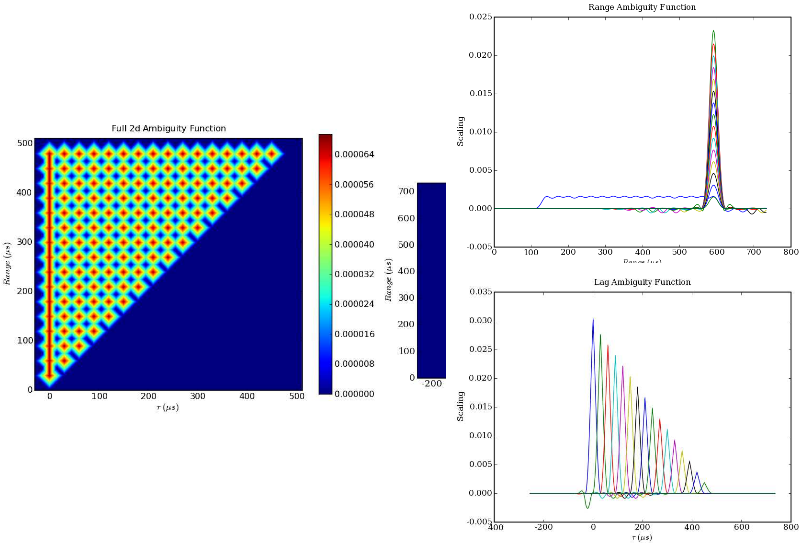
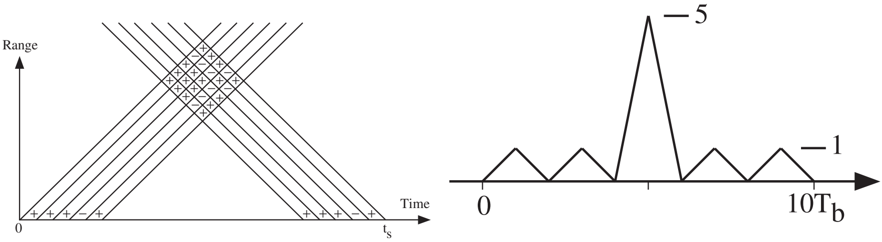
$$\langle a_0 v_0 a_1 v_1^* \rangle = \ldots$$

Standard E-region Experiment - Ambiguity Function

Ambiguity function including filter effects. 480 μs (16-baud, 30 μs baud, 32 pulse).



Standard E/F-region Power Measurement



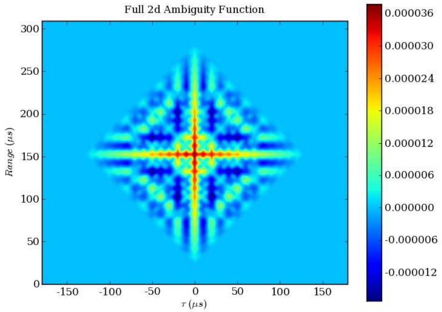
Farley and Hagfors [2005]

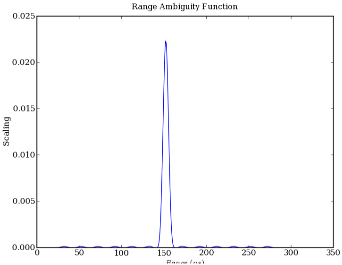
- Pulse compression code allow for high sensitivity, high range resolution power measurements.
- Plasma must remain correlated over pulse length (limits range of use for most systems).
- Typical code is 13-baud Barker code, 130 μs.

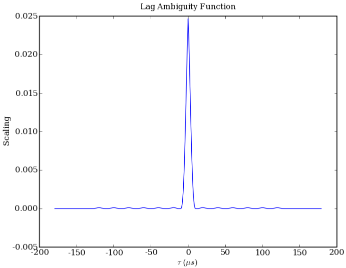
E/F-region Power Measurement - Ambiguity Function

Ambiguity function including filter effects.

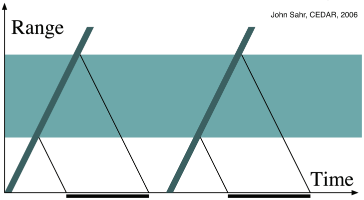
130 μs (13-baud, 10 μs baud, 5 μs sampling).







Standard *D*-region Experiments



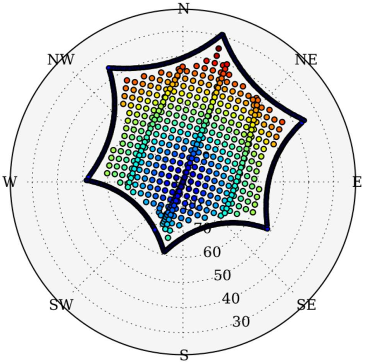
- Long correlation times (narrow spectral widths) in the *D* region require pulse-to-pulse techniques
- E.g., PFISR employs coded double-pulse techniques that give range resolutions up to 600 m and spectral resolutions up to 1 Hz.

Mode	Pulse	Baud	δR	τ	IPP	δf	Nyquist	δt
0	130 μs	10 μs	1.5 km	5 μs (0.75 km)	2 ms	2 Hz	250 Hz	1 s
1	260 μs	10 μs	1.5 km	5 μs (0.75 km)	4 ms	1 Hz	125 Hz	2.5 s
2	130 μs	10 μs	1.5 km	5 μs (0.75 km)	2 ms	2 Hz	250 Hz	1.8 s
3	280 μs	10 μs	1.5 km	5 μs (0.75 km)	3 ms	1.3 Hz	167 Hz	2.7 s
4	112 μs	4 μs	0.6 km	2 μs (0.3 km)	3 ms	1.3 Hz	167 Hz	2.7 s

PFISR System Information

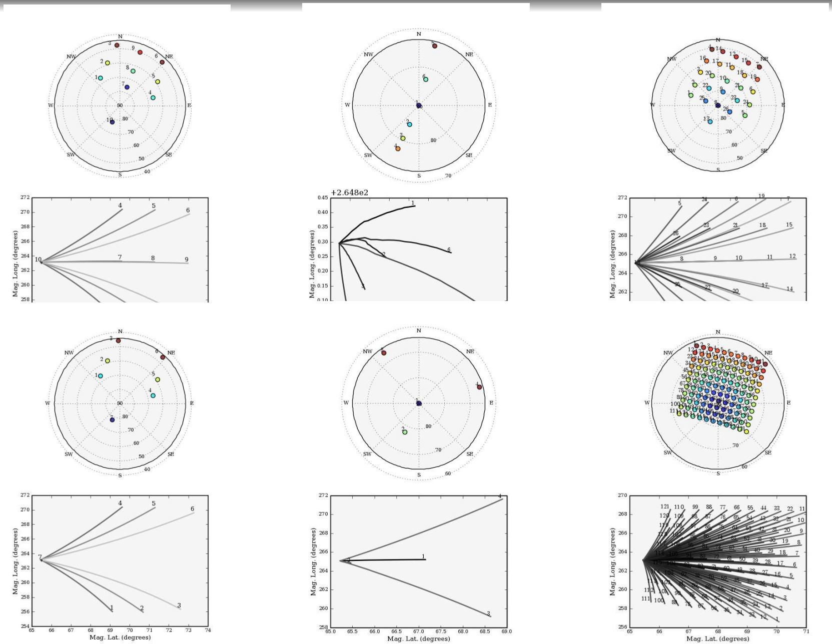
- 128-panel AMISR system (upgraded from 96 in Sep. 07)
- Pulse-to-pulse phase capability
- $\sim$1.6 MW peak Tx (upgraded from $\sim$1.3 MW)
- $\sim$10% max duty cycle
- 4 reception channels
- Tx band 449-450 MHz
- 3.5 MHz max Rx bandwidth
- 4 μs min pulsewidth (freq. allocation limitation)
- Fully programmable, remotely operable/ted
- Graceful degradation - reliable operations

Beam Pointing



- Range of pointing positions within grating lobe limits
- "Normal" experiments include ~1-10 beams
- Main limitation is integration time / sensitivity

Beam Pointing



General

A typical experiment consists of:

- Data samples
- Noise samples
- Cal pulse samples

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- Interleaving of pulses (possibly on different frequencies)
- Clutter considerations, Noise & Cal sample placement
- Maximization of duty cycle
- Beam pointing, Distribution of pulses, Integration time considerations
- All this can be very complicated

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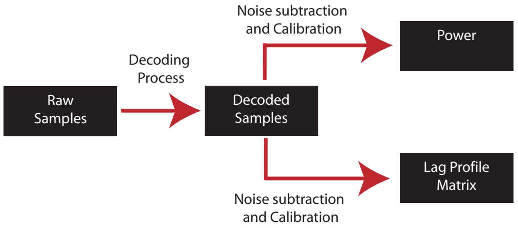

Power Estimation

Received power can be written as

$$P_r = \frac{P_t \tau_p}{r^2} K_{sys} \frac{N_e}{(1 + k^2\lambda_D^2)(1 + k^2\lambda_D^2 + T_r)} \text{ Watts}$$

where

P_r - received power (Watts)

P_t - transmit power (Watts)

τ_p - pulse length (seconds)

r - range (meters)

N_e - electron density (m^{-3})

k - Bragg scattering wavenumber (rad/m)

λ_D - Debye length (m)

T_r - electron to ion temperature ratio

K_{sys} - system constant (m^5/s)

Power Estimation

Received signal power needs to be calibrated to absolute units of Watts. To do this, we in general (a) take noise samples and (b) inject a calibration pulse (at each AEU for AMISR), which is then summed in the same way as the signal. The absolute calibration power in Watts is:

$$P_{cal} = k_B T_{cal} B \quad \mathrm{Watts}$$

where

k_B - Boltzmann constant (J/kg K)

T_{cal} - temperature of calibration source (K)

B - receiver bandwidth (Hz)

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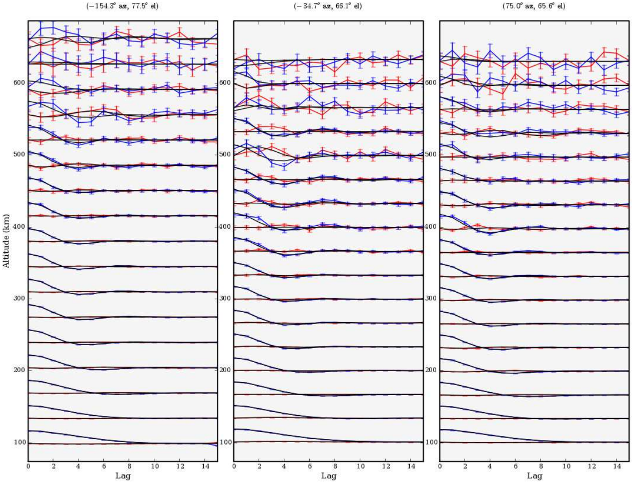
The measurement of the calibration power (after noise subtraction) can then be used as a yardstick to convert the received power to Watts. This is done as,

$$P_r = P_{cal} * (Signal - Noise)/(Cal - Noise) \quad \text{Watts}$$

ACF / Spectra Estimation - E/F region



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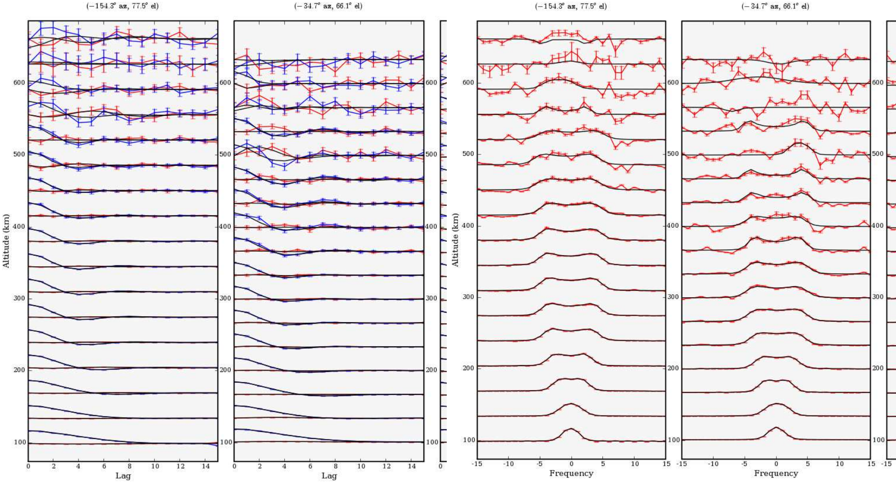

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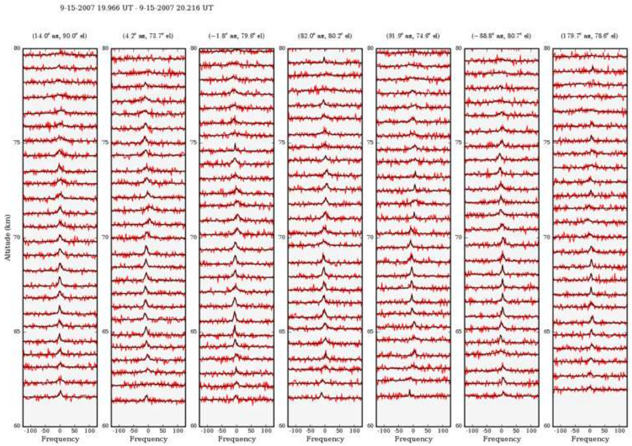

Electron Density

Recall,

$$P_r = \frac{P_t \tau_p}{r^2} K_{sys} \frac{N_e}{(1 + k^2\lambda_D^2)(1 + k^2\lambda_D^2 + T_r)} \text{ Watts}$$

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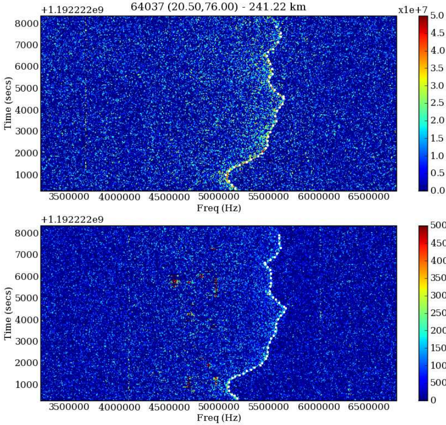
Within K_{sys} is embedded information on the gain, which for a phased-array varies with the look-angle off boresight, as well as the proximity to the grating lobe limits.

Electron Density

$$f_r^2 \approx f_p^2 + \frac{3k^2}{4\pi^2}\frac{k_B T_e}{m_e} + f_c^2 \sin^2 \alpha$$

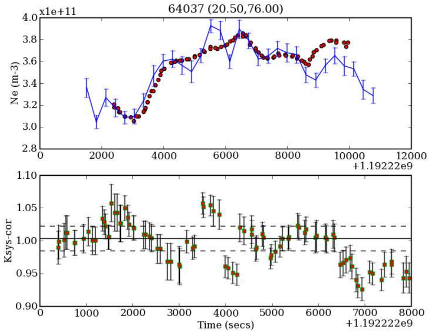
where

f_r - plasma line frequency (Hz)
f_p - plasma frequency (Hz)
T_e - electron temperature (K)
m_e - electron mass (kg)
f_c - electron cyclotron frequency (Hz)
α - magnetic aspect angle



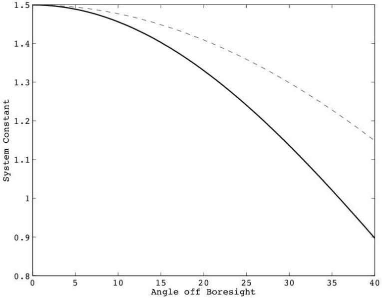
Electron Density

$$K_{sys} = A\cos^B(\theta_{BS}) \quad \mathrm{m}^5/\mathrm{s}$$

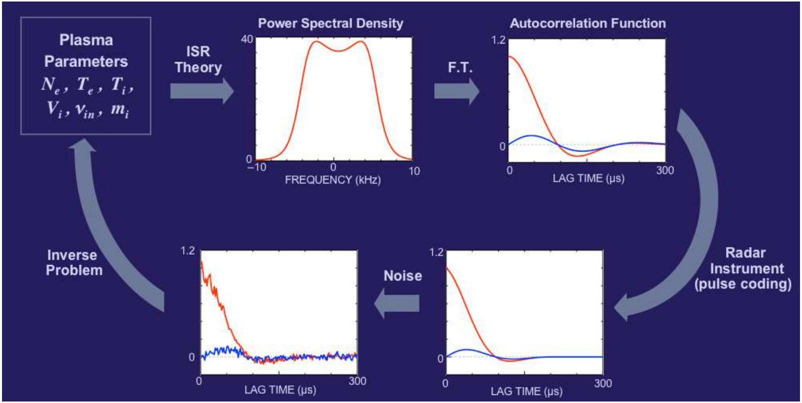


θ_{BS} - angle off boresight

A,B - constants



Fitting Spectra



Fitting Spectra

General Complicating Factors:

- Range smearing
- Lag smearing
- Pulse coding effects / "Self"-clutter
- Clutter (geophysical and not - e.g., mountains, irregularities, turbulence, non-Maxwellian)
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- *F*-region/Topside - Light ion composition
- Bottomside - Molecular ion composition
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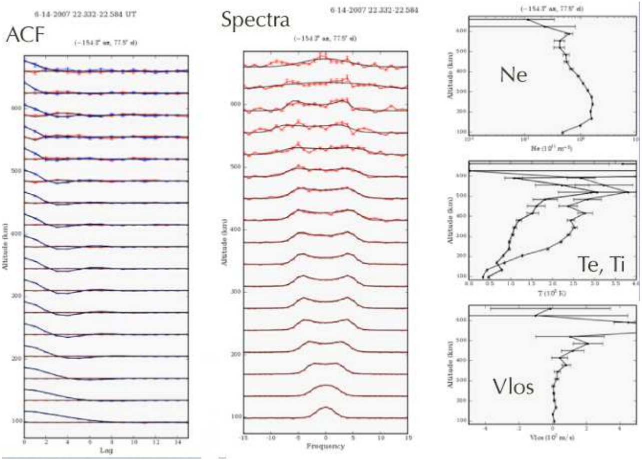
- D-region - Complete ambiguity

Approach:

- F-region - T_e, T_i, v_{los}, N_e
- Bottomside - Assume a composition profile
- E-region - $<\sim$ 105km, assume $T_e = T_i$
- D-region - Fit a Lorentzian (width, Doppler, N_e)

Fitting Spectra - Example



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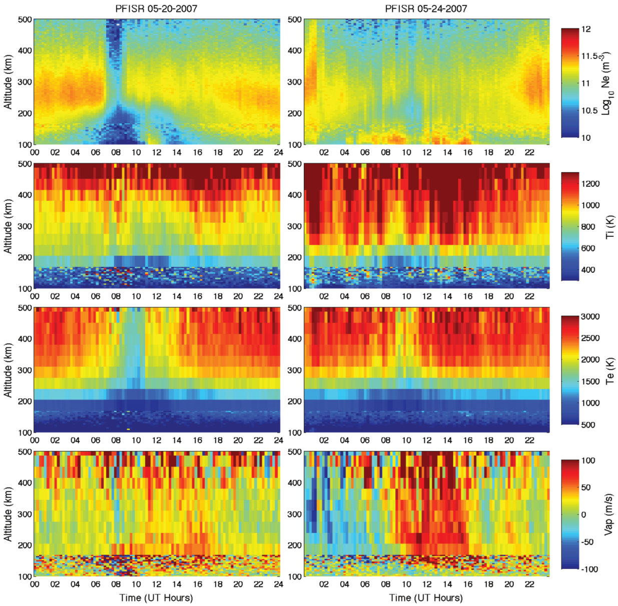

Ions: Magnetized or Unmagnetized?

Depends on ratio of gyrofrequency (qB/m_i) to collision frequency (ν_{in})

- Both winds and electric fields matter for the ions. Simple steady-state ion-momentum eqn:

$$0 = e(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - m_i \nu_{in} (\mathbf{v}_i - \mathbf{u})$$

$$C = \begin{bmatrix} (1+\kappa_i^2)^{-1} & -\kappa_i(1+\kappa_i^2)^{-1} & 0 \\ \kappa_i(1+\kappa_i^2)^{-1} & (1+\kappa_i^2)^{-1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $\kappa_i = eB/m_i\nu_{in} = \Omega_i/\nu_{in}$.

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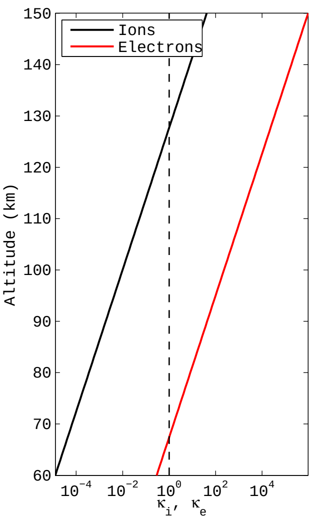
  where $\kappa_i = eB/m_i\nu_{in} = \Omega_i/\nu_{in}$. The vector velocity can then be solved for $\mathbf{v}_i = b_i C\mathbf{E} + C\mathbf{u}$ where $b_i = e/m_i\nu_{in} = \kappa_i/B$

- Whereas electrons are collisionless $\mathbf{v_e} = \mathbf{E} \times \mathbf{B}/B^2$
- Currents flow even in the absence of winds:

$$\mathbf{J} = n_e e(\mathbf{v}_i - \mathbf{v}_e) = \sigma \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$$



Vector Velocities - Preliminaries

LOS Velocity measurement can be represented as:

$$v_{los}^i = k_x^i v_x + k_y^i v_y + k_z^i v_z$$

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$$\mathbf{k} = \begin{bmatrix} k_e \\ k_n \\ k_z \end{bmatrix} = \begin{bmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} R^{-1}$$

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If we can neglect Earth curvature ("high enough" elevation angles),

$$\mathbf{k} = \begin{bmatrix} k_e \\ k_n \\ k_z \end{bmatrix} = \begin{bmatrix} \cos\theta\sin\phi \\ \cos\theta\cos\phi \\ \sin\theta \end{bmatrix}$$

where θ, ϕ are elevation and azimuth angles, respectively.

Vector Velocities - Preliminaries

For a local geomagnetic coordinate system we can use the rotation matrix,

$$R_{geo \rightarrow gmag} = \begin{bmatrix} \cos\delta & -\sin\delta & 0 \\ \sin I \sin\delta & \cos\delta \sin I & \cos I \\ -\cos I \sin\delta & -\cos I \cos\delta & \sin I \end{bmatrix}$$

where δ ($\sim 22^\circ$ for PFISR) and I ($\sim 77.5^\circ$ for PFISR) are the declination and dip angles, respectively.

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$$\mathbf{k} = \begin{bmatrix} k_{pe} \\ k_{pn} \\ k_{ap} \end{bmatrix} = \begin{bmatrix} k_e \cos\delta - k_n \sin\delta \\ k_z \cos I + \sin I (k_n \cos\delta + k_e \sin\delta) \\ k_z \sin I - \cos I (k_n \cos\delta + k_e \sin\delta) \end{bmatrix}.$$

Vector Velocities - Two Point

Two LOS velocity measurements can be written as,

$$\begin{bmatrix} v_{los}^1 \\ v_{los}^2 \end{bmatrix} = \begin{bmatrix} k_{pe}^1 & k_{pn}^1 & k_{ap}^1 \\ k_{pe}^2 & k_{pn}^2 & k_{ap}^2 \end{bmatrix} \begin{bmatrix} v_{pe} \\ v_{pn} \\ v_{ap} \end{bmatrix}$$

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Can be solved for v_{pn} and v_{pe} assuming $v_{ap} \approx 0$,

$$v_{pn} = \frac{v_{los}^1 - \frac{k_{pe}^1}{k_{pe}^2} v_{los}^2 - v_{ap}\left(k_{ap}^1 - k_{ap}^2 \frac{k_{pe}^1}{k_{pe}^2}\right)}{k_{pn}^1\left(1 - \frac{k_{pn}^2}{k_{pn}^1}\frac{k_{pe}^1}{k_{pe}^2}\right)} \approx \frac{v_{los}^1 - \frac{k_{pe}^1}{k_{pe}^2} v_{los}^2}{k_{pn}^1\left(1 - \frac{k_{pn}^2}{k_{pn}^1}\frac{k_{pe}^1}{k_{pe}^2}\right)}$$

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Implies that you need look directions with different **k** vectors.

Vector Velocities - Generalization

Multiple measurements can be written as,

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$$\mathbf{v}_{los} = A\mathbf{v}_i + \mathbf{e}_{los}$$

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$$\hat{\mathbf{v}}_i = \Sigma_v A^T (A \Sigma_v A^T + \Sigma_e)^{-1} \mathbf{v}_{los}$$

Error covariance,

$$\Sigma_{\hat{v}} = \Sigma_v - \Sigma_v A^T (A \Sigma_v A^T + \Sigma_e)^{-1} A \Sigma_v = (A^T \Sigma_e^{-1} A + \Sigma_v^{-1})^{-1}$$

Electric Fields

- While above approach can be used to resolve vectors as a function of altitude (or anything else), we often want to resolve vectors as a function of invariant latitude.

Electric Fields

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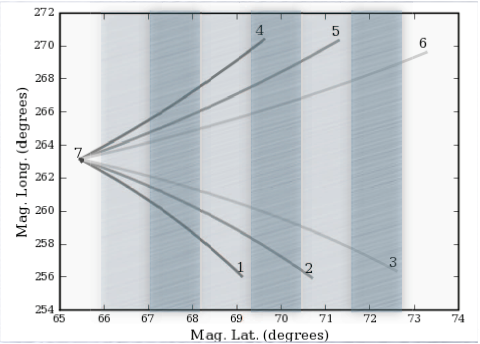
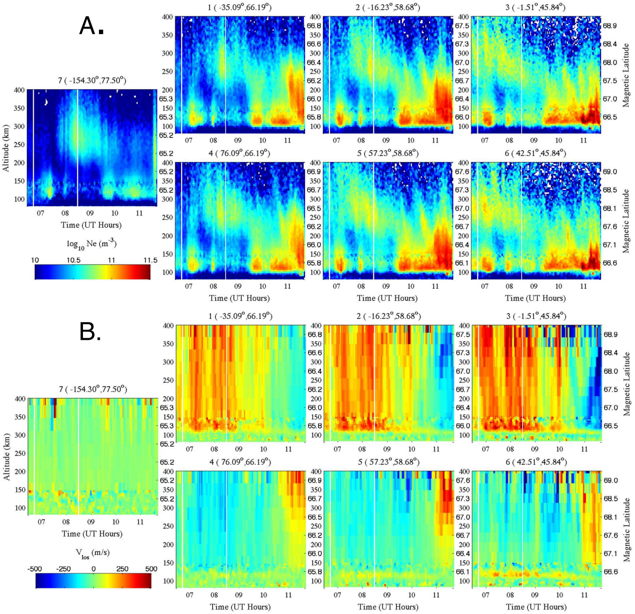


Electric Fields - Example

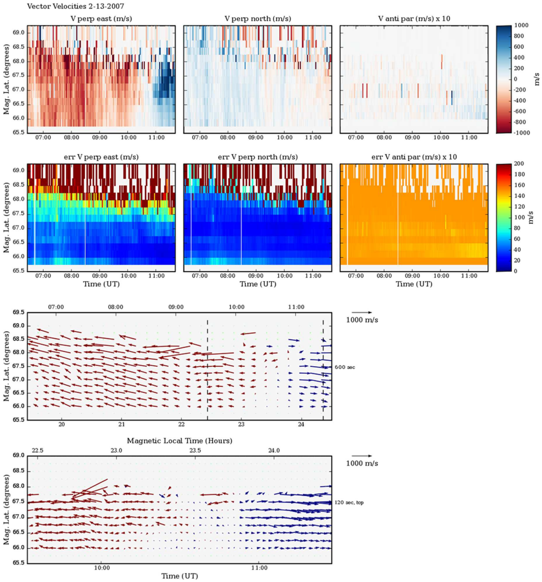


Electron Density

LOS Velocities

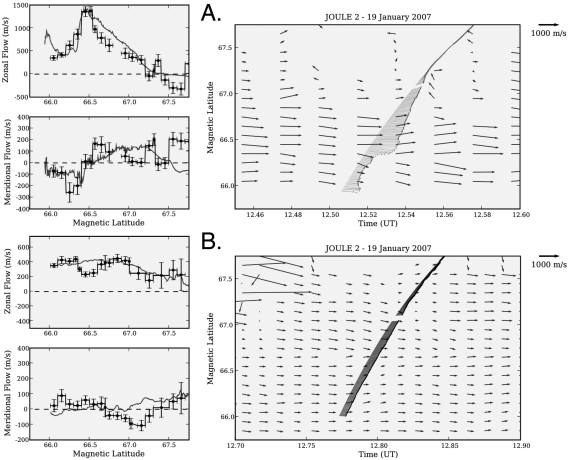
Electric Fields - Example

Resolved Vectors



Electric Fields - Example

Comparison to rocket-measured E-fields.



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$$\mathbf{v}_i = b_i C\mathbf{E} + C\mathbf{u}$$

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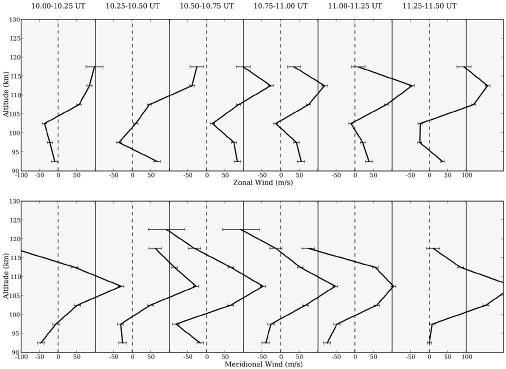
This allows for direct constraint of both the vertical wind and the parallel electric field, both of which we expect to be small.

$$\Sigma_v^{gmag} = J_{geo \to gmag} \Sigma_v^{geo} J^T_{geo \to gmag}$$

E-Region Winds - Example



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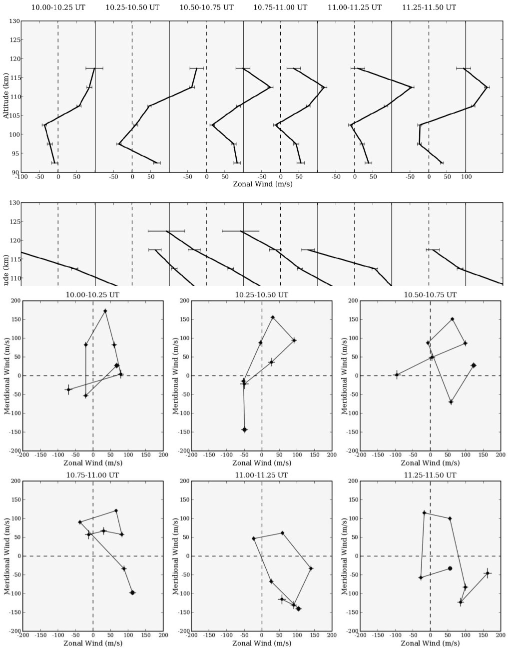

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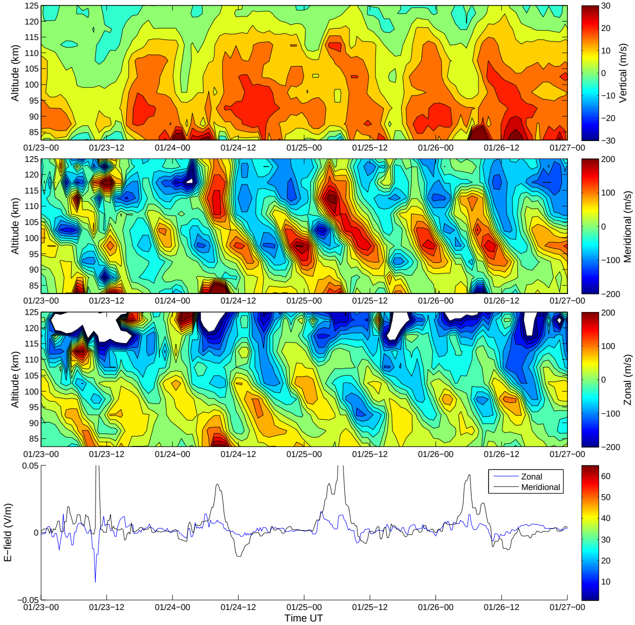

Collision Frequency

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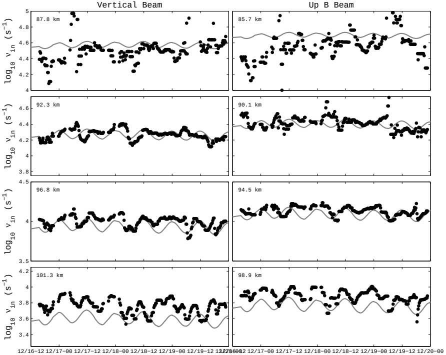
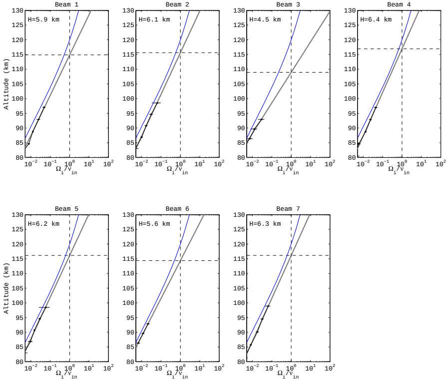
2. Examination of variation of LOS velocity with altitude

Collision Frequency - Method 1

Semi-diurnal variation over several days.



Altitude profile and extrapolation.



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Under strong convection (electric field) conditions, neglect winds

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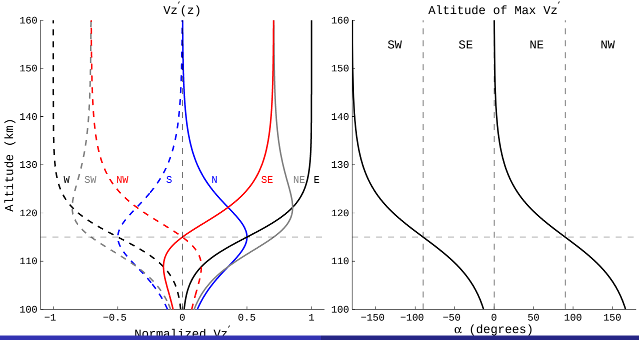

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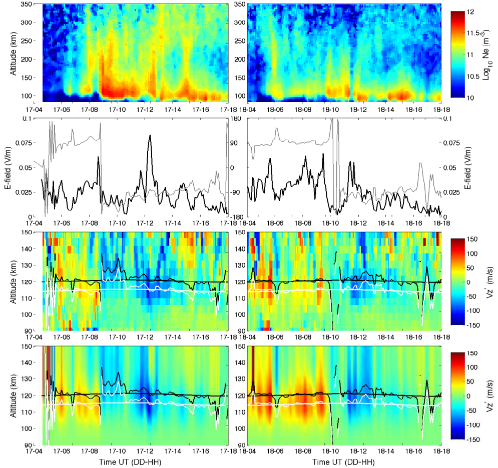
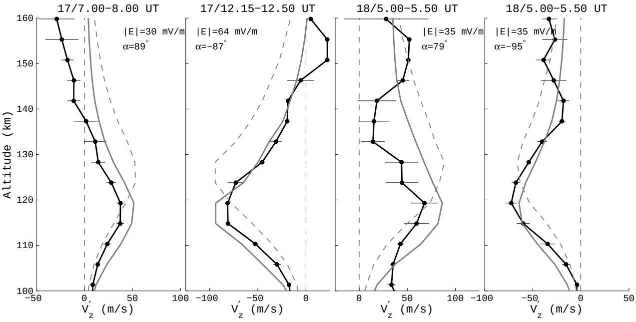


Collision Frequency - Method 2

Profiles of v_z' during high convection conditions.
Dashed - with MSIS; Solid - scaled by a factor of 2.



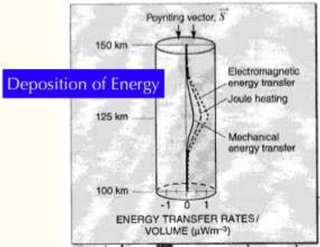
Conductivities / Currents / Joule Heating Rates

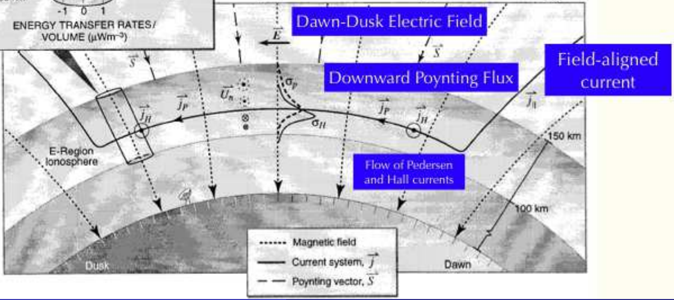
- Supplied electromagnetic energy is converted to heat (Joule heating) or momentum (Lorentz forcing)

$$q_j = \mathbf{j} \cdot \mathbf{E}' = \mathbf{j} \cdot (\mathbf{E} + \mathbf{U} \times \mathbf{B}) = \sigma_P |\mathbf{E} + \mathbf{U} \times \mathbf{B}|^2$$

$$q_m = \mathbf{U} \cdot (\mathbf{j} \times \mathbf{B})$$

- q_m negative - neutral gas is a sink (load)
- q_m positive - neutral gas is a source (generator)





Thayer [2000]

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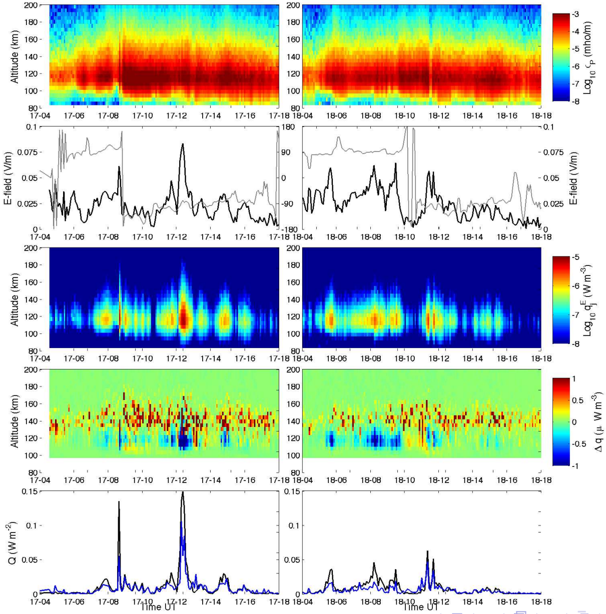

Active Areas of Reserch

1. Full profile / deconvolution techniques for IS fitting
2. Taking advantage of space and time information
3. Optimization and standardization of approaches
4. Additional parameters: molecular ion composition, height-resolved plasma lines, topside parameters, etc.
5. Additional parameters ++: *D*-region momentum fluxes, higher altitude winds, etc.
6. etc.